Quantum Simulation and Quantum Devices 2019,
ITP Beijing, Nov 18-23 2019

# `Randomized Measurements Toolbox' for Quantum Simulation

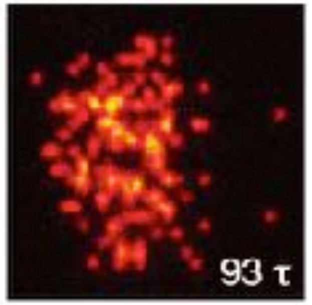
Peter Zoller

*The noise is the signal*

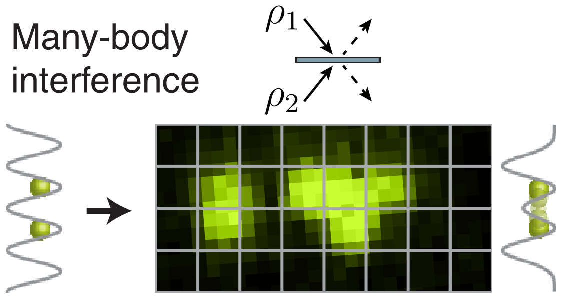
UQUAM

# The World of Quantum Simulators and Platforms

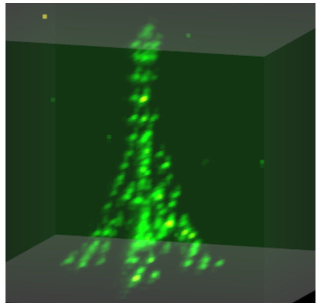
## Ultracold atoms in optical lattices, Rydberg atoms




Choi et al., Science (2016)

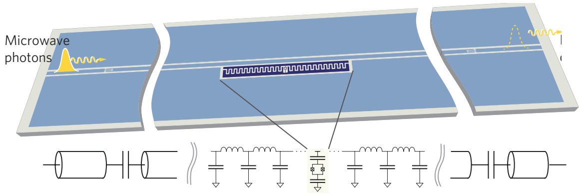



Kaufmann et al., Science (2016)



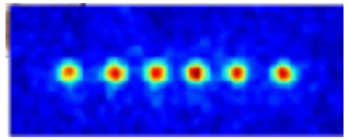
Barredo et al., Science (2016)

## Circuit QED

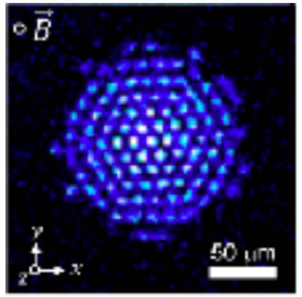



A. Houck, Princeton

## Trapped Ions



R. Blatt, Innsbruck




J. Bollinger, Boulder

## Experimental Toolbox:

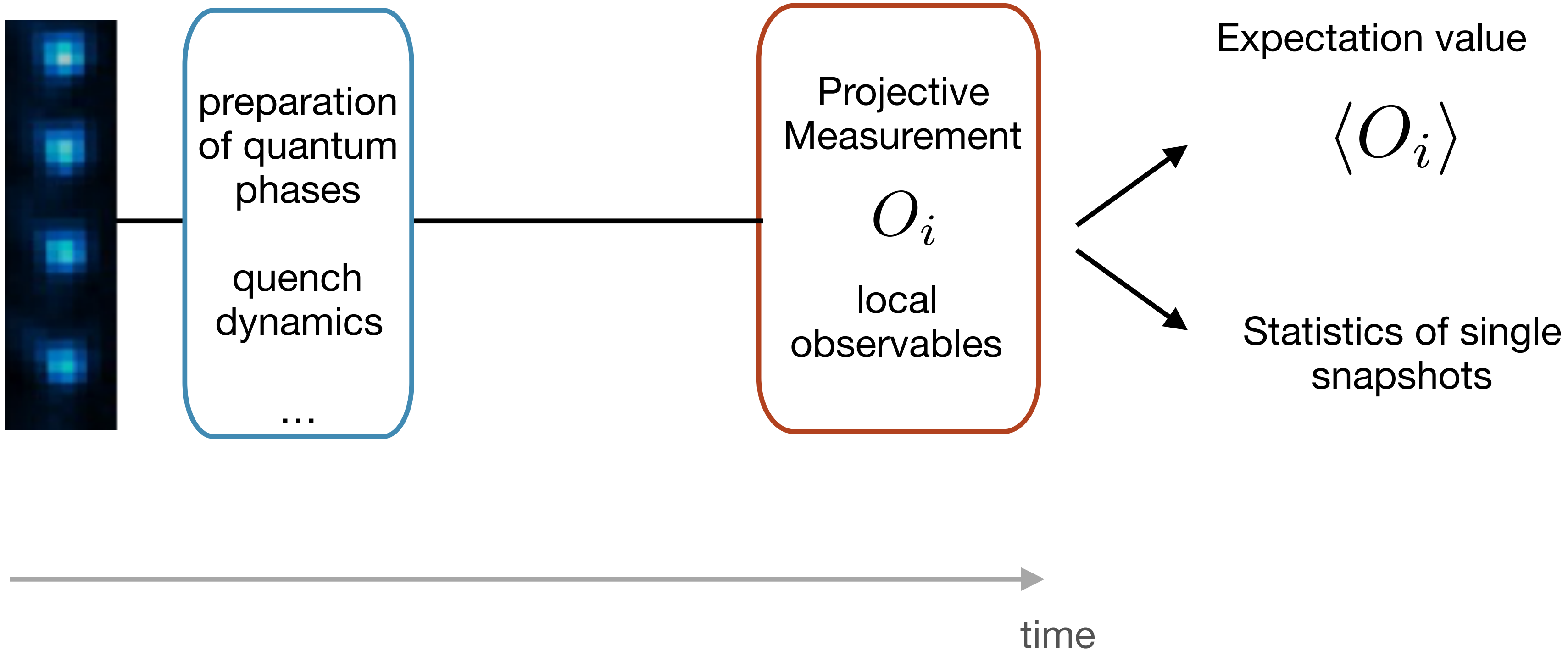
Single site quantum control & measurement

## Question / Challenge:

How to measure *complex* many-body observables beyond the usual?

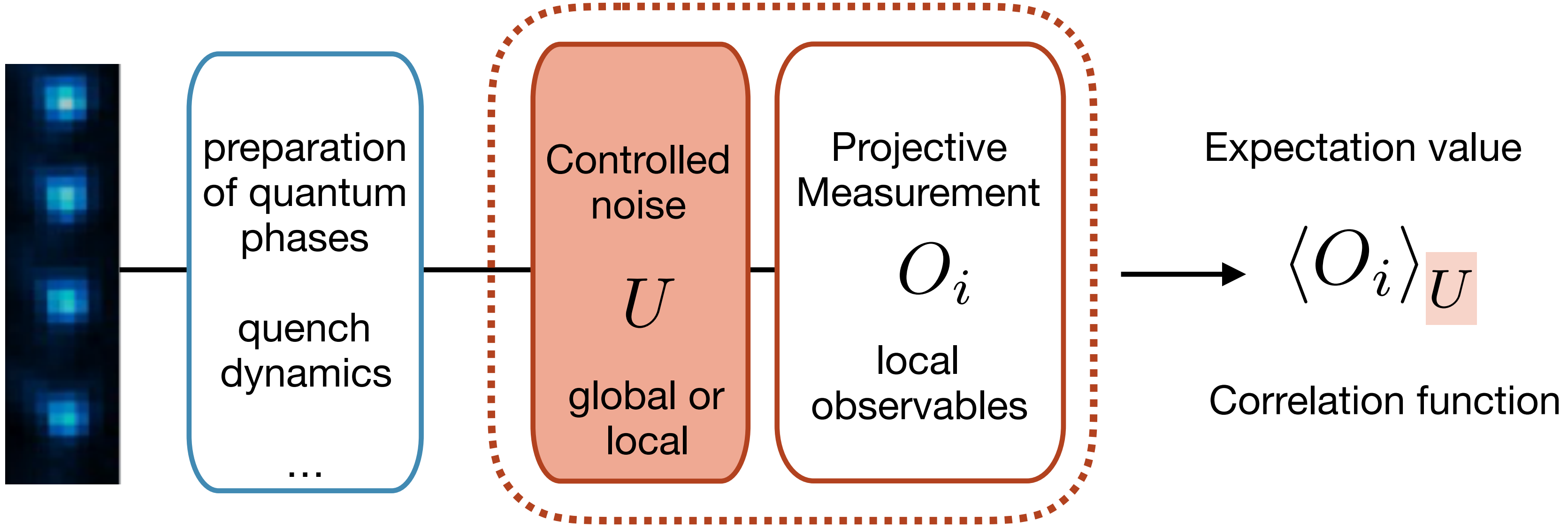
# How quantum simulation experiments are done today

## Closed many-body quantum system



# `Randomized Measurement' Toolbox

## Randomized measurements




## Statistics of expectation values

*classical* statistical correlations between *quantum* expectation values

$$\overline{\langle O_i \rangle_U \langle O_j \rangle_U}$$

'Noise' or ensemble average

Rem: 2-design in quantum information

… to measure many-body observables / properties which may be difficult to access otherwise

# Outline:

## Renyi (Entanglement) Entropies

T Brydges, A Elben, P Jurcevic, B Vermersch, C Maier, BP Lanyon, P. Z., R Blatt, CF Roos, Science 2019

## OTOCS — out-of-time-ordered correlators

B. Vermersch,* A. Elben, L. M. Sieberer, N.Y. Yao, and P.Z., PRX 2019 [theory]

M. Joshi, A Elben et al., draft [experiment]

## Cross-Platform Verification of Quantum Devices

A Elben, B Vermersch, C Kokail, R van Bijnen, T Brydges, C Maier, MK Joshi, R Blatt, CF Roos, P.Z., PRL in press [theory + experiment]

… for Intermediate Scale Quantum Devices

# Innsbruck Theory

Andreas Elben

Benoit Vermersch
→ Grenoble

Lukas Sieberer

Christian Kokail

Rick. van Bijnen

# Trapped-Ion Experiment Innsbruck

Rainer Blatt

Christian Roos

Ben Lanyon

Manoj K Joshi

Tiff Brydges

Christine Maier

…

# 10, 20 … 50 Qubit Trapped-Ion Programmable Quantum Simulator @ IQOQI-Labs

$$\hat{H}_{\text{Ising}} = \sum_{i,j} J_{ij}\hat{\sigma}_i^x\hat{\sigma}_j^x + B\sum_i \hat{\sigma}_i^z$$

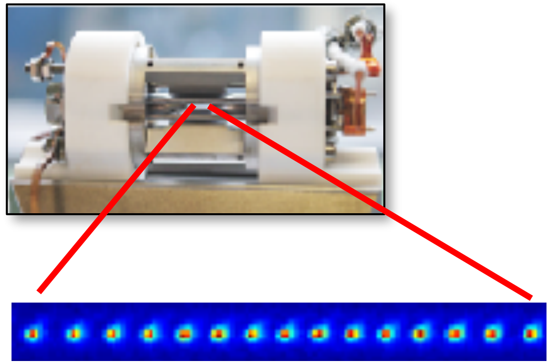
$$J_{ij} \sim \frac{1}{|i-j|^{\alpha}} \qquad \alpha = 0\ldots3$$

long range

+ single site addressing

Exp: Innsbruck, JQI …

Th: D. Poras & JI Cirac (2004), …

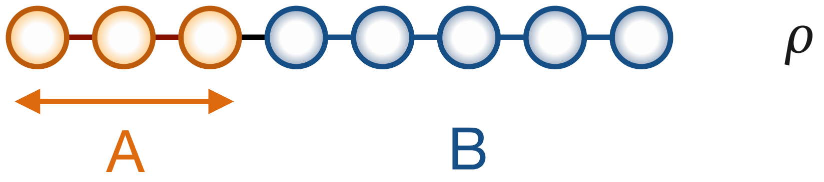


# Renyi (Entanglement) Entropies from Randomized Measurements

theory and a trapped-ion experiment

T Brydges, A Elben, P Jurcevic, B Vermersch, C Maier, BP Lanyon, P. Z., R Blatt, CF Roos, Science 2019  theory + experiment]

# Measuring Renyi Entanglement Entropy




$\mathrm{Tr}_A\, \rho_A^2$ Renyi entropy n=2

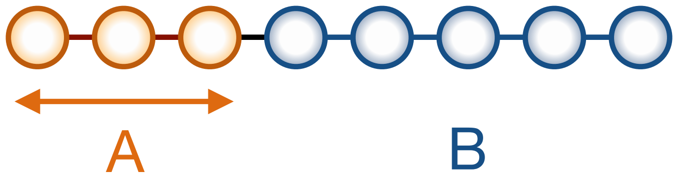
~ purity of subsystem

nonlinear functional of density matrix

but expectation values are always linear: $\langle \hat{A} \rangle = \mathrm{Tr}[\hat{A}\rho]$ :-(

# Measuring Renyi Entanglement Entropy

## Protocol 0: Tomography




$\mathrm{Tr}_A\, \rho_A^2$ Renyi entropy n=2

~ purity of subsystem

✓ expensive *

✓ if exists

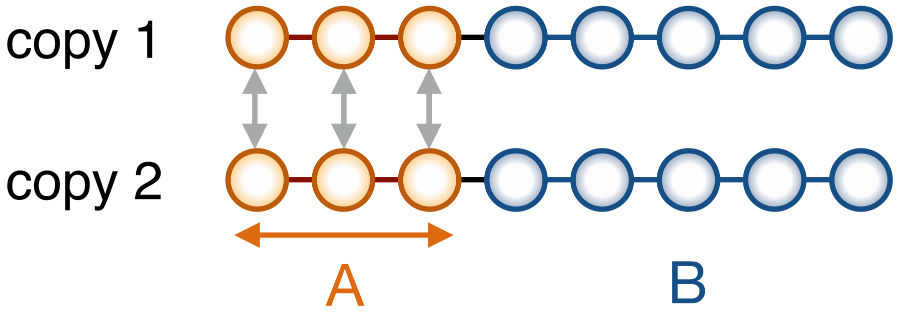
* except very small (sub)systems,
or we know something about quantum state
`importance sampling'

G Torlai, RG Melko, Machine learning quantum states in the NISQ era, arXiv:1905.04312

G Carleo, JI Cirac, K Cranmer, L Daudet, M Schuld, N Tishby, L Vogt-Maranto, L Zdeborová, Machine learning and the physical sciences, arXiv:1903.10563

# Measuring Renyi Entanglement Entropy

## Protocol 1: Copies of the quantum system




$\rho \otimes \rho$

swap as *quantum* operation between copies

$$\mathrm{Tr}_A \rho_A^2 = \mathrm{Tr}_A S (\rho_A \otimes \rho_A) \equiv \langle S \rangle$$

$$S \,|i\rangle_1 \otimes |k\rangle_2 = |k\rangle_1 \otimes |i\rangle_2$$

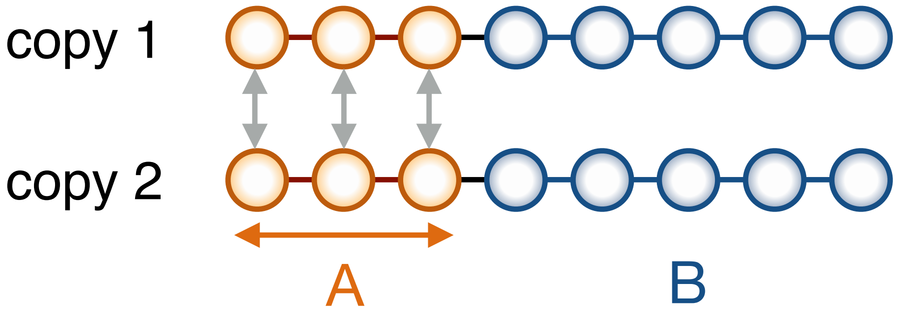
swap operator

provided we can implement, *and* measure *S*

theory: AJ Daley, H Pichler, J Schachenmayer, PZ, PRL (2012); C Moura Alves & D Jaksch, PRL (2004); A. K. Ekert PRL (2002).

