

Quantum Simulations and Quantum Devices

N-phonon bundle emission via the anti-Stokes process



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Collaborators



HUST

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Qian Bin



UW

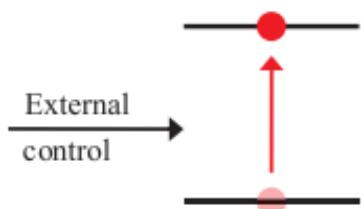
Prof. Fabrice P. Laussy

Outline

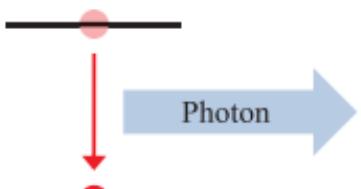
- 1) Background-bundle emission
- 2) Anti-Stokes resonance induced Super Rabi oscillation
- 3) N-phonon bundle emission with high purity
- 4) Conclusions

Single photon emission

Step 1: Excitation

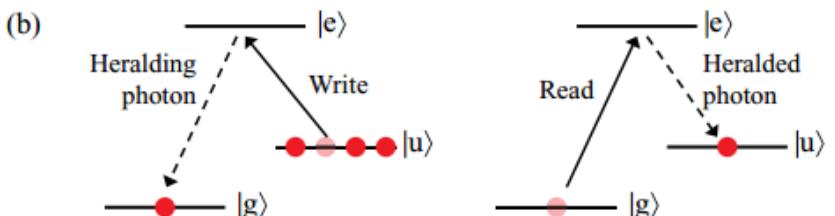
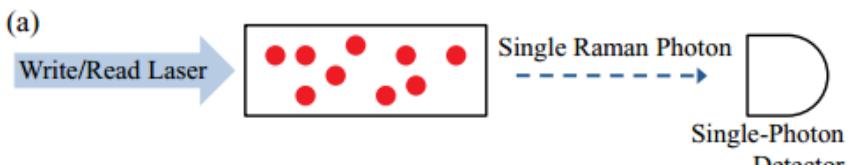


Step 2: Emission



Single emitter system

Fock state $|1\rangle$ $\text{Var} = 0$ $g^2(0) = 0$

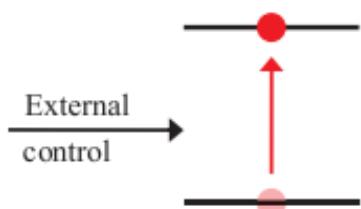


Ensemble-based emitter scheme

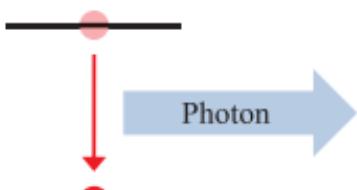
M. D. Eisaman et al., Rev. Sci. Instrum. 82, 071101 (2011)

Single photon emission

Step 1: Excitation

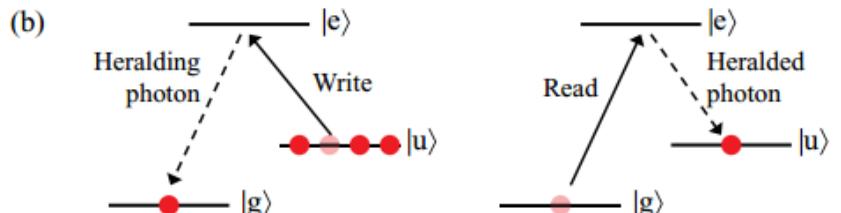


Step 2: Emission



Single emitter system

$$\text{Fock state } |1\rangle \quad \text{Var} = 0 \quad g^2(0) = 0$$

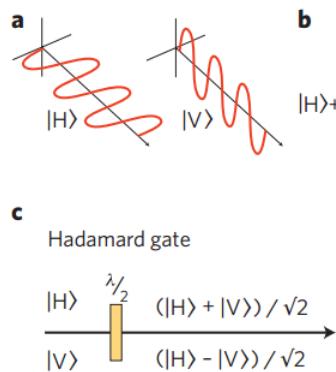


Ensemble-based emitter scheme

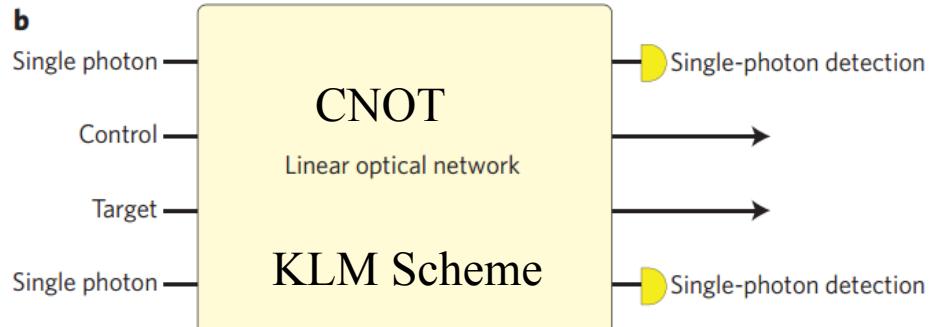
M. D. Eisaman et al., Rev. Sci. Instrum. 82, 071101 (2011)

Applications in quantum information science

Quantum Communication



Quantum Computation

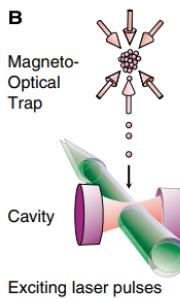


J. L. O'Brien, A. Furusawa and J. Vučković, Nature Photonics 3 , 687–695 (2009)

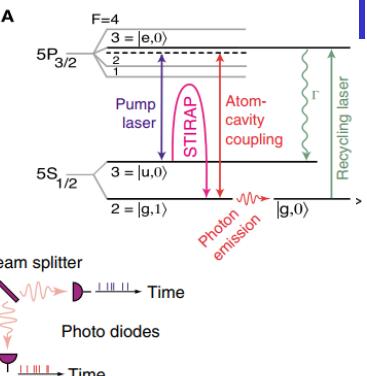
Purcell effect enhancing emission rate

M. D. Eisaman et al., Rev. Sci. Instrum. 82, 071101 (2011)

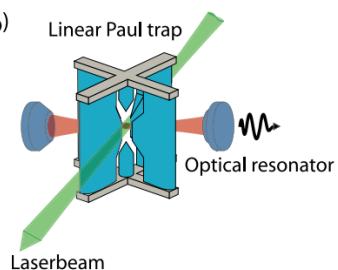
Atom



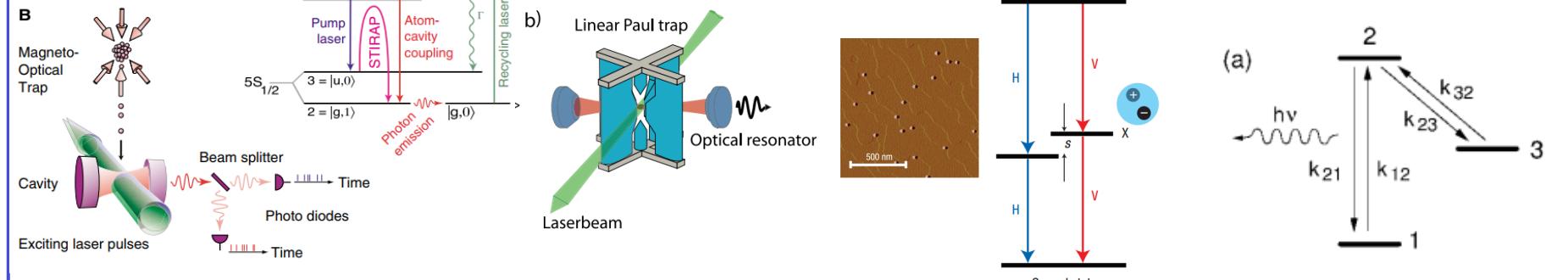
A



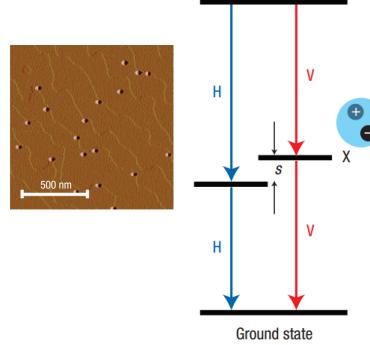
Ions



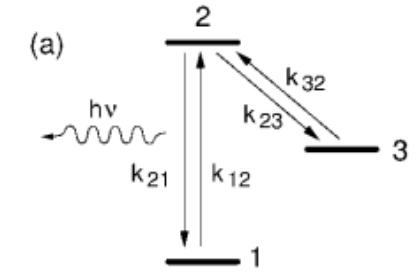
b)



QD



NV



A. Kuhn et al. PRL
89, 067901 (2002)

C. Maurer et al.,
NJP 6, 94 (2004).

A. J. Shields, Nature
Photon. 1, 215 (2007)

C. Kurtsiefer et al.,
PRL 85, 290 (2000)

Purcell effect enhancing emission rate

$$F_p = \frac{3}{4\pi^2} \left(\frac{\lambda_{free}}{n} \right)^3 \left(\frac{Q}{V} \right)$$

λ_{free} : Wavelength Q : Quality factor
 n : Refractive index V : Mode volume

Density of final states

Cavity: $\rho_c = \frac{1}{\Delta\nu V}$

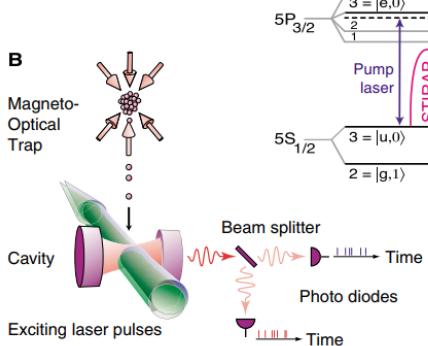
Free space: $\rho_f = \frac{8\pi n^3 \nu^2}{c^3}$

Fermi's golden rule

$$\rho_c / \rho_f = \frac{c^3}{8\pi n^3 \nu^2} \frac{Q}{\nu V} = \frac{1}{8\pi} \left(\frac{\lambda_{free}}{n} \right)^3 \left(\frac{Q}{V} \right)$$

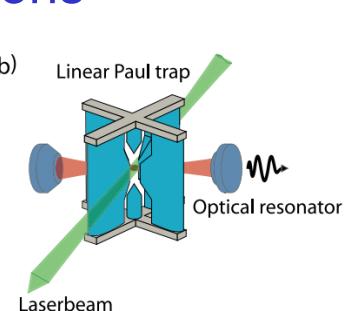
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Atom



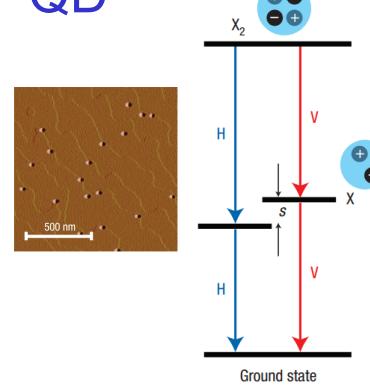
A. Kuhn et al. PRL 89, 067901 (2002)

Ions



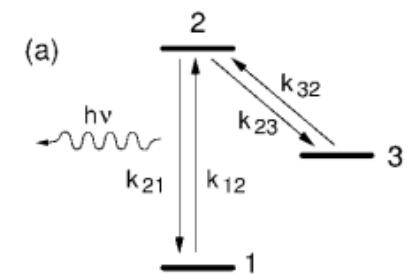
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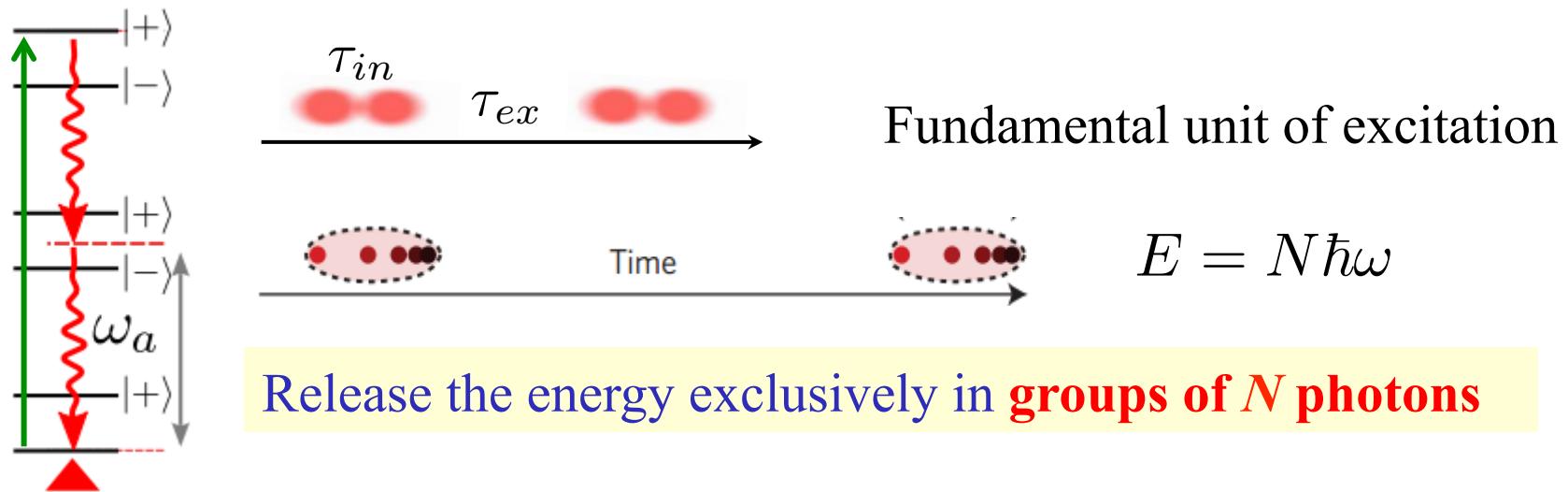
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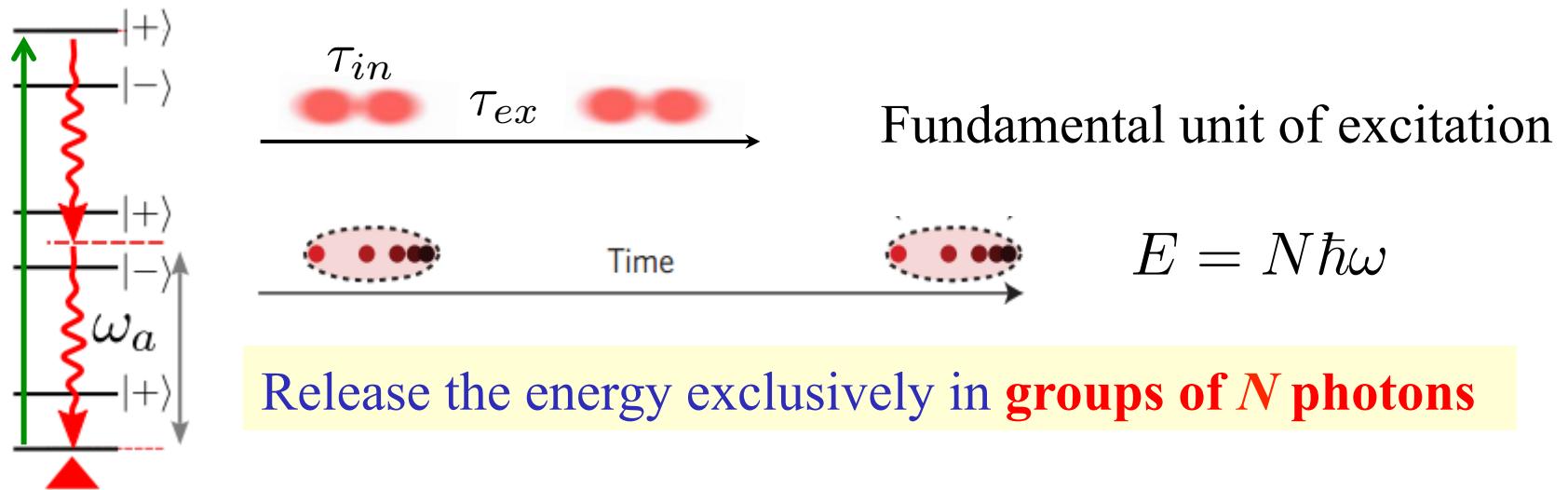


C. Kurtsiefer et al., PRL 85, 290 (2000)

From single-photon emission to N -photon bundle emission



From single-photon emission to N -photon bundle emission

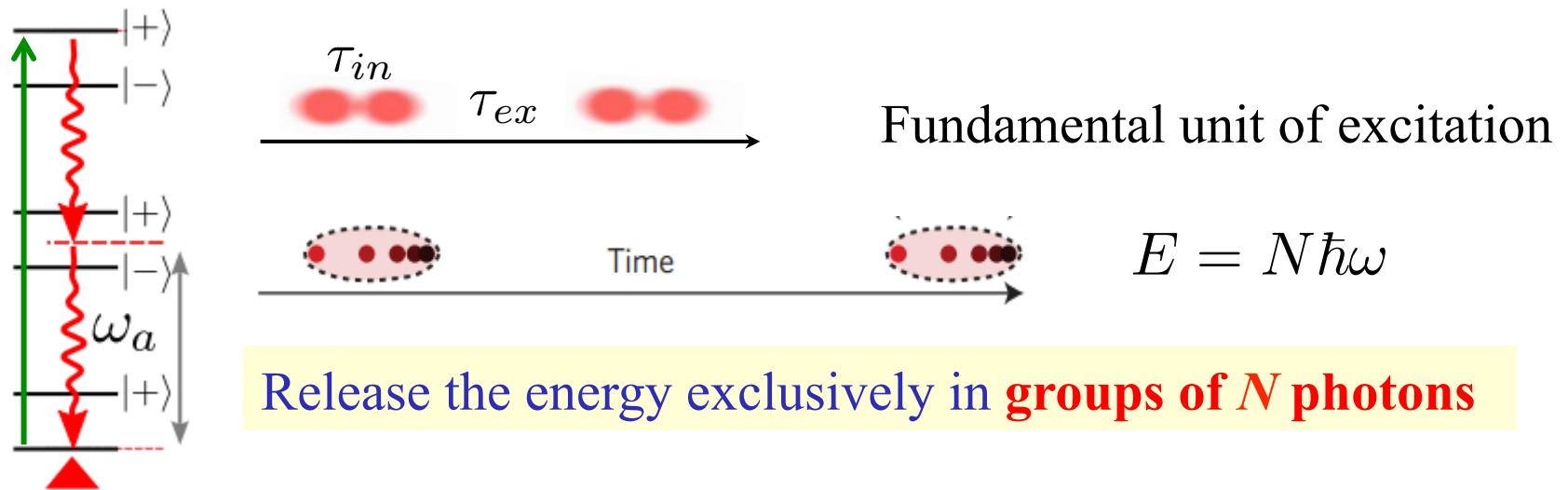


Expanding Glauber's theory by replacing the **photon** with **bundle** of N photons

$$g_N^{(n)}(t_1, \dots, t_n) = \frac{\langle \mathcal{T}_- \{ \prod_{i=1}^n [a^\dagger]^N(t_i) \} \mathcal{T}_+ \{ \prod_{i=1}^n [a]^N(t_i) \} \rangle}{\prod_{i=1}^n \langle [a^\dagger]^N [a]^N \rangle(t_i)} \quad \tau_{ex} \gg \tau_{in}$$

$$g_N^{(2)}(\tau) = \frac{\langle [a^\dagger]^N(0) [a^\dagger]^N(\tau) [a]^N(\tau) [a]^N(0) \rangle}{\langle ([a^\dagger]^N [a]^N)(0) \rangle \langle ([a^\dagger]^N [a]^N)(\tau) \rangle}$$

