Further study of $c\bar{c}c\bar{c}$ system within a chiral quark model

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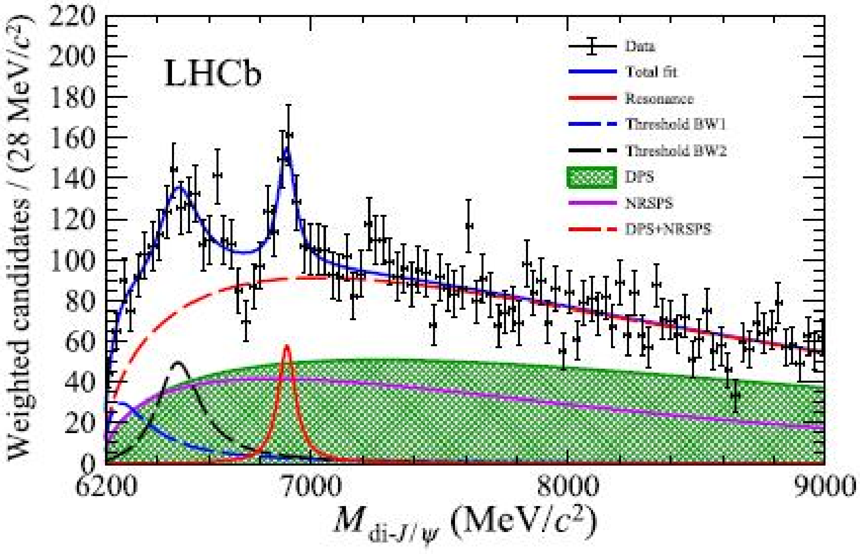
Outline

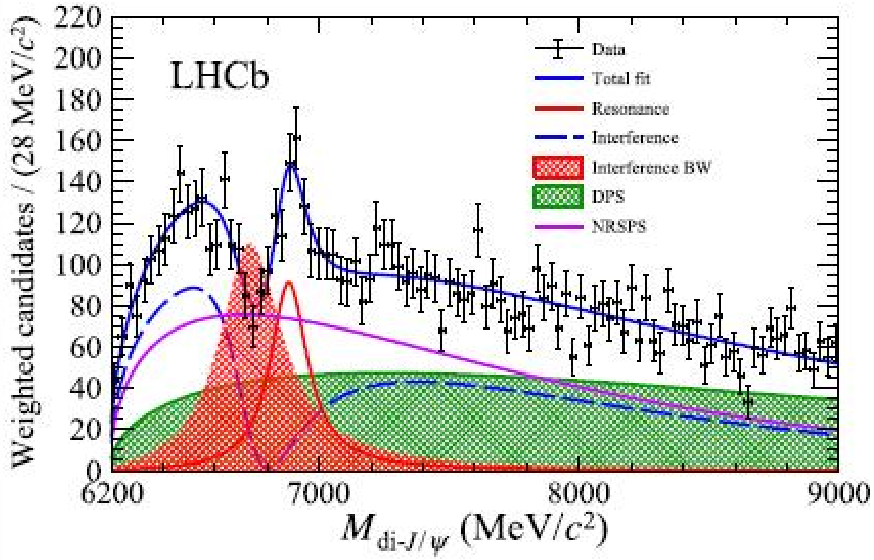
1. Introduction
2. Theoretical framework
3. Results and Discussion
4. Summary

Introduction

➢ Experimental results

LHCb Collaboration, Science Bulletin 65 (2020) 1983–1993





$m[X(6900)] = 6905 \pm 11 \pm 7\ \text{MeV}/c^2,$

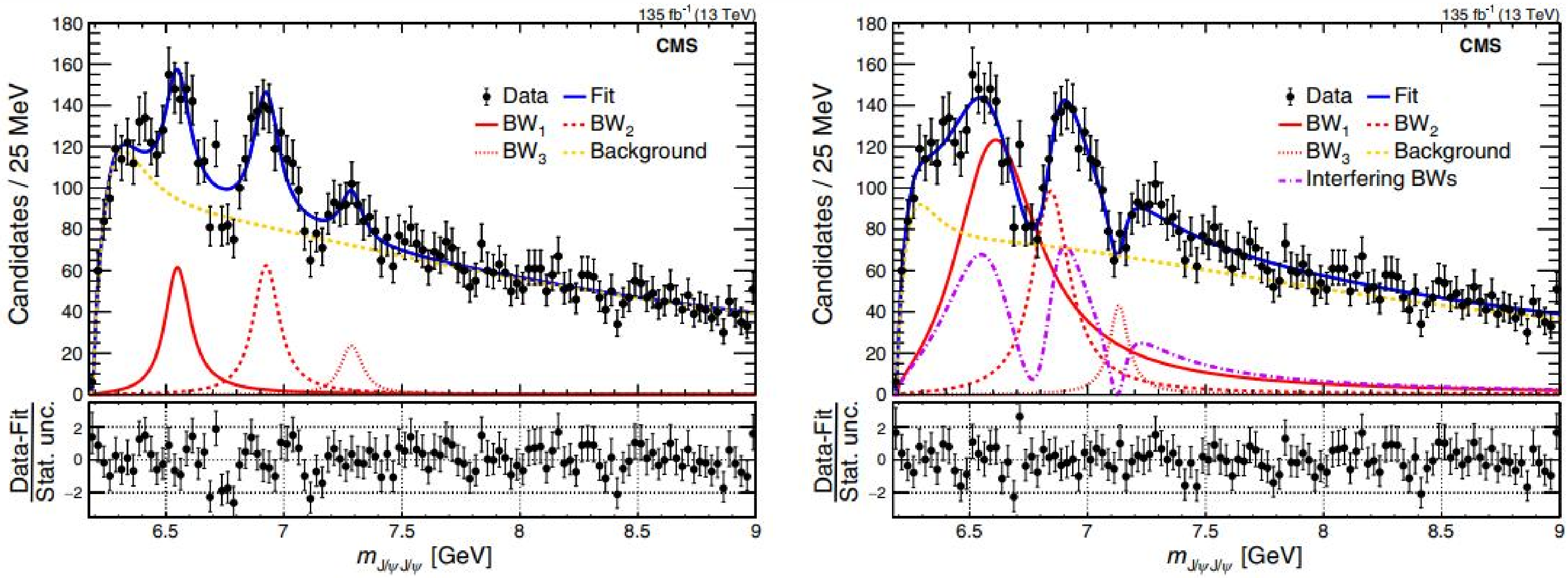
$\Gamma[X(6900)] = 80 \pm 19 \pm 33\ \text{MeV},$

$m[X(6900)] = 6886 \pm 11 \pm 11\ \text{MeV}/c^2$

$\Gamma[X(6900)] = 168 \pm 33 \pm 69\ \text{MeV}.$

Introduction

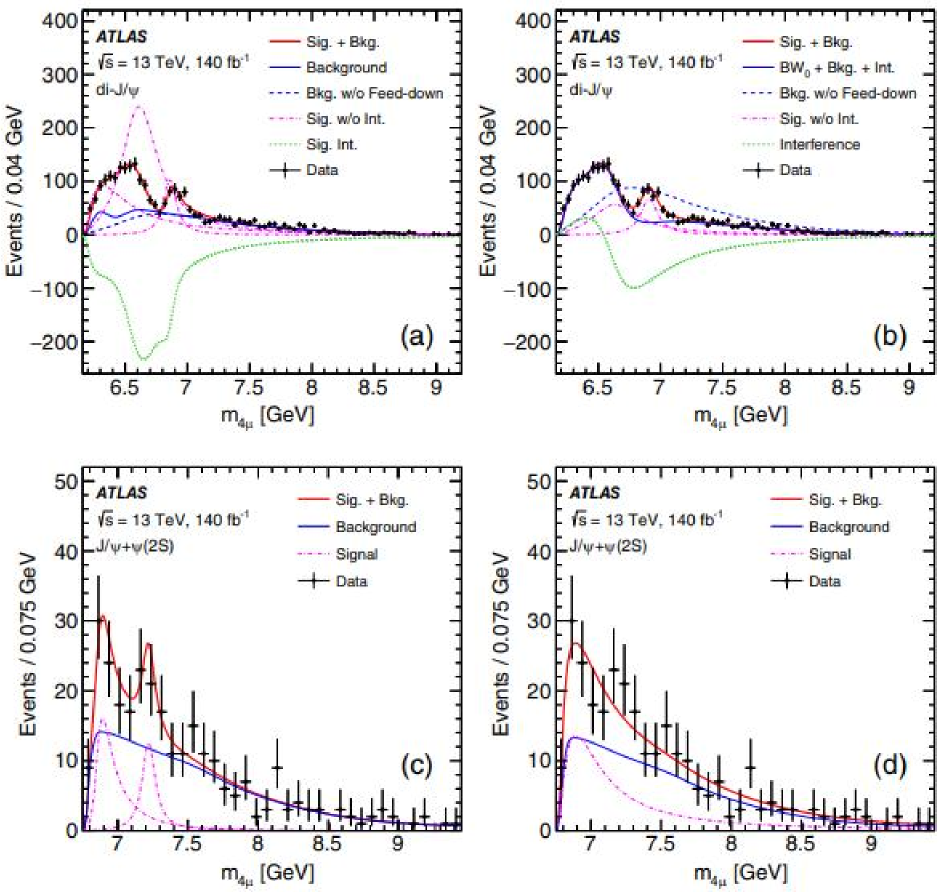
CMS Collaboration, Phys.Rev.Lett. 132 (2024) 11, 111901



