FDC partial wave analysis software and excited-baryon study at BESIII

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第七届强子谱和强子结构研讨会

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FDC, FDC-PWA, FDC-Tensorflow

- FDC: Feynman Diagram Calculation
- FDC-PWA: FDC partial wave analysis
- FDC-Tensorflow tutorial

FDC Homepage

FDC is a package to do Feynman Diagram Calculation

The Project started in 1994 and aimed at developing a package to calculate Feynman Diagram automatically. The following parts have been finished already:

Construct the Lagrangian and deduce Feynman rules automatically
Generation of all Feynman diagrams and amplitudes for a given process.
Manipulate the amplitudes of these diagrams and generation of the expression of the total squared amplitude
Deal phase space integration automatically.

This page shows part of the results generated by FDC system.

This project is in part supported by the National Natural Science Foundation of China.

FDC-PWA homepage: FDC-PWA for Partial Wave Analysis method

FDCHQHP(FDC Heavy Quarkonium HadroProduction)

Manual

Progress in FDC project #2

Jian-Xiong Wang (Beijing, Inst. High Energy Phys.) (Jul, 2004)

Published in: *Nucl.Instrum.Meth.A* 534 (2004) 241-245 • Contribution to: ACAT 03 • e-Print: hep-ph/0407058 [hep-ph]

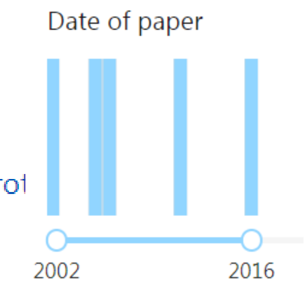
Baryonic Spectroscopy Program at BES/BEPC

Light hadron spectroscopy at BEPC

Bing-Song Zou (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.) (Aug, 2000)

Published in: *Nucl.Phys.A* 692 (2001) 362-371 • Contribution to: Biennial Conference on Low-Energy Antiprot 2000) • e-Print: hep-ph/0011174 [hep-ph]

pdf DOI cite claim reference search 5 citations

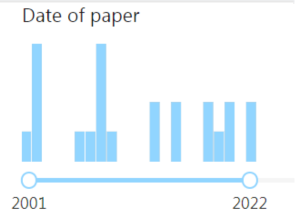


N*, Lambda*, Sigma* and Xi* resonances from J / Psi and Psi-prime decays

Bing-Song Zou (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.) (Mar, 2000)

Published in: *Nucl.Phys.A* 684 (2001) 330-332 • Contribution to: FB16 • e-Print: hep-ph/0006039 [hep-ph]

pdf DOI cite claim reference search 23 citations

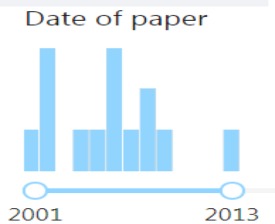


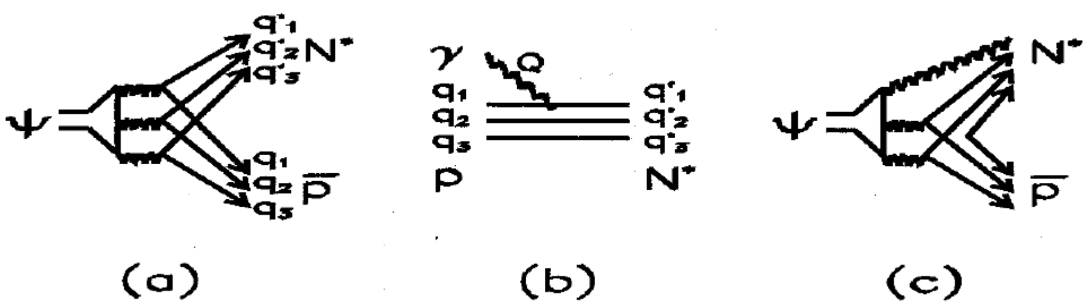
The baryon spectroscopy from J / psi decays

Bing-Song Zou (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys.) (2000)

Published in: *Nucl.Phys.A* 675 (2000) 167C-172C • Contribution to: Hadron 1999

DOI cite claim reference search 14 citations





Excited baryon in J/ψ and ψ' decays

Data sets: 10 billion J/ψ , 3 billion $\psi(2S)$

X	$Br(J/\psi \to X)$	$Br(\psi' \to X)$
$N\bar{N}\pi$	$(9.7 \pm 0.6) \times 10^{-3}$	$(7.6 \pm 0.6) \times 10^{-4}$
$p\bar{p}\pi^+\pi^-$	$(6.0 \pm 0.5) \times 10^{-3}$	$(6.0 \pm 0.4) \times 10^{-4}$
$N\bar{N}\eta$	$(4.18 \pm 0.36) \times 10^{-3}$	$(0.58 \pm 0.13) \times 10^{-4}$
$\Lambda\bar{\Lambda}\eta$	$(0.16 \pm 0.02) \times 10^{-3}$	$(0.25 \pm 0.04) \times 10^{-4}$
$pK^-\bar{\Lambda} + \mathrm{c.c.}$	$(0.86 \pm 0.11) \times 10^{-3}$	$(1.00 \pm 0.04) \times 10^{-4}$
$pK^-\bar{\Sigma}^0$	$(0.29 \pm 0.08) \times 10^{-3}$	$(0.17 \pm 0.02) \times 10^{-4}$
$\Sigma\bar{\Lambda}\pi$	$(0.83 \pm 0.07) \times 10^{-3}$	$(1.54 \pm 0.04) \times 10^{-4}$

Big challenge for PWA
Software : FDC-tf package

Advancements in FDC-PWA Development

Theoretical formalism and Monte Carlo study of partial wave analysis for J / psi --> p anti-p omega #1

W.H. Liang (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys. and Guangxi Normal U. and Natl. Lab. Heavy Ion Accel., Lanzhou), P.N. Shen (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys. and Guangxi Normal U. and Natl. Lab. Heavy Ion Accel., Lanzhou), J.X. Wang (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys. and Guangxi Normal U. and Natl. Lab. Heavy Ion Accel., Lanzhou), B.S. Zou (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys. and Guangxi Normal U. and Natl. Lab. Heavy Ion Accel., Lanzhou) (2002)

Published in: *J.Phys.G* 28 (2002) 333-343 19-23 February 2024

DOI cite claim reference search 35 citations

2.3. The effective vertices involved

$$\mathcal{L} = \bar{\psi}_1 \Gamma \psi_2 A$$

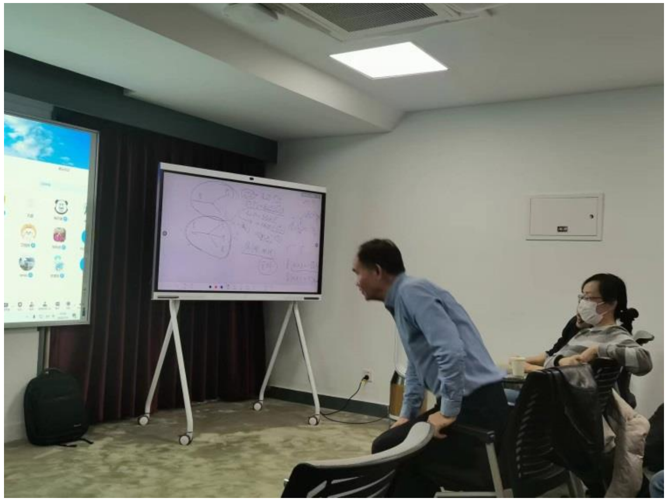
Table 2. The transformation properties of some operators.

$\Gamma =$	i	γ_5	γ_μ	$\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$	$\sigma_{\mu\nu}\gamma_5$	$g_{\mu\nu}$
$\gamma_0\Gamma^+\gamma_0 =$	$-\mathrm{i}$	$-\gamma_5$	γ_μ	$\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$	$-\sigma_{\mu\nu}\gamma_5$	$g_{\mu\nu}$
$C(\gamma_0\Gamma^+\gamma_0)^T C^{-1} =$	$-\mathrm{i}$	$-\gamma_5$	$-\gamma_\mu$	$\gamma_\mu\gamma_5$	$-\sigma_{\mu\nu}$	$\sigma_{\mu\nu}\gamma_5$	$g_{\mu\nu}$
$\gamma_0\Gamma^P\gamma_0 =$	i	$-\gamma_5$	γ_μ	$-\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$	$-\sigma_{\mu\nu}\gamma_5$	$g_{\mu\nu}$

Progress in the Development of FDC-PWA

- “To do a good job, one must first sharpen one's tools”
  - ✓ **GPU Implementation of Feynman Diagram Computation (FDC) System： FDC + Tensorflow**
  - ✓ **FDC-TF physics analyses**
    1. PWA $J/\psi \to \pi^+\pi^-\Sigma^+\bar{\Sigma}^-$
    2. PWA $J/\psi \to p\bar{p}K^+K^-$
    3. PWA $J/\psi \to \phi\eta\eta$
    4. PWA $J/\psi \to \phi\eta\eta'$
    5. PWA $\psi' \to \pi^0\Sigma^+\bar{\Sigma}^-$

    ……
  - ✓ **Upgrade of FDC and Expansion of Two Major Functions**
    1. Hyperon weak decays
    2. Radiative decays



Partial Wave Analysis Theory-Experiment Joint Workshop, 3/9/2022, China Advanced Science and Technology Center

Structure of the FDC-TF software package

FDC: generate amplitude

FDC python module

TensorFlow: MLLH calculation

Minuit in pyROOT: minimize MLLH

Signal yields, statistical uncertainty.

Matplotlib or pyROOT: Fill histograms

Dependent modules:

- Numpy, matlibplot, iminuit, Scipy, Tensorflow2.0, pyROOT
- Python version: 3.7

Development of FDC-TF Applications

- **Simultaneous fit to multiple data sets**

  Object function for data set i: $S_i = -(ln\mathcal{L}^i_{dt} - ln\mathcal{L}^i_{bkg})$

  Minimized object function: $S = \sum_i S_i$

  where S_i calculated by one GPU card, dispatched by CPU muti-threads

- **One channel decay with running width**

$$BW(s, M_0, \Gamma_0) = \frac{1}{s - M_0^2 - iM_0\Gamma(s)} \quad \text{with} \quad \Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^{2l+1} \frac{m_0}{m} B_l'^2(q, q_0, d).$$

$$B_0'(q, q_0, d) = 1,$$

$$B_1'(q, q_0, d) = \sqrt{\frac{1 + (q_0 d)^2}{1 + (qd)^2}},$$

$$B_2'(q, q_0, d) = \sqrt{\frac{9 + 3(q_0 d)^2 + (q_0 d)^4}{9 + 3(qd)^2 + (qd)^4}},$$

$$B_3'(q, q_0, d) = \sqrt{\frac{225 + 45(q_0 d)^2 + 6(q_0 d)^4 + (q_0 d)^6}{225 + 45(qd)^2 + 6(qd)^4 + (qd)^6}},$$

$$B_4'(q, q_0, d) = \sqrt{\frac{11035 + 1575(q_0 d)^2 + 135(q_0 d)^4 + 10(q_0 d)^6 + (q_0 d)^8}{11035 + 1575(qd)^2 + 135(qd)^4 + 10(qd)^6 + (qd)^8}},$$

