Time crystals
&
Quantum synchronisation

Rosario Fazio

The Abdus Salam
International Centre
for Theoretical Physics

In collaboration with

M. Dalmonte
F. Iemini
A. Russomanno

The Abdus Salam International Centre for Theoretical Physics (ICTP)

M. Schirò
O. Scarlatella

cea

J. Keeling

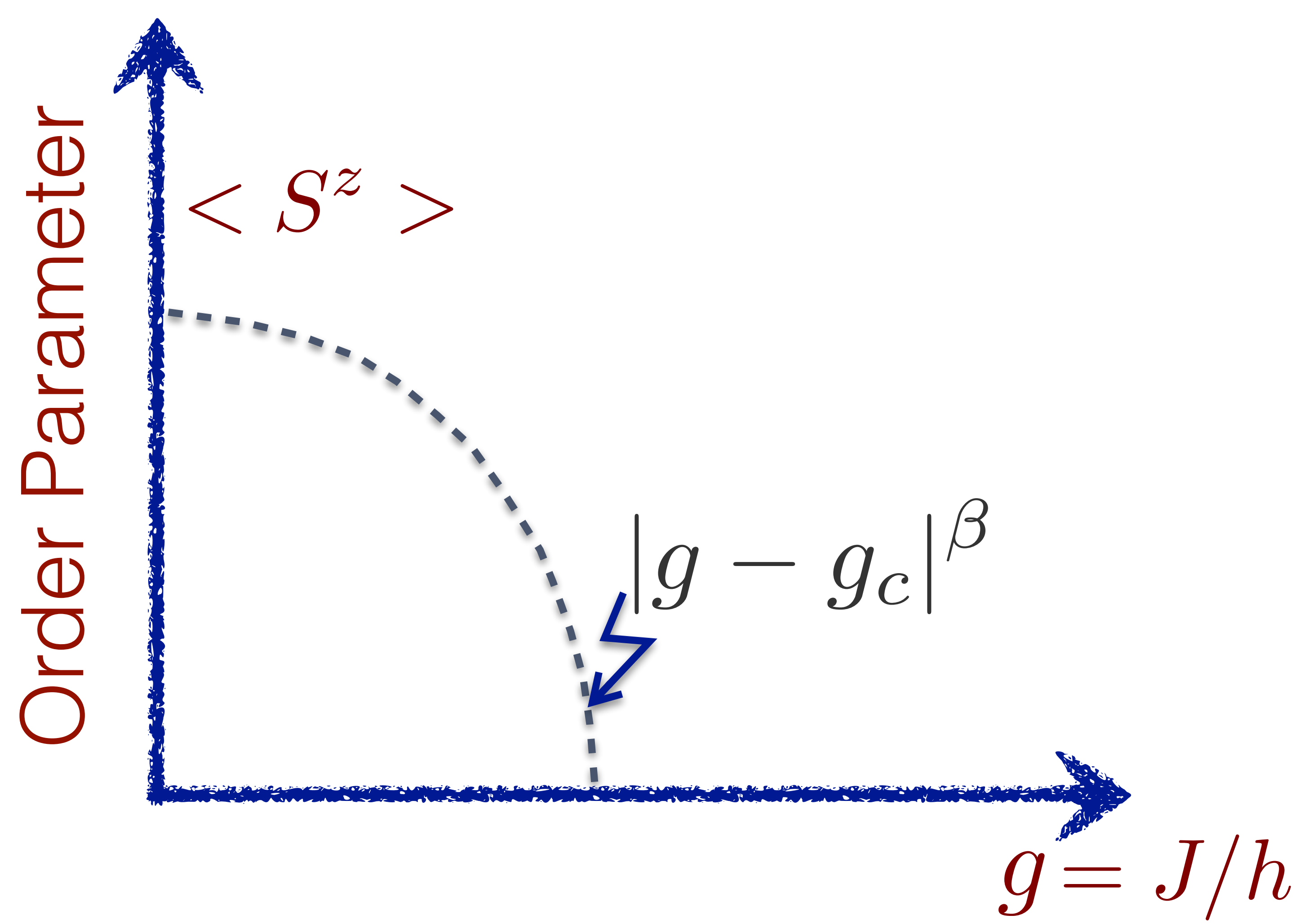
University of St Andrews

F. Iemini, A. Russomanno, J. Keeling, M. Schirò, M. Dalmonte, and R. Fazio, Phys. Rev. Lett. **121**, 035301 (2018)
O. Scarlatella, R. Fazio, and M. Schirò, Phys. Rev. B **121**, 064511, (2019)

[connections to classical synchronisation]
R. Khasseh, R. Fazio, S. Ruffo, A. Russomanno, Phys. Rev. Lett. **123**, 184301 (2019)

Spontaneous symmetry breaking *[Example]*

$$H(h) = -\,J \sum_{<ij>}^{N} S_i^z\, S_j^z - 2\,h \sum_{i}^{N} S_i^x$$



Can time-translational invariance be spontaneously broken?

Time-crystal

F. Wilczek, Physical Review Letters **109**, 160401 (2012)

- Do laws of nature allow for the existence of a time-crystalline phase? ✓✓
- …if yes, how to define/characterise a time crystal? ✓
- …where to look for it? ✓
- How much do we know of its relations to other phenomena?
- Is it "useful"?

- Time crystals
- Quantum synchronisation
- Finite frequency criticality
- …

Outline

- Brief intro to time-crystals
- Continuous (Boundary) time crystals
- Many-body limit cycles, finite-frequency criticality, quantum synchronisation
- Possible “realisation” in a system of coupled cavities
- Conclusions

Brief intro: “*A*” definition of a time crystal

$\phi(\vec{x}, t)$ local order parameter

$$\lim_{V \to \infty} \langle \phi(\vec{x}, t)\phi(\vec{x'}, t') \rangle \to f_t(t - t') \quad |\vec{x} - \vec{x'}| \to \infty$$

No-go theorem:(*) systems in the ground state or in thermal equilibrium cannot manifest any time-crystalline behaviour

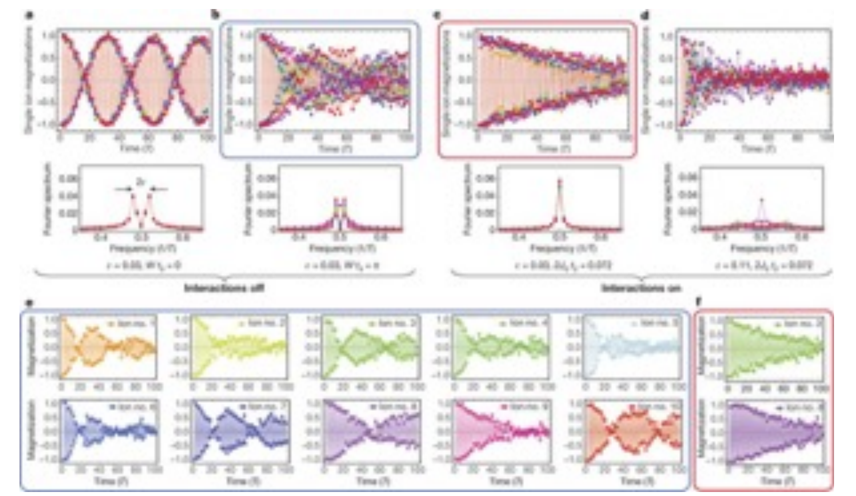
H. Watanabe and M. Oshikawa Phys. Rev. Lett. **114**, 251603 (2015)

(*) with sufficiently short-interactions

Brief intro: Floquet (discrete) time crystals

Theory
D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).
V. Khemani, A. Lazarides, R. Moessner, and S. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

Experiments
J. Zhang *et al*, Nature **543**, 217 (2017)
S. Choi *et al*, Nature **543**, 221 (2017).



$$\mathcal{H}(t+T) = \mathcal{H}(t)$$

Hamiltonian
Period T

Spontaneous breaking

Observables
Period nT

Rigidity: no fine-tuned Hamiltonian parameters.