# Measuring Renyi Entanglement Entropy

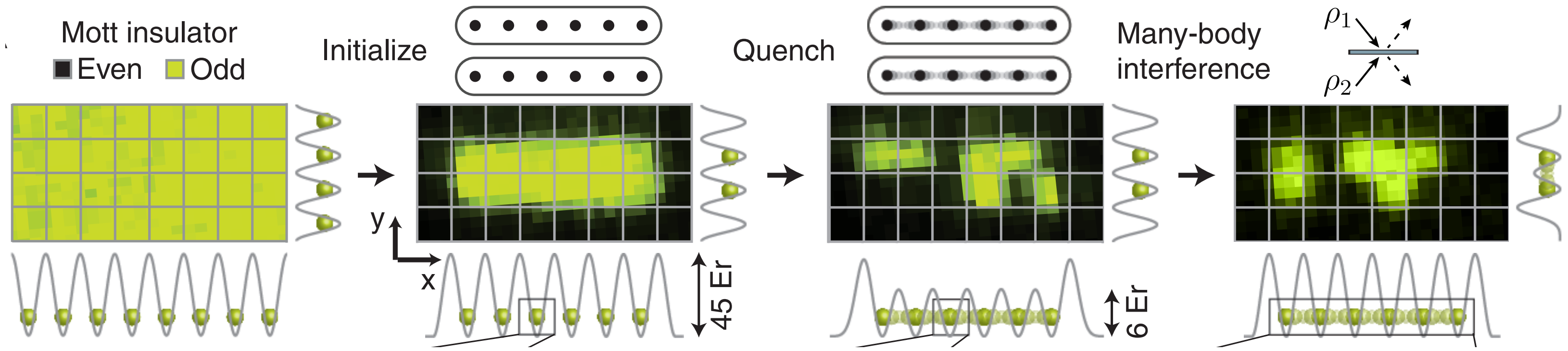
## Protocol 1: Copies of the quantum system




$\rho \otimes \rho$

*quantum* operations between copies

### Controlled few-atom systems & quantum gas microscope




experiment: R Islam *et al.*, Nature (2015); AM Kaufmann *et al.*, Science (2016) [Greiner Group]

experiment: R Islam *et al.*, Nature (2015); AM Kaufmann *et al.*, Science (2016) [Greiner Group]

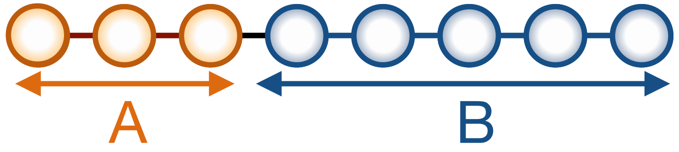
# Measuring Renyi Entanglement Entropy


A Elben


B Vermersch

## Protocol 2: Single copy of quantum system

single system




$\rho \otimes \rho$ ?

virtual copy*
(replica trick)

how?

$\mathrm{Tr}_A \, \rho_A^2$ ...

from Statistical Correlations
in Random Measurements

*signal is in the noise*

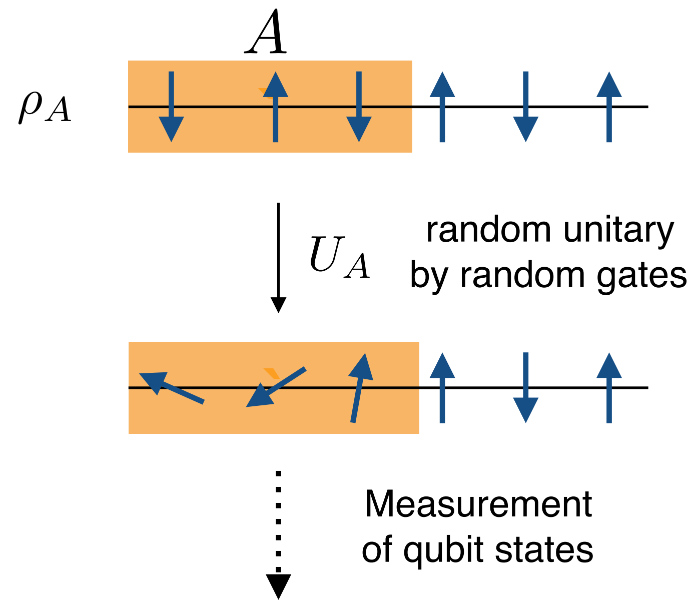
* in contrast to real copies, virtual copies are legal in quantum mechanics

# Statistical Correlations in Random Measurements

## Protocol for a chain of qubits:

### Random measurement

$$P_U(\mathbf{s}_A) = \mathrm{Tr}\left[\boldsymbol{U_A \rho_A U_A^\dagger} \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$




$$(\,0\,,\,1\,,\,0\,) = \mathbf{s}_A$$

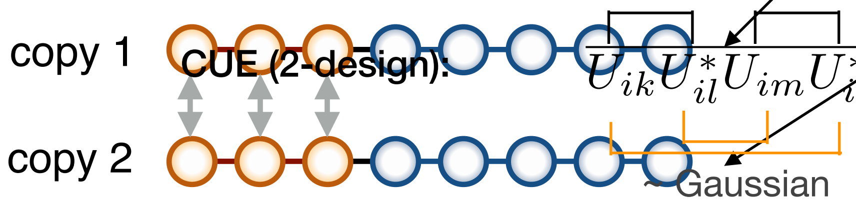
### Average over the Circular Unitary Ensemble (CUE)

$$\overline{P_U(\mathbf{s}_A)} = \frac{1}{N_{\mathcal{H}_A}} \qquad \overline{P_U(\mathbf{s}_A)^2} = \frac{1 + \mathrm{Tr}\left[\rho_A^2\right]}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$

Hilbertspace dimension of A

### Virtual copies (2-design):

$$\overline{P_U(\mathbf{s}_A)^2} = \overline{\mathrm{Tr}_{1\oplus 2}\left[\ldots U_A \rho_A U_A^\dagger \otimes U_A \rho_A U_A^\dagger\right]} = \frac{\mathrm{Tr}_{1\oplus 2}\left[(1+S)\rho_A \otimes \rho_A\right]}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$




CUE (2-design):

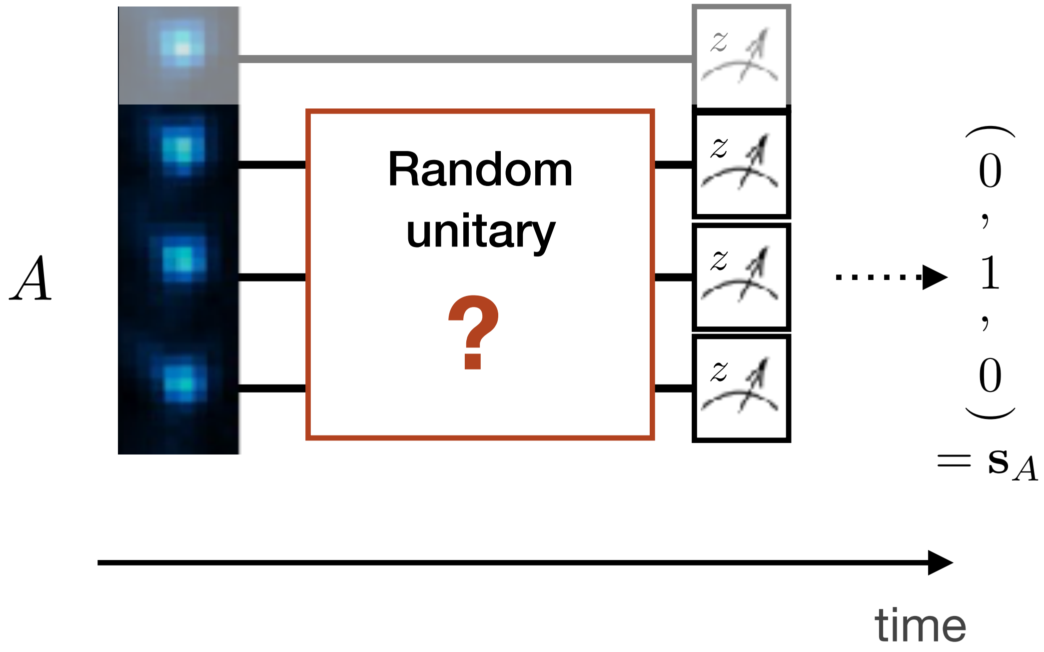
$$\overline{U_{ik} U_{il}^* U_{im} U_{in}^*} = \frac{\delta_{kl}\delta_{mn} + \delta_{kn}\delta_{ml}}{N_{\mathcal{H}_A}(N_{\mathcal{H}_A}+1)}$$

~Gaussian

S van Enk, C Beenakker (PRL 2012)

# Randomized Measurements

## Quantum Computer



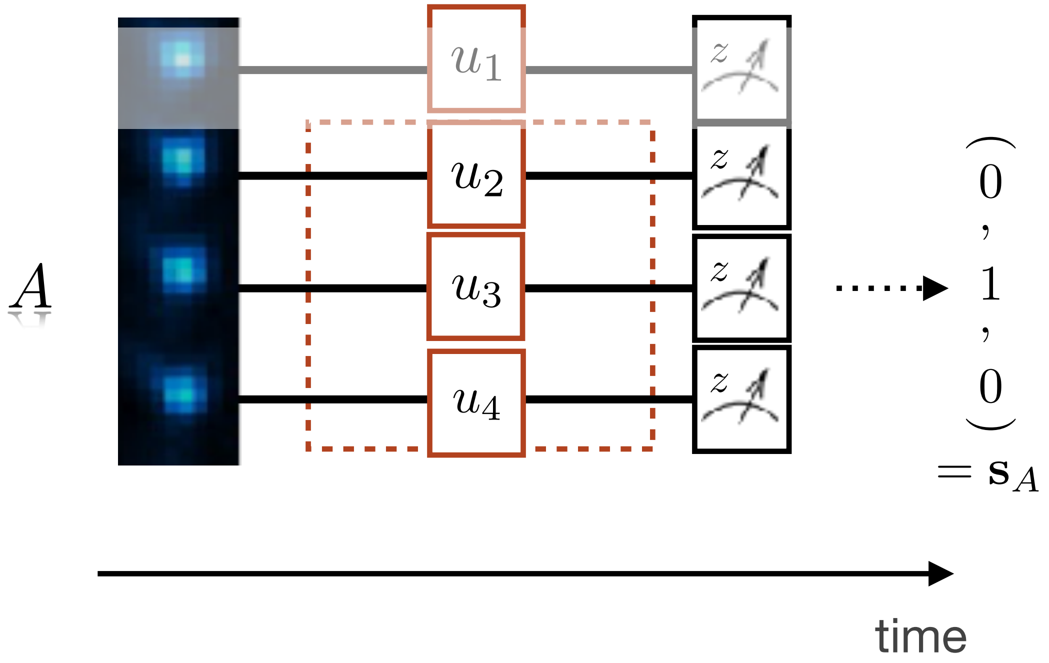

$$P_U(\mathbf{s}_A) = \mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$

with $U_A \in \mathrm{CUE}$

S van Enk, C Beenakker (PRL 2012)    A. Elben, B. Vermersch, M. Dalmonte, J. I. Cirac, and P. Zoller (PRL2018)

# Randomized Measurements

## *Programmable* Quantum Simulator




$$P_U(\mathbf{s}_A) =$$
$$\mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$

$$U_A = \bigotimes u_i \qquad u_i \in \mathrm{CUE}(d)$$

Random *single spin* rotations are sufficient!

$$\overline{P_U(s_A)P_U(s'_A)} \sim \mathrm{Tr}\rho_A^2$$

cross correlations

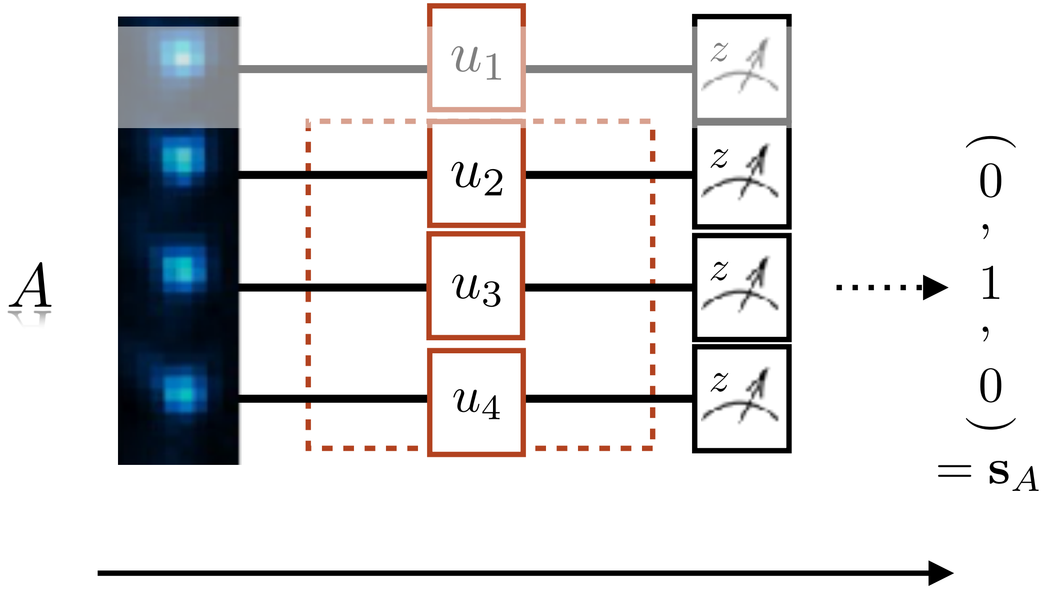
T Brydges, A Elben, P Jurcevic, B Vermersch, C Maier, BP Lanyon, P.Z., R Blatt, CF Roos, Science 2019

# Randomized Measurements

## *Programmable* Quantum Simulator


A Elben


B Vermersch




$$P_U(\mathbf{s}_A) = \mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|\mathbf{s}_A\right\rangle \left\langle \mathbf{s}_A\right|\right]$$

$$U_A = \bigotimes u_i \qquad u_i \in \mathrm{CUE}(d)$$

Random *single spin* rotations are sufficient!

$$\mathrm{Tr}\rho_A^2 = \overline{X_U}$$

with

$$X_U = 2^{N_A} \sum_{s_A, s'_A} (-2)^{-D[s_A, s'_A]} P_U(s_A) P_U(s'_A)$$

Hamming distance

random variable

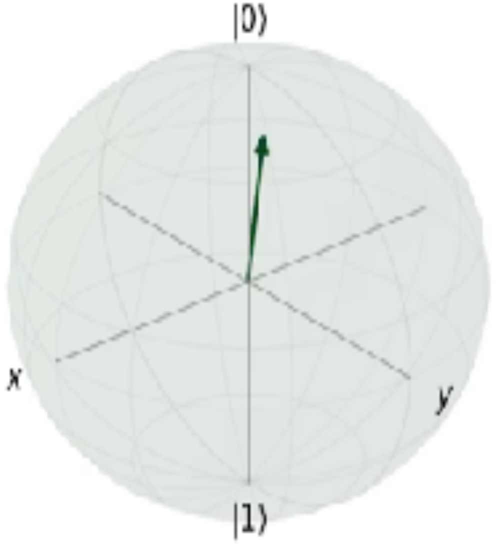
Cross correlation between configurations

