Relativistic three-body scattering and the $D^0D^{*+} - D^+D^{*0}$ system

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Based on : Xu Zhang, arXiv : 2402.02151

第七届强子谱和强子结构研讨会

4月29, 2024 成都

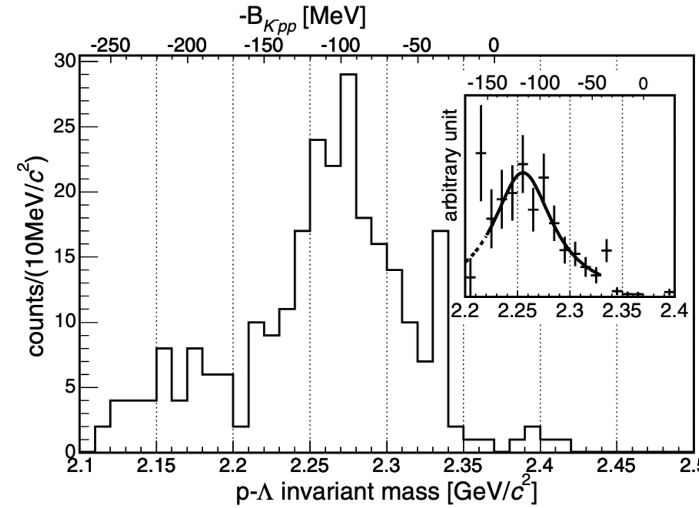
Outlines

- Relativistic three-body scattering
- The $D^0 D^{*+} - D^+ D^{*0}$ scattering
- Analytic structure of the $D^0 D^{*+} - D^+ D^{*0}$ scattering amplitude
- The $D^0 D^0 \pi^+$ decay and the pole position

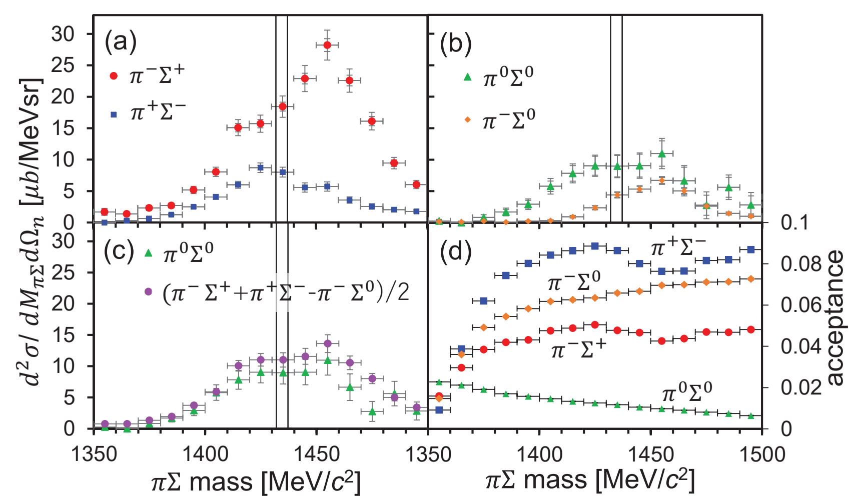
Three-body interaction

- ❖ Three-body bound state
  - Triton ^3H
  - $K^{-}pp$ bound state
- ❖ Three-body decay
  - $\pi_1(1600) \rightarrow\ 3\pi$
  - $a_1(1260) \rightarrow\ 3\pi$
  - $a_1(1420) \xrightarrow{?} K\bar{K}\pi$
- ❖ Other three-body processes

e.g. $\Lambda(1405)$ in Kd scattering



FINUDA PRL 94, 212303 (2005)

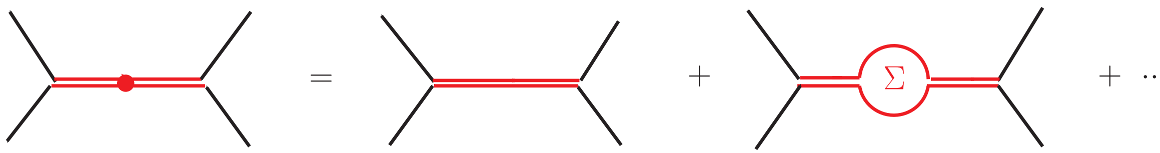


J-PARC PLB 837, 137637 (2023)

