

EW AMPLITUDES WITHOUT GAUGE CANCELLATION

JUNMOU CHEN (JINAN UNIVERSITY, GUANGZHOU)

AT [SUN YAT-SEN UNIVERSITY](#)-SHEZHEN

BASED MAINLY ON 2203.10440

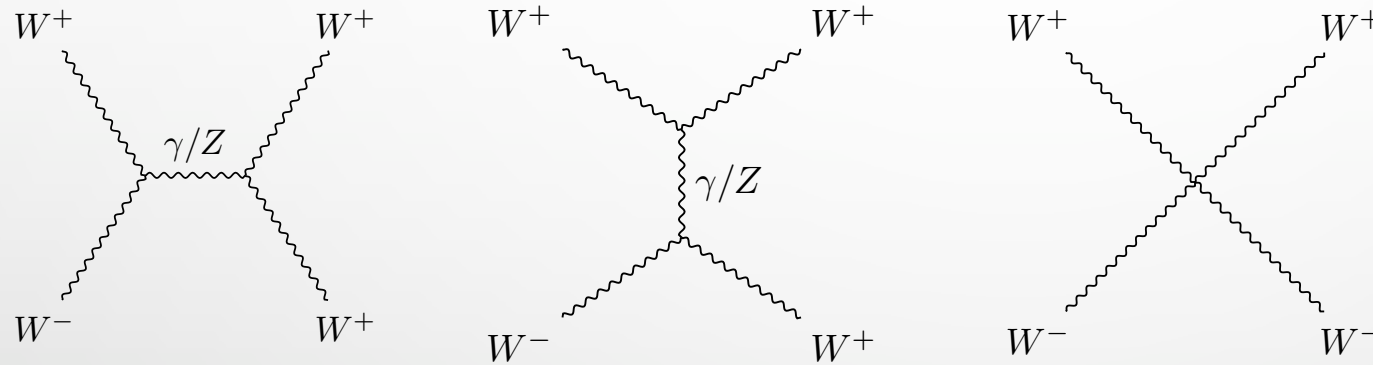
IN COLLABORATION WITH [KAORU HAGIWARA](#), [JUNICHI KANZAKI](#), [KENTAROU MAWATARI](#)

See also 1611.00788, 1902.06738

1.1 The Unitarity Problem of SM of EW Interactions

■ Ex: $WW \rightarrow WW$ with gauge couplings only

■ Longitudinal vector boson $\epsilon_L^\mu \sim k^\mu / m \sim E/m$



$$\propto (E/m_V)^4$$

■ Violate unitarity, theory not consistent.

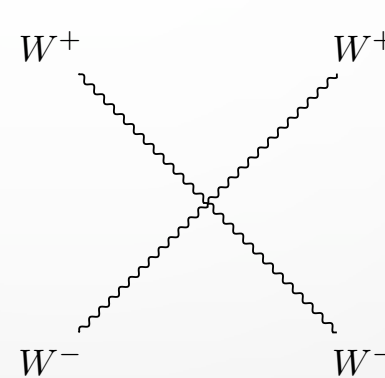
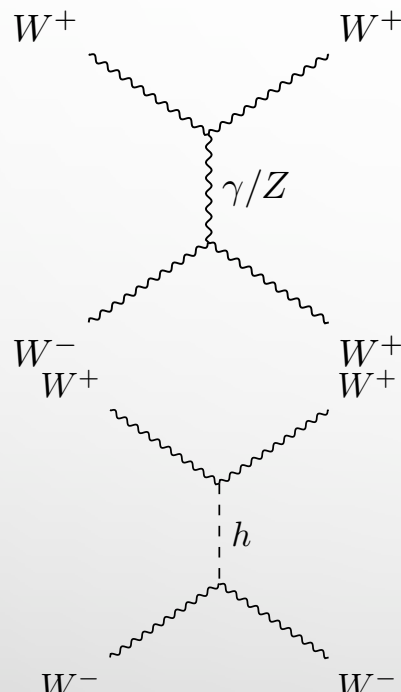
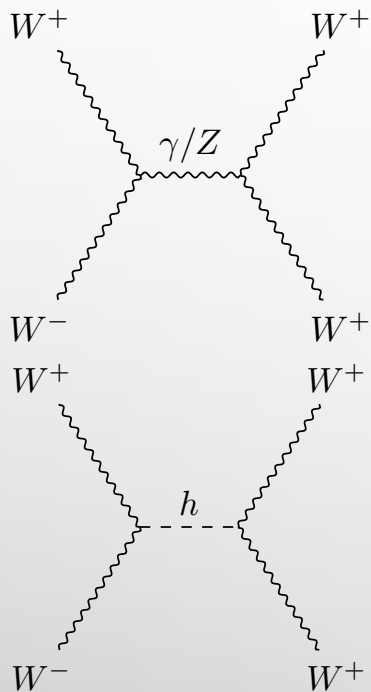
■ Also not renormalizable

■ 1. Motivation: subtle gauge cancellation

The Unitarity Problem of SM of EW Interactions

■ Solution: introduce a scalar with couplings to vector bosons

■ Vertices: hWW ; $g_{hWW} = igm_W$ (Higgs mechanism)



$$\propto (E/m_V)^4$$

$$\propto -(E/m_W)^2$$

■ Final Amplitude: $\mathcal{A} \propto \mathcal{O}(1)$

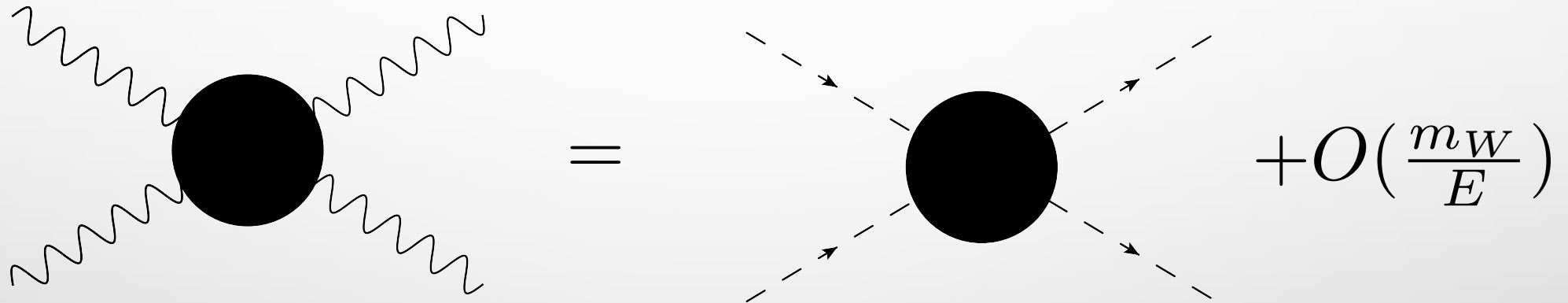
■ Unitarity restored

■ Condition: precise relation between gauge couplings and hWW

■ Reflection of gauge symmetry in S -matrix/amplitude

The Unitarity Problem of SM of EW Interactions

- Goldstone equivalence theorem(GET)
- Amplitudes of Longitudinal vector bosons approximately equal to amplitudes of the corresponding Goldstone bosons $W^\pm \simeq \phi^\pm$



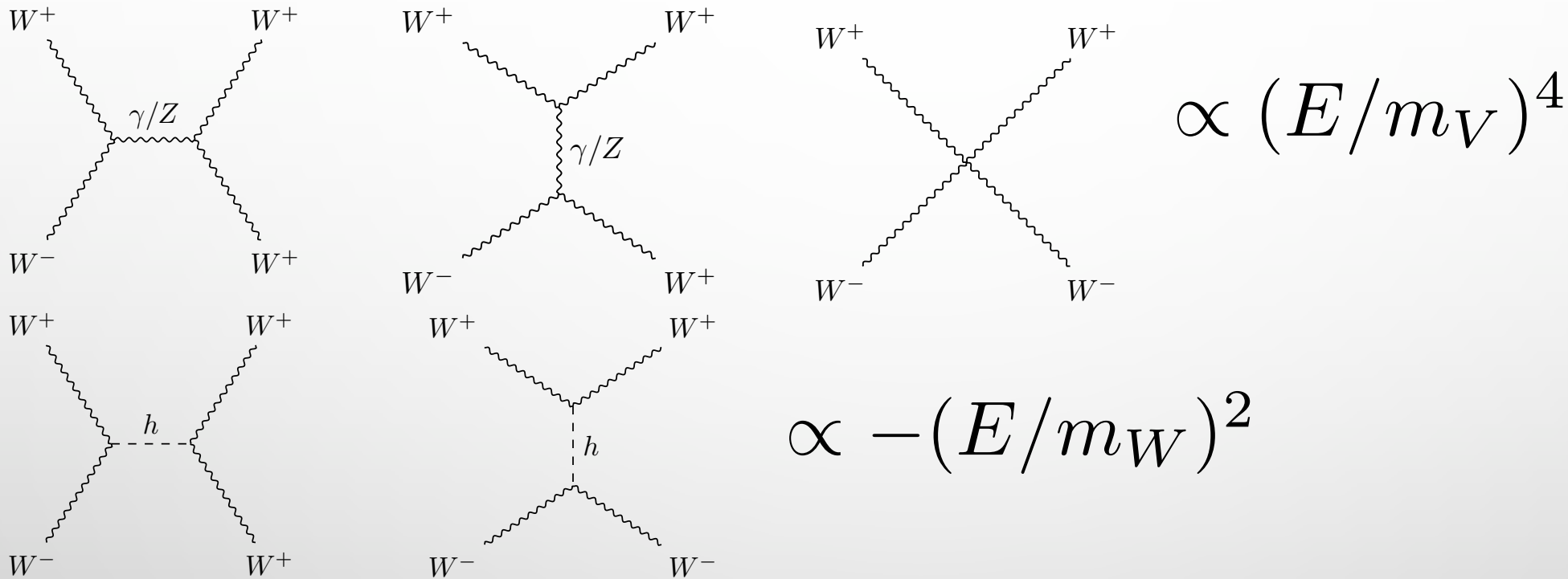
- GET is a direct consequence of “massive Ward identity”

