

Axion like particle production in $\eta \rightarrow \pi\pi a$ decay

arXiv: 2403.16064 [hep-ph]

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OUTLINE

- Introduction and motivation
- Axion SU(3) chiral perturbation theory
- Axion like particle (ALP) production in η decay
- Summary

- PQ mechanism is an elegant solution of the strong CP problem [Peccei and Quinn 1977]
- $U(1)_{PQ}$ spontaneously symmetry breaking (SSB) $\rightarrow$ Axion [Weinberg, 1978][Wilczek, 1978]

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G_{\mu\nu}^c \tilde{G}_c^{\mu\nu} + \mathcal{L}(\partial_\mu a, \psi).$$

- Axion couples to gauge bosons, quarks, leptons fields, has abundant phenomenology. [Luzio, et.al, Phys.Rept., 2020]
- Standard QCD axion mass relation

$$m_a = \sqrt{\frac{m_u m_d}{(m_u + m_d)^2} \frac{F_\pi}{f_a}} m_\pi \simeq 5.7 \left(\frac{10^6 \text{ GeV}}{f_a} \right) \text{ eV}.$$

- Visible axion model: $f_a \lesssim 10^4 \text{ GeV}$ (ruled out by astrophysical experiments) [Dicus, et.al, Phys. Rev. D, 1978][Dicus, et.al, Phys. Rev. D, 1980]
- Invisible axion model: $10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$, [Kim and Carosi, Rev. Mod. Phys., 2010]
- KSVZ axion model [Kim, Jihn E, 1979, Shifman, et al., Valentin I, 1980]
- DFSZ axion model [Dine, et al., 1981, Zhitnitsky, 1980]

➤ Heavy axions/Axion like particles (ALPs)

Holdom and Peskin, “Raising the Axion Mass”, NPB 208 (1982) 397-412.

Agrawal and Howe, “Factoring the Strong CP problem”, JHEP 12 (2018) 029.

Giorgi, et.al., “GeV ALP from TeV Vector-like Leptons”, arXiv:2402.14059 [hep-ph].

● ALPs phenomenology

Jaeckel and Spannowsky, “Probing MeV to 90 GeV axion-like particles with LEP and LHC”, PLB (2016) 482-487.

Bauer and Neubert and Thamm, “Collider Probes of Axion-Like Particles”, JHEP 12 (2017) 044.

Aloni and Soreq and Williams, “Coupling QCD-Scale Axionlike Particles to Gluons”, PRL 123 (2019) 3, 031803.

Di Luzio and Piazza, $a \rightarrow \pi\pi\pi$ decay at next-to-leading order in chiral perturbation theory”, JHEP 12 (2022) 041.

We focus on ALP production in $\eta \rightarrow \pi\pi a$ decay

● Rare $\eta^{(\prime)}$ decays provide ideal probes to detect sub-GeV new light particles.

[Precision frontier experiments in JLab, BESIII, STCF]

