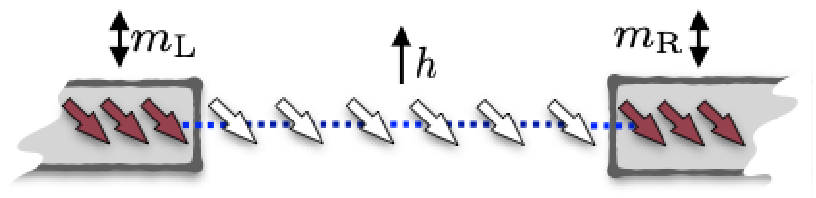
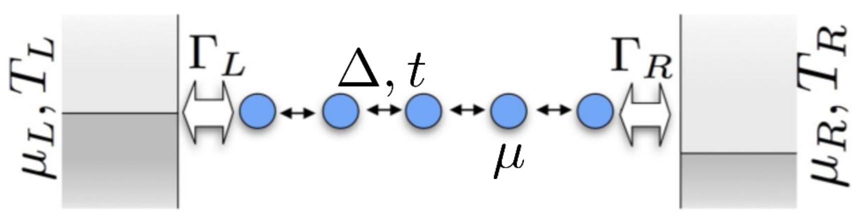
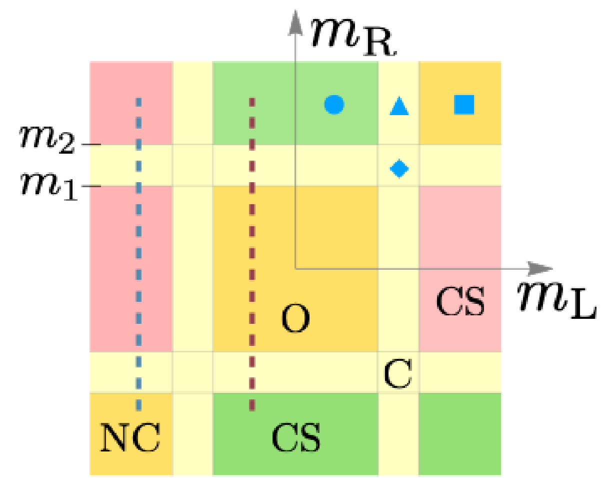
Mixed-Order Symmetry-Breaking Quantum Phase Transition Far from Equilibrium

Stefano Chesi

Beijing Computational Science Research Center (CSRC)







T. O. Puel, SC, S. Kirchner, and P. Ribeiro, PRL 122, 235701 (2019)

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Mixed-Order Symmetry-Breaking Quantum Phase Transition Far from Equilibrium

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In collaboration with:



Tharnier O. Puel
CSRC

Stefan Kirchner
Zhejiang University

Pedro Ribeiro
University of Lisbon

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1. <u>Introduction</u>
   - Non-equilibrium phase transitions
   - Model and motivations

2. <u>Non-eq transverse-field Ising model</u>
   - Energy current & phase diagram
   - Correlation functions & Mixed-order phase transition
   - Entropy scaling

3. <u>Conclusion and Outlook</u>

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3. Conclusion and Outlook

Equilibrium phase transitions

- Crystalline/amorphous materials, fluids
- Superconductivity and superfluidity
- Magnetism, anti-ferromagnetism
- Charge- and spin-density waves
- Metal-insulator transitions
- Deconfined quantum critical points
- Topological order

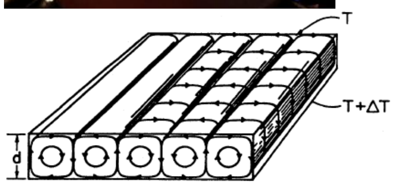
….

$$\mathcal{Z} = \mathrm{Tr}\left[e^{-H(g_1, g_2, \ldots)/k_B T}\right]$$

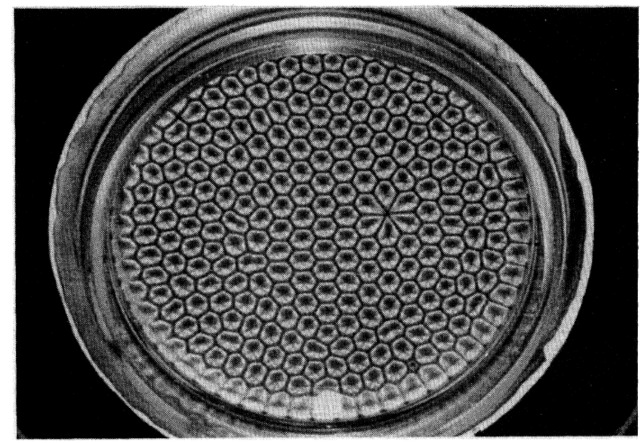
Non-equilibrium conditions

Dynamical formation of ordered structures

See, e.g., *Pattern formation outside of equilibrium*
M. Cross, P. Hohenberg, Rev. Mod. Phys. 65, 851 (1993)



Rayleigh-Bénard convection



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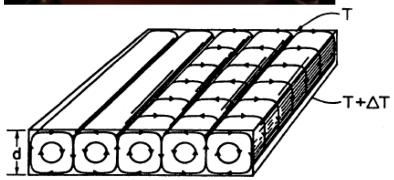
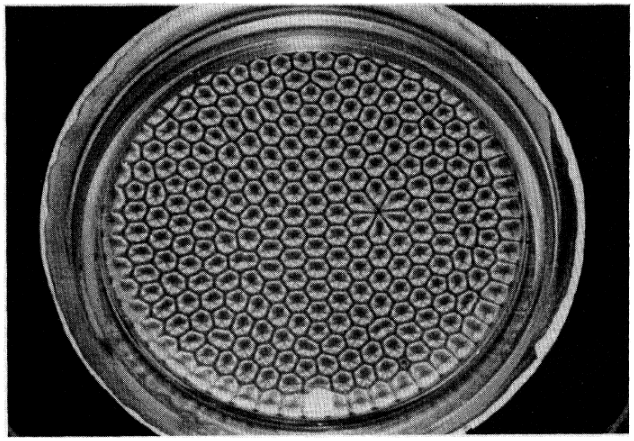
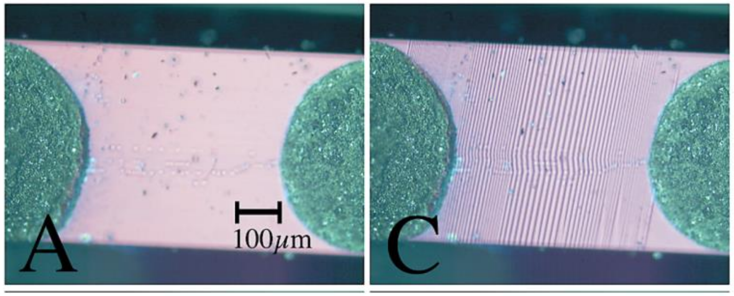


Rayleigh-Bénard convection



In quantum transport:



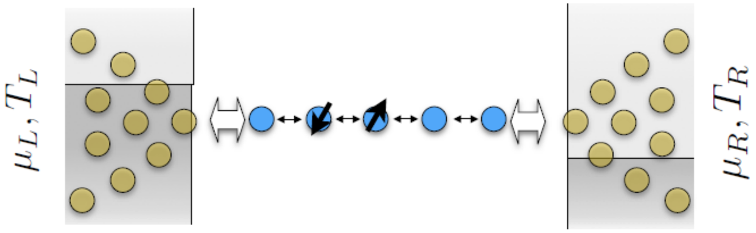
Current-induced stripes in organic molecular Mott insulators

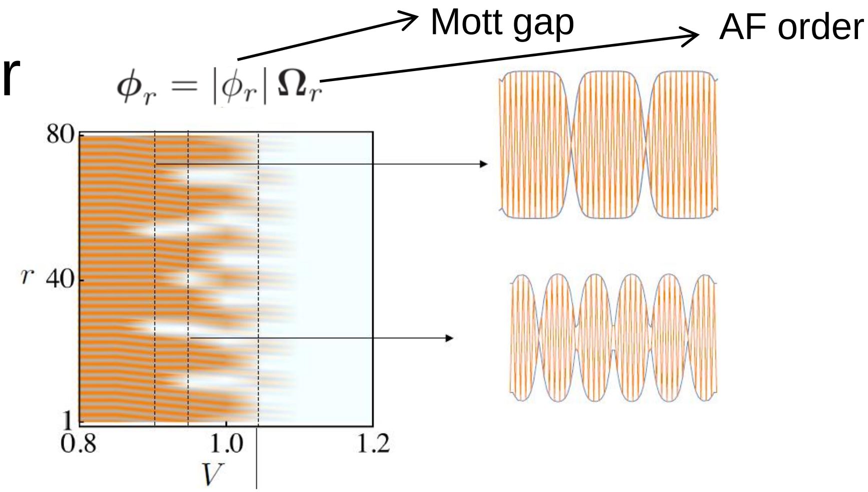
R. Kumai, Y. Okimoto, Y. Tokura
Science, 284, 1645 (1999)

Two more examples

- Non-equilibrium Mott insulator

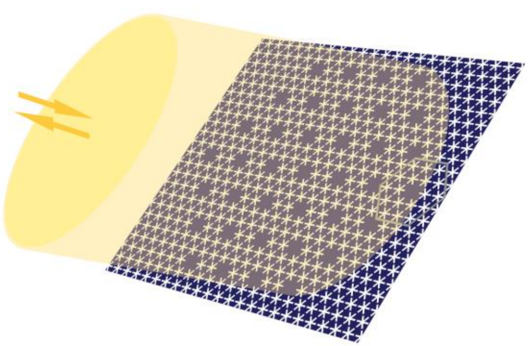
$$H_\Sigma = -\sum_{\boldsymbol{r},\boldsymbol{r}',\sigma} c^\dagger_{\boldsymbol{r}\sigma} \tilde{\boldsymbol{t}}_{\boldsymbol{r},\boldsymbol{r}'} c_{\boldsymbol{r}'\sigma} + \frac{U}{2}\sum_{\boldsymbol{r}} (n_{\boldsymbol{r}} - 1)^2$$

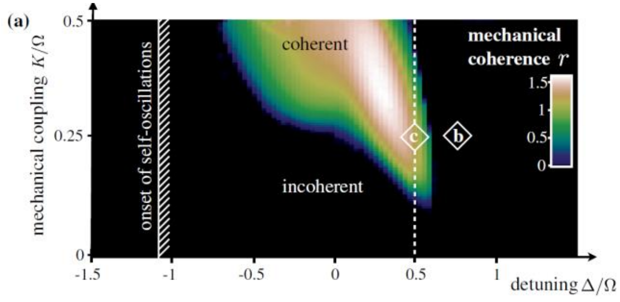


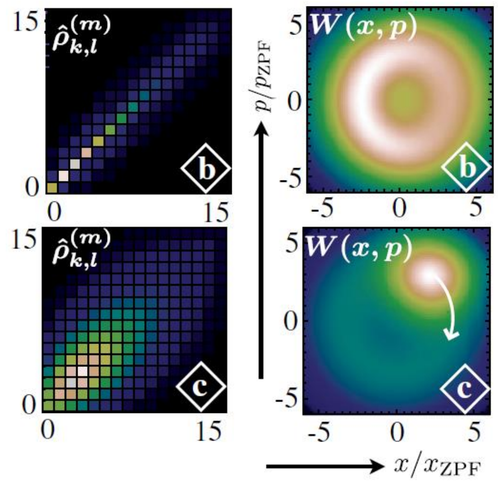


P. Ribeiro, A. E. Antipov, and A. N. Rubtsov, Phys. Rev. B 93, 144305 (2016)

- Driven optomechanical arrays







M. Ludwig and F. Marquardt, Phys. Rev. Lett. 111, 073603 (2013)

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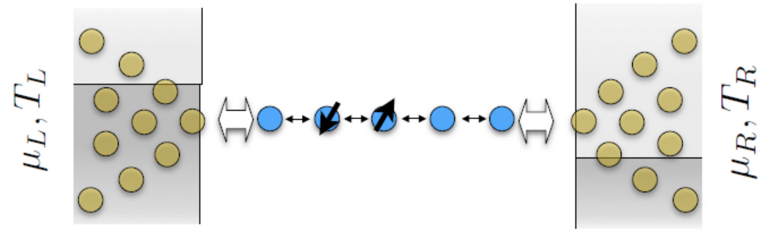

