第五届粒子物理前沿研讨会

α-generalized no-scale inflation

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Based on 2403.07333: **L Wu,** T Li, J Pei;
PRD (2022), 2205.14639: **L Wu,** T Li;
PRD (2021), 2105.07694: **L Wu,** T Li, Y Gong

Outline

1. Motivation
2. No-Scale Supergravity
   - Generic no-scale supergravity from string compactification
   - α-Generalized No-scale inflation
3. Inflation and Cosmological Predictions
4. PBH and SIGW formation
5. Summary

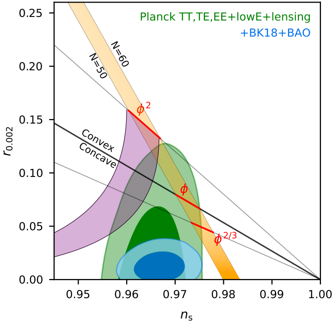
Inflation is one of leading candidates describing the evolution of the early Universe.

- The Universe undergoes a brief period of **exponential expansion** after the Big Bang;
- The generated scalar perturbations are supposed to be adiabatic, almost Gaussian and close to scale-invariant, which are consistent with the full-sky CMB measurements.

Constrains on the amplitude of PGWs:
Planck 2018 data:
$n_s = 0.9649 \pm 0.0042$ (68% C.L.),
$r_{0.002} \leq 0.10$ (95% C.L.), $A_s = 2.10 \times 10^{-9}$

- Combine with BK15: $r_{0.002} \leq 0.056$ (95% C.L.)
- Combine with BK18: $r_{0.05} \leq 0.032$ (95% C.L.)

Big challenge to inflationary models!



Planck Collaboration, 1807.06211; and BICEP/Keck Collaboration, 2110.00483, 2112.07961, 2203.16556

- Supergravity, low-energy effective theory derived from string theory, is a natural framework for inflationary model building.
- No-scale supergravity
  - can elegantly avoid η-problem
  - has a vanishing cosmological constant
  - evades the Anti-de Sitter(AdS) vacua
  - can be realized by string compactifications [1]
- The simple no-scale supergravity inspired from string compactifications is

$$K = -3\log(T + \bar{T} - 2|\varphi_i|^2)$$

where T is the Kähler moduli and φ_i denote the matter, Higgs and inflaton fields.

  - $R + R^2$ Starobinsky model can be obtained by considering a Wess-Zumino superpotential [2]
  - Detectable predictions: lower $r \simeq 12/N^2 \sim 0.001$, PBH and SIGW formation, NANOGrav data ...

[1] E. Witten, PLB 1985; T. Li, J.L. Lopez and D. V. Nanopoulos, PRD 1997

[2] J. Ellis, M. A. G. Garcia, D. V. Nanopoulos and K. A. Olive, PRL 2013

- Generalizing the factor 3 to 3α, the unified no-scale attractors[3] are studied with

$$K = -3\alpha \log(T + \overline{T} - 2|\varphi|^2)$$

  - $\alpha < 1$, may occur if not all the complex Kähler moduli contribute to driving inflation;
  - $\alpha > 1$, may occur if complex structure moduli also contribute to driving inflation

- The whole ovservational $n_s - r$ plane is covered by the complement of "exponetial α-attractor" models (left) and the "polynomial α-attractor" models (right)[4].
  - Exponetial α-attractor inflation:

$$n_s = 1 - \frac{2}{N}, \quad r = \alpha \frac{12}{N^2}.$$

  - Polynomial α-attractor inflation:

$$n_s = 1 - \frac{2}{N}\frac{k+1}{k+2}, \quad r \propto \frac{1}{N^{\frac{2(k+1)}{k+2}}}$$

[3] J. Ellis, D. V. Nanopoulos, K. A. Olive and S. Verner, JCAP 2019

[4] [illegible]

No-scale supergravity

- The $\mathcal{N}=1$ supergravity Lagrangian can be written in the form

$$\mathcal{L} = -\frac{1}{2}R + K_i^{\bar{j}}\partial_\mu \varphi^i \partial^\mu \overline{\varphi}_{\bar{j}} - V,$$

where the Kähler metric is $K_i^{\bar{j}} \equiv \partial^2 K/(\partial\varphi^i \partial\overline{\varphi}_{\bar{j}})$.