• Baryon resonance : couple channel running width

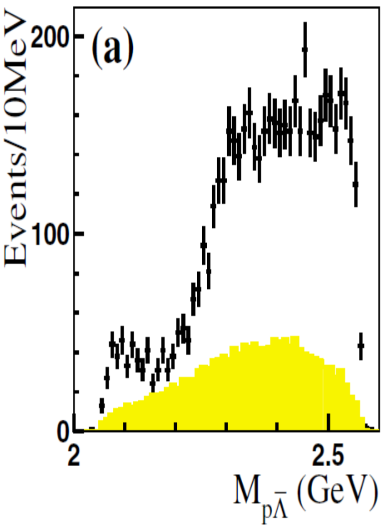
For example: $N(1535)$:

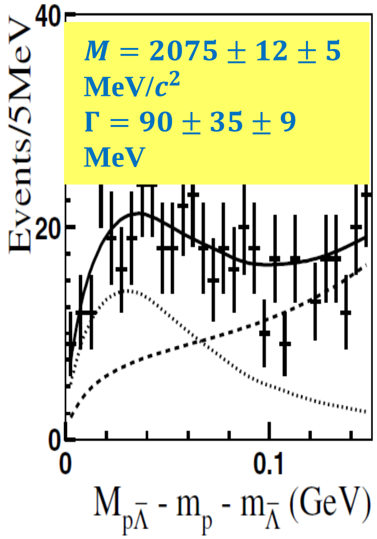
$$BW(s, M_0, \Gamma_0) = \frac{1}{s - M_0^2 - iM_0\Gamma(s)}, \text{ with } \Gamma(s) = \Gamma_0\left(0.5\ \frac{\rho_{\pi N}(s)}{\rho_{\pi N}(M_0^2)} + 0.5\,\frac{\rho_{\eta N}(s)}{\rho_{\eta N}(M_0^2)}\right)$$

关键词	N1440width	N1520width	N1535width	N1650width	N1700width
共振态	$N(1440)\ \frac{1}{2}^{+}$	$N(1520)\ \frac{3}{2}^{-}$	$N(1535)\ \frac{1}{2}^{-}$	$N(1650)\ \frac{1}{2}^{-}$	$N(1700)\ \frac{3}{2}^{-}$
关键词	N1710width	N1720width	L1380width	L1405width	L1520width
共振态	$N(1710)\ \frac{1}{2}^{+}$	$N(1720)\ \frac{3}{2}^{+}$	$\Lambda(1380)\ \frac{1}{2}^{-}$	$\Lambda(1405)\ \frac{1}{2}^{-}$	$\Lambda(1520)\ \frac{3}{2}^{-}$
关键词	L1600width	L1670width	D1232width	D1600width	D1620width
共振态	$\Lambda(1600)\ \frac{1}{2}^{+}$	$\Lambda(1670)\ \frac{1}{2}^{-}$	$\Delta(1232)\ \frac{3}{2}^{+}$	$\Delta(1600)\ \frac{3}{2}^{+}$	$\Delta(1620)\ \frac{1}{2}^{-}$
关键词	D1700width	S1385width	S1660width	S1670width	S1750width
共振态	$\Delta(1700)\ \frac{3}{2}^{-}$	$\Sigma(1385)\ \frac{3}{2}^{+}$	$\Sigma(1660)\ \frac{1}{2}^{+}$	$\Sigma(1670)\ \frac{3}{2}^{-}$	$\Sigma(1750)\ \frac{1}{2}^{-}$
关键词	S1910width	X1530width			
共振态	$\Sigma(1910)\ \frac{3}{2}^{-}$	$\Xi(1530)\ \frac{3}{2}^{+}$			

$p\bar{\Lambda}$ threshold enhancement $X(2075)$

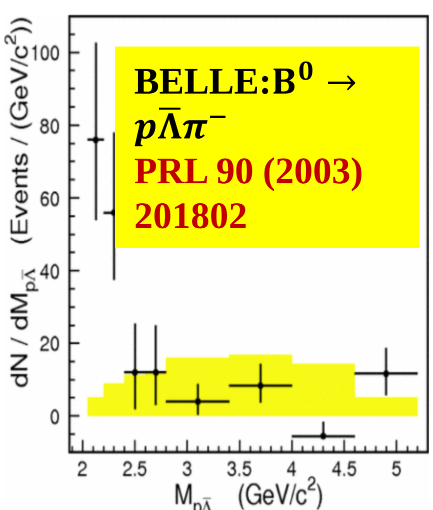
BESII: $J/\psi \to pK^-\bar{\Lambda} + c.c.$
PRL 93 (2004) 112002

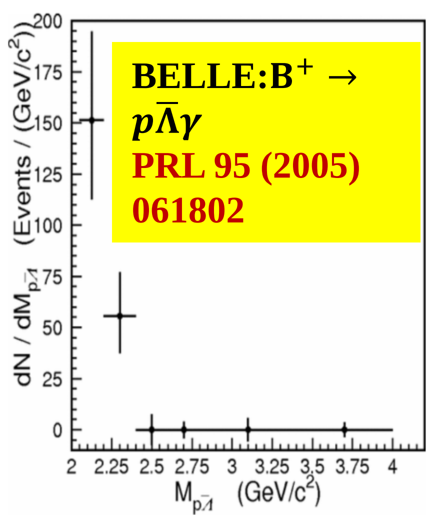


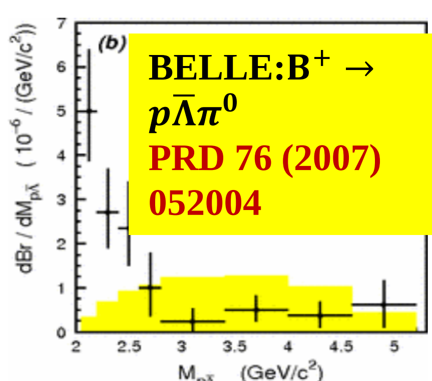


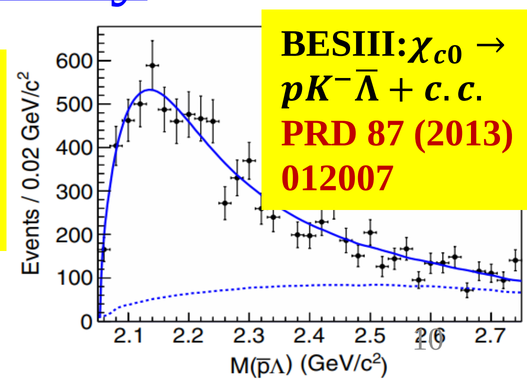
Motivation

- Observed at BESII in 2004
- Similar structure was seen in several B meson and charmonium decays
- Investigated theoretically under scenario of quark model, FSI and chiral effective theory





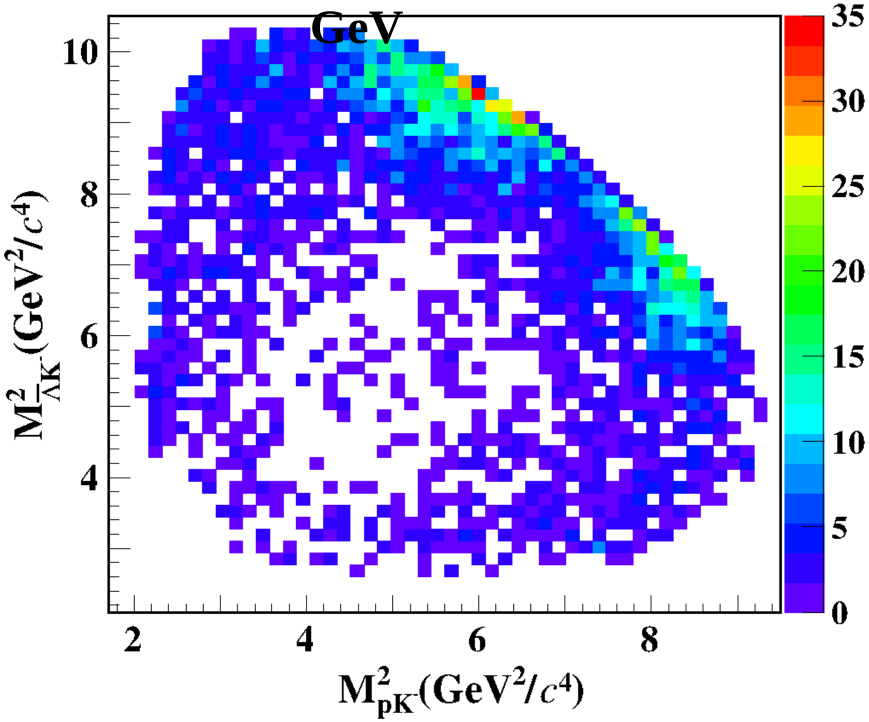




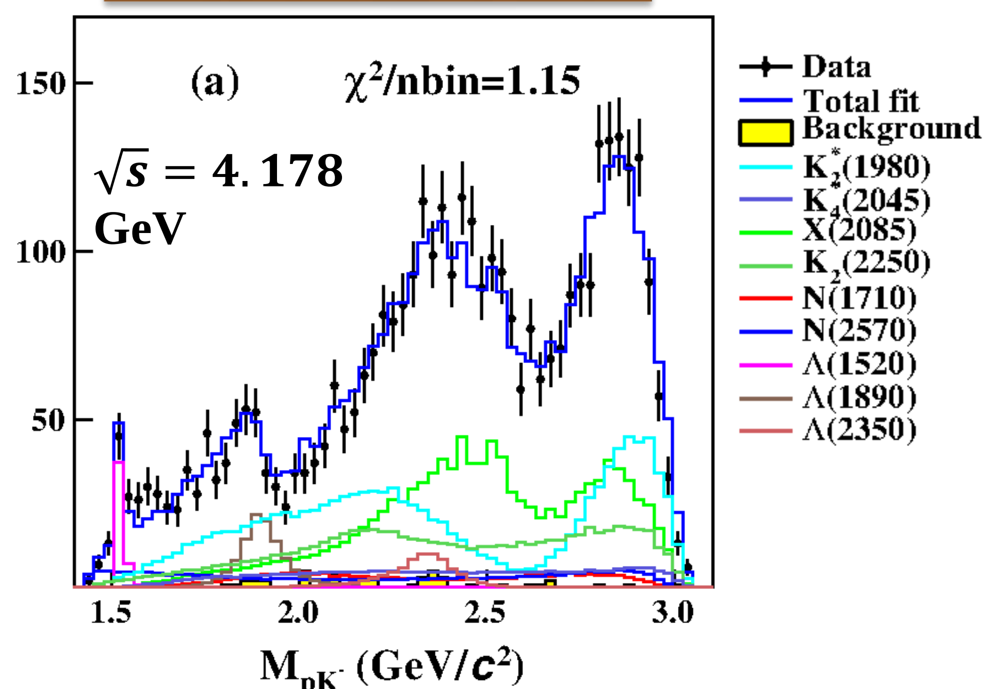
$X(2075)$ in $e^+e^- \to \gamma^* \to pK^-\bar{\Lambda}$+c.c.