Persistence: the non-trivial oscillation must persist for infinitely long time when first taking the thermodynamic limit.

(Boundary) time crystals in dissipative systems

- **Is the coupling to a bath always detrimental?**

"Many-body" limit cycles as time-crystals in open systems …

… they can be interpreted as "boundary" phases

The no-go theorem does not apply since the steady state is non-equilibrium …

… an idealised model vs possible exp realisations

These limit cycles can be understood as a macroscopic synchronised dynamics characterised by a time-dependent order parameter

Time-crystals & dissipative many-body open systems

Also investigated in …

In an ensemble of atoms collectively coupled to a leaky cavity mode.

In dissipative spin arrays

In a driven Bose-Hubbard dimer

…

Z. Gong, R. Hamazaki, and M. Ueda, Phys. Rev. Lett. **120**, 040404 (2018)
K. Tucker, B. Zhu, R. J. Lewis-Swan, J. Marino, F. Jimenez, J. G. Restrepo, and A. M. Rey, New J. Phys. **20**, 123003 (2018)
N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, F. Nori, Phys. Rev. A **98**, 063815 (2018)
B. Zhu, J. Marino, N. Y. Yao, M. D. Lukin, and E. Demler, arXiv:1904.01026 (2019)
C. Lledó, T. Mavrogordatos, and M. Szymańska, arXiv:1901.04438 (2019)
A. Riera-Campeny, M. Moreno-Cardoner, and A. Sanpera, arXiv:1909.11339 (2019)
K. Seibold, R. Rota, and V. Savona, arXiv:1910.03499 (2019)
…

$$\frac{d}{dt}\hat{\rho}_{\mathrm{b}} = \hat{\mathcal{L}}\left[\hat{\rho}_{\mathrm{b}}\right]$$

$$\hat{\mathcal{L}}[\cdot] = \sum_{\alpha}\left\{\hat{\ell}_{\alpha}\cdot\hat{\ell}_{\alpha}^{\dagger} - \frac{1}{2}\{\hat{\ell}_{\alpha}^{\dagger}\hat{\ell}_{\alpha},\cdot\}\right\}$$

A simple, solvable case …

A toy model

$$\hat{H}_{\rm b} = \omega_0 \sum_j \hat{\sigma}_j^x$$

The steady state diagram of the model has two distinct phases

$$\hat{S}^\alpha = \sum_j \hat{\sigma}_j^\alpha$$

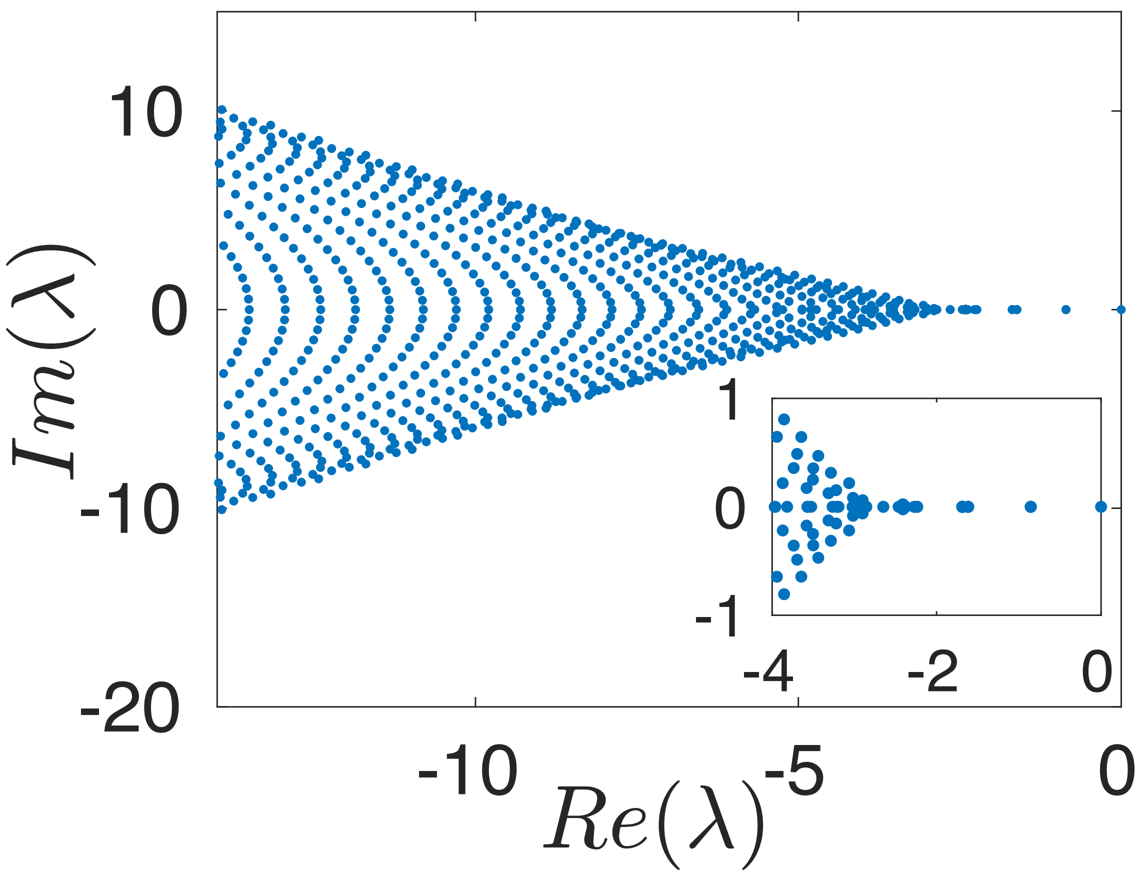
$\omega_0/\kappa < 1$	$\omega_0/\kappa > 1$
$\langle \hat{S}^z \rangle \neq 0$	$\langle \hat{S}^z \rangle = 0$