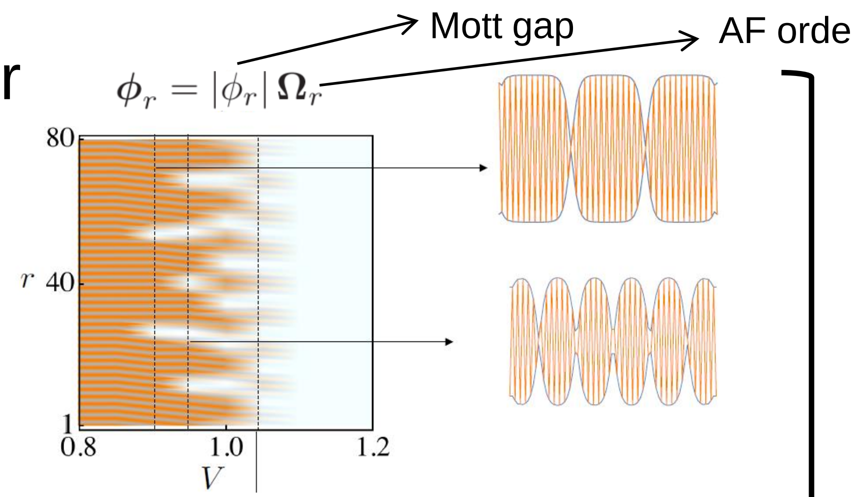

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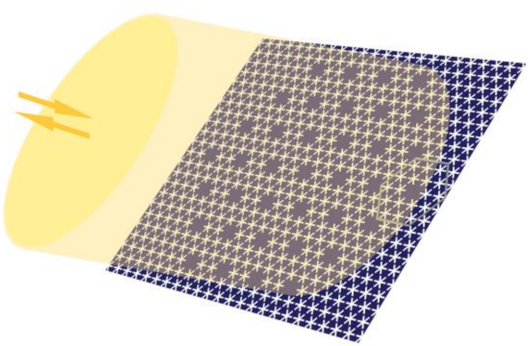

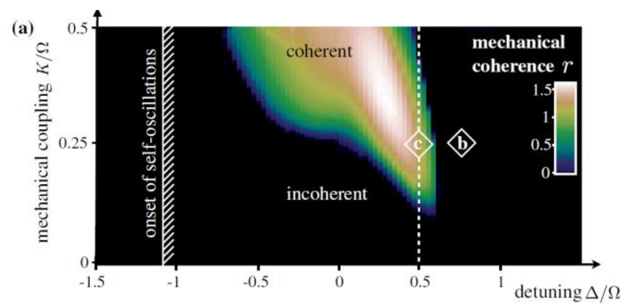

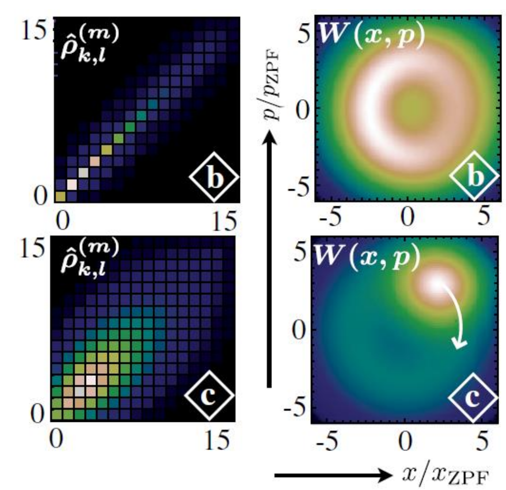

M. Ludwig and F. Marquardt, Phys. Rev. Lett. 111, 073603 (2013)

mean field theory

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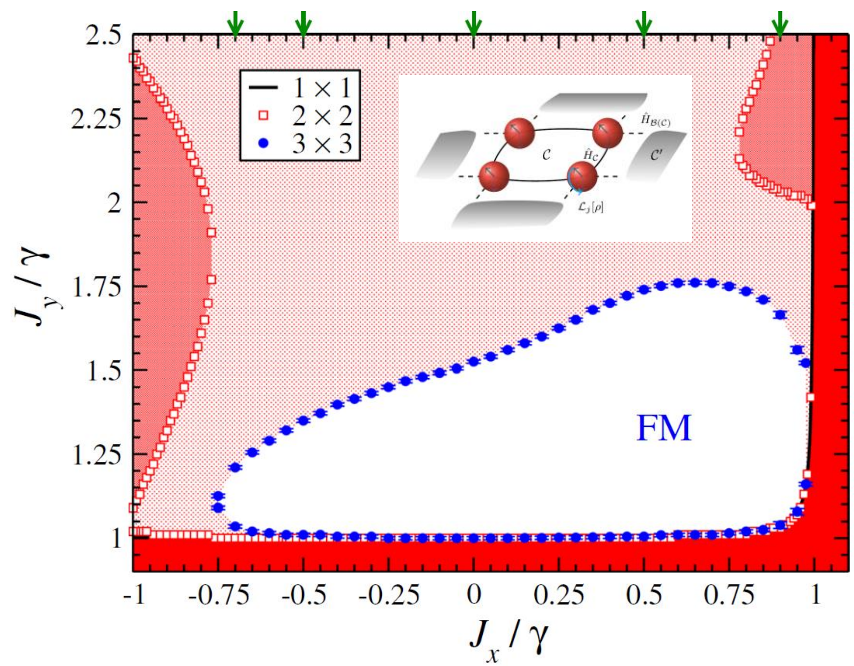
Cluster mean-field

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{H}, \rho] + \sum_j \mathcal{L}_j[\rho]$$

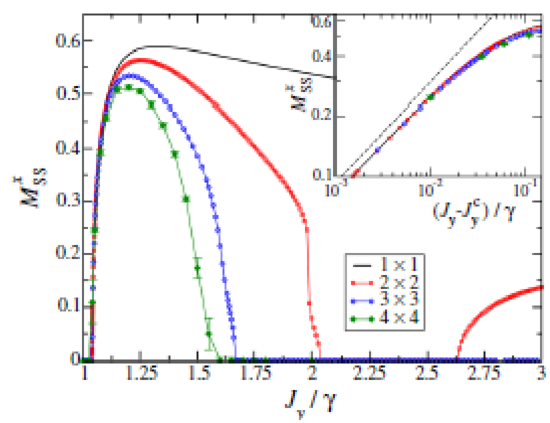
Hamiltonian

$$\hat{H} = \sum_{\langle i,j \rangle} h_{ij} = \sum_{\langle i,j \rangle} (J_x \hat{\sigma}_i^x \hat{\sigma}_j^x + J_y \hat{\sigma}_i^y \hat{\sigma}_j^y + J_z \hat{\sigma}_i^z \hat{\sigma}_j^z)$$

$$\sum_j \mathcal{L}_j[\rho] = \gamma \sum_j \left[\hat{\sigma}_j^- \rho \hat{\sigma}_j^+ - \frac{1}{2} \{ \hat{\sigma}_j^+ \hat{\sigma}_j^-, \rho \} \right]$$



max cluster size: 4x4



J. Jin, A. Biella, O. Viyuela, L. Mazza, J. Keeling, R. Fazio, and D. Rossini, Phys. Rev. X 6, 031011 (2016)

dissipative quantum many-body systems

- Interesting, diverse
- Relatively unexplored

- Theoretically challenging
- Few general principles

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Insight from exactly solvable models?

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Transverse field Ising model

$$H = -J \sum_{\langle i,j \rangle} \sigma_i^x \sigma_j^x - h \sum_i \sigma_i^z$$

Realizations: magnetic compounds (e.g., $CoNb_2O_6$), cold gases

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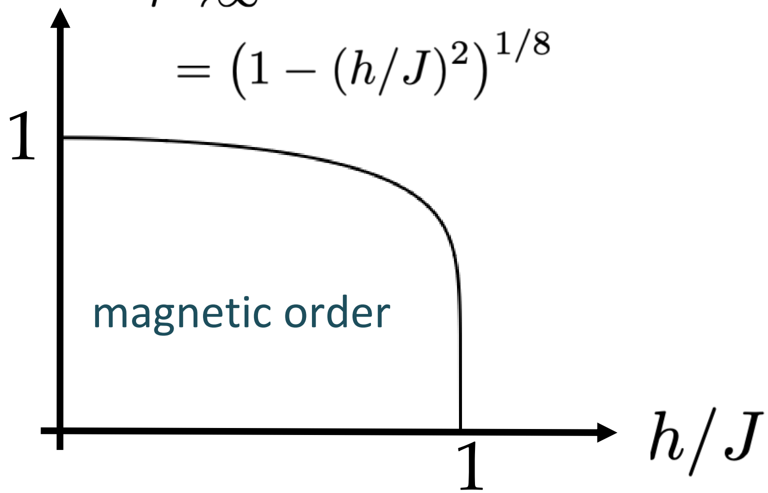
$$H = -J \sum_{\langle i,j \rangle} \sigma_i^x \sigma_j^x - h \sum_i \sigma_i^z$$

Realizations: magnetic compounds (e.g., $CoNb_2O_6$), cold gases

$$\phi \equiv \lim_{r\to\infty} \langle \sigma_i^x \sigma_{i+r}^x \rangle$$

$$= \left(1 - (h/J)^2\right)^{1/8}$$



"Drosophila" of quantum phase transitions (T=0)

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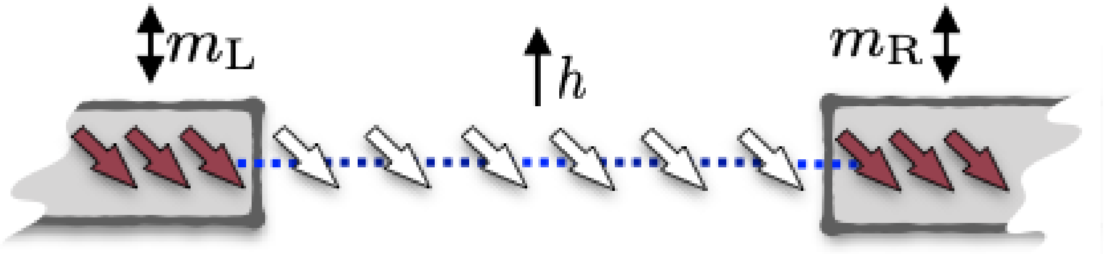
Non-equilibrium version



$$H = -J \sum_{\langle i,j \rangle} \sigma_i^x \sigma_j^x - h \sum_i \sigma_i^z + \sum_{l=\mathrm{L,R}} (H_l + H_{\mathrm{C},l})$$

$$H_l = -J_l \sum_{r \in \Omega_l} (\sigma_r^x \sigma_{r+1}^x + \sigma_r^y \sigma_{r+1}^y) - m_l M_l$$

$$H_{\mathrm{C},l} = -J_l' (\sigma_{r_l'}^x \sigma_{r_l}^x + \sigma_{r_l'}^y \sigma_{r_l}^y)$$

magnetic potentials

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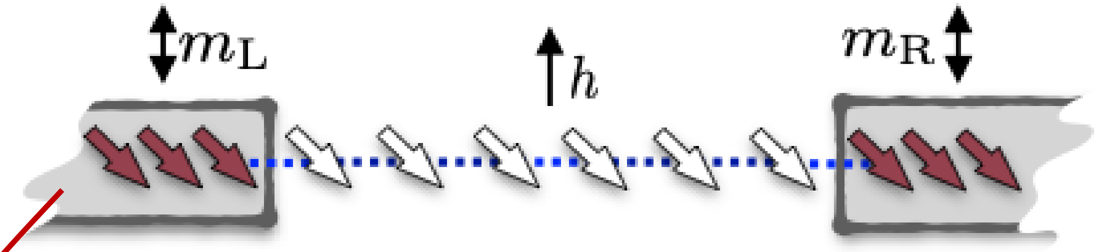
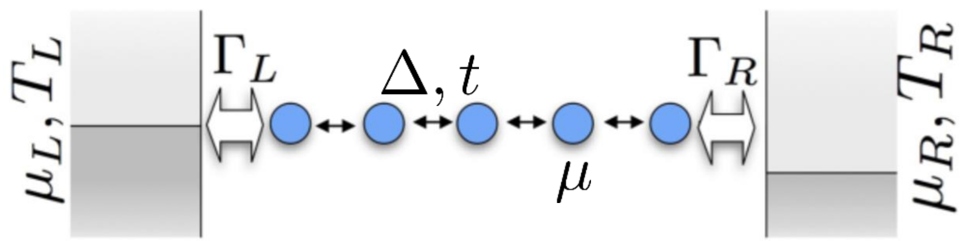
Integrability