- The effective scalar potential is

$$V = e^G \left[\frac{\partial G}{\partial \varphi^i} \left(K^{-1}\right)_{\bar{j}}^{i} \frac{\partial G}{\partial \overline{\varphi}_{\bar{j}}} - 3\right],$$

where the Kähler function is $G \equiv K + \ln|W|^2$, and $\left(K^{-1}\right)_{\bar{j}}^{i}$ is the inverse of the Kähler metric.

- Introducing Kähler covariant derivative $D_i W \equiv W_i + K_i W$, the scalar potential can be rewritten as[5]

$$V = e^K \left[D_i W \left(K^{-1}\right)_{\bar{j}}^{i} D^{\bar{j}} \overline{W} - 3|W|^2\right].$$

[5] J. Ellis etal., 2009.01709 and Refs. in there

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Generic no-scale supergravity from string compactification

We propose inflationary models in the general no-scale supergravity theories inspired by various compactifications of M-theory[6].

- **Generic Kähler potential**

$$K = -N_X \log(T_1 + \overline{T}_1 - 2|\varphi|^2) - N_Y \log(T_2 + \overline{T}_2) - N_Z \log(T_3 + \overline{T}_3),$$

where $N_X + N_Y + N_Z = 3$ and $N_{X,Y,Z}$ are integers.

- The **renormalizable superpotential** in ploynomial form

$$W = \sum_{i=0}^{3} a_i(\sqrt{2}\varphi)^i, \quad \text{with} \quad W_T = 0.$$

- The general scalar potential can be written as

$$\boxed{V = \frac{|W_\varphi|^2}{2N_X X^{N_X - 1} Y^{N_Y} Z^{N_Z}}}$$

where $X \equiv T_1 + \overline{T}_1 - 2|\varphi|^2$, $Y \equiv T_2 + \overline{T}_2$ and $Z \equiv T_3 + \overline{T}_3$.

[6] E. Witten, Nucl. Phys. B (1985); T. Li etal., hep-ph/9704247; T. Li, hep-th/9801123

CMB predicitons

- **One Modulus Model**: The E-model for φ^2 or Starobinsky inflation model can be realized. The spectrum index and tensor-to-scalar ratio are

$$n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{12}{N^2} \sim 10^{-3}.$$

- **Two Moduli Model**: The **higher order term** $\left(\frac{1+d}{1-d} e^{\chi}\right)^2$ in the numerator in potential is not small and **cannot be ignored**. Then the tensor-to-scalar ratio r is approximate to be

$$r \simeq \frac{83.60}{N^4} \sim 10^{-5}.$$

- **Three Moduli Model**: Similar to that with global supersymmetry: $V \propto |W_\varphi|^2$. The cosmological predictions n_s and r, and e-folding number are given by

$$n_s = 1 - \frac{2\left(n + \sqrt{4n^2+1} + 4nN\right)}{n + 2N\sqrt{4n^2+1} + 4nN^2} \sim 1 - \frac{2}{N},$$

$$r = \frac{16n}{n + 2N\sqrt{4n^2+1} + 4nN^2} \sim \frac{4}{N^2} \sim 10^{-4}$$

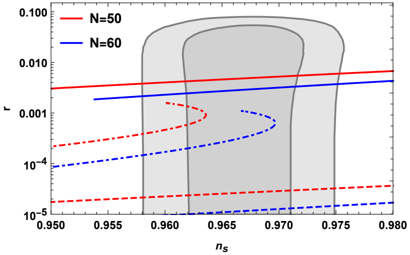
Special case

Taking the Wess-Zumino superpotential,

$$W = \frac{M}{2}\varphi^2 - \frac{\lambda}{3}\varphi^3, \quad \text{with} \quad W_T = 0.$$

the inflation potentials become ($d = \lambda/M$)

$$V_1 = \frac{M^2\varphi^2(1-d\varphi)^2}{3(c-\varphi^2)^2}, \quad V_2 = \frac{M^2\varphi^2(1-d\varphi)^2}{2c_3(c_1-\varphi^2)}, \quad V_3 = \frac{M^2\varphi^2(1-d\varphi)^2}{c_2 c_3}.$$



Top: one modulus model $r \sim 10^{-3}$
Middle: three moduli model $r \sim 10^{-4}$
Bottom: two moduli model $r \sim 10^{-5}$

The tensor-to-scalar ratios are much smaller than 0.032.

