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NEUTRINO MASS SUM RULES FROM MODULAR $\mathscr{A}_4$ SYMMETRY

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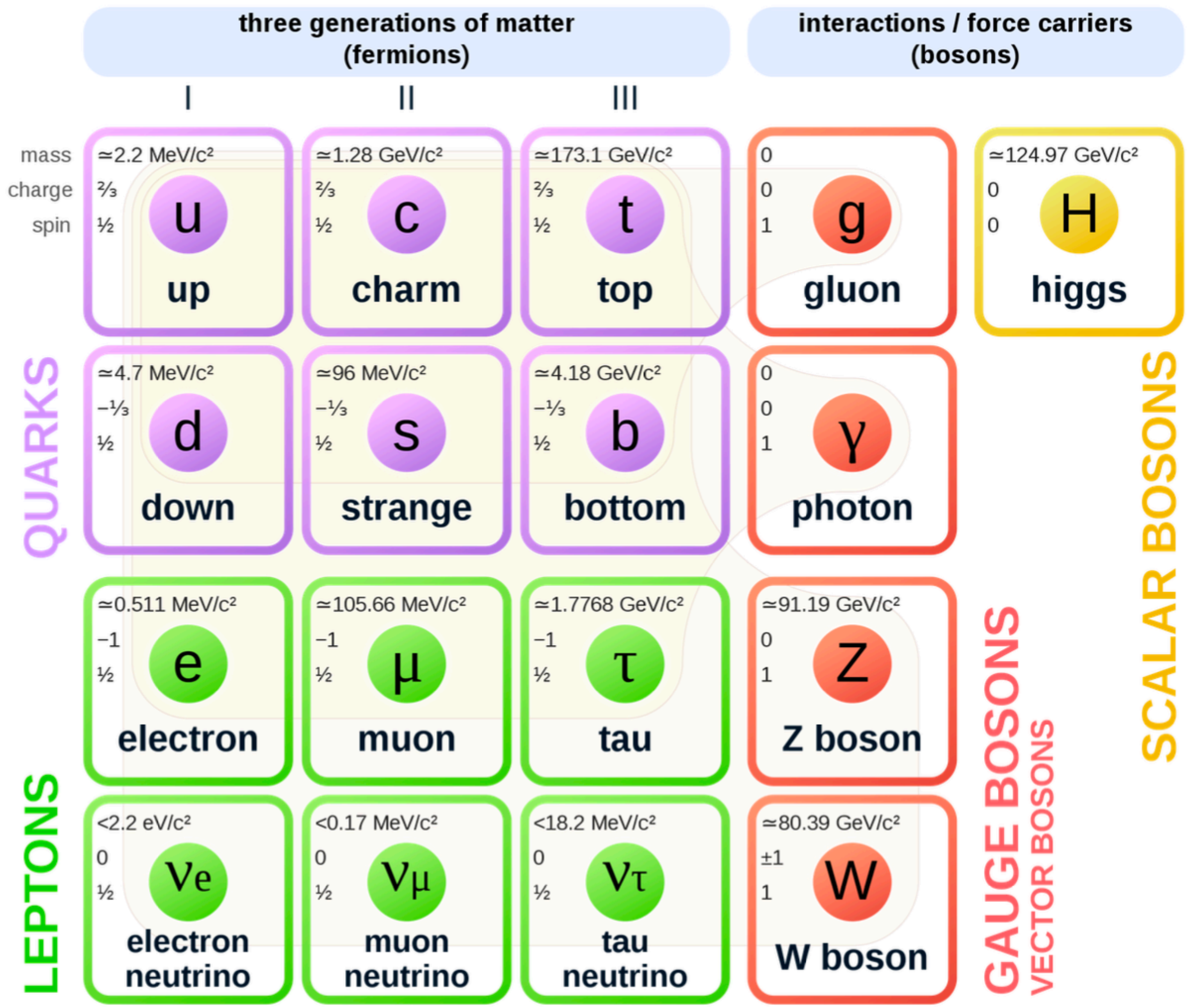
OVERVIEW

- Origin of Neutrino Masses
- Modular Symmetries as Flavour Symmetries
- $\mathscr{A}_4$ Modular Symmetric Model
- Neutrino Mass Sum Rule
- Results
- Summary

Standard Model of Elementary Particles



$$\mathbb{G}_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

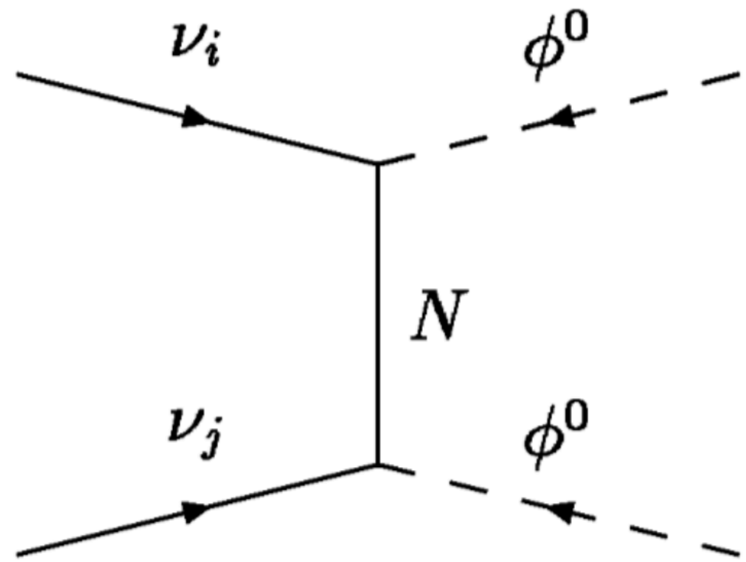
$$\mathcal{L}^Y = \bar{u} Y_u Q H + \bar{Q} Y_d d H + \text{h.c.}$$

$$\langle h^0 \rangle = v \rightarrow \quad m_f = Y_f v$$

Weinberg Operator 1979: $\frac{LHLH}{\Lambda} = \frac{(l^- H^+ - \nu H^0)(l^- H^+ - \nu H^0)}{\Lambda}$