From single-photon emission to N -photon bundle emission



Expanding Glauber's theory by replacing the **photon** with **bundle** of N photons

$$g_N^{(n)}(t_1, \dots, t_n) = \frac{\langle \mathcal{T}_- \{ \prod_{i=1}^n [a^\dagger]^N(t_i) \} \mathcal{T}_+ \{ \prod_{i=1}^n a^N(t_i) \} \rangle}{\prod_{i=1}^n \langle a^\dagger a^N \rangle(t_i)} \quad \tau_{ex} \gg \tau_{in}$$

Bunching

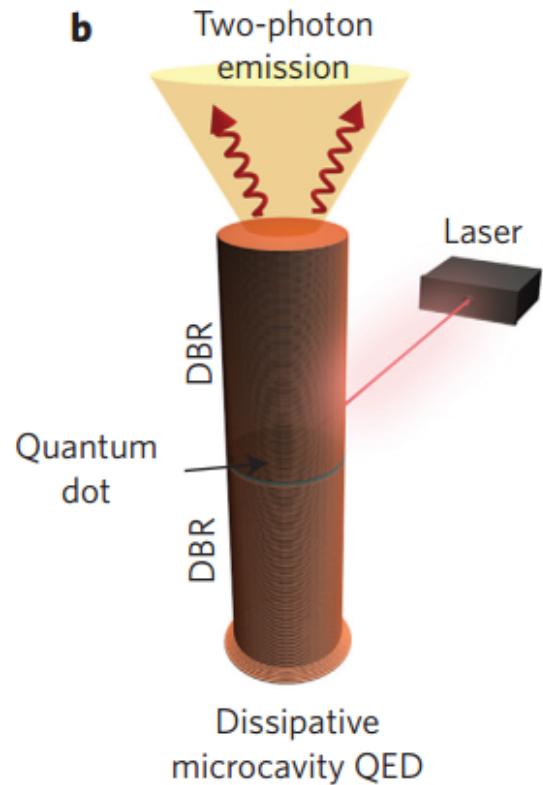
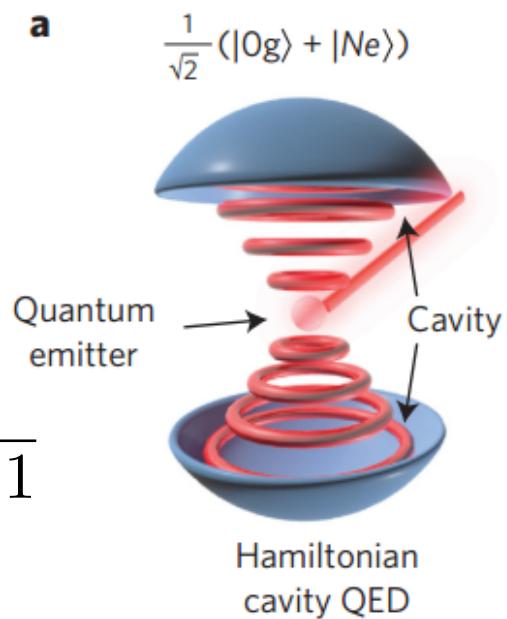
$$g_N^{(2)}(\tau) = \frac{\langle a^\dagger(0) a^\dagger(\tau) a^N(\tau) a^N(0) \rangle}{\langle (a^\dagger a^N)(0) \rangle \langle (a^\dagger a^N)(\tau) \rangle} \quad \xrightarrow{N=1} \quad g^{(2)}(\tau) = \frac{\langle a^\dagger(0) a^\dagger(\tau) a(\tau) a(0) \rangle}{\langle (a^\dagger a)(0) \rangle \langle (a^\dagger a)(\tau) \rangle}$$

N -photon bundle emission-coherent

Cavity QED system

Dispersive regime

$$|\omega_a - \omega_\sigma| \gg g\sqrt{N + 1}$$



Hamiltonian

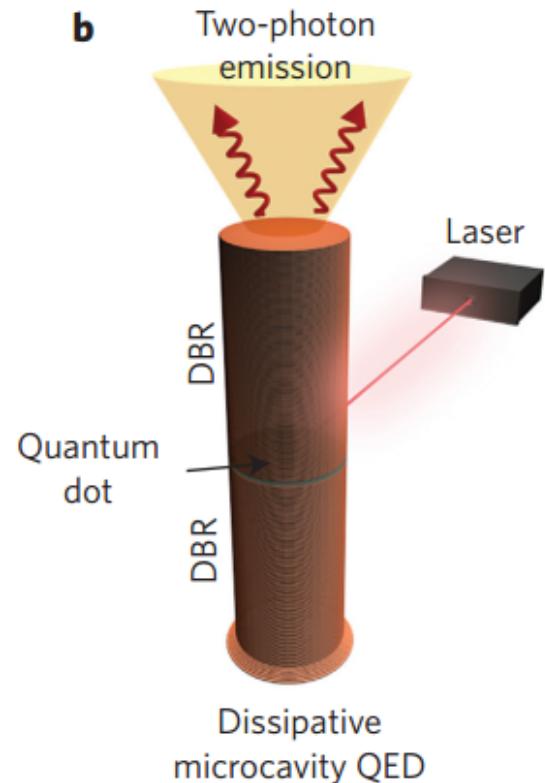
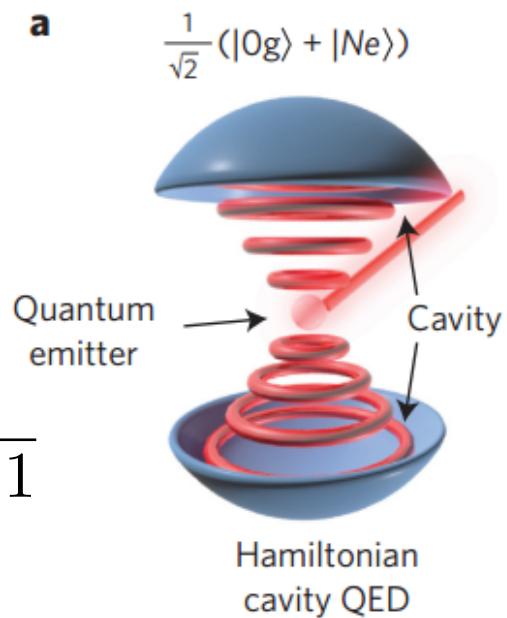
$$H = \omega_a a^\dagger a + \omega_\sigma \sigma^\dagger \sigma + g(a^\dagger \sigma + \sigma^\dagger a) + \Omega(\sigma^\dagger e^{-i\omega_L} + \sigma e^{i\omega_L})$$

N -photon bundle emission-coherent

Cavity QED system

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$$H = \omega_a a^\dagger a + \omega_\sigma \sigma^\dagger \sigma + g(a^\dagger \sigma + \sigma^\dagger a) + \Omega(\sigma^\dagger e^{-i\omega_L} + \sigma e^{i\omega_L})$$

Super Rabi oscillation

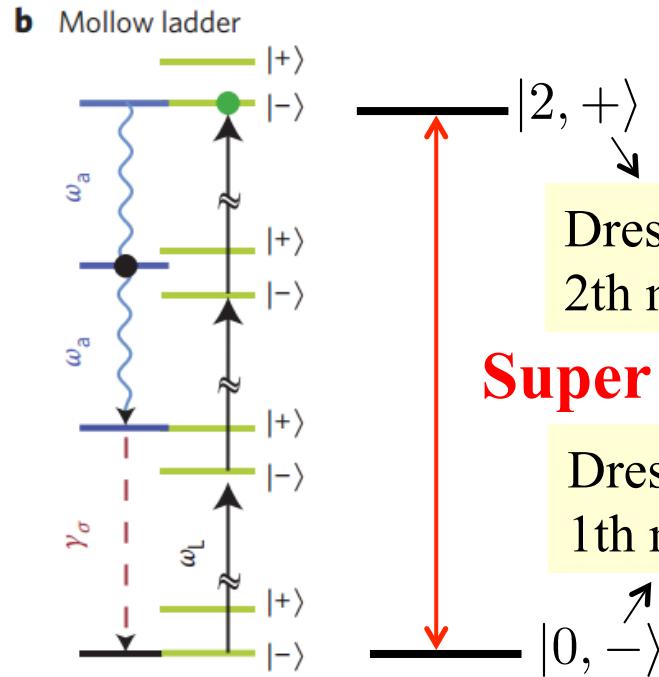
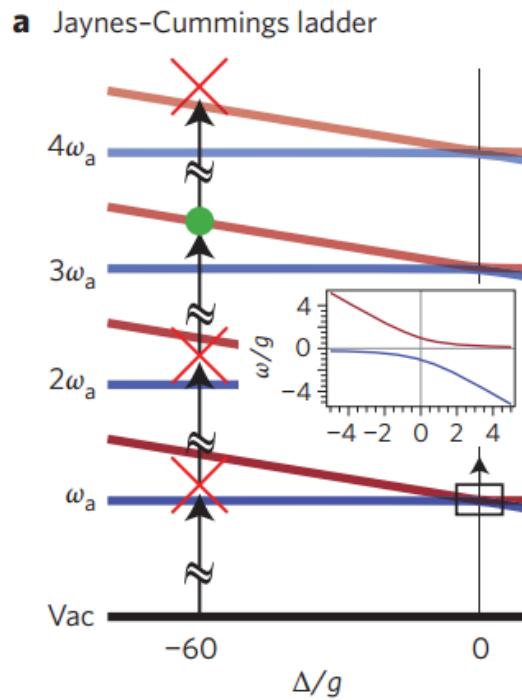


Dissipation Window

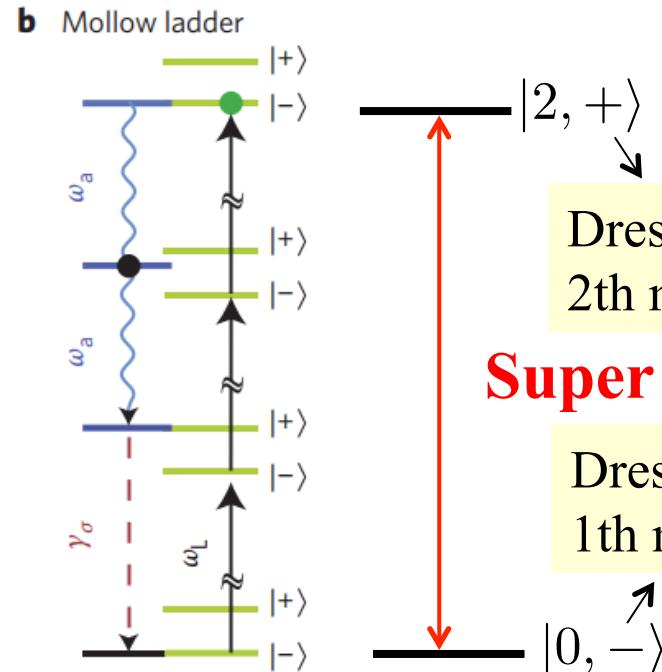
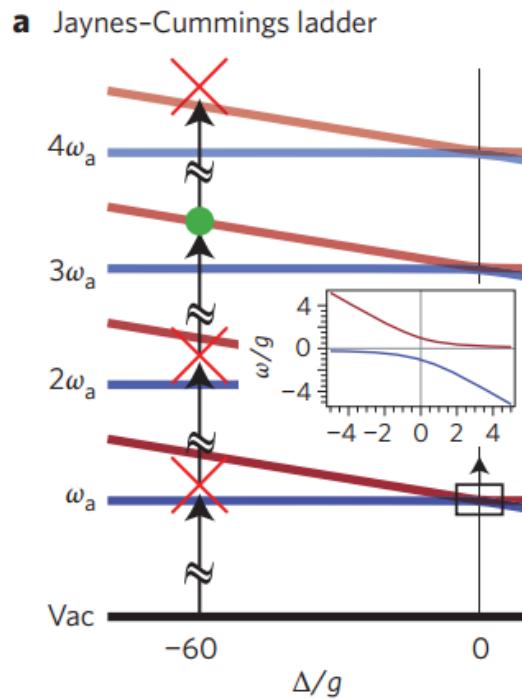
$$|0, g\rangle \leftrightarrow |N, e\rangle \text{ or } |0, -\rangle \leftrightarrow |N, +\rangle$$

$$\mathcal{L}[a] + \mathcal{L}[\sigma]$$

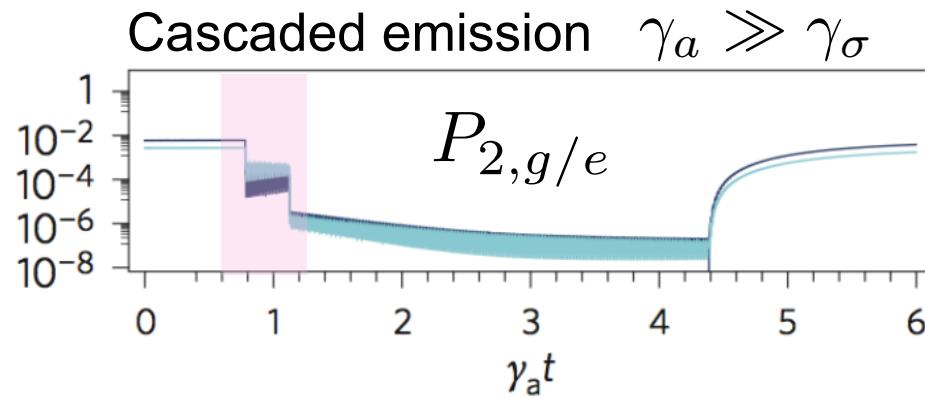
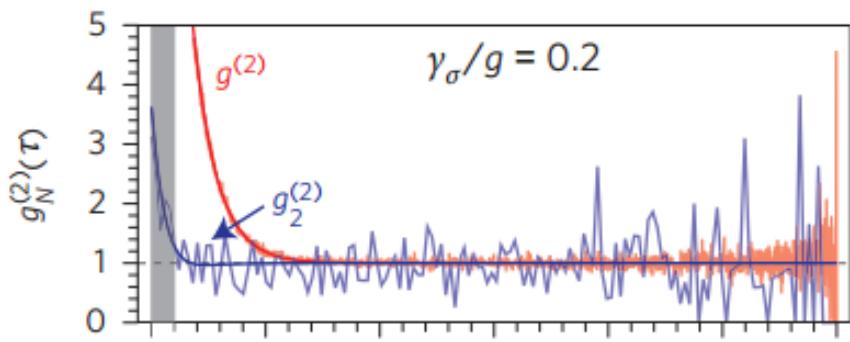
N -photon bundle emission-coherent



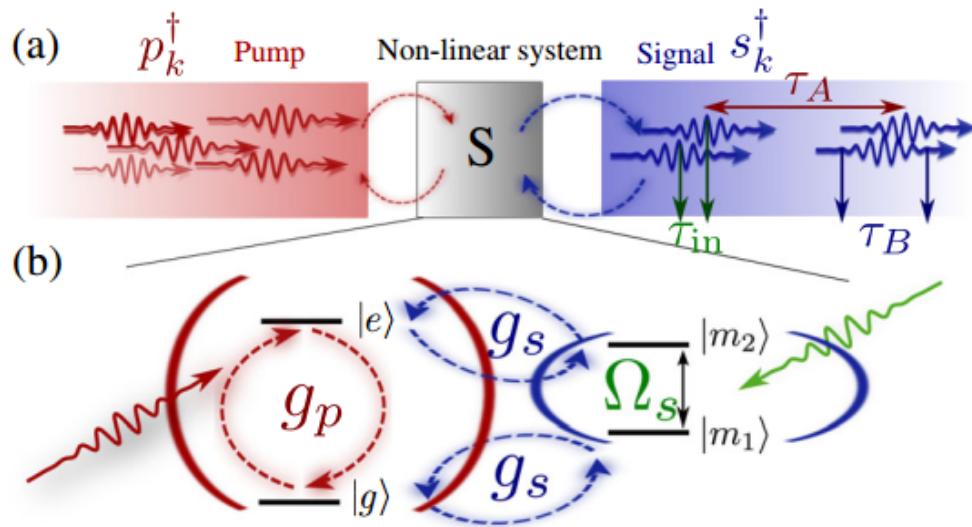
N -photon bundle emission-coherent



Super Rabi oscillation



N -photon bundle emission-antibunching in bad-cavity limit



N -photon source

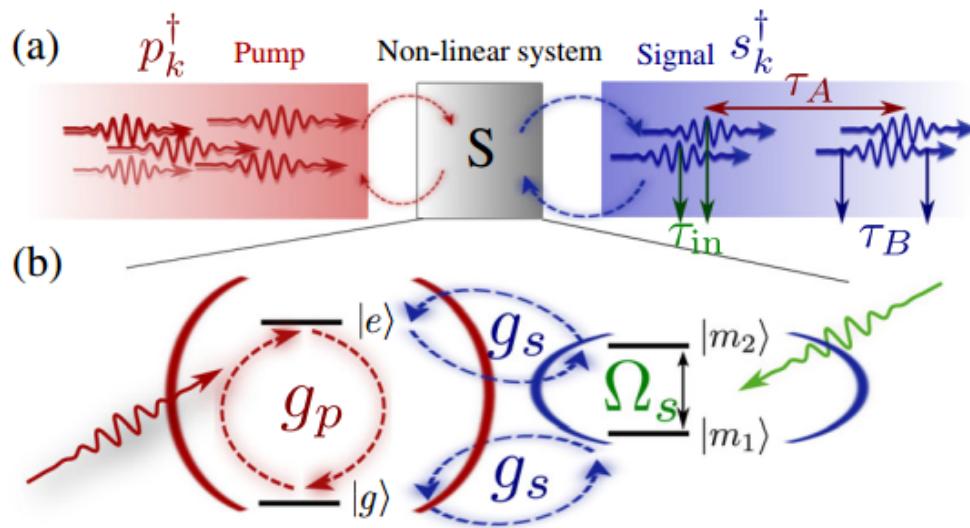
$$g^{(2)}(0) \gg 1$$

$$g_N^{(2)}(0) < g_N^{(2)}(\tau)$$

$$g_N^{(2)}(0) \rightarrow 0$$

$$H = g_p a_p^\dagger |g\rangle\langle e| + \Omega_s |m_2\rangle\langle m_1| + g_s a_s^\dagger (|m_2\rangle\langle e| + |g\rangle\langle m_1|) + H.c.$$

