$X(3872)$ Relevant $D\bar{D}^*$ Scattering in $N_f = 2$ Lattice QCD [1]

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[1] Based on: https://arxiv.org/abs/2402.14541

Outline

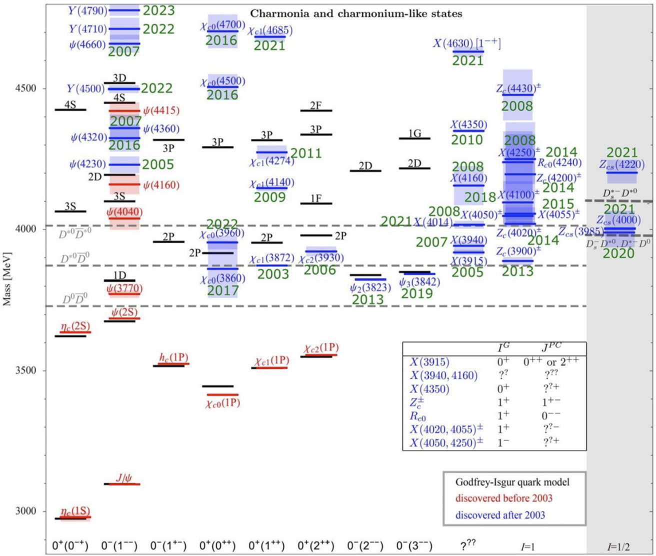
1. Introduction
2. Methodology of scattering: Lüscher's approach
3. Results: bound state and $X(3872)$
4. Discussion: a possible resonance and $\chi_{c1}(2P)$
5. Summary and Outlook

Introduction

$X(3872)$ status

1. First observation from Belle. PDG: ,
   1. $m_X = 3871.65 \pm 0.06\,\mathrm{MeV}$
   2. $\Gamma_x = 1.19 \pm 0.21\,\mathrm{MeV}$
   3. $I^G J^{PC} = 0^+ 1^{++}$

   A lot of XYZ particles observed afterwards...
2. $X(3872)$ decays mainly to $D^0 \bar{D}^{0*}$, a small fraction to $J/\psi\omega$, and also isospin violating $J/\psi\rho$
3. Intensive and extensive phenomenological studies
4. Interpreted as a $c\bar{c}$, a $D\bar{D}^*$ molecule or a tetraquark state
5. Main point of view: $D\bar{D}^* + c\bar{c}$



Introduction to Lattice QCD

Path integral quantization

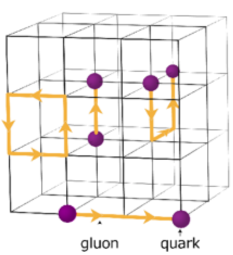
$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}] \mathcal{D}[U] \mathrm{e}^{-S_F[\psi,\bar{\psi},U] - S_G[U]} O[\psi, \bar{\psi}, U]$$

$$= \frac{1}{Z} \int \mathcal{D}[U] \mathrm{e}^{-S_G[U]} \det M[U] O[U]$$

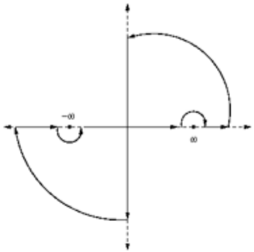
Green's function   MC simulation   Obversables

with

$$\langle Z \rangle = \int \mathcal{D}[U] \mathrm{e}^{-S_G[U]} \det M[U]$$



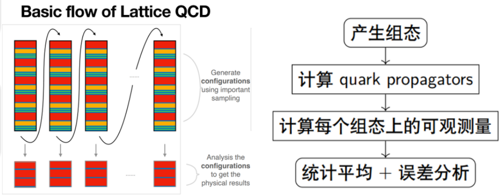
Spacetime discretization



Wick rotation

Numerical Monte Carlo Simulation

importance sampling: $\mathcal{P}[U] \propto \mathrm{e}^{-S_G[U]} \det M[U]$



Lattice QCD workflow, figure from Yi-bo's slides

Methodology of scattering: Lüscher's formulism

Finite volume scattering theory

$$\begin{cases} \text{characteristic size: } R_a \ll L \\ \text{temporal size: } L_t \ll L \\ \text{scattering energy: } E_k < m_1 + m_2 \end{cases} \xrightarrow{\text{Lüscher's formulea}} \begin{cases} \text{ERE: } a_l, k_l, \\ \text{phase shift: } \delta_l, \\ \text{dynamical pole.} \end{cases}$$

