电弱相变动力学的精确计算
——泡壁速度

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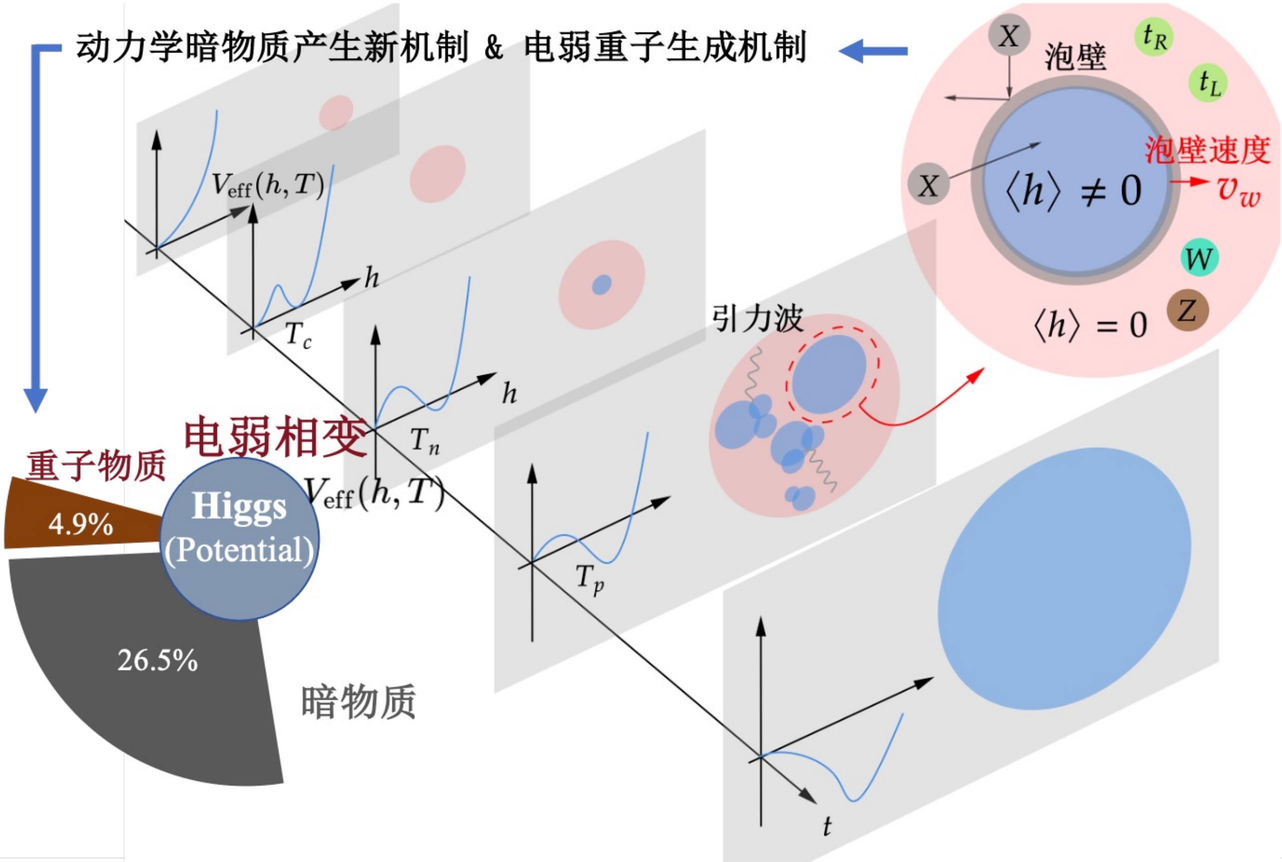
Outline

1. Motivation
2. Electroweak phase transition (SFOPT) and phase transition gravitational wave (GW) in a nutshell
3. dynamical Dark matter (DM)
4. Phase transition dynamics (bubble wall velocity)
5. Summary and outlook

Motivation Higgs particle cosmology in post-Higgs Era

What is the role of Higgs in the early universe?