N -photon bundle emission-antibunching in bad-cavity limit

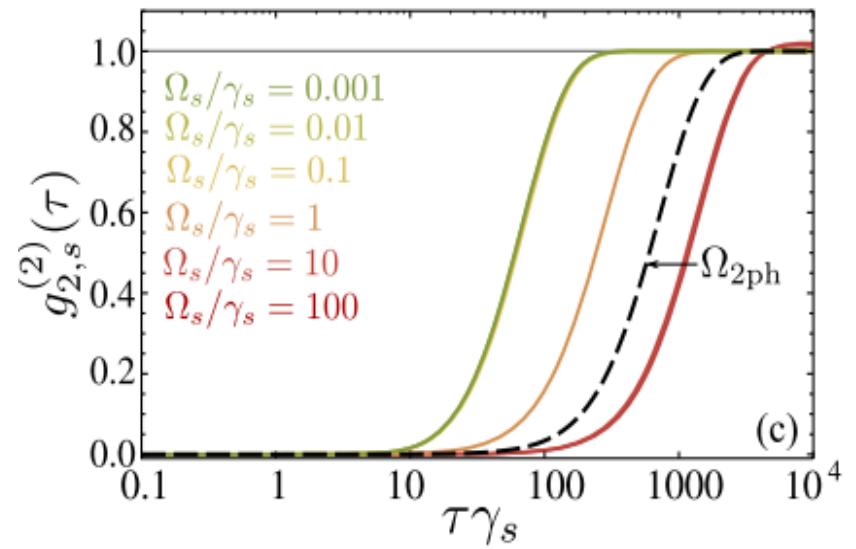
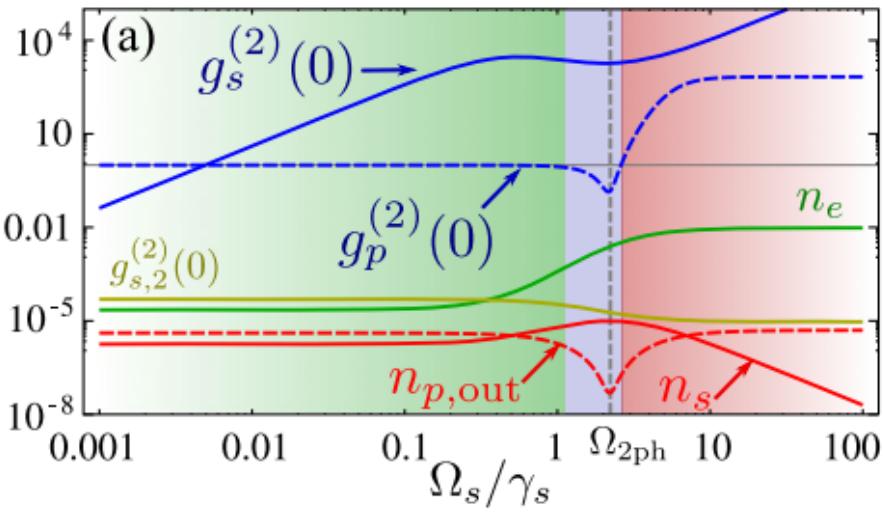


N-photon source

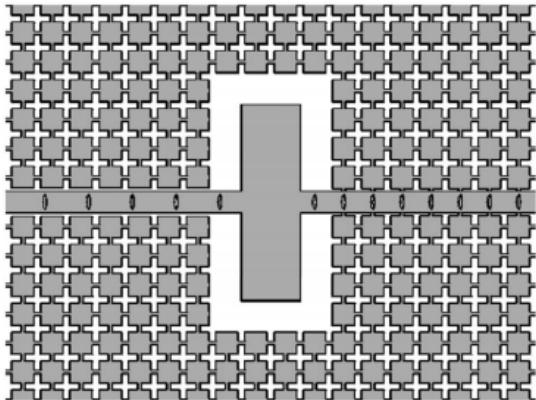
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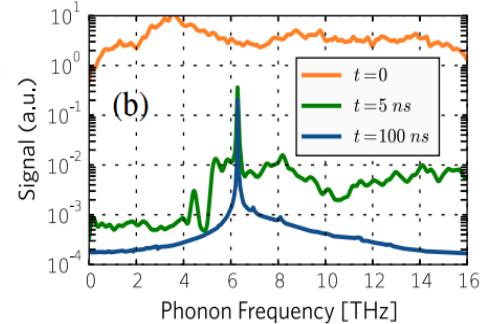
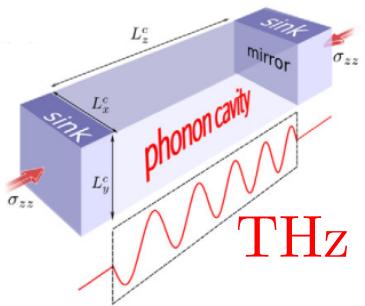


From photon to phonon



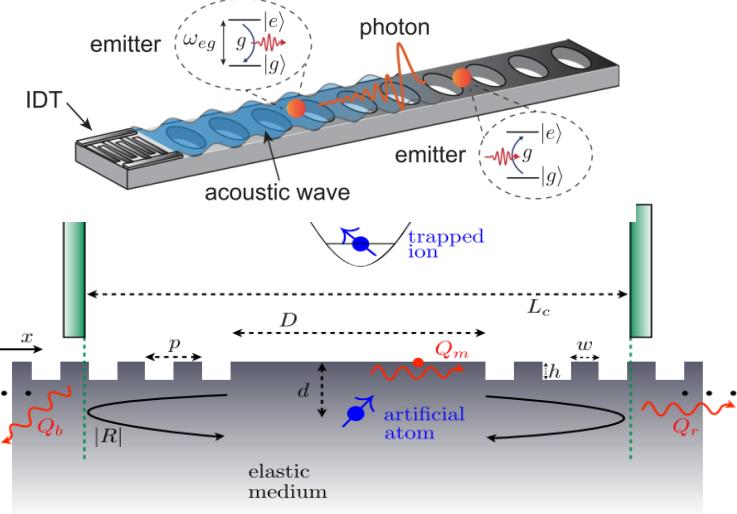
On-Chip Communication

C. Kuzyk and H. L. Wang, *Phys. Rev. X* 8, 041027 (2018)



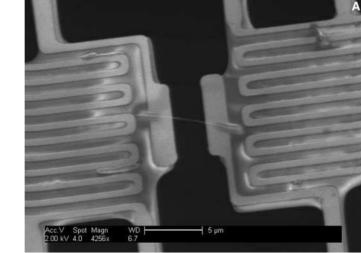
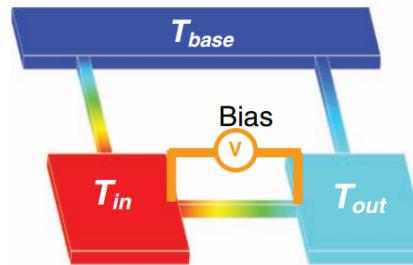
Sensing and imaging

H. Han et al., *Phys. Rev. Lett.* 114, 145501 (2015); A. J. Kent et al., *Phys. Rev. Lett.* 96, 215504 (2006);
R. Xie et al., *Adv. Funct. Mater.* 21, 1602 (2011); C. W. Chang et al., *Science* 314, 1121 (2006)



Phonon-photon Hybrid networks

M. J. A. Schuetz et al., *Phys. Rev. X* 5, 031031 (2015);
G. Calajó et al., *Phys. Rev. A* 99, 053852 (2019)

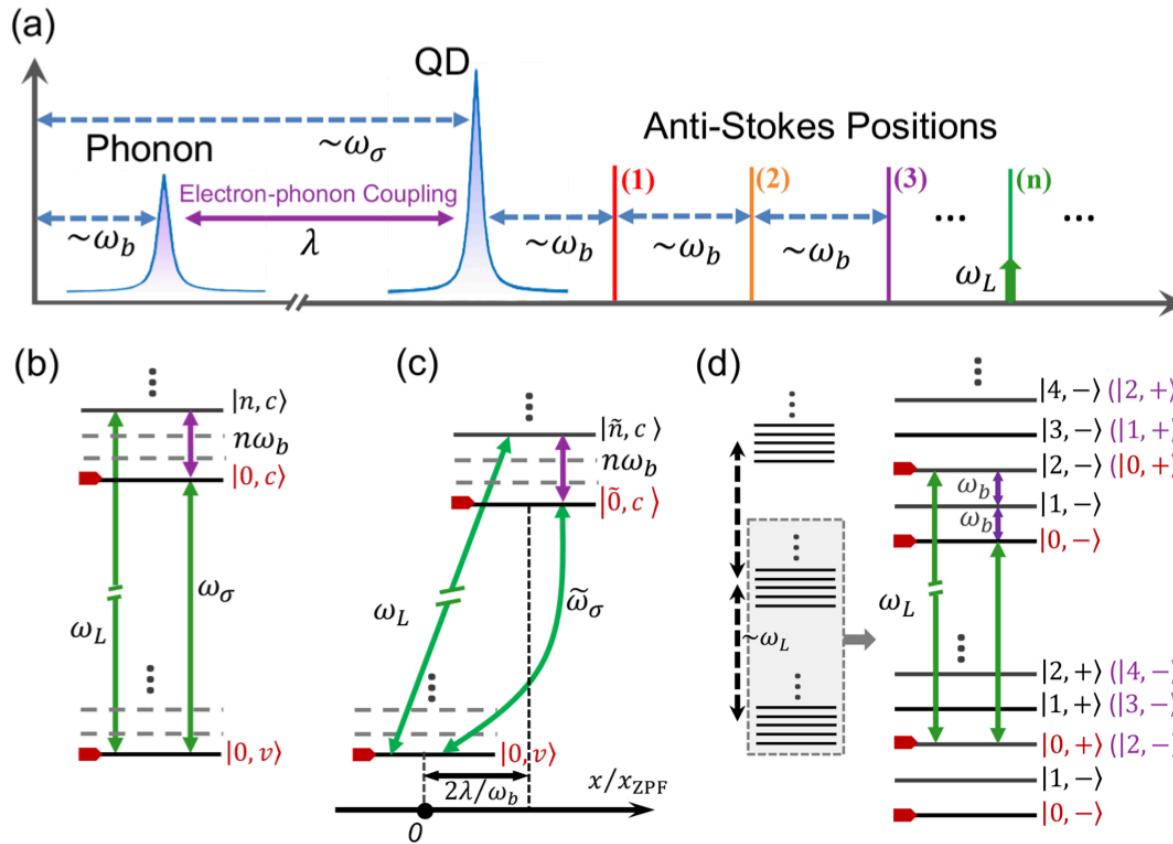


Nanophononic devices

Outline

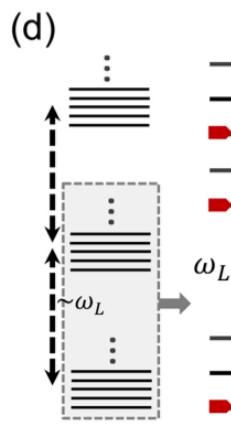
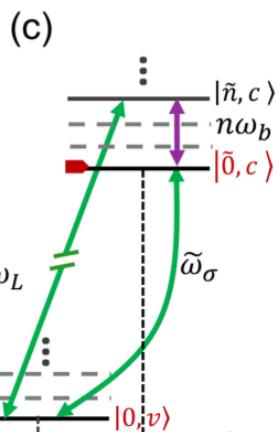
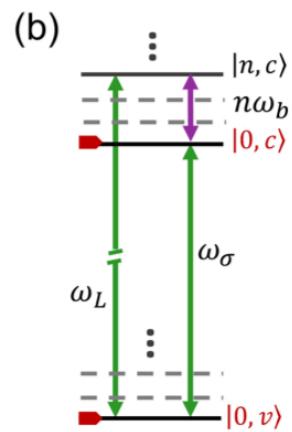
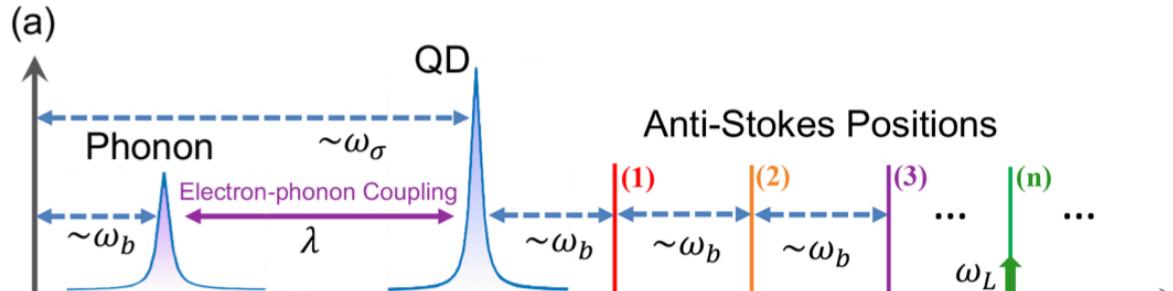
- 1) Background-bundle emission
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N -phonon Bundle Emission via the Anti-Stokes Process



$$H = \omega_b b^\dagger b + \omega_\sigma \sigma^+ \sigma + \lambda \sigma^+ \sigma (b^\dagger + b) + \Omega(e^{i\omega_L t} \sigma + h.c.)$$

N -phonon Bundle Emission via the Anti-Stokes Process



(b) Weak-driving and coupling

$$\Omega, \lambda \ll \omega_b, \omega_\sigma$$

(c) Strong coupling

$$\lambda \sim \omega_b$$

(d) Strong-driving (Mollow)

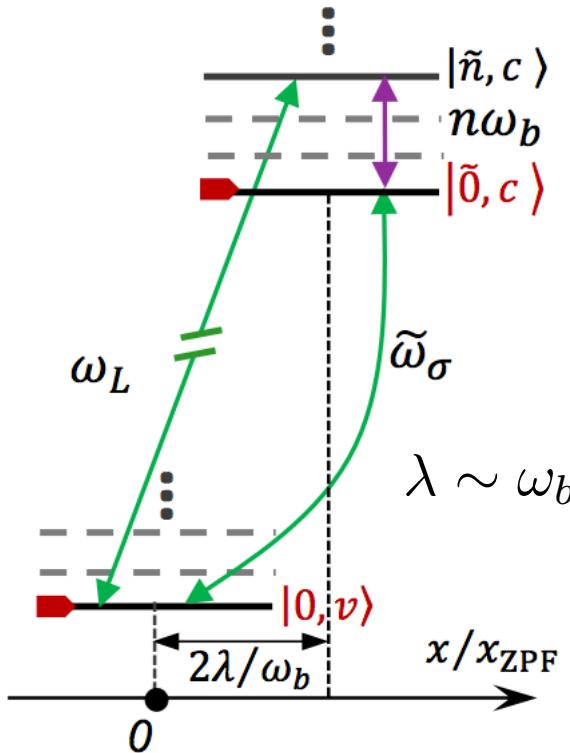
$$\Omega \sim \omega_b$$

$$H = \omega_b b^\dagger b + \omega_\sigma \sigma^+ \sigma + \lambda \sigma^+ \sigma (b^\dagger + b) + \Omega (e^{i\omega_L t} \sigma + h.c.)$$

Anti-Stokes resonance \rightarrow Super Rabi oscillation $|0, \downarrow\rangle \leftrightarrow |n, \uparrow\rangle$