Lüscher's formulism

single channel:[a]

$$p \cot \delta_0(q^2) = \frac{2}{L a_s \sqrt{\pi}} \mathcal{Z}_{00}(1, q^2). \tag{1}$$

multi-channel: [b] [c]

$$\det \left[\delta_{ij} \delta_{JJ'} + i \rho_i t_{ij}^{(J)}(s) \left(\delta_{JJ'} + i \mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L) \right) \right] = 0. \tag{2}$$

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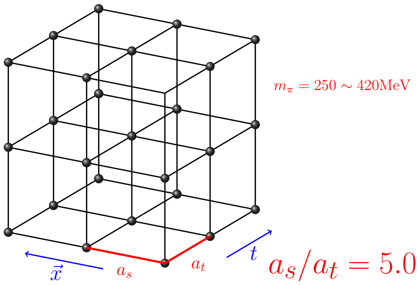
[a] Lüscher *NPB*.354,2-3,(1991)531-578

[b] L.Leskovec and S.Prelovsek *PRD*.(2015)85, 114507

[c] J.J.Dudek *et al.*(Hadron Spectrum Collab.) *PRL*.(2014)113,182001

Lattice setup

- Tadpole improved Symanzik's gauge action (C. Morningstar, PRD60(1999)034509)
- Anisotropic Lattice: $\xi = \frac{a_s}{a_t} \approx 5$
- $N_f = 2$ clover gauge ensembles with degenerate u, d sea quarks.
  Scale setting: Jiang et al, Phys.Rev.D 107 (2023) 094510
- Distillation method (M. Peardon et al. (HSC),PRD80(2009)054506).
- Calculation of disconnected diagrams of light dynamic quarks

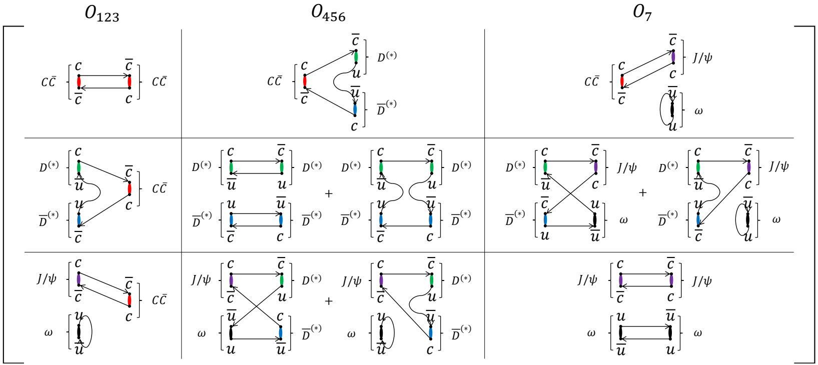


ens.	m_π (MeV)	a_t^{-1}	$N_{\rm cfg}$	$N_V^{(l)}$	$N_V^{(c)}$	$m_{\chi_{c1}}$ (MeV)	m_D (MeV)	m_{D^*} (MeV)
M245	250(3)	7.276	401	70	120	3489(3)	1873(1)	1985(2)
M305	307(2)	7.187	401	70	120	3496(2)	1881(1)	1990(2)
M360	362(1)	7.187	401	70	120	3502(2)	1884(1)	2003(2)
M415	417(1)	7.219	401	70	120	3509(2)	1896(1)	2017(1)