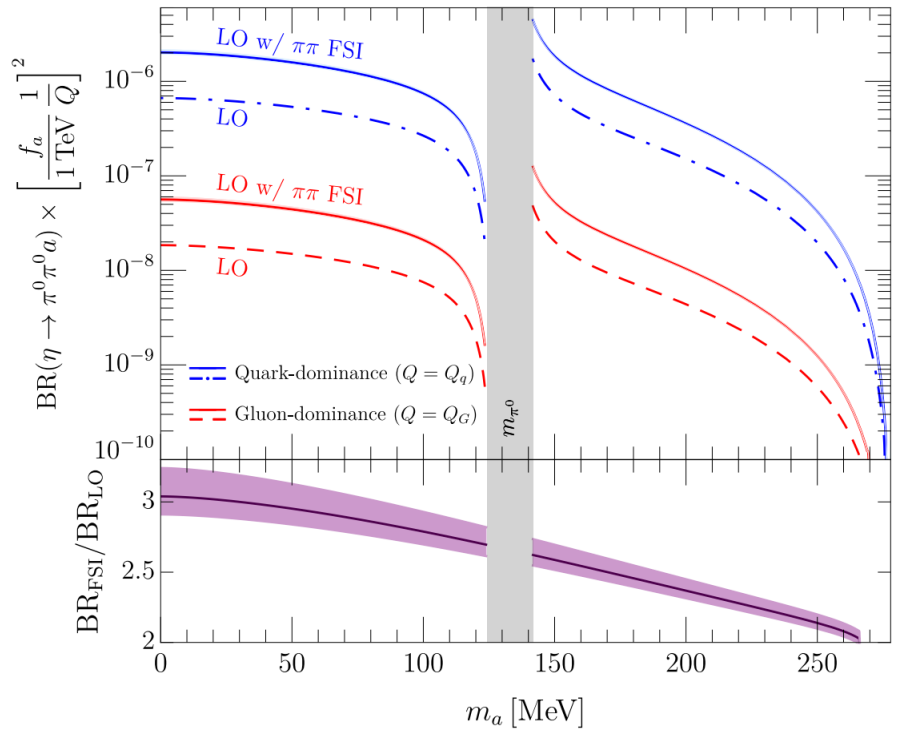
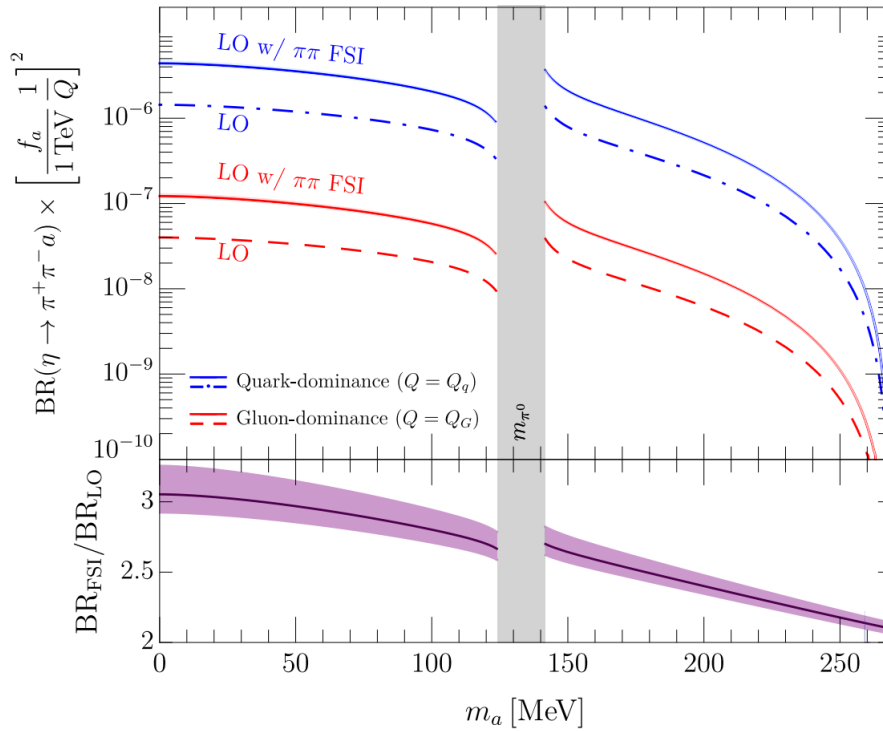
● ALPs in $\eta^{(\prime)}$ decay

Landini and Meggiolaro, “Study of the interactions of the axion with mesons and photons using a chiral effective Lagrangian model”, EPJC 80 (2020) 4, 302.

Gan, et.al., “Precision tests of fundamental physics with η and η' mesons”, Phys. Rept. 945 (2022) 1-105.

Alves and Sergi, “Final state rescattering effects in axio-hadronic η and η' decays”, arXiv:2402.02993 [hep-ph].

Alves and Sergi, arXiv:2402.02993 [hep-ph].



- **In this talk**, we discuss $\eta \rightarrow \pi\pi a$ decay in SU(3) ChPT up to NLO.
- NLO perturbative decay amplitude include s - and $t(u)$ -channel interactions perturbatively.
- The unitarized decay amplitude will also constructed to account for the s -channel $\pi\pi$ final state interaction (FSI) effect.

- We stick on the minimal axion coupling

$$\mathcal{L}_{\text{ALP}} = \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 + \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G_{\mu\nu}^c \tilde{G}_c^{\mu\nu},$$

$m_{a,0}$ the bare PQ breaking axion mass.

- Axial transformation $q \rightarrow e^{i\frac{a}{2f_a} Q_a \gamma^5} q, \quad Q_a = \frac{M^{-1}}{\langle M^{-1} \rangle},$

$$\mathcal{L}_{\text{ALP+QCD}} = \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 + \mathcal{L}_{\text{QCD}}^0 - \frac{\partial_\mu a}{2f_a} \bar{q} \gamma^\mu \gamma^5 Q_a q - (\bar{q}_R M(a) q_L + \text{H.c.}),$$

$$M(a) \equiv M e^{-i\frac{a}{f_a} Q_a}, \quad \text{work in isospin limit} \quad M = \text{diag}(\hat{m}, \hat{m}, m_s), \quad \hat{m} = \frac{m_u + m_d}{2}.$$

Study $\eta \rightarrow \pi\pi a$ decay in SU(3) chiral perturbation theory (ChPT)

[Gasser and Leutwyler, NPB, 1985]

- Effective d.o.f: eight pNGBs corresponding to SSB $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$

$$U = \exp\left(\frac{i\Phi}{F}\right), \quad \Phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta_8 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 \end{pmatrix}$$

- Effective Lagrangians can be constructed order by order

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \mathcal{L}_2 + \mathcal{L}_4 + \dots,$$

Chiral expansion by soft momentum
and light quarks masses
 $p^2, m_q \sim O(p^2)$

axion couplings can be introduced as external fields.