$$k^\mu \mathcal{M}_\mu = im_W \mathcal{M}(\phi)$$

- So applies to any theory with gauge symmetry, e.g. SMEFT

1.2 The Technique Problem of Large Cancellation

- $WW \rightarrow WW$ Large cancellation between diagrams

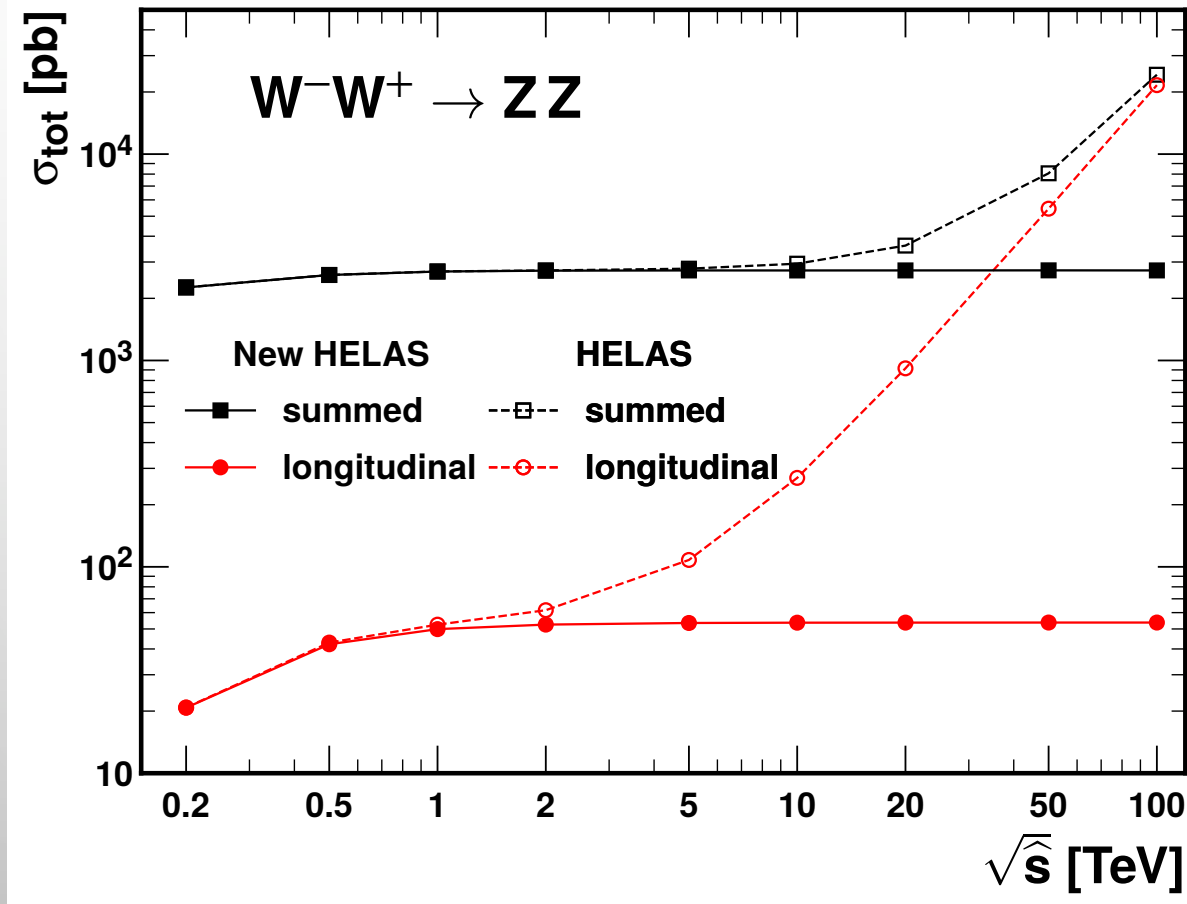


- Theory: single diagrams has unphysical energy behavior, power counting doesn't hold (compare standard way to compute $WW \rightarrow WW$ with GET)
- Numerical: large number cancellation easily leads to large error

■ 1. Motivation: subtle gauge cancellation

■ Numerical Problem

- Cancellation between large numbers(diagrams) easily leads to errors



Total cross section of $WW > ZZ$
when t/u channel propagators are
given physical decay width

(Madgraph before v3_** versions)

■ 1. Motivation: subtle gauge cancellation

- Numerical Problem: Cancellation between large numbers(diagrams)
- Computing efficiency

Madgraph v3_2_0

$pp > jj\ w+\{0\}w-\{0\}h:$

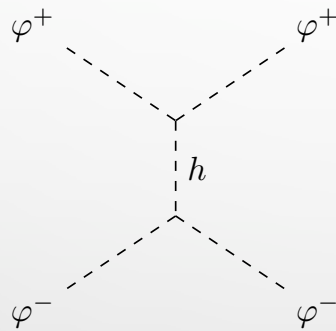
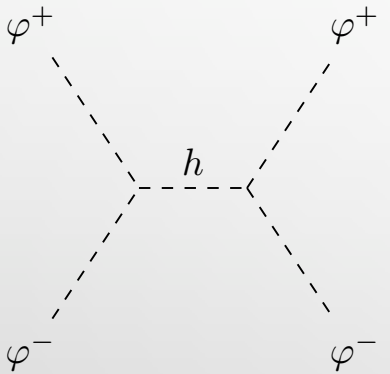
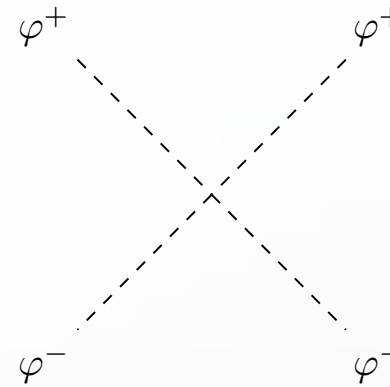
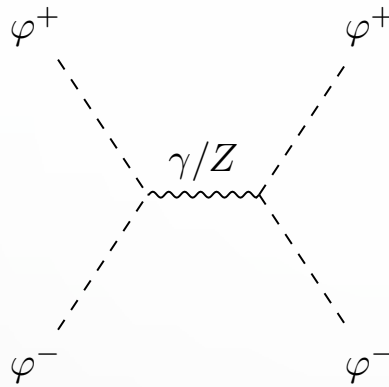
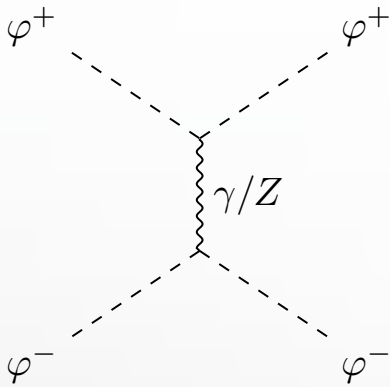
Event no: 147/10000

run_02	p p 7000.0 x 7000.0 GeV	tag_1	0.0001979 ± 2.3e-06	147	parton madevent	LHE	remove run	launch detector simulation
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$pp > jj\ w+\{T\}w-\{T\}h:$

Run	Collider	Banner	Cross section (pb)	Events	Data	Output	Action
run_01	p p 7000.0 x 7000.0 GeV	tag_1 ERROR	0.001458 ± 4.3e-06	10000	parton madevent	LHE	remove run launch detector simulation

$WW > WW$ IN GET



- Physical behavior for individual diagrams
- Power counting valid, again becomes a valuable tool for physical analysis
- However, not exact equivalent to VBS amplitude