<u>Jordan-Wigner</u> mapping:

$$\sigma_i^- = 2\mathcal{S}_{i-1}\hat{c}_i$$
$$\sigma_i^+ = 2\mathcal{S}_{i-1}\hat{c}_i^\dagger$$

with $\mathcal{S}_{i-1} = \exp\left(\pm i\pi \sum_{j=1}^{n-1} \hat{c}_j^\dagger \hat{c}_j\right)$



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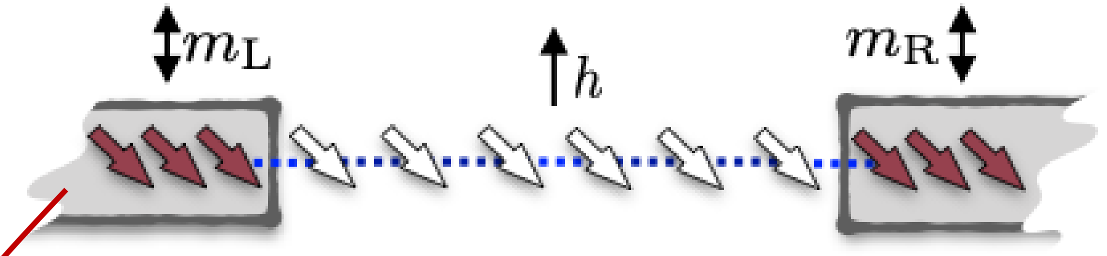

Jordan-Wigner mapping:

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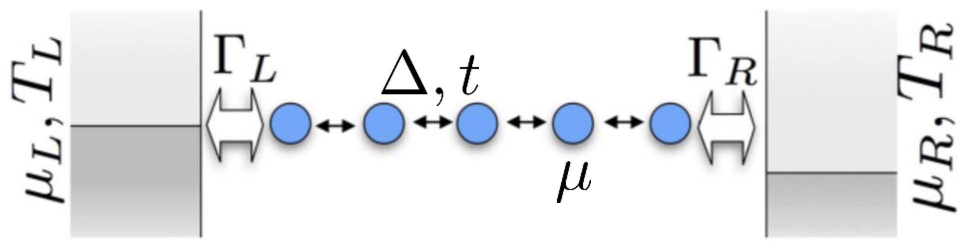
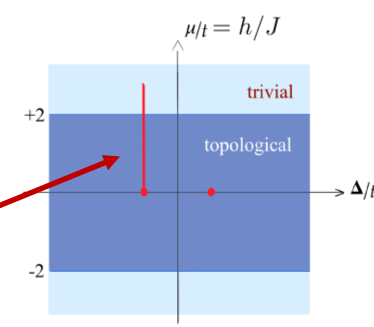


$$H = J\sum_i (\hat{c}_i - \hat{c}_i^\dagger)(\hat{c}_{i+1} + \hat{c}_{i+1}^\dagger) - 2h\sum_i \hat{c}_i^\dagger \hat{c}_i$$

here we have $\Delta = t = J$



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Method

Correlation function: $\chi \equiv \langle \mathbf{\Psi}\mathbf{\Psi}^{\dagger} \rangle$

Explicit evaluation of χ

$$\boldsymbol{\chi} = \frac{1}{2}\left[i \int \frac{d\omega}{2\pi} \boldsymbol{G}^{K}(\omega) + \mathbf{1} \right]$$

(Green's function technique)

Energy drain from (L/R) contacts: $\mathcal{J}_e = \frac{d}{dt}\langle H_L \rangle = -\frac{d}{dt}\langle H_R \rangle$

$$\mathcal{J}_e \equiv -i \left\langle [H, H_L] \right\rangle = -\frac{1}{2}\mathrm{Tr}\left[\boldsymbol{J}_r \boldsymbol{\chi} \right]$$

$$\boldsymbol{J}_r = -2ihJ[(1+\boldsymbol{S})|r-1\rangle\langle r|(1+\boldsymbol{S}) - \text{H.c.}]$$

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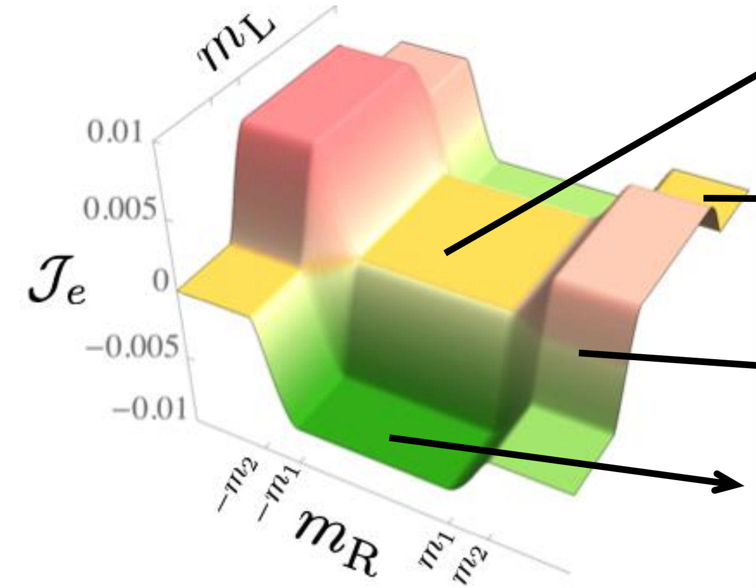
Bias-dependence



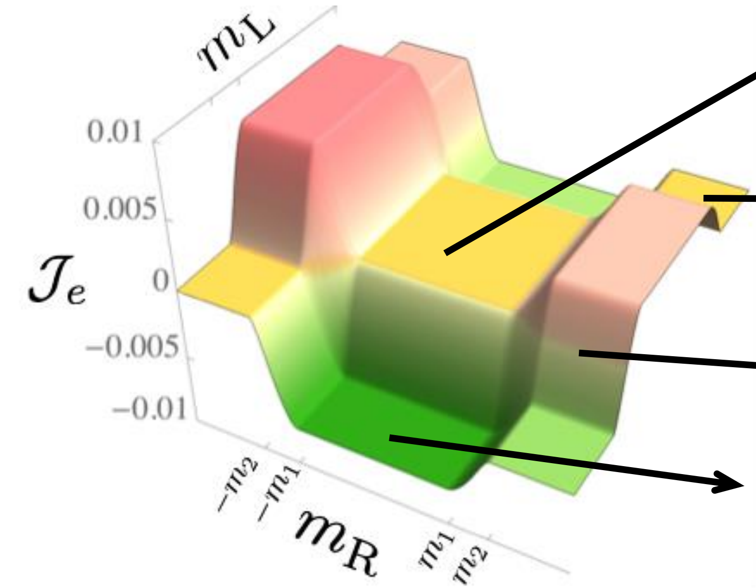
Ordered phase (O)

Non-conducting (NC)

Conducting (C)

Conducting, current-saturated (CS)

Bias-dependence



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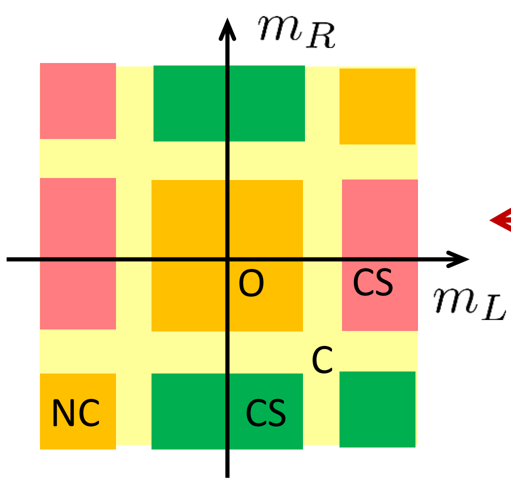
Phase diagram



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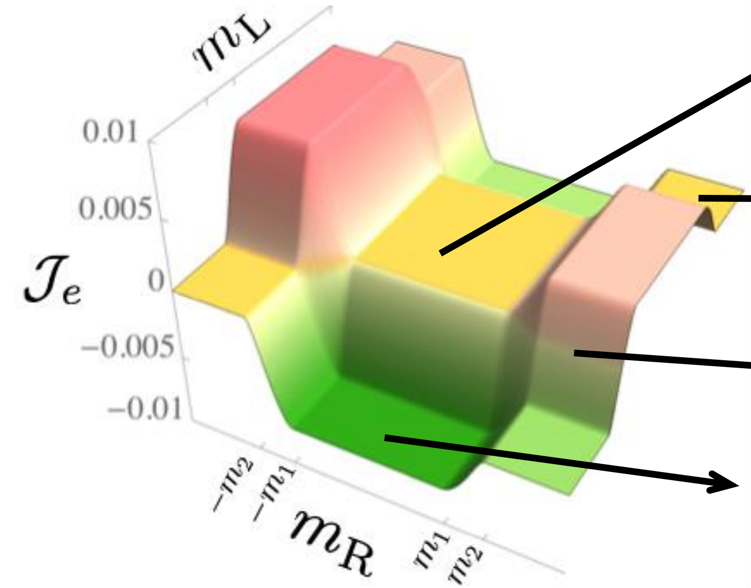

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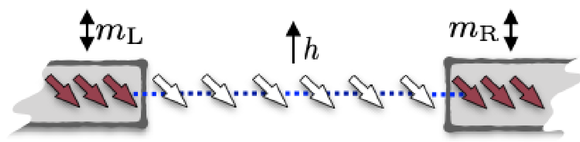