Higgs' deep connections to cosmology, such as EW baryogenesis, dark matter(DM) testable by colliders &GW signals



Motivation

Collider signals at loop level@CEPC/LHC

对撞机与引力波实验的互补探测
希格斯物理 (Higgs potential), Z-pole 物理
TeV 新物理

电弱重子生产机制
(宇宙正反物质
不对称性的起源)

电弱相变
动力学的
精确计算 v_w

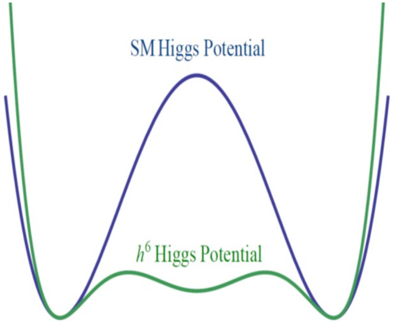
相变引力波信号的
精确预言(空间引
力波探测)

暗物质产生新机制(相变动力学暗物质)

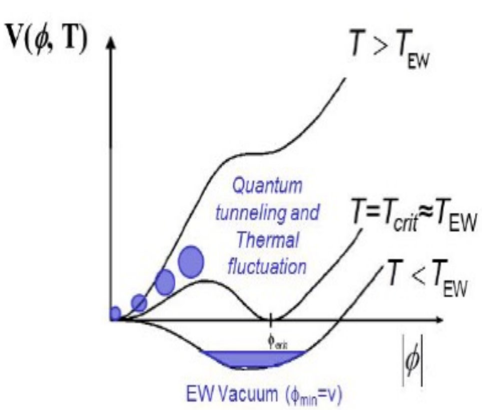
粒子物理、(有限温度)量子场论、相对论流体力学、广义相对论

Motivation

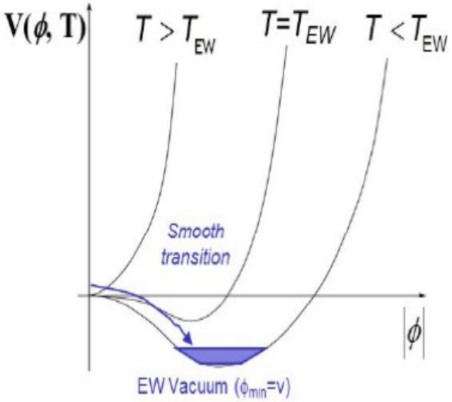
From lattice simulation



SFOPT for $m_H < 75$ GeV



Cross over for $m_H > 75$ GeV

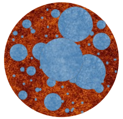


Extension of the Higgs sector is needed to SFOPT for 125 GeV Higgs boson.

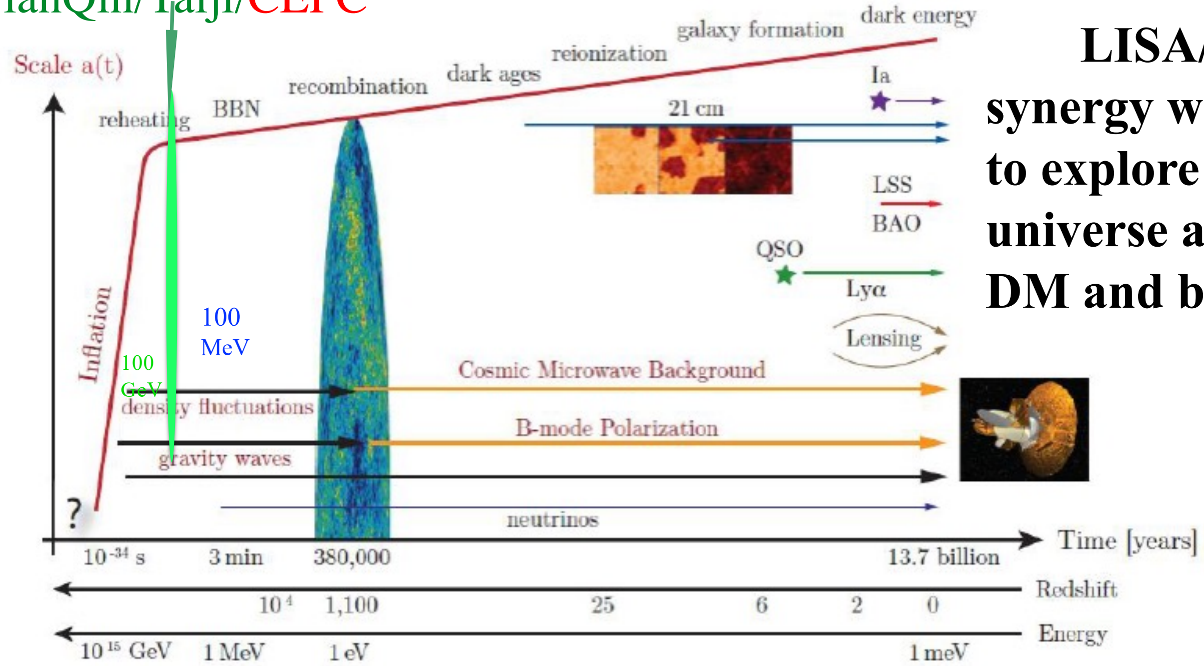
We discuss well-motivated extensions (baryogenesis, DM…) of Higgs section to realize strong first-order phase transition (SFOPT) with abundant cosmological effects.
EW phase transition and its GW signals becomes realistic after the discovery of Higgs by LHC and GW by LIGO.