J. Hannukainen and J. Larson Phys. Rev. A **98**, 042113 (2018)

$$\frac{d}{dt}\hat{\rho}_{\rm b} = i\omega_0[\hat{\rho}_{\rm b}, \hat{S}^x] + \frac{\kappa}{S}\left(\hat{S}_-\hat{\rho}_{\rm b}\hat{S}_+ - \frac{1}{2}\{\hat{S}_+\hat{S}_-, \hat{\rho}_{\rm b}\}\right)$$

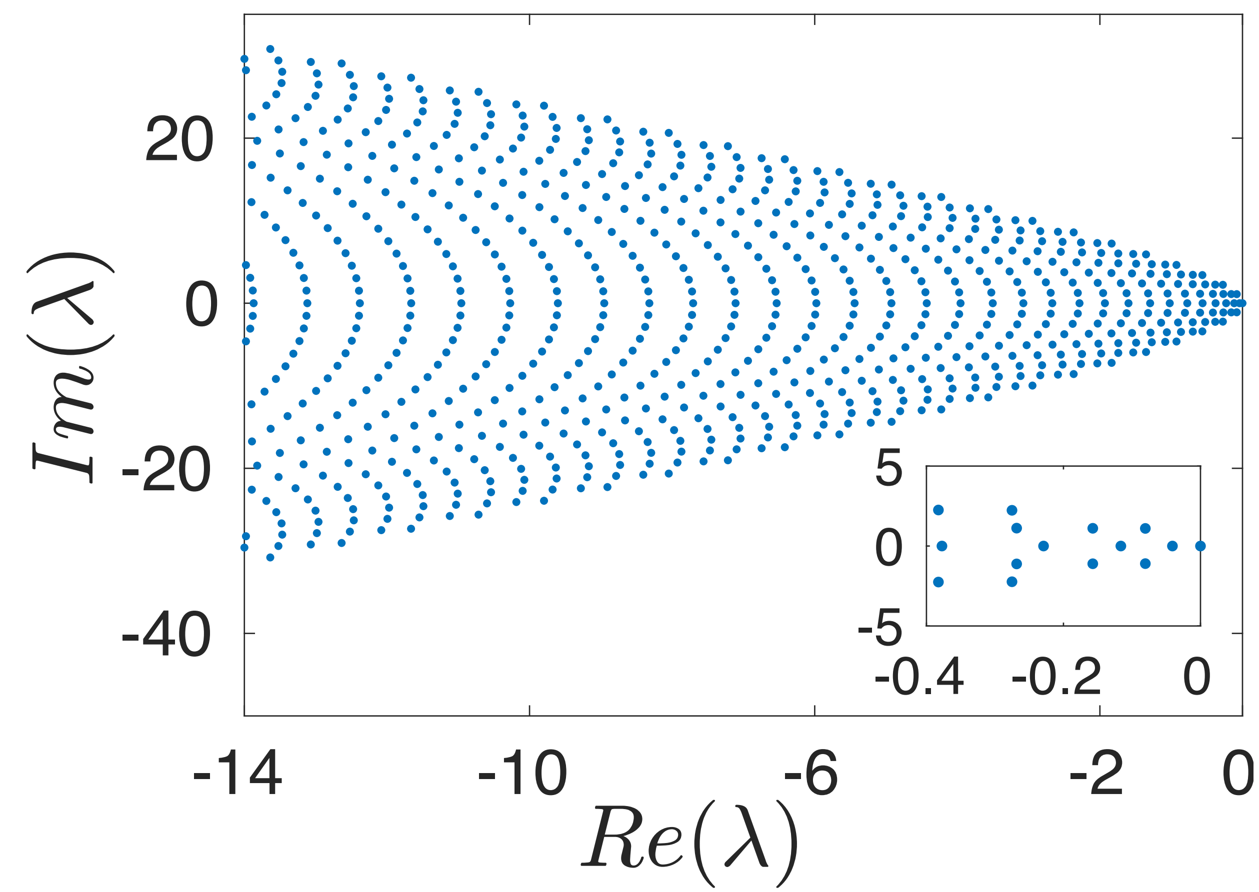
A toy model

$$\omega_0/\kappa > 1$$



The spectrum is gapped and the low-lying eigenvalues of the Liouvillian have purely real values