Relativistic three-body scattering

- Two-body subsystem interaction

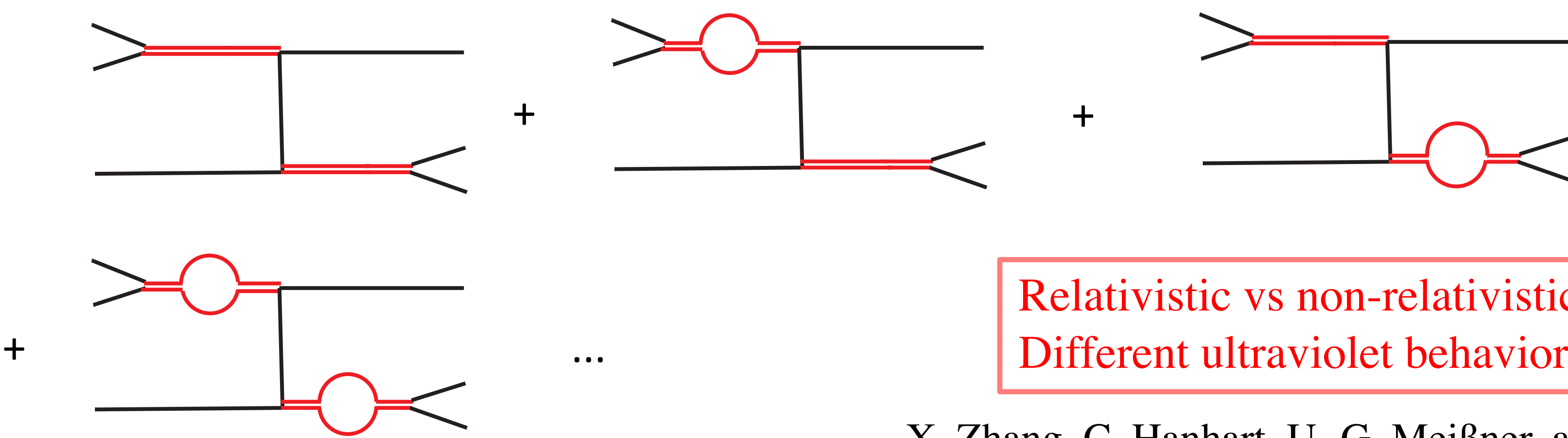
R. Aaron at al., PR 174, 2022 (1968).
R. Aaron et al., Modern three-hadron physics, 1977.



$$A = \frac{v^{\dagger} v}{\sigma - m^2 - \Sigma(\sigma) + i\varepsilon}$$

Relativistic variable

- Three-body interaction



Relativistic vs non-relativistic
Different ultraviolet behavior

X. Zhang, C. Hanhart, U.-G. Meißner, and J.-J. Xie, EPJA 58, 20 (2022).

$$T = V + V\,\tau\,T$$

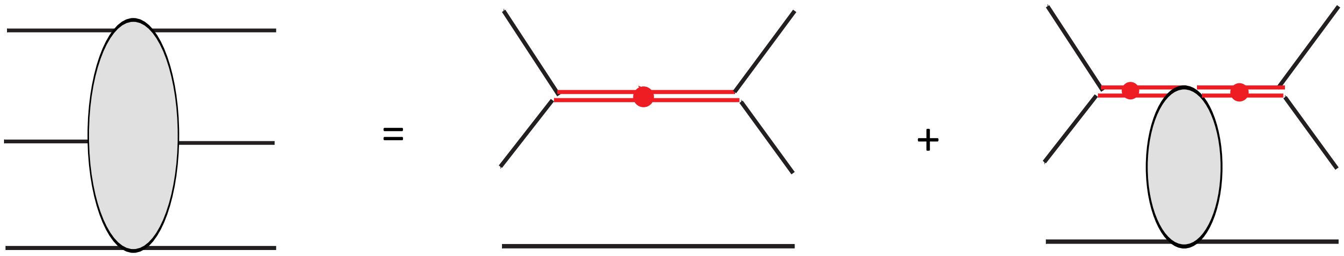
where $\tau = \frac{1}{\sigma - m^2 - \Sigma(\sigma) + i\varepsilon}$, V is the Bonn potential.

Relativistic three-body scattering

R. Aaron at al., PR 174, 2022 (1968).

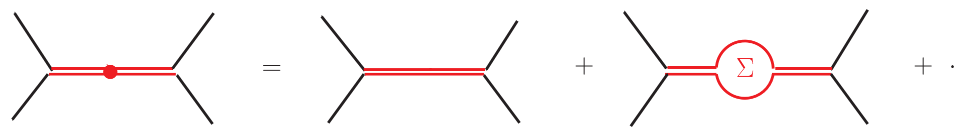
R. Aaron et al., Modern three-hadron physics, 1977.

❖ Three-body interaction



❖ Unitarity relation

$$i\,[\,M^{\dagger} - M\,] = M^{\dagger}\,M$$

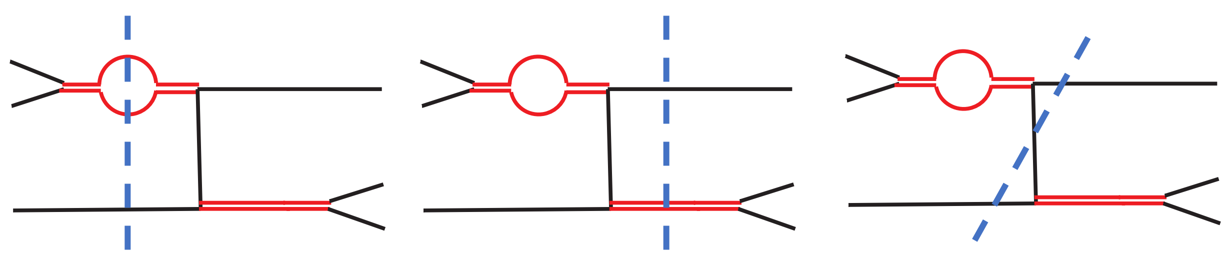


$$\Sigma \sim \int_0^{\infty} cof.\, \frac{k^2\,dk}{\sigma - [\varpi_D(k) + \varpi_\pi(k)\,]^2 + i\varepsilon}$$

❖ Multi-Riemann sheet

$$T(s,p',p) = V(s,p',p) + \int_0^{\Lambda} \frac{k^2 dk}{(2\pi)^3 2\varpi(k)} V(s,p',k)\,\tau(\sigma_k)\,T(s,k,p)$$