# Example 1: Single qubit - pure state

## State on the Bloch sphere

$$\rho = \frac{1}{2}\left(\mathbb{1}_2 + \vec{a}\cdot\vec{\sigma}\right)$$

$: |\vec{a}| = 1$



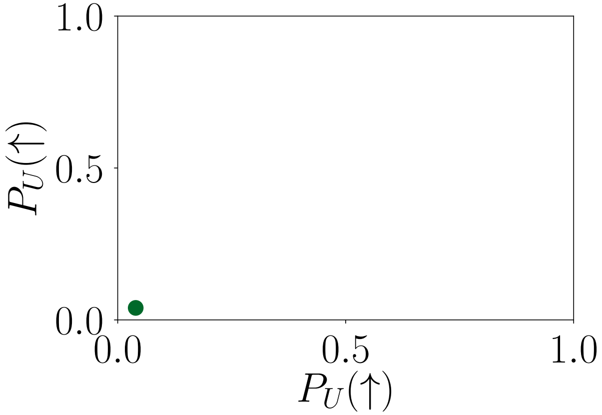

## Random Measurement

Rotation: $U\rho U^\dagger = \frac{1}{2}\left(\mathbb{1}_2 + \mathcal{R}_U(\vec{a})\cdot\vec{\sigma}\right)$

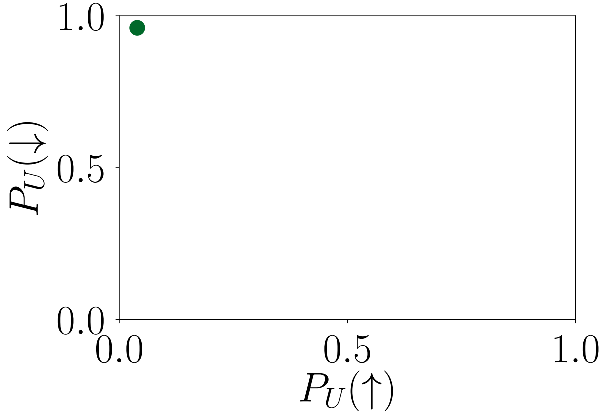
z-Measurements: $P_U(\uparrow), P_U(\downarrow)$

## Correlations of probabilities

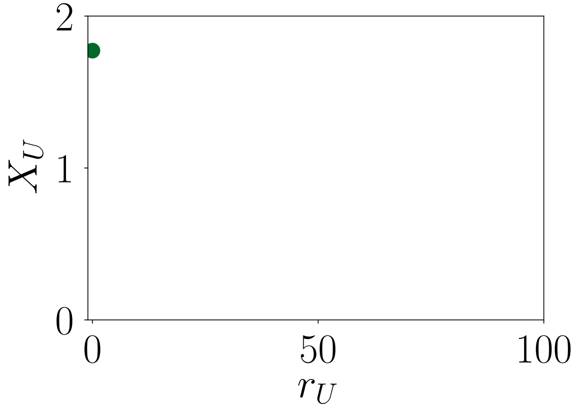
$P_U(\uparrow) \leftrightarrow P_U(\uparrow)$




$P_U(\uparrow) \leftrightarrow P_U(\downarrow)$




$$X_U = 2\left[P_U(\uparrow)^2 + P_U(\downarrow)^2 - P_U(\uparrow)P_U(\downarrow)\right]$$




# unitaries

# Example 1: Single qubit - pure state

## State on the Bloch sphere

$$\rho = \frac{1}{2}\left(\mathbb{1}_2 + \vec{a}\cdot\vec{\sigma}\right)$$

■ : $|\vec{a}| = 1$

## Random Measurement

Rotation: $U\rho U^\dagger = \frac{1}{2}\left(\mathbb{1}_2 + \mathcal{R}_U(\vec{a})\cdot\vec{\sigma}\right)$

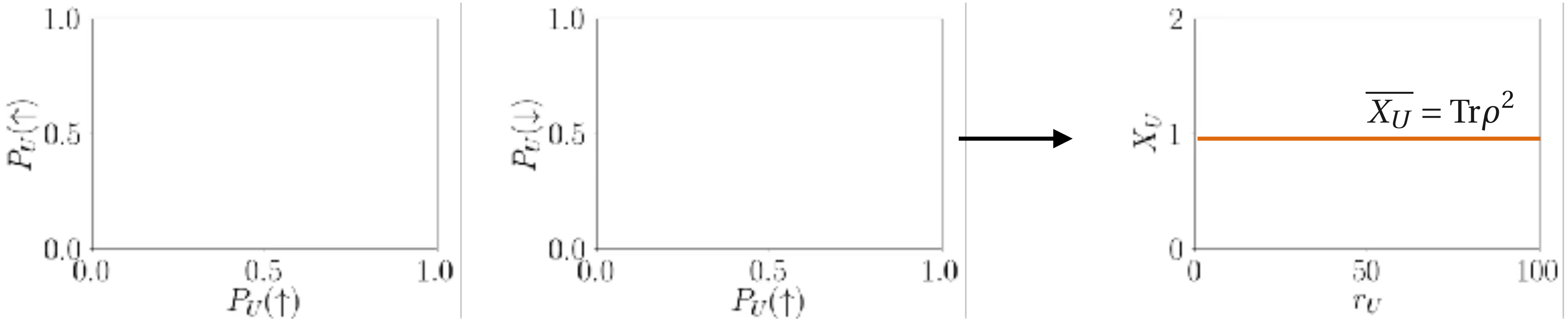
z-Measurements: $P_U(\uparrow), P_U(\downarrow)$

## Correlations of probabilities

$P_U(\uparrow) \leftrightarrow P_U(\uparrow)$

$P_U(\uparrow) \leftrightarrow P_U(\downarrow)$

$$X_U = 2\left[P_U(\uparrow)^2 + P_U(\downarrow)^2 - P_U(\uparrow)P_U(\downarrow)\right]$$




# unitaries

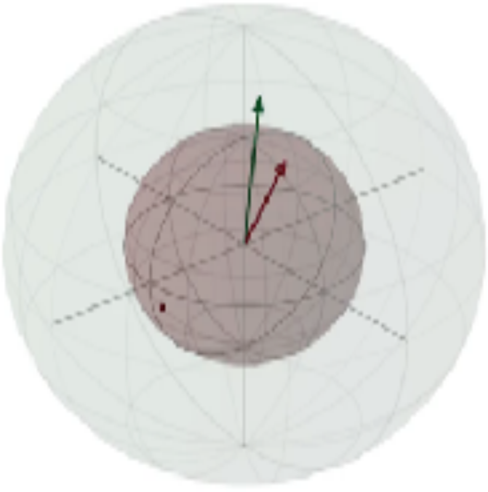
# Example 2: Single qubit - mixed state

## State on the Bloch sphere

$$\rho = \frac{1}{2}\left(\mathbb{1}_2 + \vec{a}\cdot\vec{\sigma}\right)$$

Green: $|\vec{a}| = 1$

Red: $|\vec{a}| = 0.5$



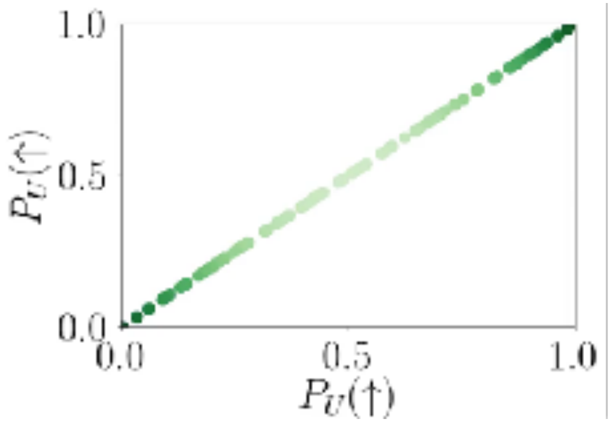
## Random Measurement

Rotation: $U\rho U^\dagger = \frac{1}{2}\left(\mathbb{1}_2 + \mathcal{R}_U(\vec{a})\cdot\vec{\sigma}\right)$

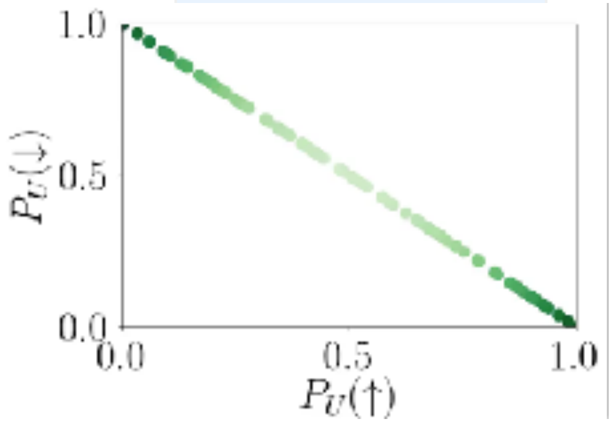
z-Measurements: $P_U(\uparrow), P_U(\downarrow)$

## Correlations of probabilities

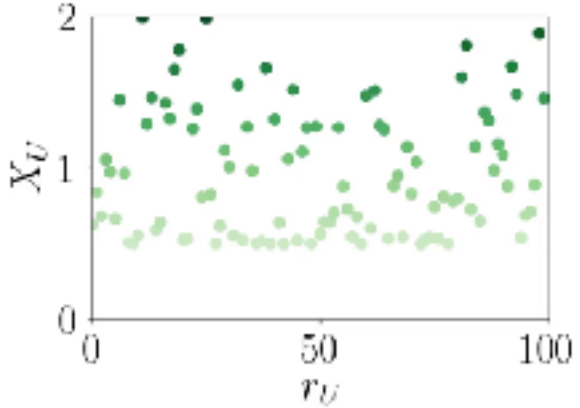
$P_U(\uparrow) \leftrightarrow P_U(\uparrow)$




$P_U(\uparrow) \leftrightarrow P_U(\downarrow)$




$$X_U = 2\left[P_U(\uparrow)^2 + P_U(\downarrow)^2 - P_U(\uparrow)P_U(\downarrow)\right]$$

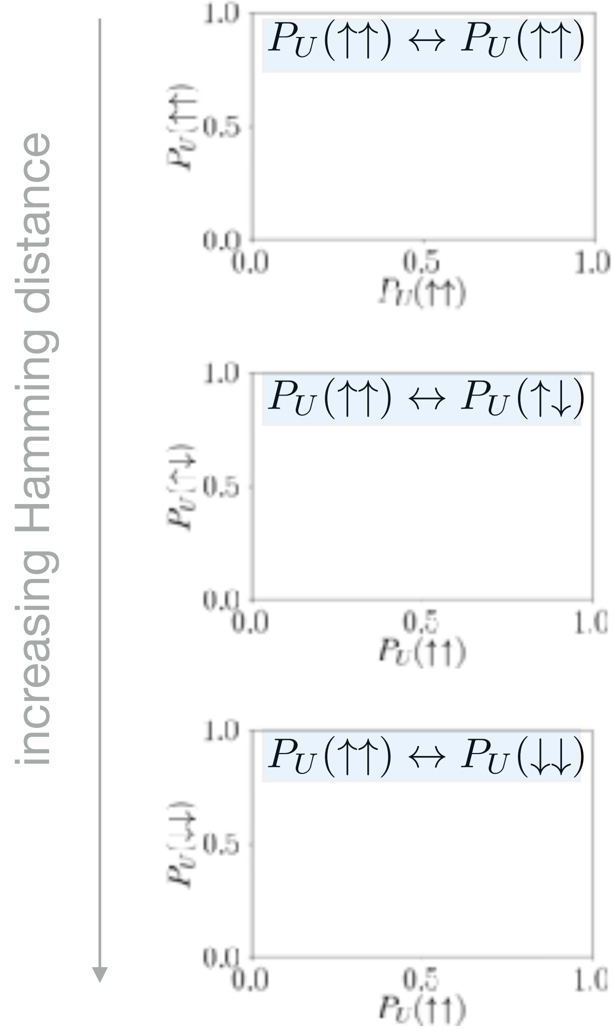



# unitaries

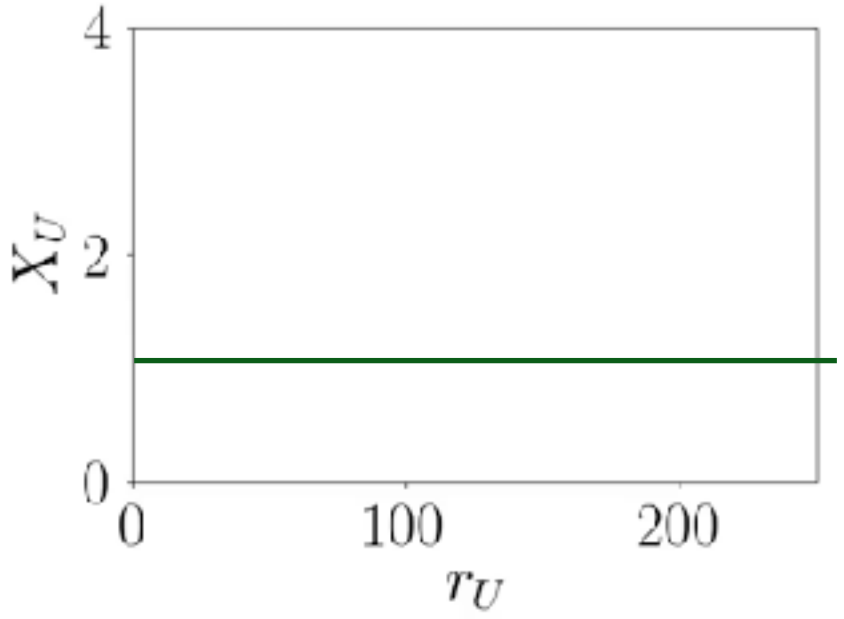
# Example 3: Two qubit — pure product state

$\rho = |\uparrow\uparrow\rangle\langle\uparrow\uparrow|$

## Correlations of probabilities




$$
\begin{aligned}
X_U =& 4\left(P_U(\uparrow\uparrow)P_U(\uparrow\uparrow) + \dots\right) \\
&- 4\left(P_U(\uparrow\uparrow)P_U(\uparrow\downarrow) + \dots\right) \\
&+ 2\left(P_U(\uparrow\uparrow)P_U(\downarrow\downarrow) + \dots\right)
\end{aligned}
$$