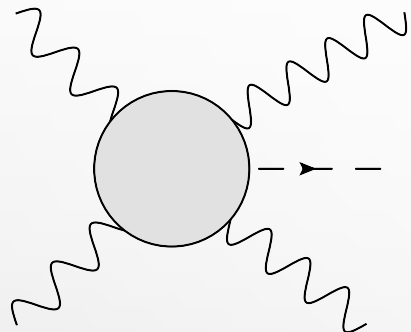
■ 1. Motivation: subtle gauge cancellation

■ Theoretical Problem

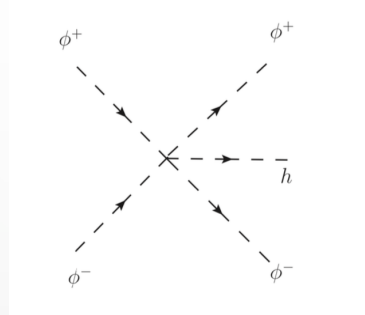
■ Ex.: SMEFT and Higgs Couplings Measurement

$$W_L W_L \rightarrow W_L W_L h / h h h$$

$$W_L^\pm / Z_L \sim \phi^\pm / \phi_0$$

$$E \gg m$$


$\simeq$



$$+ \mathcal{O}\left(\frac{m}{E}\right)$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v+h+i\phi^0}{\sqrt{2}} \end{pmatrix}$$

$$\mathcal{O}_6 = \frac{c_6}{\Lambda^2} (\Phi^\dagger \Phi)^3$$

At what exactly range is on this picture(GET) valid? Corrections?

Even simpler amplitude $WW \rightarrow WW$?

We need a method that manifests GET, but computes amplitudes exactly.

■ 1. Motivation: subtle gauge cancellation

■ Theoretical Problems

■ Ex: SMEFT and Higgs Couplings Measurement

■ Measuring Higgs self-couplings through $W_L^\pm/Z_L \sim \phi^\pm/\phi_0$ (Mainstream: double Higgs production)

$$W_L W_L \rightarrow W_L W_L h / h h h$$

$$E \gg m \quad \text{[Diagram: A circle with four external dashed lines, two incoming and two outgoing, representing a contact interaction.]} \quad \approx \quad \text{[Diagram: A contact interaction with four external dashed lines labeled } \phi^+, \phi^-, \phi^+, \phi^- \text{ and a central cross.]} \quad + \mathcal{O}\left(\frac{m}{E}\right)$$

$$\mathcal{O}_6 = \frac{c_6}{\Lambda^2} (\Phi^\dagger \Phi)^3$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v+h+i\phi^0}{\sqrt{2}} \end{pmatrix}$$

Importance is of the same level as hh production in muon colliders

- GET solution not completely satisfactory
- Need to take into account the energy suppressed terms

See, e.g. Nucl. Phys. B 261, 379 (1985)
M. S. Chanowitz and M. K. Gaillard

■ Ward identity $k^\mu \mathcal{M}_\mu = -im_V \mathcal{M}(\phi)$ (From gauge symmetry)

$$\epsilon_L^\mu = \frac{k^\mu}{m_W} - \frac{m}{E + |\vec{k}|} n^\mu \quad \mathcal{M}(W_L) = -i\mathcal{M}(\phi) - \frac{m_W}{E + |\vec{k}|} n^\mu$$

GET $\mathcal{O}(\frac{m}{E})$

■ N-point amplitudes:

$$\mathcal{M}(W_L, W_L, W_L, \dots, W_L) = (-i)^n \mathcal{M}(\phi, \phi, \phi, \dots, \phi) + (-i)^{n-1} \mathcal{M}(W_n, \phi, \phi, \dots, \phi) + \dots + \mathcal{M}(W_n, W_n, W_n, \dots, W_n) \quad (2)$$

The terms (diagrams) on the RH are 2^N times of LH (Naïve est.)

■ Problem: Too many diagrams, hard to calculate

■ 2. Solution: "New Feynman Diagram"

■ 5-component formalism

$$g_{MN} = \text{diag}(1, -1, -1, -1, -1)$$

$$\blacksquare \text{ GET } k^M = \begin{pmatrix} k^\mu \\ im_V \end{pmatrix} \quad k^M \mathcal{M}_M = k^\mu \mathcal{M}_\mu - i\mathcal{M}(\phi) = 0$$

■ Polarization vector

$$\epsilon_L^M = \begin{pmatrix} \epsilon_L^\mu \\ 0 \end{pmatrix} - k^M = \begin{pmatrix} \tilde{\epsilon}_n^\mu \\ i \end{pmatrix} \quad \text{Well behaved in high energy and a single object}$$

■ Gauge and Goldstone components combined together

$$\mathcal{M} = \epsilon^M \mathcal{M}_M = \tilde{\epsilon}^\mu \mathcal{M}_\mu - i\mathcal{M}(\phi)$$

■ N-point amplitudes(pol.vec., propagators, vertices)

$$\mathcal{M}(k_1, k_2, \dots, k_n) = \epsilon^{M_1} \epsilon^{M_2} \dots \epsilon^{M_N} \mathcal{M}_{M_1, M_2, \dots, M_N}(k_1, k_2, \dots, k_n)$$

■ 2. Solution: “New Feynman Diagram”

■ Gauge Choice and Propagators: 5-components formalism

■ Goldstone equivalence gauge(GEG)

proposed in JHEP11(2017)093

[Junmou Chen](#), [Tao Han](#) & [Brock Tweedie](#)

$$\epsilon_L^\mu = \frac{k^\mu}{m_W} - \frac{m}{E+|\vec{k}|} n^\mu$$

Gauge condition: $n \cdot V = 0$ or $n \cdot \epsilon_s = 0$

Transverse: $n \cdot \epsilon_\pm = 0$

$\epsilon_\pm$ satisfies

Longitudinal: $n \cdot \epsilon_L \neq 0$

$\epsilon_L^\mu = \frac{k^\mu}{m_V} + \tilde{\epsilon}_n^\mu$ does not satisfy

$\tilde{\epsilon}_n^\mu = -\frac{m_V}{n \cdot k} n^\mu$ with $n^\mu = (1, -\frac{\vec{k}}{|\vec{k}|})$

$$n \cdot n = 0 \rightarrow n \cdot \tilde{\epsilon}_n = 0$$

So where is k^μ term?

“Scalarized” into Goldstone mode! $k^\mu/m_V \rightarrow \phi$

■ 2. Solution: “New Feynman Diagram”

■ Gauge Choice and Propagator: 5-components formalism

■ Gauge cancellation exists also on the propagator (Rxi gauge)

■ Degrees of freedom in Feynman gauge

$$-g^{\mu\nu} + \frac{k^\mu k^\nu}{m_V^2} = \sum_{s=\pm, L} \epsilon_L^\mu \epsilon_L^\nu$$

energy increase

$$-g^{\mu\nu} = \sum_{s=\pm} \epsilon_s^\mu \epsilon_s^\nu + \epsilon_L^\mu \epsilon_L^\nu - \frac{k^\mu k^\nu}{m_V^2}$$

Physical degrees of freedom

$$= \sum_{s=\pm} \epsilon_s^\mu \epsilon_s^\nu - \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} + \frac{m^2 n^\mu n^\nu}{(n \cdot k)^2}$$

Even Feynman gauge gives unphysical energy increase!

■ 2. Solution: “New Feynman Diagram”

■ Gauge Choice and Propagators: 5-components formalism

■ Goldstone equivalence gauge(GEG)

$$\mathcal{L}_\xi = \frac{1}{2\xi} (n \cdot V)^2$$

$$\tilde{\epsilon}_n^\mu = -\frac{m_V}{n \cdot k} n^\mu$$

$$\langle (V_a, \phi_a), (V_b, \phi_b) \rangle = \frac{i\delta_{ab}}{k^2 - m_V^2} \begin{pmatrix} -g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} & -i\tilde{\epsilon}_n^\mu \\ i\tilde{\epsilon}_n^\nu & 1 \end{pmatrix}$$

When on-shell, $-g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} = \sum_{s=\pm, n} \epsilon_s^\mu \epsilon_s^\nu$

$$\begin{pmatrix} -g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k} & -i\tilde{\epsilon}_n^\mu \\ i\tilde{\epsilon}_n^\nu & 1 \end{pmatrix} = \sum_{s=\pm, L} \epsilon_s^M \epsilon_s^{N*}$$

Gauge-Goldstone mixing terms

Pol. Vec. : $\epsilon_\pm^M = \begin{pmatrix} \epsilon_\pm^\mu \\ 0 \end{pmatrix}$ $\epsilon_L^M = \begin{pmatrix} \tilde{\epsilon}_n^\mu \\ i \end{pmatrix}$

■ Vertices(SM)

Table 2. Vertex for VVV : WWZ & WWA . All momenta are outgoing, particles are final states.