Add $N_R \sim (1,1,0)$ under $\mathbb{G}_{SM}$

$$\mathcal{L}_{new} = \bar{N} Y_D LH + m_N N_R N_R + \text{h.c.}$$



$$\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \Rightarrow m_\nu \simeq \frac{-m_D^2}{m_N}.$$

$v \ll m_N \sim 10^{11}\text{GeV} \to m_\nu \ll v$ with $Y_D \sim 1$

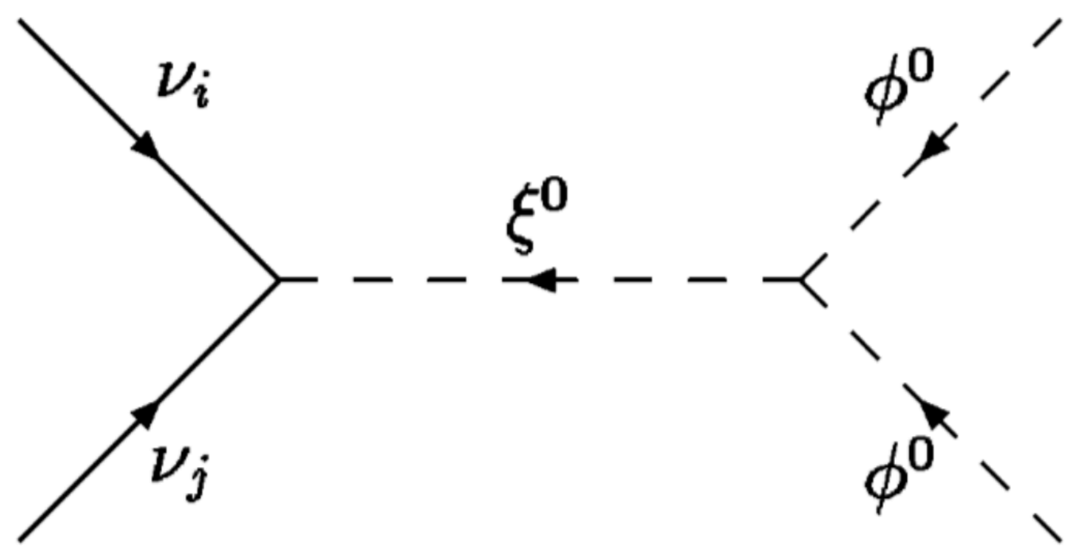
or $Y_D \ll 1 \to m_\nu \ll v$ with $m_N \sim O(10^{2-3} GeV)$

Seesaw−I

SEESAW-II

Add[1980] $\xi = (\xi^{++}, \xi^{+}, \xi^{0}) \sim (1, 3, 1)$ under $\mathbb{G}_{SM}$

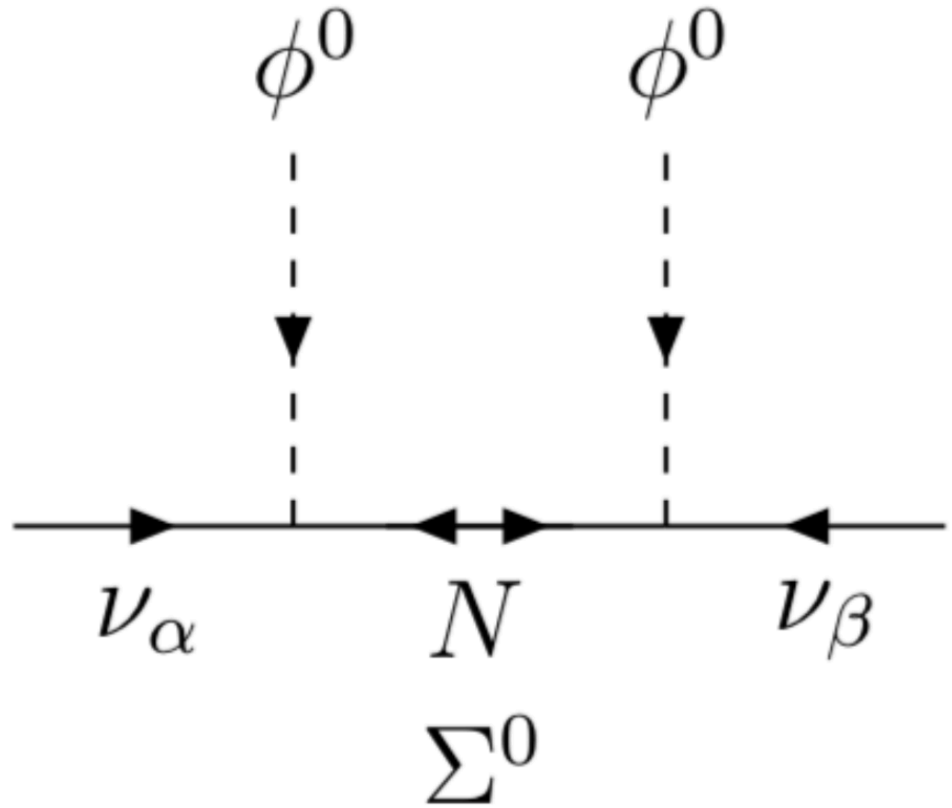
$$\mathcal{L}_{new} = YL\xi L - \mu H\xi H + \text{h.c.} \to m_\nu = Y\left\langle \xi^0 \right\rangle = -2\frac{Y\mu v^2}{M_\xi}$$