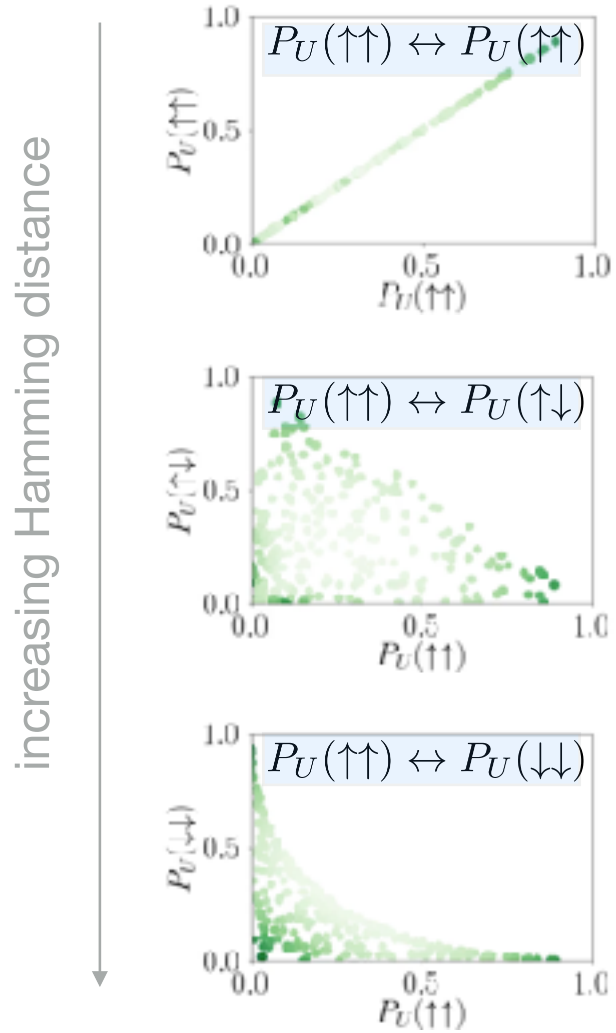



# unitaries

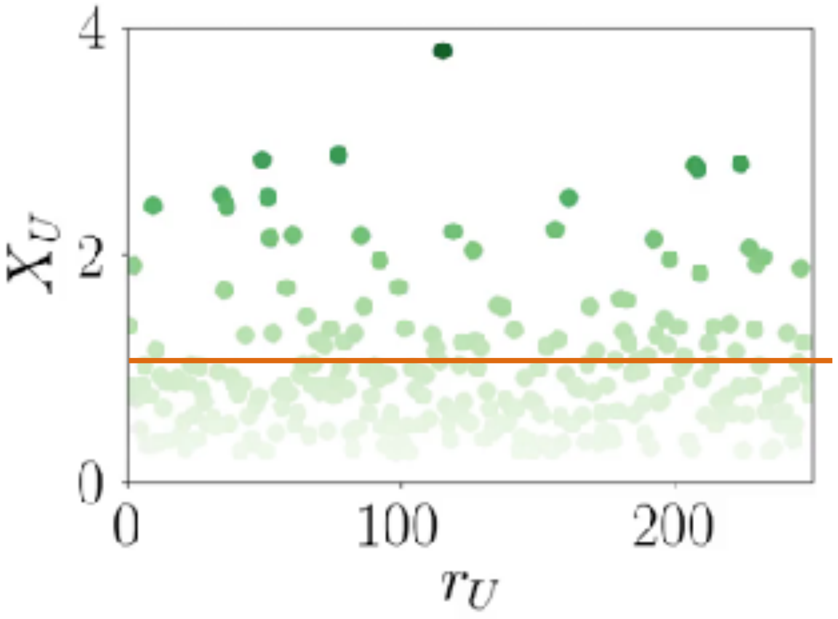
# Example 4: Two qubit — pure entangled state

- $\rho = \frac{1}{2}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)(\langle\uparrow\uparrow| + \langle\downarrow\downarrow|)$

## Correlations of probabilities




$$\begin{aligned} X_U = & \, 4\,(P_U(\uparrow\uparrow)P_U(\uparrow\uparrow) + \dots) \\ & - 4\,(P_U(\uparrow\uparrow)P_U(\uparrow\downarrow) + \dots) \\ & + 2\,(P_U(\uparrow\uparrow)P_U(\downarrow\downarrow) + \dots) \end{aligned}$$

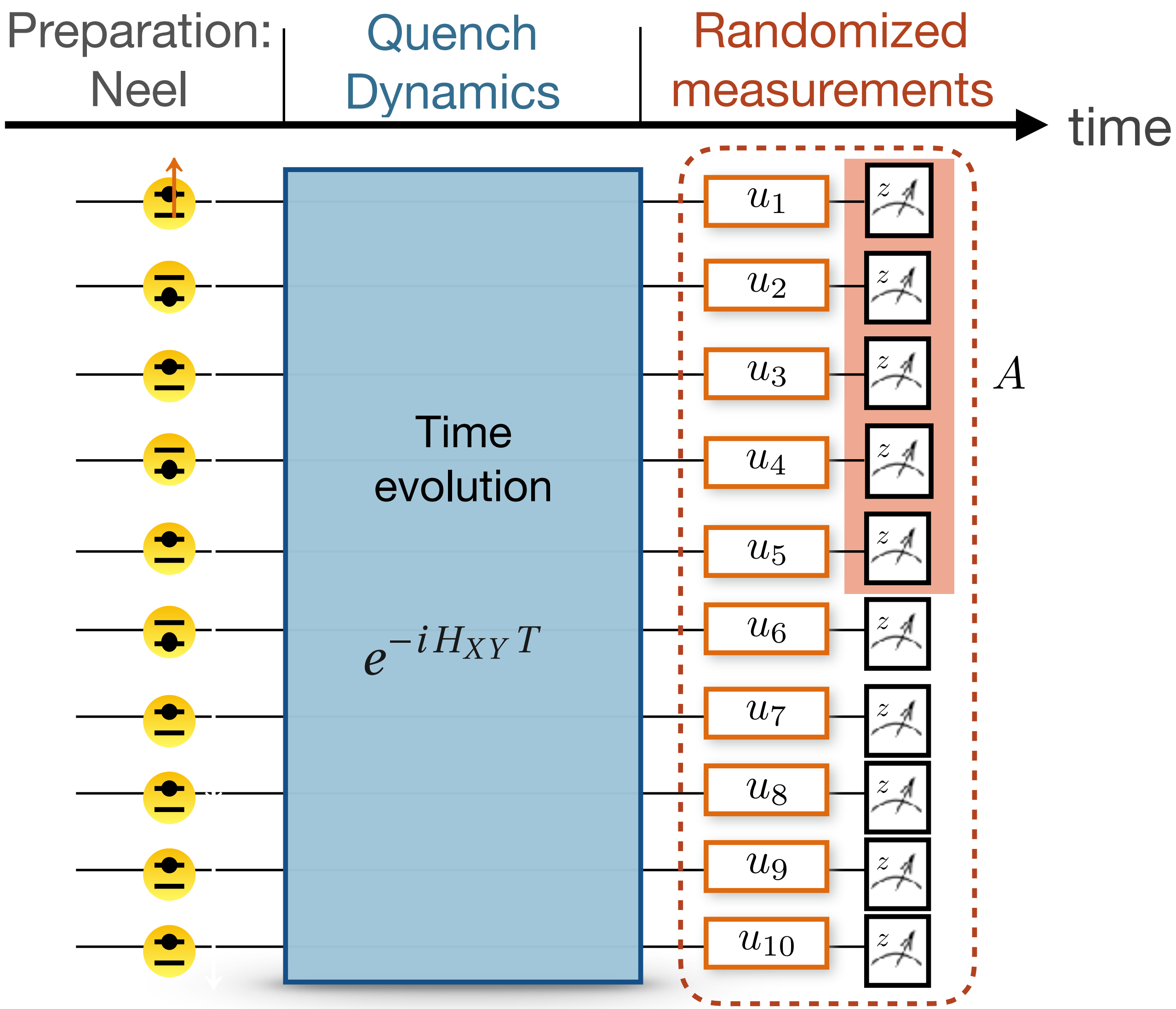



# unitaries

… reminiscent of correlation in Bell inequalities

# Entanglement in Quench Dynamics

## Quantum circuit on programmable quantum simulator




### Hamiltonian

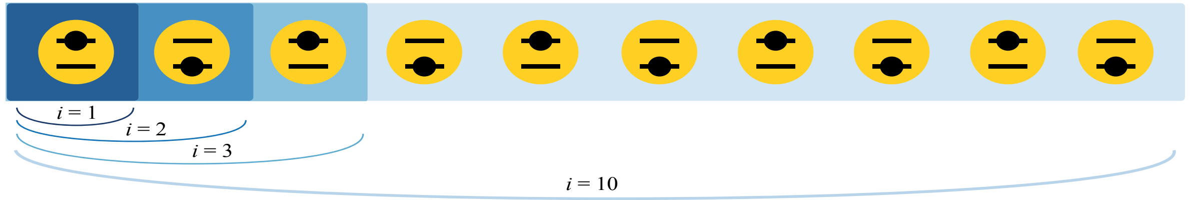
$$H_{XY} = \hbar \sum_{i<j} J_{ij} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+)$$

long range interaction

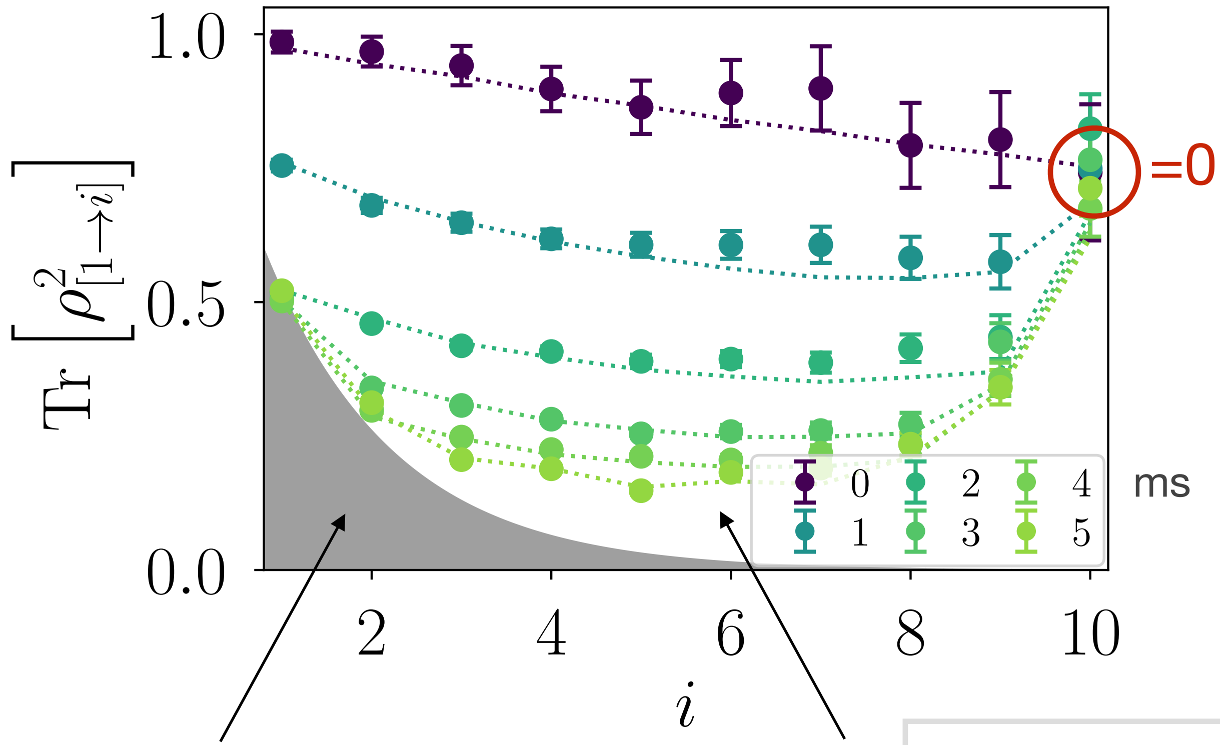
$$+ \hbar \sum_j (B + b_j) \sigma_j^z$$

local disorder potentials

# 10 Ions [no disorder]

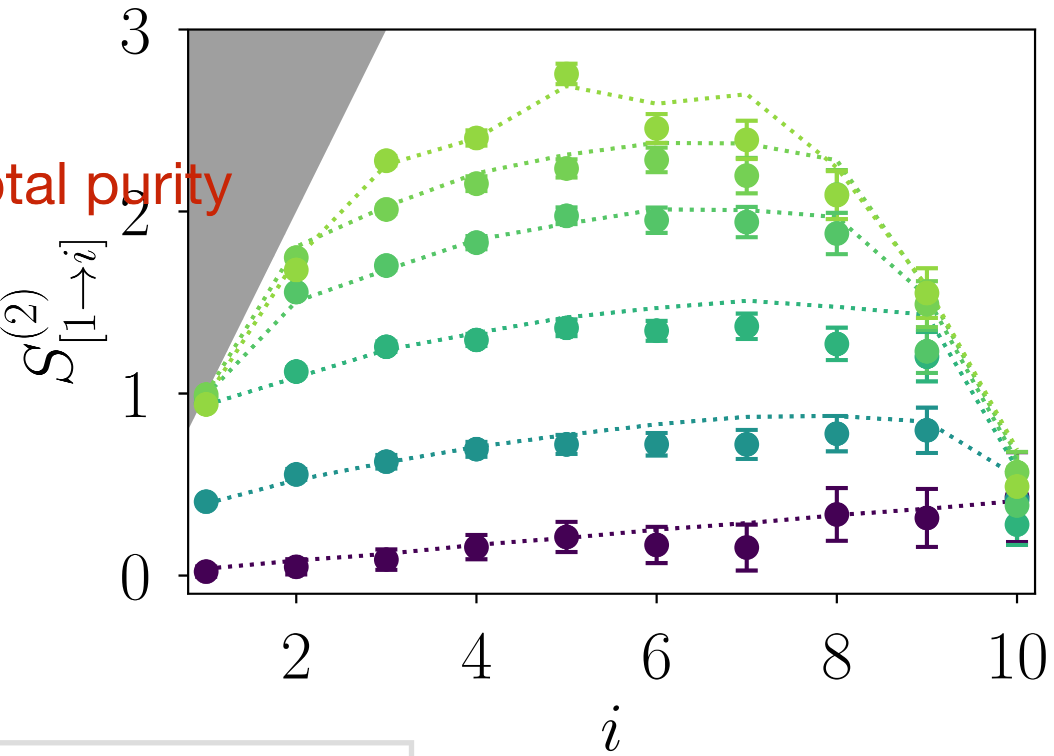



purity $\mathrm{Tr}[\rho_A^2]$

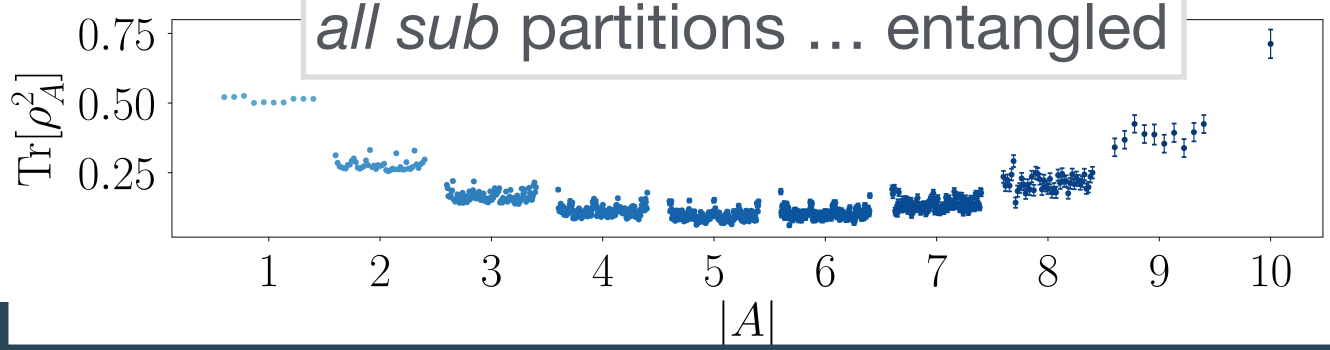



maximally mixed

Renyi entropy




*all sub* partitions … entangled




T Brydges, A Elben et al., Science 2019

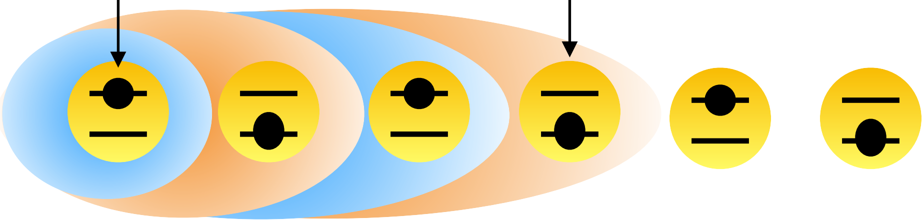
# OTOCs
# from Randomized Measurements

theory and an trapped-ion experiment

B. Vermersch,* A. Elben, L. M. Sieberer, N.Y. Yao, and P.Z., PPRX 2019
M. Joshi, A Elben et al., draft

# OTOCs & Scrambling of Quantum Information

$W = \sigma_1^z$ $\quad$ $V = \sigma_i^z$



$$W(t) = U^\dagger(t) W U(t)$$

Time-evolved Heisenberg operator

## Out-of-time ordered correlator

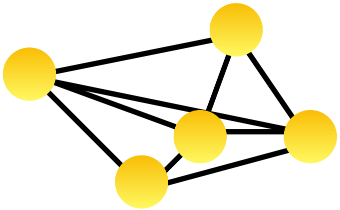
$$\begin{aligned} O(t) &= \mathrm{Tr}(\rho W(t)^\dagger V^\dagger W(t) V) \\ &\sim 1 - \mathrm{Tr}(|[W(t), V]|^2 \rho) \end{aligned}$$

$$\rho = I/\mathcal{N}_{\mathcal{H}}$$

infinite temperature

$$O(0) = 1 \longleftrightarrow O(t \gg 0) < 1$$

## Fastest scrambler in nature?



$$H_{SYK} = \frac{1}{4!} \sum_{jklm} J_{jklm} \chi_j \chi_k \chi_l \chi_m$$

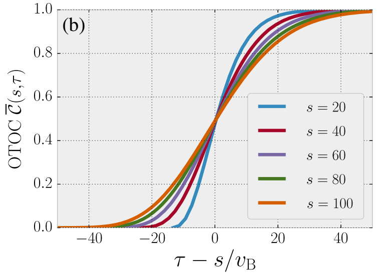
$$O(t) \sim c_0 - c_1 e^{\lambda_T t}$$

$$\lambda_T \leq \frac{2\pi}{\beta}$$

Sachdev, et al. PRL 1993,
Kitaev, A. KITP 2015,
Maldacena, et al., JHEP 2016
Hosur et al., JHEP 2016, …