- Two-body unitarity cut
- Three-body unitarity cut



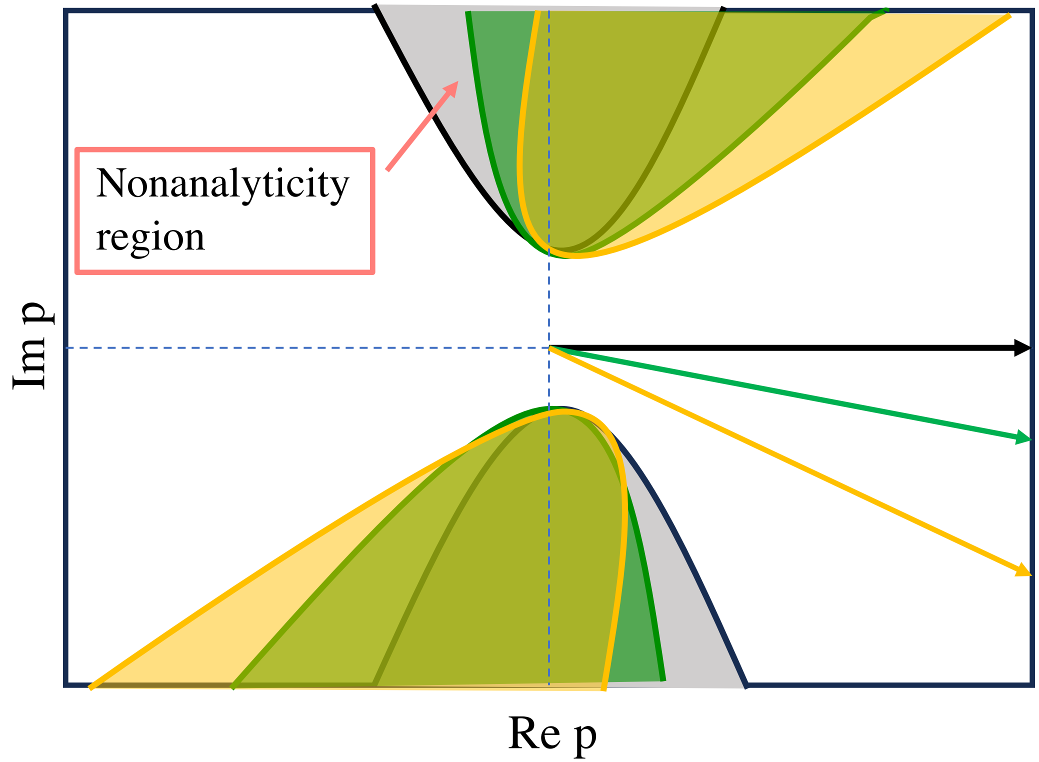
Relativistic three-body scattering

- The domains of nonanalyticity

$V(p', p) \sim$

$$\int_{-1}^{1} \frac{dx}{\sqrt{s} - [\omega_D(p') + \omega_D(p) + \omega_\pi(q)] + i\varepsilon}$$

Denominator = 0 ➡ Nonanalyticity region



W. Glockle, PRC 18, 564 (1978).

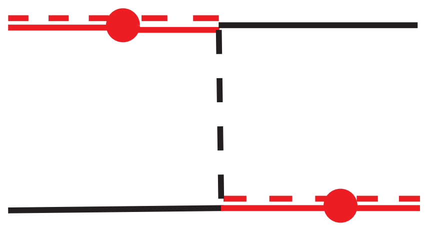
- Contour deformation

$$p \to p\, e^{-i\theta}$$
$$p' \to p'\, e^{-i\theta}$$

The analytic region is extended to the unphysical region.

DD^* scattering

❖ π-exchange

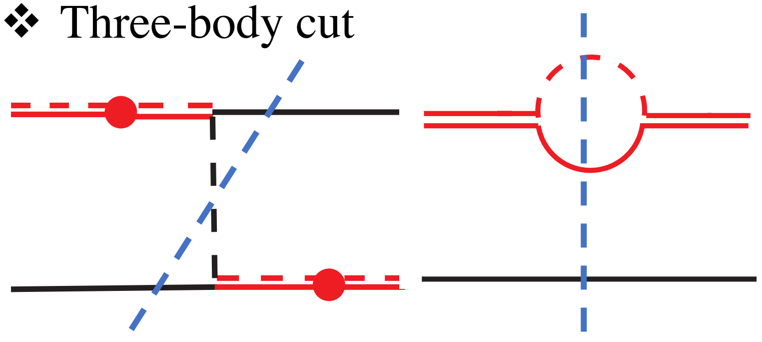


$$V \sim \frac{1}{q^2 - m_\pi^2 + i\varepsilon}$$

$$q^2 - m_\pi^2 \simeq -2m_\pi\Delta + \left(1 + \frac{m_\pi^2}{4m_D m_{D^*}}\right) q$$

$$\Delta = m_{D^*} \text{ - } m_D \text{ - } m_\pi$$

❖ Three-body cut



zero appears !

Different from the NN scattering

Three-body dynamics

M.-L. Du, V. Baru, X.-K. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, PRD 105, 014024 (2022).
L. Qiu, C. Gong, and Q. Zhao, PRD 109, 076016 (2024).
J.-Z. Wang, Z.-Y. Lin, and S.-L. Zhu, PRD 109, L071505 (2024).