- LO Lagrangian $\mathcal{L}_2 = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{LO}},$

$$\chi = 2B_0 M(a), \quad J_A^\mu|_{\text{LO}} = -i \frac{F^2}{2} \langle \bar{Q}_a \{ \partial^\mu U, U^\dagger \} \rangle.$$

- NLO Lagrangian $\mathcal{L}_4 = L_1 \langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \dots + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\text{NLO}},$

$$J_A^\mu|_{\text{NLO}} = -4iL_1 \langle \bar{Q}_a \{ U^\dagger, \partial^\mu U \} \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \dots.$$

- LECs: $F, B_0, L_{1\sim 8}.$
- $\bar{Q}_a$ denotes octet component of $Q_a.$
- We ignore the singlet component of $\bar{q}\gamma^\mu\gamma_5 Q_a q.$

There is mixing between ALP and η_8 fields

- LO mixing

$$\mathcal{L}_{\text{mix}}^{\text{LO}} = \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{1}{2} \partial_\mu \eta_8 \partial^\mu \eta_8 + \frac{F}{f_a} C_k^{a\eta} \partial_\mu a \partial^\mu \eta_8 - \frac{1}{2} \left(m_{a,0}^2 + \frac{F^2}{f_a^2} C_m^a \right) a^2 - \frac{1}{2} \bar{m}_{\eta_8}^2 \eta_8^2,$$

$$C_k^{a\eta} = \frac{1}{2} \langle \bar{Q}_a \lambda_8 \rangle = \frac{\bar{m}_{\eta_8}^2 - \bar{m}_\pi^2}{2\sqrt{3}\bar{m}_{\eta_8}^2}, \quad C_m^a = \frac{B_0}{\langle M^{-1} \rangle} = \frac{\bar{m}_\pi^2 (3\bar{m}_{\eta_8}^2 - \bar{m}_\pi^2)}{12\bar{m}_{\eta_8}^2},$$

LO contribution of pNGBs masses from light quarks masses $\bar{m}_\pi^2 = 2B_0\hat{m}$, $\bar{m}_{\eta_8}^2 = \frac{2B_0}{3}(\hat{m} + 2m_s)$

➤ Fields redefinitions up to $O(1/f_a^3)$

$$a = \left[1 + \frac{1}{2} (C_k^{a\eta})^2 \frac{m_{a,0}^2 (m_{a,0}^2 - 2\bar{m}_{\eta_8}^2)}{(m_{a,0}^2 - \bar{m}_{\eta_8}^2)^2} \frac{F^2}{f_a^2} \right] \tilde{a} - \frac{F}{f_a} C_k^{a\eta} \frac{\bar{m}_{\eta_8}^2}{\bar{m}_{\eta_8}^2 - m_{a,0}^2} \tilde{\eta},$$

$$\eta_8 = \left[1 + \frac{1}{2} (C_k^{a\eta})^2 \frac{\bar{m}_{\eta_8}^2 (\bar{m}_{\eta_8}^2 - 2m_{a,0}^2)}{(m_{a,0}^2 - \bar{m}_{\eta_8}^2)^2} \frac{F^2}{f_a^2} \right] \tilde{\eta} - \frac{F}{f_a} C_k^{a\eta} \frac{m_{a,0}^2}{m_{a,0}^2 - \bar{m}_{\eta_8}^2} \tilde{a}.$$

$$\mathcal{L}_{\text{mix}}^{\text{LO}} = \frac{1}{2} \partial_\mu \tilde{a} \partial^\mu \tilde{a} - \frac{1}{2} \bar{m}_a^2 \tilde{a}^2 + \frac{1}{2} \partial_\mu \tilde{\eta} \partial^\mu \tilde{\eta} - \frac{1}{2} \bar{m}_\eta^2 \tilde{\eta}^2 + \mathcal{O}\left(\frac{F^3}{f_a^3}\right),$$

$$\bar{m}_a^2 = m_{a,0}^2 + \frac{F^2}{f_a^2} \left(C_m^a + \frac{(C_k^{a\eta})^2 m_{a,0}^4}{m_{a,0}^2 - \bar{m}_{\eta_8}^2} \right), \quad \bar{m}_\eta^2 = \bar{m}_{\eta_8}^2 + \frac{F^2}{f_a^2} \frac{(C_k^{a\eta})^2 \bar{m}_{\eta_8}^4}{\bar{m}_{\eta_8}^2 - m_{a,0}^2}.$$

- NLO mixing and fields redefinitions for ALP and η fields

$$\mathcal{L}_{\text{mix}}^{\text{NLO}} = \frac{1}{2}(1 - \Sigma_{aa}^{(4)'})\partial_\mu \tilde{a}\partial^\mu \tilde{a} + \frac{1}{2}(1 - \Sigma_{\eta\eta}^{(4)'})\partial_\mu \tilde{\eta}\partial^\mu \tilde{\eta} - \Sigma_{a\eta}^{(4)'}\partial_\mu \tilde{a}\partial^\mu \tilde{\eta} \\ - \frac{1}{2} [\bar{m}_a^2 + \Sigma_{aa}^{(4)}(0)] \tilde{a}^2 - \frac{1}{2} [\bar{m}_\eta^2 + \Sigma_{\eta\eta}^{(4)}(0)] \tilde{\eta}^2 - \Sigma_{a\eta}^{(4)}(0) \tilde{a}\tilde{\eta}.$$

