# 第七届强子谱和强子结构研讨会

# 格点QCD上的强子谱与强子结构









国际理论物理中心-亚太地区



### **CLQCD** ensembles



### **Current status**

### $\star$ In production:

- a=0.0888(3) fm, mpi=349(2) MeV,
   L=2.49 fm;
- a-0.0683(3) fm, mpi=291(2) MeV,
   L=2.46 fm;
- Aboot 100 independent configurations each.

### +Parameter tuning:

a

- a-0.04 fm, mpi-300 MeV;
- a-0.20 fm, mpi-300 MeV;
- a-0.08 fm, mpi-135 MeV.



## Light quark mass determination







- Present CLQCD prediction of the u-d averaged light quark masses is consistent with the lattice averages within 5% uncertainty.
- Most of the uncertainties come from the nonperturbative renormalization and further improvements are in progress.
- All the finite volume, discretization and sea quark mass effects have been taken into account.







## Low energy constants determination





$$egin{aligned} m_\pi^2 &= \Lambda_\chi^2 2y \left[ 1+y \left( \ln rac{2y \Lambda_\chi^2}{m_{\pi,\mathrm{phys}}^2} -\ell_3 
ight) + \mathcal{O}(y^2) 
ight], \ F_\pi &= F \left[ 1-2y \left( \ln rac{2y \Lambda_\chi^2}{m_{\pi,\mathrm{phys}}^2} -\ell_4 
ight) + \mathcal{O}(y^2) 
ight], \end{aligned}$$





- The CLQCD prediction on the low energy constants can be more precise.
- The precision of the NLO low energy constants are higher than the present lattice averages.



## **Baryon spectrum based CLQCD ensembles**





mass using multiple interpolation fields:

- The extracted mass is independent of the interpolation fields.
- Agree with the experimental value within a few percents;
- The mass difference between octet and decuplet baryon in the  $N_f = 3$  chiral limit is 0.31(7) GeV.

Extract the ground state



### Nucleon mass based CLQCD ensembles



Parameter	Value
$M_0$	0.876(16)
$C_1$	2.13(39)
$C_2$	1.39(59)
$C_3$	-6.77(57)
$C_4$	1.85(49)
$C_5$	0.92(38)
$g_A$	0.99(27)
$g_1$	-0.03(51)
$M_{ m phys}$	0.9296(91)
$\chi^2$	0.73
Q	0.86

 Sigma term based on FH theorem:

 $\sigma_{\pi N} \equiv m_l \left\langle p \,|\, \bar{u}u + \bar{d}d \,|\, p \right\rangle = m_l \frac{\partial M_N}{\partial m_l}$ 

=48.8(6.4) MeV;

- Previous Overlap result based on FH theorem:  $\sigma_{\pi N} = 52(8) \text{ MeV};$
- Previous Overlap result based on direct ME calculation:

 $\sigma_{\pi N} = 46(7)$  MeV.

$$M\left(m_{\pi}^{\nu}, m_{\pi}^{\text{sea}}, m_{s}^{\text{sea}}, a, L\right) = \left[M_{0} + C_{1}\left(m_{\pi}^{\nu}\right)^{2} + C_{2}\left(m_{\pi}^{\text{sea}}\right)^{2} - \frac{\left(g_{A}^{2} - 4g_{A}g_{1} - 5g_{1}^{2}\right)\pi}{3\left(4\pi f_{\pi}\right)^{2}}\left(m_{\pi}^{\nu}\right)^{3} - \frac{\left(8g_{A}^{2} + 4g_{A}g_{1} + 5g_{1}^{2}\right)\pi}{3\left(4\pi f_{\pi}\right)^{2}}\left(m_{\pi}^{pq}\right)^{3} + C_{4}\frac{\left(m_{\pi}^{\nu}\right)^{2}}{L}e^{-m_{\pi}^{\nu}L} + C_{5}\left(m_{s}^{\text{sea}} - m_{s}^{\text{phys}}\right)\right]\left(1 + C_{3}a^{2}\right),$$





further suppression on the systematic uncertainties.



### **Stable hadron masses**



- ullet



The  $\eta_c$  mass extrapolated to continuum is consistent with the previous HPQCD results while slightly lower than the experimental value due to the QED and disconnected charm sea effects.

• The  $\Omega_c$  mass extrapolated to continuum is 2745(10)(20) MeV.





# **Study of** $D_0^*(2300)$

### Scattering length



At  $m_{\pi} \sim 300$  MeV, there is a virtual state pole;

When pion mass decreases, it becomes a resonance and the pole position gets close to the experiment.



H.-B. Yan, et.al., 2404.13479

## X(3872) study using anisotropic lattices







- Energy level changes after the  $D\bar{D}^*$  operator is added lacksquarewhile is insensitive to the  $J/\psi\omega$  operator.
- More details can be found in Chunjiang Shi's talk on Monday at 9:10 AM.





### **QCD** in finite volume

$10/g^2$	$m_l a$	$m_s a$	$m_c a$	V	$n_{ m cfg}$
6.784	0.000731	0.01975	0.2293	$4^3 \times 96$	238
				$8^3  imes 96$	350
				$8^2 \times 32 \times 96$	100
				$10^{3} \times 96$	148
				$12^{3} \times 96$	150
				$16^{3} \times 96$	146
				$20^{3} \times 96$	148
				$24^3 \times 96$	151
				$32^3 \times 96$	<b>98</b>
				$40^{3} \times 96$	72
				$40^{3} \times 96$	80
				$48^3 \times 48$	100

- Exploratory study of generating 2+1+1 flavor HISQ ensemble;
- Lattice spacing  $a \sim 0.055$  fm is close to that of CLQCD ensemble H48P32 (a = 0.052 fm).