Similar phenomenon happens in $D\bar{D}^*$

S. Fleming, M. Kusunoki, T. Mehen, and U. van Kolck, PRD 76, 034006 (2007).
M. Suzuki, PRD 72, 114013 (2005).
V. Baru, A. A. Filin, C. Hanhart, Y. S. Kalashnikova, A. E. Kudryavtsev, and A. V. Nefediev, PRD 84, 074029 (2011).
M. Schmidt, M. Jansen, and H. W. Hammer, PRD 98, 014032 (2018).

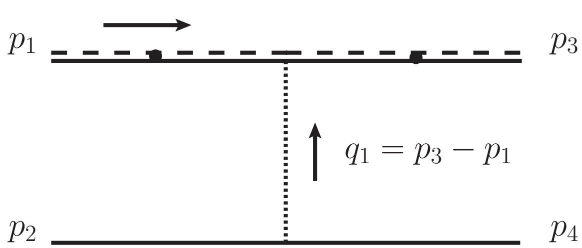
DD^* scattering

❖ Scattering equation

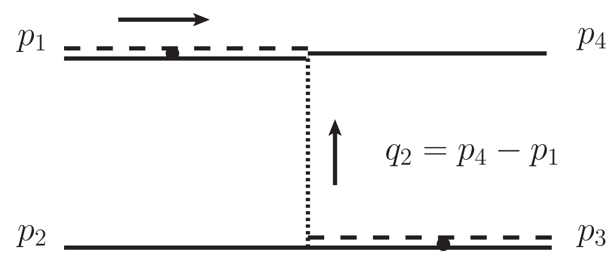
$$T(s,p',p) = V(s,p',p) + \int_0^{\Lambda} \frac{k^2 dk}{(2\pi)^3 2\varpi(k)} V(s,p',k)\, \tau(\sigma_k)\, T(s,k,p)$$

$$\sigma_k = s - 2\sqrt{s}\,\omega_D(k) + m_D^2$$

The interaction potential (including the π, σ , ρ and ω-exchange).



Tpye – A



Type – B

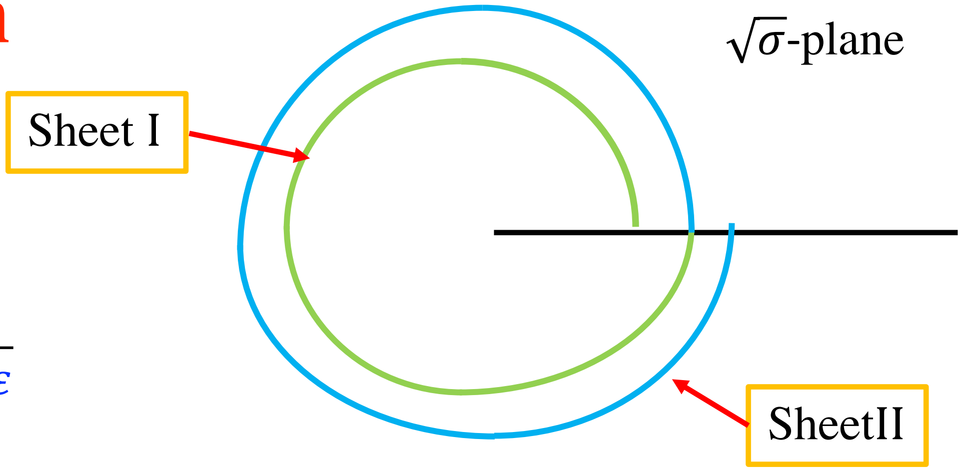
The isospin factors

<table>
<tr><th></th><th colspan="3">Type-A</th><th colspan="5">Type-B</th></tr>
<tr><td></td><td>ρ^0</td><td>$\rho^\pm$</td><td>ω</td><td>ρ^0</td><td>$\rho^\pm$</td><td>ω</td><td>π^0</td><td>$\pi^\pm$</td></tr>
<tr><td>1→ 1</td><td rowspan="2">1/2</td><td rowspan="2">…</td><td rowspan="2">-1/2</td><td rowspan="2">…</td><td rowspan="2">1</td><td rowspan="2">…</td><td rowspan="2">…</td><td rowspan="2">1</td></tr>
<tr><td>2→ 2</td></tr>
<tr><td>1→ 2</td><td>…</td><td>-1</td><td>…</td><td>-1/2</td><td>…</td><td>1/2</td><td>-1/2</td><td>…</td></tr>
</table>