$\Sigma_{ij}^{(4)}(p^2)$ is the $O(p^4)$ two-point 1PI amplitude

$$\Sigma_{aa}^{(4)} = O(p^4/f_a^2), \quad \Sigma_{a\eta}^{(4)} = O(p^4/f_a), \quad \Sigma_{\eta\eta}^{(4)} = O(p^4).$$

$\Sigma_{ij}^{(4)'}$ denotes $d\Sigma_{ij}^{(4)}(p^2)/dp^2$, which is momentum independent up to NLO.

➤ Fields redefinitions at NLO

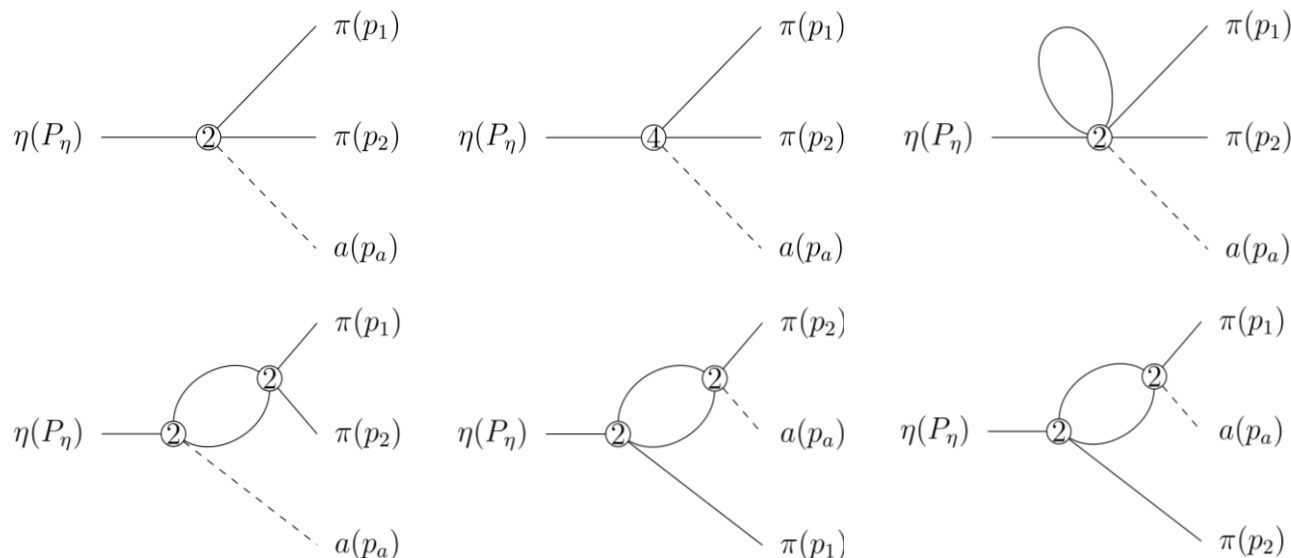
$$\tilde{a} = \left(1 + \frac{1}{2}\Sigma_{aa}^{(4)'}\right) a_{\text{phy}} + \frac{\Sigma_{a\eta}^{(4)}(m_\eta^2)}{m_\eta^2 - m_a^2} \eta_{\text{phy}}, \quad \tilde{\eta} = \left(1 + \frac{1}{2}\Sigma_{\eta\eta}^{(4)'}\right) \eta_{\text{phy}} + \frac{\Sigma_{a\eta}^{(4)'}(m_a^2)}{m_a^2 - m_\eta^2} a_{\text{phy}}. \\ \mathcal{L}_{\text{mix}}^{\text{NLO}} = \frac{1}{2}\partial_\mu a_{\text{phy}}\partial^\mu a_{\text{phy}} - \frac{1}{2}m_a^2 a_{\text{phy}}^2 + \frac{1}{2}\partial_\mu \eta_{\text{phy}}\partial^\mu \eta_{\text{phy}} - \frac{1}{2}m_\eta^2 \eta_{\text{phy}}^2$$

Perturbative calculation of the amplitude is performed by using fields variables a_{phy} and η_{phy} .

$$\langle a(p_a) \pi^i(p_1) \pi^j(p_2) | \hat{T} | \eta(P_\eta) \rangle = (2\pi)^4 \delta^4(P_\eta - p_1 - p_2 - p_a) \mathcal{M}_{\eta; \pi^i \pi^j a}(s, t, u),$$

$$s = (p_1 + p_2)^2 = m_{\pi\pi}^2, \quad t = (p_a + p_2)^2 = m_{a\pi}^2, \quad u = (p_a + p_1)^2$$