### Hadron masses in finite volume



- $1/L \sim 200 \, {\rm MeV}$
- The  $m_N 3/2m_{\pi}$  (or  $m_N 3/2m_{a_0}$  at small L) remains at small L.



- All the hadron masses increas at small L and chiral symmetry restored at  $L \sim 1$  fm (corresponds to



### **Transversity PDF of nucleon**



Ensemble	$a({\rm fm})$	$L^3 \times T$	$m_{\pi}({ m MeV})$	$m_{\pi}L$	$N_{ m con}$
X650	0.098	$48^3 \times 48$	338	8.1	150
H102	0.085	$32^3 \times 96$	354	4.9	500
H105		$32^3  imes 96$	281	3.9	500
C101		$48^3 \times 96$	222	4.6	500
N203	0.064	$48^3 \times 128$	348	5.4	500
N302	0.049	$48^3 \times 128$	348	4.2	500

F. Yao, et. al., LPC, PRL131(2023) 261901



Correlation between transverse polarized nucleon and its component quarks:

- Iso vector quark contribution only (u-d);
- International collaboration including ulletmembers from China, US and Germany;
- First PDF prediction extrapolated to the continuum, infinite momentum and physical quark mass limits.









## **Transversity PDF of nucleon**



C. Alexandrou, et. al., PRD98(2018)091503



Comparing to the previous Transversity calculation:

- Proper renormalization  $\bullet$ to remove all the power divergences;
- First attempt of using  $\bullet$ the joint fit to do the controllable extrapolations;
- Include the systematic uncertainties from renormalization scale and scheme, and also kinds of extrapolations.

$$\begin{aligned} \delta q\left(x, P_{z}, a, m_{\pi}\right) &= \frac{1 - g' m_{\pi}^{2} \ln(m_{\pi}^{2}/\mu_{0}^{2}) + m_{\pi}^{2} k(x)}{1 - g' m_{\pi}^{2} \ln(m_{\pi}^{2}/\mu_{0}^{2})} \\ &\times \left[\delta q_{0}(x) + a^{2} f(x) + a^{2} P_{z}^{2} h(x) + \frac{g(x, a)}{P_{z}^{2}}\right] \end{aligned}$$

 $\delta u(x,\mu) - \delta d(x,\mu)$ 







## From quasi-TMD to TMD

Besides the quasi-TMD  $\tilde{f}$  and perturbative calculable hard kernel H, one also need the following nonperturbative quantities to reach the physical TMD f: • Intrinsic soft function  $S_I$ ;





• Rapidity evolution kernel (CS-kernel) K.



### First prediction on physical **TMD PDF and WF**









$$\widetilde{f}(x, b_{\perp}, \zeta_{z}, \mu) \sqrt{S_{I}(b_{\perp}, \mu)} = H(r) e^{\frac{1}{2} \ln\left(\frac{\zeta_{z}}{\zeta}\right) K(b_{\perp}, \mu)}$$
$$\times f(x, b_{\perp}, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^{2}}{\zeta_{z}}, \frac{M^{2}}{(P^{z})^{2}}, \frac{M}{b}\right)$$

- Based on intrinsic soft function  $S_I$ and CS-kernel *K*, first predictions on physcial TMD PDF and TMD WF are provided;
- Control of the systematic  $\bullet$ uncertainties is in progress.





# **Electroweak box diagram and** $V_{ud}$



Ensemble	$m_{\pi}  [{ m MeV}]$	L $T$	$a^{-1}  [{ m GeV}]$	$N_{ m conf}$
24D	142.6(3)	$24 \ 64$	1.023(2)	207
32D-fine	143.6(9)	$32 \ 64$	1.378(5)	69



- The first lattice QCD calculation the universal axial yW-box contribution to both superallowed nuclear and neutron beta decays;
- Results shows mild lattice spacing dependence using DW fermion, yields  $V_{ud}$  = 0.97386(11)<sub>exp</sub>(9)<sub>RC</sub>(27)<sub>NS</sub> and slightly reduces the tension with the CKM unitarity.



# $D_{\varsigma}^*$ radiative decay and $V_{\varsigma}$

$$\langle 0|\mathcal{O}_{D_s}(0)|D_s(\vec{p})\rangle = Z_{D_s}$$
  
$$\langle D_s(p)|J_{\nu}^{\text{em}}(0)|D_{s,\mu}^*(p')\rangle = \frac{2V_{\text{eff}}(q^2)}{m_{D_s} + m_{D_s^*}}\epsilon_{\mu\nu\alpha\beta}p_{\alpha}p_{\beta}'$$

$$V_{\text{eff}}(q^2) = \frac{-(m_{D_s} + m_{D_s^*})E_{D_s}}{2Z_{D_s}m_{D_s^*}}e^{E_{D_s}t}$$
$$\times \int d^3\vec{x} \frac{j_1(|\vec{p}||\vec{x}|)}{|\vec{p}||\vec{x}|} \epsilon_{\mu\nu\alpha0}x_{\alpha}H_{\mu\nu}(\vec{x},t)$$





• Predict  $\Gamma(D_s^* \to D_s \gamma)$ =0.0549(54) KeV and then suppress the uncertainty of the previous HPQCD calculation by a factor of 4;

Combining the recent experiment, one can obtain  $f_{D_s^*}|V_{cs}| = 190.5^{+55.1}_{-41.7} \pm 12.6$  MeV.

More details can be found in Yu Meng's talk on Monday at 9:50 AM.



### Sumary

- Light quark mass and low energy constants have been properly extracted using the CLQCD ensembles, and study on the charm quark and also hadron spectrums are on going.
- Systematic hadron spectrum and structure studies can be carried out using the CLQCD ensembles.
- We expect more LQCD studies will use the CLQCD ensembles in the near future, based on the techniques we developed on the other ensembles.