SEESAW-III

Add $\Sigma_R \sim (1,3,0)$ under $\mathbb{G}_{SM}$

$$\mathcal{L}_{new} = \bar{\Sigma} Y_D L H + m_N \Sigma_R \Sigma_R + \text{h.c.}$$



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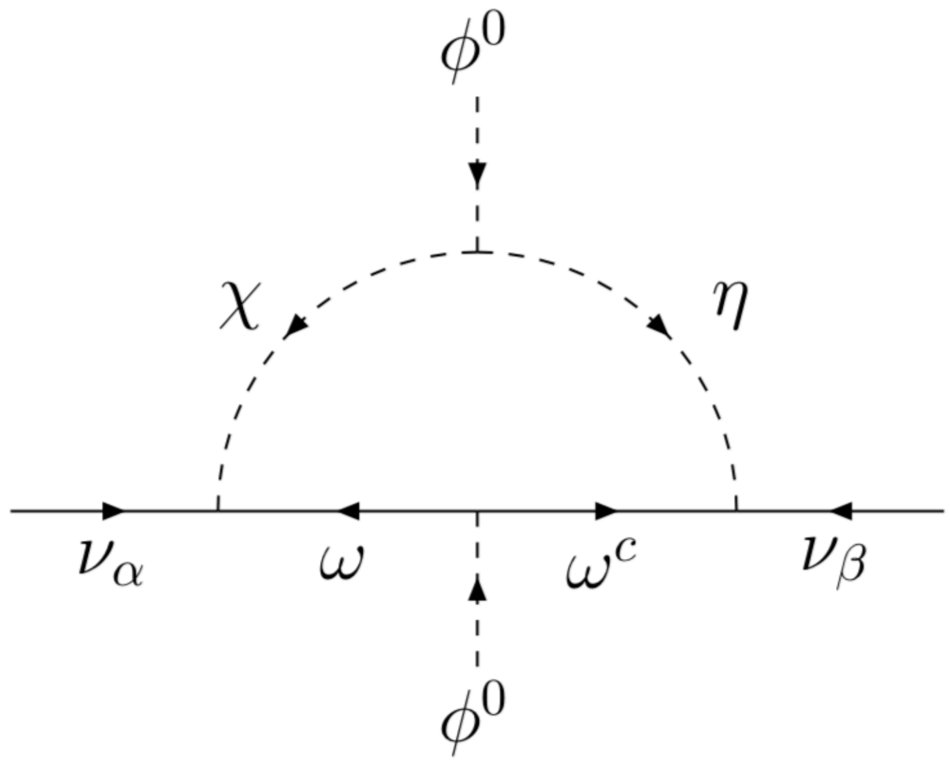
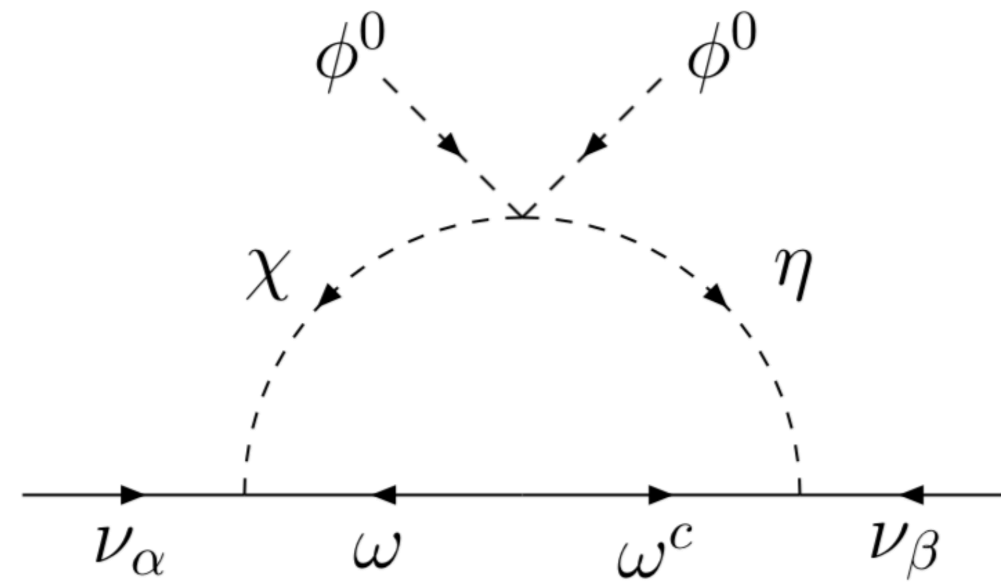
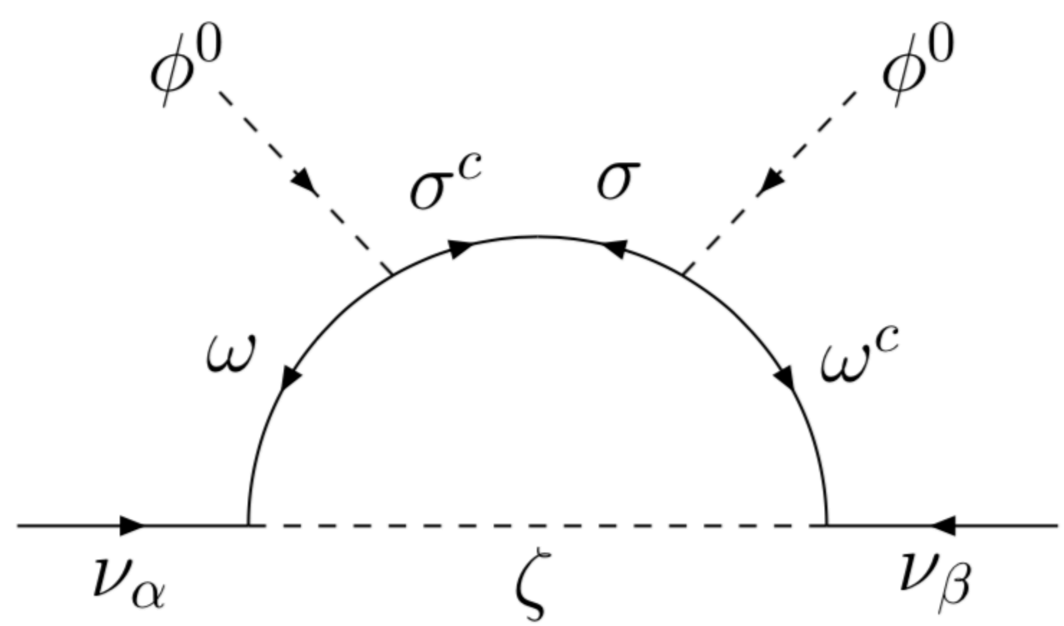
RADIATIVE MAJORANA NEUTRINO MASSES