- $X(2075)$[K(2075)] first observed in $J/\psi \to pK^-\Lambda + c.c.$, but spin-parity not measured.
- Similar evidence found in $B \to p\Lambda\pi$, and $\psi', \chi_{cJ} \to pK^-\Lambda$.
- Near $p\Lambda_c$ threshold, an enhancement also observed in $B^- \to \Lambda_c^+\bar{p}\pi^-$

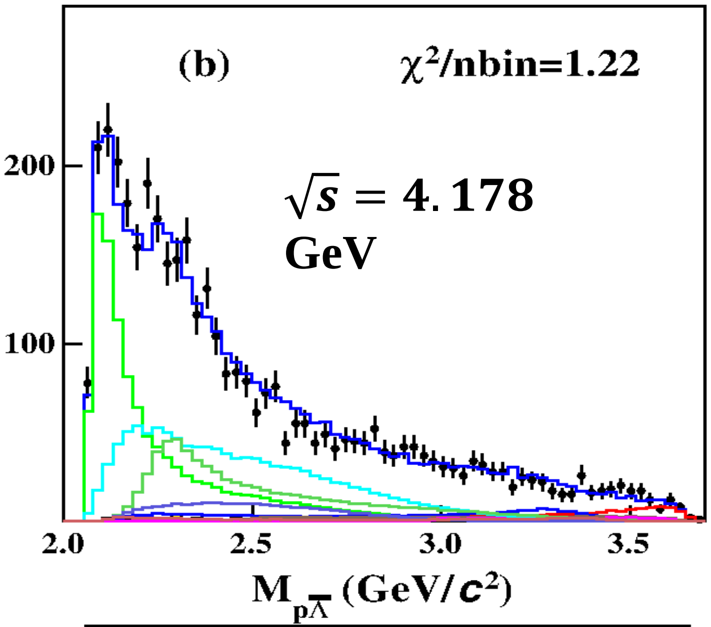
$\sqrt{s} = 4.178$ GeV

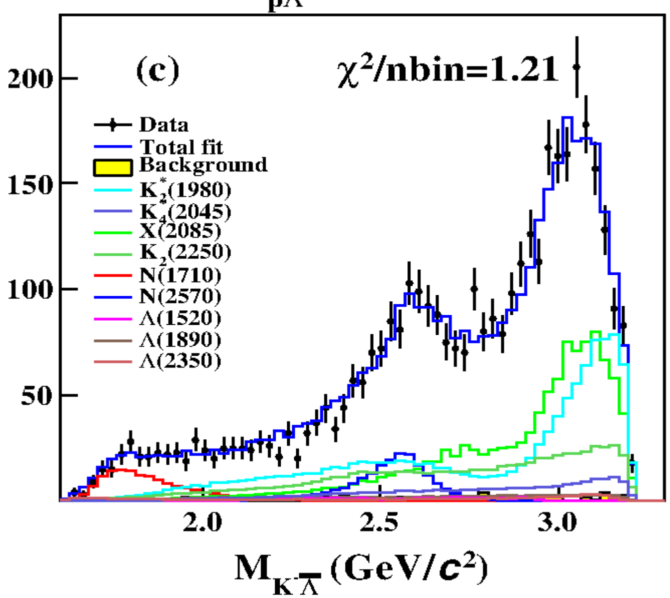


PRL131, 151901 (2023)



$X(2075)$ in $e^+e^- \to \gamma^* \to pK^-\bar{\Lambda} + c.c.$





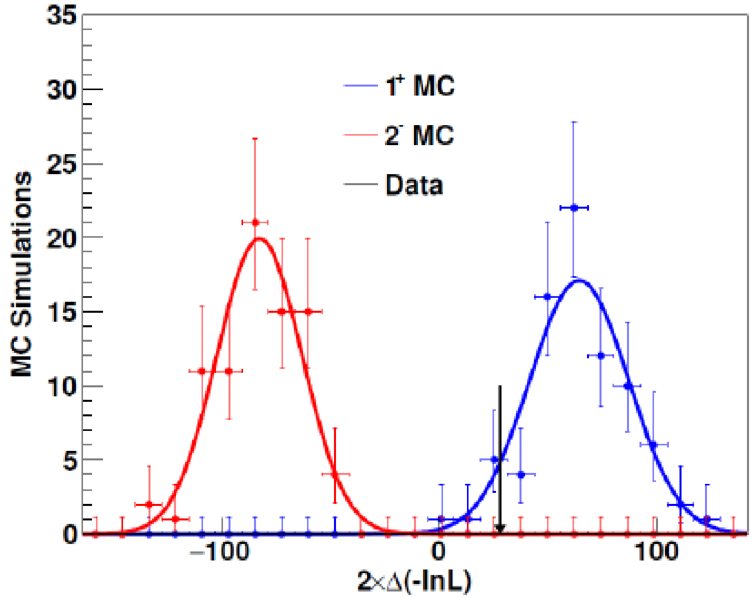
PRL131, 151901 (2023)

- $X(2075)$, $J^P = 1^+$
- $M_{pole} = (2084^{+4}_{-2} \pm 9)$ MeV
- $\Gamma_{pole} = (58^{+4}_{-3} \pm 25\,)$ MeV

	$\Delta \ln \mathcal{L}$	Δndf	Significance
1^+ over 0^-	40.6	4	8.3
1^+ over 1^-	30.2	2	7.5
1^+ over 2^+	44.8	2	9.2
1^+ over 2^-	13.8	0	5.3

$\sqrt{s}$ (GeV)	$M_{\rm pole}$ (MeV/c^2)	$\Gamma_{\rm pole}$ (MeV)
4.008	2085±14	50±16
4.178	2085±6	62±10
4.226	2088±10	68±12
4.258	2083±11	48±10
4.416	2088±13	56±12
4.682	2092±10	54±10
Average	2086±4	56±5

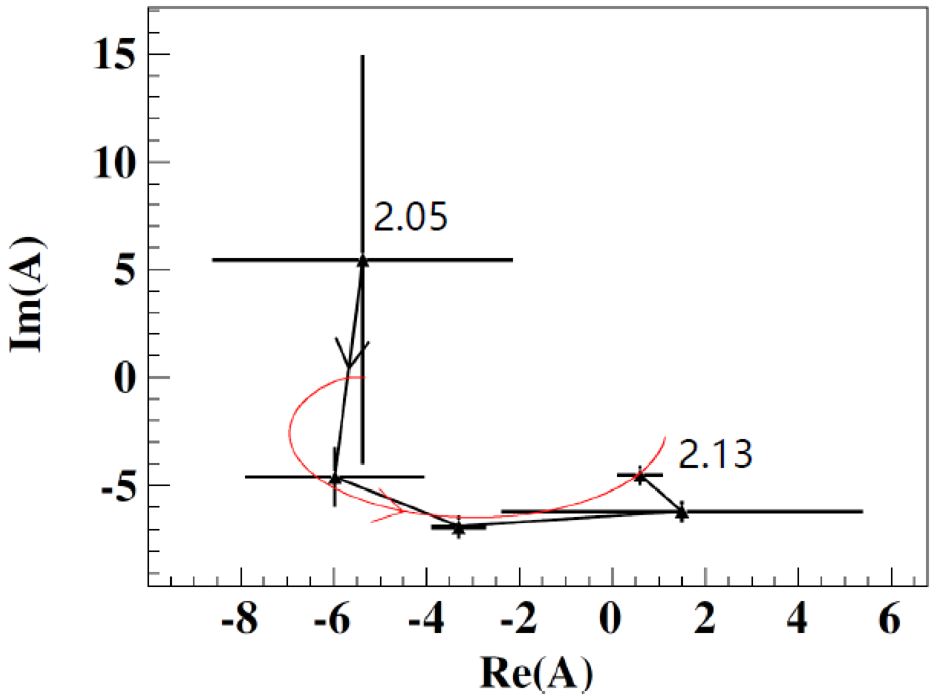
Check significance with toy MC method



	$\Delta \ln \mathcal{L}$	Δndf	Significance
1^+ over 0^-	40.6	4	8.3
1^+ over 1^-	30.2	2	7.5
1^+ over 2^+	44.8	2	9.2
1^+ over 2^-	13.8	0	5.3

- ✓ The statistical significances of 1^+ over $0^-, 1^-$, and 2^+ are obtained with $\Delta \ln L = \ln L^{1^+} - \ln L^{J^P}$ and Δndf
- ✓ For 1^+ over 2^-, the Δndf is assumed to be 1, following PRL 115 (2015), 072001.
- ✓ The approach based on MC simulation is checked. The $t = -2\ln(L^{2^-}/L^{1^+})$ distribution suggests a statistical significance $5.6\,\sigma$

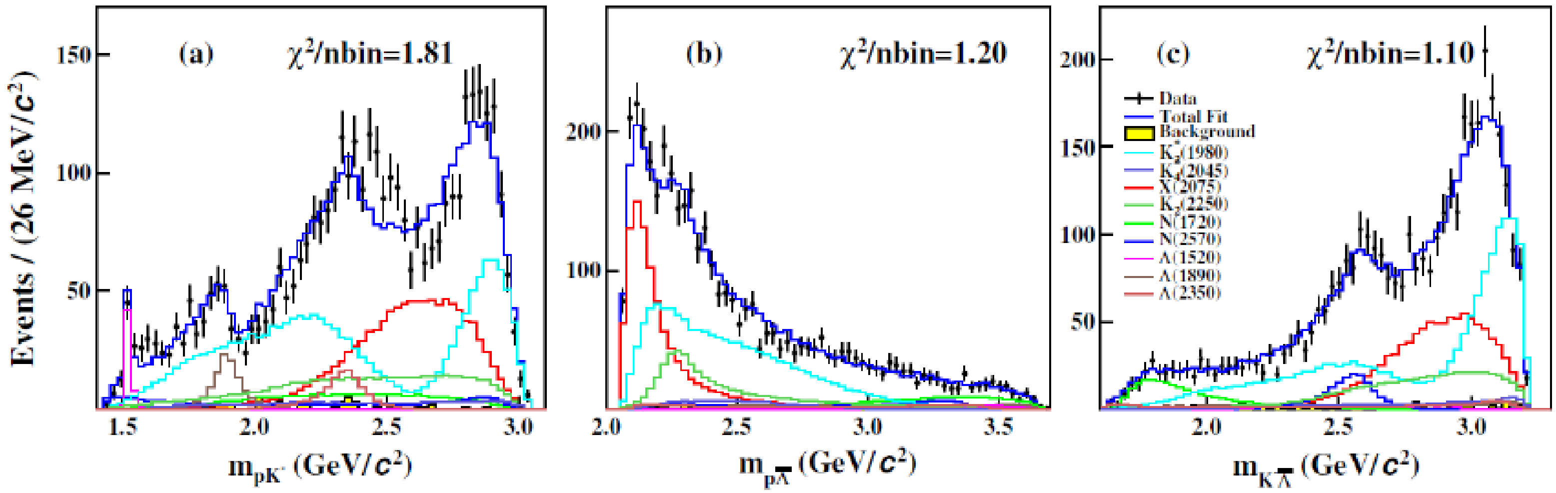
X(2075) Argand plot



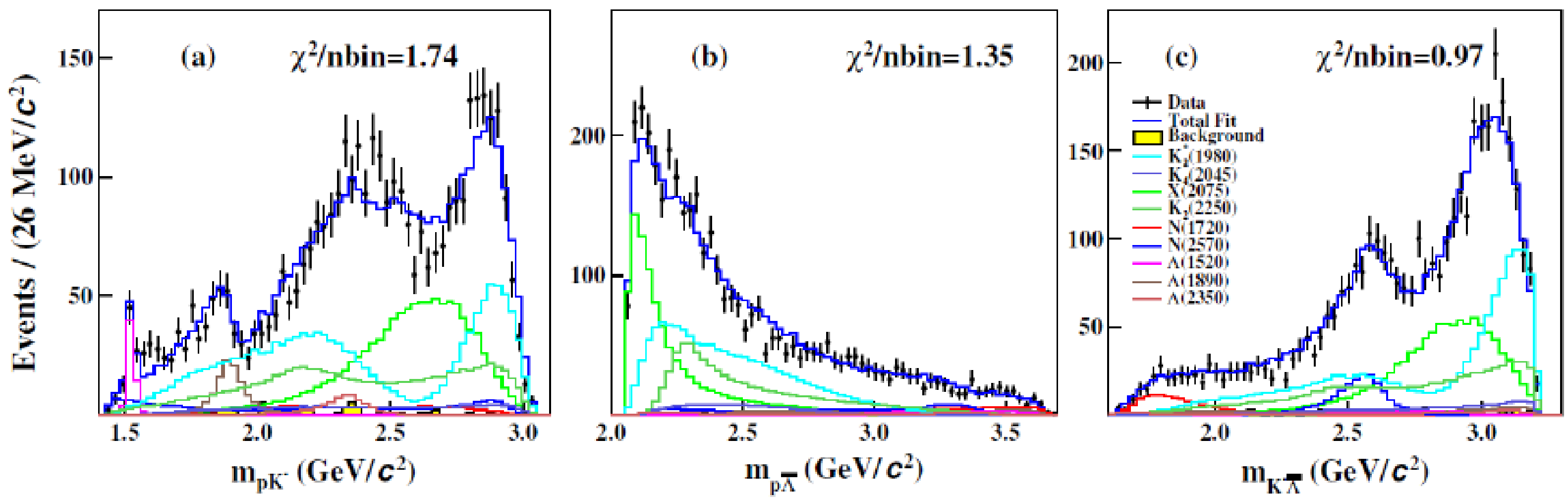
- From 2.05 – 2.13 GeV with step size 20 MeV
- BW function replaced with a complex number A_i in point i
- Cubic interpolation between two points
- No conclusion whether $X(2075)$ exhibits the characteristics of a resonance or not.