Analytic continuation

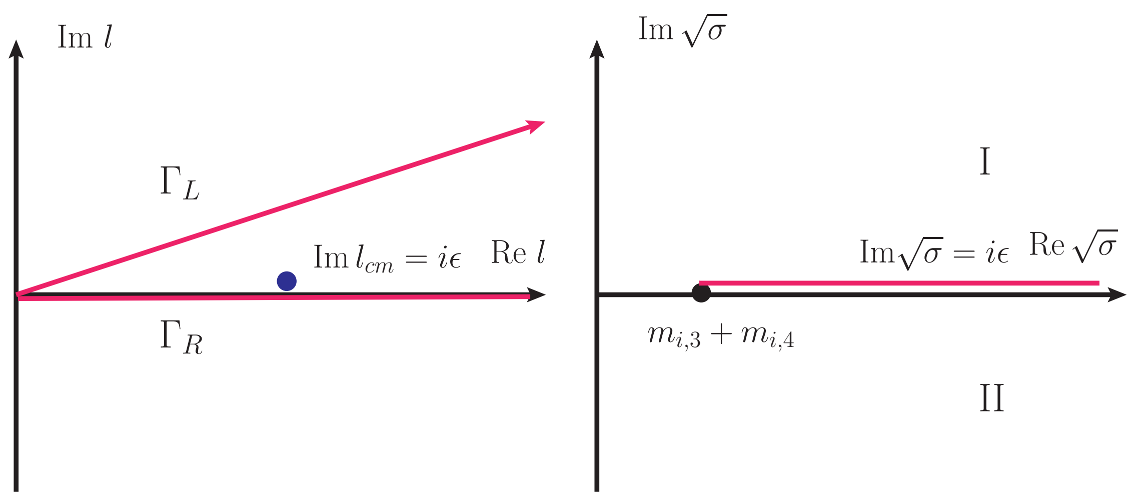
- Self-energy
  - First Riemann sheet

$$\Sigma \sim \int_0^\infty cof.\ \frac{k^2\, dk}{\sigma - [\varpi_D(k) + \varpi_\pi(k)\,]^2 + i\varepsilon}$$



$$\sqrt{\sigma} = (\, m_D + m_\pi) + re^{i\theta}$$

Integral contour



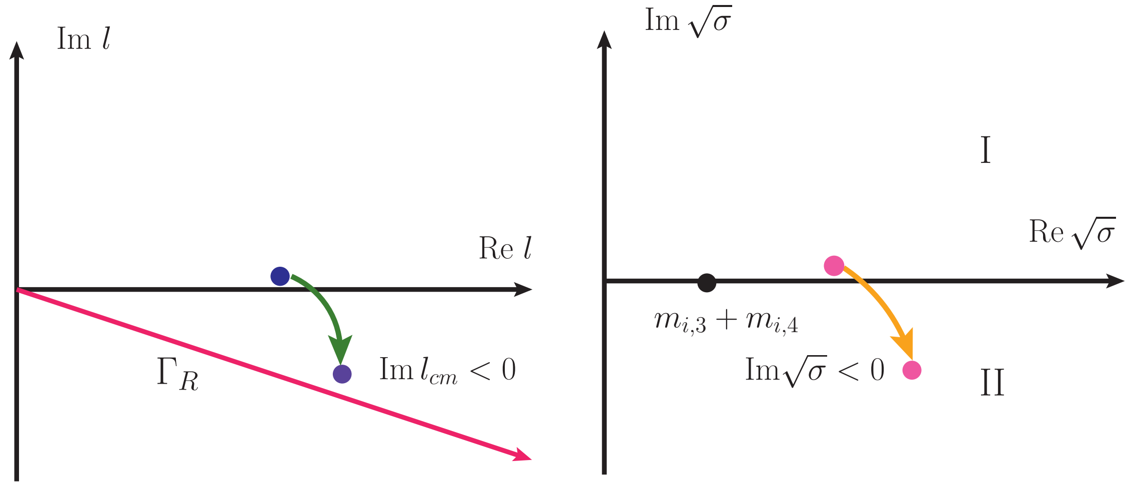
Analytic continuation

❖ Self-energy

$$\Sigma \sim \int_0^{\infty} cof.\ \frac{k^2\, dk}{\sigma - [\varpi_D(k) + \varpi_\pi(k)]^2 + i\varepsilon}$$

- Second Riemann sheet (resonance)

Integral contour

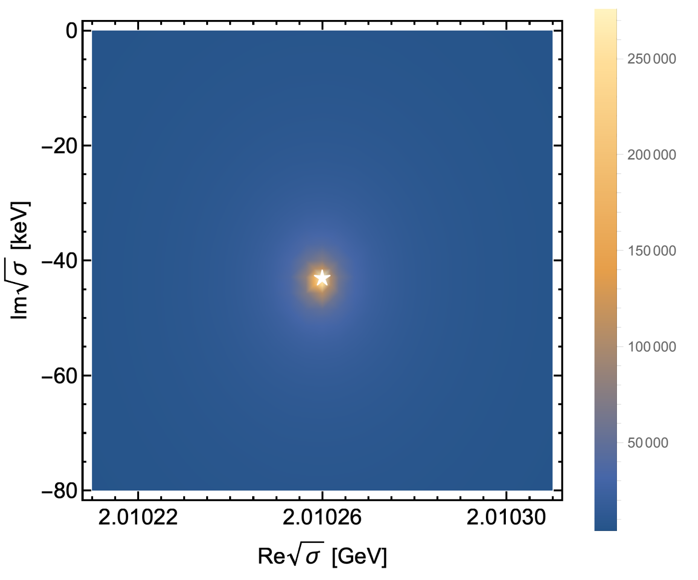


$$\tau = \frac{1}{\sigma - m^2 - \Sigma_1^R(\sigma) - \Sigma_2^R(\sigma) + i\varepsilon}$$