		BW_1	BW_2	BW_3
No interference	m (MeV)	$6552 \pm 10 \pm 12$	$6927 \pm 9 \pm 4$	$7287^{+20}_{-18} \pm 5$
	Γ (MeV)	$124^{+32}_{-26} \pm 33$	$122^{+24}_{-21} \pm 18$	$95^{+59}_{-40} \pm 19$
	N	470^{+120}_{-110}	492^{+78}_{-73}	156^{+64}_{-51}
Interference	m (MeV)	6638^{+43+16}_{-38-31}	6847^{+44+48}_{-28-20}	7134^{+48+41}_{-25-15}
	Γ (MeV)	$440^{+230+110}_{-200-240}$	191^{+66+25}_{-49-17}	97^{+40+29}_{-29-26}

Introduction

ATLAS Collaboration Phys.Rev.Lett. 131 (2023) 15, 151902



Di-J/ψ	Model A	Model B
m_0	$6.41 \pm 0.08^{+0.08}_{-0.03}$	$6.65 \pm 0.02^{+0.03}_{-0.02}$
Γ_0	$0.59 \pm 0.35^{+0.12}_{-0.20}$	$0.44 \pm 0.05^{+0.06}_{-0.05}$
m_1	$6.63 \pm 0.05^{+0.08}_{-0.01}$	...
Γ_1	$0.35 \pm 0.11^{+0.11}_{-0.04}$	
m_2	$6.86 \pm 0.03^{+0.01}_{-0.02}$	$6.91 \pm 0.01 \pm 0.01$
Γ_2	$0.11 \pm 0.05^{+0.02}_{-0.01}$	$0.15 \pm 0.03 \pm 0.01$
$\Delta s/s$	$\pm 5.1\%^{+8.1\%}_{-8.9\%}$	...

$J/\psi + \psi(2S)$	Model α	Model β
m_3	$7.22 \pm 0.03^{+0.01}_{-0.04}$	$6.96 \pm 0.05 \pm 0.03$
Γ_3	$0.09 \pm 0.06^{+0.06}_{-0.05}$	$0.51 \pm 0.17^{+0.11}_{-0.10}$
$\Delta s/s$	$\pm 21\%^{+25\%}_{-15\%}$	$\pm 20\% \pm 12\%$

Introduction

- **Theoretical studies**

- **Before LHCb's results**

Y. Iwasaki, Prog. Theor. Phys. 54, 492 (1975)

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A. Esposito, A.D. Polosa, Eur. Phys. J. C 78(9), 782 (2018)

W. Chen, H.X. Chen, X. Liu, T.G. Steele, S.L. Zhu, Phys. Lett. B 773, 247 (2017)

and others

Introduction

- After 2020

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M. S. Liu, F. X. Liu, X. H. Zhong, Q. Zhao, Phys.Rev.D 109 7, 076017 (2024)

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and others.

Introduction

- Our work

arXiv:2403.10375

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- Bound state calculation

  meson-meson

  diquark-antidiquark

- A stabilization method (real scaling method)

Theoretical framework

- **The chiral quark model: The most used QM**

  Describes properties of hadrons， hadron-hadron interactions well

- **In ChQM:**

  Confinement: confining potential (phenomenology)

  Asmptotic freedom: single-gluon-exchange potential

  Chiral symmetry spontaneous breaking: Goldstone boson exchange potential

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- L.Ya Glozman, Z. Papp, W. Plessas, Physics Letters B 381 (1996) 311-316
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The chiral quark model (ChQM)

$$H = \sum_{i=1}^{n}(m_i + \frac{p_i^2}{2m_i}) - T_{cm} + \sum_{i=1<j}^{n}(V_{con}(r_{ij}) + V_{oge}(r_{ij})),$$

$$V_{CON}^{C} = [-a_c r_{ij}^2 - \Delta]\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c,$$

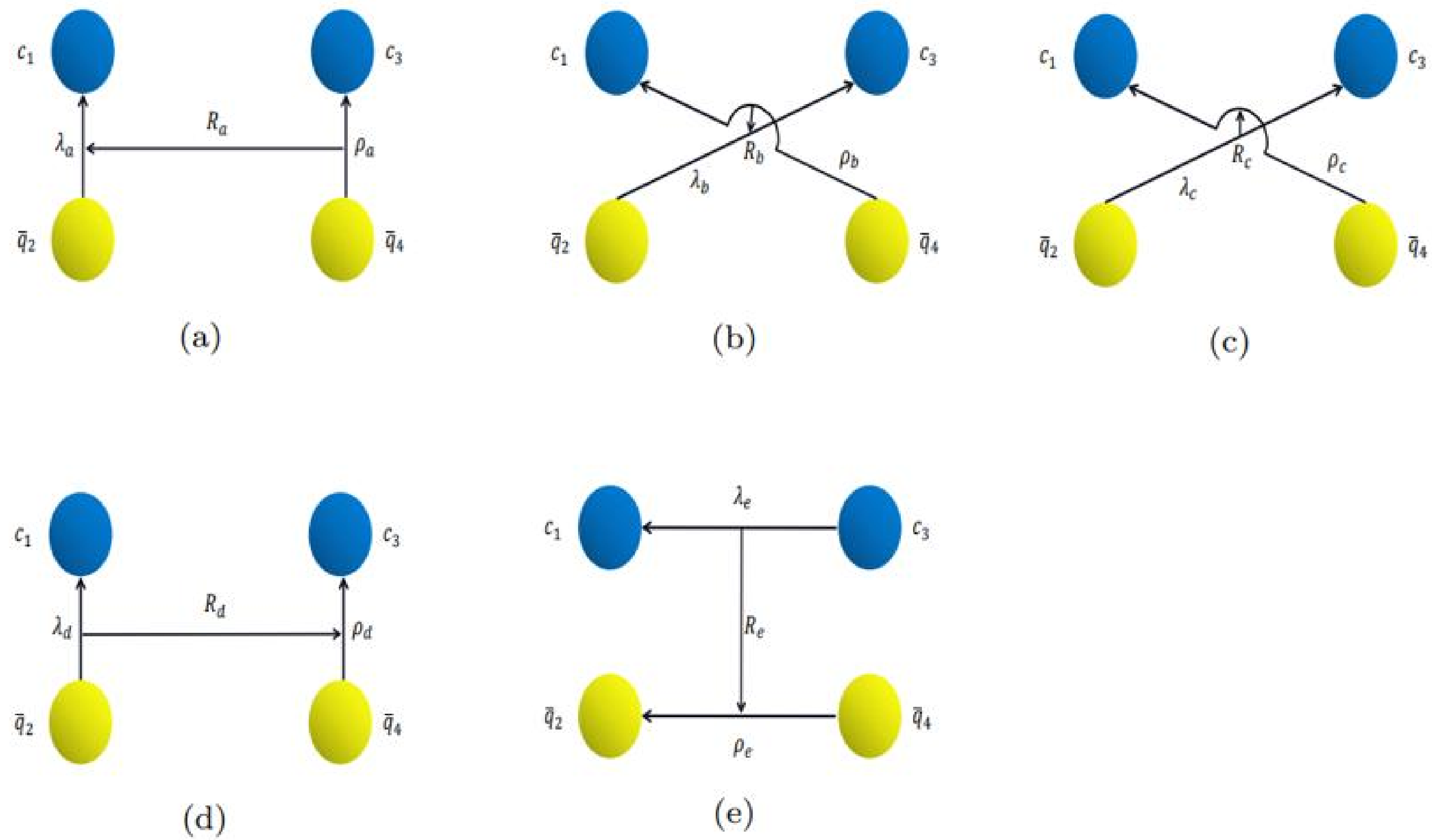
$$V_{CON}^{SO} = -\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \frac{a_c}{4m_i^2 m_j^2}[((m_i^2 + m_j^2)(1-2a_s) + 4m_i m_j(1-a_s)) \times (\boldsymbol{S}_+ \cdot \boldsymbol{L}) + (m_j^2 - m_i^2)(1-2a_s)(\boldsymbol{S}_+ \cdot \boldsymbol{L})]$$