$V_{VVV}^{M_1 M_2 M_3}$		$W^- W^+ Z$	$W^- W^+ A$
$V^{\mu_1 \mu_2 \mu_3}$		$-ia(g^{\mu_1 \mu_2}(p_1 - p_2)^{\mu_3} + g^{\mu_2 \mu_3}(p_2 - p_3)^{\mu_1} + g^{\mu_3 \mu_1}(p_3 - p_1)^{\mu_2})$	
$V^{4\mu_i \mu_j}$	$V^{4\mu_2 \mu_3}$	$b_1 \eta^{\mu_2 \mu_3}$	$a = gc_W$
	$V^{\mu_1 4\mu_3}$	$b_2 g^{\mu_1 \mu_3}$	$a = gs_W$
	$V^{\mu_1 \mu_2 4}$	$b_3 g^{\mu_1 \mu_2}$	
$V^{44\mu_i}$	$V^{44\mu_3}$	$c_1 (p_1 - p_2)^{\mu_3}$	$b_1 = -\frac{g^2 s_W^2 v}{2c_W}$
	$V^{\mu_1 44}$	$ic_2 (p_2 - p_3)^{\mu_1}$	$b_2 = \frac{g^2 s_W^2 v}{2c_W}$
	$V^{4\mu_2 4}$	$ic_3 (p_3 - p_1)^{\mu_2}$	$b_3 = 0$
V^{444}		0	0

Full List see 2203.10440

■ 3.1 Numerical Implementation

■ (Old) HELAS(HELicity Amplitudes Subroutines) Introduction

HELAS compute amplitudes for Madgraph See e.g. 1106.0522

HELAS Basic setup: subroutines

Vertices

External Lines

External line	Subroutine
Flowing-In Fermion	IXXXXX
Flowing-Out Fermion	OXXXXX
Vector Boson	VXXXXX
Scalar Boson	SXXXXX

Table 2.3: List of the vertex subroutines in HELAS system.

Vertex	Inputs	Output	Subroutine
FFV	FFV	Amplitude	IOVXXX
	FF	V	JIOXXX, J3XXXX
	FV	F	FVIXXX, FVOXXX
FFS	FFS	Amplitude	IOSXXX
	FF	S	HIOXXX
	FS	F	FSIXXX, FSOXXX
VVV	VVV	Amplitude	VVVXXX
	VV	V	JVVXXX
VVS	VVS	Amplitude	VVSXXX
	VS	V	JVSXXX
	VV	S	HVVXXX

■ 3.1 Numerical Implementation

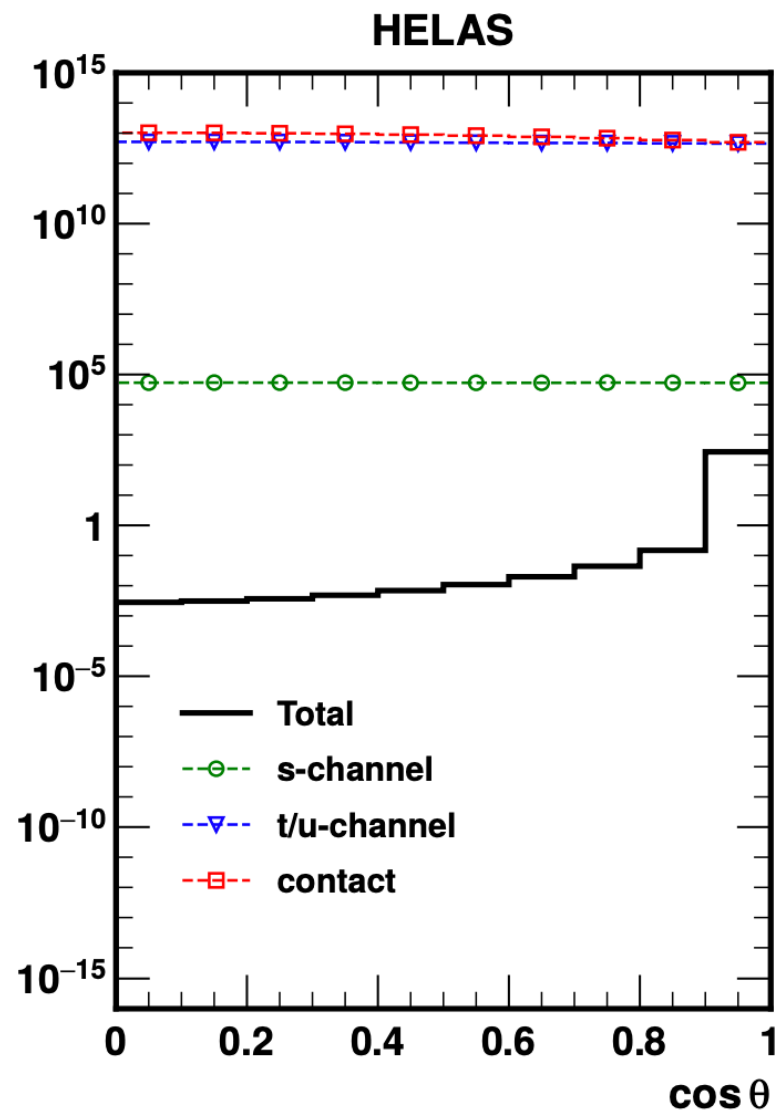
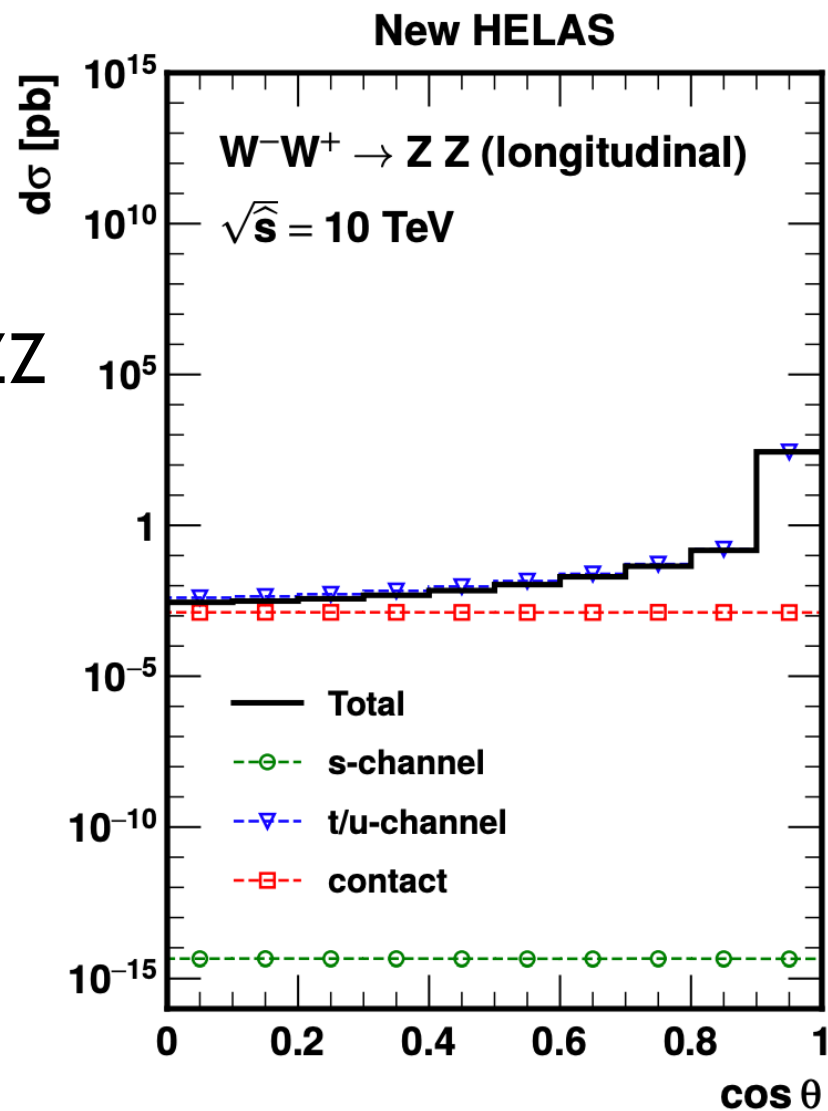
■ New HELAS (implementing “New Feynman Diagrams”)

1. Replace polarization vectors of W/Z to 5-component form
2. Replace propagators of W/Z to GEG in 5-component form
3. Extend vertices involving W/Z to incorporate Goldstone components
- (4). Replace propagators of photon and gluons to parton shower gauge (counter part of GEG for massless case.)
5. Add vertices not existing in Old Feynman Rules(ZZZZ, WWZh,WWAh)