check alternative J^P

$J^P = 0^-$



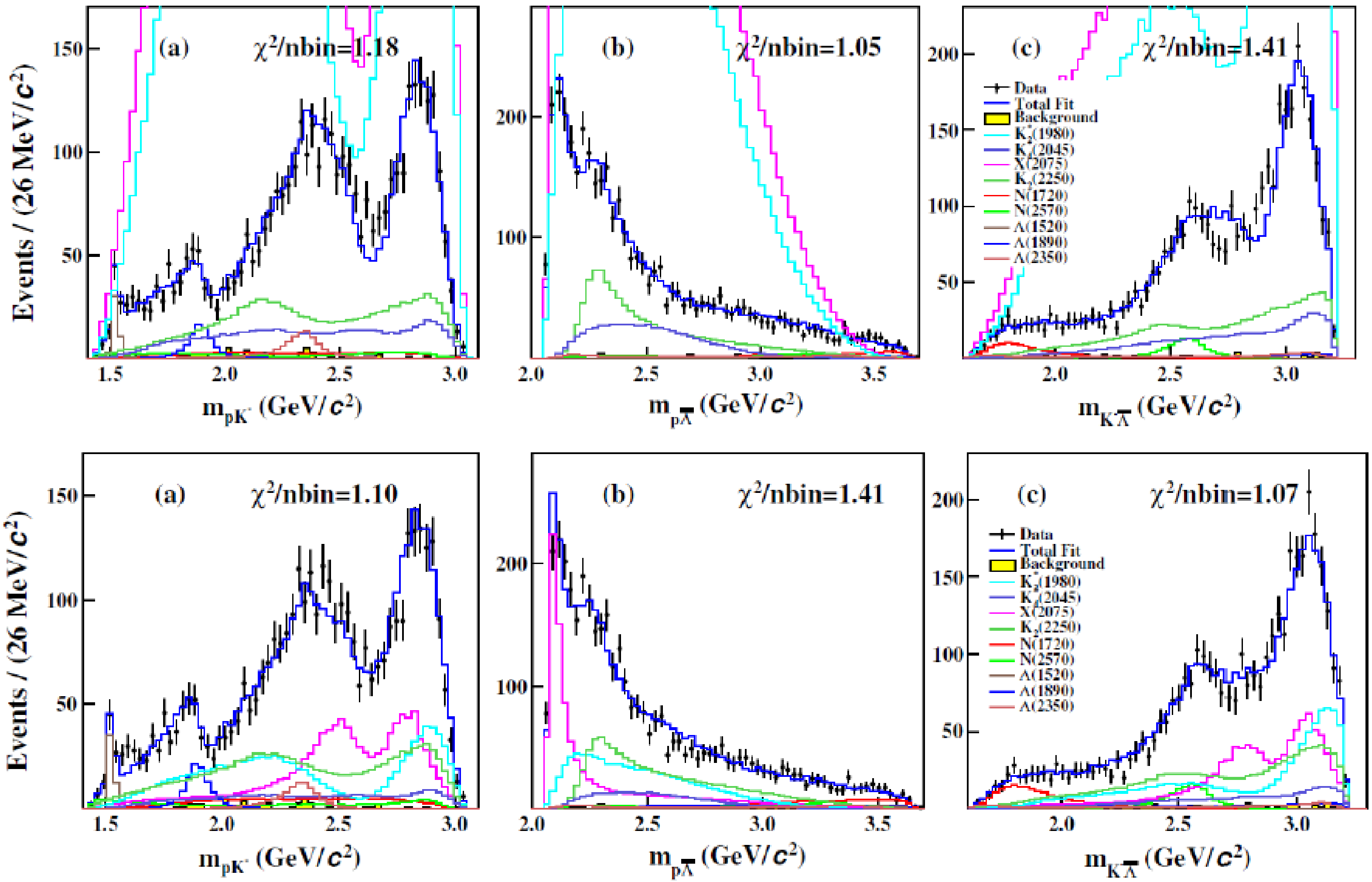
$J^P = 1^-$



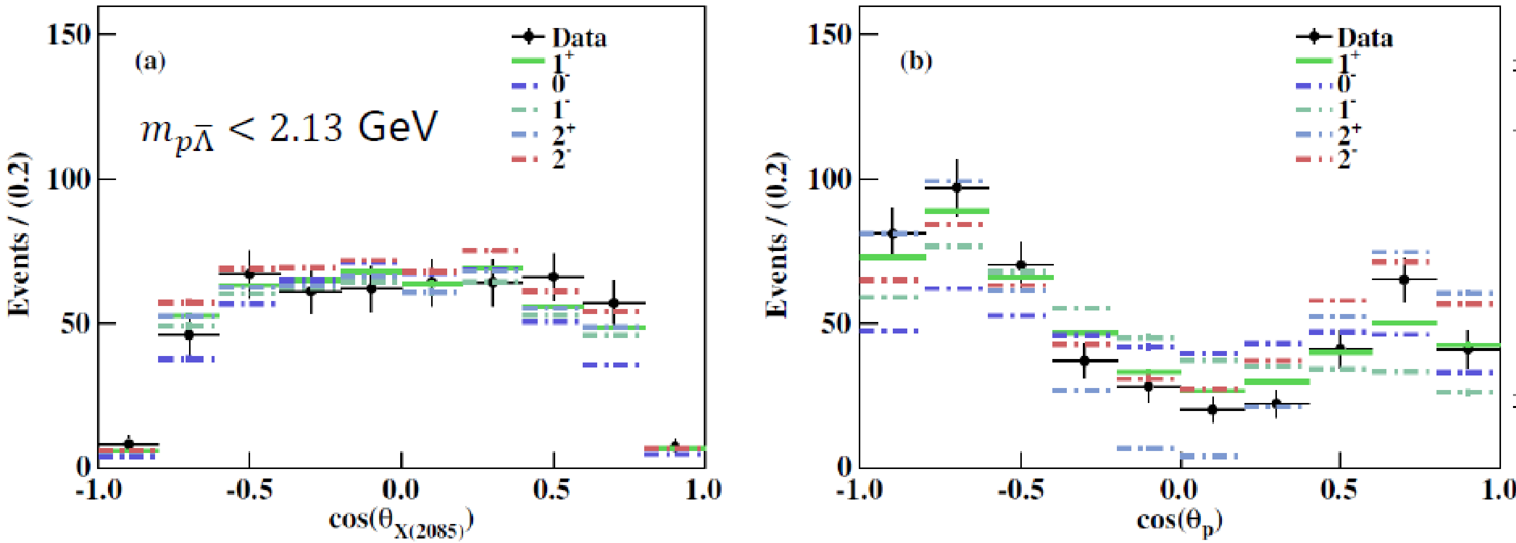
check alternative J^P

$J^P = 2^+$

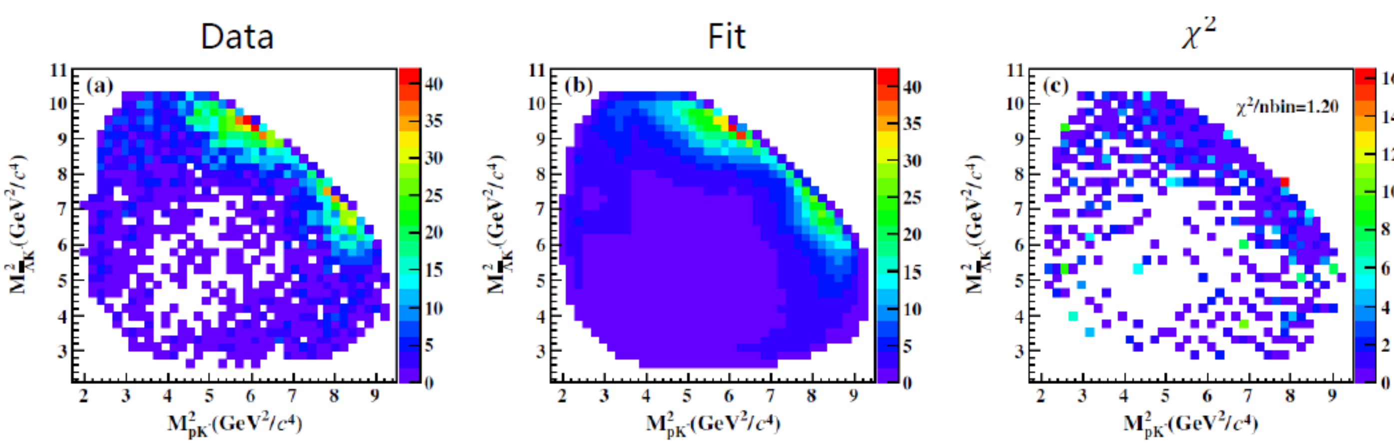
$J^P = 2^-$



Data favor 1^+ hypothesis

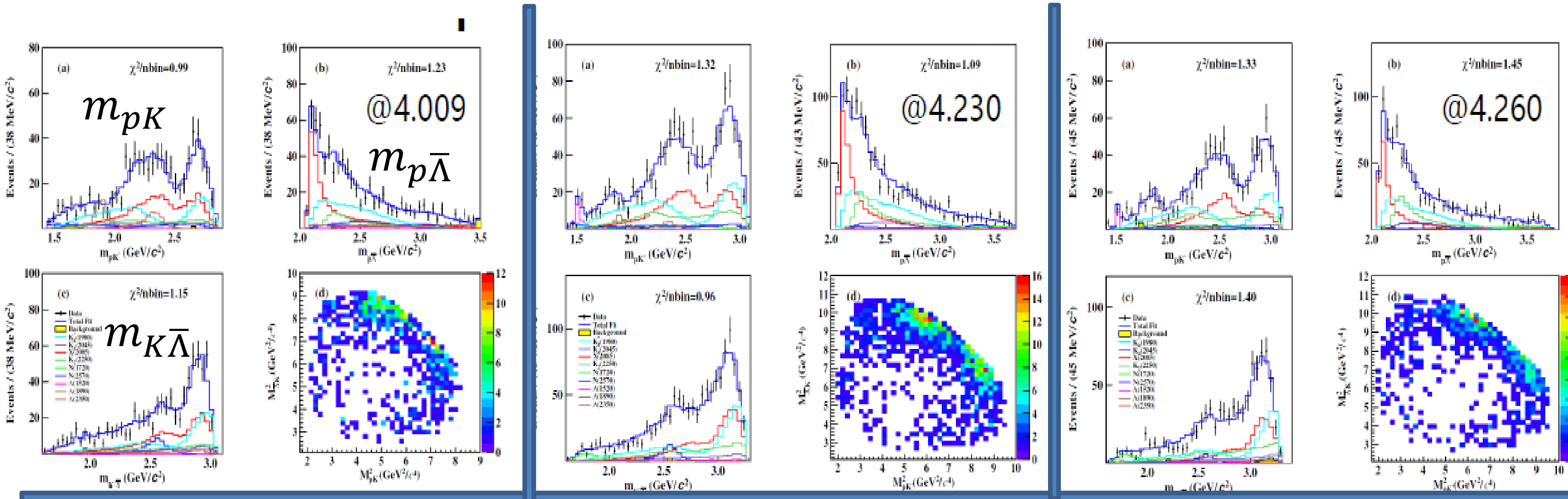


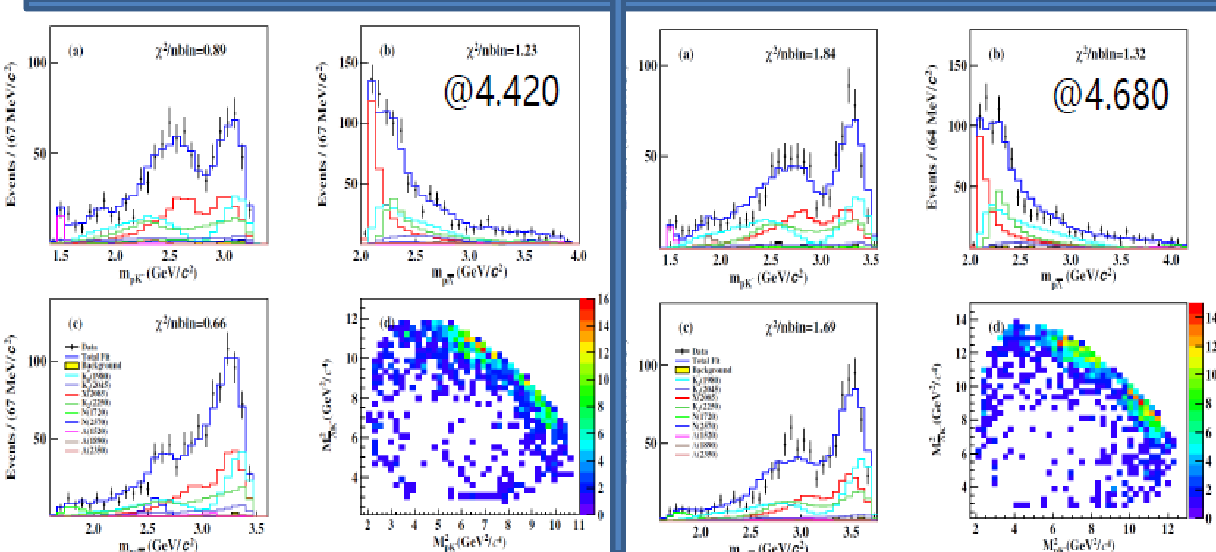
J^P	$\chi^2_{\text{angle}}/\text{nbin}$
1^+	19.4/20
0^-	107.2/20
1^-	80.4/20
2^+	53.2/20
2^-	41.2/20



J^P	$\chi^2_{\text{Dalitz}}/\text{nbin}$
1^+	490.5/409
0^-	602.7/409
1^-	632.1/409
2^+	549.1/409
2^-	502.7/409

Data at other five points





Data luminosity

$\sqrt{s}$	$\mathcal{L}_{\text{int}}$	Year
4.008	482.0 ± 4.7	2011
4.178	3189.0 ± 31.9	2016
4.226	1100.9 ± 7.0	2013
4.258	828.4 ± 5.5	2013
4.416	1090.7 ± 7.2	2014