$$V_{OGE}^{C} = \frac{1}{4}\alpha_s \boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \{\frac{1}{r_{ij}} - \frac{1}{6m_i m_j}\boldsymbol{\sigma_i} \cdot \boldsymbol{\sigma_j} \frac{e^{-\frac{r_{ij}}{r_0\mu}}}{r_{ij} r_0(\mu)^2}\},$$

$$V_{OGE}^{T} = -\frac{1}{16}\frac{\alpha_s}{m_i m_j}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c [\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}}(\frac{1}{r_{ij}^2} + \frac{1}{3r_g^2(\mu)} + \frac{1}{r_{ij} r_g(\mu)})]S_{ij},$$

$$V_{OGE}^{SO} = -\frac{1}{16}\frac{\alpha_s}{m_i^2 m_j^2}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c [\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3}(1 + \frac{r_{ij}}{r_g(\mu)})] \times [((m_i + m_j)^2 + 2m_i m_j)(\boldsymbol{S}_+ \cdot \boldsymbol{L}) + (m_j^2 - m_i^2)(\boldsymbol{S}_- \cdot \boldsymbol{L})]$$

Gaussian Expansion Method



(a)-(d) mean meson-meson structure, while (d) represents diquark-antidiquark structure

Gaussian Expansion Method

$$\psi_{JM} = \mathcal{A}\,\phi^{(1)}_{JM}(\lambda_a,\rho_a,R_a) = \phi^{(1)}_{JM}(\lambda_a,\rho_a,R_a) + \phi^{(2)}_{JM}(\lambda_b,\rho_b,R_b) + \phi^{(3)}_{JM}(\lambda_c,\rho_c,R_c) + \phi^{(4)}_{JM}(\lambda_d,\rho_d,R_d)$$

$$\phi^{(c)}_{JM}(r_c,R_c) = \sum_{nl,NL} \underline{\boldsymbol{C_{NL,lm}}}\,\phi^{(c)}_{nl}(r_c)\psi^{(c)}_{NL}(R_c)[Y_{lm}(\widehat{\boldsymbol{r_c}}) \otimes Y_{LM}(\widehat{\boldsymbol{R_c}})\,]_{JM}$$

Determined by diagonalizing H

Radial part Gaussian function:

$$\phi_{nl}(r) = \sum_{n=1}^{n_{max}} c_n\,N_{nl}\,r^l\,e^{-\nu_n r^2} \quad , \qquad N_{nl} = \left[\frac{2^{l+2}(2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi}\,(2l+1)!!}\right]^{\frac{1}{2}}$$

Wave functions

- **The spin-orbit wave function**

$$\chi^{\sigma}_{11} = \alpha\alpha, \quad \chi^{\sigma}_{10} = \frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha),$$
$$\chi^{\sigma}_{1-1} = \beta\beta, \quad \chi^{\sigma}_{00} = \frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha).$$

$$\psi_{J_1,m_{J_1}}(\boldsymbol{\lambda}) = \phi_{n_1L_1m_1}(\boldsymbol{\lambda}) \otimes \chi_{S_1,m_{S_1}},$$
$$\psi_{J_2,m_{J_2}}(\boldsymbol{\rho}) = \phi_{n_2L_2m_2}(\boldsymbol{\rho}) \otimes \chi_{S_2,m_{S_2}}.$$

$$\psi_{J_{12},m_{J_{12}}} = \psi_{J_1,m_{J_1}}(\boldsymbol{\lambda}) \otimes \psi_{J_2,m_{J_2}}(\boldsymbol{\rho}),$$
$$\psi^{SO}_i(\mathbf{r}) = \psi_{J_{12},m_{J_{12}}} \otimes \phi_{n_3L_3m_3}(\mathbf{R}),$$
$$i \equiv \{L_1, S_1, J_1, L_2, S_2, J_2, J_{12}, L_3\}.$$

TABLE III. Different combinations of J-J coupling.

$L_3 = 0$	$J_1 = 0$	$J_2 = 0$	index(i)
	$L_1 = 0$ $S_1 = 0$	$L_2 = 0$ $S_2 = 0$	1
	$L_1 = 1$ $S_1 = 1$	$L_2 = 1$ $S_2 = 1$	2
	$J_1 = 1$	$J_2 = 1$	index(i)
	$L_1 = 0$ $S_1 = 1$	$L_2 = 0$ $S_2 = 1$	3
	$L_1 = 1$ $S_1 = 0$	$L_2 = 1$ $S_2 = 0$	4
	$L_1 = 1$ $S_1 = 1$	$L_2 = 1$ $S_2 = 1$	5
	$L_1 = 1$ $S_1 = 1$	$L_2 = 1$ $S_2 = 0$	6
	$J_1 = 2$	$J_2 = 2$	index(i)
	$L_1 = 1$ $S_1 = 1$	$L_2 = 1$ $S_2 = 1$	7

Wave functions

- **The color wave function:**

$$1 \otimes 1 \quad |C_1\rangle = \sqrt{\frac{1}{9}}(r_1\bar{r}_2r_3\bar{r}_4 + r_1\bar{r}_2g_3\bar{g}_4 + r_1\bar{r}_2b_3\bar{b}_4 + g_1\bar{g}_2r_3\bar{r}_4 + g_1\bar{g}_2g_3\bar{g}_4 + g_1\bar{g}_2b_3\bar{b}_4 + b_1\bar{b}_2r_3\bar{r}_4 + b_1\bar{b}_2g_3\bar{g}_4 + b_1\bar{b}_2b_3\bar{b}_4)$$

$$8 \otimes 8 \quad |C_2\rangle = \sqrt{\frac{1}{72}}(3r_1\bar{b}_2b_3\bar{r}_4 + 3r_1\bar{g}_2g_3\bar{r}_4 + 3g_1\bar{b}_2b_3\bar{g}_4 + 3b_1\bar{g}_2g_3\bar{b}_4 + 3g_1\bar{r}_2r_3\bar{g}_4 + 3b_1\bar{r}_2r_3\bar{b}_4 + 2r_1\bar{r}_2r_3\bar{r}_4 + 2g_1\bar{g}_2g_3\bar{g}_4 + 2b_1\bar{b}_2b_3\bar{b}_4 - r_1\bar{r}_2g_3\bar{g}_4 - g_1\bar{g}_2r_3\bar{r}_4 - b_1\bar{b}_2g_3\bar{g}_4 - b_1\bar{b}_2r_3\bar{r}_4 - g_1\bar{g}_2b_3\bar{b}_4 - r_1\bar{r}_2b_3\bar{b}_4).$$