## Quantum many-body: thermalization vs. localization

### Chaotic many-body systems

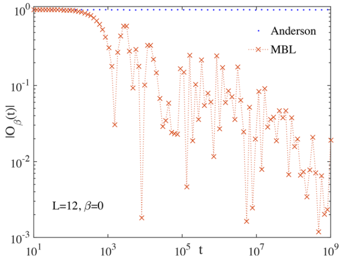



Bohrdt et al., New J. Phys. 2017,
Nahum et al., PRX 2018,
C. W. von Keyserlingk et al., PRX 2018,
M. C. Tran, et al. arXiv:1808.05225,
Lewis-Swan et al., Nat. Comm. 2019, …

operator wavefront

$$O(t) \sim c_0 - c_1 e^{\lambda_T (t - r/v_B)}$$

### (Many-body) localized systems



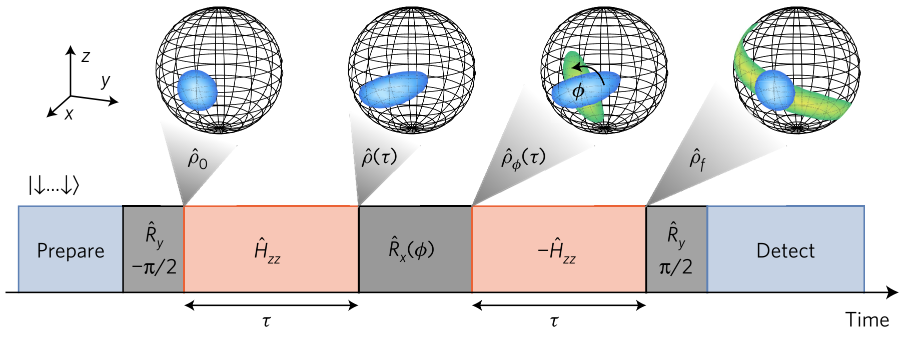

Fan, et al. Sci. Bul. 2017
Chen, X et al. Ann. Phys. 2017, …

slow powerlaw decay with scale

$$t_0 \sim e^{-|i-1|/\xi}$$

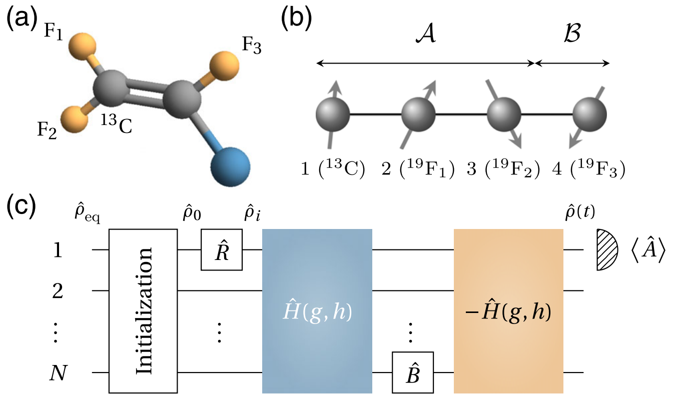
# OTOC Measurement Protocols

## Trapped ions (all-to-all interactions)



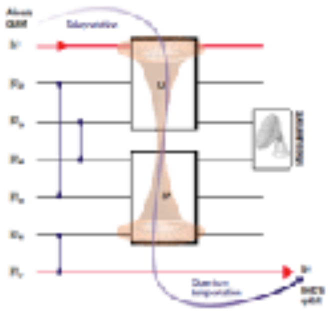

M. Gärttner, J. G. Bohnet, A. Safavi-Naini, M. L. Wall, J. J. Bollinger, and A. M. Rey, Nat. Phys. 2017

## NMR: Trotter evolution




J. Li, R. Fan, H. Wang, B. Ye, B. Zeng, H. Zhai, X. Peng, and J. Du, Phys. Rev. X 7, 2017

## Trapped ions (Gates+Teleportation)



K. A. Landsman, C. Figgatt, T. Schuster, N. M. Linke, B. Yoshida, N. Y. Yao, C. Monroe, Nature 2019

**Key challenges:** → **Implementing time-reversal** → **The role of decoherence**

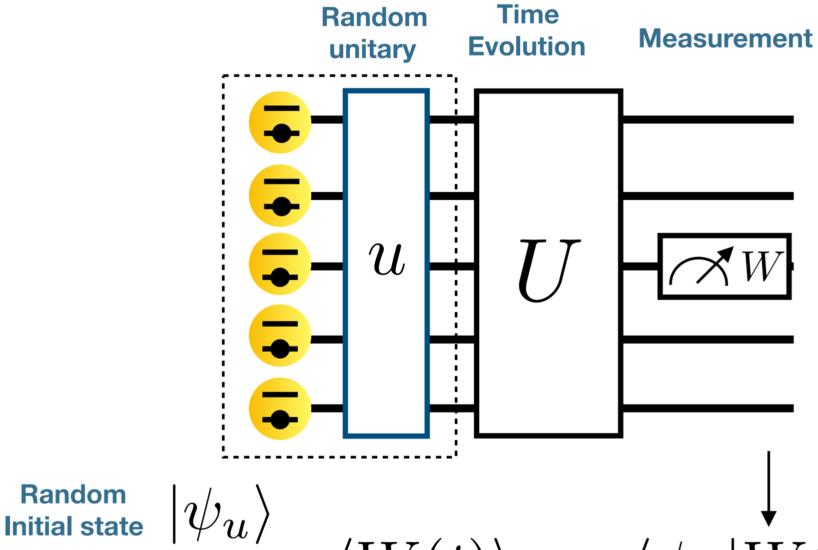


**How to measure OTOCs in a many-body system with local interactions - without time-reversal?**

see also: Viewpoint on Physics by Yao and Swingle

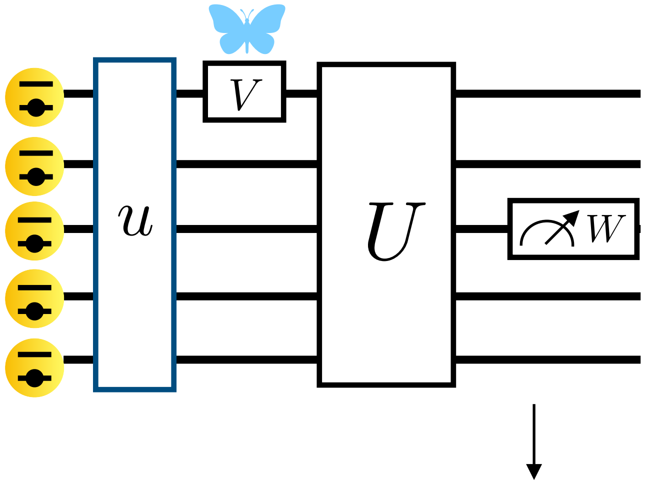
# Protocol 1: *Global* Random Unitary

## Experiment 1



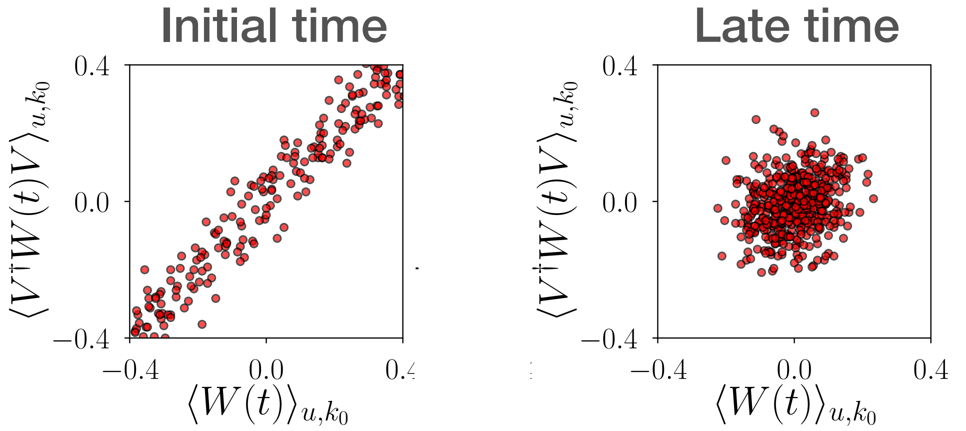

$$\langle W(t)\rangle_u = \langle \psi_u | W(t) | \psi_u \rangle$$

## Experiment 2




$$\langle V W(t) V\rangle_u = \langle \psi_u V | W(t) | V \psi_u \rangle$$

## Correlations of the two experiments



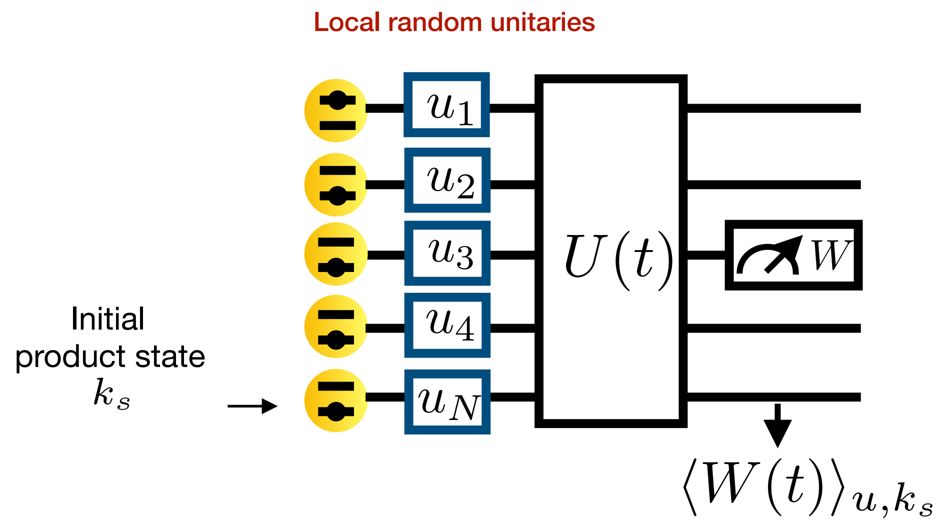

## OTOC (infinite temperature)

$$\frac{1}{\mathcal{N}} \overline{\langle W(t)\rangle_u \langle V W(t) V\rangle_u}$$

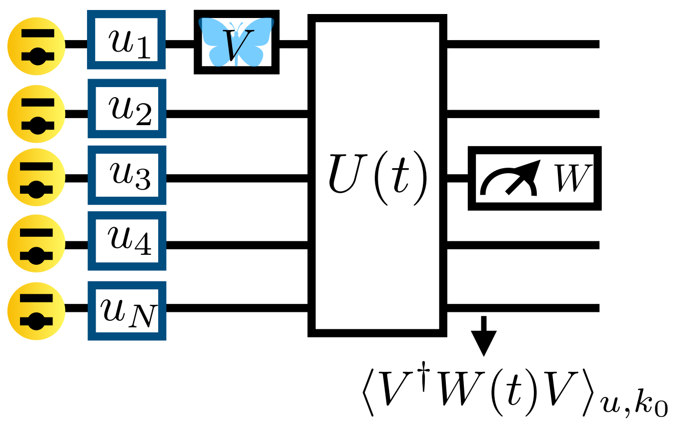
$$= \mathrm{Tr}\left[W(t) V W(t) V\right]$$

# Protocol 2: *Local* Random Unitary

## Experiment 1




## Experiment 2




## Statistical correlations of locally randomized measurements map to Modified OTOCs:

$$O_n(t) = \frac{1}{\mathcal{D}_n^{(\mathrm{L})}} \sum_{k_s \in E_n} c_{k_s} \overline{\langle W(t)\rangle_{u,k_s} \langle V^\dagger W(t) V\rangle_{u,k_0}},$$

Set of $2^n$ initial states, n=0,..,N

$$= \frac{\sum_{A,B_n \subseteq A} \mathrm{Tr}_A\left(W(t)_A (VW(t)V)_A\right)}{\sum_{A,B_n \subseteq A} \mathrm{Tr}_A\left(W(t)_A W(t)_A\right)}$$

Sum of (out-of-time ordered) correlation functions

$B_n = \{1, \ldots, n\}$

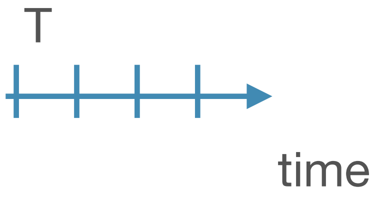
→ $O_N(t) = O(t) = \mathrm{Tr}\left[W(t)VW(t)V\right]$

Fast converging series: n=0,1,2
is generically sufficient

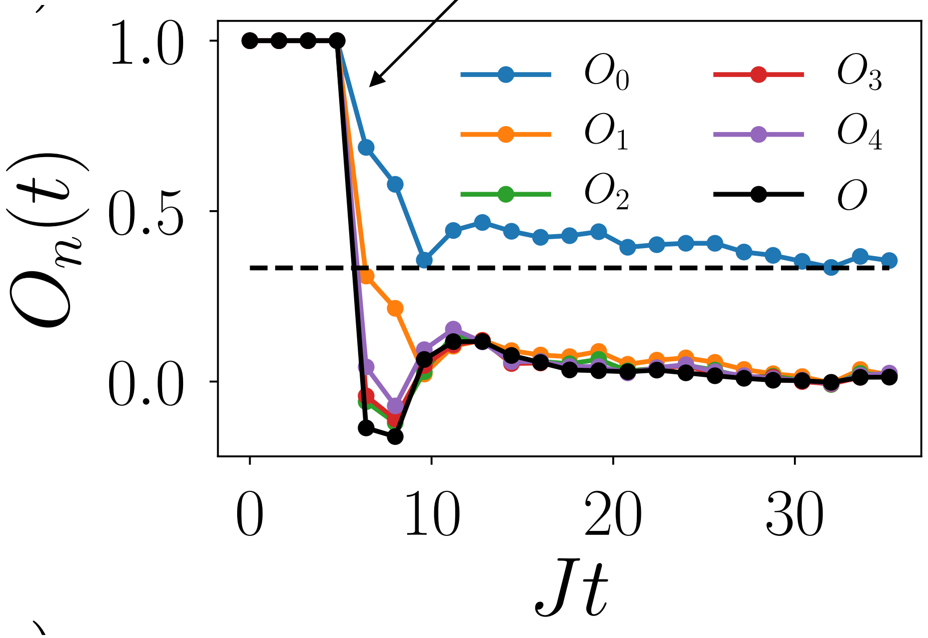
# Example: Quantum Many-Body Chaos

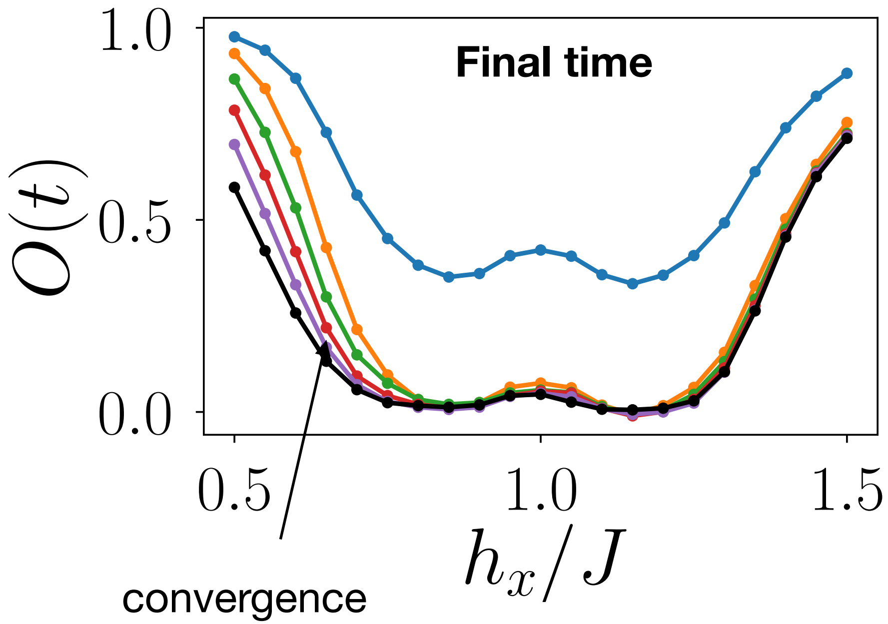
## Kicked Ising model on N=8 sites

$$U(t=nT) = \left[\exp\left(-i\frac{T}{2}[\sum_i \sigma_i^z\sigma_{i+1}^z + h_z\sum_i \sigma_i^z]\right)\exp\left(-i\frac{T}{2}h_x\sum_i \sigma_i^x\right)\right]^n$$




decay time identified for all n=0,...