4.682	1669.3 ± 9.0	2020

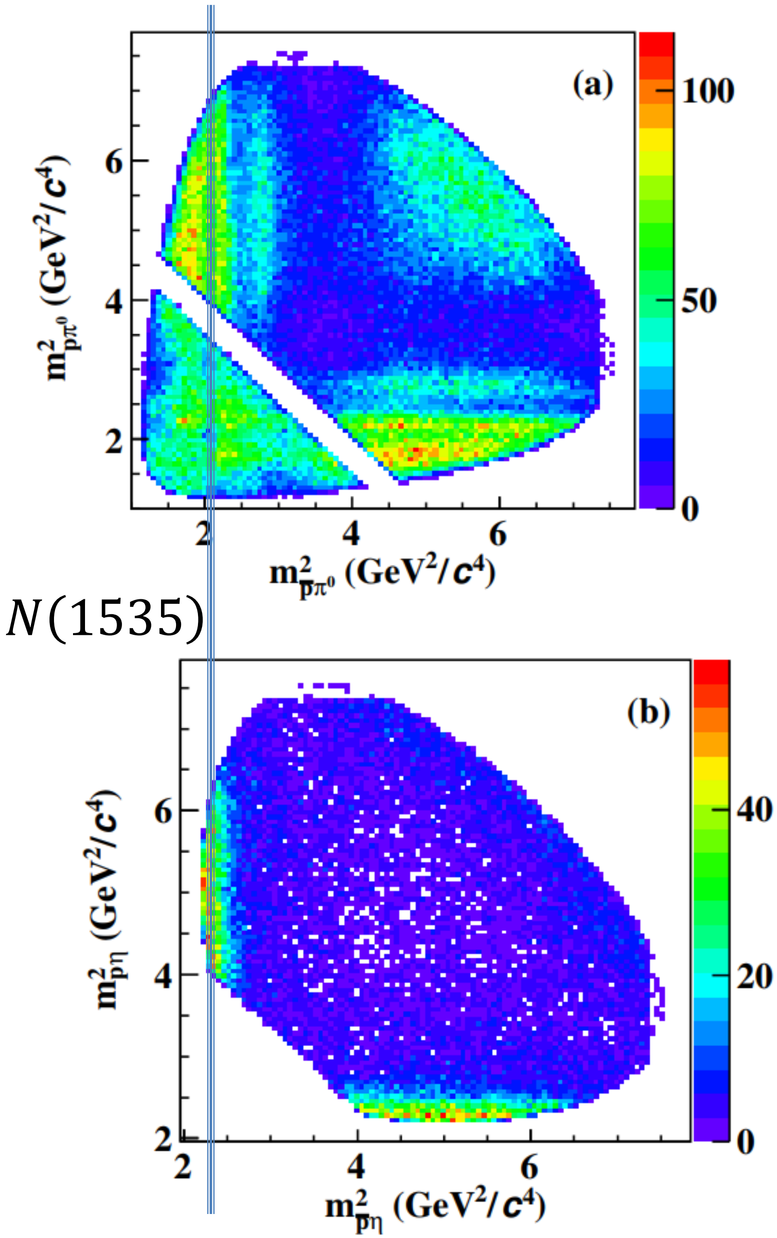
The exotic properties of $N(1535)$

- $N(1535)$: $\Gamma_{N\eta}/\Gamma_{N\pi} = 1.0 \pm 0.4$, $m_{K\Lambda} \sim 1.61$ GeV

Resonance	Br($N^* \to N\pi$)	Br($N^* \to N\eta$)	Br($N^* \to N\pi\pi$)
$N(1440)\,\frac{1}{2}^{+}$ ****	(55-75) %	$< 1\%$	$(17-50)\%$
$N(1520)\,\frac{3}{2}^{-}$ ****	(55-65) %	(0.05-0.07) %	(25-35) %
$N(1535)\,\frac{1}{2}^{-}$ ****	(32-52) %	(30-55) %	(4-31) %

- Inverted mass order
  $N(1440)$: $\sim$ 1.37 GeV, $N(1535)$: $\sim$ 1.51GeV
- Theoretical investigations
  - dynamical generated bound states in $K\Lambda - K\Sigma$ channel
  - Pentaquark $uuds\bar{s}$
  - Lattice QCD simulation

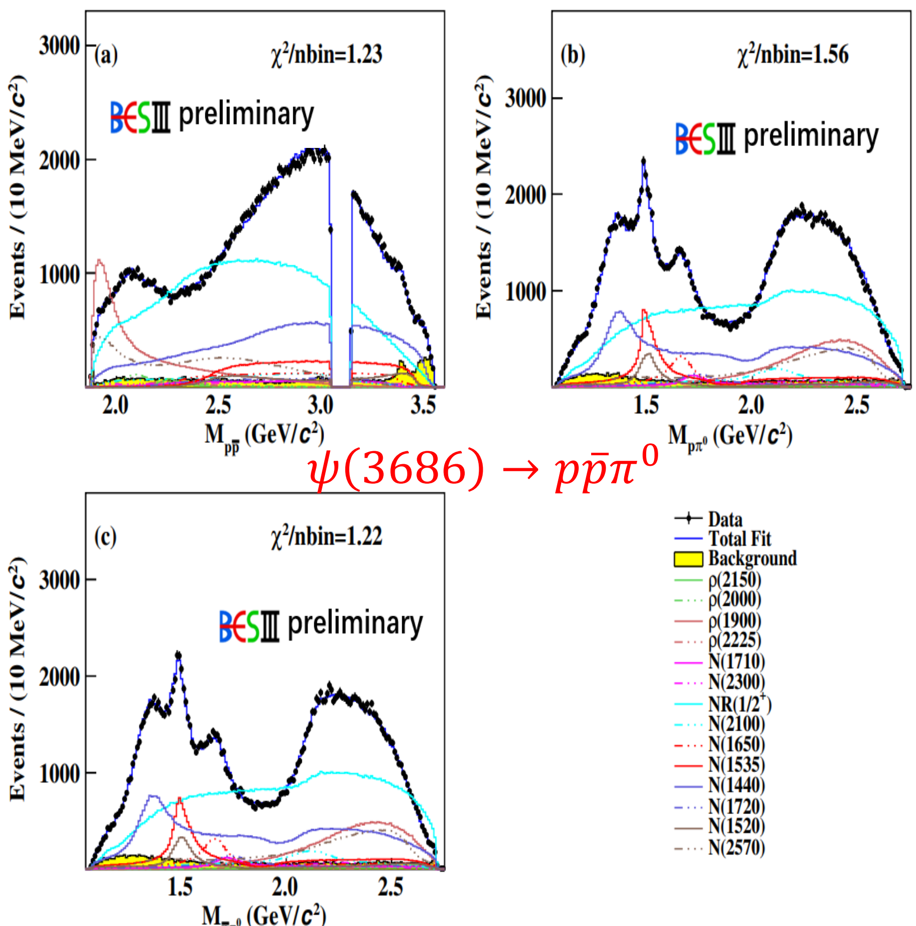
$N(1535)$ in $\psi' \to p\bar{p}\pi^0, p\bar{p}\eta$