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α-generalized no-scale inflation

The Kähler potential is given by

$$K = -3\alpha \ln\left(T + \overline{T} - |\varphi|^2\right) - 3(1-\alpha)\ln\left(T' + \bar{T}'\right).$$

Here, the parameter $0 < \alpha \leq 1$ will continuously connect the above three models with each other.

- Given the Wess-Zumino superpotential, $\beta = \lambda/M$
- Stabilize the muduli fields, $2\langle \mathrm{Re}(T_i)\rangle = c_i$ and $\langle \mathrm{Im}(T_i)\rangle = 0$
- Assume that the inflation goes along the real components of φ

The scalar potential becomes

$$V_J(\varphi) = V_0 \varphi^2 (1 - \beta\varphi)^2 \left(c_1 - \varphi^2\right)^{1-3\alpha}$$

The kinetic term in Lagrangian is **noncanonical**, so we need to define a new canonical field χ, which satisfies

$$\frac{1}{2}\partial_\mu\chi\partial^\mu\chi = K_{\varphi\bar{\varphi}}\partial_\mu\varphi\partial^\mu\bar{\varphi}$$

By integrating the above equation, we get the **field transformation**

$$\chi = \sqrt{6\alpha}\tanh^{-1}\left(\frac{\varphi}{\sqrt{c_1}}\right) \ , \quad \varphi = \sqrt{c_1}\tanh\left(\frac{\chi}{\sqrt{6\alpha}}\right).$$

Then the potential in Einstein Frame is

$$V_E(x) = V_0 \tanh^2\left(\frac{x}{\sqrt{6\alpha}}\right)\operatorname{sech}^2\left(\frac{x}{\sqrt{6\alpha}}\right)^{1-3\alpha}\left(1-\beta\tanh\left(\frac{x}{\sqrt{6\alpha}}\right)\right)^2$$

with $\chi = (x + \mathrm{i}y)/\sqrt{2}$

Starobinsky-like inflation model

$$K = -3\alpha \ln(T + \bar{T} - |\varphi|^2) - 3(1-\alpha)\ln(T' + \bar{T}'),$$
$$W = \sqrt{3\alpha}(2T - \varphi^2)^{3\alpha} f(\varphi).$$

Then the scalar potential becomes

$$V = (1 - \varphi^2)^{3\alpha+1} f'(\varphi)$$

Field transformations:

$$\varphi = \sqrt{2}\tanh(\chi/\sqrt{6\alpha}), \ \chi = \sqrt{6\alpha}\tanh^{-1}(\varphi/\sqrt{2})$$

The Starobinsky-like inflation potential:

$$\boxed{V = \frac{1}{12}M^2(1 - e^{-\sqrt{\frac{2}{3\alpha}}\chi})^2}$$

Here, we choose $f'(\phi) = M\varphi(1+\varphi)(1-\varphi)^{-\frac{1+3\alpha}{2}}$

Slow-roll phase

- Background evolution of the canonical scalar field

$$\ddot{x} + 3H\dot{x} + V'(x) = 0$$
$$3H^2 = \frac{1}{2}\dot{x} + V(x)$$

Here, we set $M_{\rm Pl} = 1$.

- Hubble slow-roll parameters

$$\epsilon_H = -\dot{H}/H^2, \;\; \eta_H = \dot{\epsilon}_H/(H\epsilon_H)$$

- Slow-roll conditions

$$\epsilon_H, \eta_H \ll 1$$

- Number of e-foldings

$$N = \int_{t_*}^{t_e} H(t)\mathrm{d}t \sim 50 - 60$$

- Mukhanov-Sasaki equation: equation of motion for the inflaton fluctuation $v = a\delta x$,

$$\frac{\mathrm{d}^2 v_k}{\mathrm{d}\tau^2} + \left(k^2 - \frac{1}{z}\frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2}\right) v_k^2 = 0$$