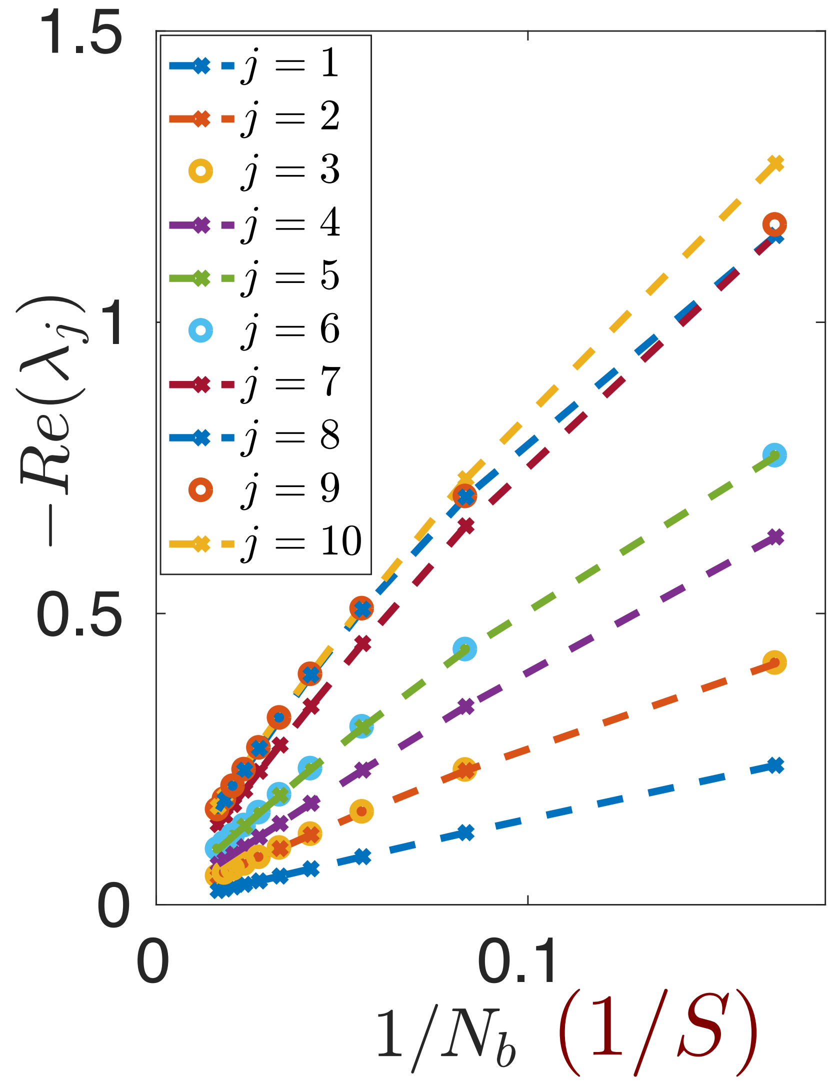
$$\omega_0/\kappa < 1$$

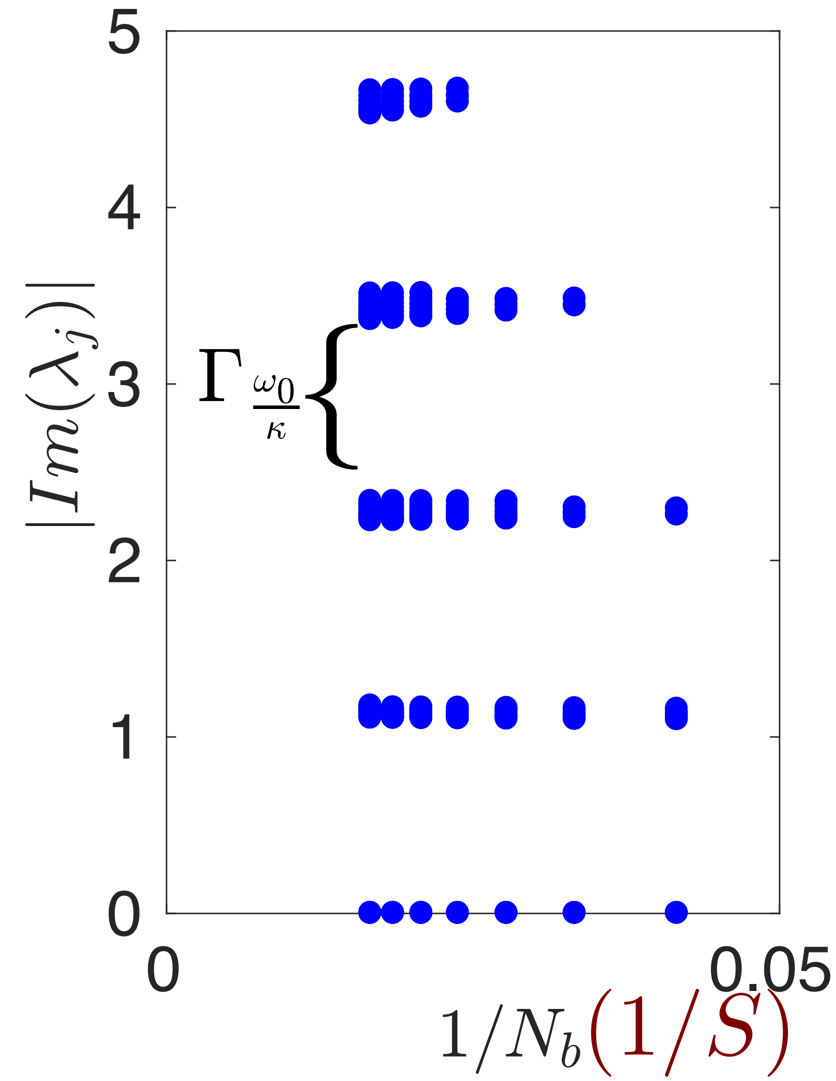


The spectrum becomes gapless and the low-lying excited eigenvalues have a non zero imaginary part

A toy model

Finite size scaling of the Liouvillian eigenvalues in the BTC

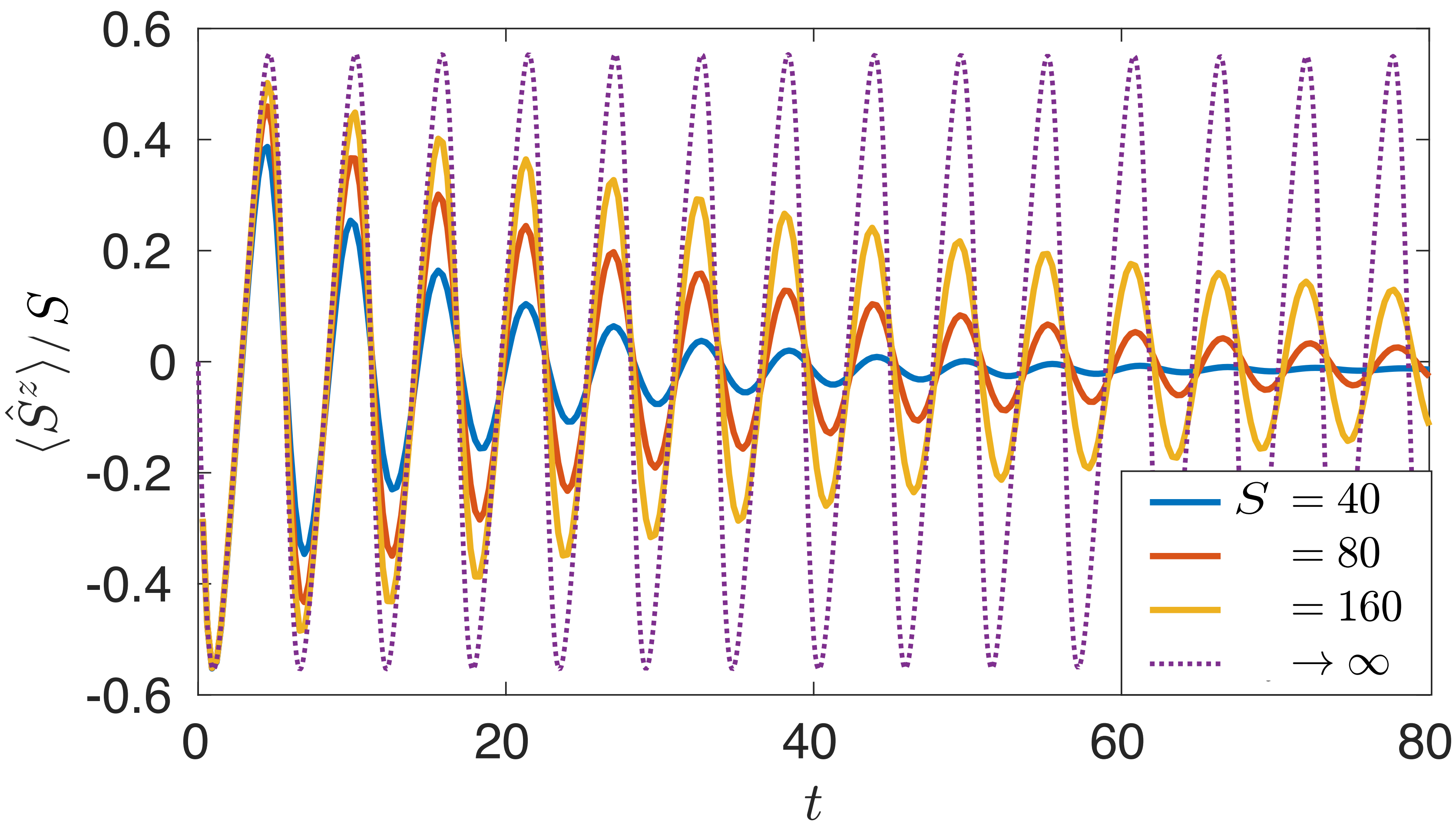




The real parts scale to zero as a power-law of the inverse system size.