■ New HELAS Results: $2 \rightarrow 2$ VBS

■ 3.2 Numerical Results

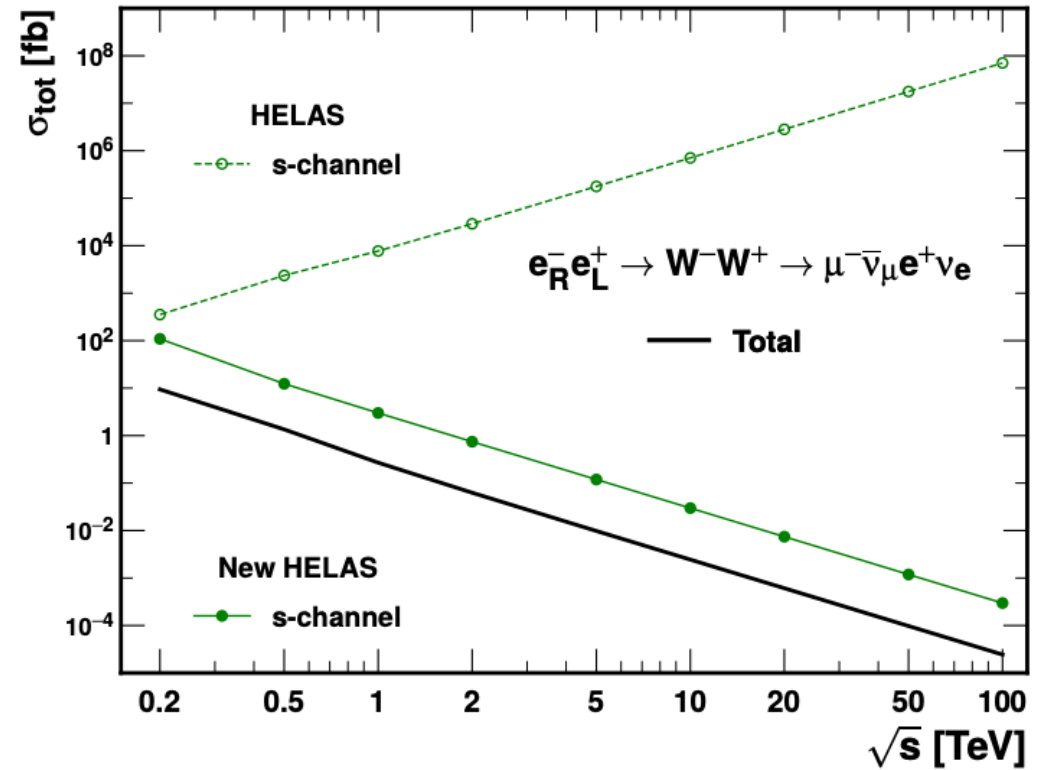
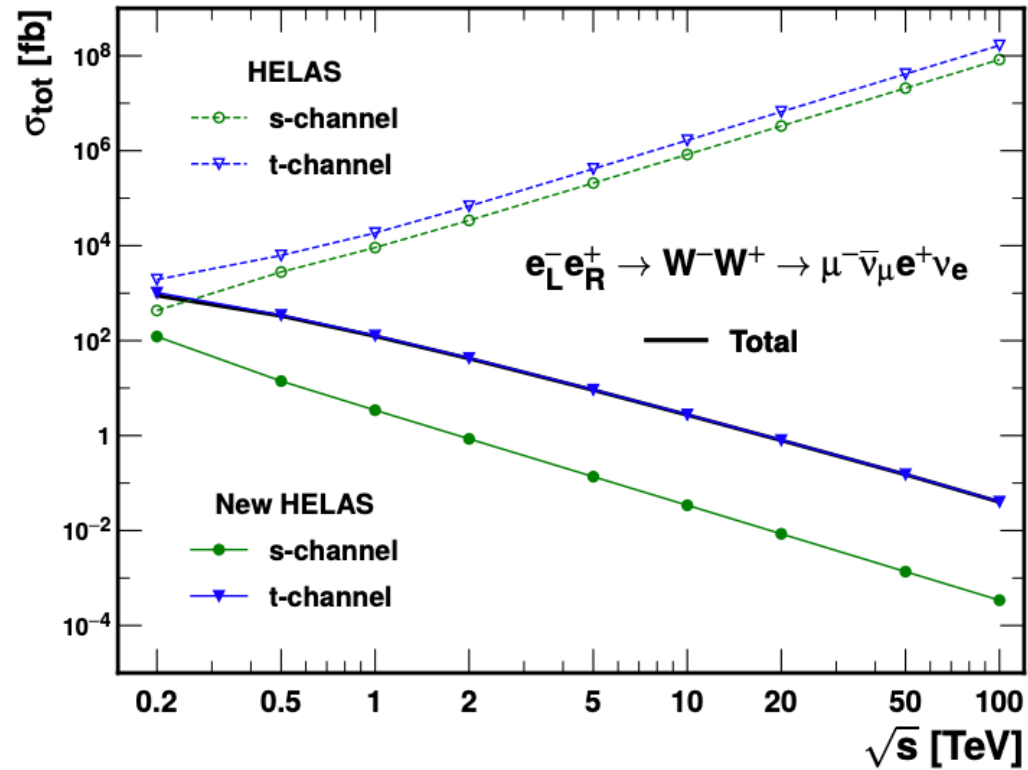
$WW > ZZ$



■ 3.2 Numerical Results

■ New HELAS Results: WW production at lepton colliders

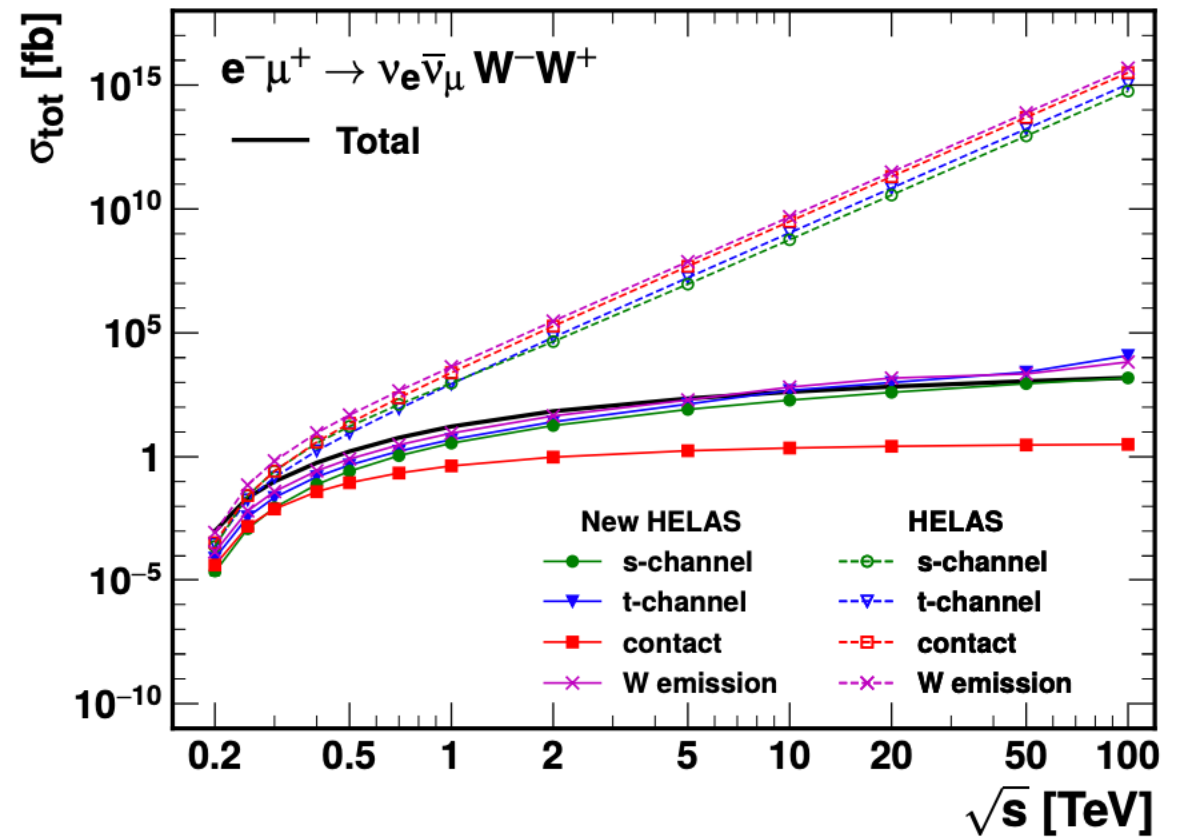
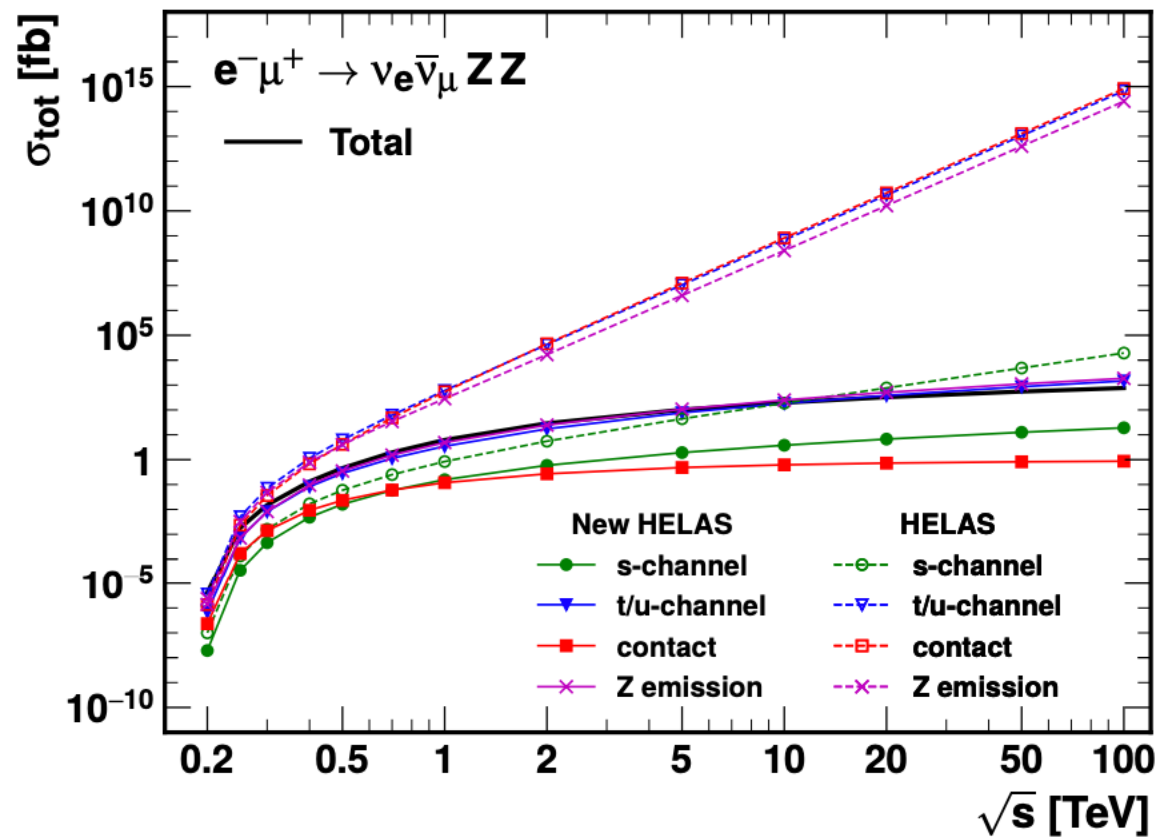
$e^+e^- \rightarrow W^+W^- \rightarrow \mu^-\bar{\nu}_e e^+ \nu_e$: total cross section