- Bunch-Davis vacuum:

$$v_k|_{k\tau\to-\infty} = e^{-ik\tau}/\sqrt{2k}$$

- Power spectrum

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z}\right|^2_{k\ll aH}$$

- CMB observations

$$n_s = 1 + 2\eta_H - 4\epsilon_H, \;\; r = 16\epsilon_H,$$
$$P_{\mathcal{R}} = H^2/(8\pi^2\epsilon_H)$$

Ultra-slow-roll phase

- The Friedman equation and the Klein-Gordon function can be written as

$$x'' + (3 - \varepsilon_H)x' = 0 \ , \quad H^2 = \frac{2V}{6 - x'^2} . \quad (' \to N = \ln a)$$

- The velocity of inflaton $x' = e^{-3N}$, and then the Hubble slow-roll parameters become

$$\varepsilon_H = \frac{1}{2}x'^2 \ , \quad \eta_H = \varepsilon_H - \frac{1}{2}\frac{\varepsilon_H'}{\varepsilon_H} .$$

- When **potential is perfectly flat**, the slow-roll conditions are violated and the power spectrum experiences a notable enhancement:

$$\varepsilon_H \simeq \varepsilon_{SR} \sim 0, \ \eta_H \simeq 3, \ \mathcal{P}_{\mathcal{R}} = \frac{H^2}{8\pi^2 \varepsilon_H} \sim e^{6N} .$$

Inflationary predictions

- $\alpha = 1$ and $\beta = 1$: Straobinsky inflation, $V = \frac{V_0}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}x}\right)^2$.
- $0 < \alpha < 1/3$: Define $Y \equiv \cosh\frac{2x}{\sqrt{6\alpha}}$, then the predictions are

$$ns(Y) \simeq 1 - 4\left(1 + \frac{1}{3\alpha}\right)\frac{1}{1+Y}\ , r(Y) \simeq \frac{64}{3\alpha\left(Y^2 - 1\right)}$$

or

$$n_s(N) = 1 - \frac{4\left(9\alpha^2 - 1\right)}{3\alpha\left(6\alpha + e^{\left(2-\frac{2}{3\alpha}\right)N} - 4\right)}$$

When $Y \sim 10^2 - 10^3$, we have $n_s \sim 0.92 - 0.99$ and $r \sim 10^{-5} - 10^{-3}$.

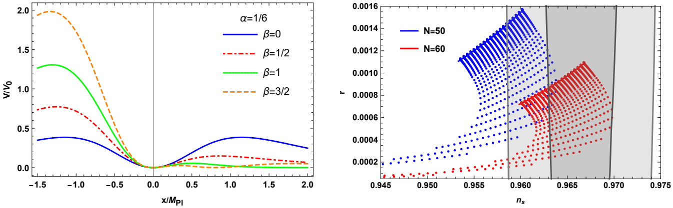


Figure 1: (a) Potential with $\alpha = 1/6$. (b) The observations n_s vs r

The dependence on parameters α and β

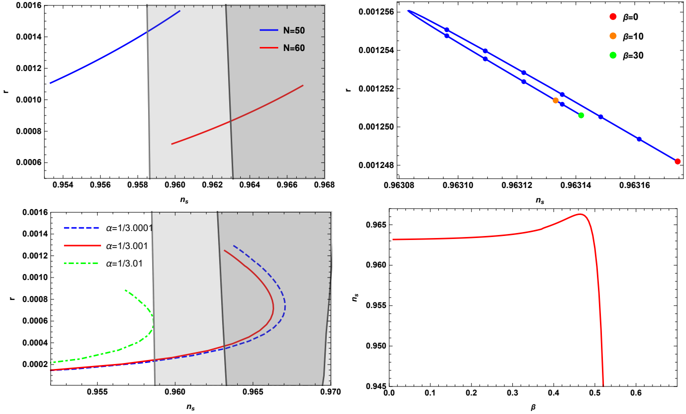


Figure 2: The observations n_s vs r.
Upper: $\alpha \lesssim 1/3$ and $\beta = 0$(left), $\alpha = 1/3.001$ and $N = 55$ (right);
Down: For given α (left) and the dependence of n_s on β (right).