The imaginary parts of the eigenvalues show a band structure

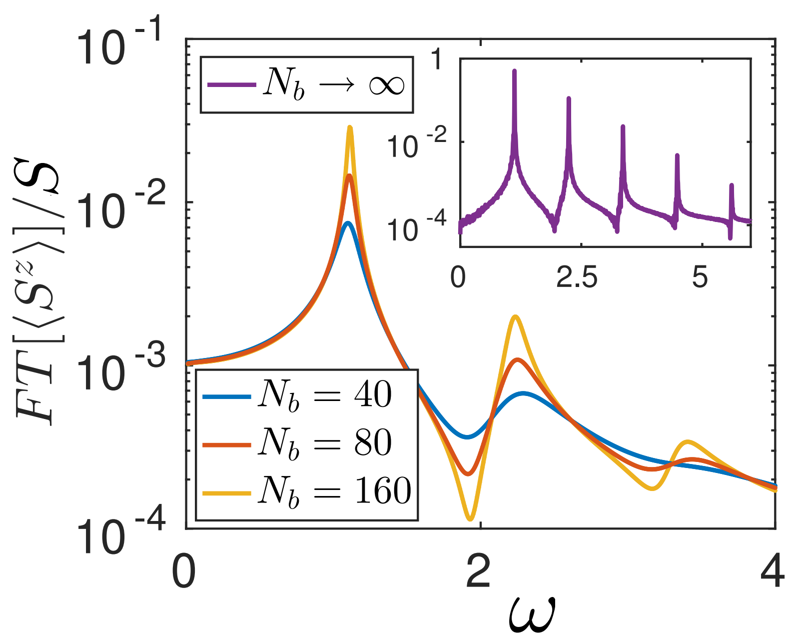
A toy model

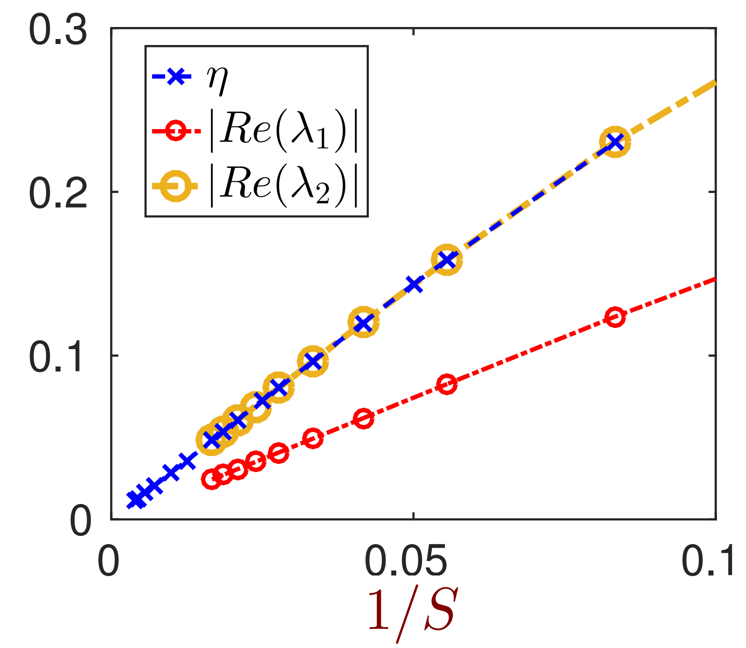


The peaks in the Fourier transform are associated to the band separations in the imaginary part of the Lindblad eigenvalues (in the inset the thermodynamic limit where the oscillations persist indefinitely.

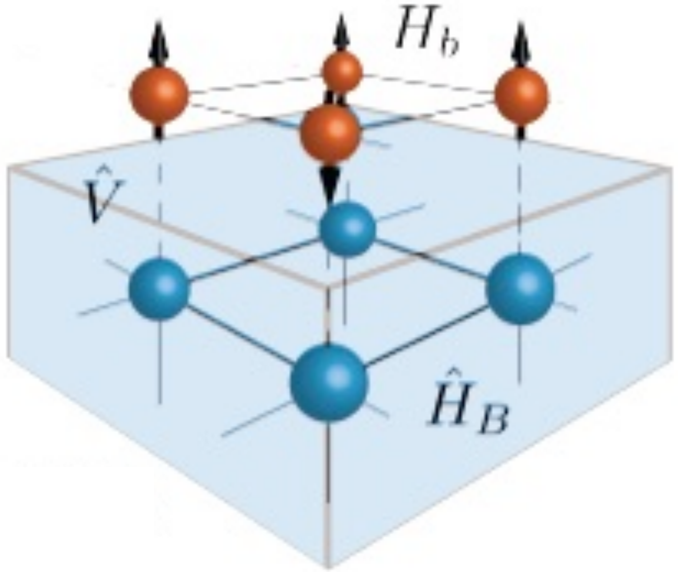
The decay rate of the oscillations scales as

$1/S$

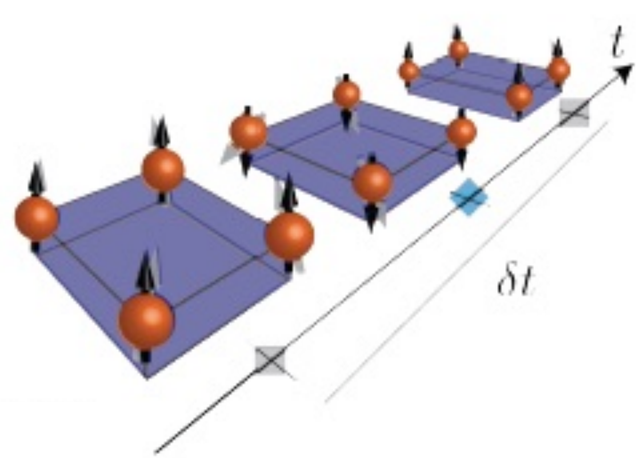




$$\hat{H} = \hat{H}_{\mathrm{B}} + \hat{H}_{\mathrm{b}} + \hat{V}$$



$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle$$



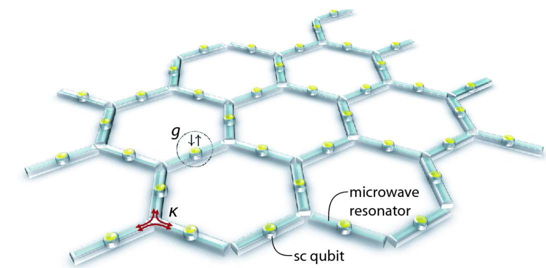
$$\hat{\rho}_{\mathrm{b}} = \mathrm{Tr}_{\mathrm{B}}\left(|\psi(t)\rangle\langle\psi(t)|\right)$$

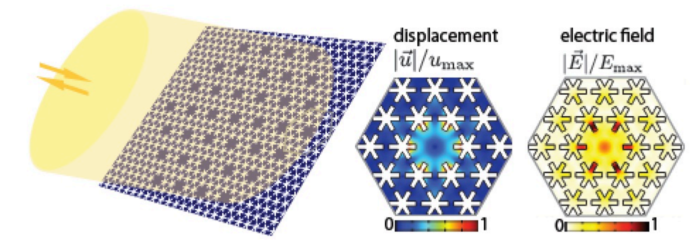
$$\frac{d}{dt}\hat{\rho}_{\mathrm{b}} = \hat{\mathcal{L}}\left[\hat{\rho}_{\mathrm{b}}\right]$$

“Many-body limit cycles”/ finite frequency criticality

A finite frequency mode can become critical, giving rise to genuine time-domain instabilities of the quantum dynamics and to an associated breaking of time-translational invariance

.. seen (at mean-field level) in model systems of interacting Rydberg atoms, opto-mechanical arrays, coupled cavity arrays and interacting spin-systems.