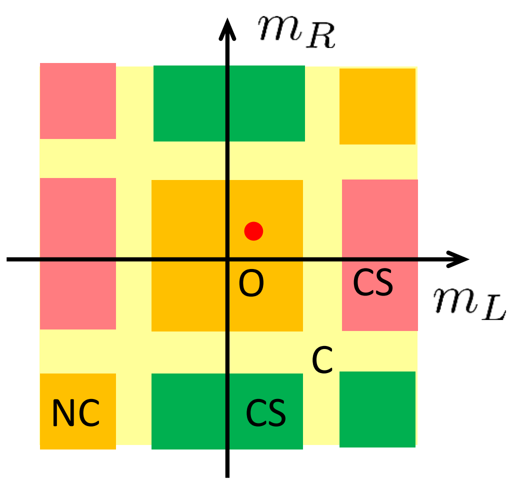

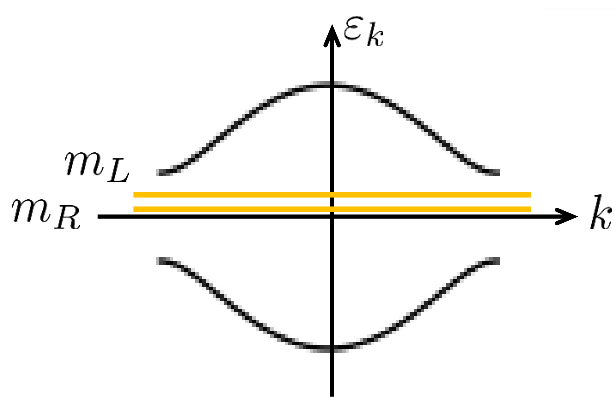

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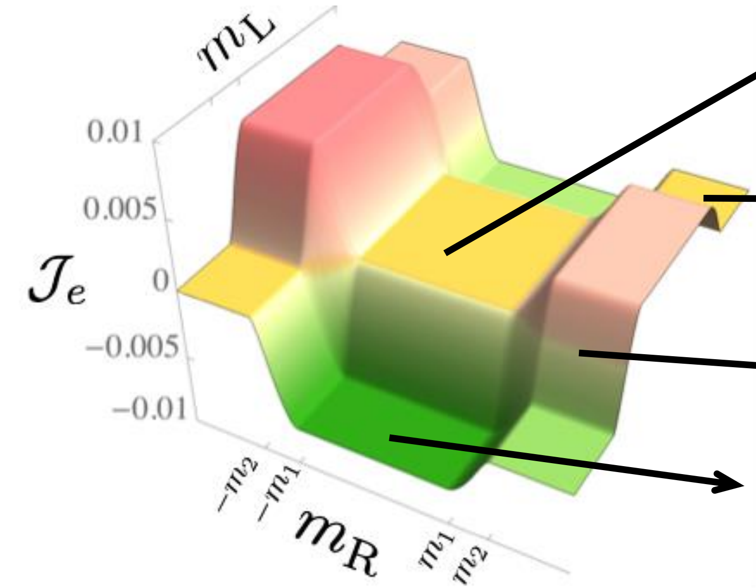

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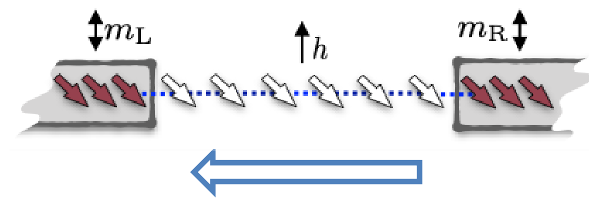

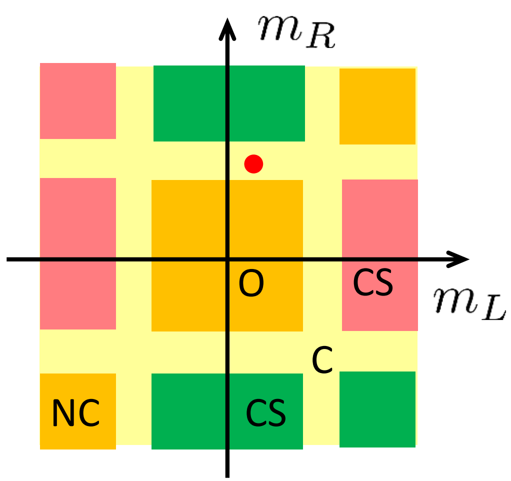

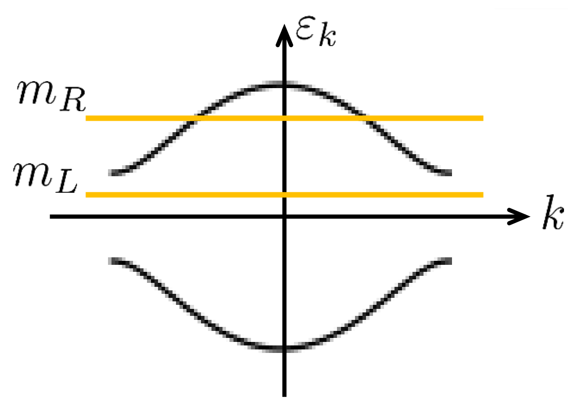

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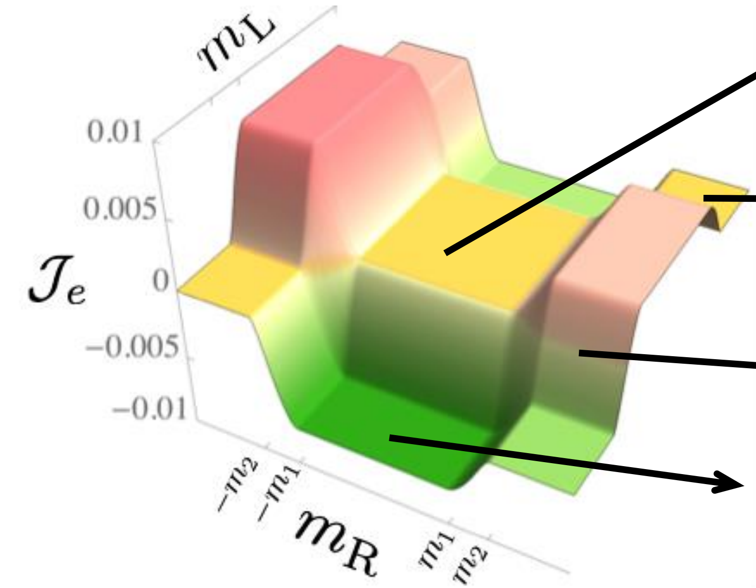

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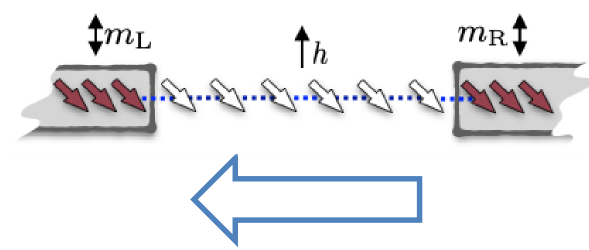

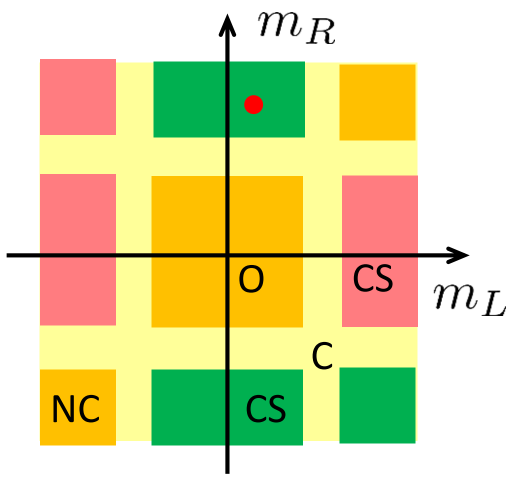

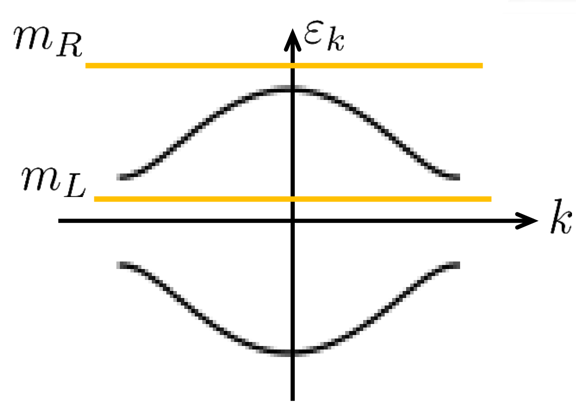

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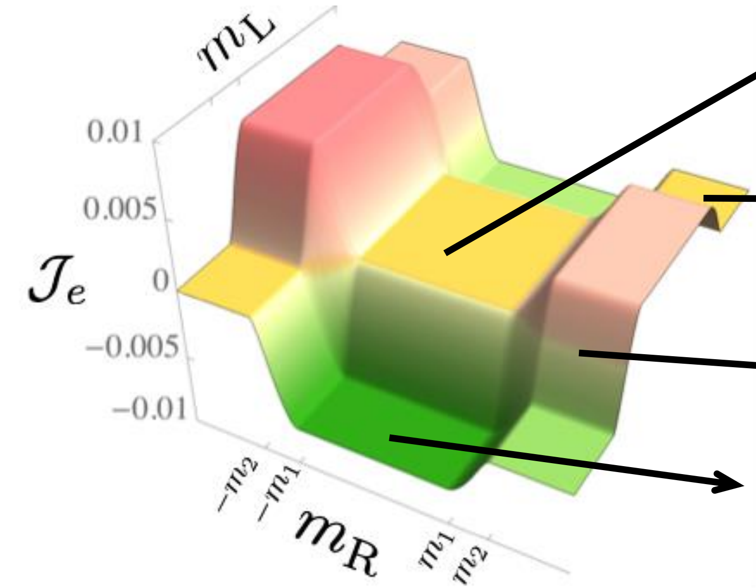

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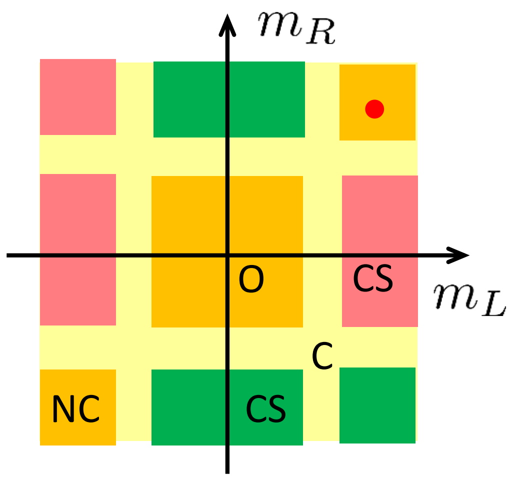