$$\bar{3} \otimes 3 \quad |C_3\rangle = \sqrt{\frac{1}{12}}(r_1g_3\bar{r}_2\bar{g}_4 - r_1g_3\bar{g}_2\bar{r}_4 + g_1r_3\bar{g}_2\bar{r}_4 - g_1r_3\bar{r}_2\bar{g}_4 + r_1b_3\bar{r}_2\bar{b}_4 - r_1b_3\bar{b}_2\bar{r}_4 + b_1r_3\bar{b}_2\bar{r}_4 - b_1r_3\bar{r}_2\bar{b}_4 + g_1b_3\bar{g}_2\bar{b}_4 - g_1b_3\bar{b}_2\bar{g}_4 + b_1g_3\bar{b}_2\bar{g}_4 - b_1g_3\bar{g}_2\bar{b}_4).$$

$$6 \otimes \bar{6} \quad |C_4\rangle = \sqrt{\frac{1}{24}}(2r_1r_3\bar{r}_2\bar{r}_4 + 2g_1g_3\bar{g}_2\bar{g}_4 + 2b_1b_3\bar{b}_2\bar{b}_4 + r_1g_3\bar{r}_2\bar{g}_4 + r_1g_3\bar{g}_2\bar{r}_4 + g_1r_3\bar{g}_2\bar{r}_4 + g_1r_3\bar{r}_2\bar{g}_4 + r_1b_3\bar{r}_2\bar{b}_4 + r_1b_3\bar{b}_2\bar{r}_4 + b_1r_3\bar{b}_2\bar{r}_4 + b_1r_3\bar{r}_2\bar{b}_4 + g_1b_3\bar{g}_2\bar{b}_4 + g_1b_3\bar{b}_2\bar{g}_4 + b_1g_3\bar{b}_2\bar{g}_4 + b_1g_3\bar{g}_2\bar{b}_4).$$

Wave functions

- **The flavor wave function**

$$|F_1\rangle = c_1\bar{c}_2c_3\bar{c}_4$$
$$|F_2\rangle = c_1c_3\bar{c}_2\bar{c}_4$$

Results and Discussion

parameters:

m_c(MeV)	a_c(MeV)	Δ(MeV)	$\hat{r}_0$(MeV)	$\hat{r}_g$(MeV)	α_{cc}	a_s
4978	98	-18.1	81.0	100.6	0.56	0.77

masses of mesons (unit:MeV):

Meson	This work	ChQM2[70]	LP[73]	EXP.(PDG)
η_c	2980	2990	2983	2983.9±0.4
$\eta_c(2S)$	3637	3627	3635	3637.7±1.1
$\eta_c(3S)$	4132	-	4048	-
J/ψ	3100	3097	3097	3096.9±0.006
$\psi(2S)$	3713	3685	3679	3686.1±0.06
h_c	3508	3507	3522	3525.37±0.14
χ_{c0}	3428	3436	3415	3414.71±0.30
χ_{c1}	3493	3494	3516	3510.67±0.05
χ_{c2}	3543	3526	3552	3556.17±0.07

Results and Discussion

bound state calculation in cccc system with $J^P=0^+$ (unit:MeV)

Channel	$\|[LS]_i F_j C_k\rangle$	E_{th}	E	Mixed
$\eta_c \eta_c$	$\|111\rangle$	5960	5962	5961
$J/\psi J/\psi$	$\|321\rangle$	6200	6201	
$\chi_{c0}\chi_{c0}$	$\|221\rangle$	6856	6858	
$\chi_{c1}\chi_{c1}$	$\|521\rangle$	6986	6988	
$\chi_{c2}\chi_{c2}$	$\|712\rangle$	7086	7089	
$\chi_{c1}h_c$	$\|612\rangle$	7001	7004	
$h_c h_c$	$\|412\rangle$	7016	7018	
$[\eta_c]_8[\eta_c]_8$	$\|112\rangle$		6454	6300
$[J/\psi]_8[J/\psi]_8$	$\|222\rangle$		6430	
$[cc]^0_{\bar{6}}[\bar{c}\bar{c}]^0_6$	$\|134\rangle$		6445	
$[cc]^1_3[\bar{c}\bar{c}]^1_{\bar{3}}$	$\|243\rangle$		6411	
Complete coupled-channels:				5960

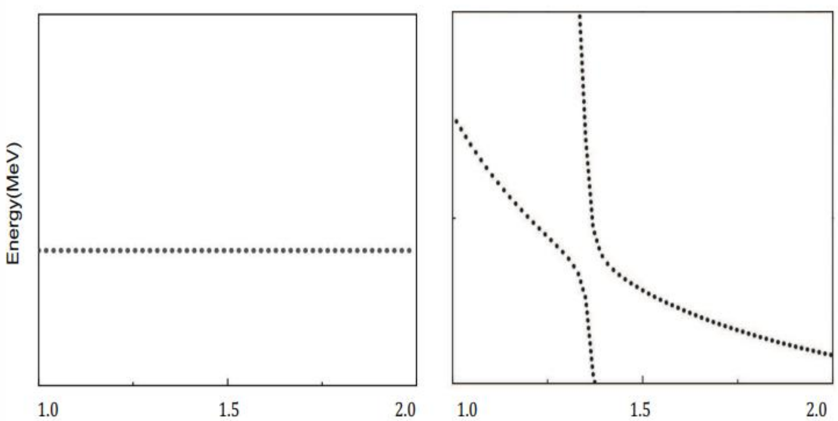
There is no bound state below the below the minimum threshold.

Results and Discussion

Resonance state test and decay width

- A stabilization method (real scaling method)

J. Simon, J. Chem, Phys. 75, 2465 (1981).



(a) (b)

- Gaussian size parameters rn are scaled by multiplying one factor α : rn −→ αrn.
- A compact resonance should not be affected by the variation of α while other continuum state will fall off towards its threshold.
- A resonance state shall emerge as avoid-crossing structures periodically.

