Partial wave effects in the heavy quarkonium radiative electromagnetic decays

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CONTENTS

Bethe-Salpeter 方程

$$(\not{p}_1 - m_2)\chi(q)\not{p}_2 + m_2) = i\int \frac{d^4k}{(2\pi)^4} V(P,k,q)\chi(k) \tag{1}$$

$$\begin{aligned} p_1 &= \alpha_1 P + q, \alpha_1 = \frac{m_1}{m_1+m_2} \\ p_2 &= \alpha_2 P - q, \alpha_2 = \frac{m_2}{m_1+m_2} \end{aligned} \tag{2}$$

$$V(r) = \frac{\lambda}{\alpha}(1-e^{-\alpha r}) + V_0 - \gamma_0 \otimes \gamma^0 \frac{4}{3}\frac{\alpha_s}{r}e^{-\alpha r} \tag{3}$$

Total momentum P and relative momentum q of mesons

E.E.Salpeter and H.A.Bethe, Phys. Rev. 84 (1951)1232.

The positive energy wave function of 1^{--} state

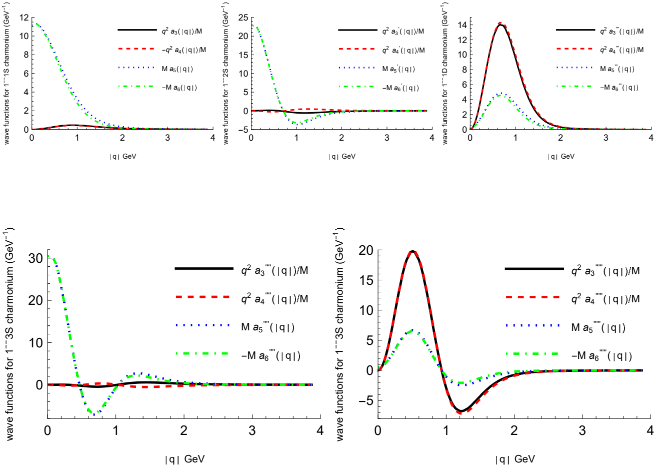
$$\begin{aligned}
\varphi^{++}_{1^{--}}(q_\perp) &= q_\perp.\epsilon\left(A_1+\frac{\not P}{M}A_2\right)+\left(q_\perp.\epsilon\not q_\perp-\frac{1}{3}q_\perp^2\not\epsilon\right)\left(\frac{1}{M}A_3-\frac{\not P}{M^2}A_4\right)\\
&+M\not\epsilon\left(A_5+\frac{\not P}{M}A_6\right)+\frac{1}{3}q_\perp^2\not\epsilon\left(\frac{1}{M}A_3-\frac{\not P}{M^2}A_4\right)+M\not\epsilon\frac{\not P\not q_\perp}{M^2}A_7\\
&= q_\perp.\epsilon\left(D_1+\frac{\not P}{M}D_2+\frac{\not q_\perp}{M}D_3+\frac{\not P\not q_\perp}{M^2}D_4\right)\\
&+M\not\epsilon\left(D_5+\frac{\not P}{M}D_6+\frac{\not P\not q_\perp}{M^2}D_7\right), \qquad (4)
\end{aligned}$$

A_i is the function of the radial part of the relativistic wave function a_i

	$\psi\ (1S)$	$\psi\ (2S)$	$\psi\ (1D)$	$\psi\ (3S)$	$\psi\ (2D)$
$S : P : D$	1 : 0.126 : 0.0551	1 : 0.148 : 0.0647	0.0631 : 0.171 : 1	1 : 0.170 : 0.0705	0.0711 : 0.199 : 1
	$\Upsilon\ (1S)$	$\Upsilon\ (2S)$	$\Upsilon\ (1D)$	$\Upsilon\ (3S)$	$\Upsilon\ (2D)$
$S : P : D$	1 : 0.0395 : 0.0205	1 : 0.0434 : 0.0236	0.0202 : 0.0443 : 1	1 : 0.0486 : 0.0259	0.0244 : 0.0495 : 1