Zee[1986]

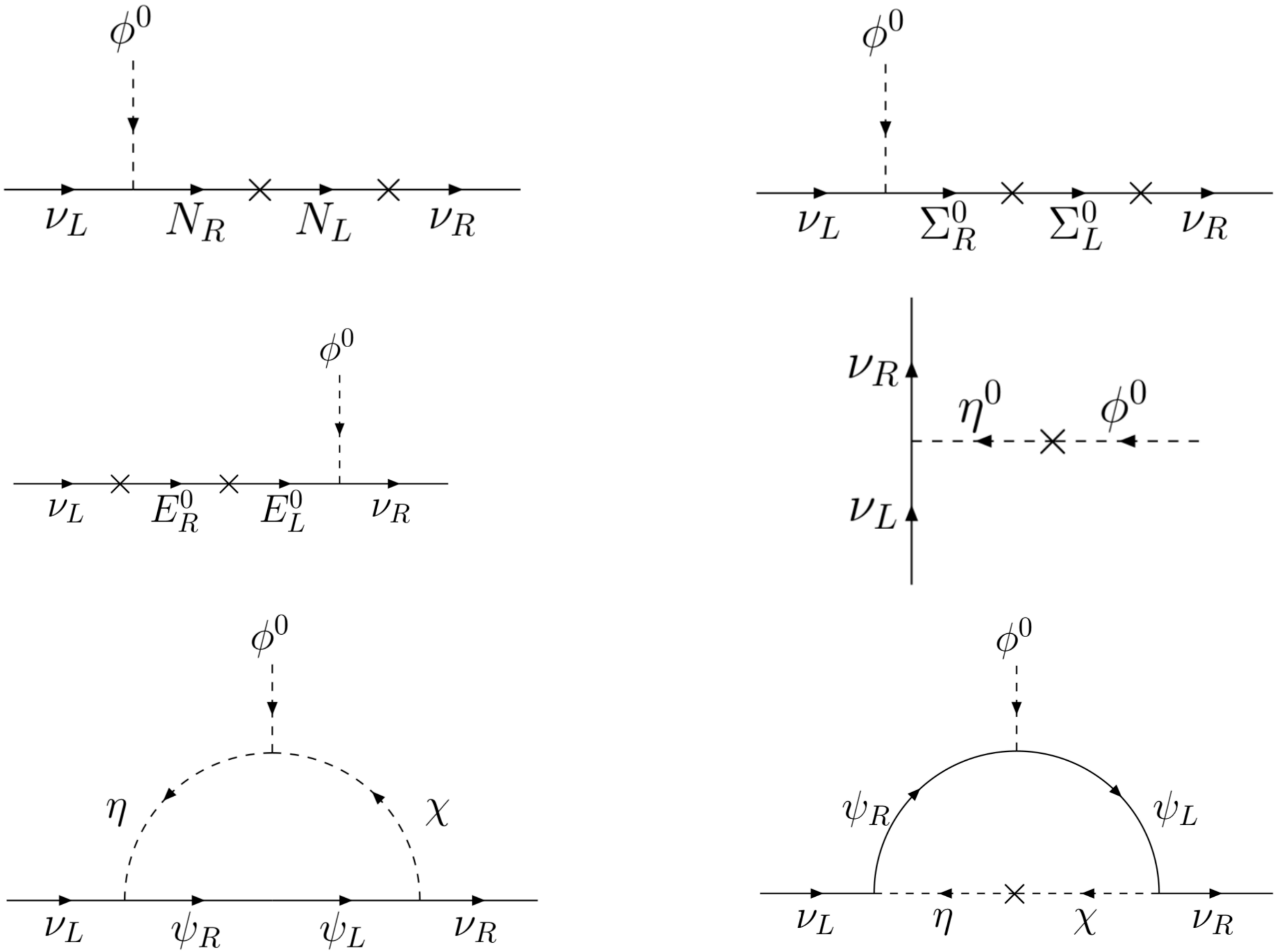


Ma[2006]



Fraser,Ma,OP[2014]