■ 3.2 Numerical Results

■ New HELAS Results: VBS at lepton colliders

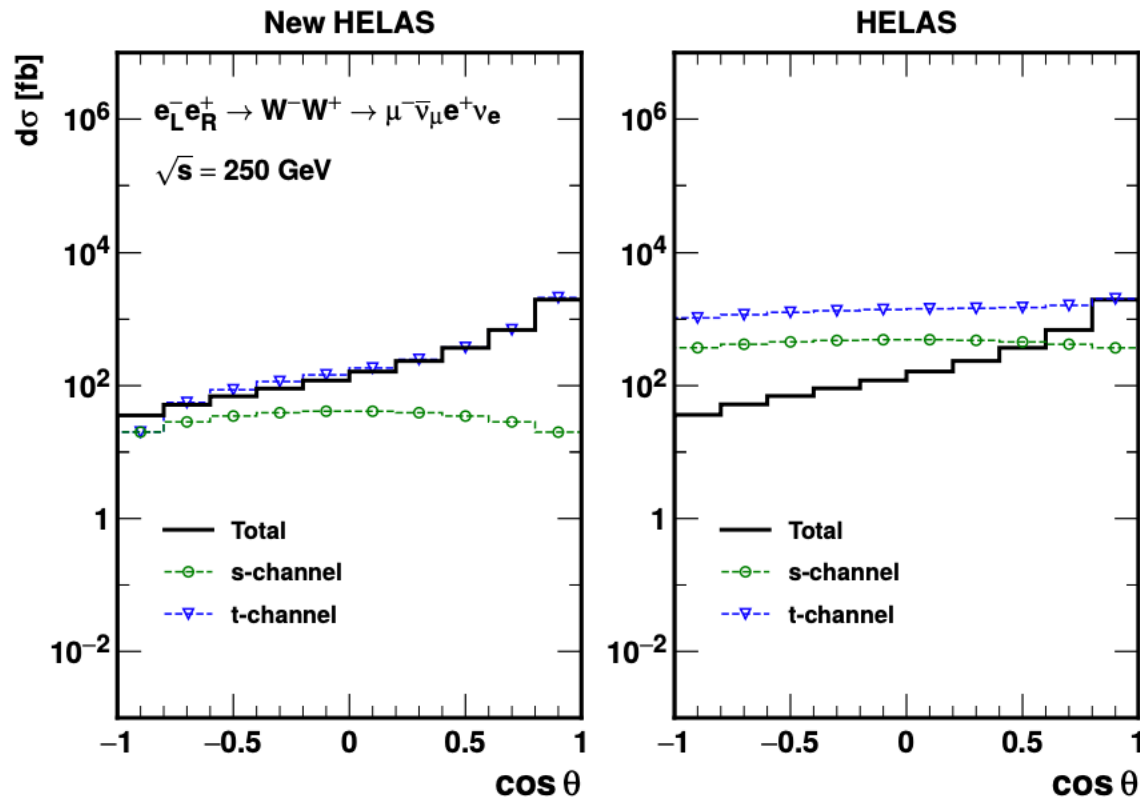
○ $e^- \mu^+ \rightarrow \nu_e \bar{\nu}_\mu ZZ / WW (WW \rightarrow ZZ / WW)$: total cross section



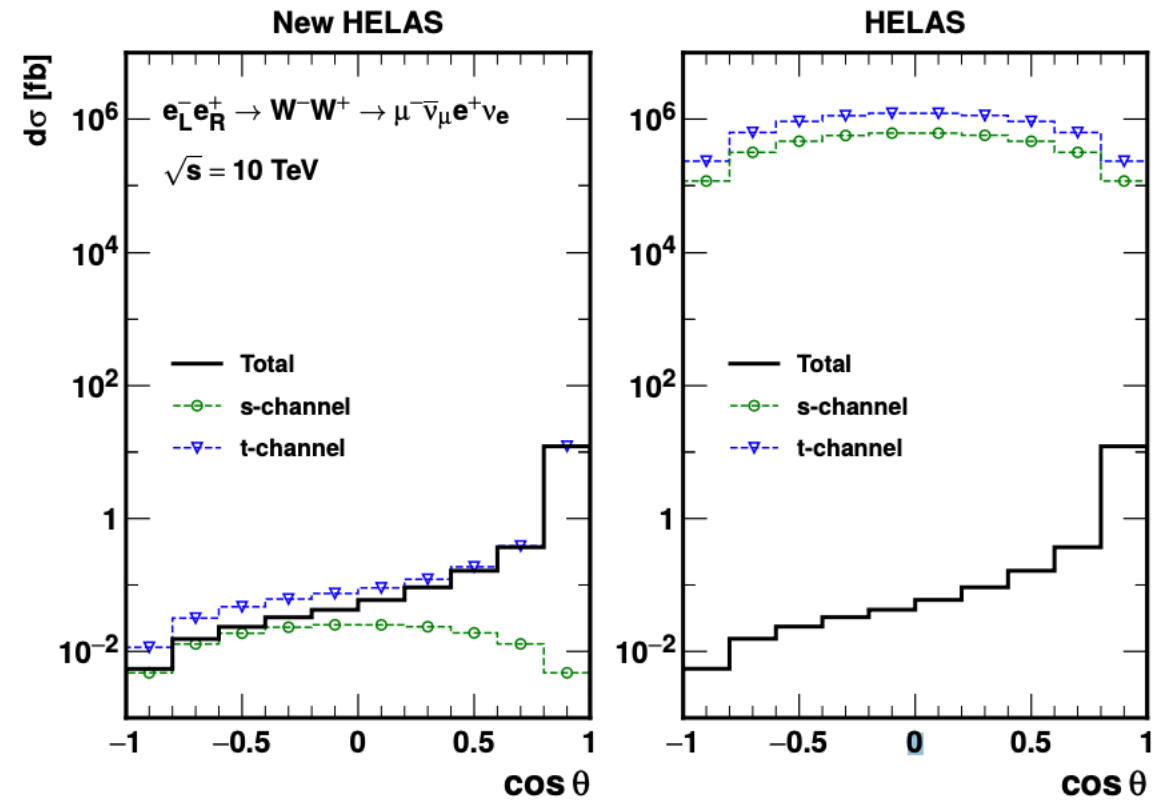
■ 3.2 Numerical Results

■ New HELAS Results: WW production at lepton colliders

$e^+e^- \rightarrow W^+W^- \rightarrow \mu^-\bar{\nu}_\mu e^+\nu_e$: angular distribution



(a)



(b)

CONCLUSION

1. We propose a formalism (New Feynman Diagram) to solve gauge cancellation of weak scattering amplitudes, including:
5-component pol. vec. , propagators (in GEG), vertices
2. We implement NFD to HELAS(SM), and demonstrate some numerical results
3. Future: Needing deeper understanding, integrated into Madgraph(?),
Other models(?)