N -phonon associated Anti-Stokes resonances

$$(c) \omega_L = \omega_\sigma - \lambda^2/\omega_b + n\omega_b$$



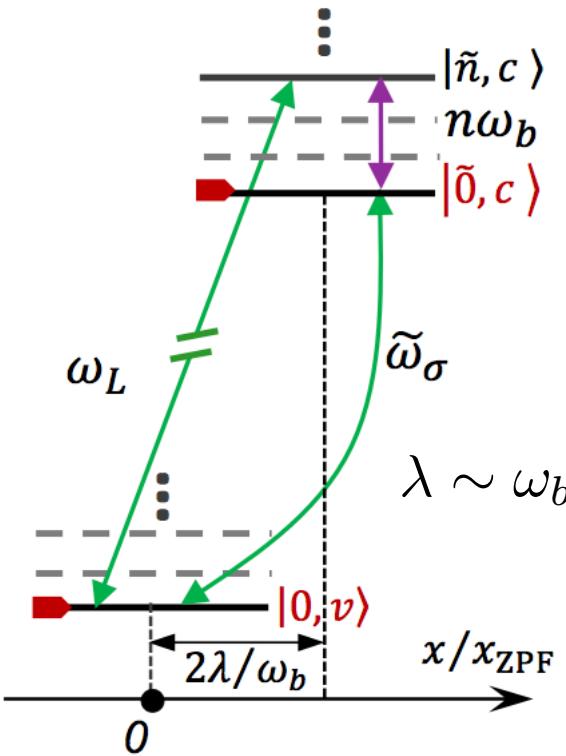
$$|n, v\rangle, |\tilde{n}, c\rangle = D^\dagger |n, c\rangle$$

$$D = \exp[(\lambda/\omega_b)\sigma^\dagger \sigma(b^\dagger - b)]$$

$$\tilde{\omega}_\sigma = \omega_\sigma - \lambda^2/\omega_b$$

N -phonon associated Anti-Stokes resonances

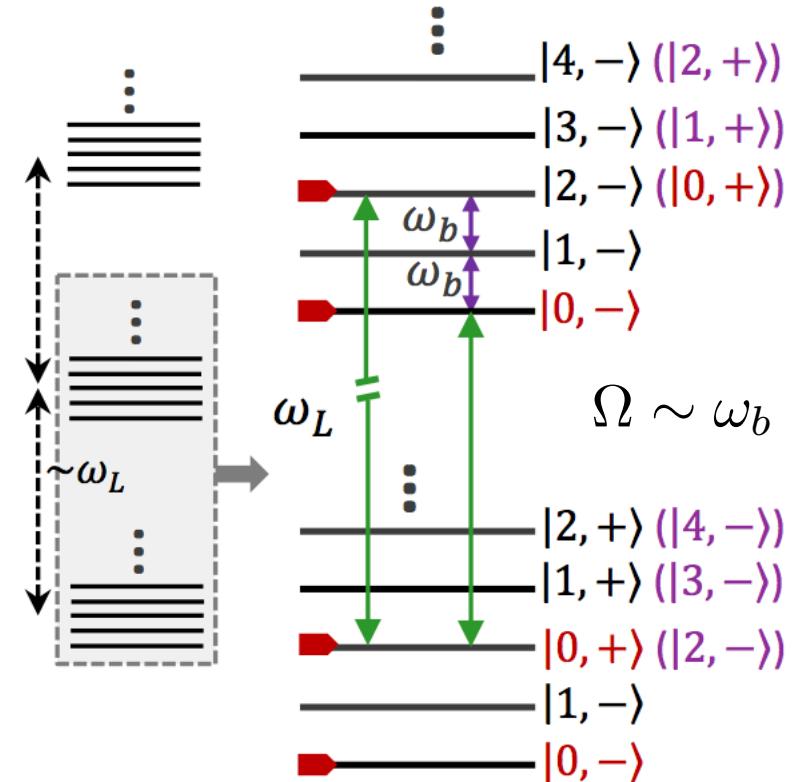
$$(c) \omega_L = \omega_\sigma - \lambda^2/\omega_b + n\omega_b \quad (d) \omega_L = \omega_\sigma + \sqrt{(n\omega_b)^2 - 4\Omega^2}$$



$$|n, \nu\rangle, |\tilde{n}, c\rangle = D^\dagger |n, c\rangle$$

$$D = \exp[(\lambda/\omega_b)\sigma^\dagger \sigma(b^\dagger - b)]$$

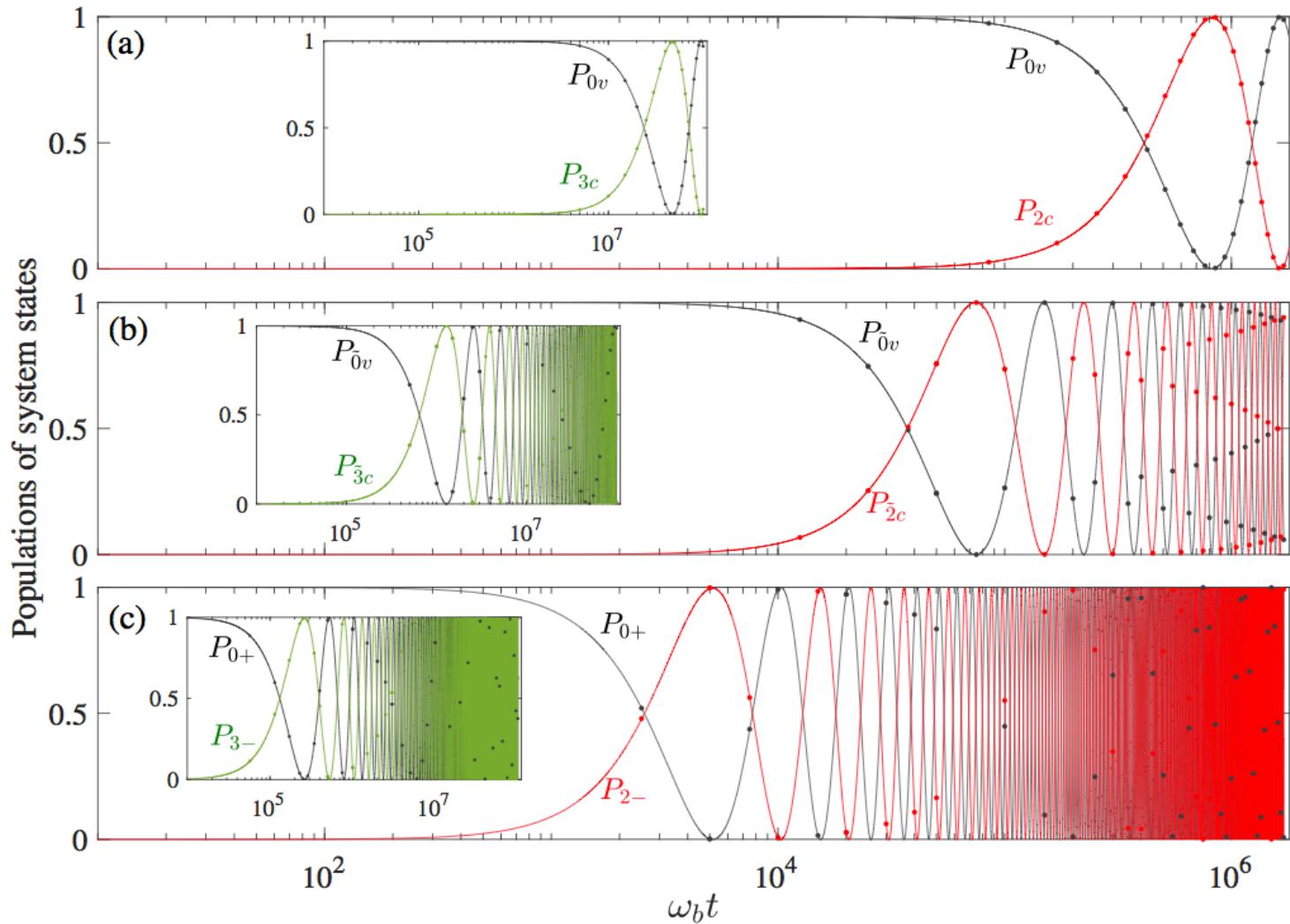
$$\tilde{\omega}_\sigma = \omega_\sigma - \lambda^2/\omega_b$$



$$|n, \pm\rangle, |\pm\rangle = c_\pm |\nu\rangle \pm c_\mp |c\rangle$$

$$E_{|n, \pm\rangle} = n\omega_b + \Delta/2 \pm \sqrt{\Delta^2 + 4\Omega^2}/2$$

Super-Rabi Oscillations

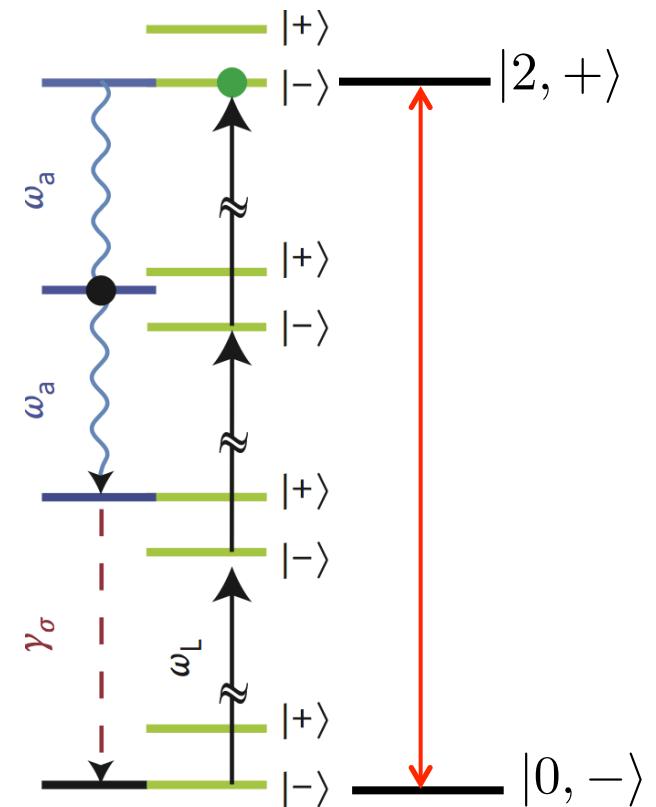
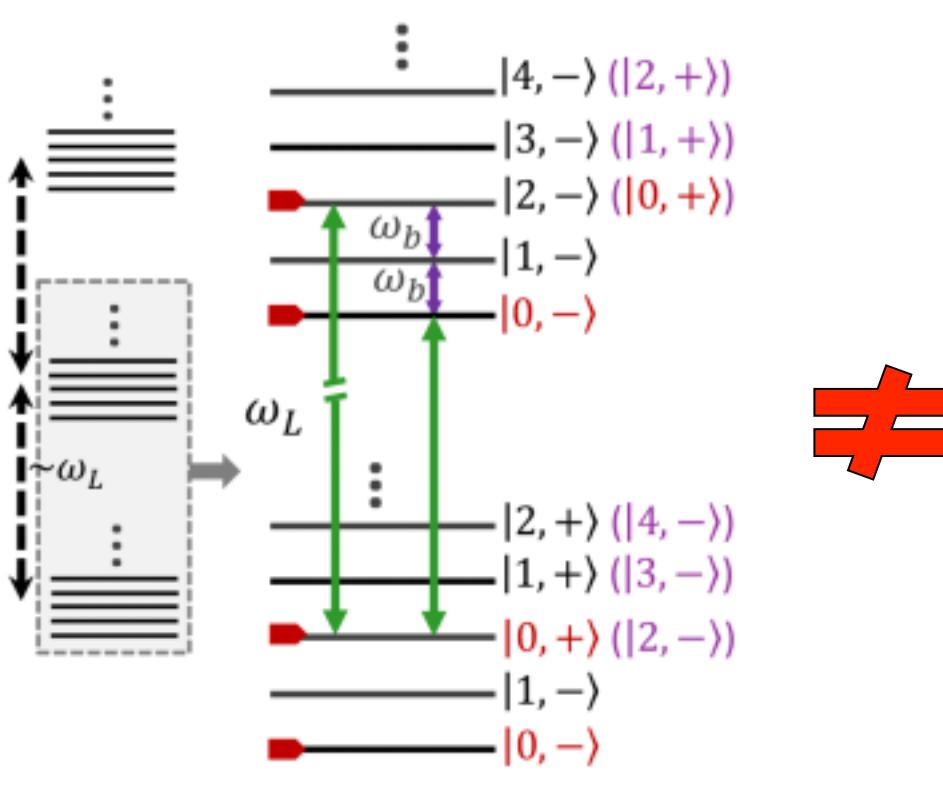


$$\Omega_{\text{eff}}^{(2)} = \frac{\sqrt{2}\Omega\lambda^2}{(2\Omega^2 - \lambda^2 + 2\omega_b^2)}$$

$$\Omega_{\text{eff}}^{(2)} = \frac{\Omega\lambda^2}{\sqrt{2}\omega_b^2} e^{-\lambda/2\omega_b}$$

$$\Omega_{\text{eff}}^{(2)} = -\sqrt{2}c_+c_-\lambda^2\dots$$

Different from the JC regime



$$H = \omega_b b^\dagger b + \omega_\sigma \sigma^+ \sigma + \lambda \sigma^+ \sigma (b^\dagger + b) + \Omega(e^{i\omega_L t} \sigma + h.c.)$$

$$H = \omega_b b^\dagger b + \omega_\sigma \sigma^+ \sigma + g(b^\dagger \sigma + b \sigma^+) + \Omega(e^{i\omega_L t} \sigma + h.c.)$$

Super Rabi Oscillation



Proper Dissipation Channel



N-phonon bundle emission

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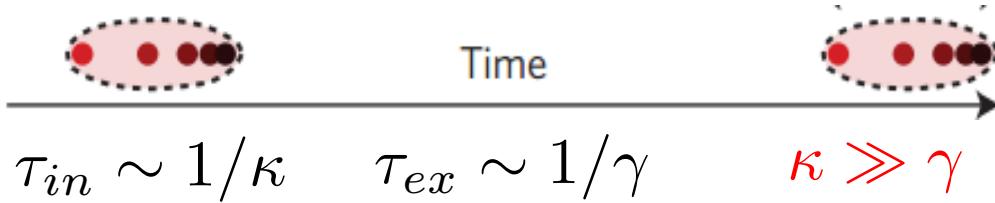
Dissipation dynamics with master equation

$$\frac{d\rho}{dt} = -i[H, \rho] + \kappa\mathcal{L}[b] + \gamma\mathcal{L}[\sigma]$$

Cavity decay rate: κ

$$\mathcal{L}[O] = (2O\rho O^\dagger - \rho O^\dagger O - O^\dagger O\rho)/2$$

QD decay rate: γ



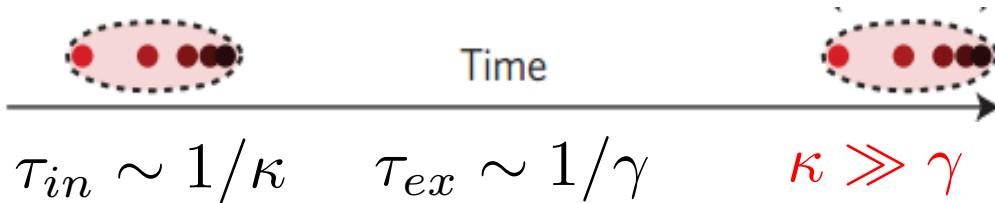
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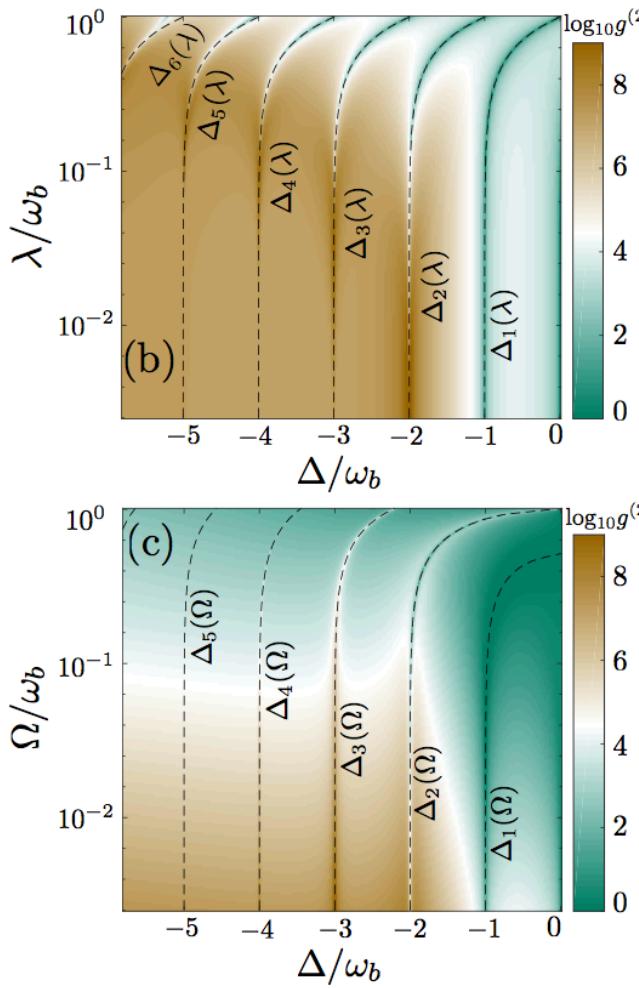
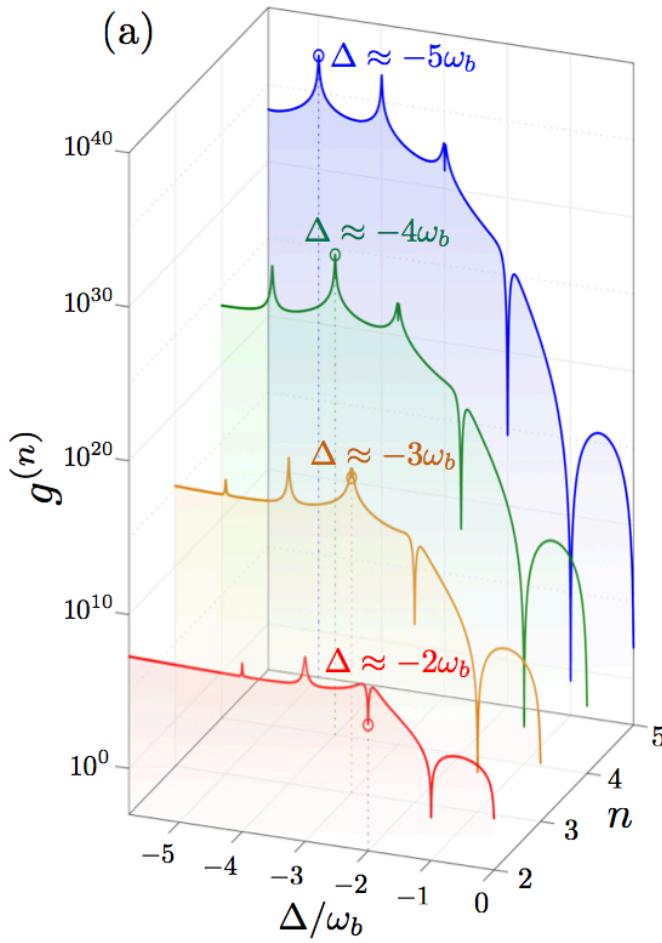
Phonons correlation functions