DIRAC NEUTRINO MASSES



E. Ma&OP, [Phys.Lett.B 764 (2017) 142-144]

MODULAR SYMMETRY

$$\tau \longrightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d} \qquad S^2 = \mathcal{I} \, , \qquad (ST)^3 = \mathcal{I} \, .$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad S : \tau \longrightarrow -\frac{1}{\tau} \, , \qquad T : \tau \longrightarrow \tau + 1 \, .$$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) \, , \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\mathrm{mod} N) \right\} \qquad \Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N)$$

$$f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau) \, , \quad \gamma \in \Gamma(N) \qquad \Gamma_3 \simeq \mathscr{A}_4$$

$$y_1(\tau) = 1 + 12q + 36q^2 + 12q^3 + \ldots$$

$$y_2(\tau) = -6q^{1/3}(1 + 7q + 8q^2 + \ldots) \qquad q \equiv e^{i2\pi\tau}$$

$$y_3(\tau) = -18q^{2/3}(1 + 2q + 5q^2 + \ldots)$$

$$Y_3^{(4)} = (y_1^2 - y_2 y_3, y_3^2 - y_1 y_2, y_2^2 - y_1 y_3)$$

$$Y_1^{(4)} = y_1^2 + 2y_2 y_3$$

$$Y_{1'}^{(4)} = y_3^2 + 2y_1 y_2$$

$$Y_{1''}^{(4)} = y_2^2 + 2y_1 y_3$$

$$Y_1^{(6)} = y_1^3 + y_2^3 + y_3^3 - 3y_1 y_2 y_3$$

$$Y_{3a}^{(6)} = (y_1^3 + 2y_1 y_2 y_3, y_1^2 y_2 + 2y_2^2 y_3, y_1^2 y_3 + 2y_3^2 y_2)$$

$$Y_{3b}^{(6)} = (y_3^3 + 2y_1 y_2 y_3, y_3^2 y_1 + 2y_1^2 y_2, y_3^2 y_2 + 2y_2^2 y_1)$$

$$Y_{3c}^{(6)} = (y_2^3 + 2y_1 y_2 y_3, y_2^2 y_3 + 2y_3^2 y_1, y_2^2 y_1 + 2y_1^2 y_3)$$

$\mathscr{A}_4$ MODULAR SYMMETRIC MODEL

Fields	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$\Gamma_3 \simeq A_4$	$-k$
L_i	1	2	$-\frac{1}{2}$	$\mathbf{3}$	-3
E_i^c	1	1	1	$\mathbf{1}, \mathbf{1}', \mathbf{1}''$	-1
H_u	1	2	$\frac{1}{2}$	$\mathbf{1}$	0
H_d	1	2	$-\frac{1}{2}$	$\mathbf{1}$	0
Δ	1	3	1	$\mathbf{1}$	0

$\mathscr{A}_4$ MODULAR SYMMETRIC MODEL

$$\mathscr{W} = \alpha_1 \left(Y_e L\right)_1 E_1^c H_d + \alpha_2 \left(Y_e L\right)_{1''} E_2^c H_d + \alpha_3 \left(Y_e L\right)_{1'} E_3^c H_d$$

$$+\alpha \left(Y_{\nu,1} (LL)_{3_S}\right)_1 \Delta + \beta \left(Y_{\nu,2} (LL)_{3_S}\right)_1 \Delta$$

$$+\mu H_u H_d + \mu_\Delta H_d H_d \Delta$$

Yukawas	$\Gamma_3 \simeq \mathcal{A}_4$	$\boldsymbol{k}$
$\boldsymbol{Y_e = Y_3^{(4)}}$	$\mathbf{3}$	4
$\boldsymbol{Y_{\nu,1} = Y_{3a}^{(6)}}$	$\mathbf{3}$	6
$\boldsymbol{Y_{\nu,2} = Y_{3b}^{(6)}}$	$\mathbf{3}$	6

NEUTRINO AND CHARGED LEPTON MASS MATRIX

$$M_\ell = v_H \begin{pmatrix} Y^{(4)}_{3,1} & Y^{(4)}_{3,2} & Y^{(4)}_{3,3} \\ Y^{(4)}_{3,3} & Y^{(4)}_{3,1} & Y^{(4)}_{3,2} \\ Y^{(4)}_{3,2} & Y^{(4)}_{3,3} & Y^{(4)}_{3,1} \end{pmatrix} \begin{pmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{pmatrix}.$$

$$M_\nu = v_\Delta \begin{pmatrix} 2Y_1 & -Y_3 & -Y_2 \\ -Y_3 & 2Y_2 & -Y_1 \\ -Y_2 & -Y_1 & 2Y_3 \end{pmatrix}$$

where $Y_i \equiv \alpha\, Y^{(6)}_{3a,i} + \beta\, Y^{(6)}_{3b,i}$ and $i \in \{1,2,3\}$

NEUTRINO MASS SUM RULE

The neutrino mass matrix features the interesting neutrino mass ordering independent sum rule

$$m_{\text{heaviest}} = \frac{1}{2}\sum_i m_i, \ ^{\dagger} \quad i = 1,2,3.$$

m_i: the three physical masses of the neutrinos

m_{heaviest}: the heaviest out of the three light neutrinos

[†]:S.C.Centelles et.al., JHEP 10 (2022) 080

Neutrino Mass Sum Rule + Neutrino Oscillations = the Neutrino Masses are fixed

NO:

$$m_3 = m_1 + m_2$$

$$\Delta m^2_{21} = 7.5 \times 10^{-5}\,\text{eV}^2, \quad \Delta m^2_{31} = 2.55 \times 10^{-3}\,\text{eV}^{2\star}$$

$$m_1 = 0.0282\,\text{eV}, \quad m_2 = 0.0295\,\text{eV}, \quad m_3 = 0.0578\,\text{eV}$$

IO:

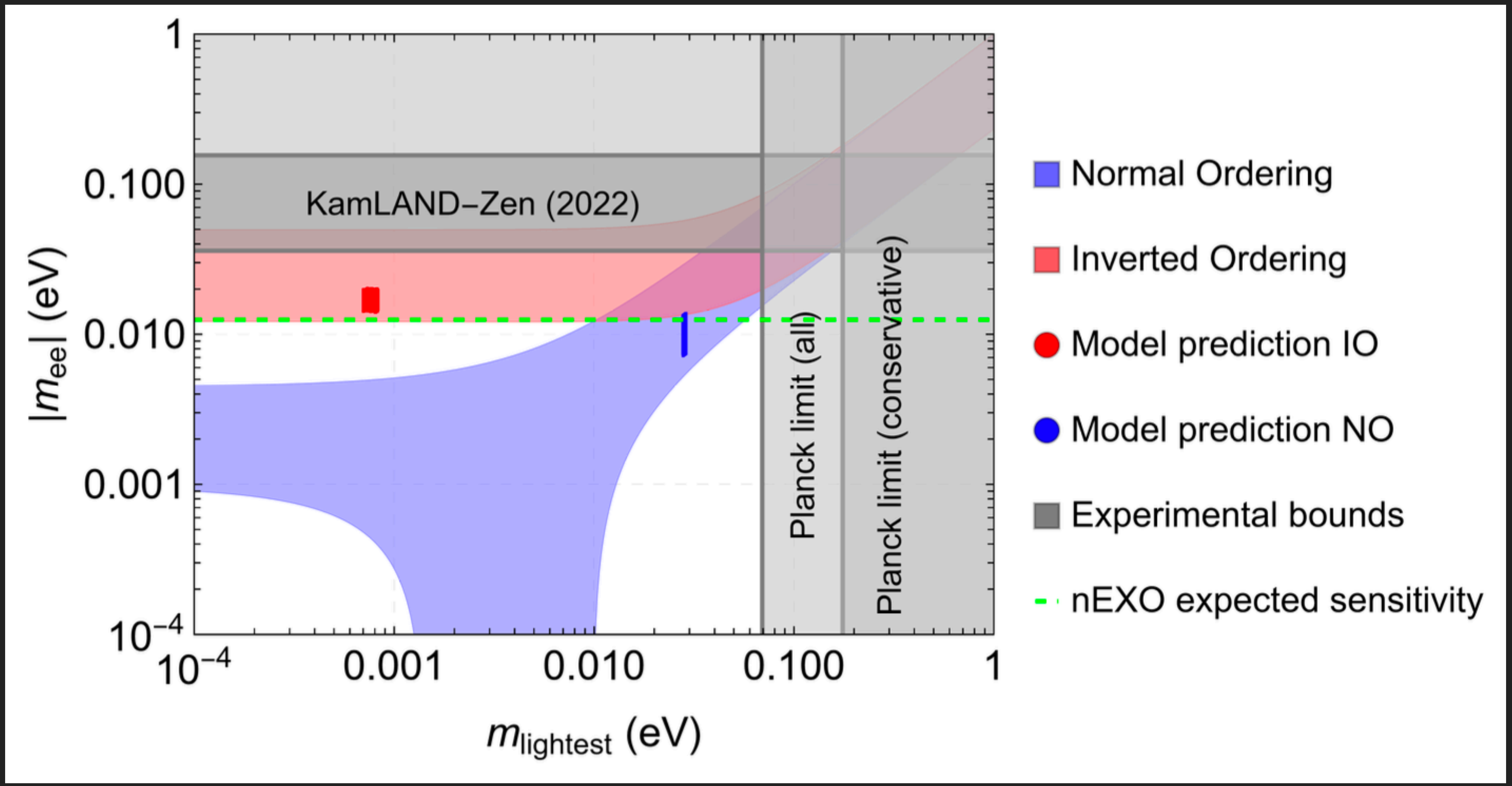
$$m_2 = m_1 + m_3$$

$$\Delta m^2_{21} = 7.5 \times 10^{-5}\,\text{eV}^2, \quad \Delta m^2_{31} = -2.45 \times 10^{-3}\,\text{eV}^{2\star}$$

$$m_3 = 7.5 \times 10^{-4}\,\text{eV}, \quad m_1 = 0.049\,\text{eV}, \quad m_2 = 0.050\,\text{eV}$$

*:P.F.Salas *et.al.*, JHEP 02 (2021) 071; P.F.Salas *et.al.*, 10.5281/zenodo.4726908.