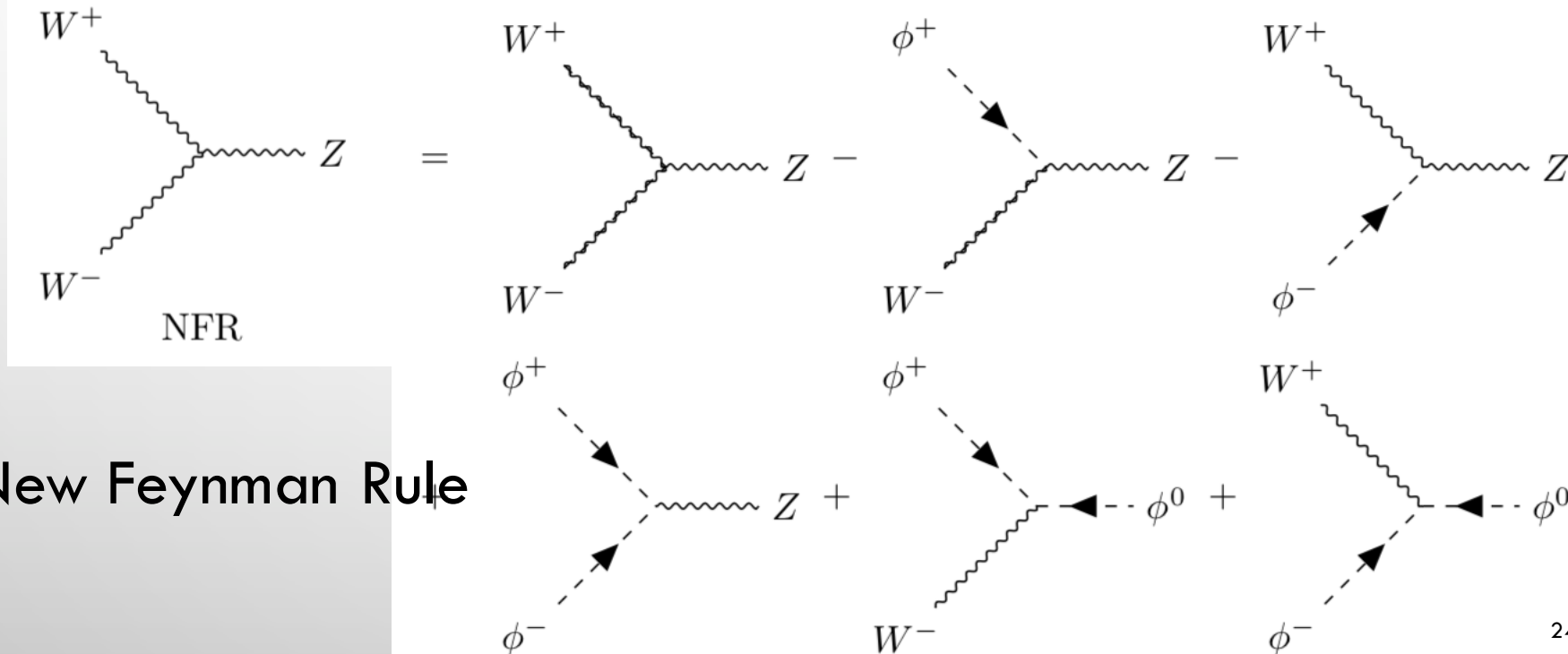
■ 2. Solution: “New Feynman Diagram”

■ New Feynman Rule(SM)

Propagator:

$$M \overset{W^\pm/Z}{\sim} N \underset{\text{NFR}}{=} \begin{pmatrix} \mu \sim \text{wavy} \nu & \mu \sim \text{dashed} \\ \text{dashed} \sim \nu & \text{dashed} \end{pmatrix}$$

WWZ:



NFR=New Feynman Rule

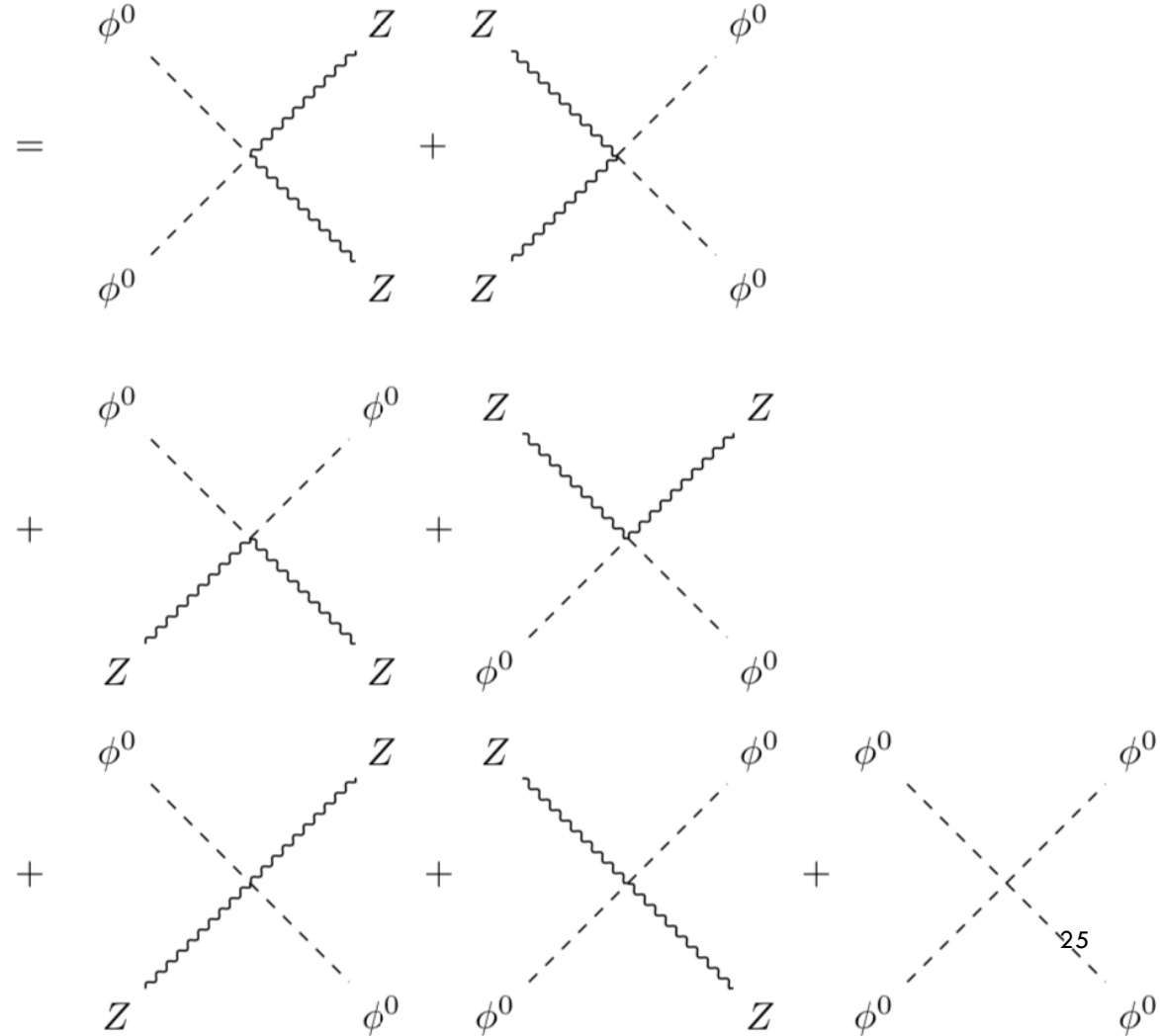
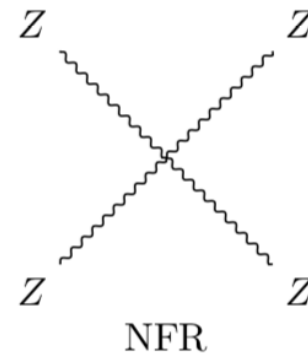
“wave function” of ϕ : i for initial, $-i$ for final

■ 2. Solution: “New Feynman Diagram”

■ New Feynman Rule(SM)