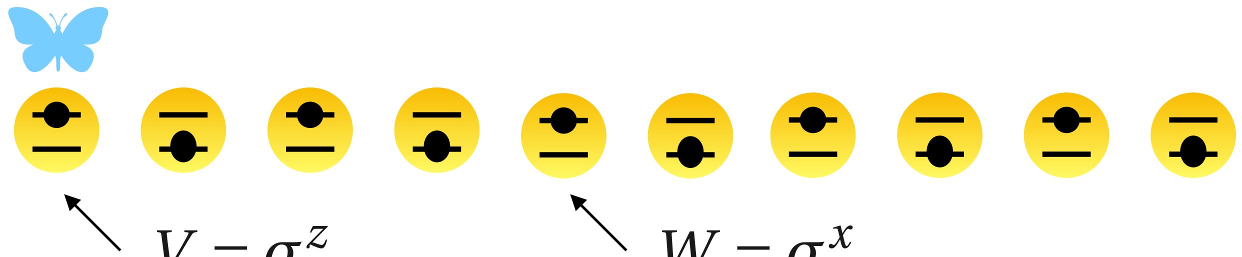



convergence with n

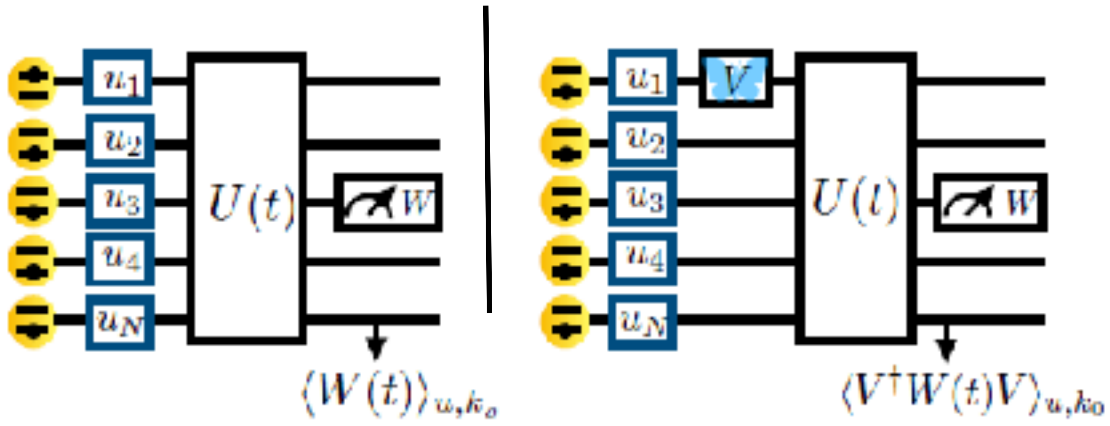
Vermersch et al., PRX 2019

# Trapped-Ion Experiment

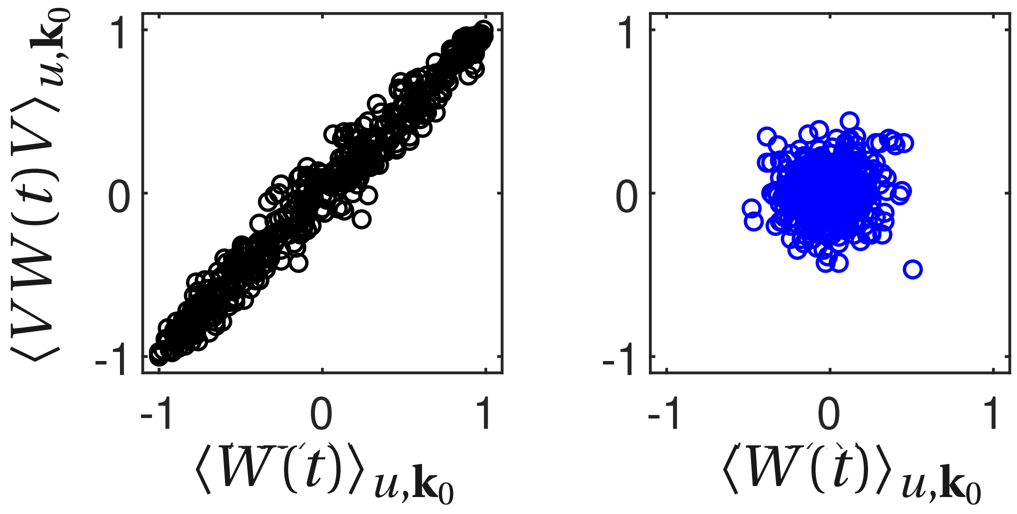


$V = \sigma_1^z$ $\quad W = \sigma_{j=5}^x$

Experiment 1 | Experiment 2




Cross correlations




At t=0 | at long time

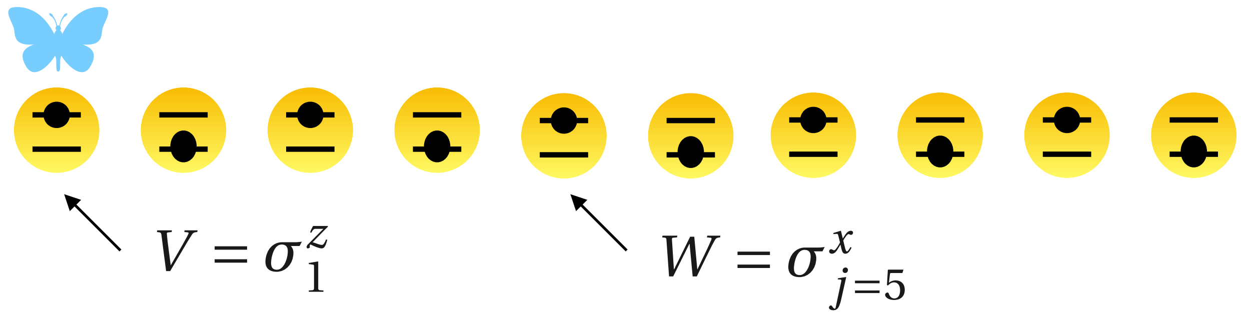
Correlations preserved | Loss of correlations

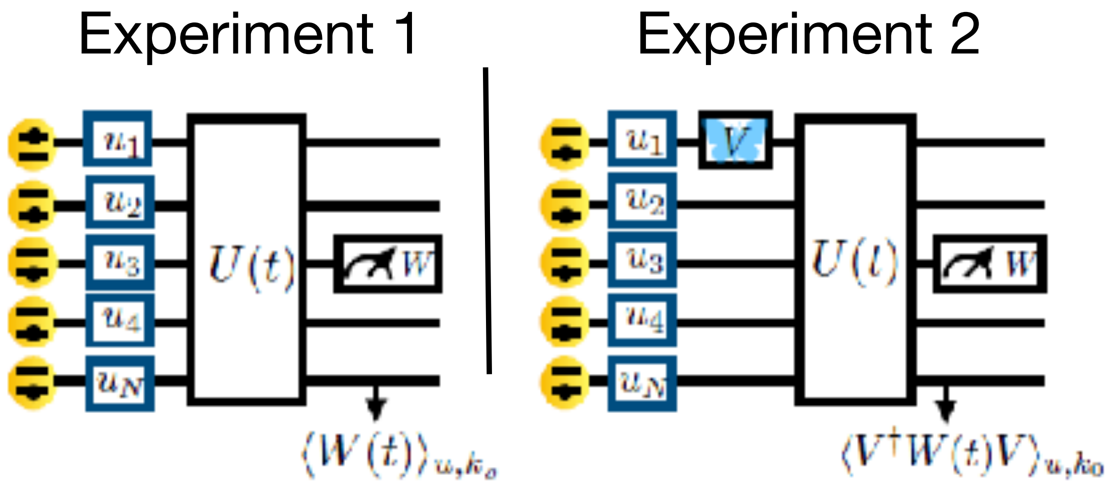
$\alpha \approx 1.21$

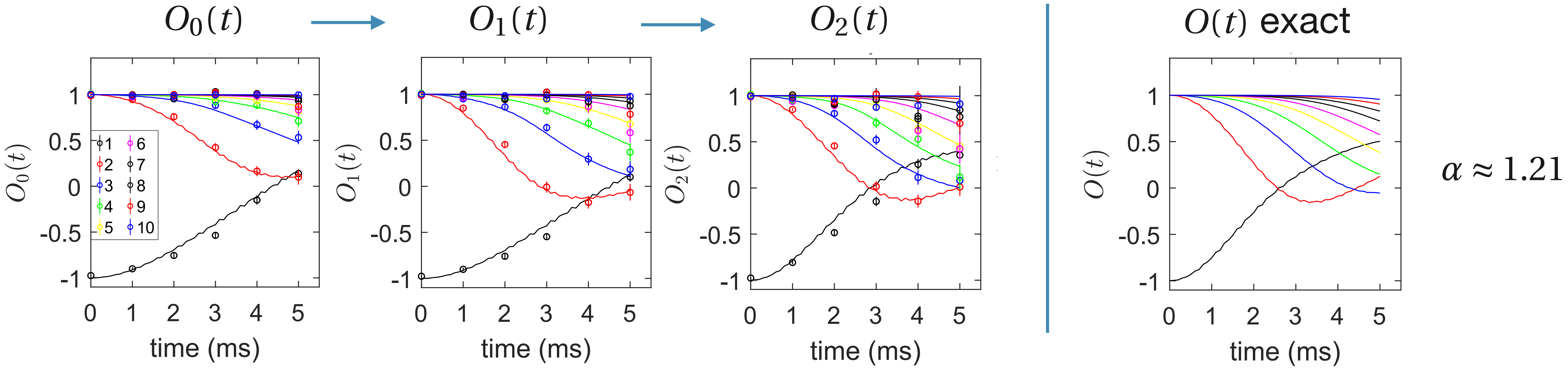
$N_U = 500$ unitaries

$N_M = 150$ measurements

# Trapped-Ion Experiment



# Statistical Errors and Decoherence

## Statistical Errors

$L(t) \approx v_B t$ "Scrambling length"

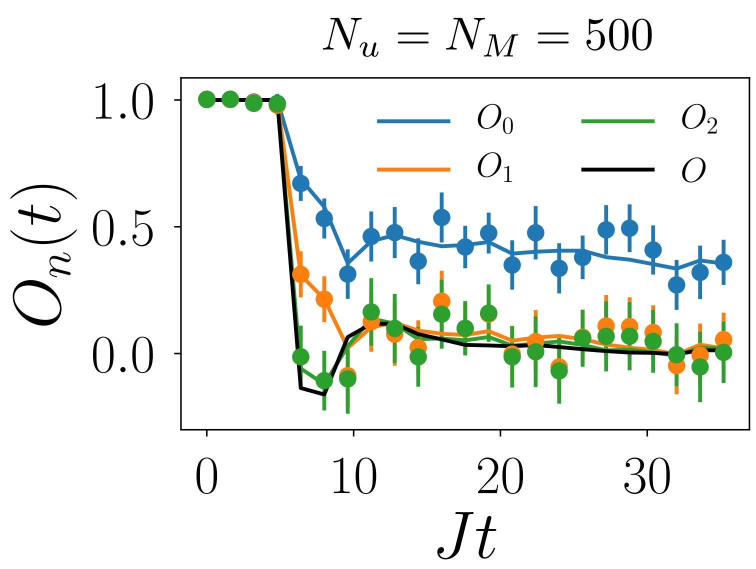
$N_M = 2^{L(t)} \longrightarrow$ error $1/\sqrt{N_u}$

# projective measurements

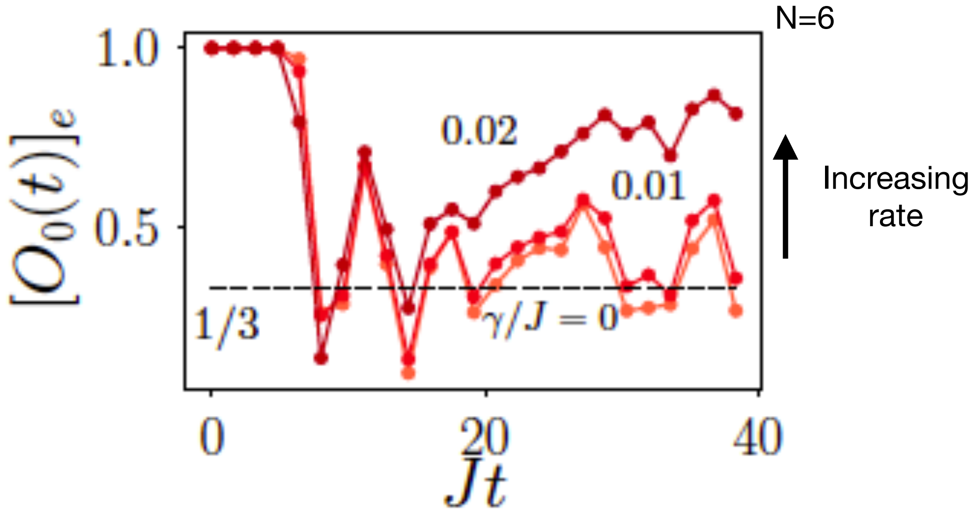
# unitaries

**Independent of System Size**

All N operators W are measured simultaneously




Kicked Ising model with spontaneous emission




## Decoherence

`robust'

- OTOCs from *normalized* statistical correlations

  ➡ e.g. Depolarization is scaled out

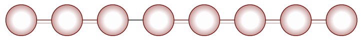
- Decoherence increases correlations (unique steady state)