Motivation

EW phase transition/ baryogenesis:



LISA/TianQin/Taiji/CEPC



LISA/Tianqin/Taiji in synergy with CEPC helps to explore the early universe around 100 GeV, DM and baryogenesis.

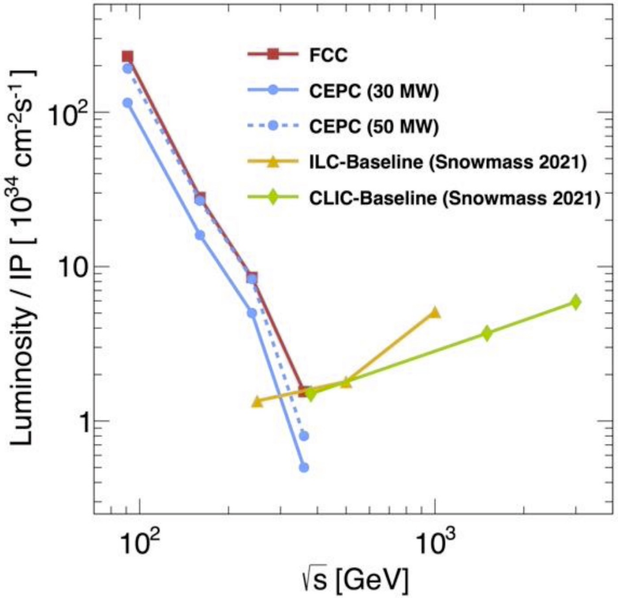
credit:D.Baumann

Motivation

Complementary

Particle approach

CEPC/SppC, FCC etc.

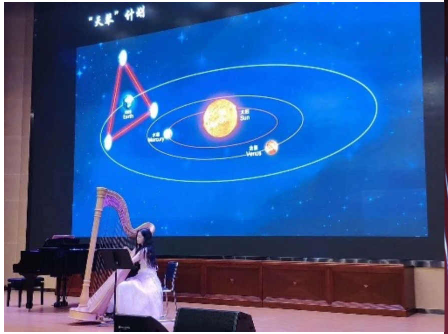


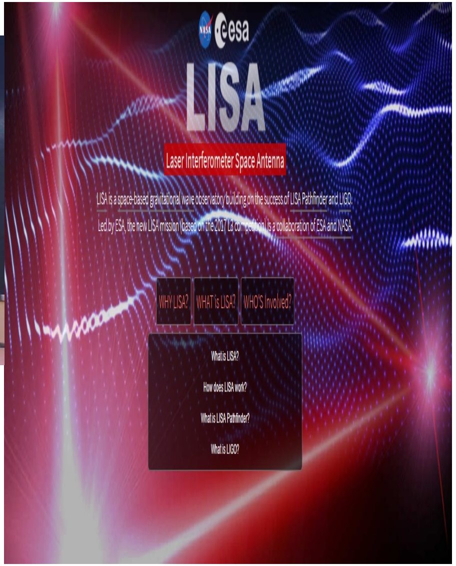
Relate by Higgs physics:EW phase transition/ baryogenesis