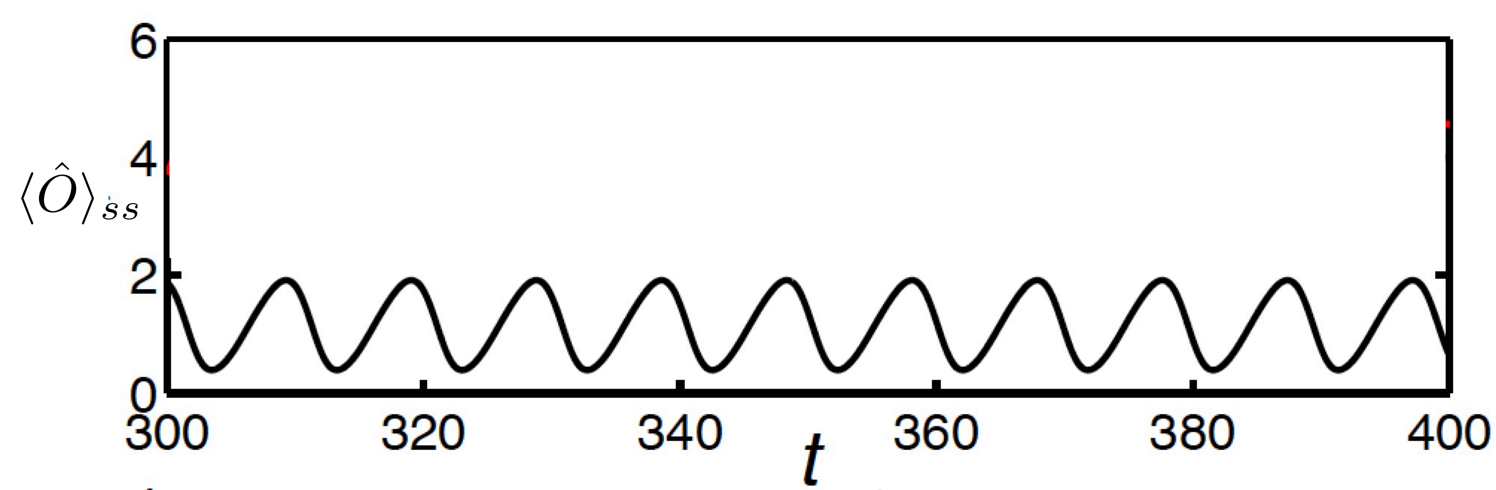
- T. E. Lee, H. Häffner, and M. C. Cross, Phys. Rev. A **84**, 031402 (2011)
- M. Ludwig and F. Marquardt, Phys. Rev. Lett. **111**, 073603 (2013)
- J. Jin, D. Rossini, R. Fazio, M. Leib, and M. J. Hartmann, Phys. Rev. Lett. **110**, 163605 (2013)
- C.-K. Chan, T. E. Lee, and S. Gopalakrishnan, Phys. Rev. A **91**, 051601 (2015)
- M. Schiró, C. Joshi, M. Bordyuh, R. Fazio, J. Keeling, and H. E. Türeci, Phys. Rev. Lett. **116**, 143603 (2016)
- M. Schiró, O. Scarlatella, and R. Fazio, Phys. Rev. B **99**, 064511 (2019)

…

In the case of cavity arrays/ optomechanical arrays

$$\langle a \rangle_{ss}$$

$$\langle \hat{O} \rangle_{ss} = \mathrm{Tr} \rho_{ss} \hat{O} = f(t)$$



Connection to quantum synchronisation

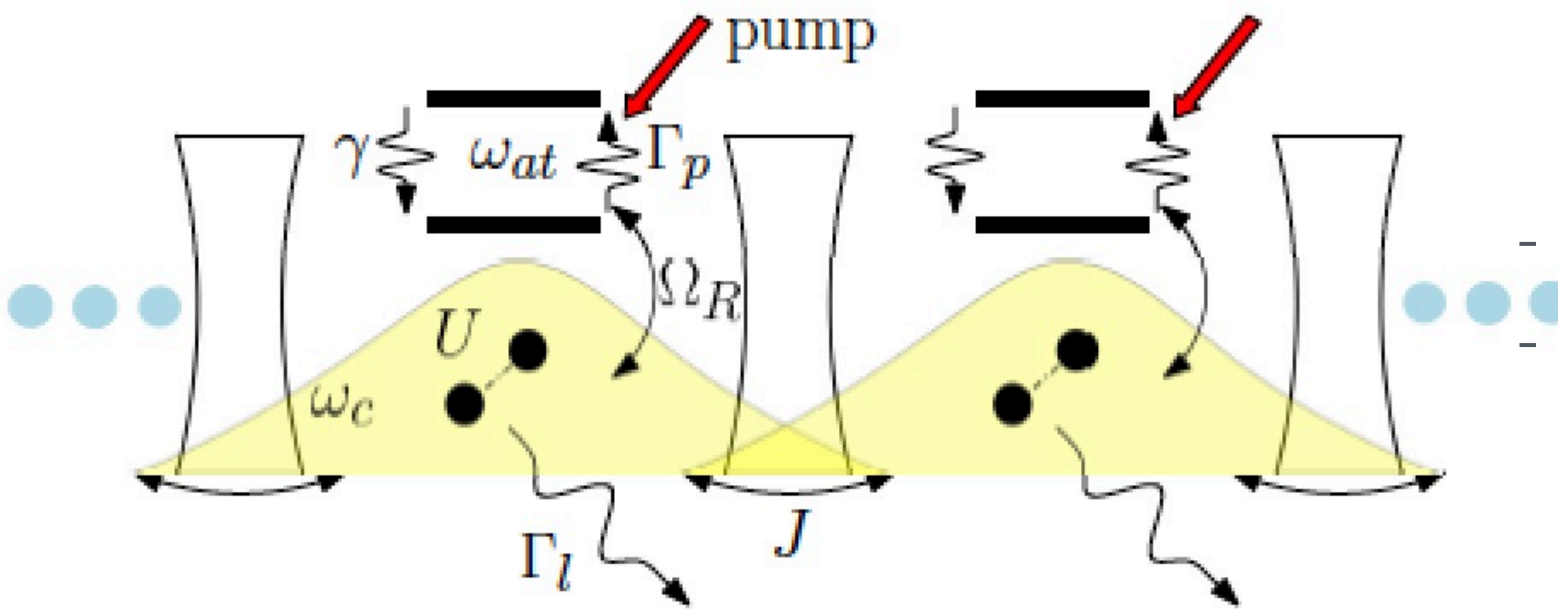
- Phase synchronization in a quantum many-body systems
- Dynamical locking of the phase across the whole array
- “Kuramoto-like” dynamics in a quantum setting

Photonic phases in presence of incoherent driving and dissipation

A Mott to Superfluid transition?