● Feynman diagrams up to NLO



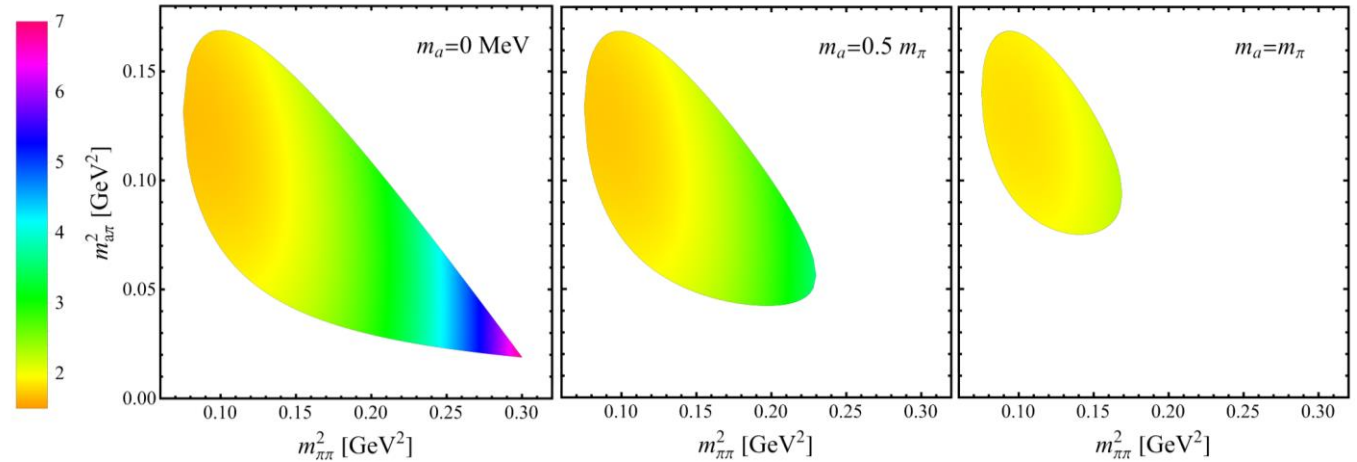
● Parameters

Masses and F_π [MeV]				LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of 10^{-3})							
m_π	m_K	m_η	F_π	L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
137	496	548	92.1	1.0(1)	1.6(2)	-3.8(3)	0.0(3)	1.2(1)	0.0(4)	-0.3(2)	0.5(2)

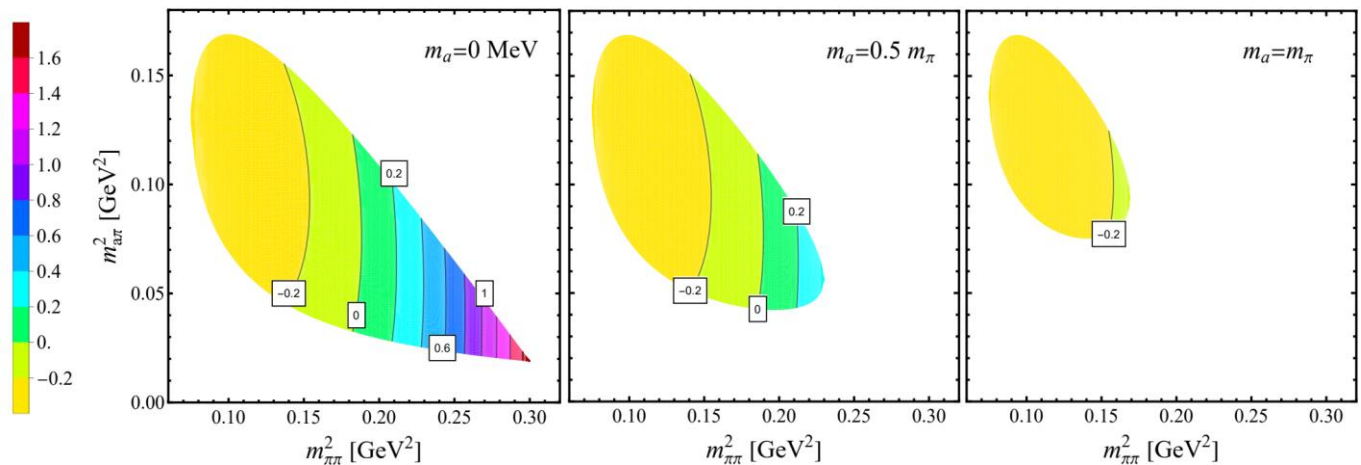
[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

In isospin limit, the decay amplitudes for $\pi^+\pi^-$ channel and $\pi^0\pi^0$ channel turn out to be the same up to NLO.