  ➡ Opposite signal compared to unitary scrambling

# Cross-platform verification [Intermediate Scale Quantum Devices]

**Quantum machine 1**

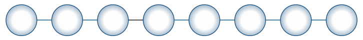
$|\Psi_1\rangle$



*Verify* that both have prepared the *same* state

**Quantum machine 2**

$|\Psi_2\rangle$



experiment vs. theory

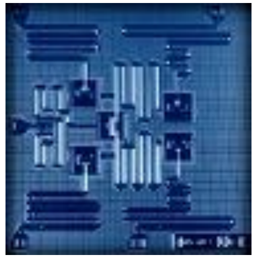
**Theory**

*known* quantum state

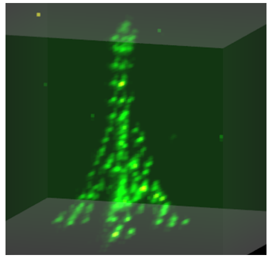
here: fidelities from statistical correlations between randomized measurements:
*no a-priori knowledge of quantum state*

Fidelity estimation from few Pauli measurements, ST Flammia & YK Liu (PR:L 2011):
for known theory state

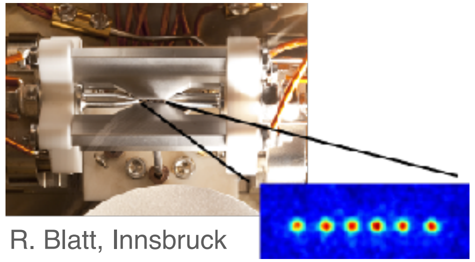
## Quantum computer / simulator / quantum sensor & different platforms



IBM chip



Barredo et al., Science (2016)



R. Blatt, Innsbruck

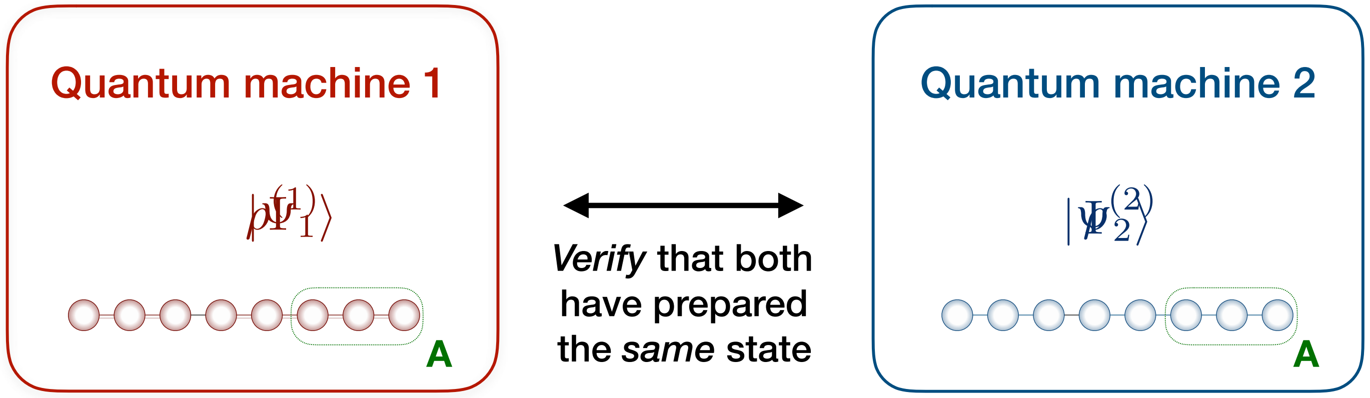
...

## Classical simulation

(if possible)

Classical computer (Jülich)

# Cross-verification




## Fidelity

For pure states $\mathcal{F}\left(|\Psi_1\rangle, |\Psi_2\rangle\right) = |\langle\Psi_1|\Psi_2\rangle|^2$

For mixed states

$$\mathcal{F}(\rho_A^{(1)}, \rho_A^{(2)}) = \frac{\mathrm{Tr}\left[\rho_A^{(1)}\rho_A^{(2)}\right]}{\max\left(\mathrm{Tr}\left[(\rho_A^{(1)})^2\right], \mathrm{Tr}\left[(\rho_A^{(2)})^2\right]\right)}$$

**Normalization:**

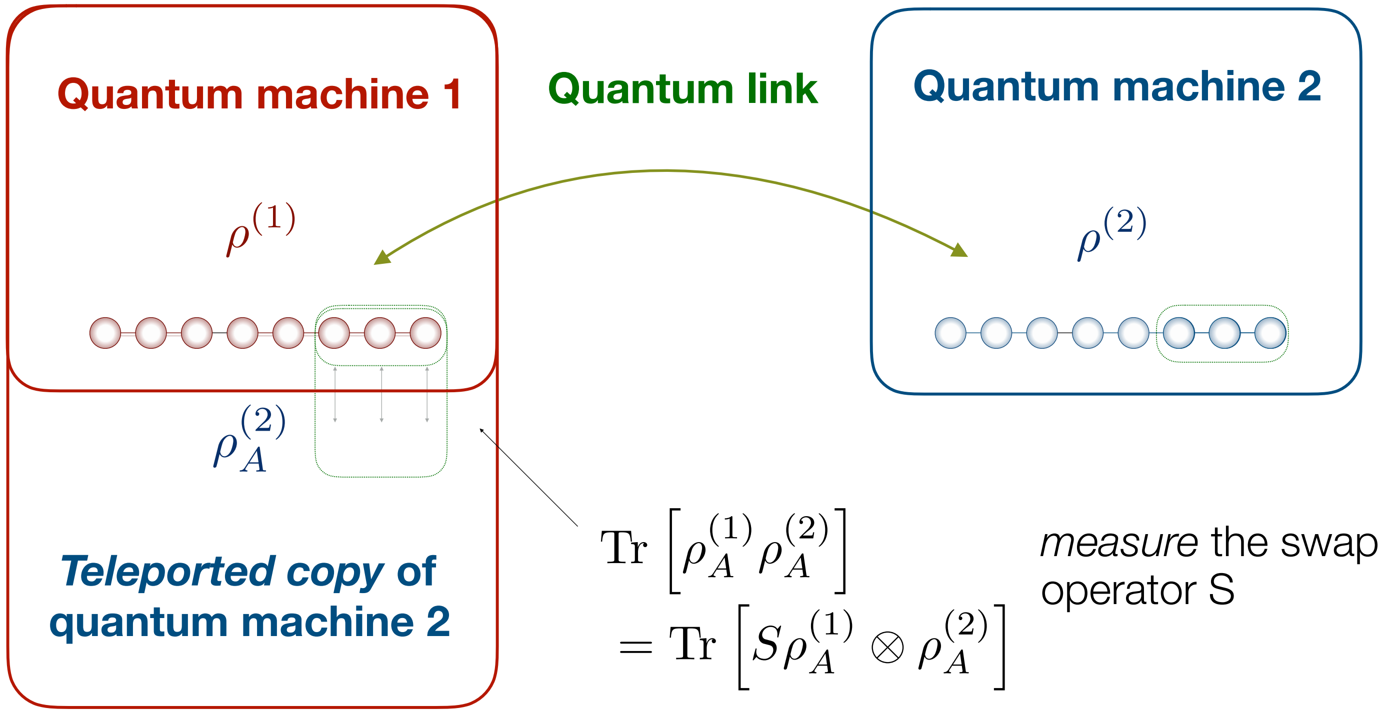
- max
- geometric mean

How to measure in many-body systems?

Review: Liang et al., arXiv:1810.08034
Josza-axioms

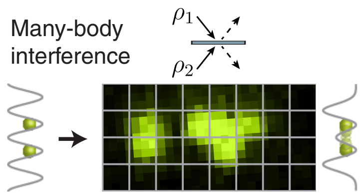
# Measuring many-body fidelities

## Version 1: Quantum Link




$$\mathrm{Tr}\left[\rho_A^{(1)}\rho_A^{(2)}\right] = \mathrm{Tr}\left[S\rho_A^{(1)} \otimes \rho_A^{(2)}\right]$$

*measure* the swap operator S




**Renyi-Entropies:**
Experiment [Greiner Group]:
Islam *et al*., Nature (2015);
Kaufmann *et al.*, Science (2016)
Theory: Daley et al. PRL 20121

## Version 2: Classical Link

Perform **tomography**, communicate results via **classical link**, compute

**How to measure *without* quantum link but more efficiently than tomography*?**

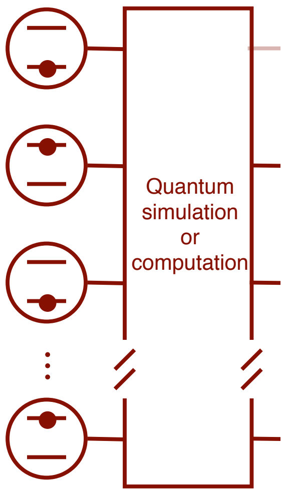
… statistical correlations between randomized measurements

here: no assumption on quantum state

G Torlai & RG Melko, *Machine learning quantum states in the NISQ era*,,arXiv:1905.04312

# Cross-correlations of randomized measurements

Platform 1

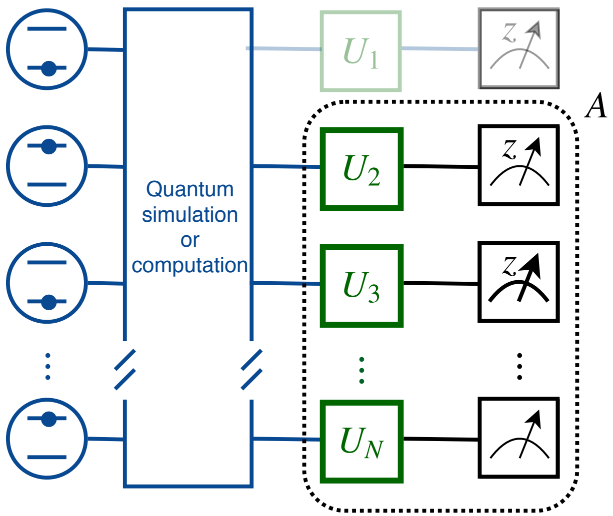



$$P_U^{(1)}(\mathbf{s}_A) = \mathrm{Tr}\left[U\rho_1 U^\dagger \left|\mathbf{s}_A\right\rangle\left\langle \mathbf{s}_A\right|\right]$$

**Classical link**
**+**
**more 'efficient' than tomography**

Platform 2

**Same random unitaries**




$$P_U^{(2)}(\mathbf{s}_A) = \mathrm{Tr}\left[U\rho_2 U^\dagger \left|\mathbf{s}_A\right\rangle\left\langle \mathbf{s}_A\right|\right]$$

**Purity 1**

$$\overline{\left(P_U^{(1)}(s_A)\right)^2} \sim \mathrm{Tr}[\rho_{1,A}^2]$$

$$\mathrm{Tr}[\rho_{1,A}\rho_{2,A}] = 2^{N_A}\sum_{s_A,s'_A}(-2)^{-D[s_A,s'_A]}\,\overline{P_U^{(1)}(s'_A)P_U^{(2)}(s_A)}$$

**Overlap**

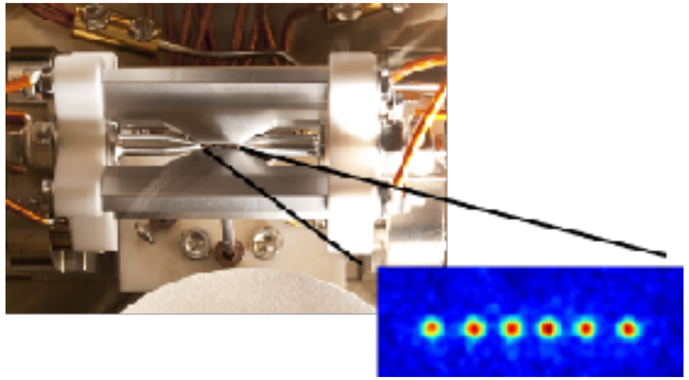
**Purity 2**

$$\overline{\left(P_U^{(2)}(s_A)\right)^2} \sim \mathrm{Tr}[\rho_{2,A}^2]$$

# Theory vs. experiment fidelities: `emulating x-platform’

## Experiment

$\rho_E$



Quench dynamics within
the long-range XY-model

↔

## Classical simulation (Theory)

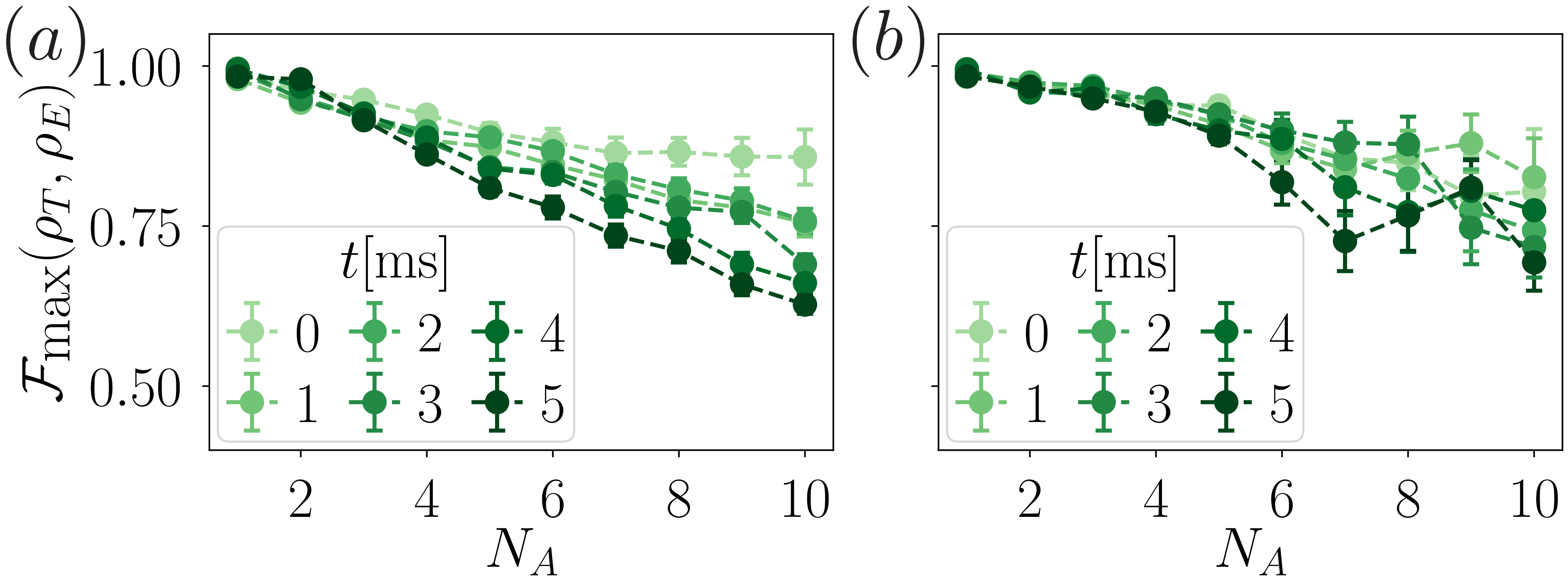
$\rho_T$

Unitary time evolution simulated with
exact diagonalization

Note for theory-experiment mode:
compare to ‘direct fidelity estimation’ a la
Silva et al., PRL 2011, Liu, Flammia, PRL 2011

# Theory vs. Experiment Fidelities: `Emulating X-Platform’

Measured fidelities $\mathscr{F}_{\max}(\rho_E, \rho_T)$ vs. partition size $N_A$ (total system $10$ qubits) for Neel states evolved with $H_{XY}$ ($J_0 = 420 s^{-1}, \alpha = 1.24$) for various times.




Theory states $\rho_T$ are obtained with (a) unitary dynamics and including (b) decoherence effects.