G.-L. Wang, Phys. Lett. B 633, 492 (2006).

the relativistic wave function for the state 1^{--}



The positive energy wave function of 0^{++} $(^3P_0)$ state

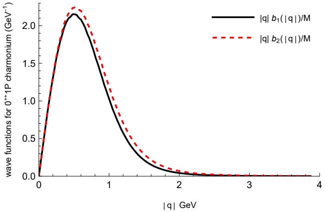
$$\varphi^{++}_{0^{++}}(q_\perp) = B_1 \not{q}_\perp + B_2 \frac{\not{P}\not{q}_\perp}{M} + B_3, \quad (5)$$

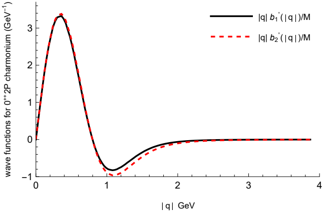
B_i is the function of the radial part of the relativistic wave function

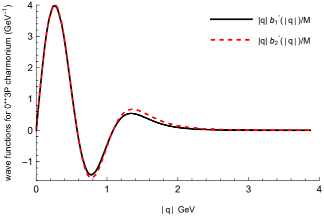
	$\chi_{c0}(1P)$	$\chi_{c0}(2P)$	$\chi_{c0}(3P)$
P : S	1 : 0.127	1 : 0.142	1 : 0.157
	$\chi_{b0}(1P)$	$\chi_{b0}(2P)$	$\chi_{b0}(3P)$
P : S	1 : 0.0361	1 : 0.0399	1 : 0.0444

G.-L. Wang, Phys. Lett. B 650, 15 (2007).

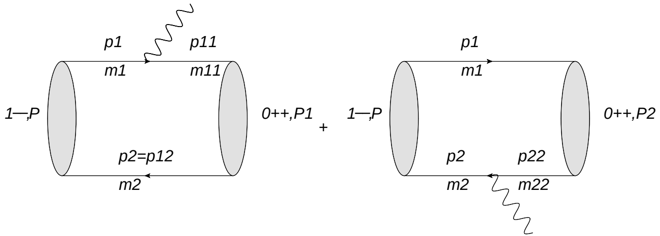
the relativistic wave function for the state 0^{++}







Feynman diagrams for the process $1^{--} \to 0^{++}\gamma$



The transition amplitude for one photon radiative decay of a quarkonium is written as:

$$T = < P_f \epsilon_f, k\epsilon_0 |S| P\epsilon > = (2\pi)^4 e e_q \delta^4(P_f + k - P)\epsilon_{0\mu}\mathcal{M}^{\mu}, (6)$$

$M_{\psi(1S)} = 3096 MeV$	$M_{\psi(2S)} = 3686 MeV$	$M_{\psi(1D)} = 3773 MeV$	$M_{\psi(3S)} = 4039 MeV$	$M_{\psi(2D)} = 4191 MeV$
$M_{\Upsilon(1S)} = 9460 MeV$	$M_{\Upsilon(2S)} = 10023 MeV$	$M_{\Upsilon(1D)} = 10129 MeV$	$M_{\Upsilon(3S)} = 10355 MeV$	$M_{\Upsilon(2D)} = 10434 MeV$
$M_{\chi_{c0}(1P)} = 3414 MeV$	$M_{\chi_{c0}(2P)} = 3862 MeV$	$M_{\chi_{c0}(3P)} = 4140 MeV$		
$M_{\chi_{b0}(1P)} = 9859 MeV$	$M_{\chi_{b0}(2P)} = 10232 MeV$	$M_{\chi_{b0}(3P)} = 10524 MeV$		
$M_{\chi_{c1}(1P)} = 3510 MeV$	$M_{\chi_{c1}(2P)} = 3872 MeV$	$M_{\chi_{c1}(3P)} = 4228 MeV$		
$M_{\chi_{b1}(1P)} = 9892 MeV$	$M_{\chi_{b1}(2P)} = 10255 MeV$	$M_{\chi_{b1}(3P)} = 10513 MeV$		
$M_{\chi_{c2}(1P)} = 3556 MeV$	$M_{\chi_{c2}(2P)} = 3922 MeV$	$M_{\chi_{c2}(1F)} = 4037 MeV$		
$M_{\chi_{b2}(1P)} = 9912 MeV$	$M_{\chi_{b2}(2P)} = 10268 MeV$	$M_{\chi_{b2}(1F)} = 10374 MeV$		