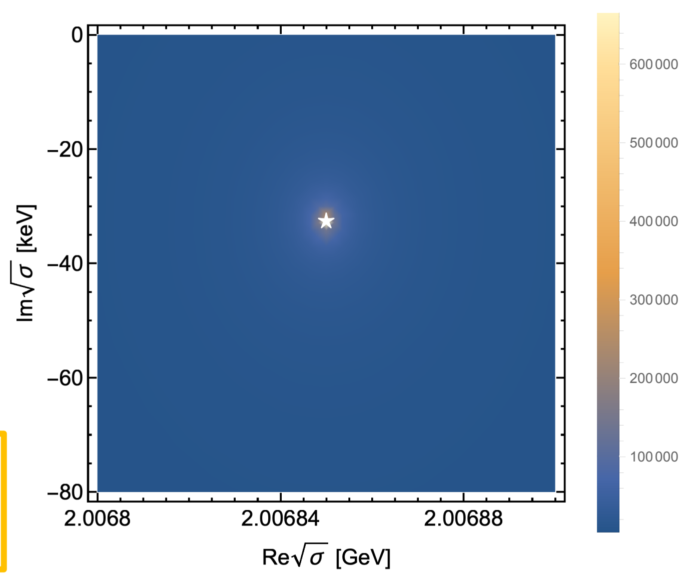
$$\Sigma_i^R(\sigma) = \Sigma_i(\sigma) - \Sigma_1^{\rm sub}(\sigma)$$

$D\pi$ interaction in p -wave
Twice subtraction

D^{*+} pole position



D^{*0} pole position

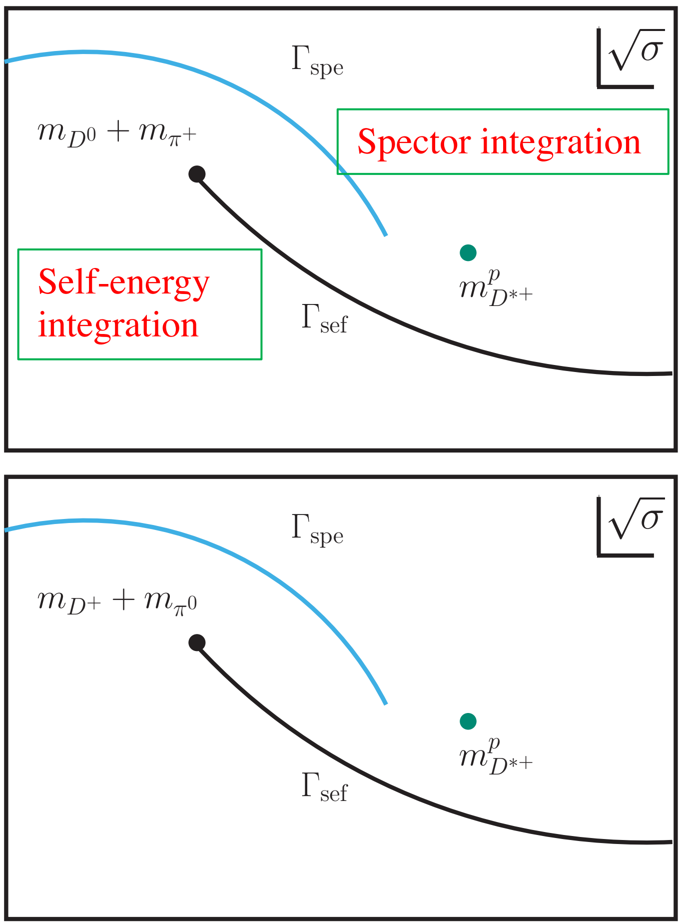


Analytic continuation

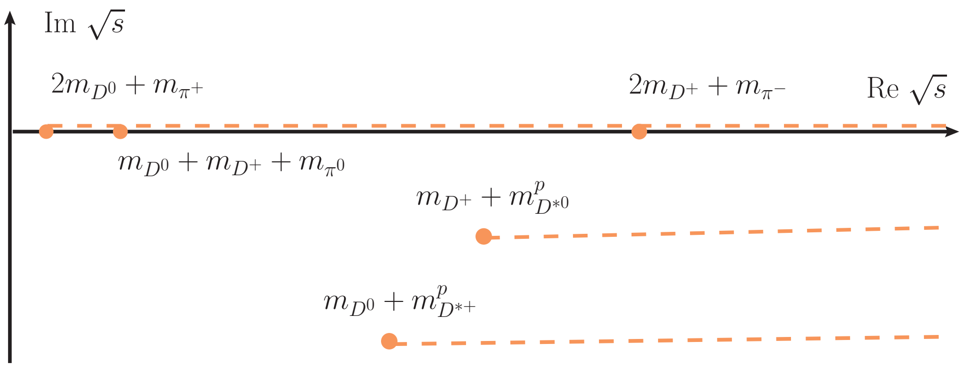
$$T(s,p',p) = V(s,p',p) + \int_0^{\Lambda} \frac{k^2 dk}{(2\pi)^3 2\varpi(k)} V(s,p',k)\, \tau(\sigma_k)\, T(s,k,p)$$