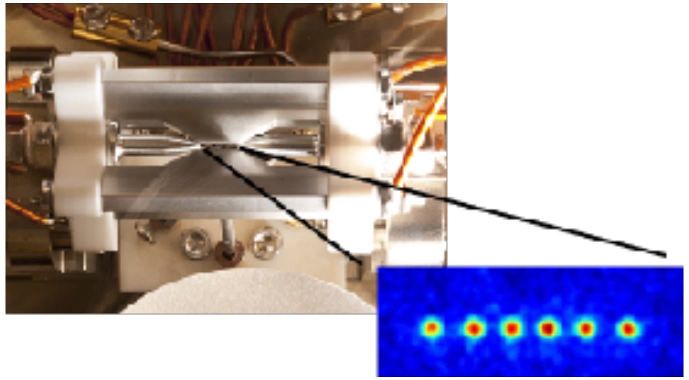
$N_U = 500, \ N_M = 150$

Data taken in context of: T Brydges, A Elben, P Jurcevic, B Vermersch, C Maier, BP Lanyon, P. Z., R Blatt, CF Roos, Science 2019

# Experiment vs. Experiment Fidelities: `Emulating X-Platform'

## Experiment 1

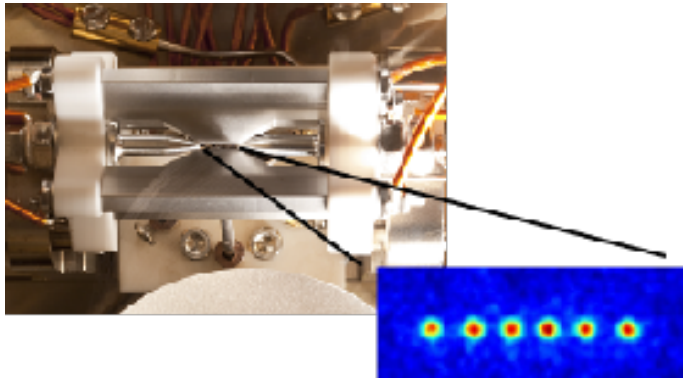
$\rho_{E_1}$



Quench dynamics within
the long-range XY-model -
**Day 1**

## Experiment 2

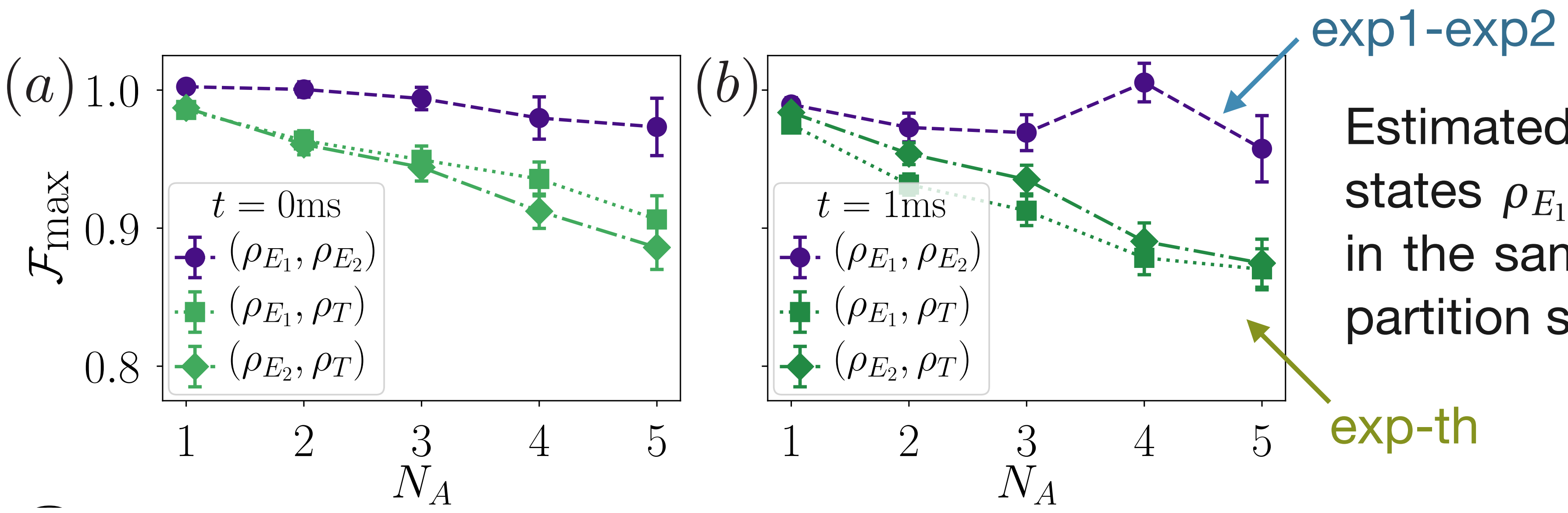
$\rho_{E_2}$



Quench dynamics within
the long-range XY-model -
**Day 2**

… what we really want to do is to compare Innsbruck & Maryland etc.

# Experiment *self-verification* in a trapped ion quantum simulator



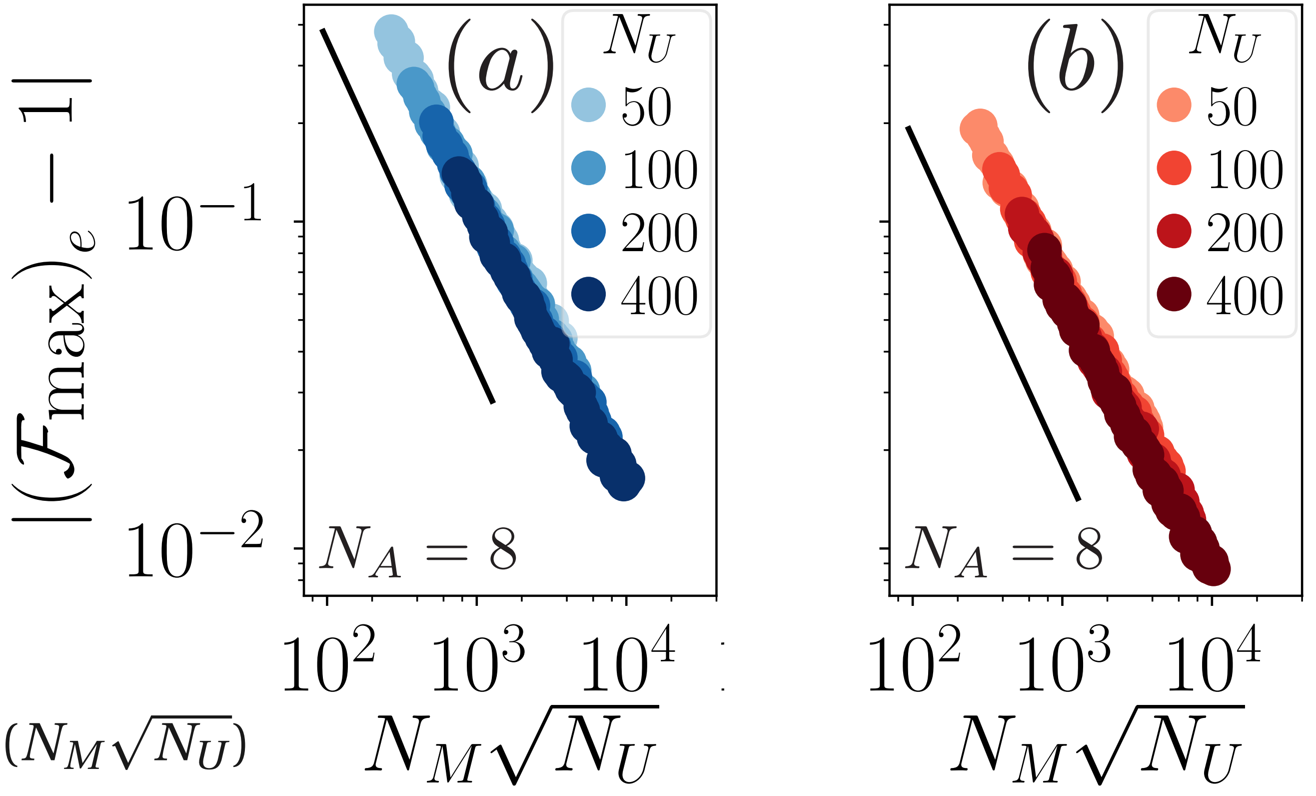

Estimated fidelities $\mathscr{F}_{\max}$ of two reduced states $\rho_{E_1}$ and $\rho_{E_2}$ prepared sequentially in the same experiment as a function of partition size, $[1 \rightarrow N_A]$.

Data taken in context of: T Brydges, A Elben, P Jurcevic, B Vermersch, C Maier, BP Lanyon, P. Z., R Blatt, CF Roos, Science 2019

# Scaling of the required number of measurements [numerical results]

A statistical error of the estimated fidelity $\mathcal{F}_e$ arises from a finite number of measurements $N_M$ and unitaries $N_U$

Average statistical error for $\rho_{A,1} = \rho_{A,2}$ and $N_A = 8$ qubits




$\sim 1/(N_M\sqrt{N_U})$

pure product state (PP) pure Haar random state (PR)

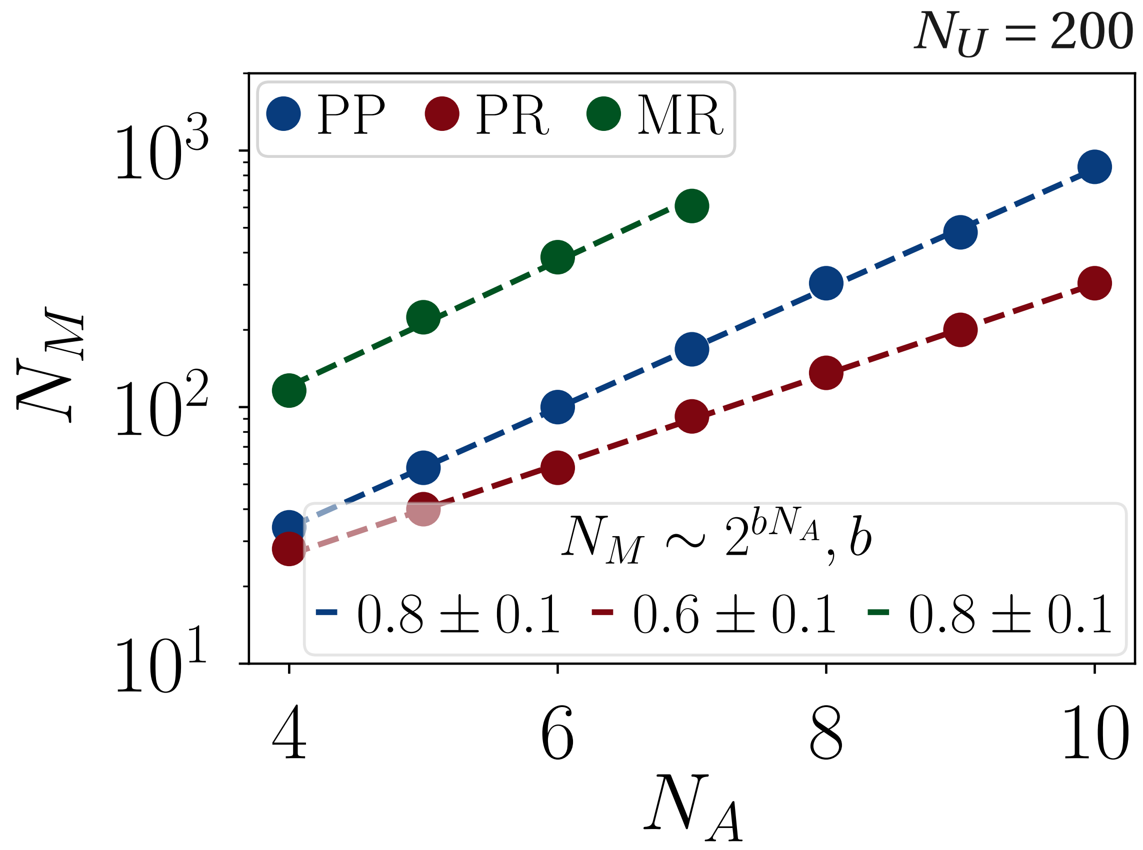
**Q1.:** For given measurement budget $N_M \times N_U$, how many runs to reduce fidelity error below $\epsilon$

**Q2.:** Optimal allocation of $N_M$ vs. $N_U$ for given total budget.

**A2.:** Depends on quantum state; iterative optimal allocation of $N_M$ vs. $N_U$.

# Scaling of the required number of measurements [numerical results]

Minimal number of required measurements $N_M$ to estimate $(\mathscr{F}_{\mathrm{max}}(\rho_A, \rho_A))_e$ for error $\epsilon = 0.05$ vs. number qubits $N_A$ for $N_U = 100$.




**PP:** pure product state
**PR:** pure Haar random state
**MR:** mixed random states

Results:

- Scaling statistical error

$$|[\mathscr{F}_{\mathrm{max}}(\rho_A, \rho_A)]_e - 1| \sim 1/(N_M \sqrt{N_U})$$

for $N_M \lesssim D_A = 2^{N_A}$ and $N_U \gg 1$,

- Scaling experimental runs

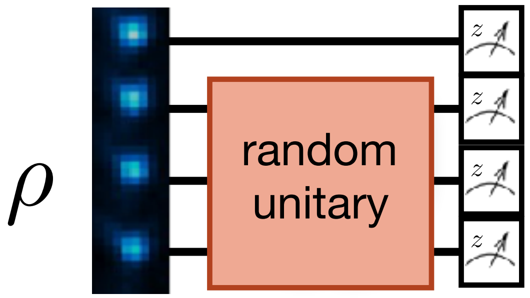
$$N_U N_M \sim 2^{bN_A}$$

with $b \lesssim 1$ vs. full tomography $b \geq 2$

# Conclusions: `Randomized Measurement Toolbox’

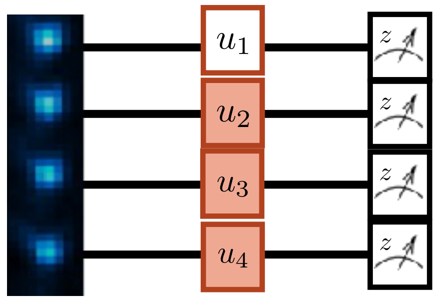
- Randomized Measurements

**global**




with $U_A \in \mathrm{CUE}$

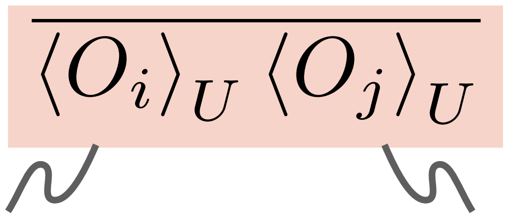
**local random unitaries**




$\longrightarrow \langle O_i \rangle_U$

$U_A = \bigotimes u_i \quad u_i \in \mathrm{CUE}(d)$

**self/cross-correlations**




system 1
day 1

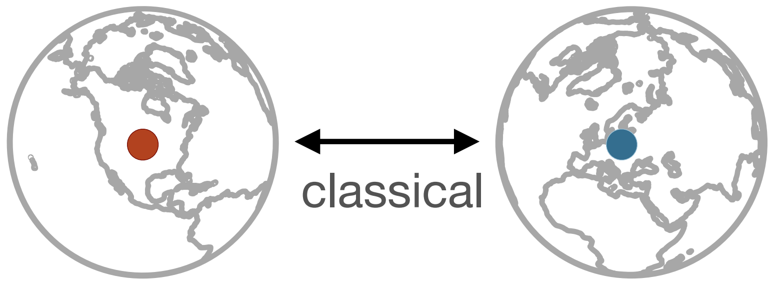
system 2
day 2

- Examples:

  ✓Renyi (entanglement) entropies
  ✓OTOCs
  ✓X-platform verification

Different locations, times & platforms

$\mathcal{F} \sim \mathrm{Tr}(\rho_A^{(1)} \rho_A^{(2)})$




- Scaling #qubits as friendly exponential ~ intermediate scale quantum devices
- *No assumption* on quantum state: Importance sampling