C.-H. Chang and G.-L. Wang, Sci. China Phys. Mech. Astron. 53, 2005 (2010).

Process	Ours	NR[18]	R[39]	NR[40]	R[41]	R[42]	NRa[43]	NRb[43]	PDG
$\psi(2S) \to \gamma\chi_{c0}(1P)$	39.9	50	26.3	47.0	25.2	26	22	22	28.8 ± 1.4
$\psi(2S) \to \gamma\chi_{c1}(1P)$	35.6	45	22.9	42.8	29.1	29	42	45	28.7 ± 1.5
$\psi(2S) \to \gamma\chi_{c2}(1P)$	24.5	29	18.2	30.1	25.2	24	38	46	28.0 ± 1.4
$\psi(3770) \to \gamma\chi_{c0}(1P)$	290		355	299	243.9	213	272	261	188 ± 23
$\psi(3770) \to \gamma\chi_{c1}(1P)$	90.8		135	99.0	104.9	77	138	135	67.7 ± 8.7
$\psi(3770) \to \gamma\chi_{c2}(1P)$	24.5		6.9	3.88	1.9	3.3	7.1	8.1	< 17.4

Process	Ours	R[41]	[17]	R[42]	[33]	[38]	R[39]	R[35]	PDG
$\Upsilon(2S) \to \gamma\chi_{b0}(1P)$	1.13	1.62	1.29	0.74	1.19	0.91	1.09	1.09	1.22 ± 0.23
$\Upsilon(2S) \to \gamma\chi_{b1}(1P)$	1.80	2.45	2.00	1.40	2.28	1.63	1.84	2.17	2.21 ± 0.23
$\Upsilon(2S) \to \gamma\chi_{b2}(1P)$	1.83	2.46	2.04	1.67	2.58	1.88	2.08	2.62	2.29 ± 0.30
$\Upsilon(1D) \to \gamma\chi_{b0}(1P)$	15.5	23.4	20.1	12.5		16.5	20.98	19.8	
$\Upsilon(1D) \to \gamma\chi_{b1}(1P)$	7.94	12.7	10.7	7.59		9.7	12.29	13.3	
$\Upsilon(1D) \to \gamma\chi_{b2}(1P)$	0.416	0.69	0.564	0.44		0.56	0.65	1.02	

1^{--} \ 0^{++}	Whole $(P' + S')$	P' wave	S' wave
Whole $(S + P + D)$	39.9	33.6	0.269
S wave	34.6	34.6	0
P wave	0.215	4.2×10^{-3}	0.279
D wave	8.5×10^{-4}	3.9×10^{-4}	9.0×10^{-5}

表: The contributions of different partial waves to the decay width (keV).

The relativistic effect of $\psi(2S) \to \gamma\chi_{c0}(1P)$,

$$\frac{\Gamma_{rel} - \Gamma_{non-rel}}{\Gamma_{rel}} = 13.8\%, \tag{7}$$

$\Gamma_{non-rel}$ is obtained only using the nonrelativistic wave functions

0^{++} \ 1^{--}	Whole ($P' + S'$)	P' wave	S' wave
Whole ($S + P + D$)	290	257	0.925
D wave	255	250	0.0210
S wave	4.0×10^{-3}	4.0×10^{-3}	0
P wave	0.944	0.0235	0.670

表: The contributions of different partial waves to the decay width (keV).