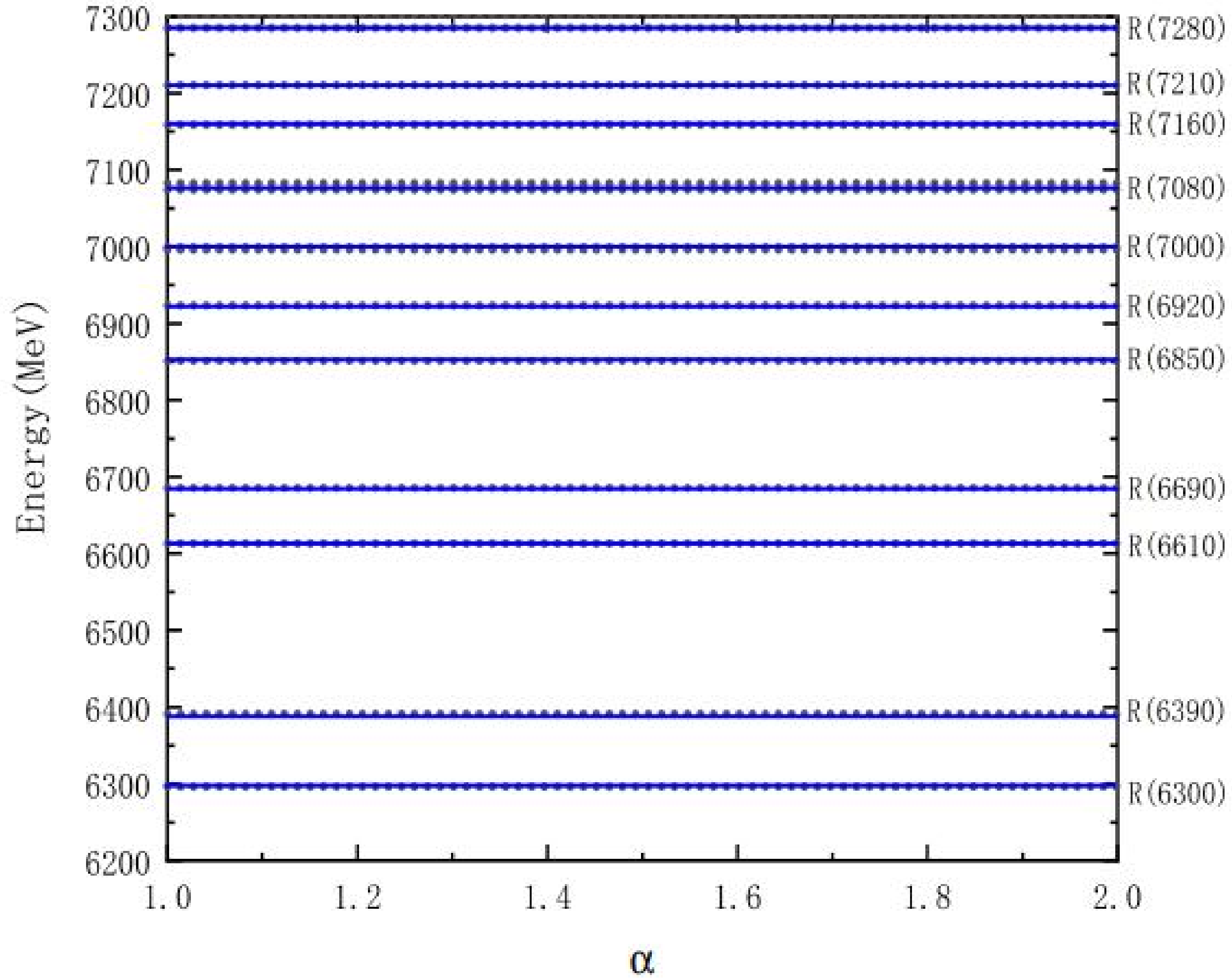
- Decay width

$$\Gamma = 4V(\alpha)\frac{\sqrt{k_r k_c}}{|k_r - k_c|}$$

- V(α) is the minimum energy difference between contimuum state and resonance.
- k_c and k_r stand for the slopes of scattering state and resonance state at the avoid-crossing point respectively.