Bundle correlation functions

$$g^{(n)} = \frac{\langle b^{\dagger n} b^n \rangle}{\langle b^\dagger b \rangle^n}$$

$$g_N^{(n)}(t_1, \dots, t_n) = \frac{\langle \mathcal{T}_- \{ \prod_{i=1}^n b^{\dagger N}(t_i) \} \mathcal{T}_+ \{ \prod_{i=1}^n b^N(t_i) \} \rangle}{\prod_{i=1}^n \langle b^{\dagger N} b^N \rangle(t_i)}$$

Bundles of strongly correlated phonons

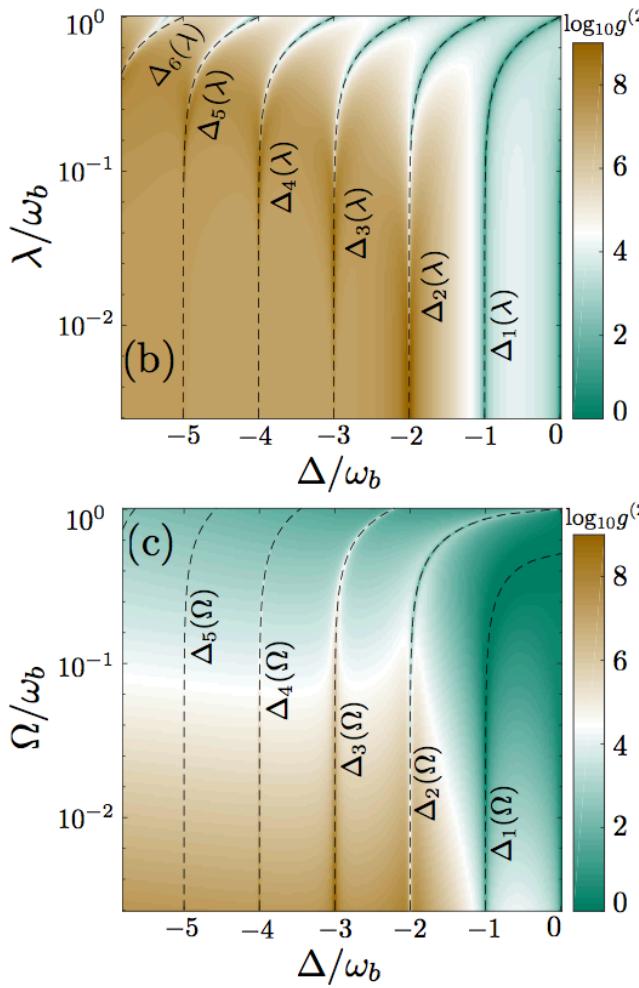
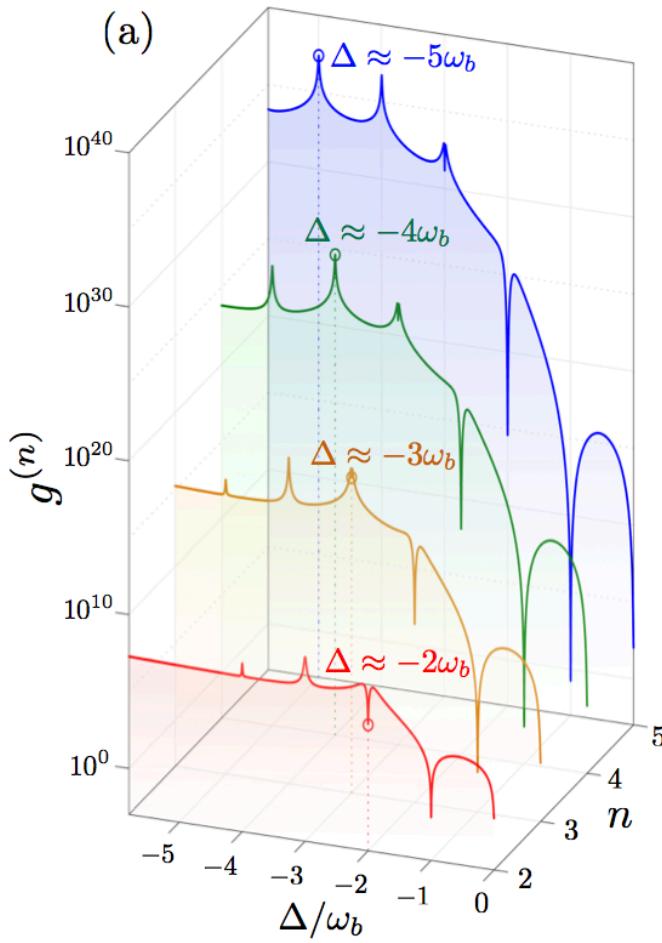


$$g^{(n)} = \frac{\langle b^\dagger^n b^n \rangle}{\langle b^\dagger b \rangle^n}$$

Strong bunching at n -phonon associated anti-Stokes resonances

(a) $\Delta = -n\omega_b$ (b) $\Delta = \Delta_n(\lambda)$ (c) $\Delta = \Delta_n(\Omega)$

Bundles of strongly correlated phonons



$$g^{(n)} = \frac{\langle b^\dagger^n b^n \rangle}{\langle b^\dagger b \rangle^n}$$

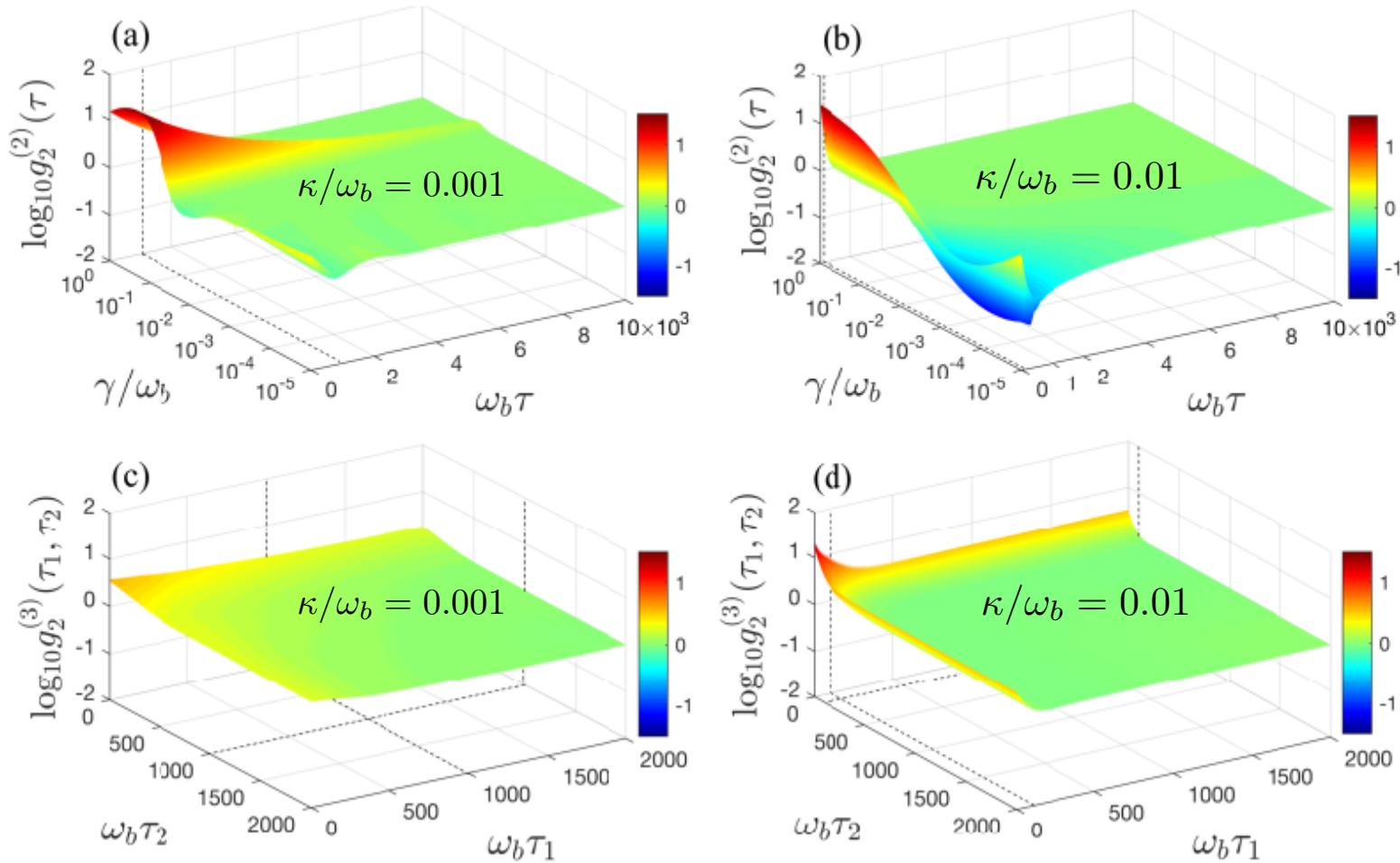


is robust to
varying system
parameters

Strong bunching at n -phonon associated anti-Stokes resonances

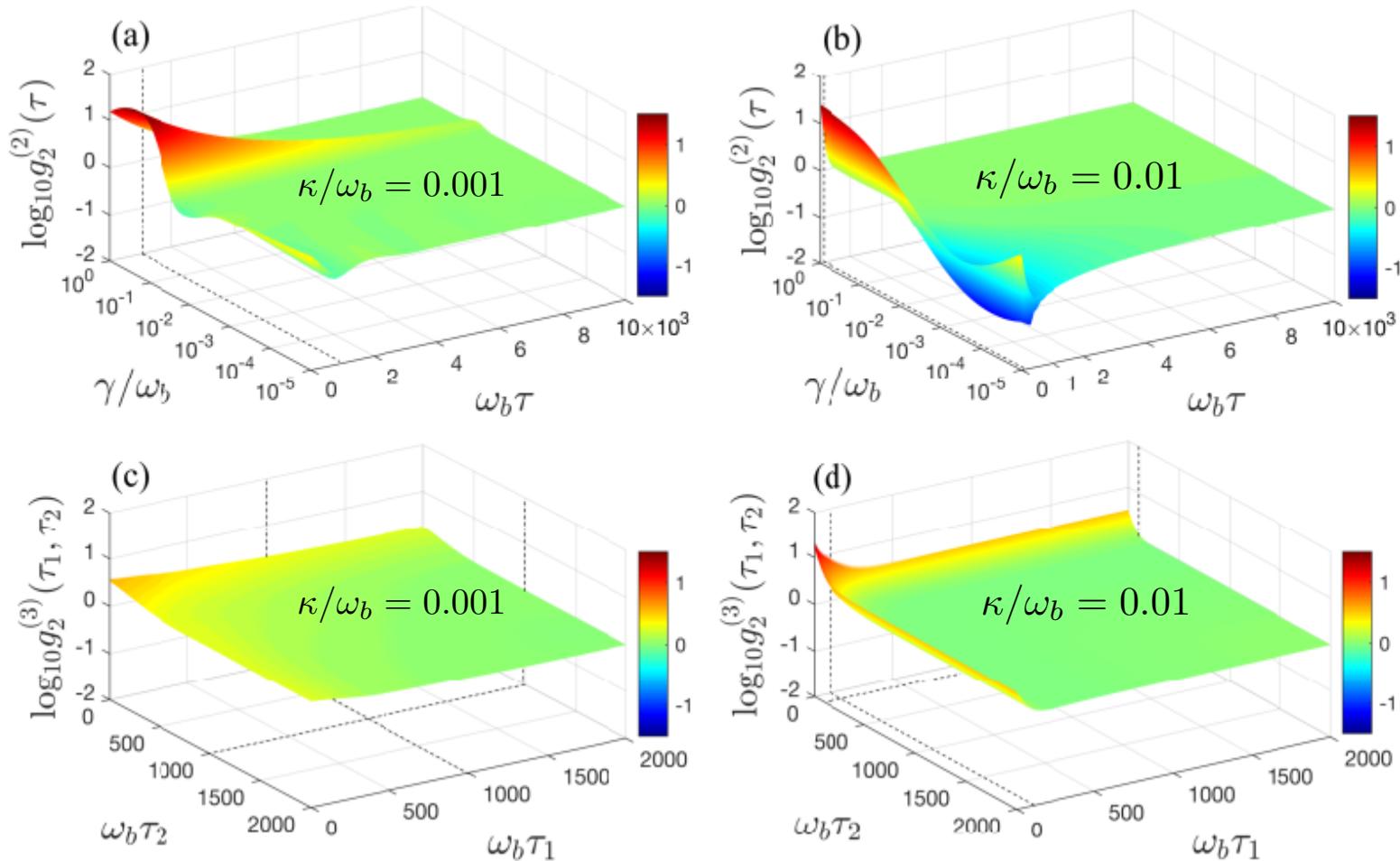
(a) $\Delta = -n\omega_b$ (b) $\Delta = \Delta_n(\lambda)$ (c) $\Delta = \Delta_n(\Omega)$

Statistic characteristics of emitted n -phonon bundles



Valid regime: $\tau > 1/\kappa$

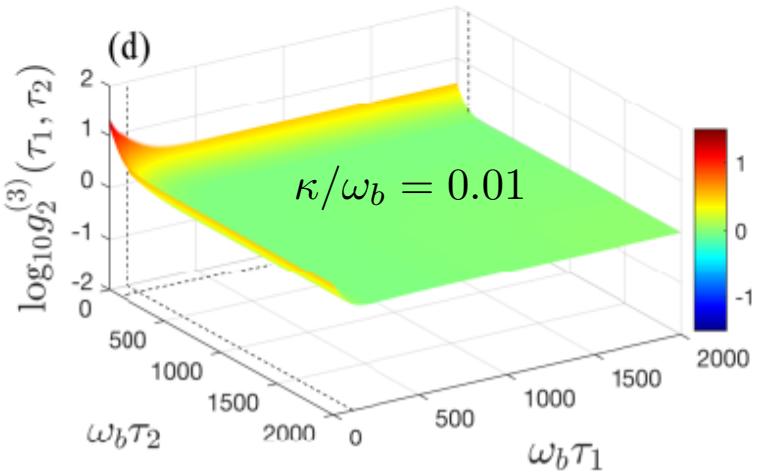
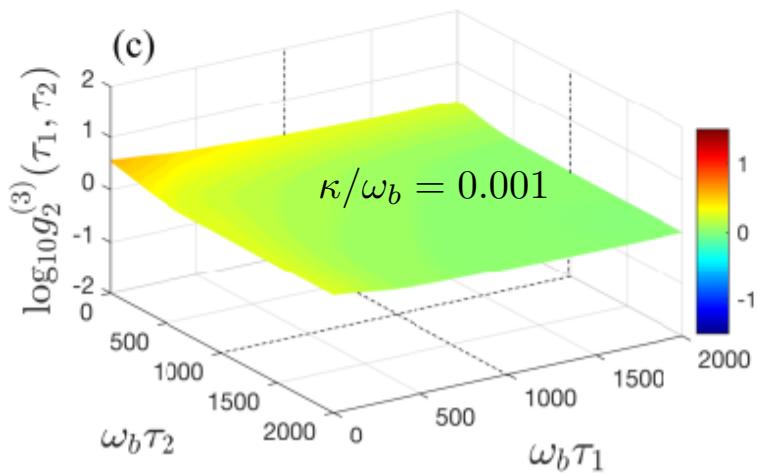
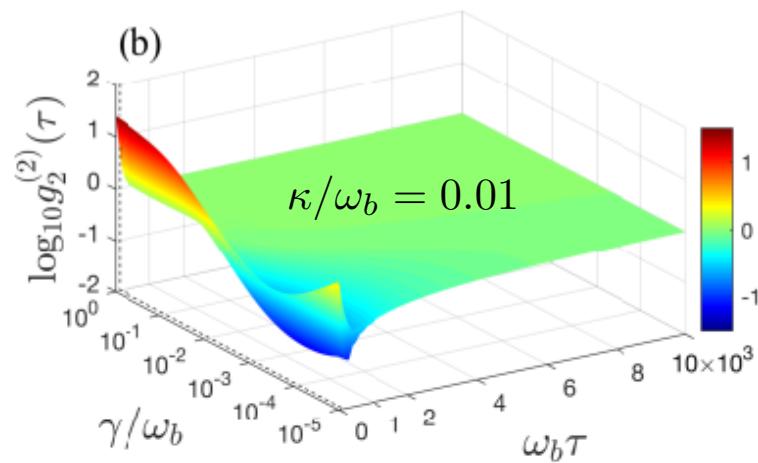
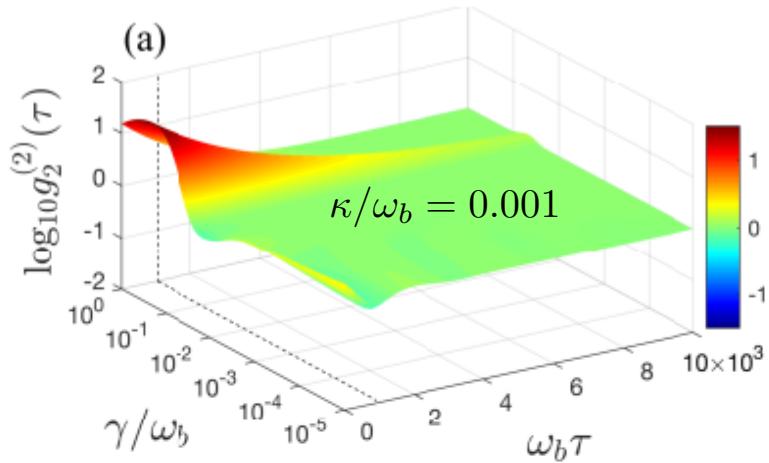
Statistic characteristics of emitted n -phonon bundles