Models with $1/3 < \alpha < 1$: whole $n_s - r$ plane

$$P1: \quad V_x(\beta, x) = 0 \ , \quad V_{xx}(\beta, x) = 0 \ , \quad \text{with } x > 0$$
$$P2: \quad V_x(\alpha, \beta, x) \simeq 0 \ , \quad V_{xx}(\alpha, \beta, x) = 0 \ , \quad \text{with } x > 0$$

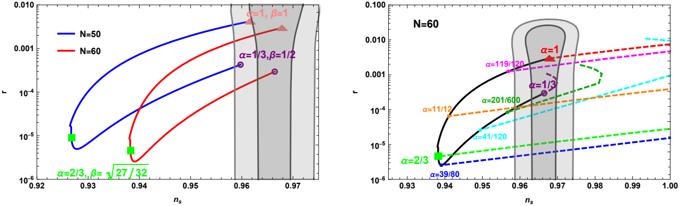
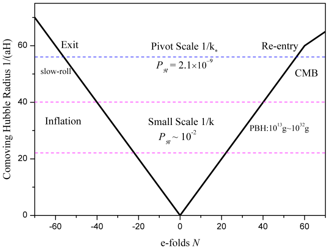
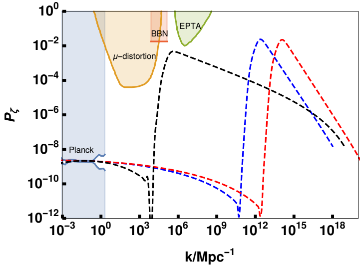


Figure 3: The observations n_s *vs* r for models with parameters from space P1(left) and P2(right) .

- PBH will give useful information for early Universe and produce various astrophysical consequences
  - primordial density perturbations, ...
  - seed for supermassive BHs, generation of large-scale structure, ...
- PBH as cold dark matter candidates.
  - during inflation, pre/re-heating, phase transition, domain wall, cosmic string, peak theory, ...
- An important topic: formation of PBH during inflation:
  - Inflection inflation model
  - Multi-scalar inflation model
  - Framework of non-minimal derivative coupling
- Near the inflection point: the slow-roll conditions are violated ($\varepsilon_H \sim 10^{-7}$; $\eta_H \sim 3$), so the primordial power spectrum is enhanced, at the same time the number of e-folds also increases dramatically.

Inflection point model



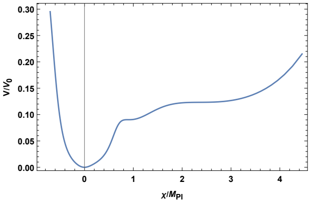


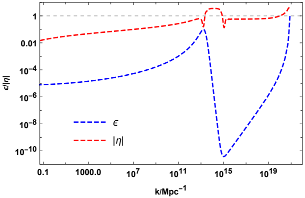
- Power spectrum at different scale:

  Large scale: Scale invariant, CMB; **Small scale**: Enhancement, PBHs
- Ultra-slow-roll phase: An exponential term is added to bring an inflection point into the Kähler potential,

$$-ae^{-b(\varphi^2+\bar{\varphi}^2)}(\varphi^2+\bar{\varphi}^2)$$

The potential of the cannonical scalar field χ





- There is an inflection point at $\chi_p = 0.877\ \mathrm{M_{Pl}}$, where the slow-roll conditions are no longer satisfied: $\epsilon \sim 10^{-7}$ and $\eta \sim 3$.
- Near the inflection, the primordial curvature perturbations are enhanced, which cause gravitational collapse in the overdense region at the horizon re-entry during the radiation-dominated era.
- If the density fluctuation is large than a centain threshold $\delta_c(0.07 - 0.7)$, the gravity can overcome the pressure and hence PBH forms.