$$10^6 f_a^2 \frac{d^2\Gamma_{\eta \rightarrow \pi^+\pi^- a}}{ds dt} = \frac{10^6 f_a^2}{32(2\pi)^3 m_\eta^3} \left(|\mathcal{M}_{\eta;\pi\pi a}^{(2)}|^2 + 2\mathcal{M}_{\eta;\pi\pi a}^{(2)} \text{Re}(\mathcal{M}_{\eta;\pi\pi a}^{(4)}) + |\mathcal{M}_{\eta;\pi\pi a}^{(4)}|^2 \right) \quad \text{in unit of GeV}^{-1}$$



$$\left(2\mathcal{M}_{\eta;\pi\pi a}^{(2)} \text{Re}(\mathcal{M}_{\eta;\pi\pi a}^{(4)}) + |\mathcal{M}_{\eta;\pi\pi a}^{(4)}|^2 \right) / |\mathcal{M}_{\eta;\pi\pi a}^{(2)}|^2$$



We focus on s -channel $\pi\pi$ two-body FSI in unitarization.

- Isospin amplitude

$$\begin{aligned}\mathcal{M}_{\eta;\pi\pi a}^{I=0} &= -\frac{1}{\sqrt{3}}\mathcal{M}_{\eta;\pi^+\pi^-a} - \frac{1}{\sqrt{3}}\mathcal{M}_{\eta;\pi^-\pi^+a} - \frac{1}{\sqrt{3}}\mathcal{M}_{\eta;\pi^0\pi^0a} = -\sqrt{3}\mathcal{M}_{\eta;\pi\pi a}, \\ \mathcal{M}_{\eta;\pi\pi a}^{I=1} &= -\frac{1}{\sqrt{2}}\mathcal{M}_{\eta;\pi^+\pi^-a} + \frac{1}{\sqrt{2}}\mathcal{M}_{\eta;\pi^-\pi^+a} = 0,\end{aligned}$$

- Partial waves (PWs) in $\pi\pi$ c.o.m frame

$$\mathcal{M}_{\eta;\pi\pi a}^{0J}(s) = \frac{1}{2(\sqrt{2})} \int_{-1}^{+1} d\cos\theta P_J(\cos\theta) \mathcal{M}_{\eta;\pi\pi a}^0(s, t, u), \quad J = 0, 2, 4, \dots,$$

- Unitary relation consider s -channel $\pi\pi$ contribution

$$\text{Im}\mathcal{M}_{\eta;\pi\pi a}^{0J}(s + i0^+) = \rho_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{0J}(s) T_{\pi\pi \rightarrow \pi\pi}^{0J*}(s), \quad (2m_\pi < \sqrt{s} < 2m_K),$$

$$\text{Im}\mathcal{M}_{\eta;\pi\pi a}^{00,(4)}(s + i0^+) = \rho_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s), \quad (2m_\pi < \sqrt{s} < 2m_K).$$

$$\rho_{\pi\pi}(s) = \frac{1}{16\pi} \sqrt{1 - \frac{4m_\pi^2}{s}}, \quad T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s) = \frac{2s - m_\pi^2}{2F_\pi^2}.$$

- Unitarized PW amplitude

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) = \frac{\mathcal{M}_{\eta;\pi\pi a}^{00,\text{L}}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)},$$

$$G_{\pi\pi}(s) = -\frac{1}{(4\pi)^2} \left(\log \frac{m_\pi^2}{\mu^2} - \sigma_\pi(s) \log \frac{\sigma_\pi(s) - 1}{\sigma_\pi(s) + 1} - 1 \right),$$

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{L}}(s) = \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) + \mathcal{M}_{\eta;\pi\pi a}^{00,(4)}(s) - G_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s) T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s).$$