RESULTS



$$\sum_i m_i^{\text{NO}} \in [0.1138, 0.1176] \text{ eV}^\star$$

$$\sum_i m_i^{\text{IO}} \in [0.1007, 0.1041] \text{ eV}^\star$$

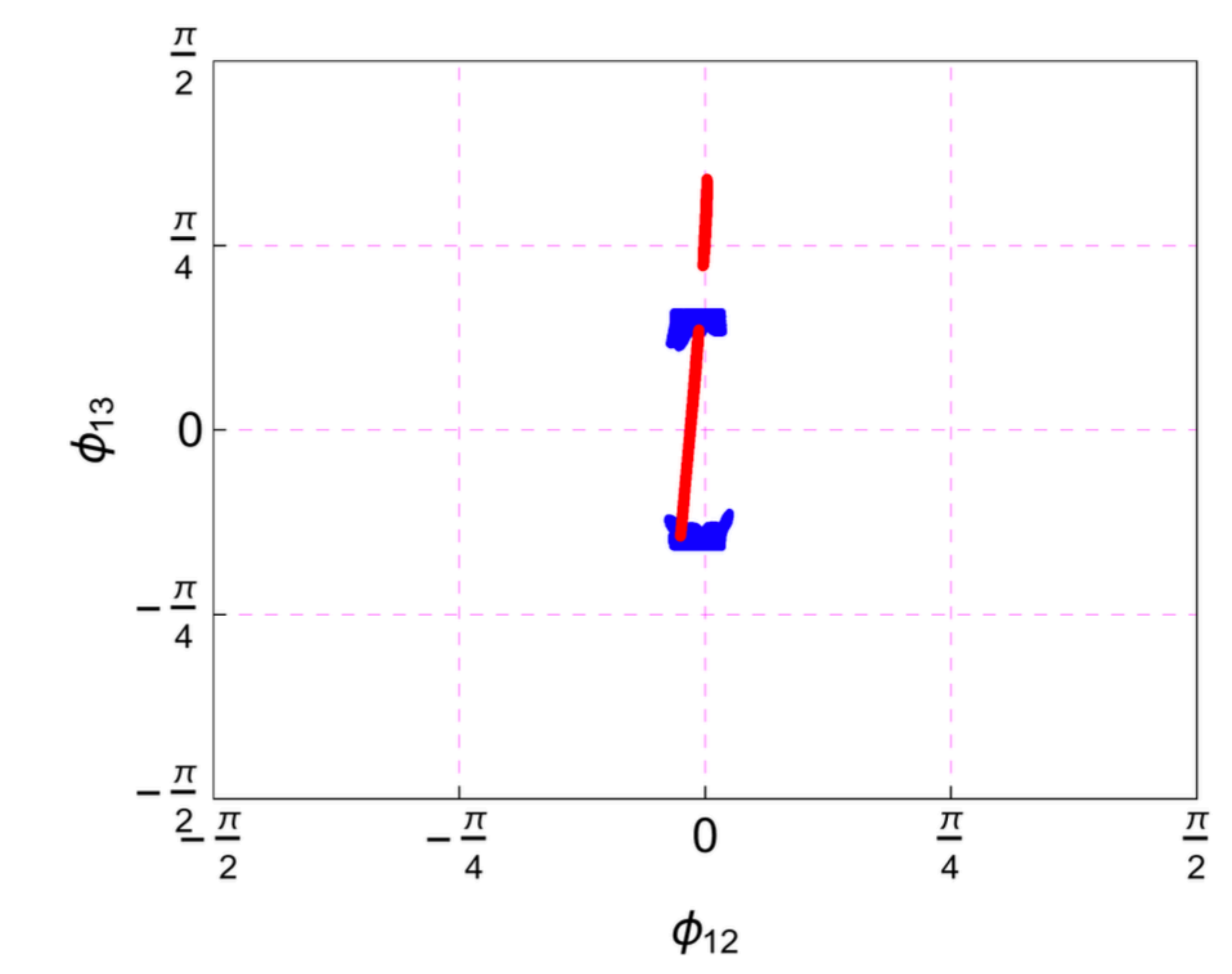
$$|m_{ee}| = \left| \sum_i U_{ei}^2 m_i \right|^a$$

$$= \left| c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 e^{2i\phi_{12}} m_2 + s_{13}^2 e^{2i\phi_{13}} m_3 \right|$$

*:N. Aghanim *et.al.*, Astron. Astrophys. 641 (2020) A6; a: W. Rodejohann *et.al.*, Phys. Rev. D 84 (2011) 073011

RESULTS

Correlation between Majorana phases



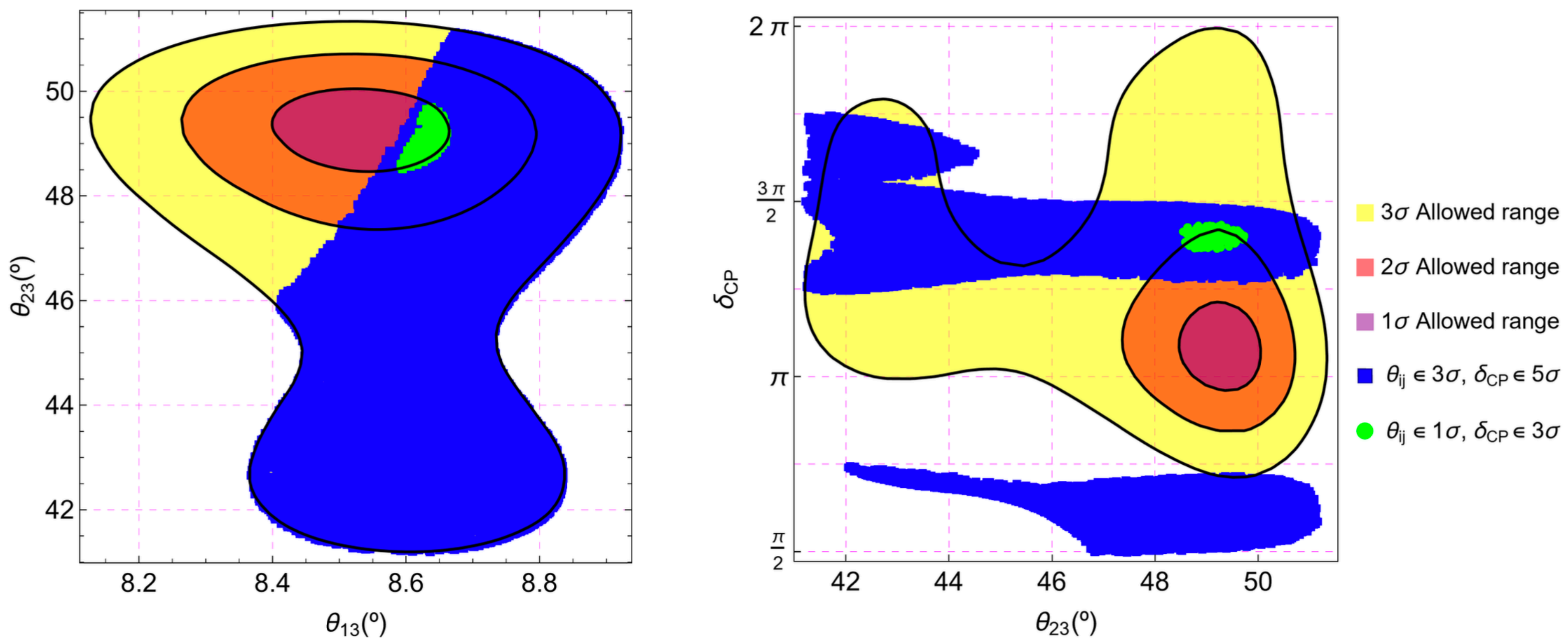
$$m_{\nu_e}^{\text{eff}} = \sqrt{\sum_i |U_{ei}|^2 m_i^2},$$