- Assuming the primordial perturbations obey Gaussian statistics, the fractional energy density of PBHs at their formation time is given by the Press-Schechter formalism

$$\beta(M) \equiv \frac{\rho_{PBH}}{\rho_{tot}} \simeq \sqrt{\frac{2}{\pi}} \frac{\sqrt{P_\zeta}}{\mu_c} \exp\left(-\frac{\mu_c^2}{2P_\zeta}\right), \quad \text{with } \mu_c = 9\delta_c/2\sqrt{2}$$

- The fractional energy density of PBHs with the mass M to DM is[7]

$$f_{PBH}(M) \equiv \frac{\Omega_{PBH} h^2}{\Omega_{DM} h^2}$$
$$= \frac{\beta(M)}{3.94 \times 10^{-9}} \left(\frac{\gamma}{0.2}\right)^{\frac{1}{2}} \left(\frac{g_*}{3.36}\right)^{-\frac{1}{4}} \left(\frac{0.12}{\Omega_{DM} h^2}\right) \left(\frac{M}{M_\odot}\right)^{-\frac{1}{2}},$$

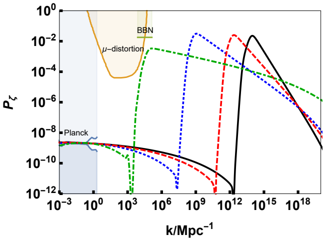
where $\gamma = 0.2$, $\Omega_{DM} h^2 = 0.12$ and $g_* = 106.75$.

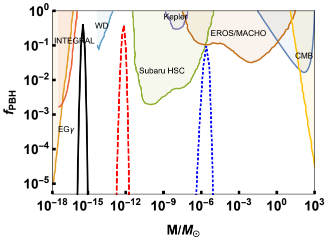
- The mass of PBHs is

$$\frac{M(k)}{M_\odot} = 3.68 \times \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*}{3.36}\right)^{-\frac{1}{6}} \left(\frac{k}{10^6 \ \mathrm{Mpc}^{-1}}\right)^{-2}.$$

[7] B. Carr etal., 1607.06077; B.Carr etal., 2002.12778

Primordial curvature perturbations and PBH abundances





Four benchmark points where the PBH mass is around $\mathcal{O}(10^{-16}M_\odot)$, $\mathcal{O}(10^{-13}M_\odot)$, $\mathcal{O}(10^{-6}M_\odot)$ and $\mathcal{O}(10^{2}M_\odot)$.

- $\Rightarrow$ almost all DM with $Y_{PBH} \simeq 1$;
- $\Rightarrow$ part of dark matter with $Y_{PBH} \simeq 0.07$;
- $\Rightarrow$ hard to explain DM because of the significantly small value of $Y_{PBH} \simeq 10^{-83}$.

Primordial curvature perturbations and PBH abundances

Model	β	a	b	$x_*/M_{\rm Pl}$	n_s	r	N
M_1	0.8181	0.4564032	5.4	2.034	0.9685	9.1×10^{-5}	51.6
M_2	0.8185	0.4528304	5.3	2.035	0.9642	1.4×10^{-4}	52.7
M_3	0.818	0.4497767	5.2	2.041	0.9689	5.6×10^{-4}	52.4
M_4	0.814	0.447051	5.05	2.050	0.9737	5.3×10^{-3}	56.5

Table 1: The parameters for the model with $\alpha=39/80$ and $c_1=1$.

Model	k/Mpc^{-1}	$\mathcal{P}_{\mathcal{R}}$	$M_{PBH}/M_\odot$	f_{PBH}	f_{GW}/Hz
M_1	9.6×10^{13}	0.023	4.0×10^{-16}	0.41	1.1
M_2	2.3×10^{12}	0.025	7.2×10^{-13}	0.39	0.026
M_3	1.2×10^{9}	0.031	2.5×10^{-6}	0.096	1.6×10^{-6}
M_4	1.7×10^{5}	3.3×10^{-3}	-	-	1.6×10^{-9}

Table 2: The peak values of the power spectrum, the mass and abundance of PBHs, and the frequency of SIGWs.

Scalar induced gravitational waves

Since the scalar perturbations and tensor perturbations are coupled at the second order, the large primordial curvature perturbation on small scales will induce second order tensor perturbations.