Different from equilibrium

competition U vs J is not sufficient



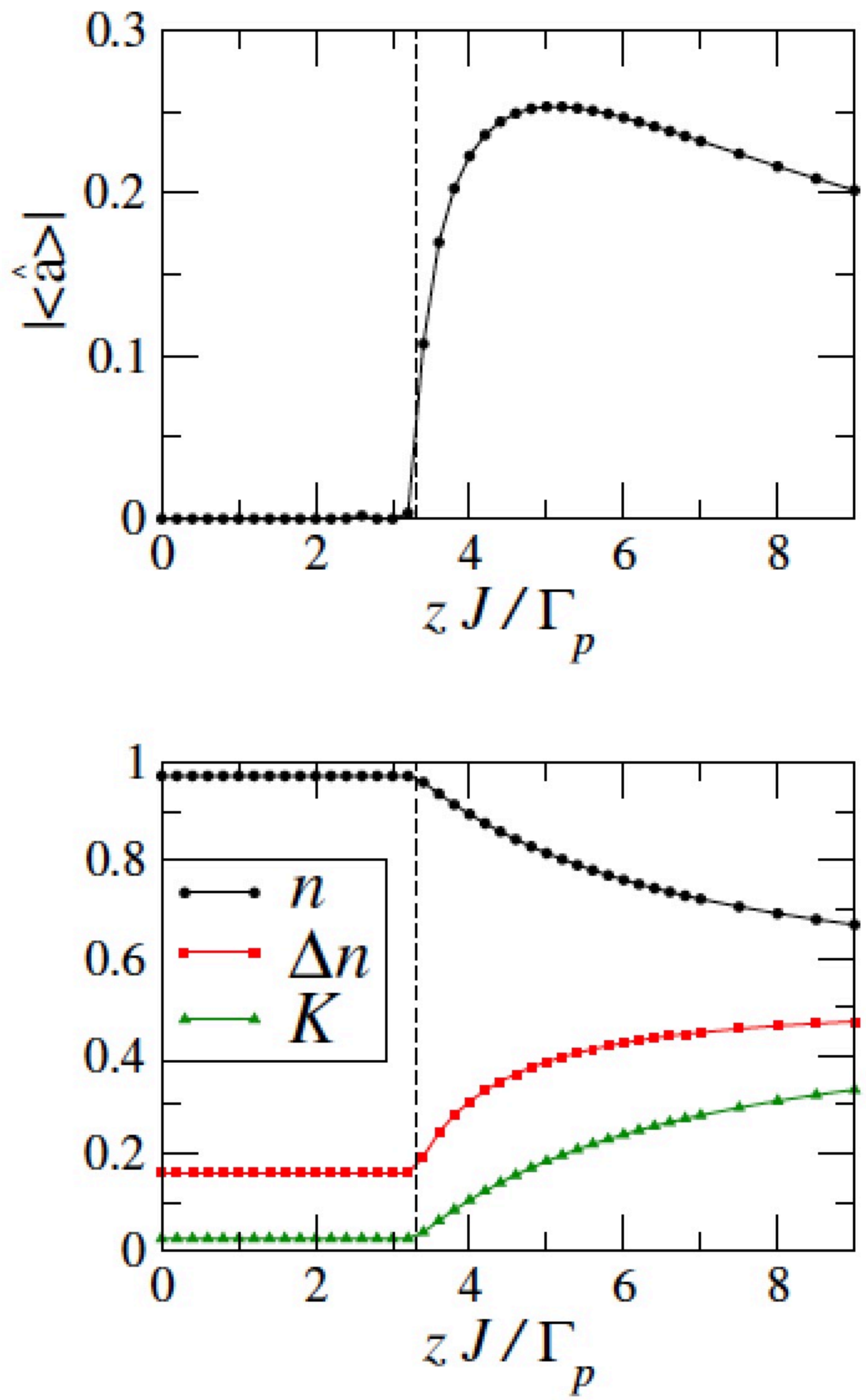
- Interacting photons [U]
- Pumped two-level systems

Non-markovian incoherent pumping

J. Lebreuilly, M. Wouters, and I. Carusotto, Comptes Rendus Physique **17**, 836 (2016)
A. Biella, F. Storme, J. Lebreuilly, D. Rossini, R. Fazio, I. Carusotto, and C. Ciuti, Phys. Rev. A **96**, 023839 (2017)
J. Lebreuilly, A. Biella, F. Storme, D. Rossini, R. Fazio, C. Ciuti, and I. Carusotto, Phys. Rev. B **97**, 035103 (2018).

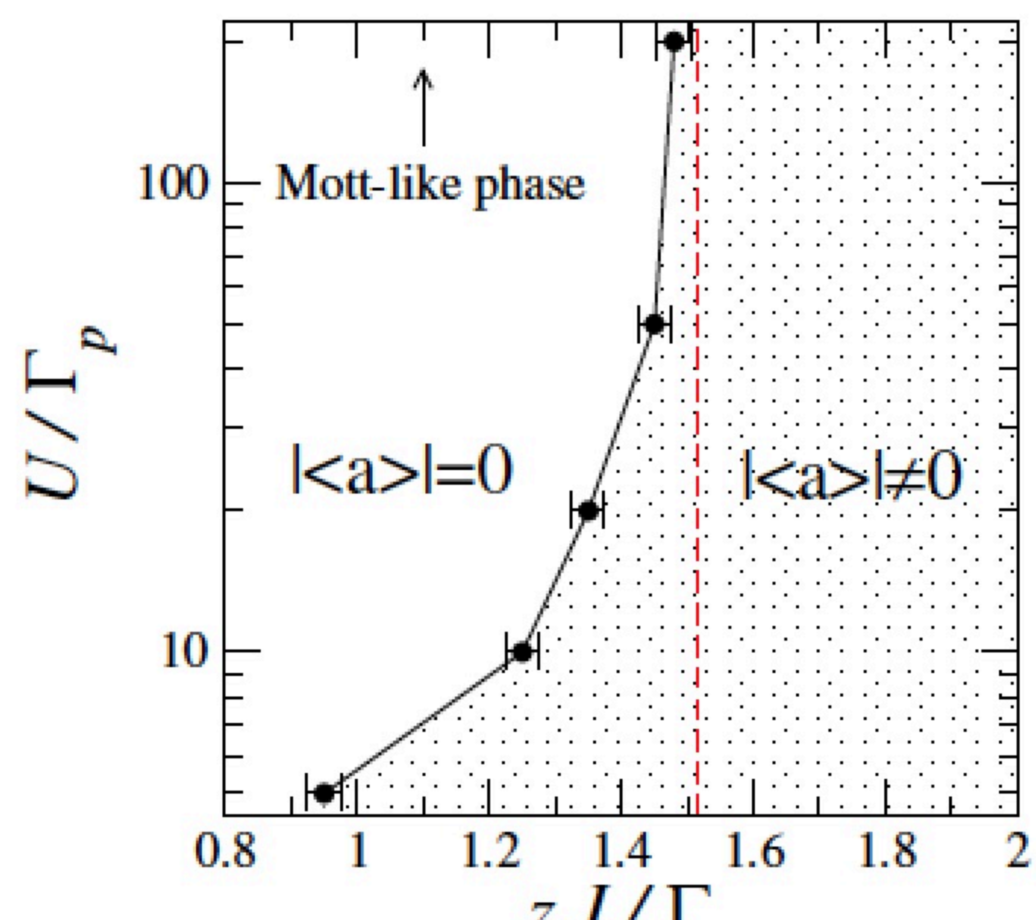
Finite J - Transition to a superfluid state