Double test on the Higgs potential and baryogenesis, DM

Wave approach

LISA/TianQin/Taiji ~2034

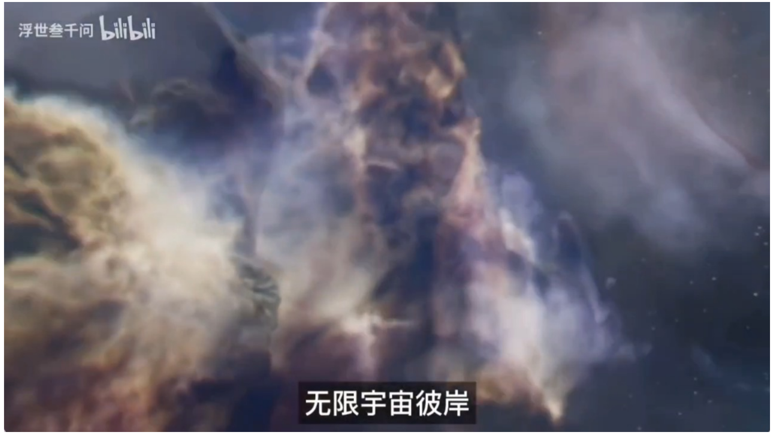




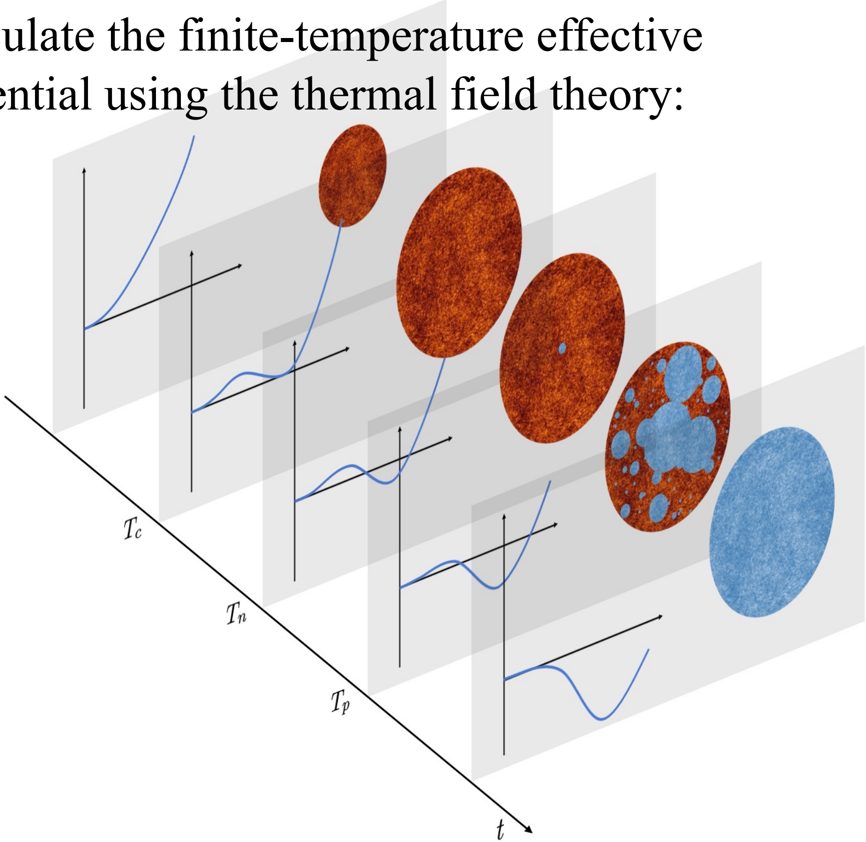
“天琴”

“Harpe in space”

Phase transition GW in a nutshell



calculate the finite-temperature effective potential using the thermal field theory:



这世上的热闹，源自隧穿 $\Gamma = \Gamma_0 e^{-S(T)}$

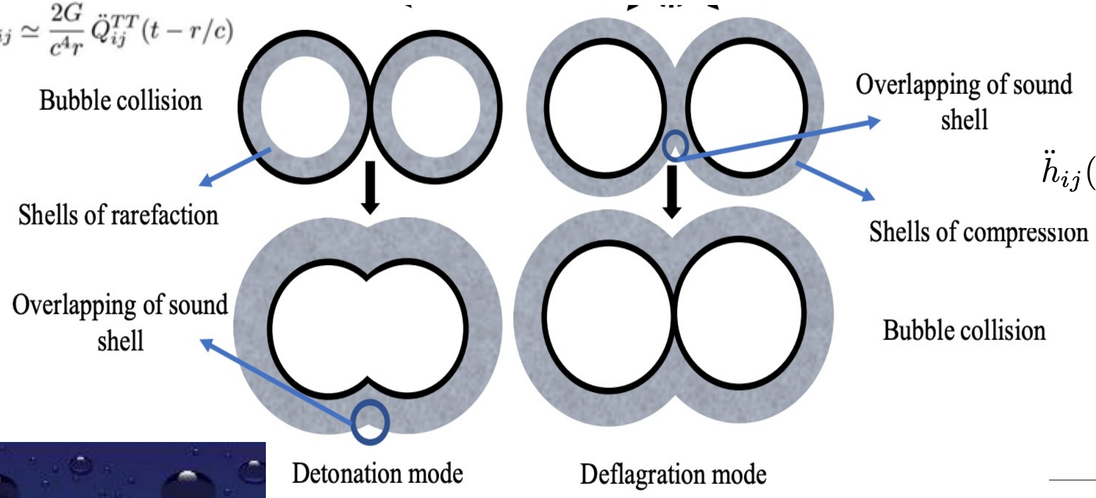
$$S(T) = \int d^4x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial x} \right)^2 + V_{\text{eff}}(\phi, T) \right]$$

$$V_{\text{eff}}^{(1)}(\bar{\phi}) = \sum_i n_i \left[\int \frac{\mathrm{d}^D p}{(2\pi)^D} \ln\left(p^2 + m_i^2(\bar{\phi})\right) + J_{\mathrm{B,F}}\left(\frac{m_i^2(\bar{\phi})}{T^2} \right) \right]$$

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell

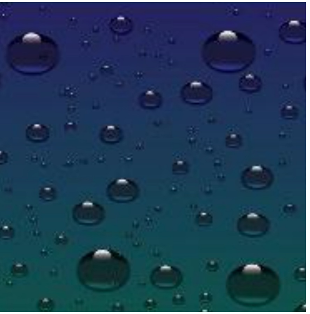
$$h_{ij} \simeq \frac{2G}{c^4 r} \ddot{Q}_{ij}^{TT}(t - r/c)$$

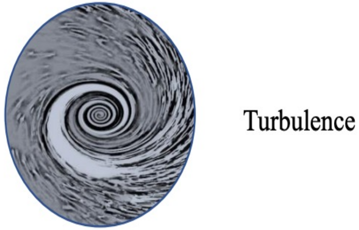


$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\ddot{h}_{ij}(\mathbf{x},t) + 3H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2} h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

各向异性
剪切应力张量
产生引力波





General form Π_{ij}
$[\partial_i \phi \partial_j \phi]^{TT}$
$[\gamma^2(\rho + p) v_i v_j]^{TT}$
$[-E_i E_j - B_i B_j]^{TT}$
$\partial_i \Psi, \partial_i \Phi$

E. Witten, Phys. Rev. D 30, 272 (1984)
C. J. Hogan, Phys. Lett. B 133, 172 (1983);
M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49, 2837 (1994))

EW phase transition GW becomes more interesting and realistic after the discovery of Higgs by LHC and GW by LIGO.

Xiao Wang, FPH, Xinmin Zhang, JCAP05(2020)045

Phase transition GW in a nutshell

characteristic frequency of the GW signal

$$f_* = \frac{1}{\ell_*} \geq H_*$$

$$\epsilon_* = \ell_* H_*$$

Ratio of the typical length-scale of the GW sourcing process (size of the anisotropic stresses) and the Hubble scale at the generation time

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\epsilon_*} \left(\frac{g(T_*)}{100} \right)^{1/6} \frac{T_*}{\mathrm{GeV}} \,\mathrm{Hz}$$

电弱相变对应的峰值频率在mHz附近，刚好在空间引力波实验(LISA、天琴、太极)的探测区间

Phase transition dynamics

Theory: 相变引力波信号、相变暗物质、早期宇宙电弱重子生成机制**最核心却最难计算**的是泡泡膨胀速度

v_b

Experiment: 实验上最重要的相变参数也是泡泡膨胀速度

arXiv:2404.xxxxx

Finite-temperature effective potential

$V_{eff}(\phi, T)$ α T_p $R_* H_*$

(1). Daisy resummation problem: Pawani scheme vs. Arnold scheme
(2). Gauge dependence problem: see Michael J. Ramsey-Musolf's works
(3). No perturbative calculations: lattice calculations and dim-reduction method: by D. Weir, Michael J. Ramsey-Musolf et.al

Bubble wall velocity
v_b ***this talk***

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang ， arXiv:2007.10343 ,
Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith , arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, FPH, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