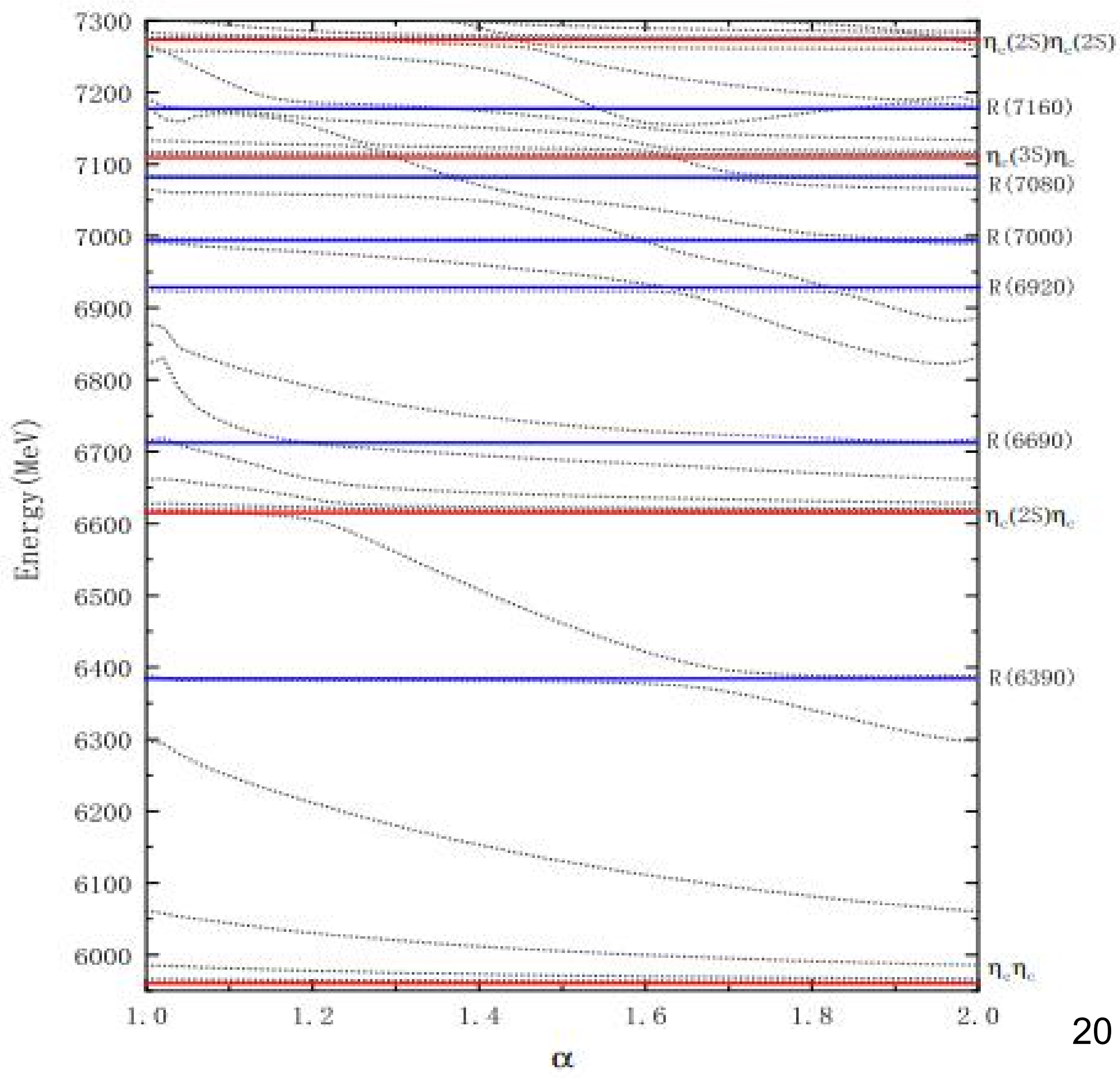
Results and Discussion

- Four colorful channels



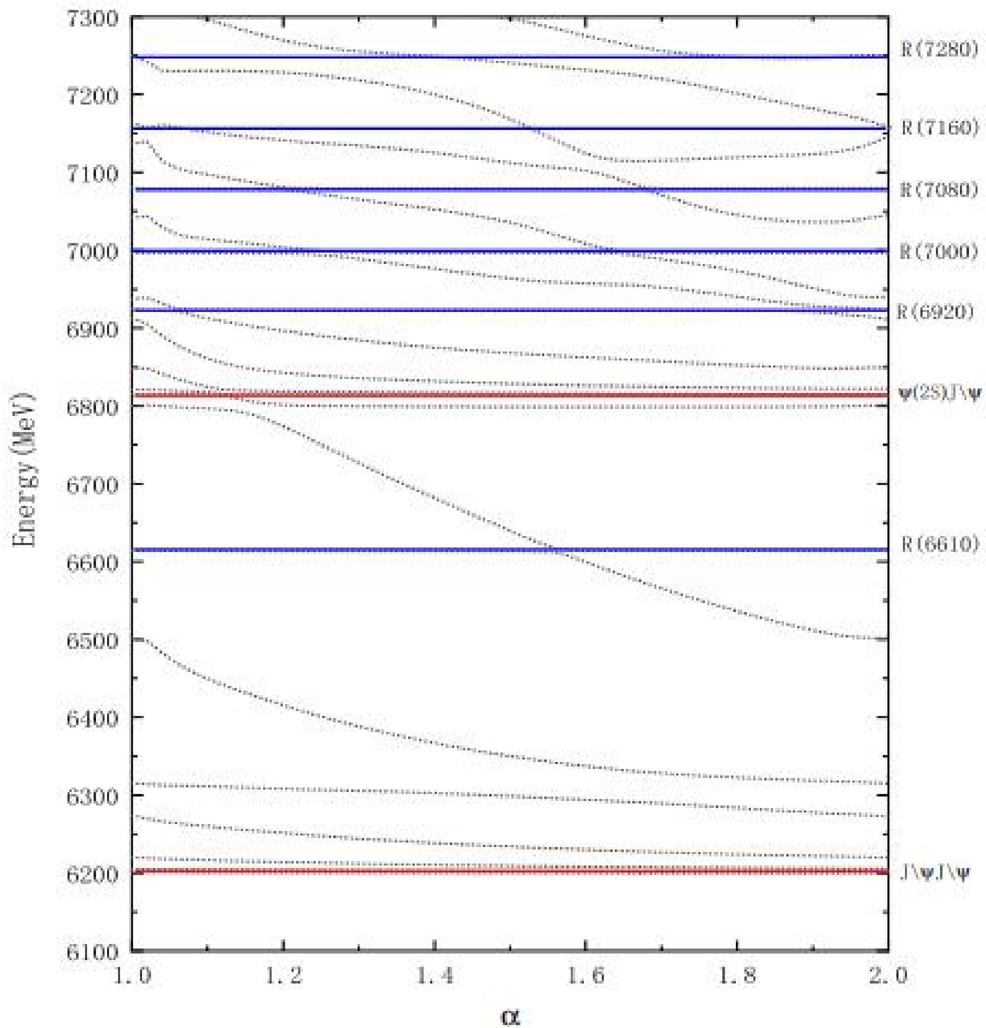
Results and Discussion

- Four colorful channels
  + open channel $\eta_c\eta_c$



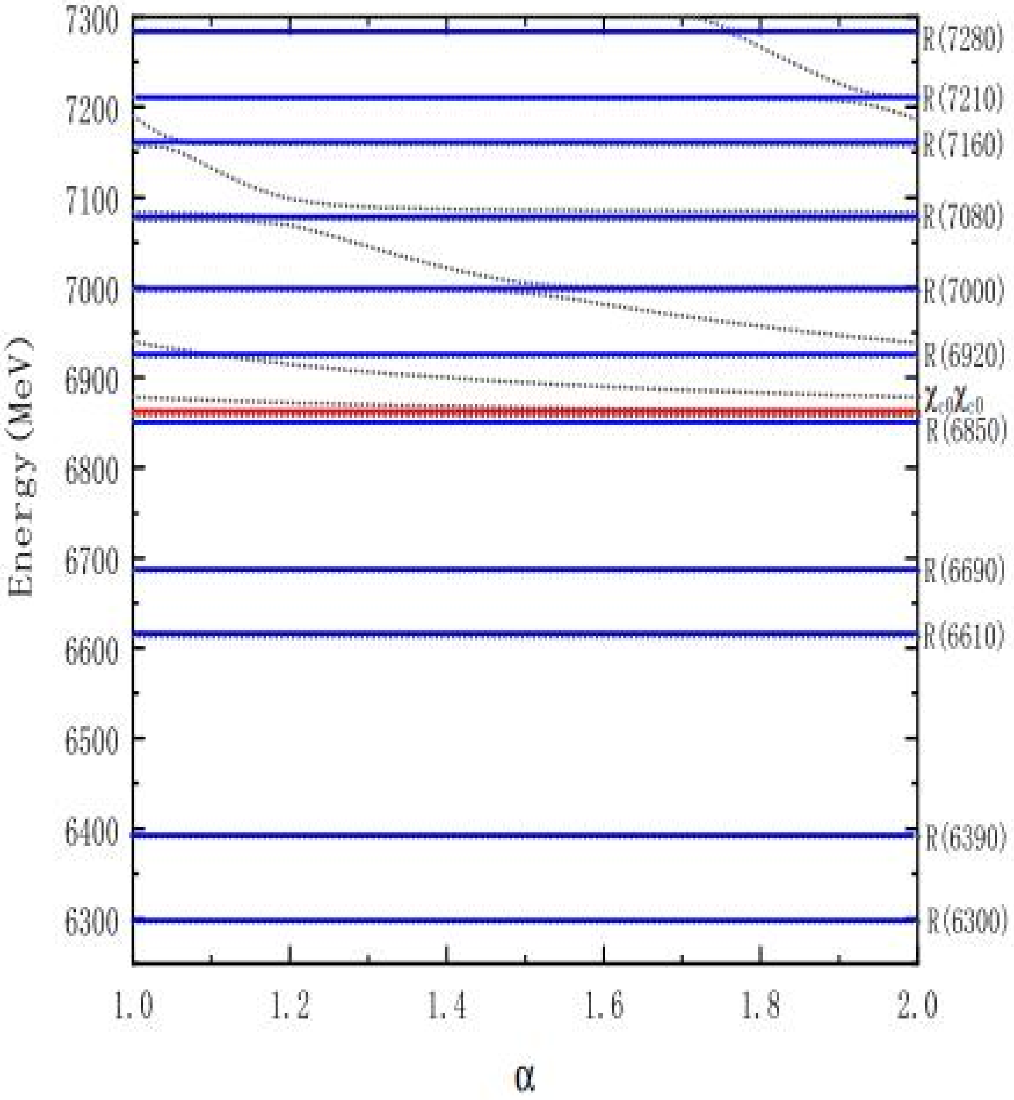
Results and Discussion

- Four colorful channels
  + open channel $J/\psi J/\psi$



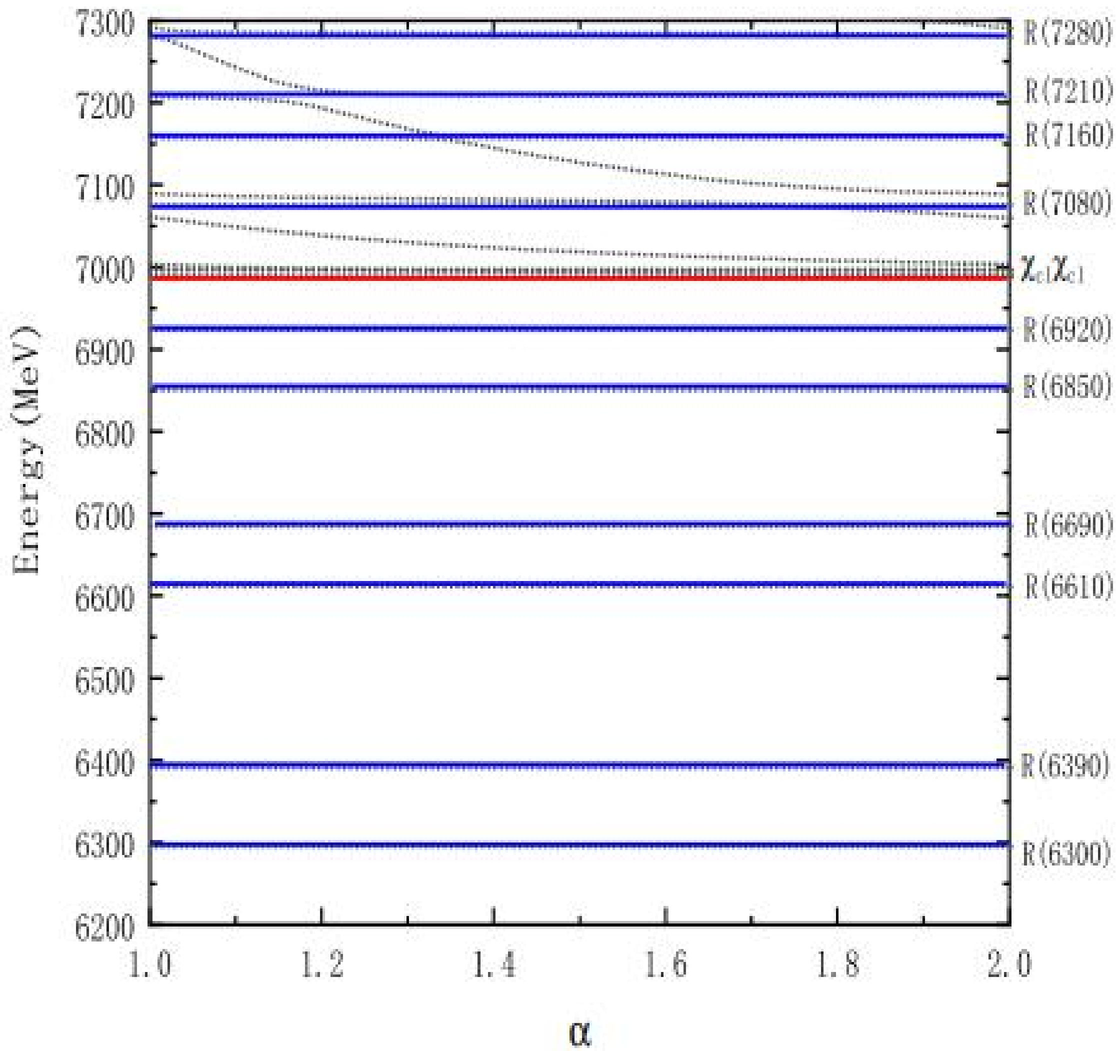
Results and Discussion

- Four colorful channels
  + open channel $\chi_{c0}\chi_{c0}$



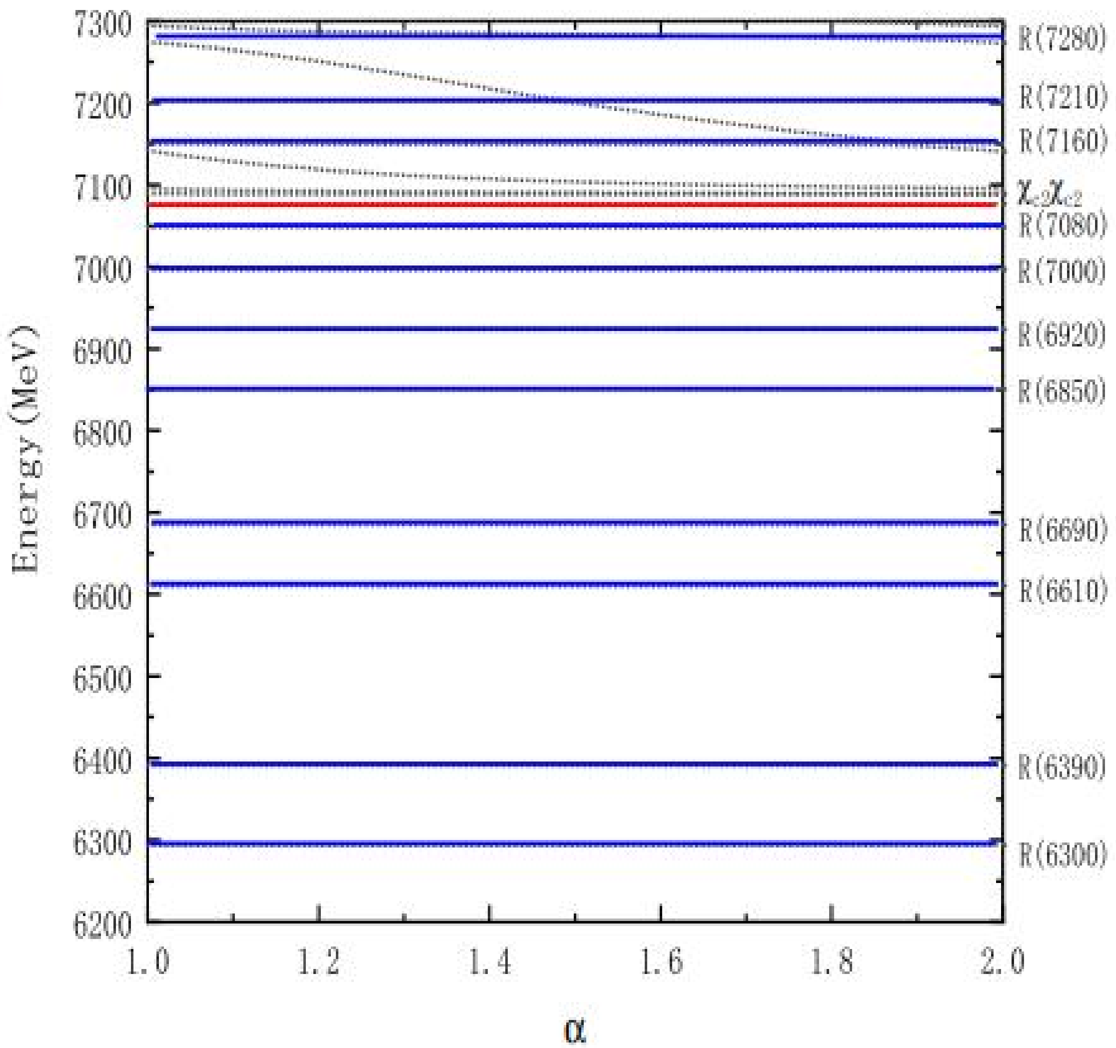
Results and Discussion

- Four colorful channels + open channel $\chi_{c1}\chi_{c1}$