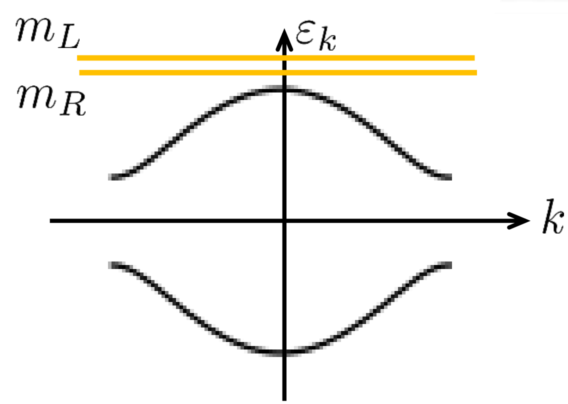

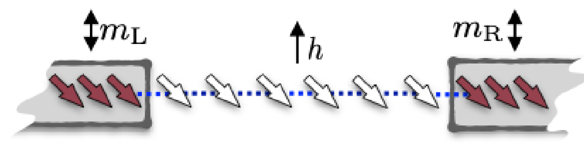

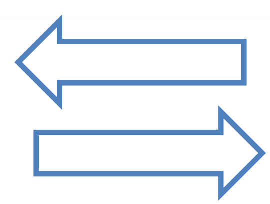

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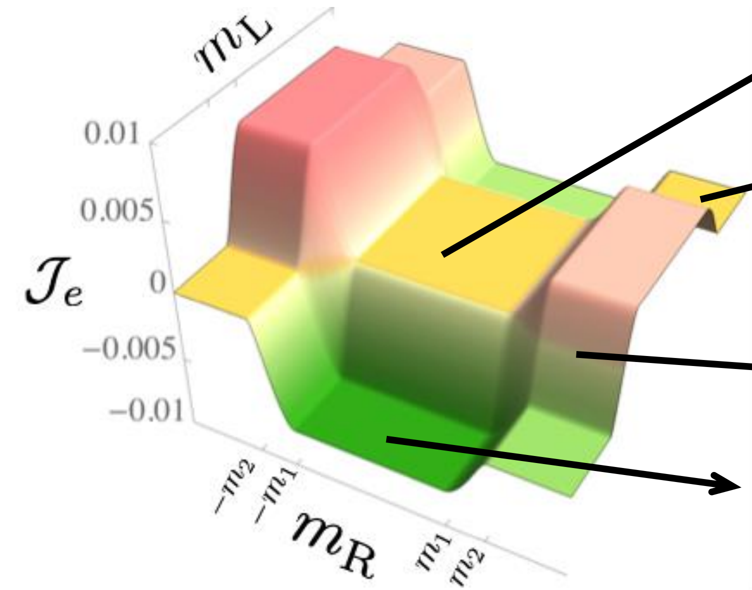
Occupation numbers



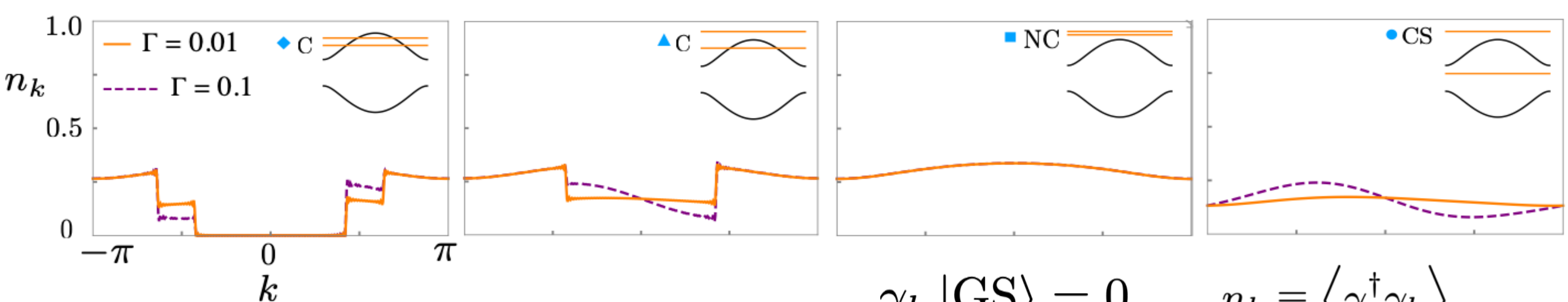
Ordered phase (O)

Disordered phase, non-conducting (NC)

Disordered, conducting (C)

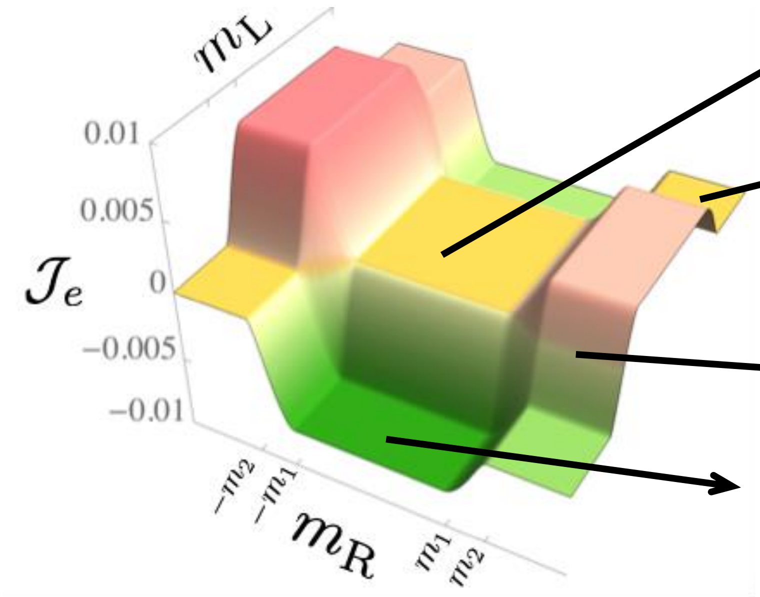
Conducting, current-saturated (CS)

Examples of band occupation (also from χ !)



$$\gamma_k \,|\mathrm{GS}\rangle = 0$$

$$n_k \equiv \left\langle \gamma_k^\dagger \gamma_k \right\rangle_S$$



What about magnetic order?

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Magnetic order

$$\mathbb{C}^{\alpha\beta}_{r,r'} = \langle \sigma^{\alpha}_{r}\sigma^{\beta}_{r'}\rangle - \langle \sigma^{\alpha}_{r}\rangle\langle \sigma^{\beta}_{r'}\rangle = \det\left[i(2\chi_{[r,r']} - 1)\right]^{\frac{1}{2}}$$

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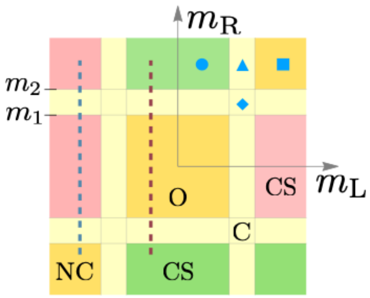
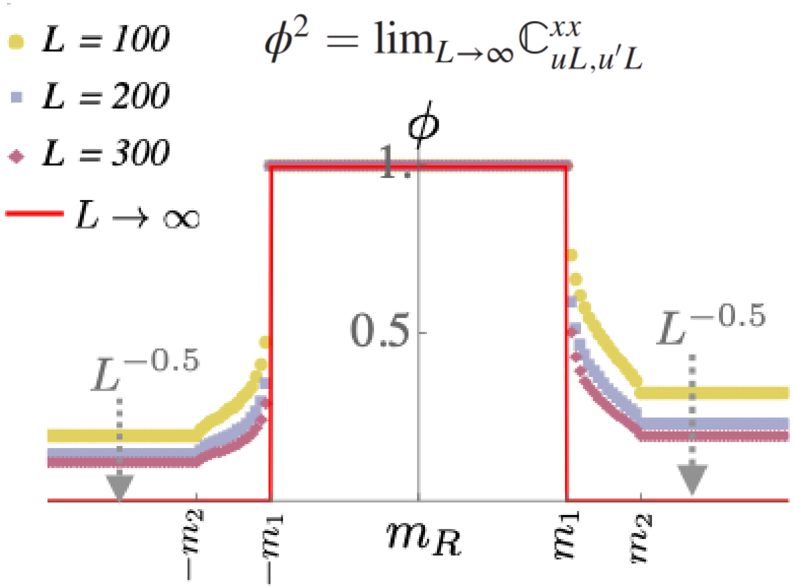
Magnetic order