Schematic calculation of $I^G J^{PC} = 0^+ 1^{++}$ system

flavor structure of 0^+1^{++} $D\bar{D}^*$ system

$$\left| D\bar{D}^* \right\rangle^{Q=0}_{I=0} = \frac{1}{2}\left(\left| D^+ \bar{D}^{*-} \right\rangle + \left| D^0 \bar{D}^{*0} \right\rangle - \left| \bar{D}^0 D^{*0} \right\rangle - \left| D^- D^{*+} \right\rangle \right) \quad (3)$$

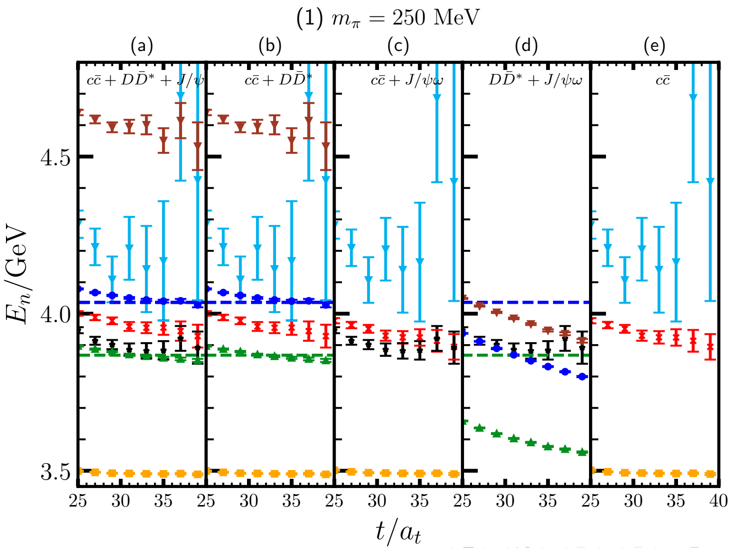


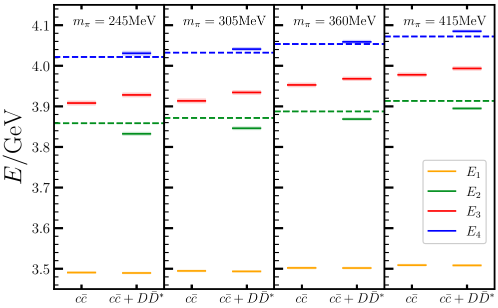
$$\mathcal{S} = \{\mathcal{O}_\alpha | \alpha = 1, \cdots, 7\} = \{\mathcal{O}^{r=0}_{c\bar{c}}, \mathcal{O}^{r=1}_{c\bar{c}}, \mathcal{O}^{r=2}_{c\bar{c}}, \mathcal{O}^{q=0,\gamma_5}_{D\bar{D}}, \mathcal{O}^{q=0,\gamma_4\gamma_5}_{D\bar{D}}, \mathcal{O}^{q=1}_{D\bar{D}}, \mathcal{O}^{q=0}_{J/\psi\omega}\}.$$

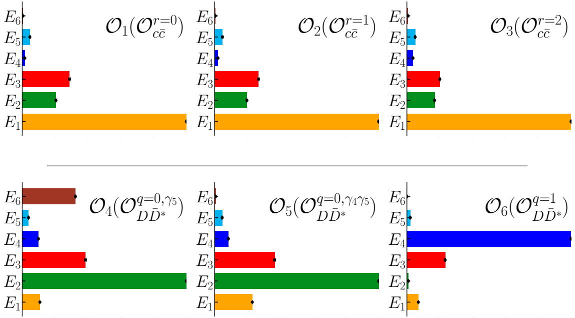
Energy levels of $I^G J^{PC} = 0^+ 1^{++}$ system

- $c\bar{c} + D\bar{D}^* + J/\psi\omega$ operators.
- $c\bar{c} + D\bar{D}^*$: black points (correspond to $J/\psi\omega$ state) disappear.
- $c\bar{c} + J/\psi\omega$: Energy levels close to non-interacting $D\bar{D}^*$ energies disappear.
- $D\bar{D}^* + J/\psi\omega$: Energy levels close to non-interacting $D\bar{D}^*$ and $J/\psi\omega$ states.
- $c\bar{c}$: Energy levels close to χ_{c1} states.