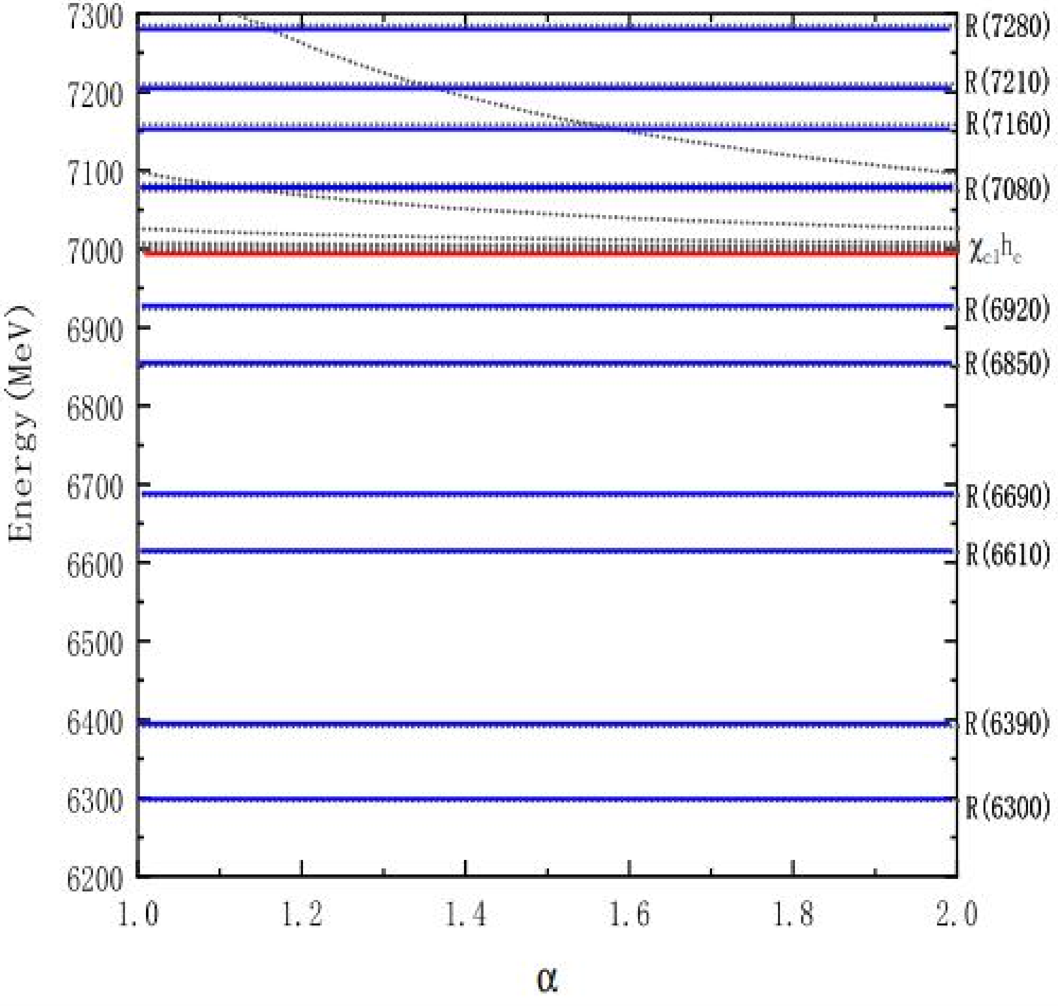
Results and Discussion

- Four colorful channels
  + open channel $\chi_{c2}\chi_{c2}$



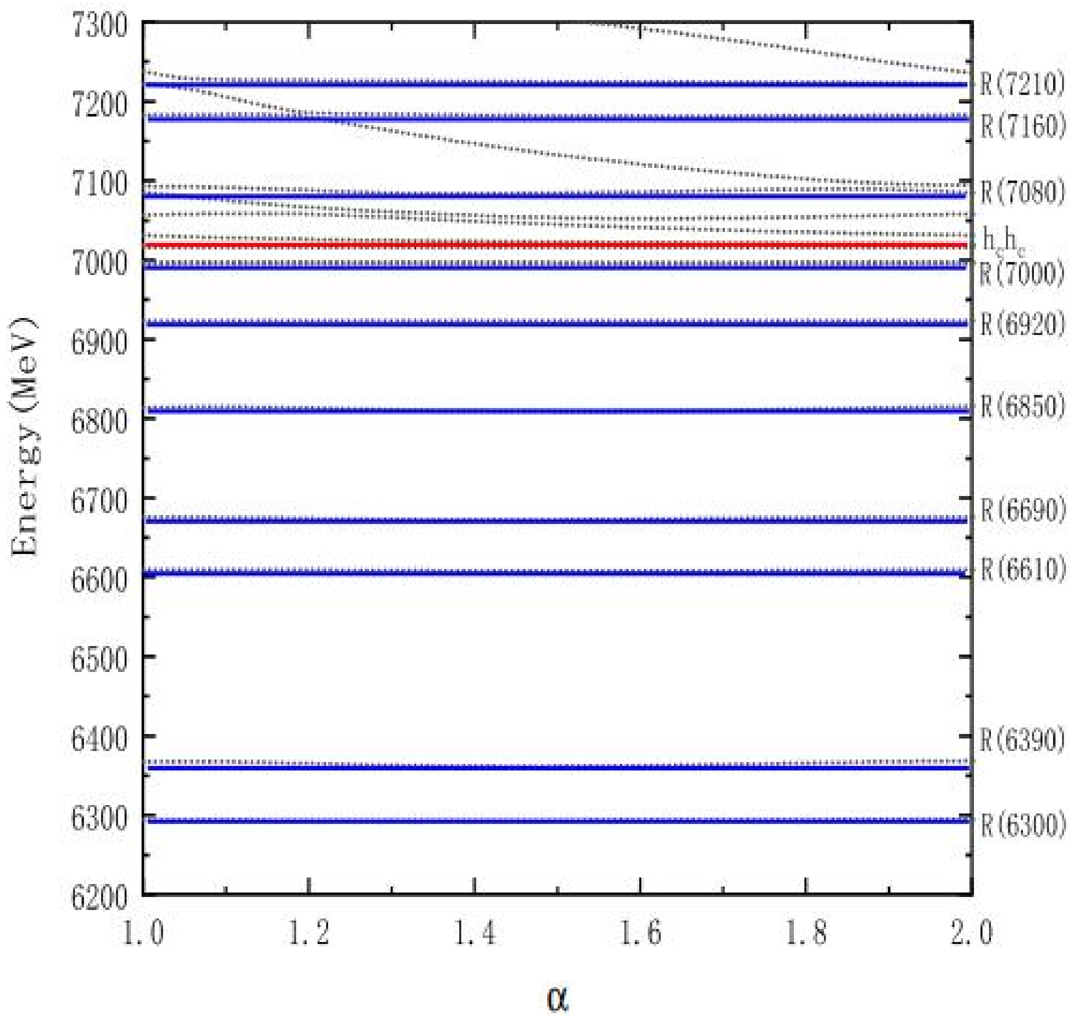
Results and Discussion

- Four colorful channels
  \+ open channel $\chi_{c1}h_c$



Results and Discussion

- Four colorful channels
  + open channel $h_c h_c$



Results and Discussion

- Four colorful channels
  + all open channels

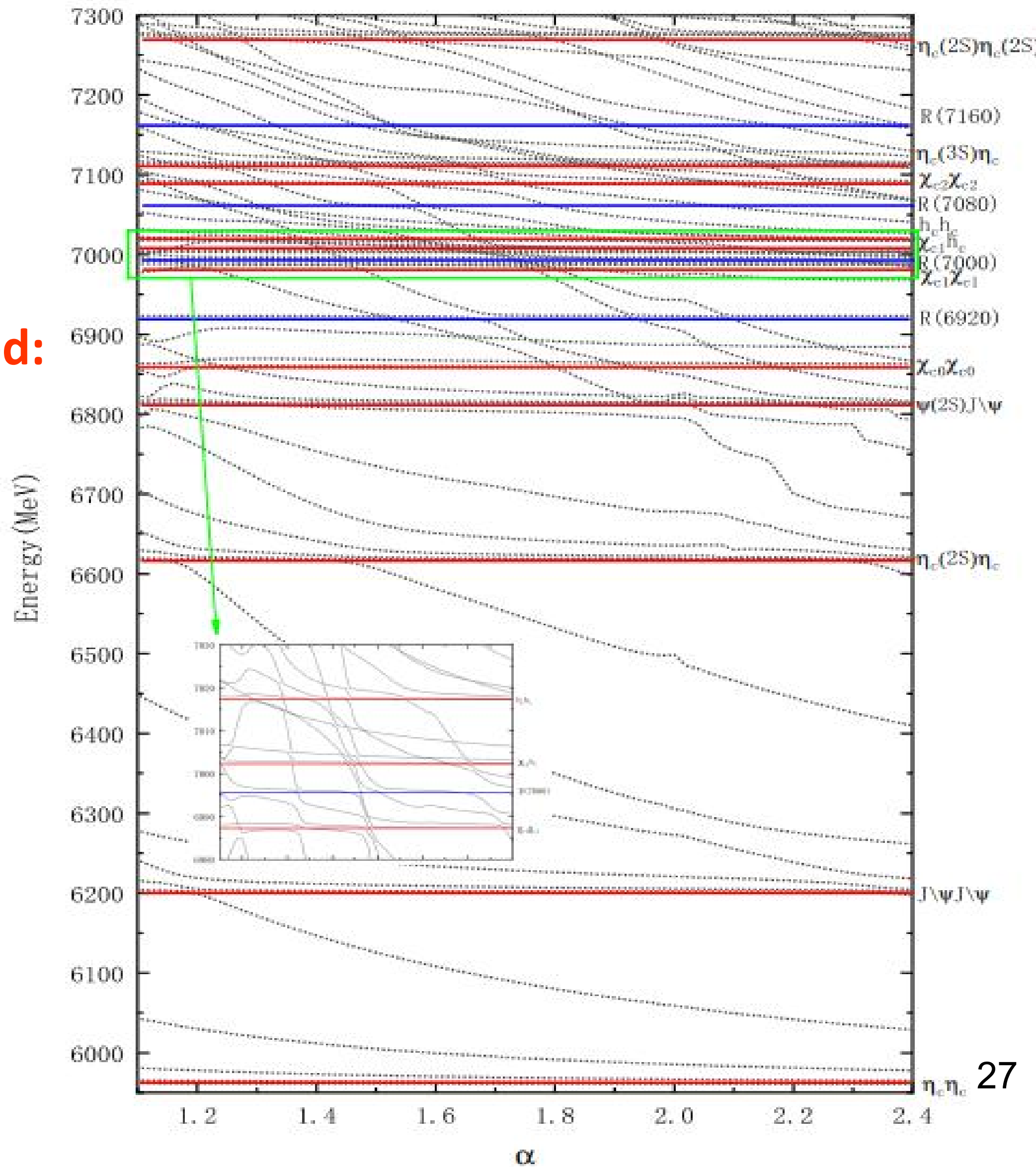
Only four resonance states survived:

R(6920)

R(7000)

R(7080)

R(7160)



Results and Discussion

TABLE VI. Various decay channels and corresponding decay widths of the obtained resonances. (unit: MeV)

Decay channels	$R(6920)$	$R(7160)$	$R(7000)$	$R(7080)$
$\eta_c\eta_c$	1.1	9.3	7.9	12.4
$J/\psi J/\psi$	9.8	22.9	10.3	1.9
$\chi_{c0}\chi_{c0}$	0.2	35.1	26.2	34.8
$\chi_{c1}\chi_{c1}$	-	2.7	0.2	1.8
$\chi_{c2}\chi_{c2}$	-	2.5	-	-
$\chi_{c1}h_c$	-	0.4	-	0.3
$h_c h_c$	-	4.9	-	8.8
Total	10.1	77.8	44.6	60.0

Summary

- There is no bound state below the below the minimum threshold in $c\bar{c}c\bar{c}$ system.
- **Four resonance states are obtained:**

resonance	width
R(6920)	Γ =10.1 MeV
R(7000)	Γ =77.8 MeV
R(7080)	Γ =44.6 MeV
R(7160)	Γ =60.0 MeV

Thanks for your attention！