$m_{\nu_e}^{\text{eff}} < 0.8$ eV, 90% CL (2023),

$m_{\nu_e}^{\text{eff}} < 0.2$ eV, 90% CL (2025 expected sensitivity)

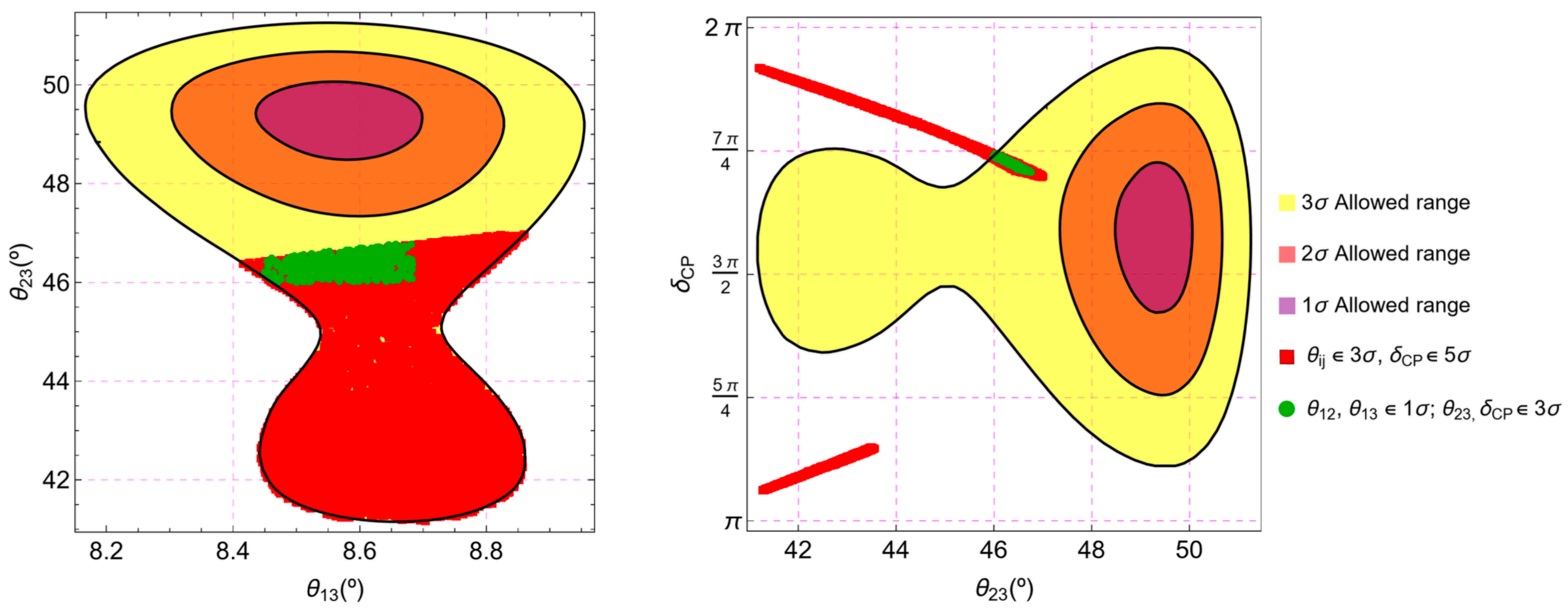
†: M. Aker *et.al.*, [Nature Phys. 18 no. 2, (2022) 160–166]

RESULTS



- NO case: the blue dots satisfy all the 2D 3σ mixing angles correlations, while the CP violation phase δ_{CP} is in its 5σ region. The green dots instead have the mixing angles inside their 1σ regions and δ_{CP} in their 3σ ones.
- Left:Correlation between θ_{13} and θ_{23}. This correlation implies an lower bound on $\theta_{13} > 8.36^{\circ}$.
- Right: Correlation between θ_{23} and δ_{CP}

RESULTS



- IO case: the red dots satisfy all the 2D 3σ mixing angles correlations, while the CP violation phase δ_{CP} is in its 5σ region. The dark green dots instead have the mixing angles θ_{12} and θ_{13} inside their 1σ regions, while δ_{CP} and θ_{23} are in their 3σ ones.
- Left: θ_{13} vs θ_{23}. The model predicts an upper bound on $\theta_{23} < 46.8^\circ$.
- Right: θ_{23} vs δ_{CP}.

SUMMARY

- Naturally small neutrino masses and Minimal MSSM extension
- Remarkably predictive model
- Sum Rule + Neutrino Oscillations = Fixed Neutrino Masses
- NO: $\theta_{13} > 8.36^{\circ}$, correlation between θ_{23} and δ_{CP}
- IO: $\theta_{23} < 46.8^{\circ}$, correlation between θ_{23} and δ_{CP}
- The combination of current and future neutrino oscillation experiments will reduce the parameter space even further and will potentially rule out the inverted ordering case.

谢谢！