The unitarized amplitude satisfies the relation

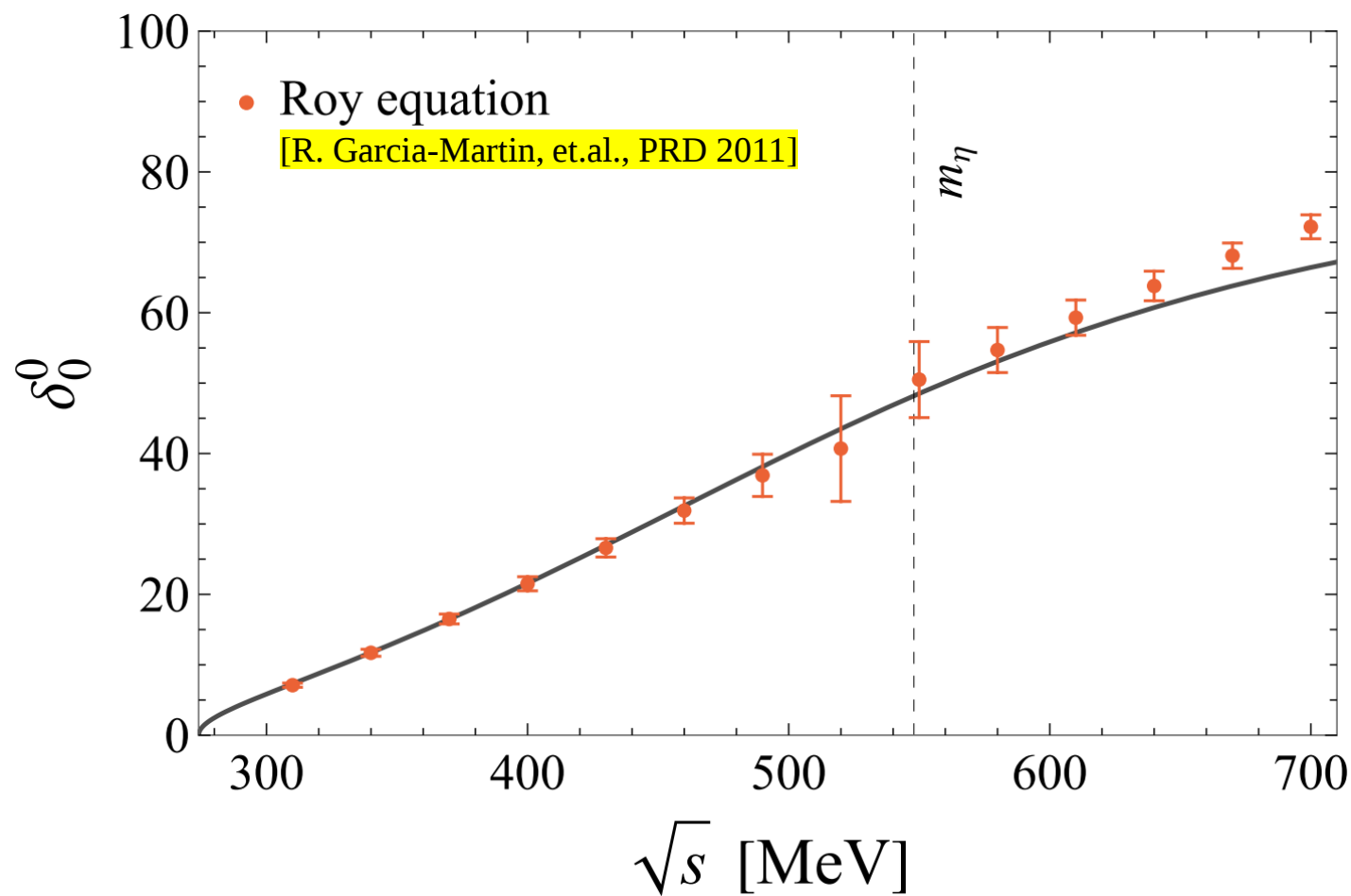
$$\text{Im} \mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) = \rho_{\pi\pi}(s) \mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni}}(s) (T_{\pi\pi \rightarrow \pi\pi}^{00,\text{Uni}}(s))^*, \quad (2m_\pi < \sqrt{s} < 2m_K)$$

if we define the unitarized $\pi\pi$ PW

$$T_{\pi\pi \rightarrow \pi\pi}^{00,\text{Uni}}(s) = \frac{T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}$$

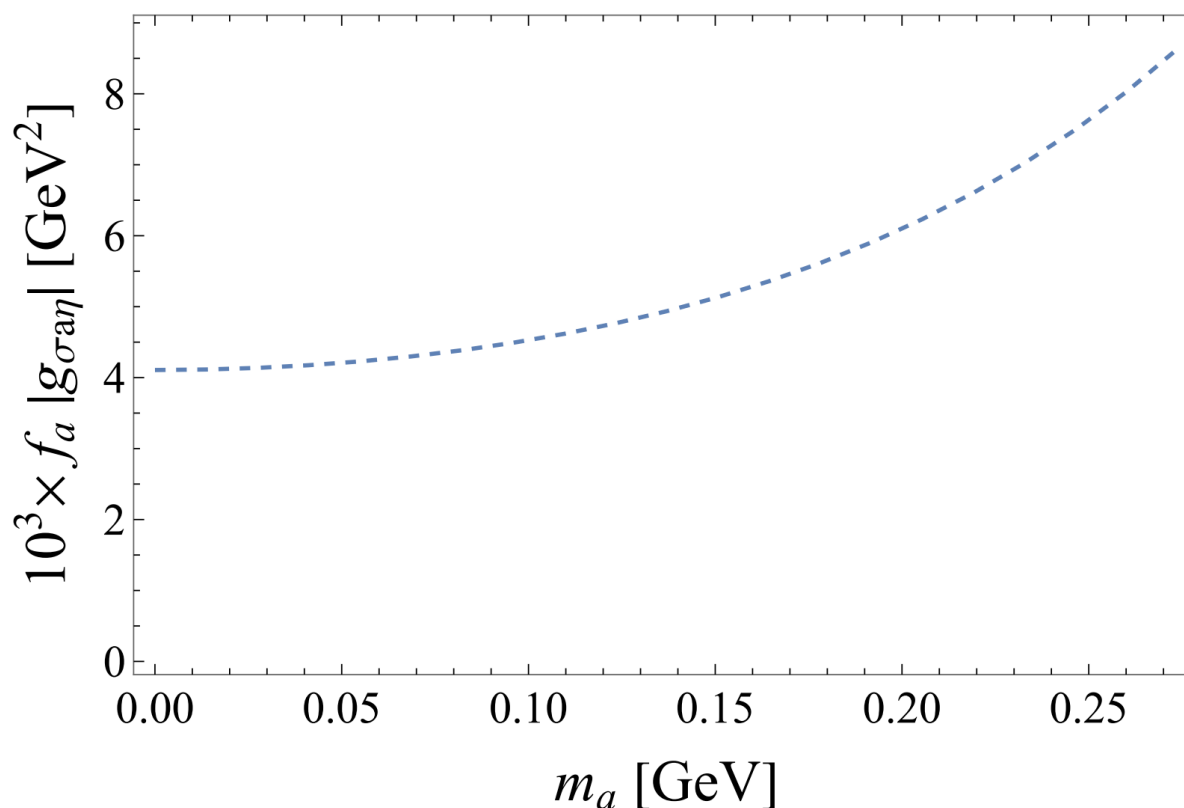
- Unitarized PW amplitude based on LO amplitude