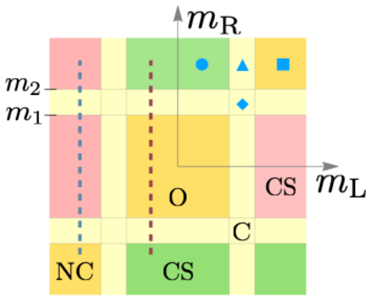


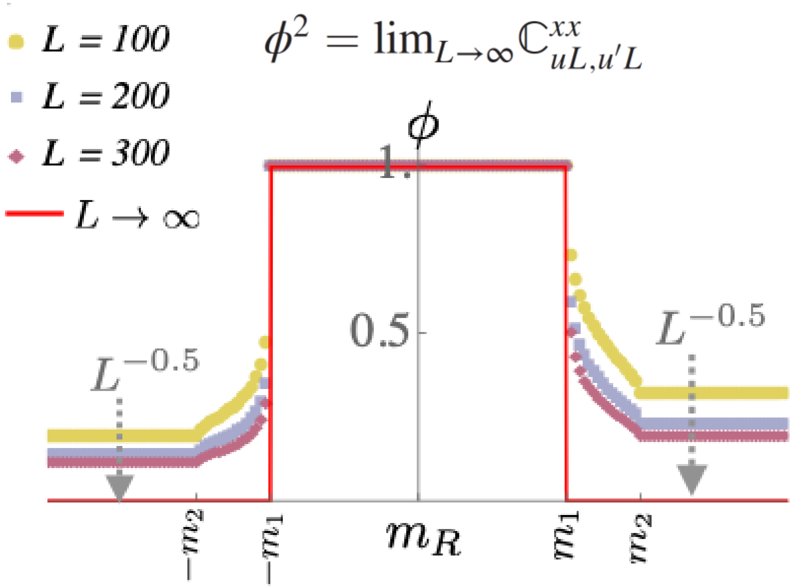


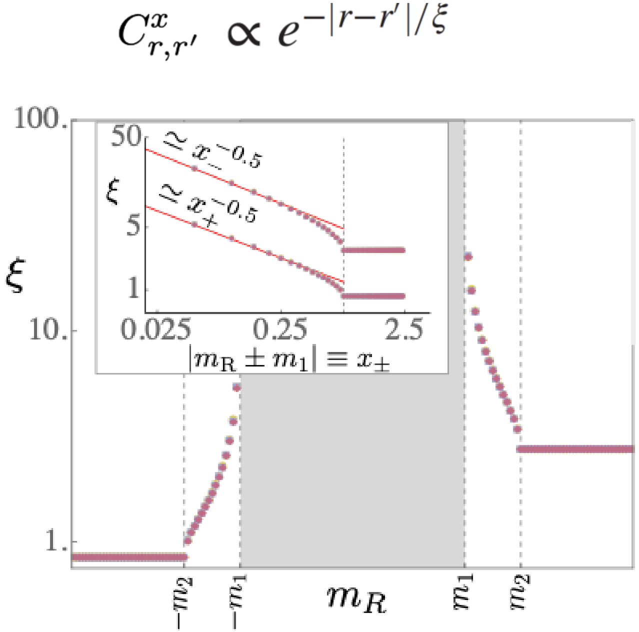
discontinuous
order parameter

CSRC

Magnetic order







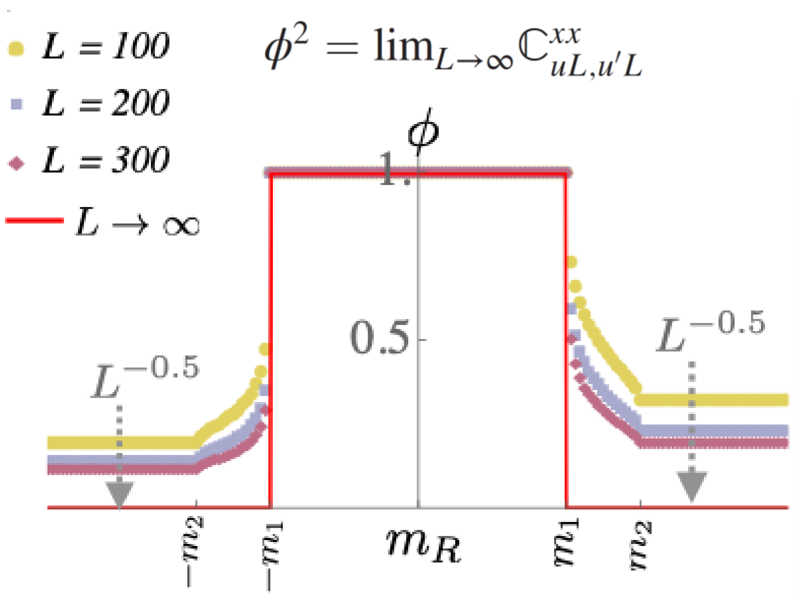
discontinuous
order parameter

diverging
correlation length

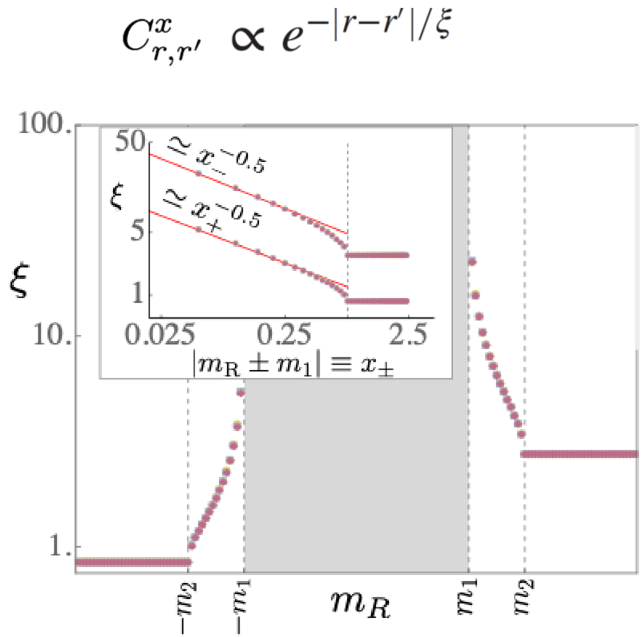
CSRC

Coexistence of defining features of 1st and 2nd order phase transitions

→ **mixed-order phase transition**



discontinuous
order parameter



diverging
correlation length

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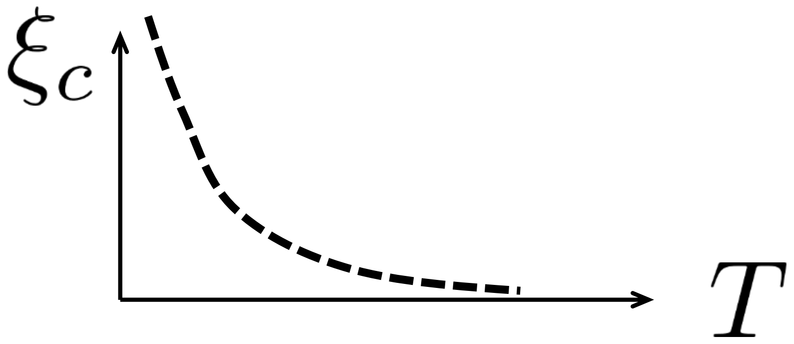
Intuition from equilibrium (T>0)

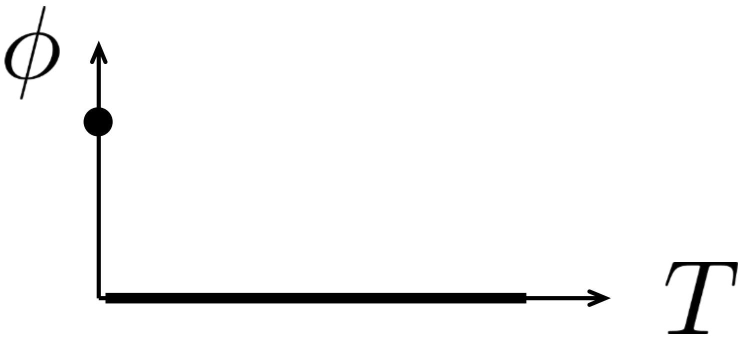
Ordered phase (h<J) at finite T

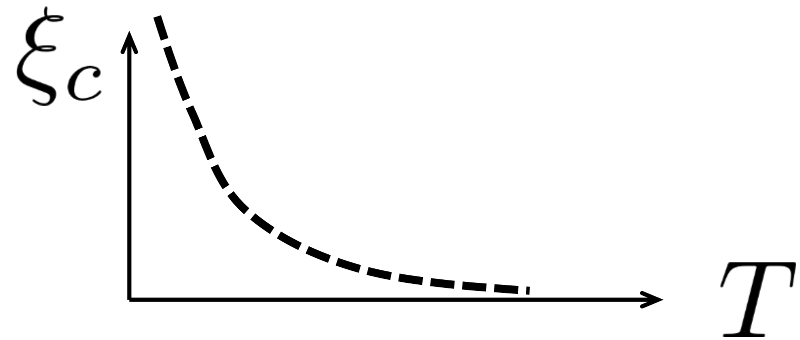


- Exponentially small density of excitation destroys order
- Correlation length is diverging at T=0

Intuition from equilibrium (finite T)

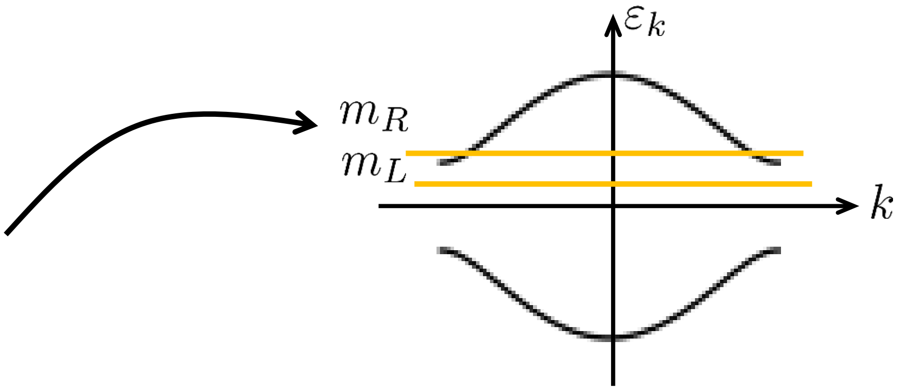
Ordered phase (h<J) at finite T





- Exponentially small density of excitation destroys order
- Correlation length is diverging at T=0

excitations from
the right contact

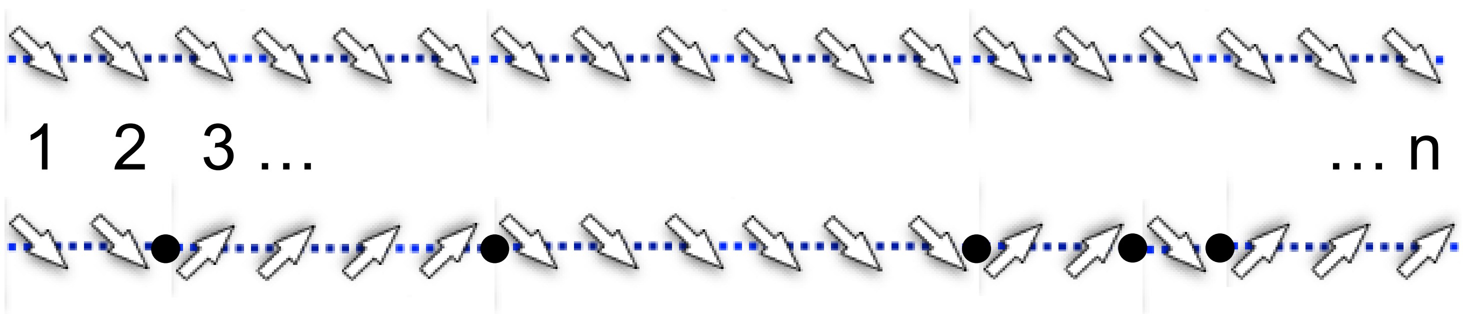


Density of excitations ρ

Intuition: at T<<0 excitations ~ domain walls

$n \ll \rho^{-1}$

$n \gg \rho^{-1}$



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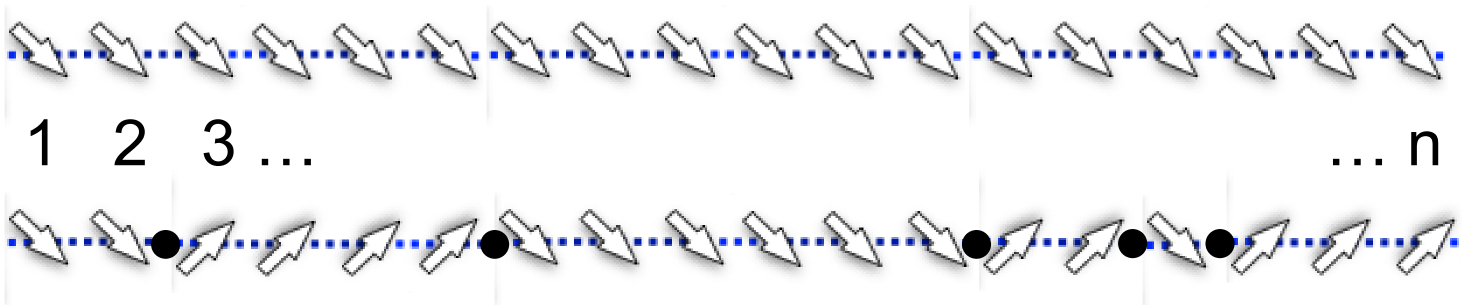
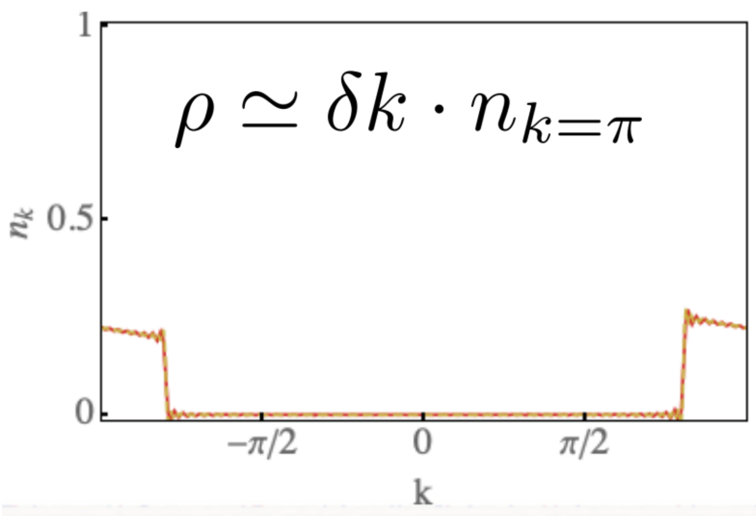


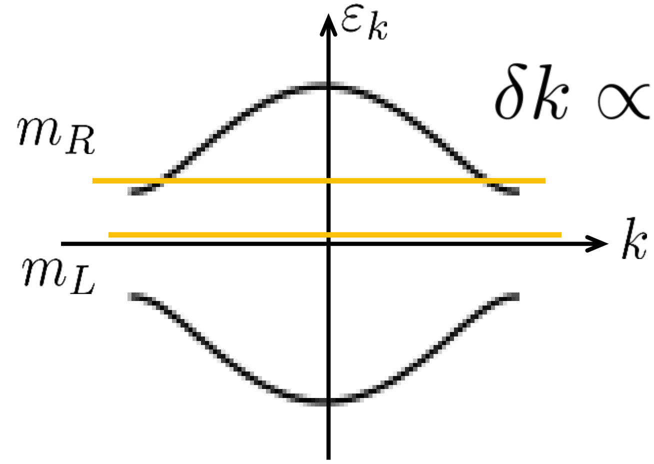
Total number of domain walls: $M = \rho L$