$$\psi \quad \longrightarrow \quad \langle a_i \rangle$$



The phase transition we observe is **not** related to the competition between photon hopping and nonlinearity.

On increasing J effective pumping becomes less efficient.



A. Biella, F. Storme, J. Lebreuilly, D. Rossini, R. Fazio, I. Carusotto, and C. Ciuti, Phys. Rev. A **96**, 023839 (2017)

Finite-frequency criticality $\psi \longrightarrow \langle a_i \rangle$

Question: related to whether finite frequency modes can become critical, giving rise to genuine time-domain instabilities of the quantum dynamics and to an associated breaking of time-translational invariance,

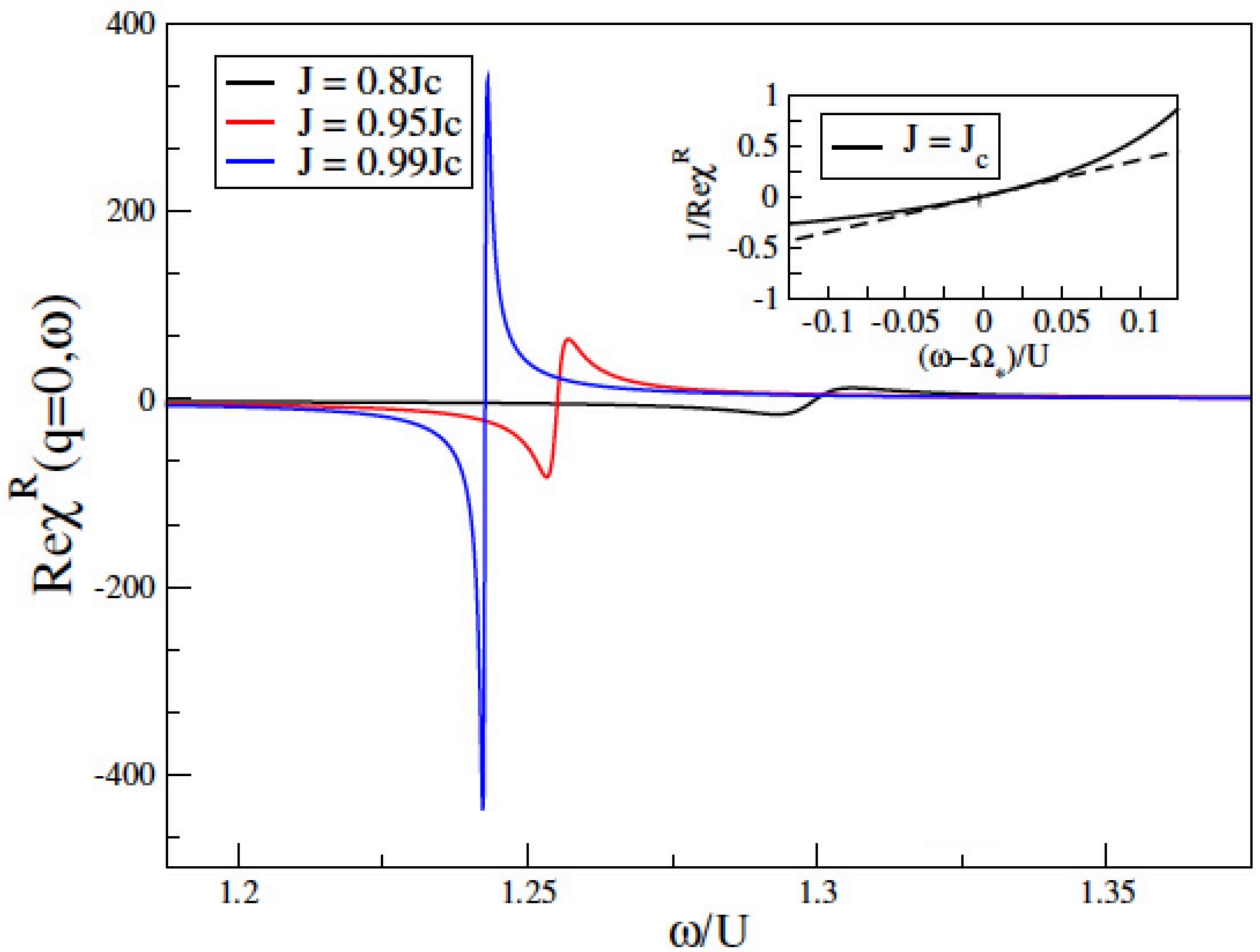
$$\chi_R = \frac{1}{-2J\sum_\alpha \cos q_\alpha - G^R_{loc}(\omega)}$$

$$G^R_{loc}(\omega) = \int dt e^{i\omega t} \langle [a(t), a(t')] \rangle$$

Exact single-site retarded local Green's function in presence of interaction, drive and dissipation (J=0)

O. Scarlatella, R. Fazio, and M. Schirò, PRB (2019)

Finite-frequency criticality



Well defined resonance structure:
The real part going through a zero while the imaginary part showing a peak, which gets sharper and narrower as the hopping is increased, and eventually turns into a genuine finite frequency pole when a critical value Jc is reached

$$\chi_R \sim \frac{\chi_0}{(\omega - \Omega^*)^\gamma}$$

The frequency Ω^* is an emergent scale arising out of the interplay between non-equilibrium effects and interactions,

Lowest lying modes can be obtained from the pole of the susceptibility - Near the critical point Jc one of the two branches at q=0 becomes soft, with both the real and imaginary part of the spectral gap going to zero

O. Scarlatella, R. Fazio, and M. Schirò, to be submitted

Conclusions

- I introduced **boundary time crystals**. The phenomenon is analogous to surface critical phenomena.

- BTCs are intimately linked to the emergence of a periodic dynamics in some macroscopic observable of an open quantum many-body system (**quantum synchronisation**).