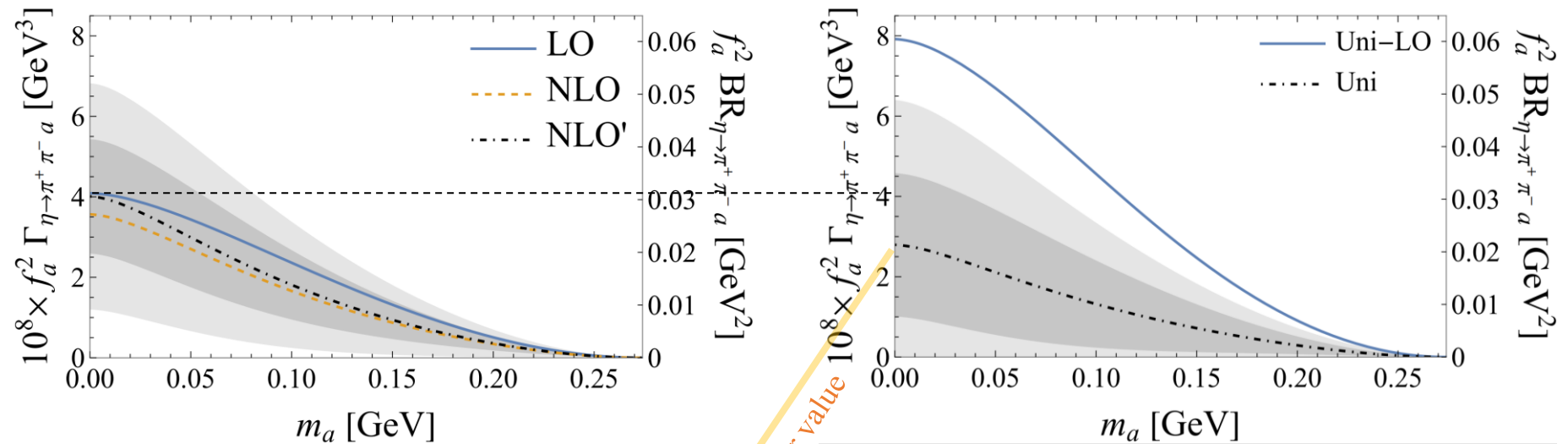
$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni-LO}}(s) = \frac{\mathcal{M}_{\eta;\pi\pi a}^{00,(2)}(s)}{1 - G_{\pi\pi}(s)T_{\pi\pi \rightarrow \pi\pi}^{00,(2)}(s)}.$$



- Pole position of $f_0(500)/\sigma$ on the second Riemann sheet $\sqrt{s_\sigma} = 457 \pm i251$ MeV
- σ - $a\eta$ coupling varying with m_a

$$\mathcal{M}_{\eta;\pi\pi a}^{00,\text{Uni,II}}(s) \Big|_{s \rightarrow s_\sigma} \sim - \frac{g_{\sigma\pi\pi} g_{\sigma a\eta}}{s - s_\sigma}$$





Center value

$$\text{BR}_{\eta \rightarrow \pi^+ \pi^- a} \Big|_{m_a \rightarrow 0} = 2.1 \times 10^{-2} \left(\frac{\text{GeV}^2}{f_a^2} \right)$$

SUMMARY

- We study $\eta \rightarrow \pi\pi a$ decay up to NLO in SU(3) ChPT.
- The unitarized decay amplitudes were further constructed to consider the s -channel $\pi\pi$ final state interaction effect, which can cause 30% correction in the estimation of partial decay width.
- We predict the branch ratio for $\eta \rightarrow \pi\pi a$ decay by varying m_a from 0 to $m_\eta - 2m_\pi$, but the result has a large uncertainty due to the error of LECs. To further pin down the error bars of LECs will be helpful to give more precise predictions to $\eta \rightarrow \pi\pi a$ decay process.

Thank you for listening