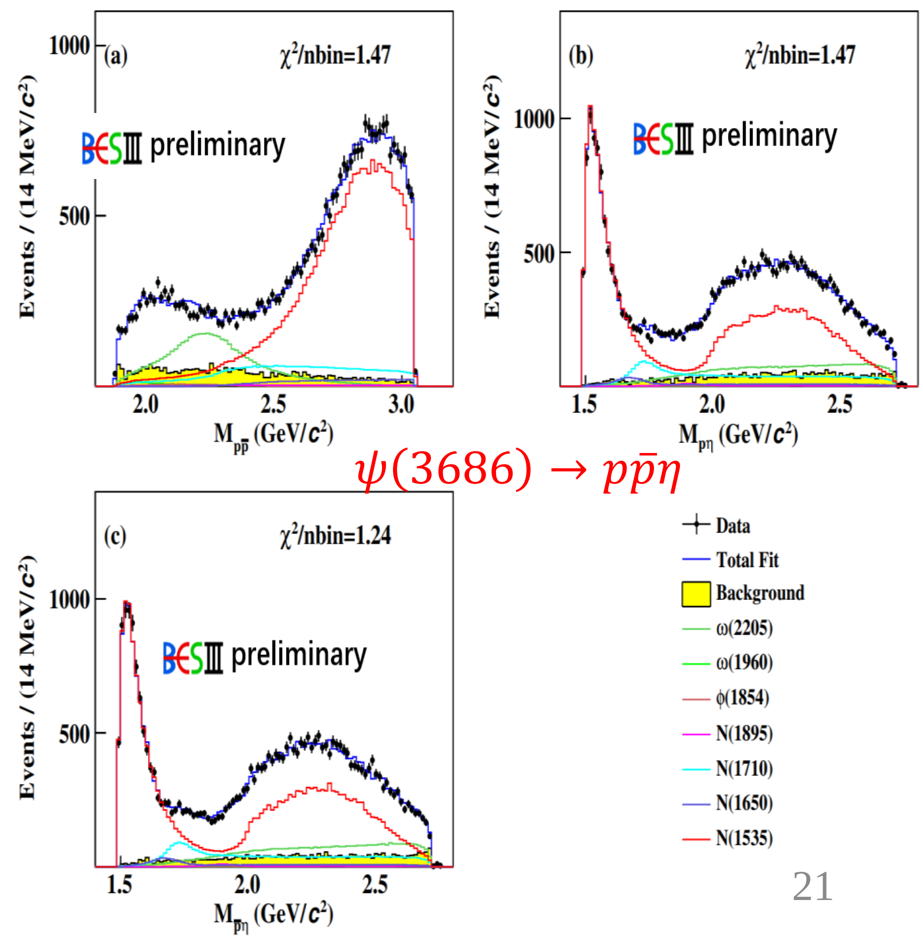


Resonance state	Mass (MeV/c^2)	Width (MeV)	$I^G(J^{PC})$
$\rho(1900)$	1880±30	130±30	$1^+(1^{--})$
$\rho(2000)$	2078±6	149±21	$1^+(1^{--})$
$\rho(2150)$	2254±22	109±76	$1^+(1^{--})$
$\rho(2225)$	2225±35	335±100	$1^+(2^{--})$
$\phi_3(1850)$	1854±7	87±28	$0^-(3^{--})$
$\omega(1960)$	1960±25	195±60	$0^-(1^{--})$
$\omega(2205)$	2205±30	350±90	$0^-(1^{--})$
NR($\frac{1}{2}^+$)	—	—	$\frac{1}{2}(\frac{1}{2}^+)$
$N(1440)$	1406±3	314±9	$\frac{1}{2}(\frac{1}{2}^+)$
$N(1520)$	1512±2	121±3	$\frac{1}{2}(\frac{3}{2}^-)$
$N(1535)$	1525±2	147±5	$\frac{1}{2}(\frac{1}{2}^-)$
$N(1650)$	1666±3	133±7	$\frac{1}{2}(\frac{1}{2}^-)$
$N(1710)$	1710±30	140±60	$\frac{1}{2}(\frac{1}{2}^+)$
$N(1720)$	1720±30	250±100	$\frac{1}{2}(\frac{3}{2}^+)$
$N(1895)$	1895±25	120±80	$\frac{1}{2}(\frac{1}{2}^-)$
$N(2100)$	2100±50	260±60	$\frac{1}{2}(\frac{1}{2}^+)$
$N(2300)$	2300±116	340±114	$\frac{1}{2}(\frac{1}{2}^+)$
$N(2570)$	2570±39	250±70	$\frac{1}{2}(\frac{5}{2}^-)$

PWA of $\psi(3686) \to p\bar{p}\pi^0/\eta$

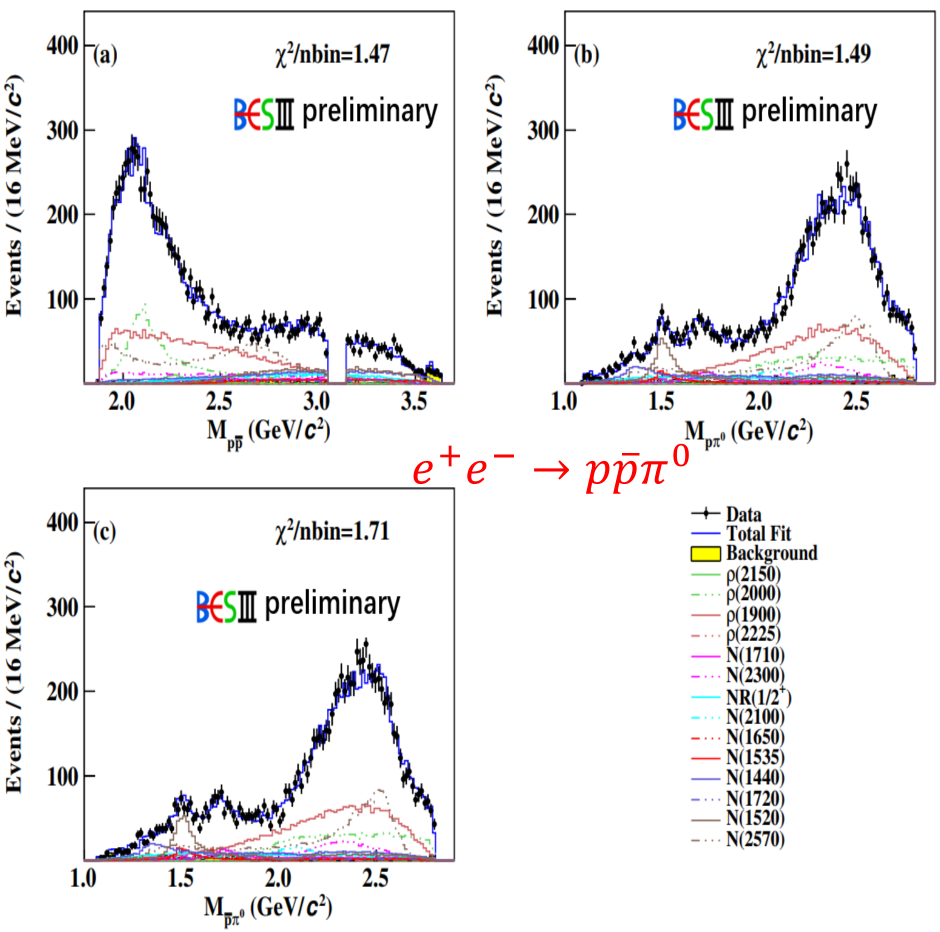
- ✓ Data can be well described with Several N^*& $\rho^*/\omega^*/\phi^*$ states.
- ✓ Three lowest lying N^* is described with KSU model[10]: $\Gamma(\sqrt{s}) = \Gamma_0 \times \sum r_i \times \frac{\rho_i(\sqrt{s})}{\rho_i(m_0)}$
- ✓ **$\Gamma_{N\eta}/\Gamma_{N\pi}$ of $N(1535)$ is determined to be $0.99 \pm 0.05_{sta.}$**



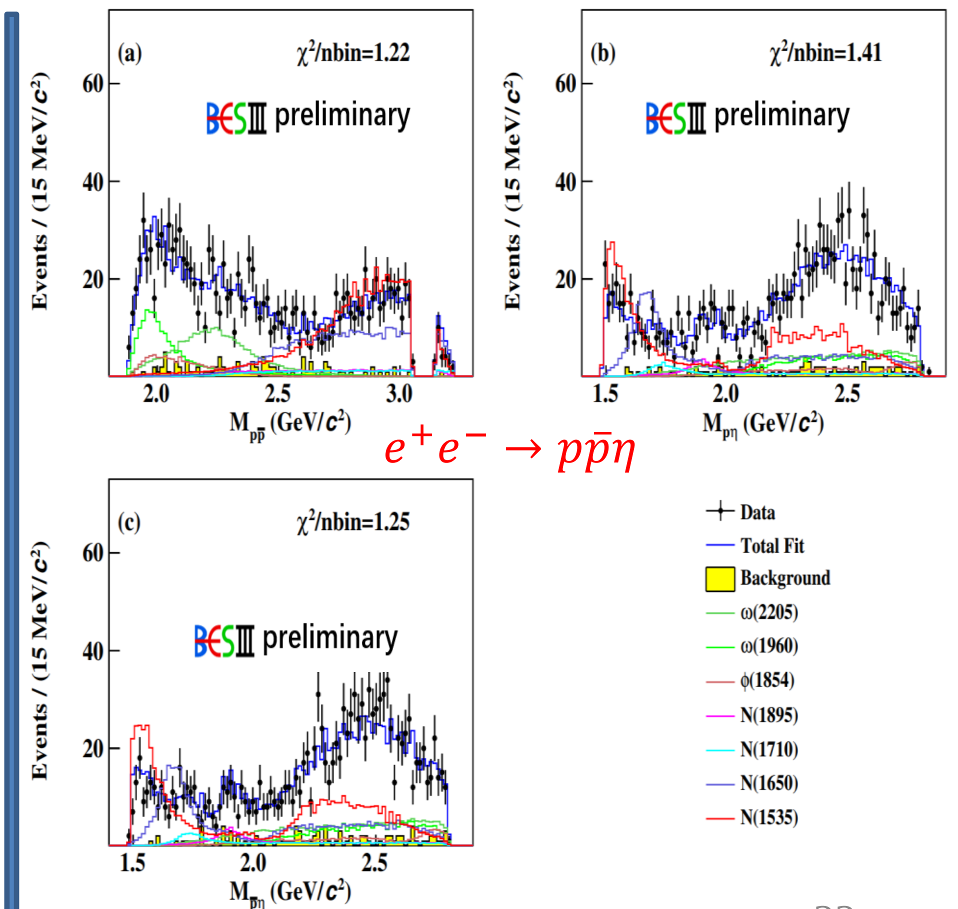


$e^+e^- \to p\bar{p}\pi^0/\eta$ @3.773 GeV

- ✓ **PWA fit is performed @$\sqrt{s} = 3.773$ GeV with 2.93 fb^{-1} to estimate continuum background**
- ✓ **Ignore potential contributions from $\psi(3773) \to p\bar{p}\pi^0/\eta$[12]**
- ✓ **The same sets of intermediate states are considered as @$\sqrt{s} = 3.686$ GeV**