Probability of *j* domain walls in [1,*n*]: $p_j = (\frac{n}{L})^j (1 - \frac{n}{L})^{M-j}$

$$\xi_c \simeq \frac{1}{2\rho}$$

Application: critical exponent



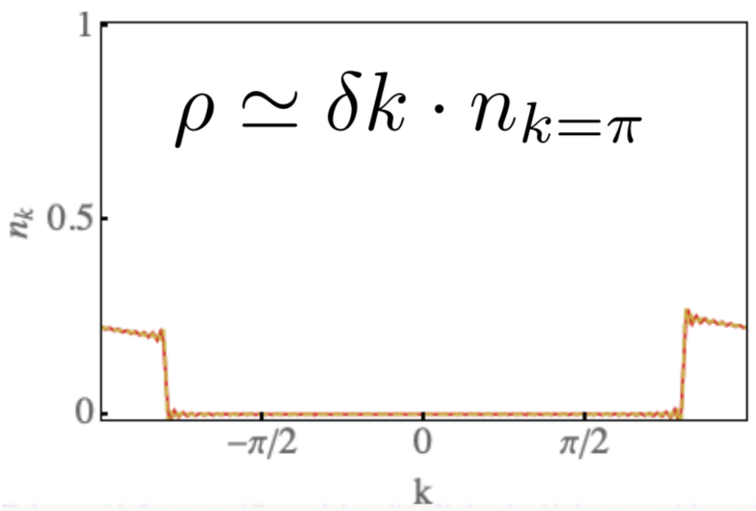


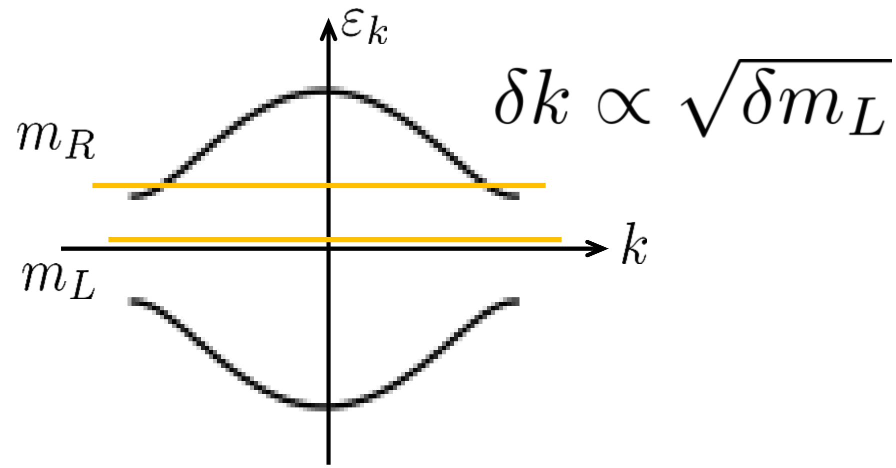
$$\delta k \propto \sqrt{\delta m_L}$$

$$\xi_c \simeq \frac{1}{2\rho} \propto \frac{1}{\sqrt{\delta m_L}}$$

critical exponent 1/2

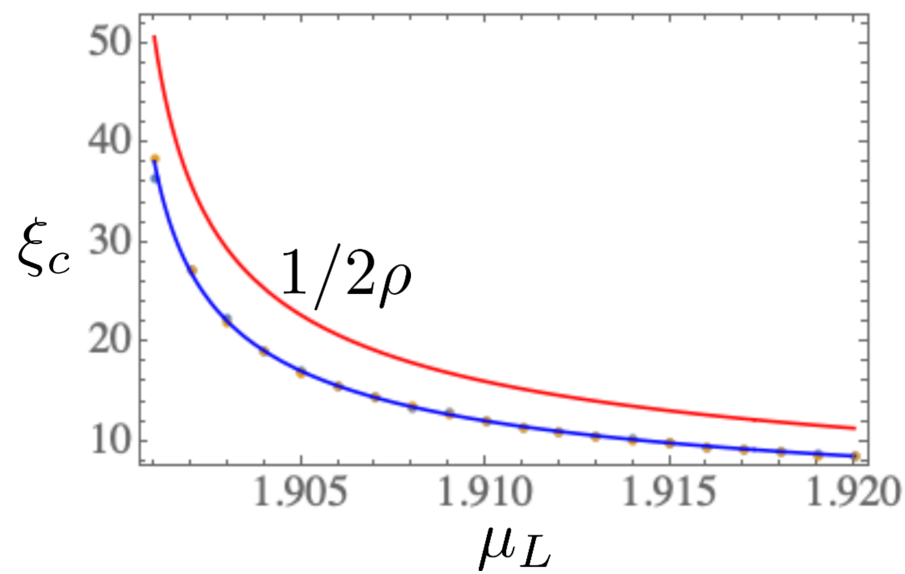
Application: critical exponent





$$\xi_c \simeq \frac{1}{2\rho} \propto \frac{1}{\sqrt{\delta m_L}}$$

critical exponent 1/2



Direct calculation

E. Lieb, I. Schultz and D. Mattis, Ann. of Phys. 16, 406 (1961)
S. Sachdev, Nucl. Phys. B 464, 576 (1996)

$$\langle \sigma_i^z \sigma_{i+n}^z \rangle = \begin{vmatrix} D_0 & D_{-1} & . & . & D_{-n+1} \\ D_1 & D_0 & . & . & . \\ . & . & . & . & . \\ . & . & . & D_0 & D_{-1} \\ D_{n-1} & . & . & D_1 & D_0 \end{vmatrix}$$

Toeplitz determinant

$$D_n = \int \frac{dk}{2\pi} e^{-ink} \, \phi(k)$$

Asymptotic dependence:

$$\langle \sigma_i^z \sigma_{i+n}^z \rangle \propto \exp\left[\frac{n}{2\pi} \int_0^{2\pi} \log \phi\left(\theta\right) d\theta \right]$$

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Direct calculation

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Toeplitz determinant

$$D_n = \int \frac{dk}{2\pi} e^{-ink} \sqrt{\frac{1 - ge^{ik}}{1 - ge^{-ik}}} (1 - n_k - n_{-k})$$

$$\xi_c^{-1} = -\int_0^{2\pi} \frac{dk}{2\pi} \log\left[1 - n_k - n_{-k}\right]$$

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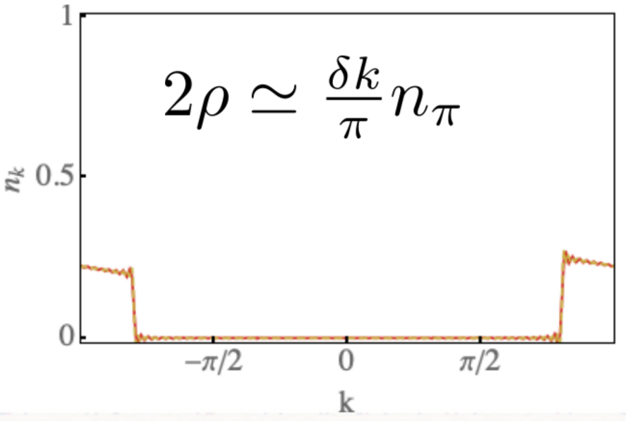
Scaling behavior

$$\xi_c^{-1} = -\int_0^{2\pi} \frac{dk}{2\pi} \log\left[1 - n_k - n_{-k}\right]$$

$$\simeq -\frac{\delta k}{2\pi} \log[1 - 2n_\pi]$$



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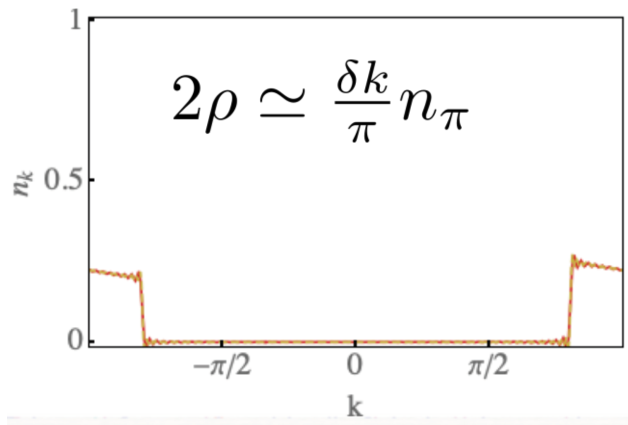

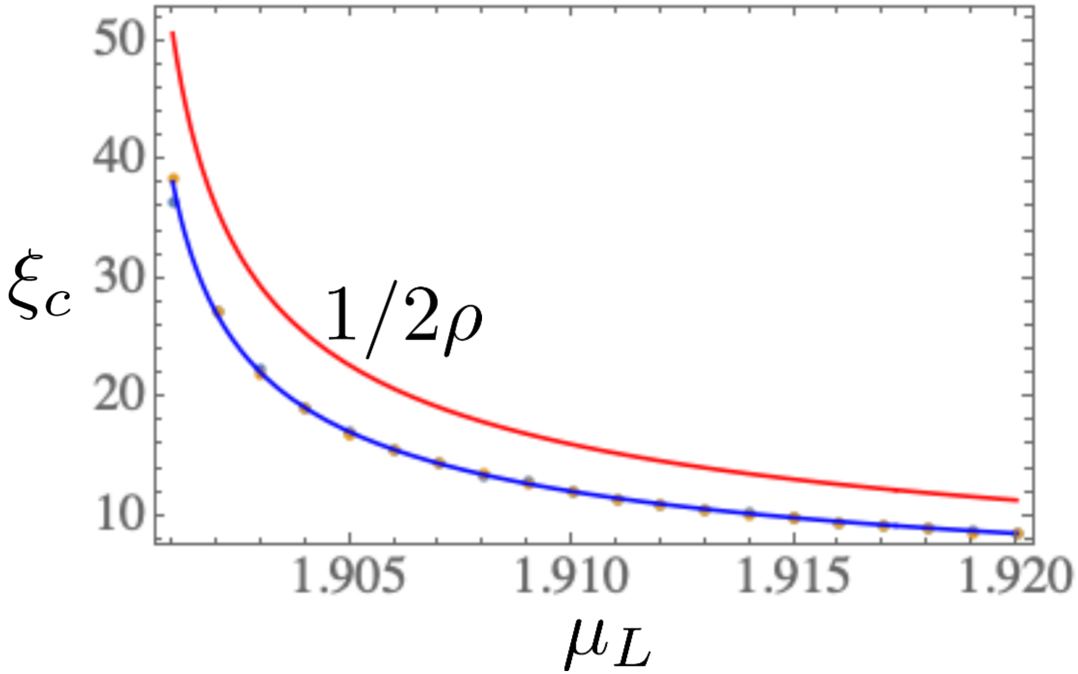

1. Introduction
    - Non-equilibrium phase transitions
    - Model and motivations

2. Non-eq transverse-field Ising model
    - Energy current & phase diagram
    - Correlation functions & mixed-order phase transition
    - Entropy scaling

3. Conclusion and Outlook

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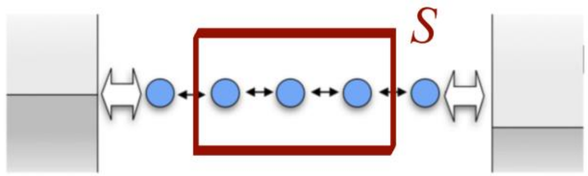
Scaling law of entropy