Valid regime: $\tau > 1/\kappa$

Bunching $\xrightarrow{\gamma}$ Uncorrelated $\xrightarrow{\gamma}$ Antibunching

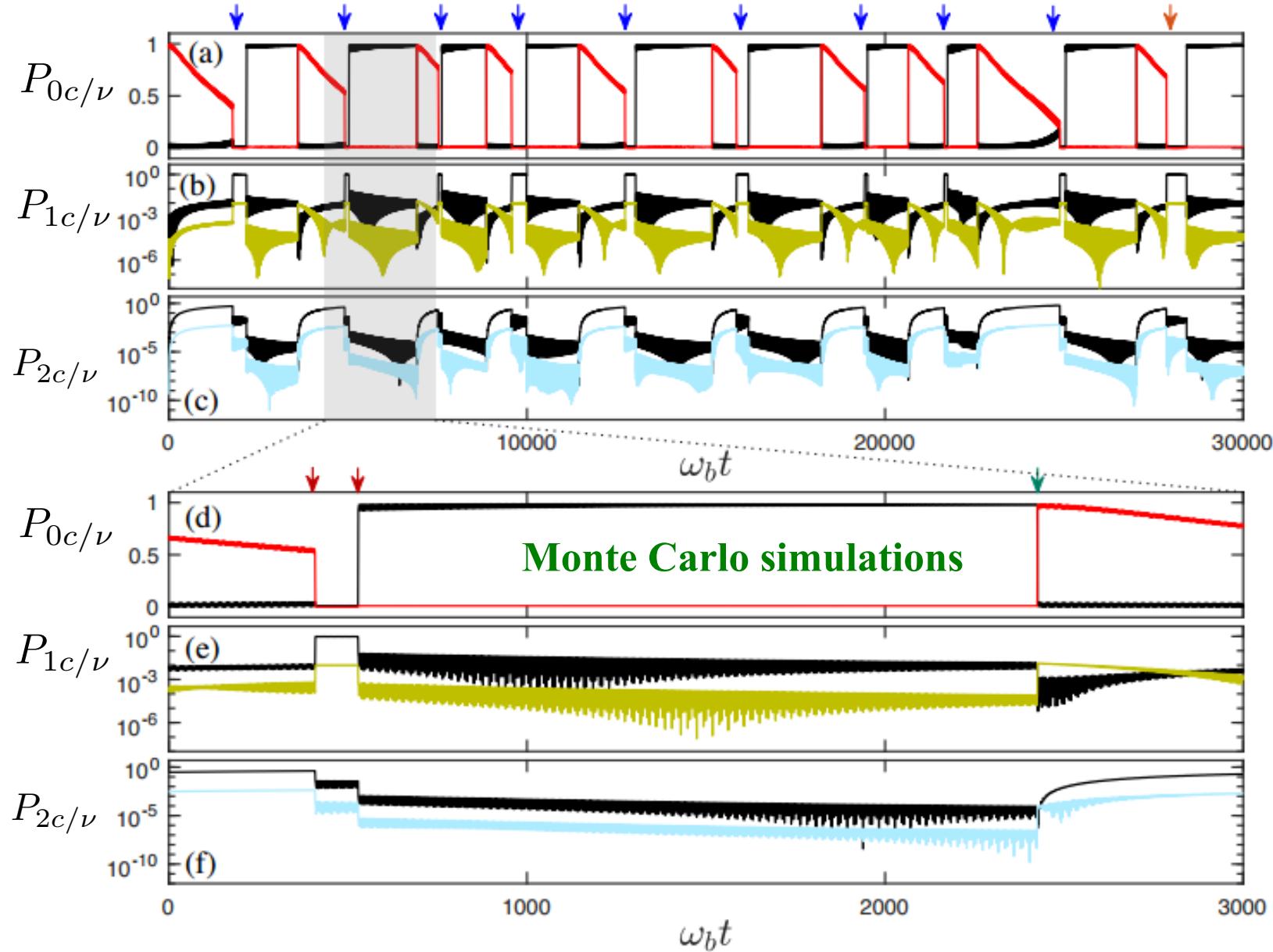
Statistic characteristics of emitted n -phonon bundles



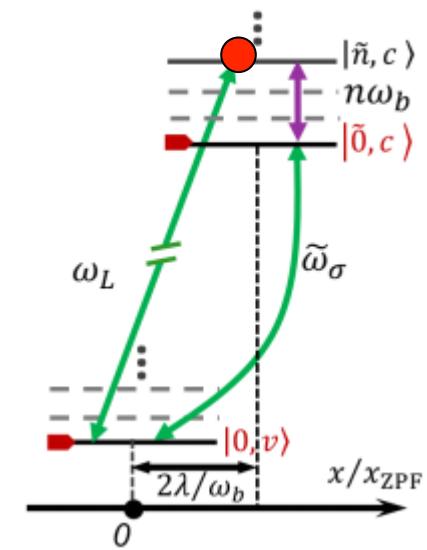
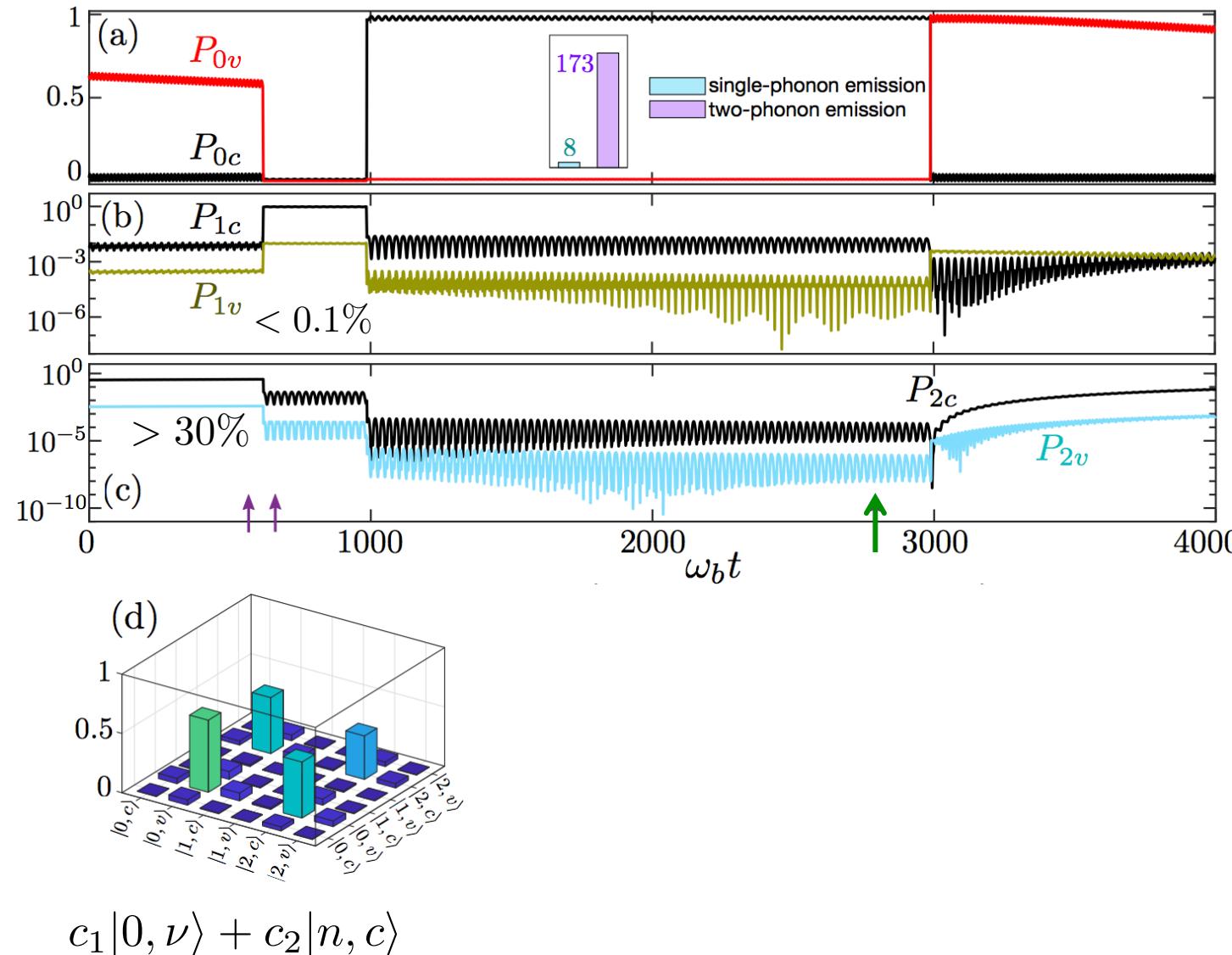
Valid regime: $\tau > 1/\kappa$

Bunching $\xrightarrow{\gamma}$ Uncorrelated $\xrightarrow{\gamma}$ Antibunching Two-phonon laser $g^{(2)}(\tau) \approx 1, g^{(3)}(\tau_1, \tau_2) \approx 1$

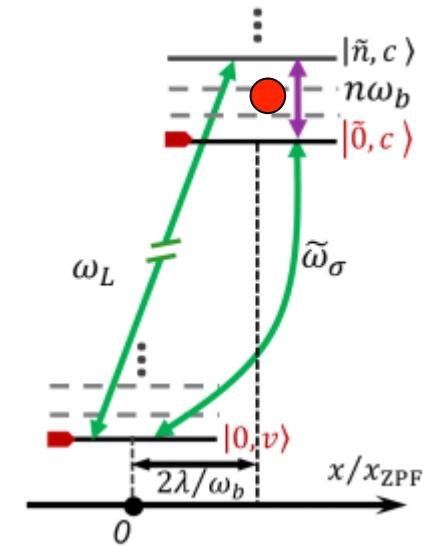
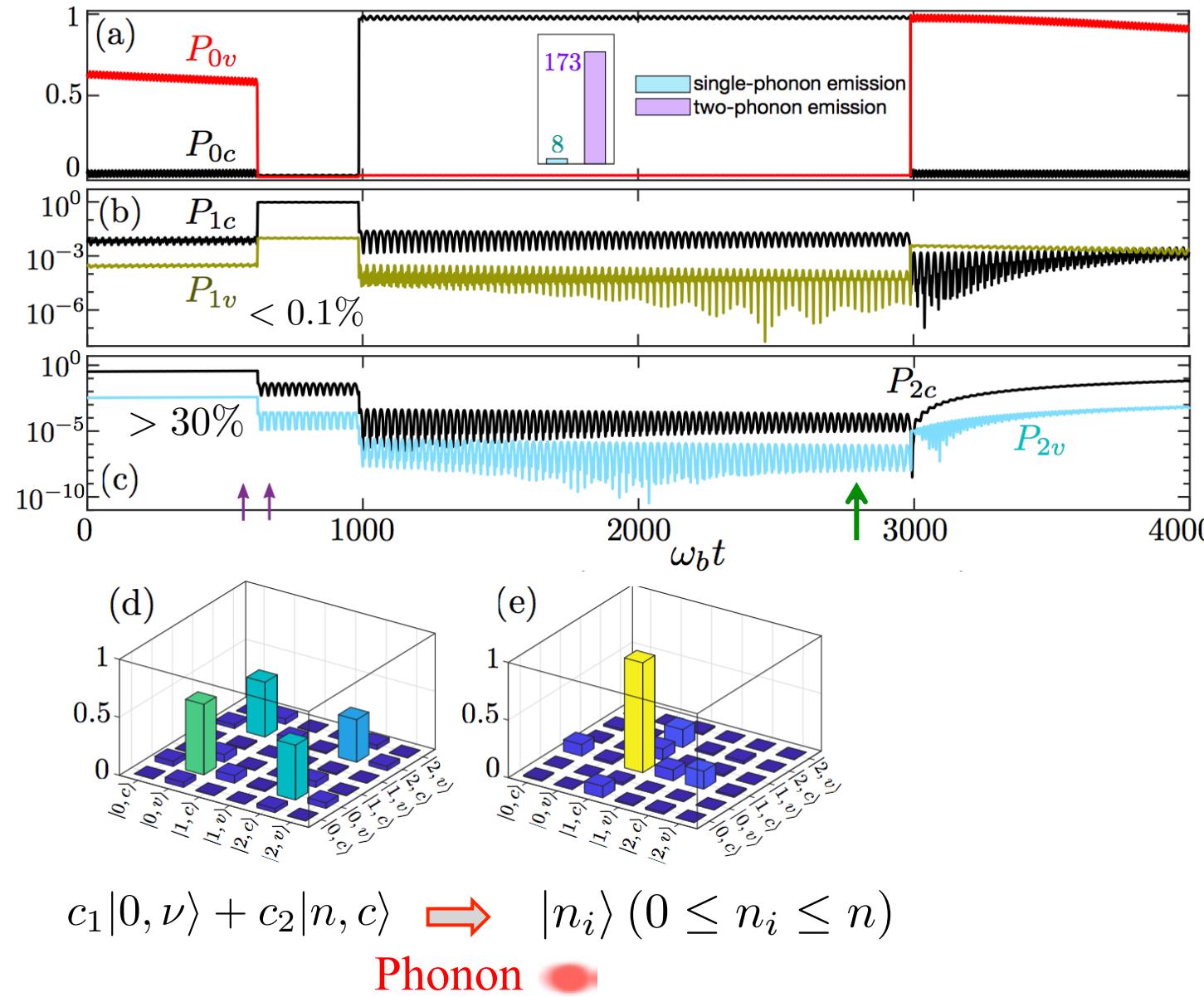
N -phonon bundle emission- dynamics trajectories



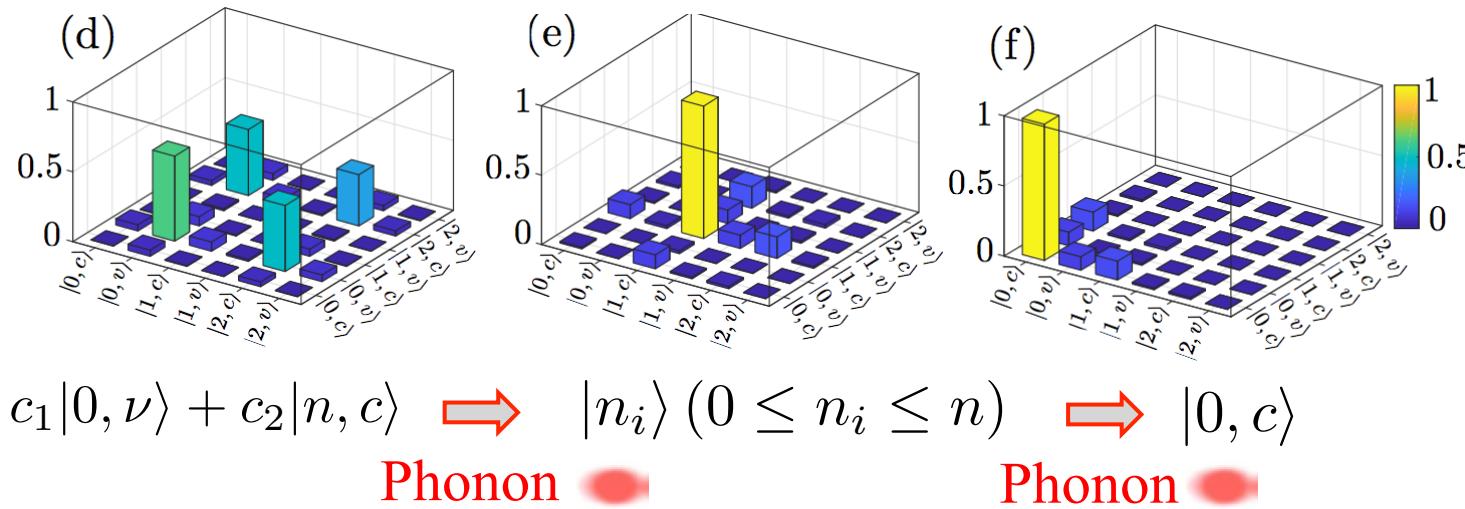
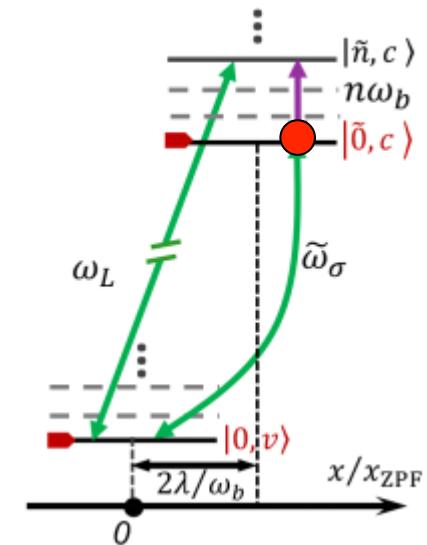
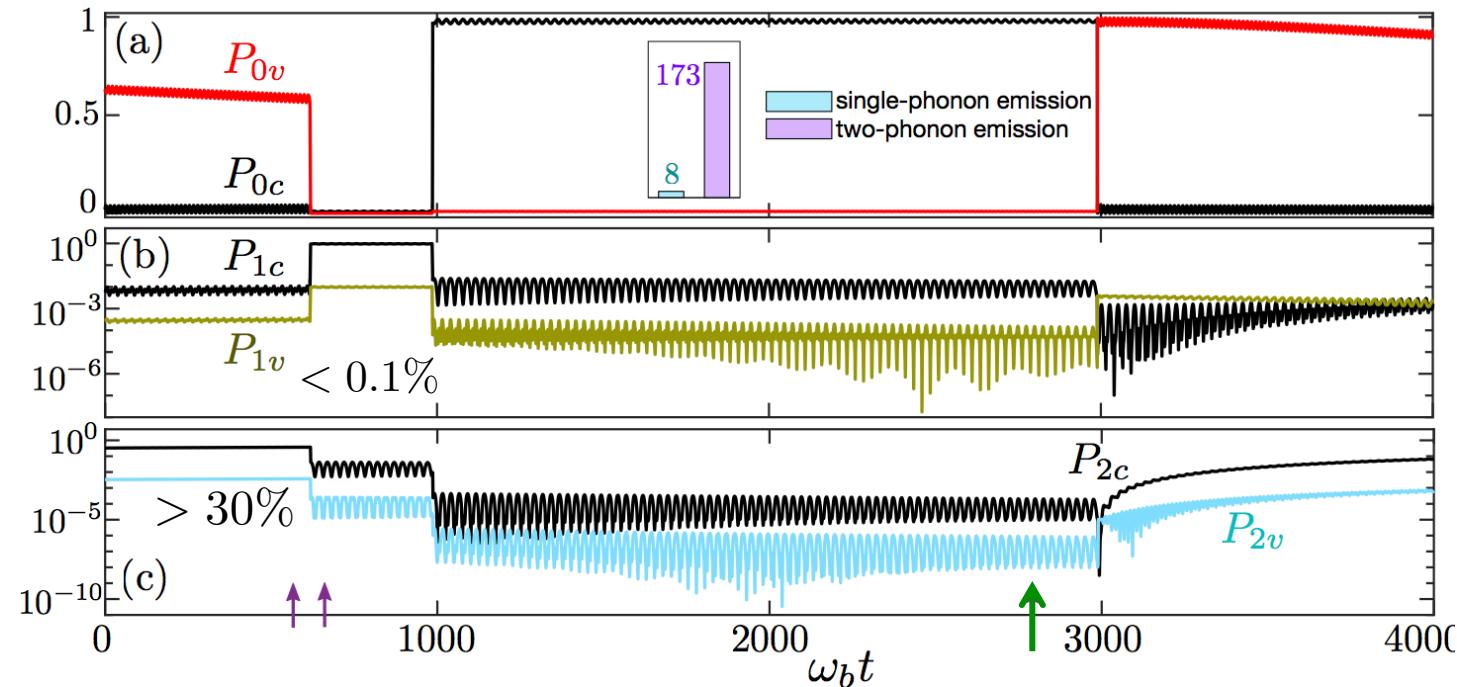
Cascade emission of Fock states in a short time window