In summary: In all the cases, $J/\psi\omega$ energy has no sizable changes w/o $c\bar{c}$ and $D\bar{D}^*$ operators. So $J/\psi\omega$ almost decouples from other states and is neglected from the discussion in this work.







Identify the energy levels:

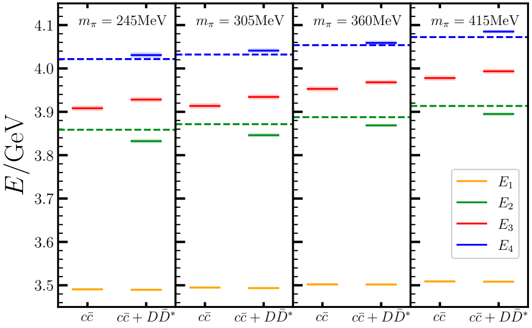
- E_1 : ~ 3.5 GeV, should be χ_{c1}.
- E_2 : close but below the $D\bar{D}^*$ threshold.
- E_3 : far from and in middle of the non-interacting $D\bar{D}^*$ energies $E_{D\bar{D}^*}^{q=0}$ and $E_{D\bar{D}^*}^{q=1}$
- E_4 : close but above $E_{D\bar{D}^*}^{q=1}$

Operator couplings

- E_1 : coupled most by $c\bar{c}$ operators.
- E_2 : coupled most by $\mathcal{O}_{D\bar{D}^*}^{q=0}$ and substantially by $c\bar{c}$ operators.
- E_3 : coupled substantially by $\mathcal{O}_{D\bar{D}^*}^{q=0}$, $\mathcal{O}_{D\bar{D}^*}^{q=1}$ and $c\bar{c}$ operators.
- E_4 : coupled most by $\mathcal{O}_{D\bar{D}^*}^{q=1}$ and a little by $\mathcal{O}_{D\bar{D}^*}^{q=0}$ and $c\bar{c}$ operators.

Existence of a bound state below $D\bar{D}^*$ threshold

- **Lüscher's formulism** for S-wave single channel
  $p\cot\delta_0(q^2) = \frac{2}{La_s\sqrt{\pi}}\mathcal{Z}_{00}(1, q^2).$
- $E_n(p_n) = \sqrt{m_D^2 + p_n^2} + \sqrt{m_{D^*}^2 + p_n^2},$
  $q^2 \equiv \left(\frac{L}{2\pi}\right)^2 p^2$



Effective Eange Expansion(ERE)

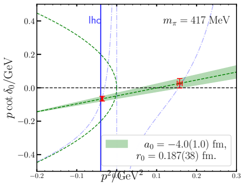
$$p\cot\delta_0(p) = \frac{1}{a_0} + \frac{1}{2}r_0p^2 + \mathcal{O}(p^4).$$

Solving ERE with E_2 and E_3, we can obtain the parameters (a_0, r_0)

pole singularity in $\mathcal{T}$ matrix

$$\mathcal{T} \propto (p\cot\delta_0(p) - ip)^{-1}.$$

- taking (a_0, r_0) as the approximation in the $V \to \infty$ limit, the pole equation $p_B\cot\delta_0(p_B) - ip_B = 0$ gives the banding energy $E_B = E_{D\bar{D}^*}(p_B) - (m_D + m_{D^*})$.