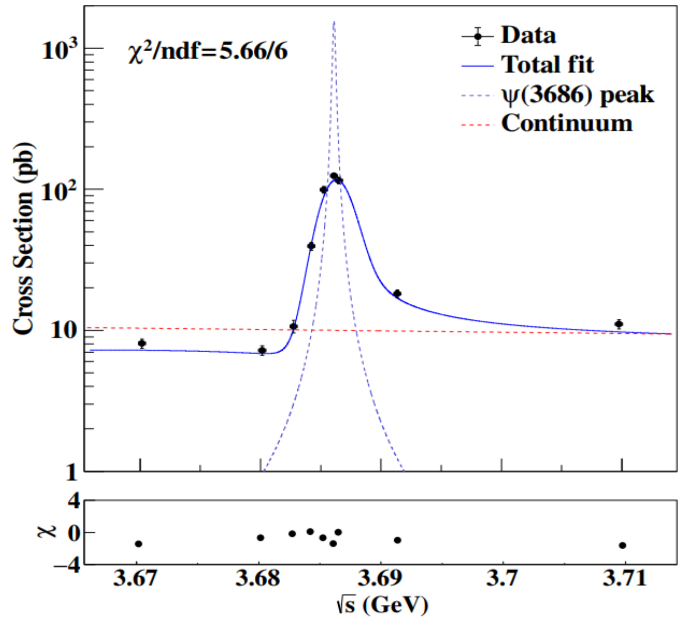
$e^+e^- \to p\bar{p}\pi^0$

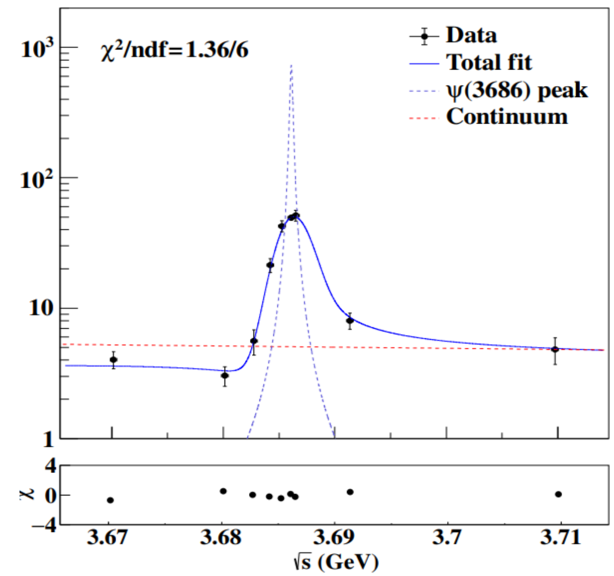


$e^+e^- \to p\bar{p}\eta$

$e^+e^- \to p\bar{p}\pi^0/\eta$ @$\sqrt{s} = 3.67 \sim 3.709$ GeV

$\sqrt{s}$ (MeV)	$\mathcal{L}$ (pb^{-1})
3670.16	84.7
3680.14	84.8
3682.75	28.7
3684.22	28.7
3685.26	26.0
3686.10	3877.1
3686.50	25.1
3691.36	69.4
3709.76	70.3





- ✓ The $\frac{\Gamma_{N(1535)\to N\eta}}{\Gamma_{N(1535)\to N\pi}}$ **is determined to be $0.99 \pm 0.05_{sta.}$**, confirms the fixed target experiment results listed by PDG.
- ✓ The strong coupling to Nη suggest a **significant $s\bar{s}$ component** in $N(1535)$, possibly $uud + s\bar{s}$? Need to be confirmed in $K\Lambda$ final state!
- ✓ The other N^* structures in $\psi' \to p\bar{p}\pi^0/\eta$ are studied & branching fraction measured.
- ✓ $\frac{B(\psi(3686))}{B(J/\psi)}$ ~12% confirmed by $p\bar{p}\pi^0$ channel, but violated by $p\bar{p}\eta$ channel.

Program to study baryon spectroscopy at BESIII

- $\psi(3686) \rightarrow \Lambda\bar{\Lambda}\eta, \Lambda\bar{\Lambda}\pi^0$, BESIII, Phys. Rev., D106, 072006 (2022)
- $\psi(3686) \rightarrow \mathrm{p}\bar{\mathrm{p}}\eta, \mathrm{p}\bar{\mathrm{p}}\pi^0$, being reviewed in BESIII
- $\psi(3686) \rightarrow \mathrm{p}\bar{\mathrm{p}}K^+K^-$, being reviewed in BESIII
- $\psi(3686), J/\psi \rightarrow \bar{\Lambda}\Sigma^0\pi^0 + c.c.$, being reviewed in BESIII
- $\psi(3686) \rightarrow \mathrm{K}^+\bar{\Lambda}\Xi^-, K\bar{\Sigma}^0\Xi^-$, $K_S^0\bar{\Sigma}^+\Xi^-$, being analysis
- $\psi(3686) \rightarrow \pi^0\Sigma^+\bar{\Sigma}^-$, being analysis
- $\psi(3686) \rightarrow p\bar{p}\phi$, being analysis
- $\psi(3686) \rightarrow pK^-\bar{\Sigma}^0 + c.c.$, being analysis
- $\chi_{c0} \rightarrow pK\bar{\Lambda} + c.c.$, $p\bar{p}K^+K^-$, being analysis
- $J/\psi \rightarrow \Sigma^+\bar{\Sigma}^-\pi^+\pi^-$, being analysis
- $J/\psi \rightarrow pK^-\bar{\Lambda} + c.c.$, being analysis
- $J/\psi \rightarrow p\bar{p}\omega$, being analysis
- $J/\psi \rightarrow \eta\Lambda\bar{\Sigma}^0 + c.c.$, being analysis
- ……

Summary and Outlook

- FDC-TF package availability
- Ongoing baryonic analyses
- Resonance lineshape: BW, running-width,K-matrix
- Continued development of multiple applications for FDC-TF
- Prospects for FDC-PWA generation code:
  - Examination of multi-step decays involving hyperons
  - Analysis of χ_{cJ} decays
  - Investigation of radiative decays

Thanks for your attention

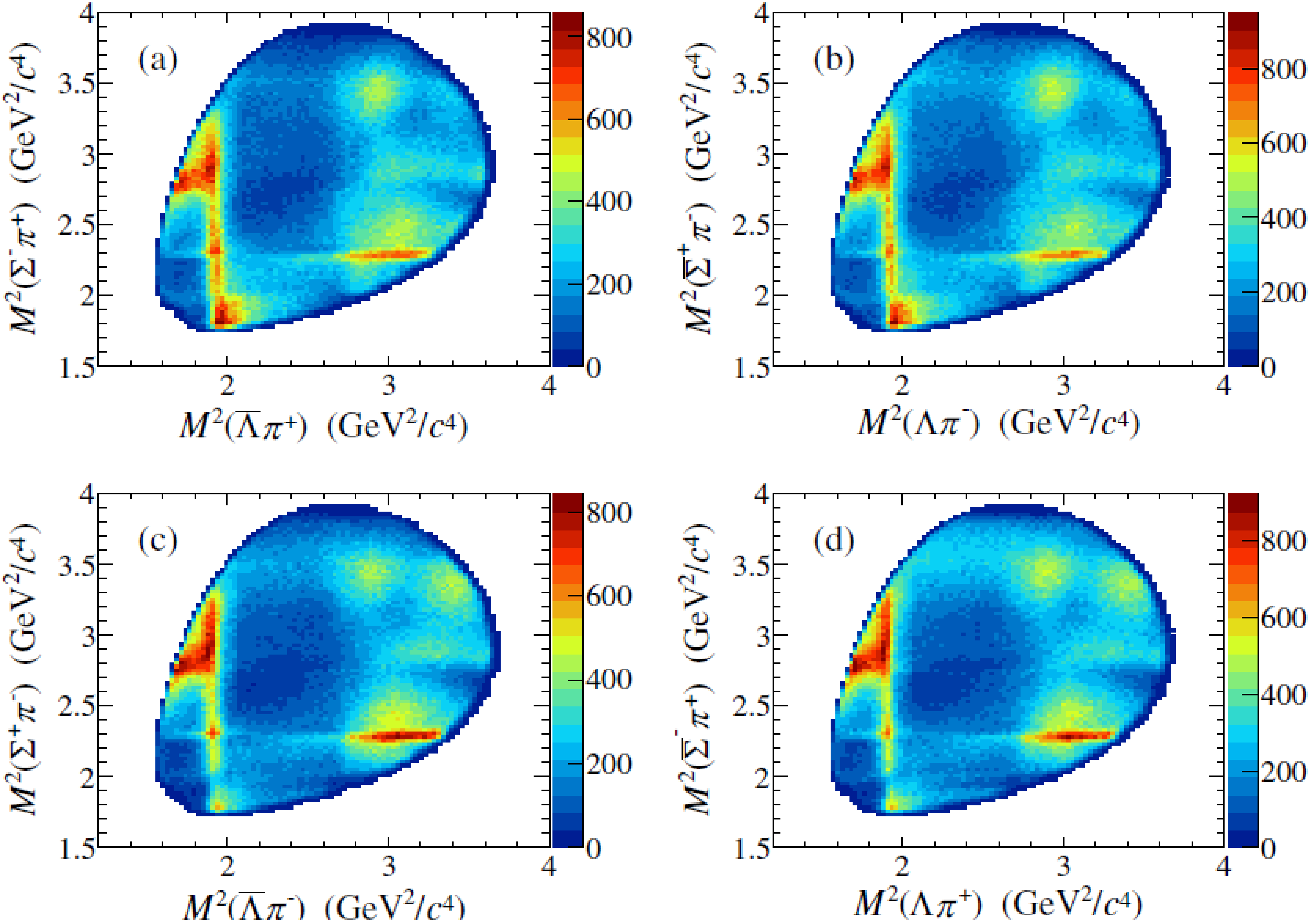
backup

Σ^* and Λ^* in $J/\psi \to \bar{\Lambda}\pi^{\pm}\Sigma^{\mp} + c.c.$

BESIII, PRD108, 112012 (2023)

$\Lambda(1380)$
$\Lambda(1405)$
$\Lambda(1520)$
$\Lambda(1600)$
$\Lambda(1670)$
$\Lambda(1690)$
$\Lambda(1820)$
$\Lambda(1830)$
$\Lambda(1890)$
$\Sigma(1385)$
$\Sigma(1660)$
$\Sigma(1670)$
$\Sigma(1750)$