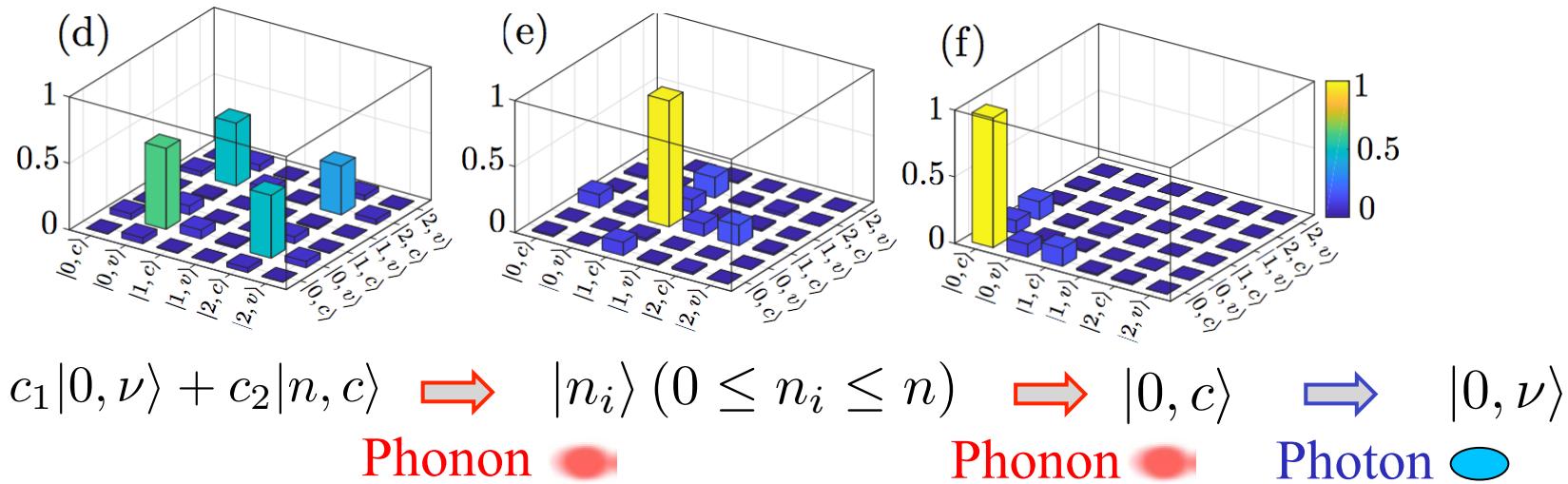
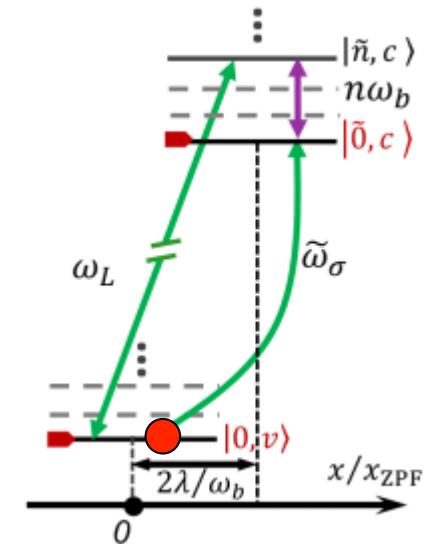
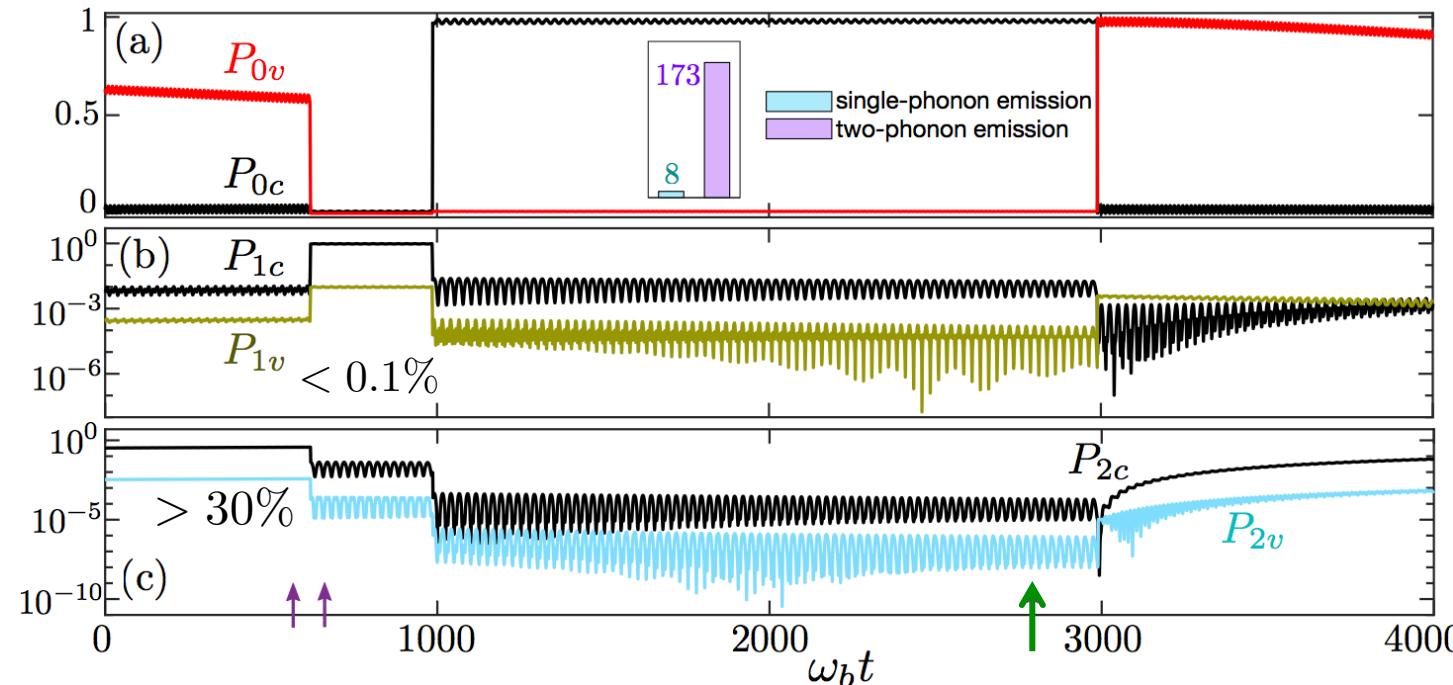
Cascade emission of Fock states in a short time window



Cascade emission of Fock states in a short time window



Cascade emission of Fock states in a short time window



Single cascade-emission process of two-phonon

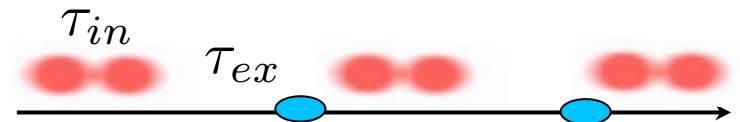
Initially: $P_2 > 30\%$, $P_1 < 0.1\%$

$$\tau_{in} < 1/\kappa < \tau_{ex} \approx 1/\gamma$$

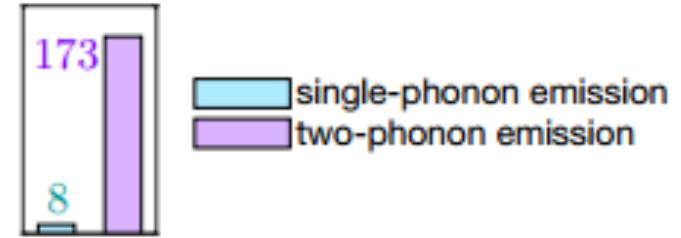
Emit a first phonon: $P_1 \approx 1$

$$\tau_{in} < 1/\kappa$$

Emit the second phonon: $P_{0c} \approx 1$



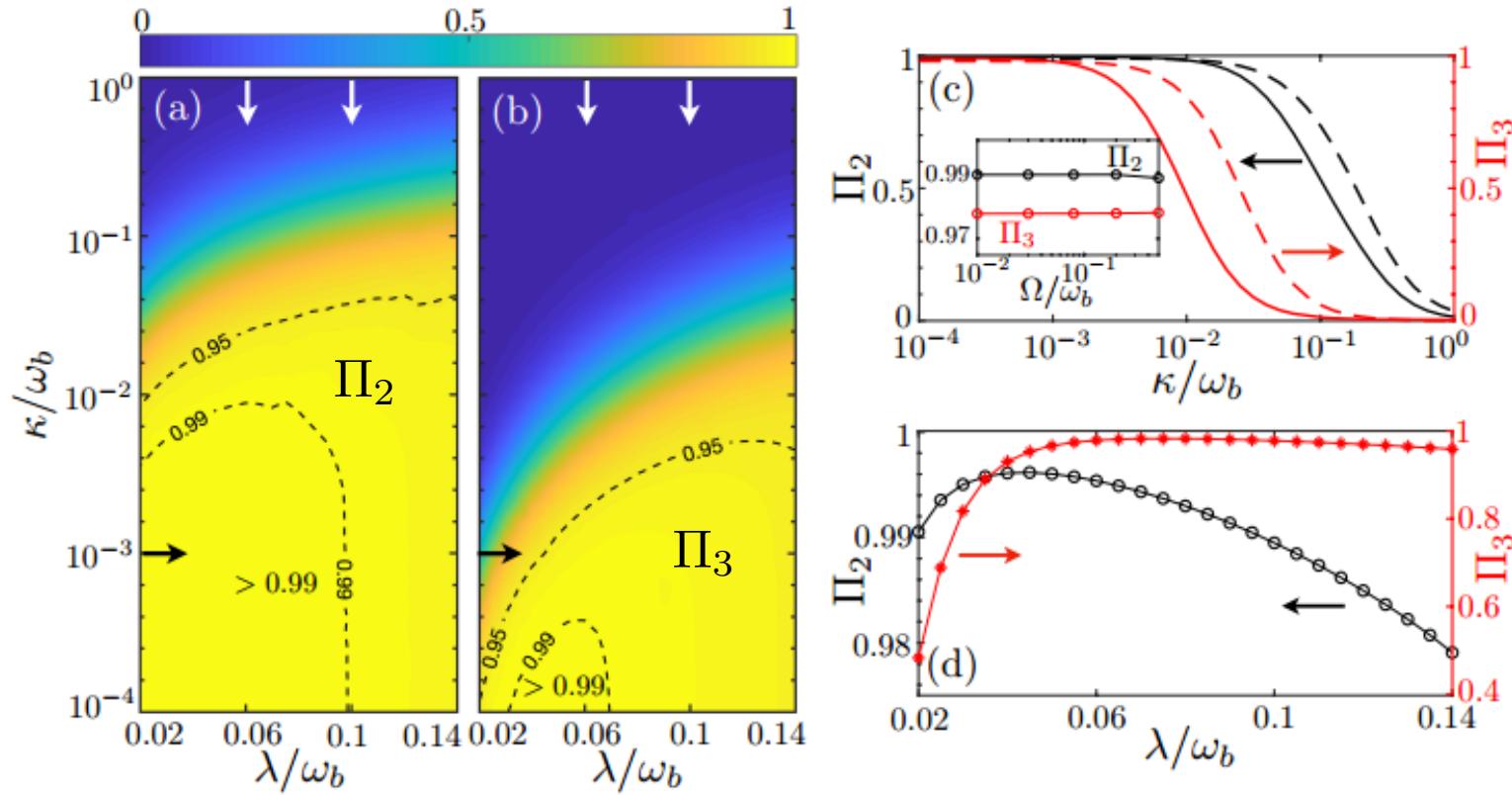
Photon emission QD flip $|0, c\rangle \rightarrow |0, \nu\rangle$



Optical heralded two-phonon bundle emission

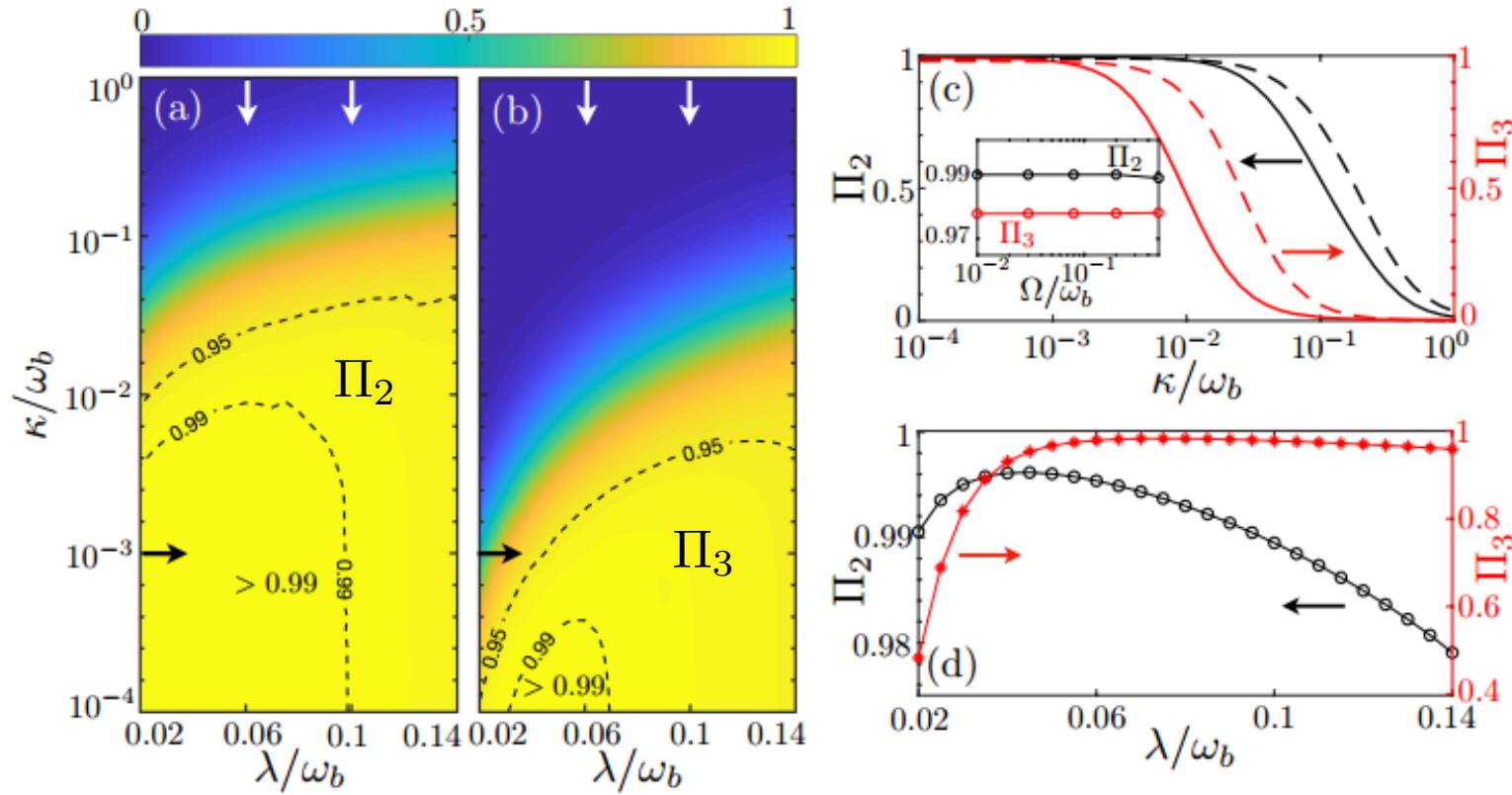
Two-phonon emission rates $\approx 1.8 \times 10^9 / s$

The purity of n -phonon emission



Definition: $\Pi_n = n\tilde{P}_n / \sum_{n_i=1}^n n_i \tilde{P}_{n_i} \rightarrow n$ -phonon state population in the time-span T