The relativistic effect of $\psi(1D) \to \gamma\chi_{c0}(1P)$,

$$\frac{\Gamma_{rel} - \Gamma_{non-rel}}{\Gamma_{rel}} = 4.43\%, \tag{8}$$

$\psi(2S)$	$\psi(1D)$	$\psi(3S)$	$\psi(2D)$
$\psi(2S)\to\gamma\chi_{c0}(1P)$	$\psi(3770)\to\gamma\chi_{c0}(1P)$	$\psi(4040)\to\gamma\chi_{c0}(1P)$	$\psi(4160)\to\gamma\chi_{c0}(1P)$
$\psi(2S)\to\gamma\chi_{c1}(1P)$	$\psi(3770)\to\gamma\chi_{c1}(1P)$	$\psi(4040)\to\gamma\chi_{c1}(1P)$	$\psi(4160)\to\gamma\chi_{c1}(1P)$
$\psi(2S)\to\gamma\chi_{c2}(1P)$	$\psi(3770)\to\gamma\chi_{c2}(1P)$	$\psi(4040)\to\gamma\chi_{c2}(1P)$	$\psi(4160)\to\gamma\chi_{c2}(1P)$
		$\psi(4040)\to\gamma\chi_{c0}(3860)$	$\psi(4160)\to\gamma\chi_{c0}(3860)$
		$\psi(4040)\to\gamma\chi_{c1}(3872)$	$\psi(4160)\to\gamma\chi_{c1}(3872)$
		$\psi(4040)\to\gamma\chi_{c2}(3930)$	$\psi(4160)\to\gamma\chi_{c2}(3930)$
			$\psi(4160)\to\gamma\chi_{c2}(1F)$

$\Upsilon(2S)$	$\Upsilon(1D)$	$\Upsilon(3S)$	$\Upsilon(2D)$
$\Upsilon(2S)\to\gamma\chi_{b0}(1P)$	$\Upsilon(1D)\to\gamma\chi_{b0}(1P)$	$\Upsilon(3S)\to\gamma\chi_{b0}(1P)$	$\Upsilon(2D)\to\gamma\chi_{b0}(1P)$
$\Upsilon(2S)\to\gamma\chi_{b1}(1P)$	$\Upsilon(1D)\to\gamma\chi_{b1}(1P)$	$\Upsilon(3S)\to\gamma\chi_{b1}(1P)$	$\Upsilon(2D)\to\gamma\chi_{b1}(1P)$
$\Upsilon(2S)\to\gamma\chi_{b2}(1P)$	$\Upsilon(1D)\to\gamma\chi_{b2}(1P)$	$\Upsilon(3S)\to\gamma\chi_{b2}(1P)$	$\Upsilon(2D)\to\gamma\chi_{b0}(2P)$
		$\Upsilon(3S)\to\gamma\chi_{b0}(2P)$	$\Upsilon(2D)\to\gamma\chi_{b1}(2P)$
		$\Upsilon(3S)\to\gamma\chi_{b1}(2P)$	$\Upsilon(2D)\to\gamma\chi_{b2}(2P)$
		$\Upsilon(3S)\to\gamma\chi_{b2}(2P)$	$\Upsilon(2D)\to\gamma\chi_{b2}(1F)$

Conclusion

Result1

All the mesons are mixing states.

Result2

For the S and P wave dominated states, the dominant S and P waves provide the main and non-relativistic contribution, while the partial waves of the small components mainly contribute to the relativistic correction.

Result3

For the S wave in $\Psi(nD)$ and $\Upsilon(nD)$ (n = 1, 2) and the P wave in $\chi_{c2}(1F)$ and $\chi_{b2}(1F)$, the two sources have large proportions separately, while their contributions cancel out and are not significant in this article.

Thank you all !