Energy budget
κ

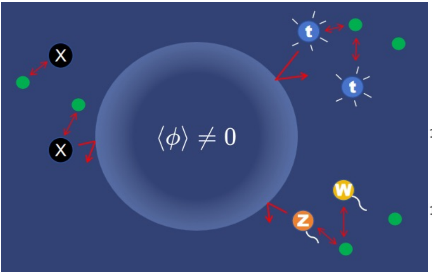
F. Giese, T. Konstandin, K. Schmitz and J. van de Vis ,arXiv:2010.09744
Xiao Wang, **FPH** and Xinmin Zhang, Phys.Rev.D 103 (2021) 10, 103520
Xiao Wang, Chi Tian, **FPH**, JCAP 07 (2023) 006

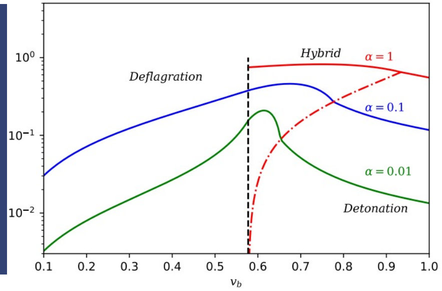
泡壁速度至关重要

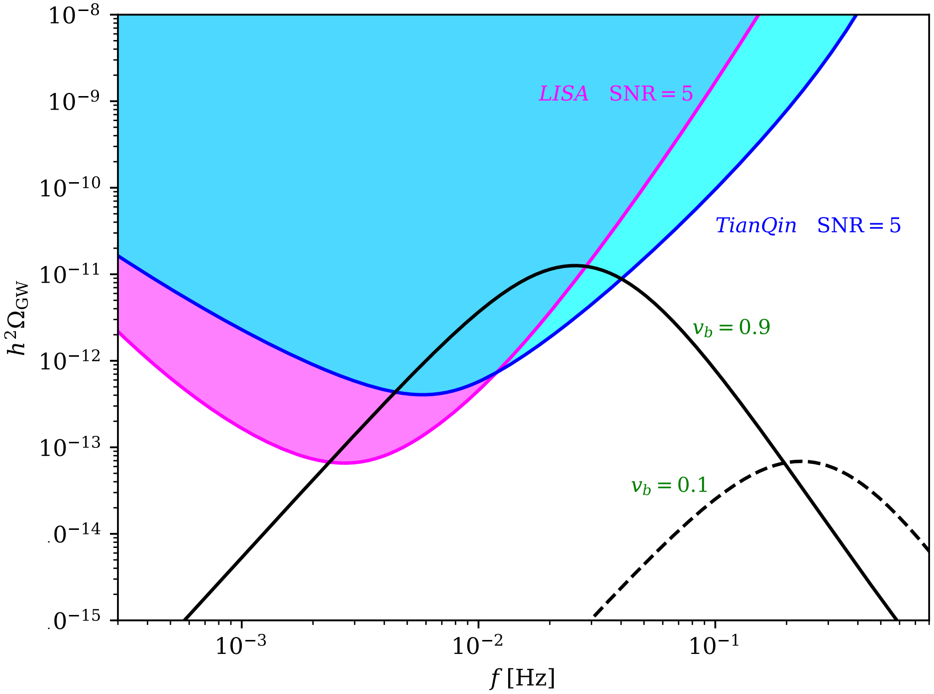
相变引力波信号、相变暗物质以及早期宇宙电弱重子数产生机制最核心的参数是泡泡膨胀速度 v_w

GW detection favor lager v_w
EW baryogenesis favor smaller v_w
Dynamical DM is sensitive to v_w

S. Hoche, J. Kozaczuk, A. J. Long, J. Turner and Y. Wang, arXiv:2007.10343, Avi Friedlander, Ian Banta, James M. Cline, David Tucker-Smith, arXiv:2009.14295v2
Xiao Wang, **FPH,** Xinmin Zhang,arXiv:2011.12903
Siyu Jiang, **FPH**, xiao wang, Phys.Rev.D 107 (2023) 9, 095005