- The equation of motion for the tensor mode is

$$h_{\mathbf{k}}'' + 2\mathcal{H} h_{\mathbf{k}}' + k^2 h_{\mathbf{k}} = 4\mathcal{S}_{\mathbf{k}}$$

- Using Green's function, the power spectrum of the tensor perturbation can be written as

$$\mathcal{P}_h(k,\eta) = 4\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 I_{RD}^2(u,v,x)\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

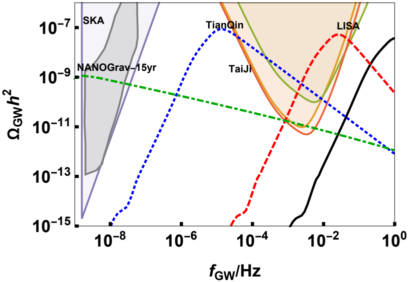
and the fractional energy density of the induced GWs[8] is

$$\Omega_{GW}(\eta,k) = \frac{\rho_{GW}(\eta,k)}{\rho_{tot}(\eta)} = \frac{1}{24}\frac{k^2}{\mathcal{H}^2}\overline{\mathcal{P}_h(\eta,k)},$$

where the overline denotes oscillation average.

[8] J.R. Espinosa etal., 1804.07732; F. Zhang etal., 2008.12961

The energy densities of SIGWs



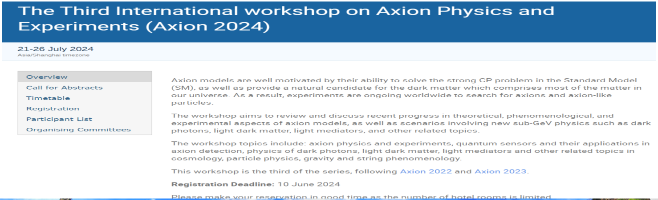
- Model M_4: wide peak at $[10^{-10},\ 10^2]$ Hz
- Model M_3: peak at 10^{-5} Hz
- Model M_2: peak at 10^{-2} Hz
- Model M_1: peak at 1 Hz

- The generated SIWGs will be tested by the space-based or ground-based GW detector.
- The wide bands can be interpreted as the stochastic GW background observed by NANOGrav.

- We have studied three classes of no-scale inflation models with one, two, and three moduli which can be realized naturally via **string compactifications**.
    - $n_s \simeq 1 - 2/N \sim 0.965$ for all models;
    - r is $r \simeq 12/N^2$, $83/N^4$ and $4/N^2$ for the one, two and three moduli models, respectively
- We have studied a novel class of α-generalized no-scale inflation models, where the parameter $0 < \alpha \leq 1$ will continuously connect the above three models with each other.
- Formation of PBHs and SIGWs are investigated by introducing an exponential term into Kähler potential.

The Third International workshop on Axion Physics and Experiments (Axion 2024), 21-26 July 2024

- Axion 2022: Online
- Axion 2023: Xi'an, Shaanxi province
- **Axion 2024: Zhangjiajie, Hunan province https://indico.itp.ac.cn/event/244/overview**



The Third International workshop on Axion Physics and Experiments (Axion 2024)

21-26 July 2024
Asia/Shanghai timezone

- Overview
- Call for Abstracts
- Timetable
- Registration
- Participant List
- Organising Committees

Axion models are well motivated by their ability to solve the strong CP problem in the Standard Model (SM), as well as provide a natural candidate for the dark matter which comprises most of the matter in our universe. As a result, experiments are ongoing worldwide to search for axions and axion-like particles.

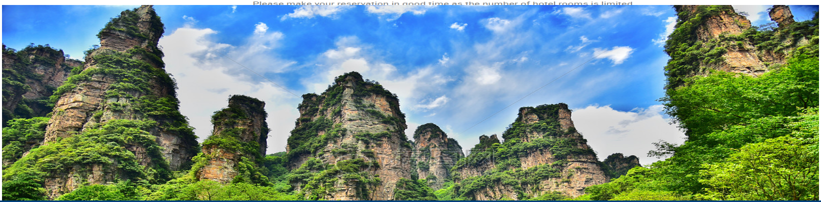
The workshop aims to review and discuss recent progress in theoretical, phenomenological, and experimental aspects of axion models, as well as scenarios involving new sub-GeV physics such as dark photons, light dark matter, light mediators, and other related topics.

The workshop topics include: axion physics and experiments, quantum sensors and their applications in axion detection, physics of dark photons, light dark matter, light mediators and other related topics in cosmology, particle physics, gravity and string phenomenology.

This workshop is the third of the series, following Axion 2022 and Axion 2023.

Registration Deadline: 10 June 2024

Please make your reservation in good time as the number of hotel rooms is limited.



Thank you!