■ Vertices doesn't exist in “Old” Feynman Diagrams

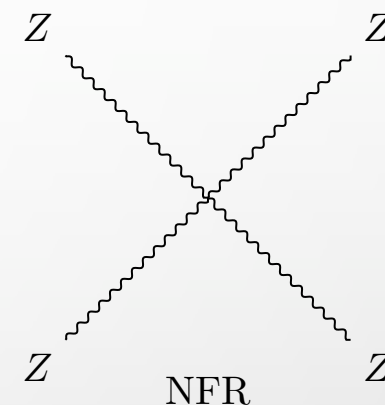
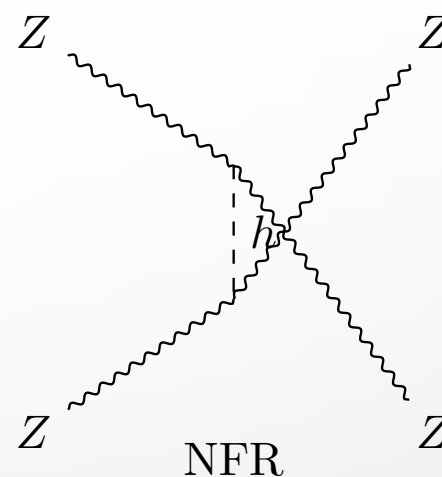
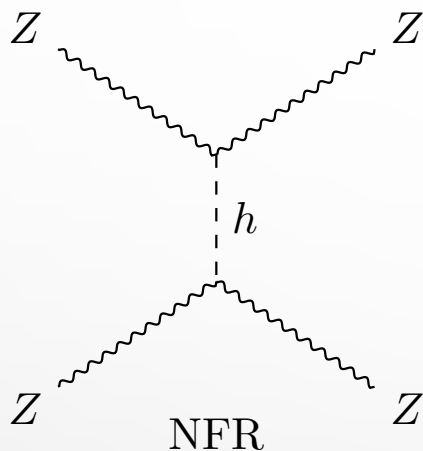
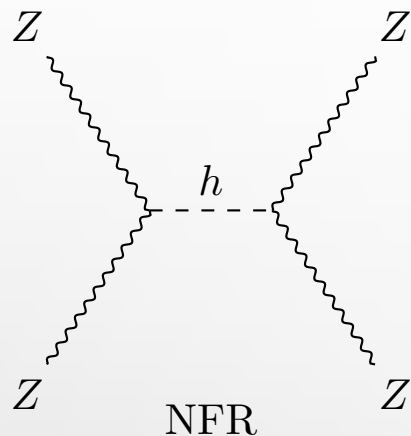
ZZZZ:



NFR=New Feynman Rule

■ New Feynman Rule(SM)

■ Ex2: $ZZ \rightarrow ZZ$



Doesn't exist in
"Old" Feynman Rules

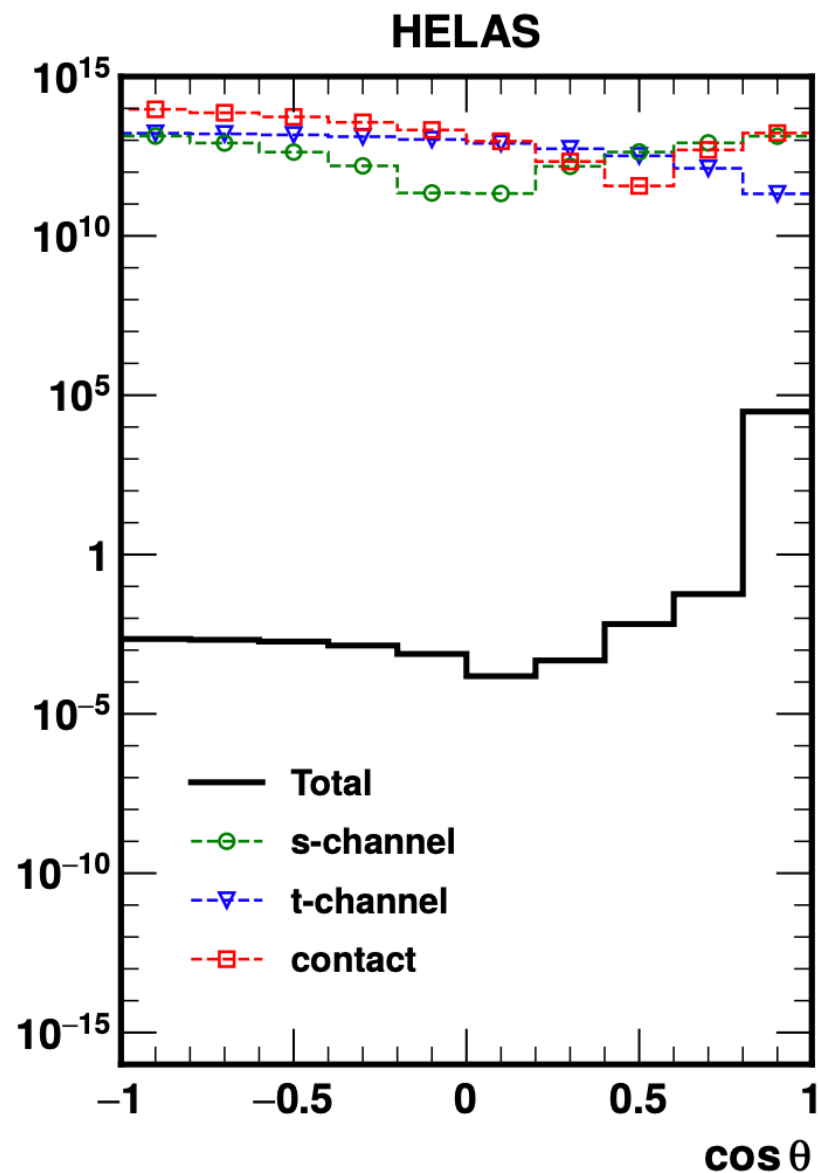
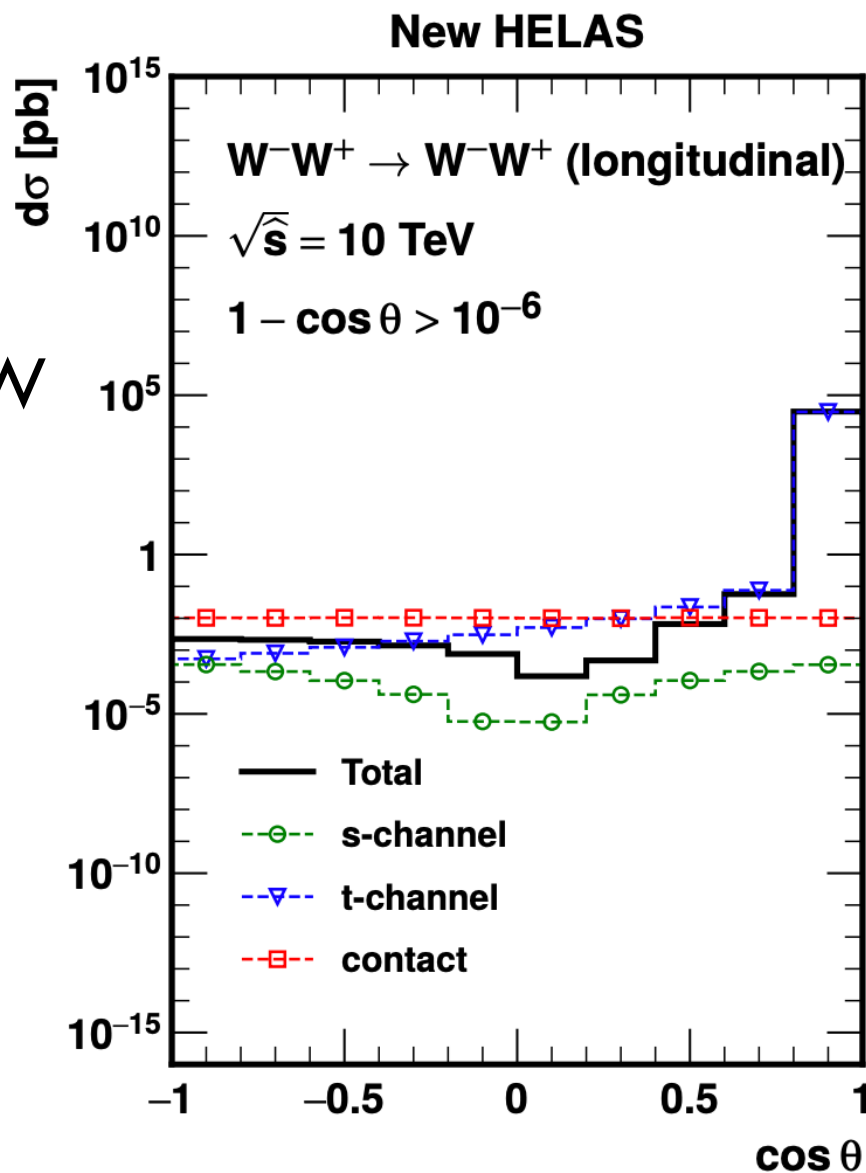
NFR=New Feynman Rule

■ 2. Solution: "New Feynman Diagram"

■ New HELAS Results: $2 \rightarrow 2$ VBS

$WW > WW$

■ 4. Numerical Results



■ New HELAS Results: $2 \rightarrow 2$ VBS

$ZZ > ZZ$

■ 4. Numerical Results