❖ Across the three-body cuts

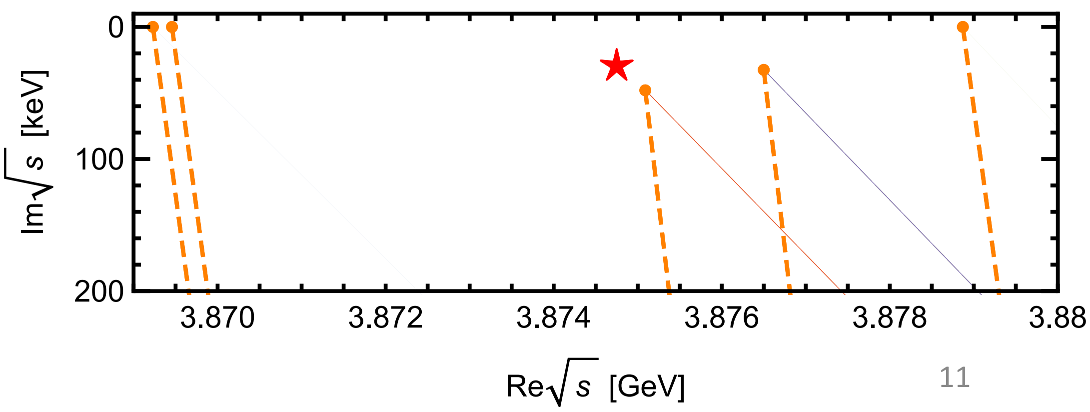
- Contour mapped in the $\sqrt{\sigma}$ plane.



- The cut structure in complex $\sqrt{s}$ plane



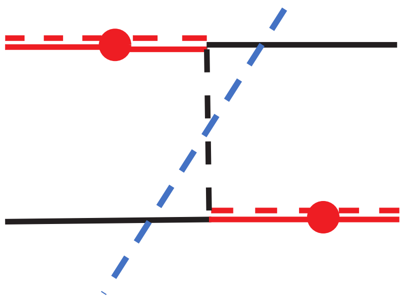
- The cut structure after contour deformation



Solution for real momentum

- Three-body singularities

X. Zhang, H.-L. Fu, F.-K. Guo, and H.-W. Hammer, PRC 108, 044304

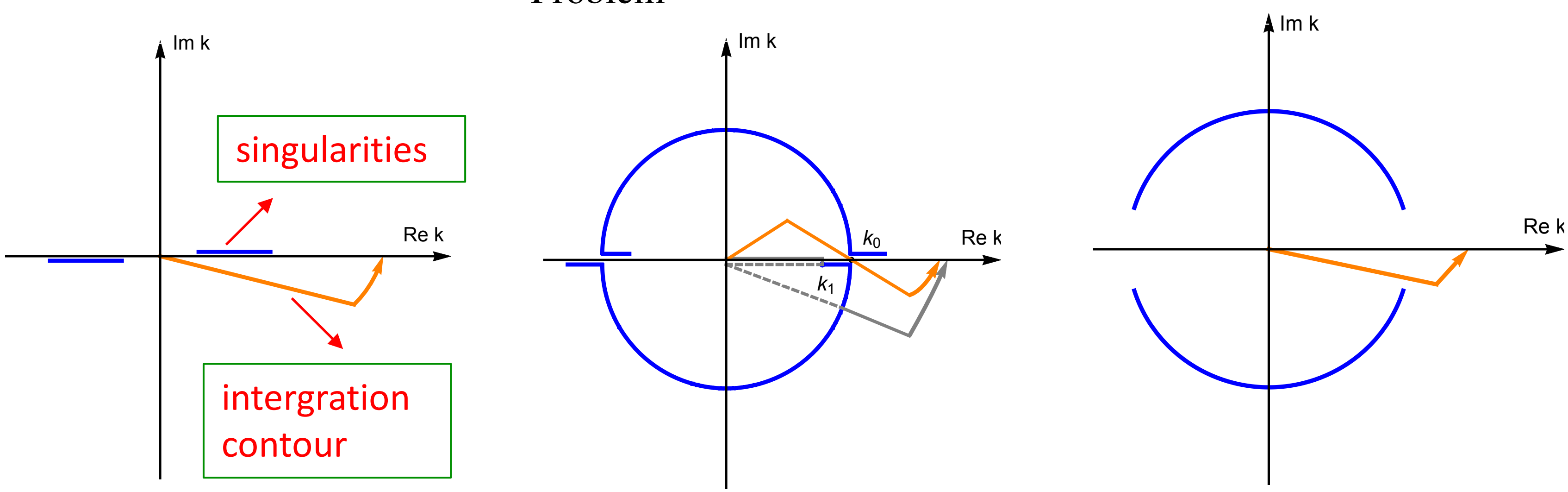


$$V \sim \int_{-1}^{1} \frac{dx}{\sqrt{s} - [\omega_D(p') + \omega_D(p) + \omega_\pi(q)] + i\varepsilon} \Rightarrow \textbf{Zero}$$