The purity of n -phonon emission

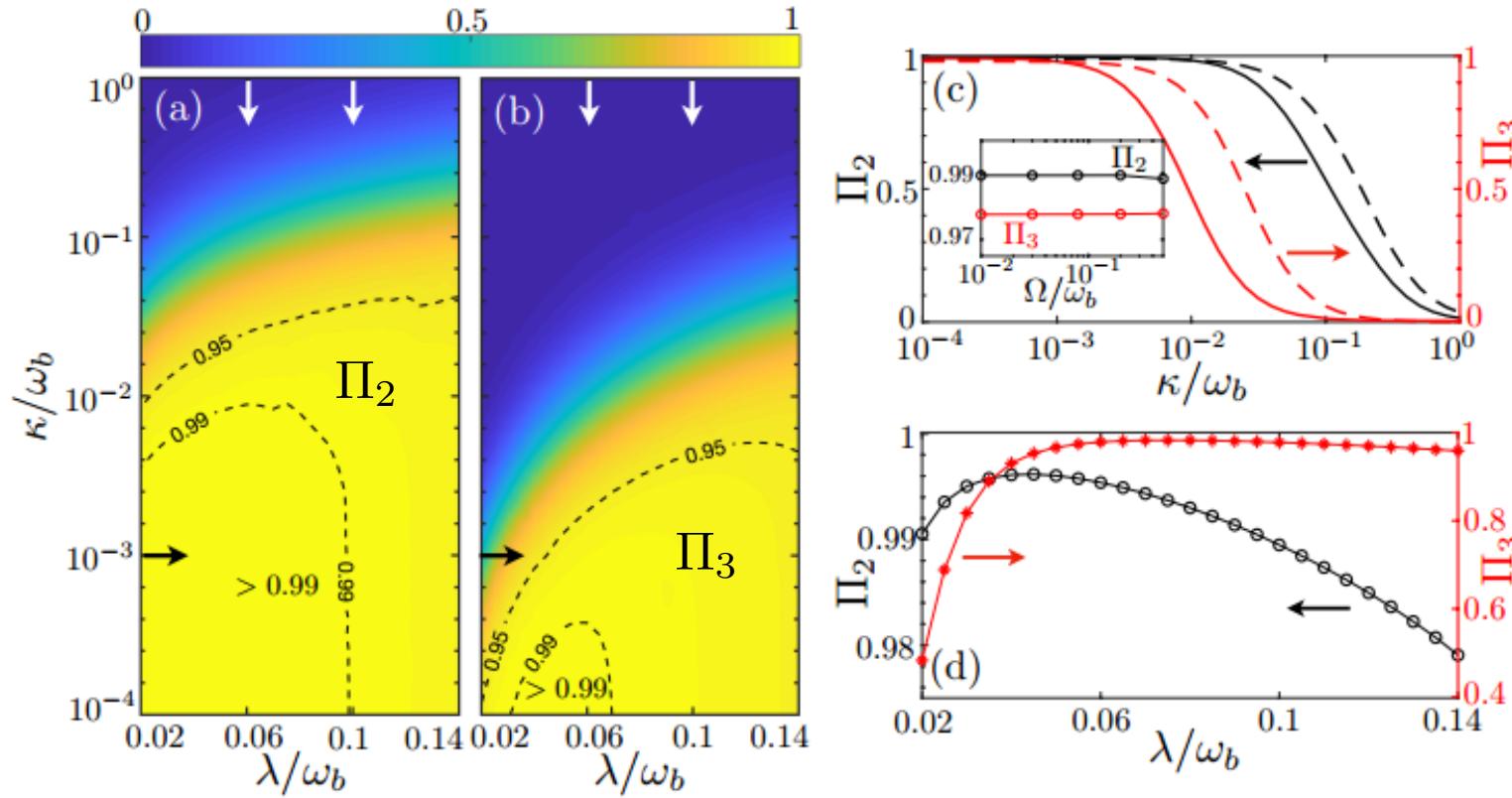


Definition: $\Pi_n = n\tilde{P}_n / \sum_{n_i=1}^n n_i \tilde{P}_{n_i} \rightarrow n$ -phonon state population in the time-span T

Wide range
 $\Pi_{2,3} > 95\%$

Π_n is robust with Ω

The purity of n -phonon emission



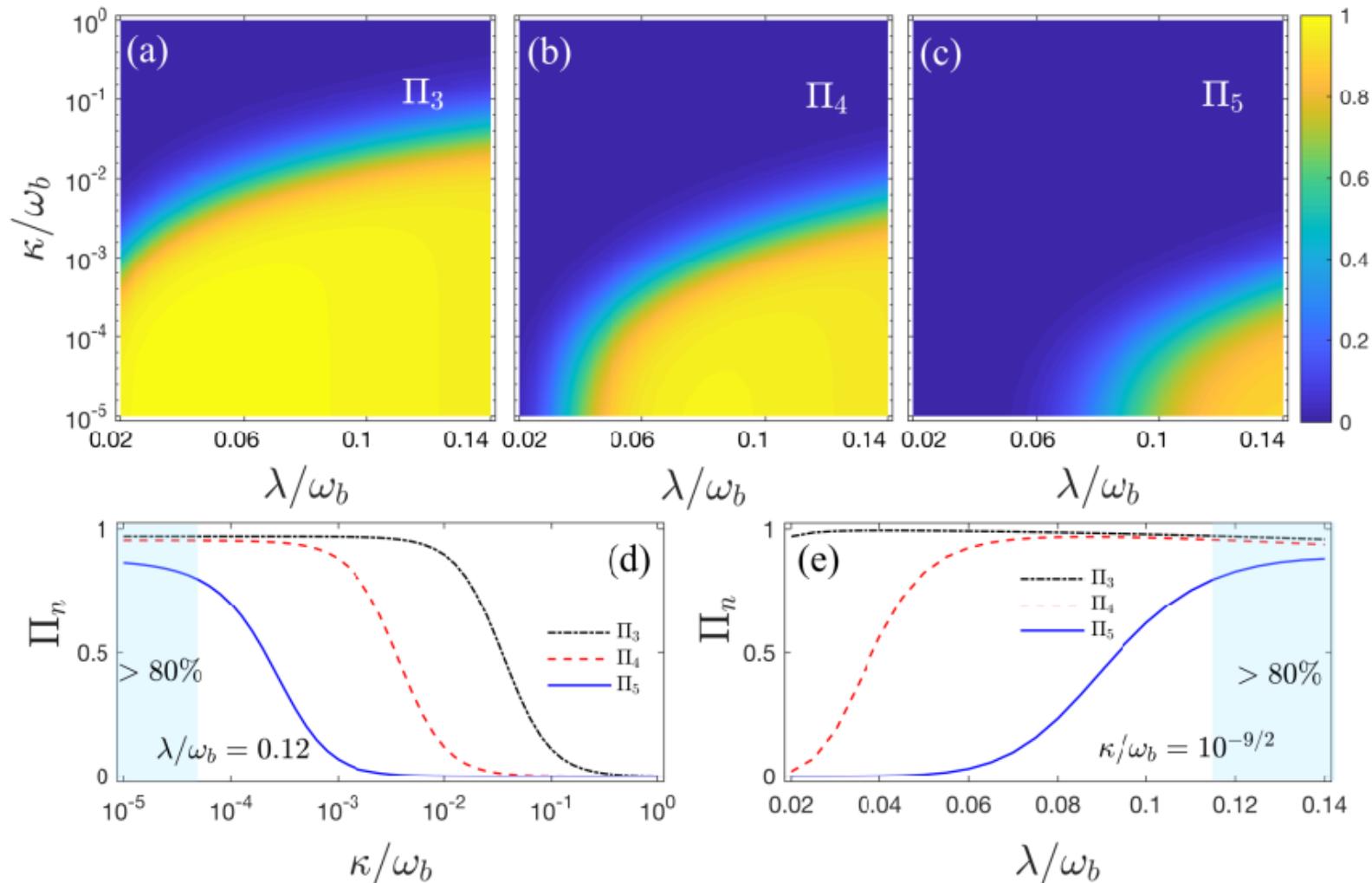
Definition: $\Pi_n = n\tilde{P}_n / \sum_{n_i=1}^n n_i \tilde{P}_{n_i} \rightarrow n$ -phonon state population in the time-span T

Increasing λ can enhance (decrease) the Π_3 (Π_2) by enhancing the high-order phonon sideband processes.

Wide range
 $\Pi_{2,3} > 95\%$

Π_n is robust with Ω

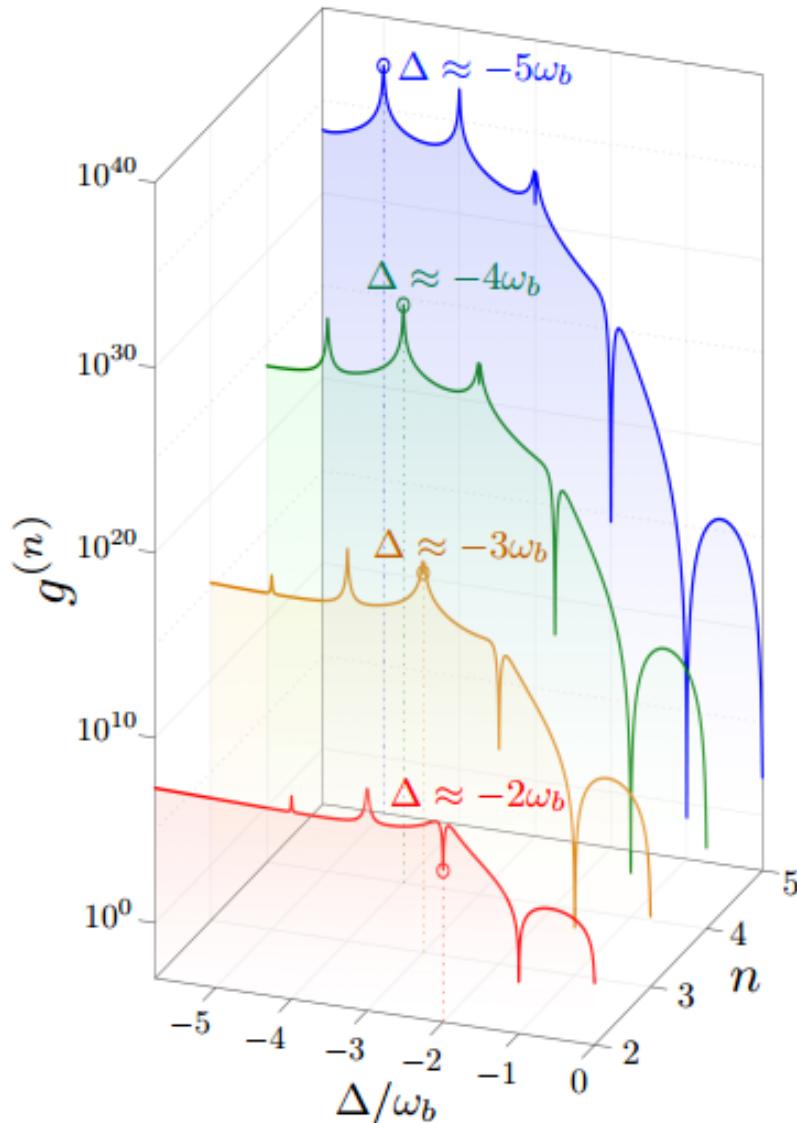
The purity of n -phonon emission



The purity is less robust for larger n

$\Pi_4 > 95\%$, $\Pi_5 > 80\%$ with feasible experimental parameter

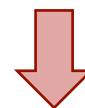
The mechanism deciding high purity for large n



Anti-Stokes resonances



Frequency differences between the n and $(n+1)$ -phonon resonances are almost independent of n

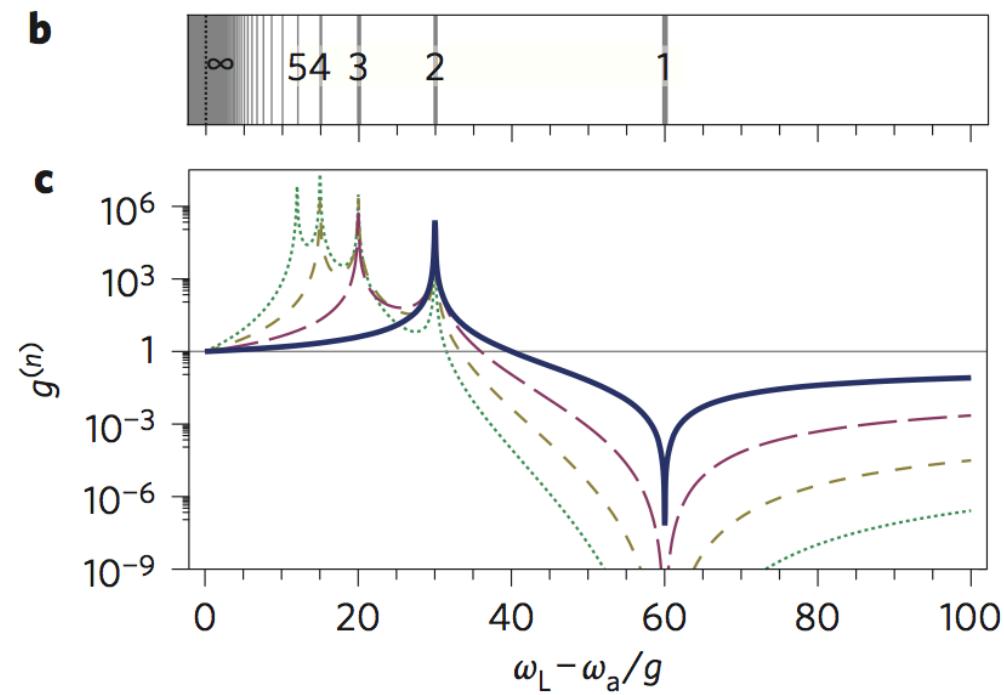


Even for large n , the optimum pumping frequency can be well resolved when $\omega_b \gg \kappa, \gamma$

Electron-phonon coupling

The mechanism of previous work

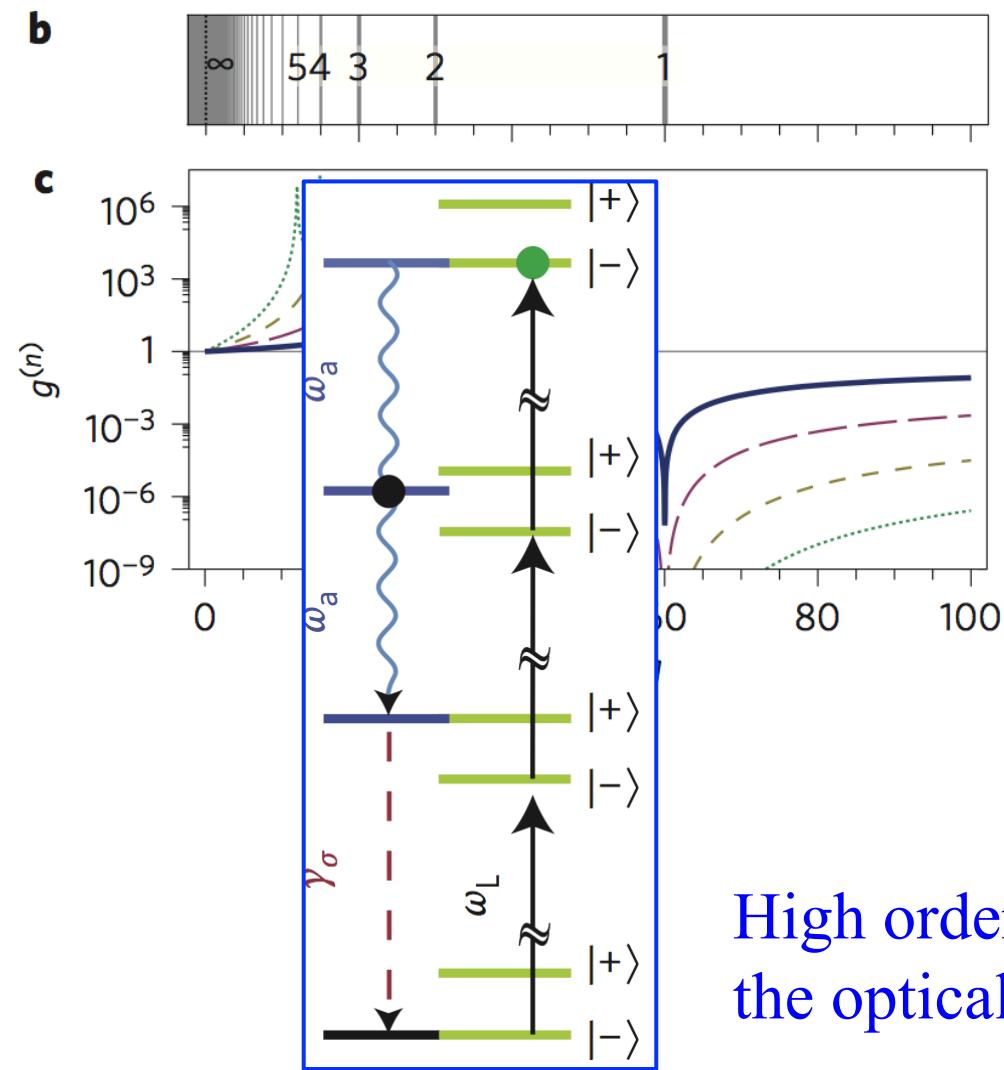
JC coupling



Frequency differences between the n and $(n+1)$ photon resonances becomes small as increasing n

The mechanism of previous work

JC coupling

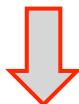


Frequency differences between the n and $(n+1)$ photon resonances becomes small as increasing n

High order process for the optical driving

Advantages and applications of our work

The mechanism of anti-Stokes processes

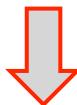


Expands the theory of n -quanta bundle emission

Broadens the family of anti-Stokes processes

Advantages and applications of our work

The mechanism of anti-Stokes processes



Expands the theory of n -quanta bundle emission

Broadens the family of anti-Stokes processes

Mixed phonon-photon emission



Optical heralded phonon laser/guns

On chip quantum communications and metrology

Acknowledgments



National Natural Science
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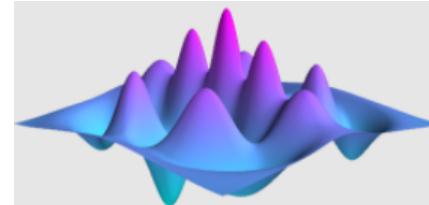
Xin-You thanks Dr.
Carlos Sánchez Muñoz



中华人民共和国科学技术部

Ministry of Science and Technology of the People's Republic of China

国家重点研发计划-量子调控与量子信息



QuTiP <http://qutip.org/>
Quantum Toolbox in Python



Fabrice P. Laussy
thanks Dr. E. del Valle

Franco Nori thanks



RIKEN-AIST Challenge
Research Fund

Conclusions

- We introduced the basic knowledge of n -photon bundle emission.
- We proposed n -phonon bundle emission via anti-Stokes process.
- We discussed the purity of n -phonon bundle emission and the corresponding applications.

Thanks for your attention!