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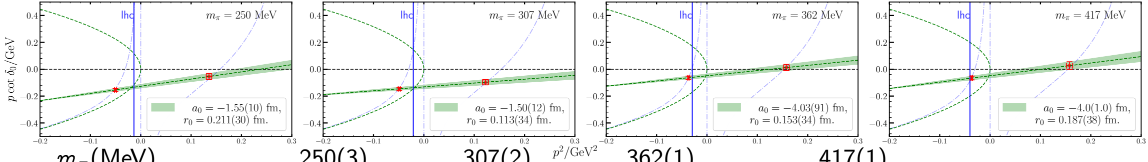


m_π(MeV)	250(3)	307(2)	362(1)	417(1)
$(p^{1\pi}_{\mathrm{lhc}})^2(\mathrm{GeV}^2)$	-0.0135(4)	-0.0210(4)	-0.0292(3)	$-0.0400(3)$(★)
a_0 (fm)	-1.55(10)	-1.50(12)	-4.03(91)	$-4.0(1.0)$(★)
r_0 (fm)	0.211(30)	0.113(34)	0.153(34)	0.187(38)(★)
E_B (MeV)	$-9.7^{+2.1}_{-2.2}$ (*)	$-9.7^{+1.9}_{-2.0}$ (*)	$-1.3^{+0.6}_{-0.8}$ (*)	$-1.3^{+0.8}_{-1.0}$(★)

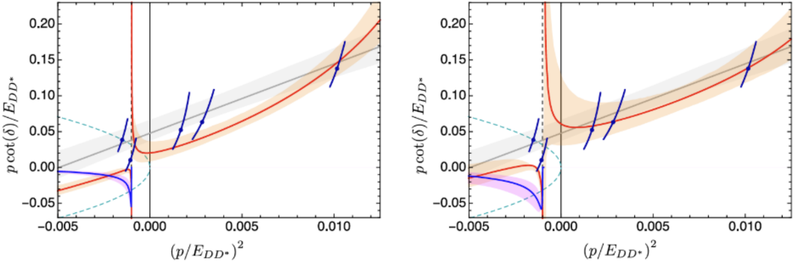
- E_2 : The lattice energy is lower than the $D\bar{D}^*$ threshold by $20\mathrm{MeV}$ or even more.
- a_0 : Large negative: existence of a bound state.
- r_0 : Small positive: the compositeness $X \sim 1$ up to a $\mathcal{O}\left(p^2\right)$ correction (Y. Li et al., PRD105(2022)L071502) .
- The bound state is predominantly a $D\bar{D}^*$ molecule.
- For $m_\pi \lesssim 360$ MeV, our results suffering from the Left-Hand Cut (lhc) issue. (M.-L. Du et al., PRL131(2023)131901, L. Meng et al., arXiv:2312.01930 [hep-lat])

$$\left(p^{1\pi}_{lhc}\right)^2 \approx \frac{1}{4}\left[(\Delta M)^2 - m_\pi^2\right]$$

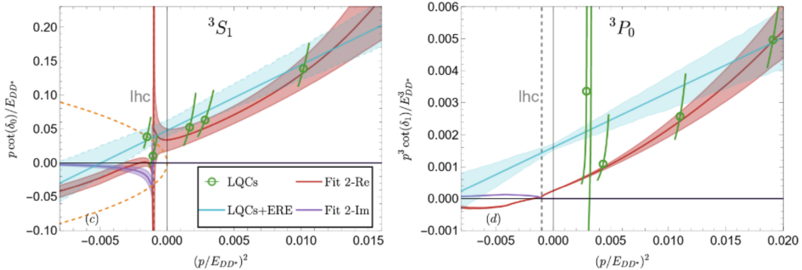
.

Left-hand cut issue: example of $T_{cc}^+(3875)$

- M.-L. Du et al.,PRL131(2023)131901
  Case studies on $T_{cc}^+(3875)$ relevant $D\bar{D}^*$ scattering. The data points are from lattice QCD calculation (M. Padmanath et al. PRL129(2022)032002)



- L. Meng et al., arXiv:2312.01930 [hep-lat]
  Similar to the discussion above. But the lattice finite volume energy levels are used to fix the parameters in the EFT involved. The prediction of the EFT (red curves) are compared with ERE with out OPE.