1^- NR



Toy MC with 9 Λ^*, 4 Σ^* and 1^- NR states

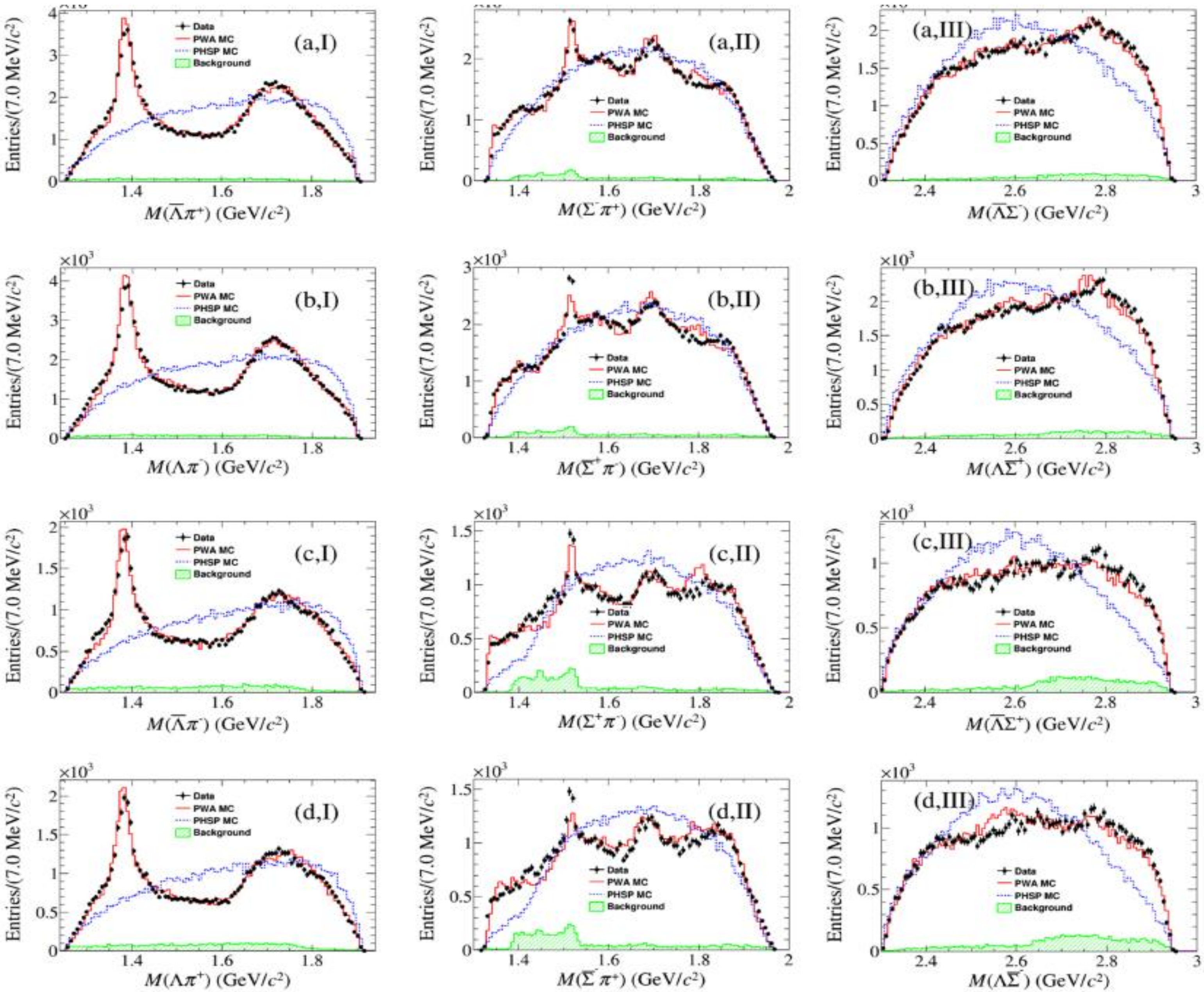
N^{obs}

1.32 M

1.38 M

1.26 M

1.31 M



Performance test

- GPU : Tesla V100-SXM2-32GB
- decay: $\psi' \to pK^-\overline{\Lambda} + c.c.$
  - 5969 data events, and 80,000 PHSP events. 179 parameters in the fit.
- 24 resonances included in the fit

$$N^*(1710), N^*(1870), N^*(1720), \Lambda(1810),$$
$$\Lambda(1800), \Lambda(1670), \Lambda(1600), \Lambda(1405), \mathrm{K}_1(2075)$$
$$\mathrm{N}^*(2060), \Lambda(2325), \Lambda(1890), \Lambda(1690), \Lambda(1520)$$
$$\mathrm{K}_2(2250), \mathrm{N}^*(1990), \mathrm{N}^*(2190), \Lambda(2110), \Lambda(1830)$$
$$\Lambda(1820), \mathrm{N}^*(2250), \Lambda(2100), \Lambda(2020), \Lambda(2350)$$

Performance test (cont.)

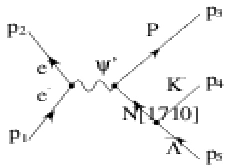


Fig. 1

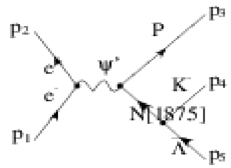


Fig. 2

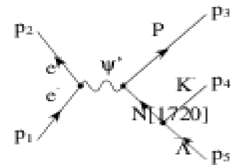


Fig. 3

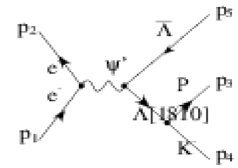


Fig. 4

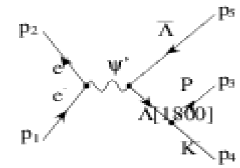


Fig. 5

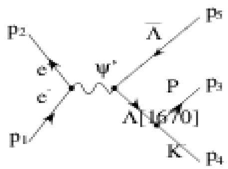


Fig. 6

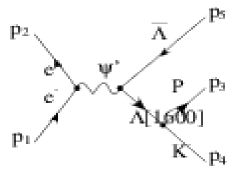


Fig. 7

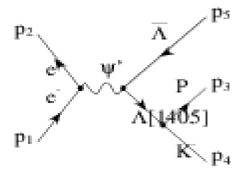


Fig. 8

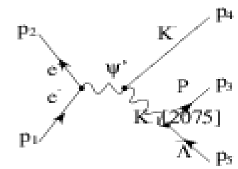


Fig. 9

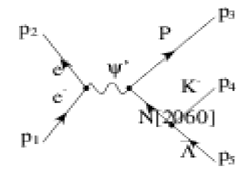


Fig. 10

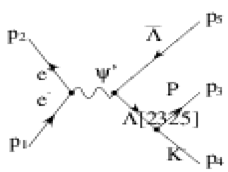


Fig. 11

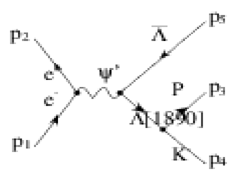


Fig. 12

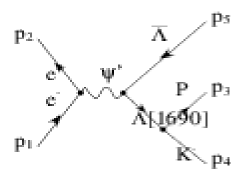


Fig. 13

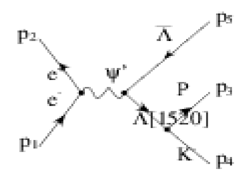


Fig. 14

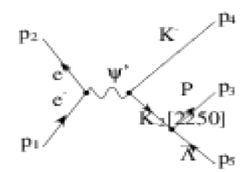


Fig. 15

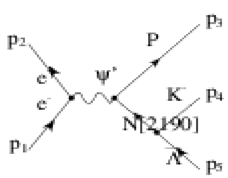


Fig. 16

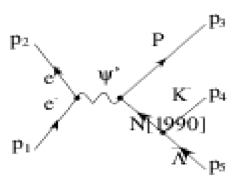


Fig. 17

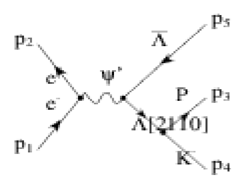


Fig. 18

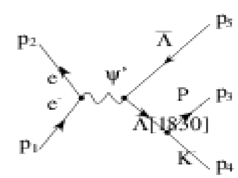


Fig. 19

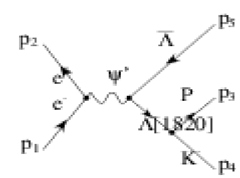


Fig. 20

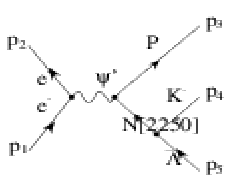


Fig. 21

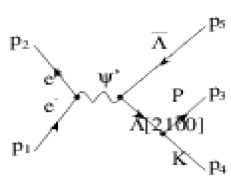


Fig. 22

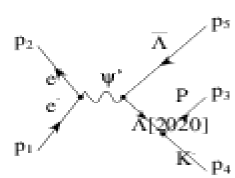


Fig. 23

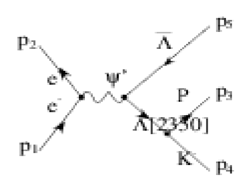
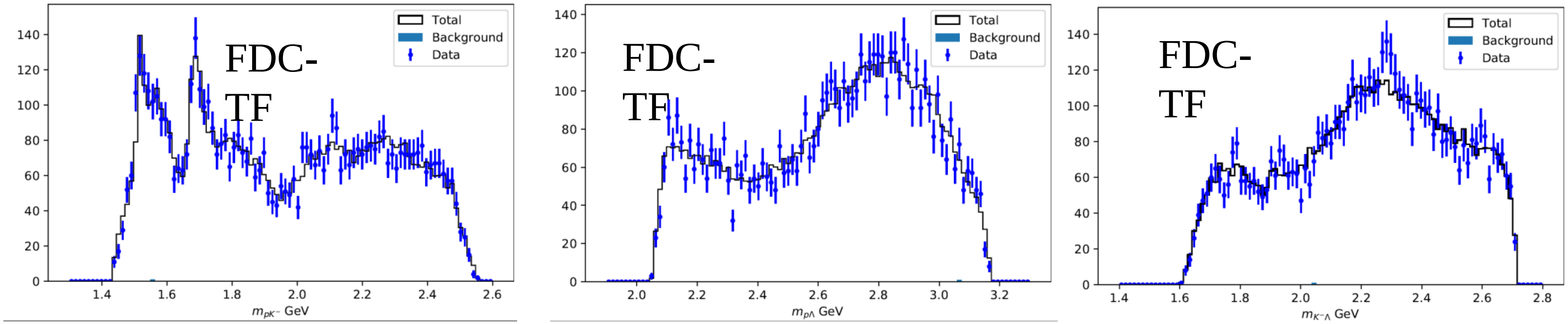


Fig. 24

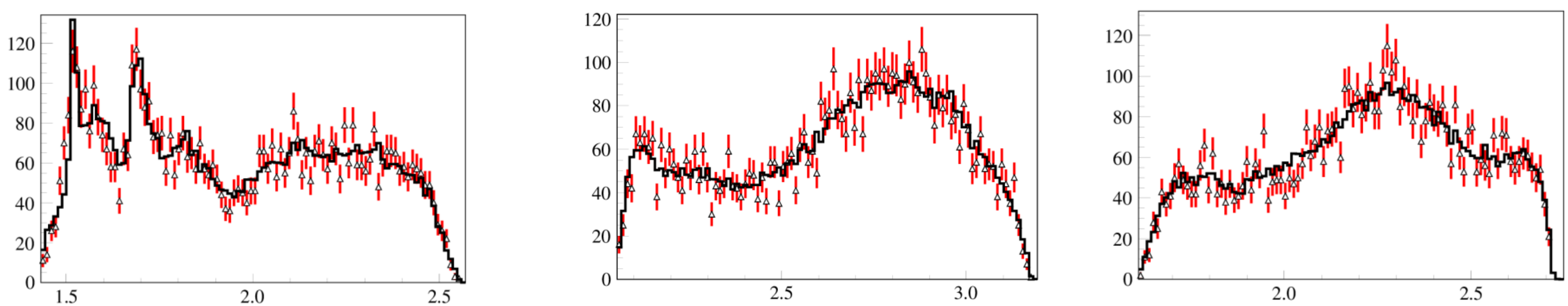
Performance test (cont.)

- $\frac{t_c}{t_g} = \frac{30}{0.07} \approx 430 : t_c(t_g)$ times cost for CPU (GPU) calculation per iteration

FDC-TF: $-\ln L = -789.45$



FDC: $-\ln L = -789.10$



Performance test (cont.)

- Check on yields ratios

Mode	FDC	FDC-tf
1	0.034	0.034
2	0.007	0.007
3	0.187	0.187
4	0.069	0.069
5	0.116	0.116
6	0.018	0.018
7	**0.122**	**0.123**
8	0.050	0.050
9	0.051	0.051
10	0.041	0.041
11	0.035	0.035
12	0.040	0.040

Mode	FDC	FDC-tf
13	0.042	0.042
14	0.021	0.021
15	0.056	0.056
16	0.523	0.523
17	0.005	0.005
18	0.614	0.611
19	0.034	0.034
20	0.023	0.023
21	0.003	0.003
22	0.137	0.138
23	0.009	0.009
24	0.011	0.006

@ /hpcfs/bes/gpupwa/pingrg/fdc/pkl2/process/fort_dbg