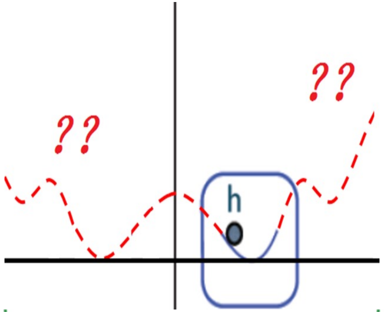


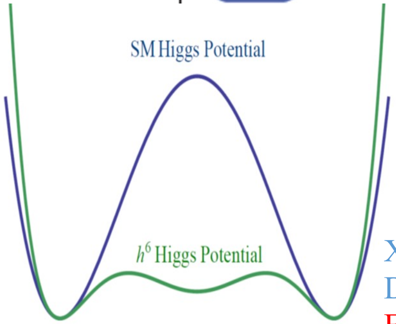




SFOPT and Higgs potential

What is the shape of Higgs potential?





Current data tells us nothing but the quadratic oscillation around the VEV 246 GeV with 125 GeV mass.

$$V(h) = \frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4} h^4$$

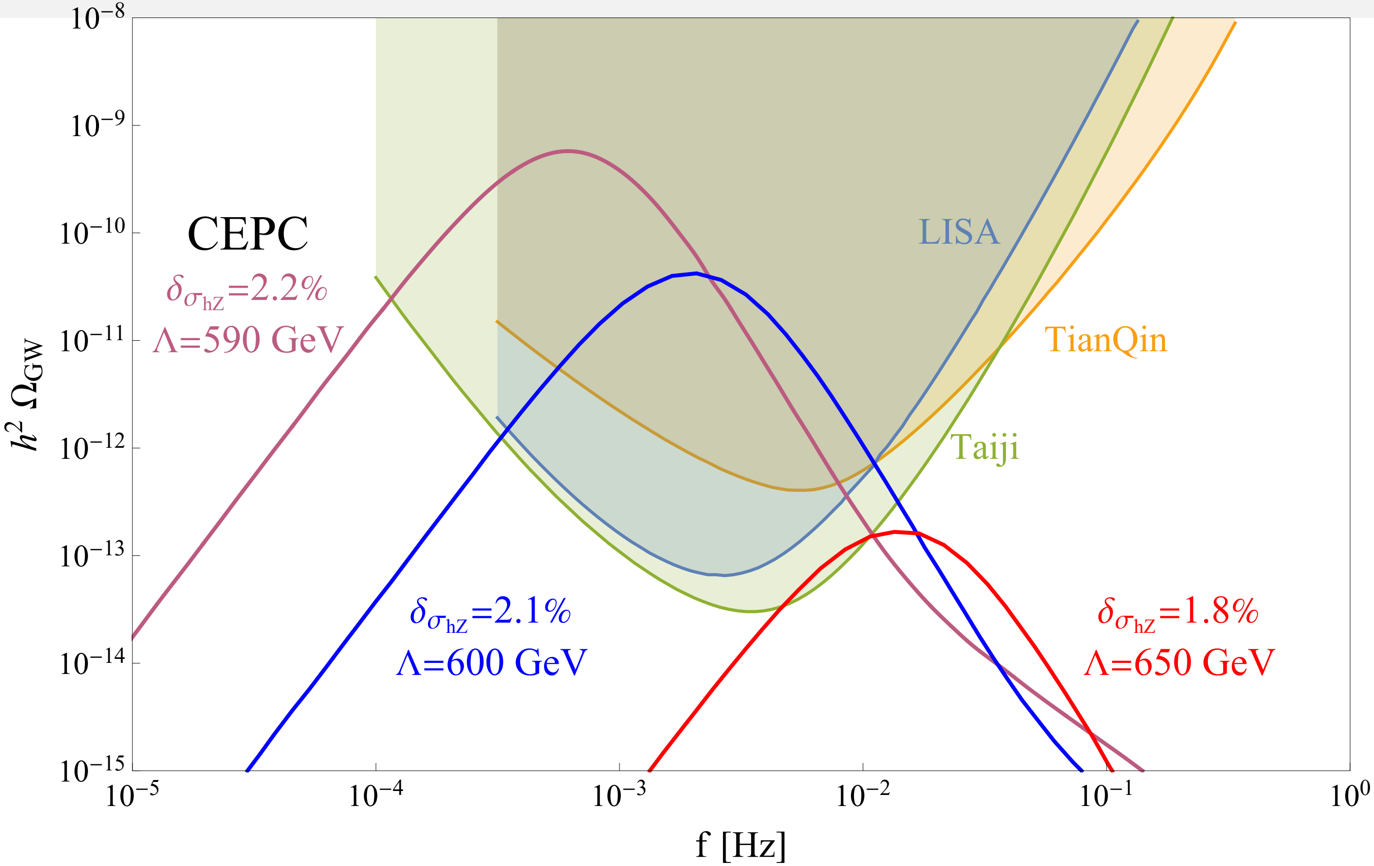
or $$V(h) = \frac{1}{2}\mu^2 h^2 - \frac{\lambda}{4} h^4 + \frac{1}{\Lambda^2} h^6$$