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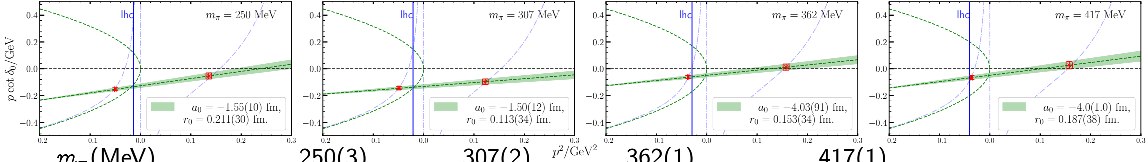

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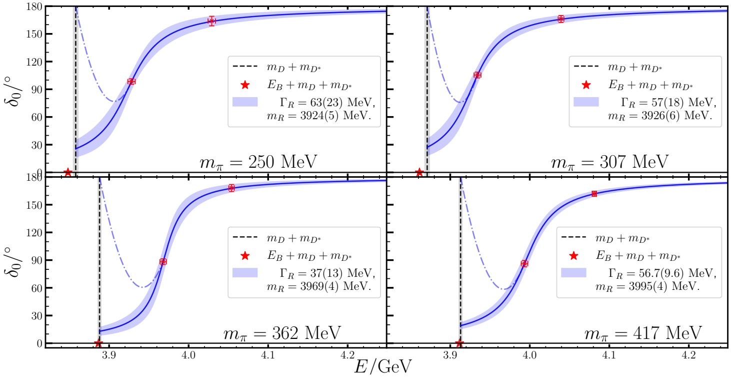
$$\left(p^{1\pi}_{lhc}\right)^2 \approx \frac{1}{4}\left[(\Delta M)^2 - m_\pi^2\right]$$

.

A possible resonance below 4.0 GeV



Generalized Levinson's theorem

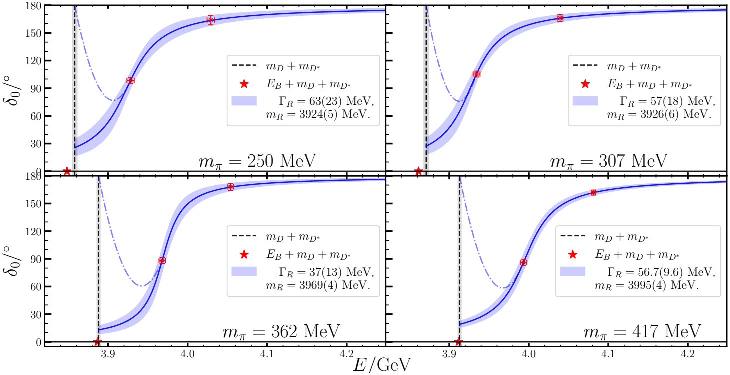
$$\delta_l(0^+) - \delta_l(p_{\max}) = (n_l - n_b)\pi$$

- n_l : # of bound states,
- n_b : # of bare states below the energy corresponding to p_{max}. (F. Vidal et al., PRC45(1992)418, Y. Li et al., PRD105(2022)116024)

- E_3 : a scattering phase around $\delta(E_3) \sim 90°$
- E_4 : a scattering phase close to $\delta(E_4) \sim 180°$
- Exactly as the expectation of the Generalized Levinson's theorem
- Hint at the existence of a resonance

A possible resonance below 4.0 GeV

- Breit-Wigner ansatz for a resonance : $\mathcal{T} \approx \frac{1}{\cot\delta_0 - i} \sim \frac{1}{(m_R - E) - \frac{i\Gamma_R}{2}}$
- Resonance parameters through $\delta_0(E) = \arctan\left(\frac{\Gamma_R}{2(m_R - E)}\right)$ by using E_3 and E_4 .
- Caution: The parameters (m_R, Γ_R) may change , since they are only two energy levels.