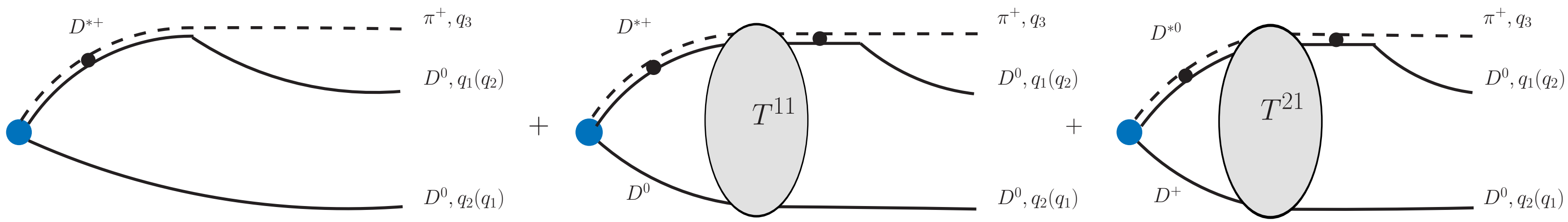
$$x = \vec{p}\,\vec{p}'/pp'$$

- Contour deformation

E. Schmid, H. Ziegelmann, The Quantum Mechanical Three-Body Problem



Three-body $D^0 D^0 \pi^+$ dacay



❖ The mass distribution

$$\frac{d\Gamma(\sqrt{s})}{d\sqrt{s}} = \int \frac{1}{(2\pi)^5} \frac{1}{16\, s} \left(\frac{1}{3} \sum_\Lambda \left| \sum_\lambda M_{\Lambda\lambda}(\vec{q}_1, \vec{q}_2, \vec{q}_3) \right|^2 \right) q_3^* \, q_1 \, dm_{23} \; d\Omega_3^* \, \mathrm{d}\Omega_1$$

Decay amplitude

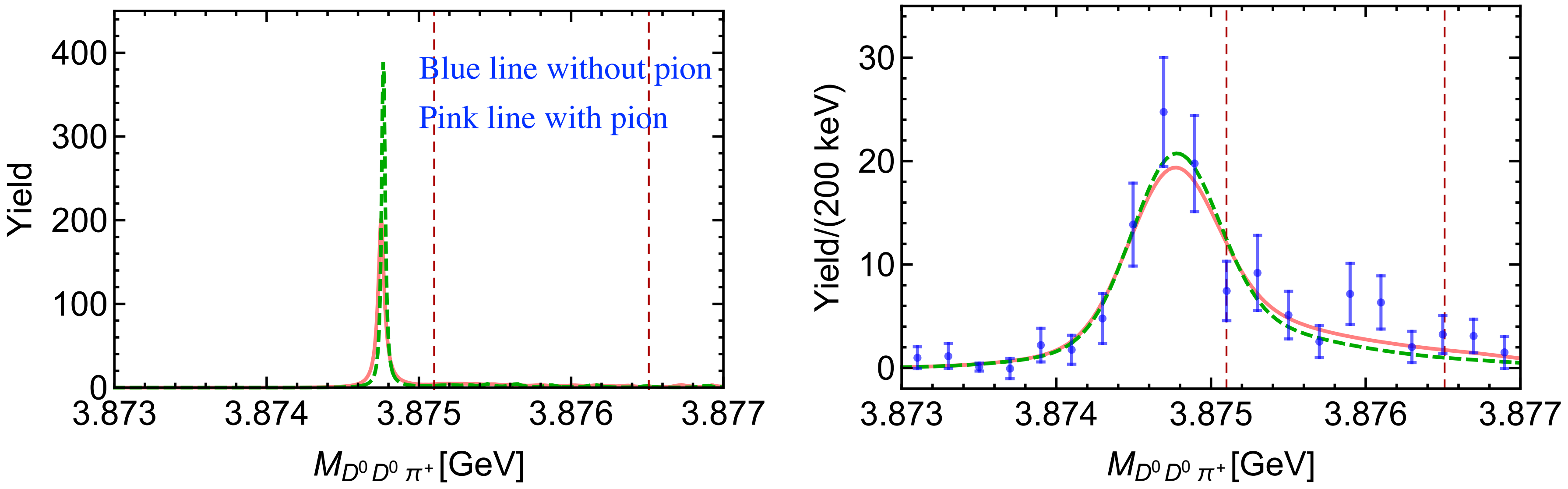
$$M_{\Lambda\lambda}(\vec{q}_1, \vec{q}_2, \vec{q}_3) = \frac{\mathcal{F}}{\sqrt{2}} \left[\sqrt{\frac{3}{4\pi}} \, D^{1*}_{\Lambda\lambda}(\phi_1, \theta_1, 0) M_L(q_1) \, U_{L\lambda} \, v_\lambda(\vec{q}_2, \vec{q}_3) + \vec{q}_1 \leftrightarrow \vec{q}_2 \right]$$

❖ Short-distance interaction is absorbed into $\mathcal{F}$.

Numerical results

Cut off Λ and the pole positions from fitting of the $D^0D^0\pi^+$ line shape obtained by the LHCb Collaboration.

Scheme	$\chi^2/d.o.f$	Λ GeV	$\sqrt{s_{pole}^{thr}}$ keV
I	18.11/(20−1) = 0.95	0.4551 ± 0.0018	$-332^{+37}_{-36} - i(18 \pm 1)$
II	14.47/(20−1) = 0.76	0.3701 ± 0.0017	$-351^{+37}_{-35} - i(28 \pm 1)$



Fitting results of the $D^0D^0\pi^+$ line shapes before (left panel) and after (right panel) convolution with the energy resolution function.

Summary

- ❖ The analytic continuation of the coupled-channel $D^0D^{*+} - D^+D^{*0}$ scattering amplitude is studied.
- ❖ The π-exchange term has a signification on the pole position of the T_{cc}^+. Including the π-exchange term, the width of T_{cc}^+ will be increased by a factor of 1.5.
- ❖ We will extend our framework to calculate $3\pi - K\bar{K}\pi$ coupled system.

Thank you for your attention!