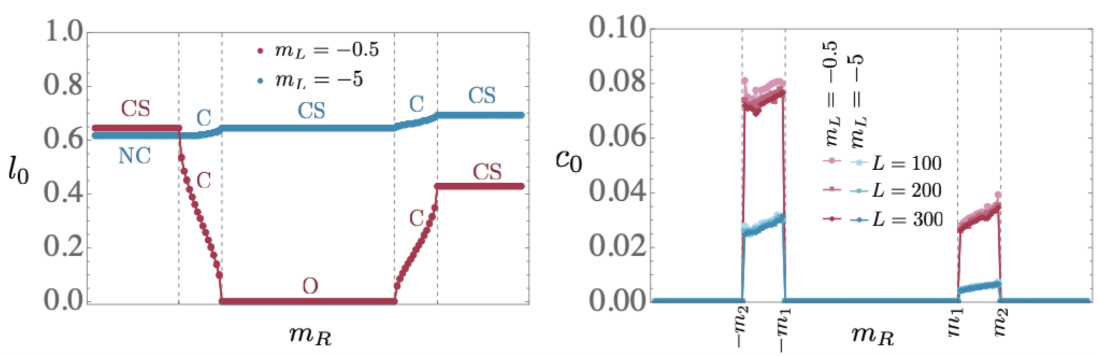
$$E(\rho_S) = -\mathrm{Tr}\left[\rho_S \ln(\rho_S)\right]$$

Scaling law:

$$E \simeq l_0 S + c_0 \ln(S)$$

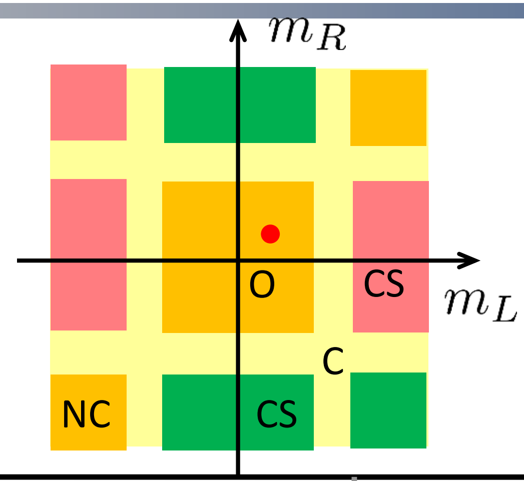
"normal" behaviors:

gapped systems	metals	finite temperature
$l_0 = 0$	$l_0 = 0$	$l_0 \neq 0$
$c_0 = 0$	$c_0 = 1/3$ (1D)	$c_0 = 0$



Scaling law of entropy

Ordered phase



Scaling law:

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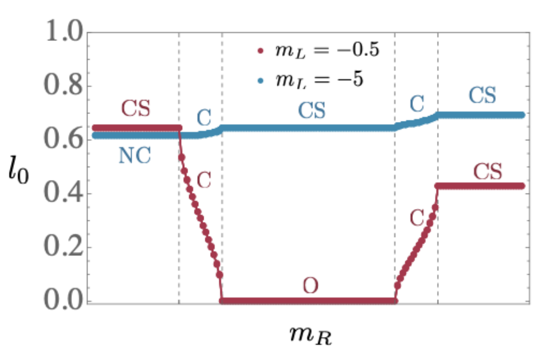
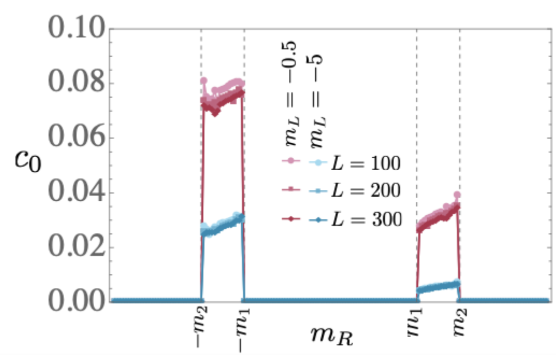
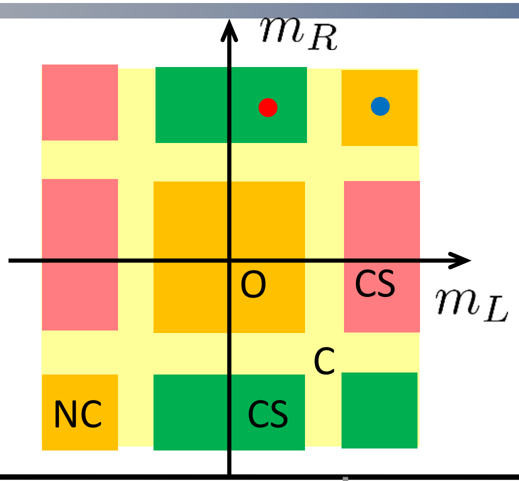




Scaling law of entropy

Current-saturated

Non-conducting



Scaling law:

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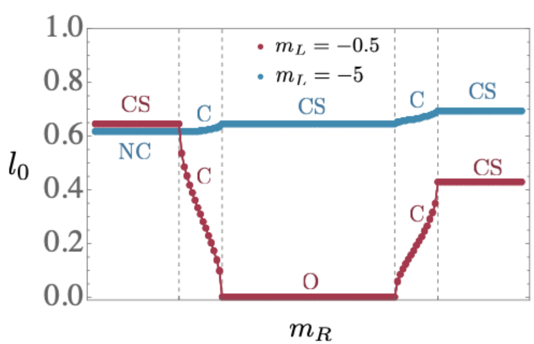

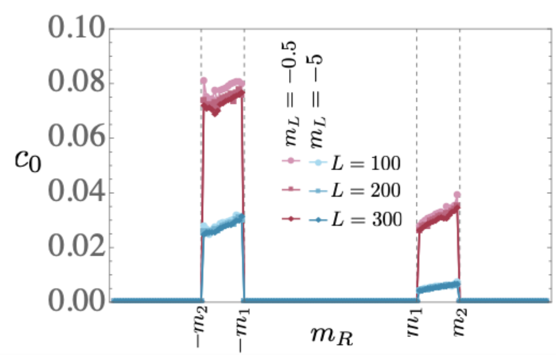

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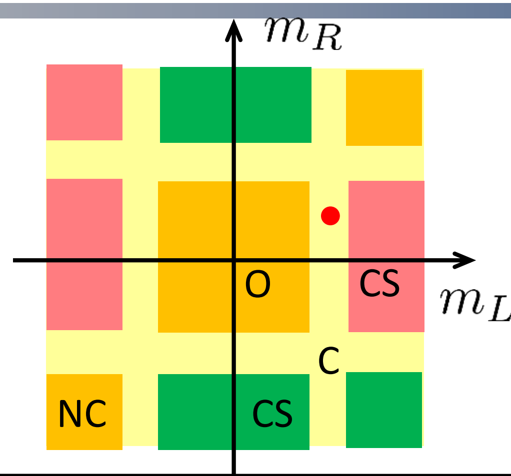
Scaling law of entropy

Conducting



Scaling law:

$$E \simeq l_0 S + c_0 \ln(S)$$

gapped systems	metals	finite temperature
$l_0 = 0$	$l_0 = 0$	$l_0 \neq 0$
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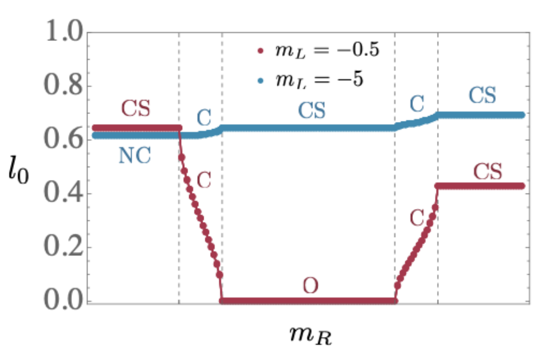

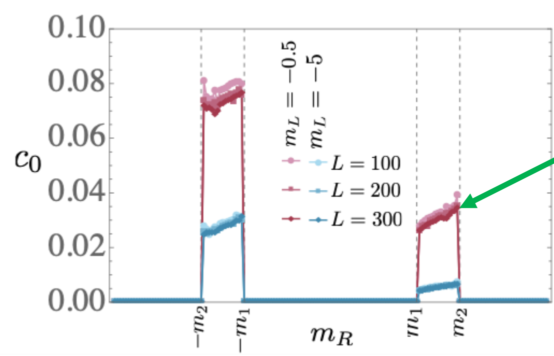

$c_0 \neq 0 \neq 1/3$

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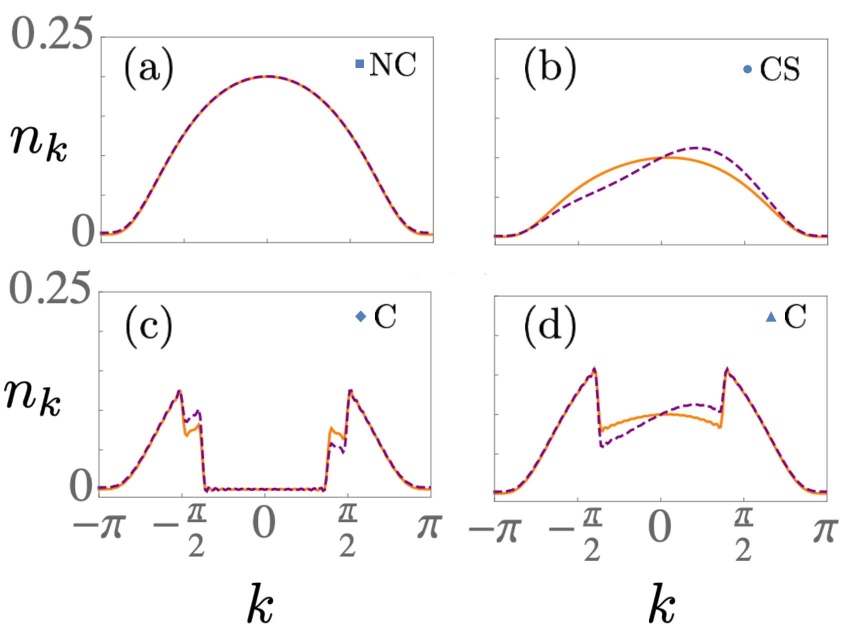
Connection to occupation numbers



Scaling law:

$$E \simeq l_0 S + c_0 \ln(S)$$

- $n_k \neq 0,1$ implies non-trivial scaling law
- Discontinuities on n_k give rise to logarithmc corrections

O	C	CS, NC
$l_0 = 0$ $c_0 = 0$	$l_0 \neq 0$ $c_0 \neq 0 \neq 1/3$	$l_0 \neq 0$ $c_0 = 0$

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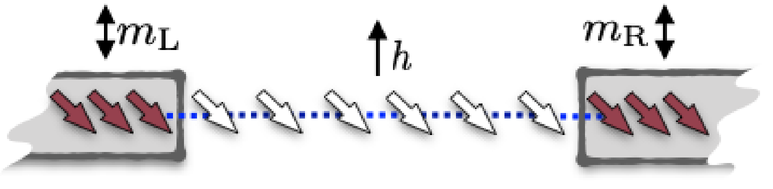
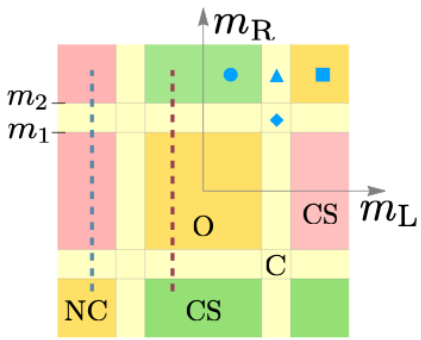
Summary

- Rich non-equilibrium phase diagram
- Mixed-order QPT, under nonequilibrium conditions
- Violation to the equilibrium area law in the conducting phases





- Extension to anisotropic XY interactions? (sensitive phases)
- General theory? Field-theoretical description?

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T. O. Puel, SC, S. Kirchner, and P. Ribeiro, PRL 122, 235701 (2019)