potential experimental observation

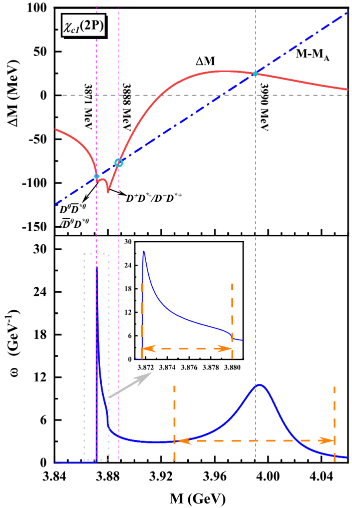
X(3940): $m_X = 3942(9)\mathrm{MeV}$

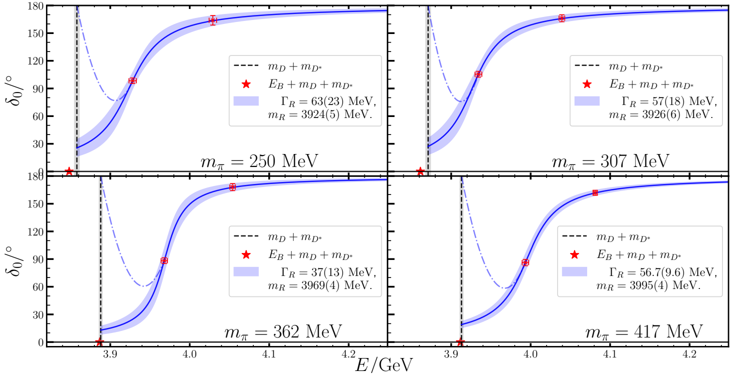
$\Gamma_X = 37^{+27}_{-17}$ MeV

(Belle, PRL98(2007)082001; PRL100(2008)20200)

m_π(MeV)	250(3)	307(2)	362(1)	417(1)
m_R(MeV)	3924(5)	3926(6)	3969(4)	3995(4)
Γ_R(MeV)	63(23)	57(18)	37(13)	57(10)

A possible resonance below 4.0 GeV





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Q. Deng et al., 2312.10296

Summary & Outlook

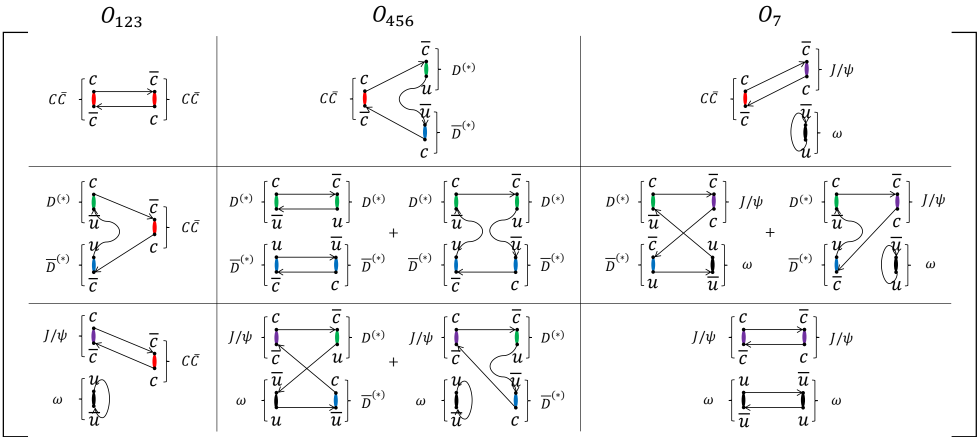
Summary

1. The pion mass m_π dependence (from 250 MeV to 417 MeV) of the S-wave $D\bar{D}^*(I=0)$ scattering process was studied in $N_f = 2$ lattice QCD.
2. A state below the $D\bar{D}^*$ threshold was observed at each pion mass.
3. Particularly for $m_\pi = 417$ MeV, this state is considered as a definite bound state in the infinite volume limit, located about 1 MeV below the $D\bar{D}^*$ threshold. This bound state is primarily a $D\bar{D}^*$ molecular component, possibly corresponding to the physical $X(3872)$ state.
4. The possibility of a resonance existing below 4.0 GeV was proposed for the first time from lattice QCD perspective.

Thank You

Backup slides

Backup for $X3872$ study



Backup for $X3872$ study