SMEFT

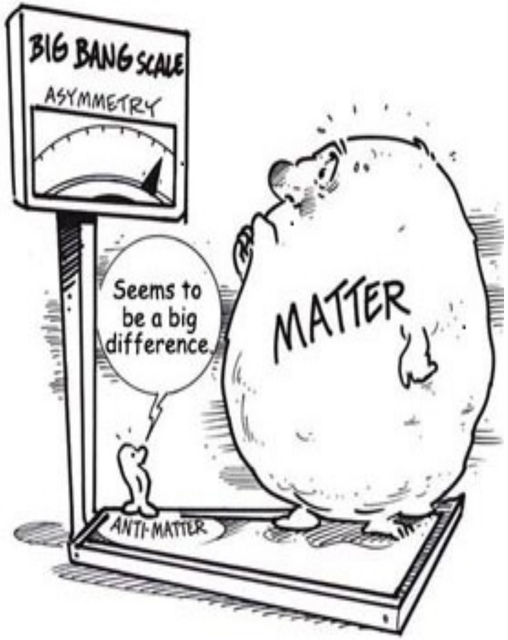
Produce a SFOPT, large deviation of Higgs trilinear coupling, and GW

Xinmin Zhang Phys.Rev. D47 (1993) 3065-3067;C. Grojean, G. Servant, J. Well PRD71(2005)036001
D.J.H. Chung, A. J. Long, Lian-tao Wang Phys.Rev. D87(2013) 023509
FPH, et.al, Phys.Rev.D94(2016)no.4,041702 ; **FPH**, et.al, Phys.Rev.D93 (2016) no.10,103515
arXiv:1511.06495, Nima Arkani-Hamed et. al.; PreCDR of CEPC; arXiv: 1811.10545,CDR of CEPC

SFOPT and Higgs potential



SFOPT and EW baryogenesis



$\eta_B = n_B/n_\gamma = 5.8 - 6.6 \times 10^{-10}$

(CMB, BBN)

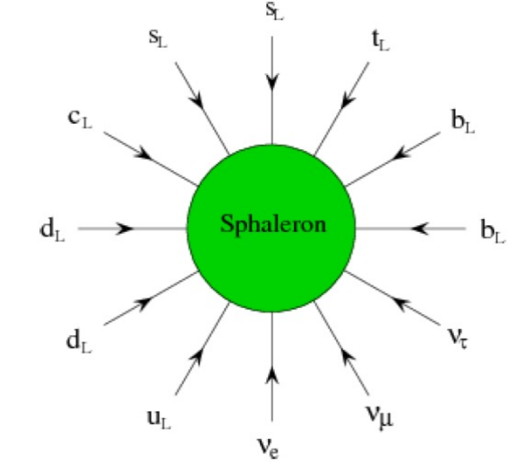
A long standing problem in particle cosmology is the origin of baryon asymmetry of the universe.

After discovery of Higgs@LHC & GW @aLIGO, EW baryogenesis becomes a testable scenario.

SM technically has all the 3 elements for baryogenesis(Sakharov conditions)

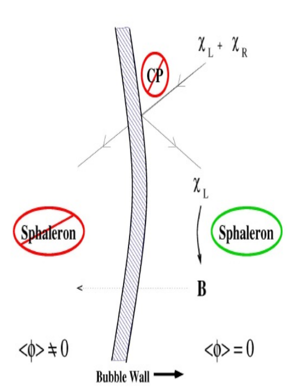
- B violation from anomaly in B+L current;
- C and CP-violation: CKM matrix, but too weak, need new CP-violating sources;
- Departure from thermal equilibrium: SFOPT with expanding Higgs bubble wall

SFOPT and EW baryogenesis



$$\Gamma_S \sim T^4$$

$$\Gamma_S \sim \mathrm{Exp}(-\phi_C/T_C)$$



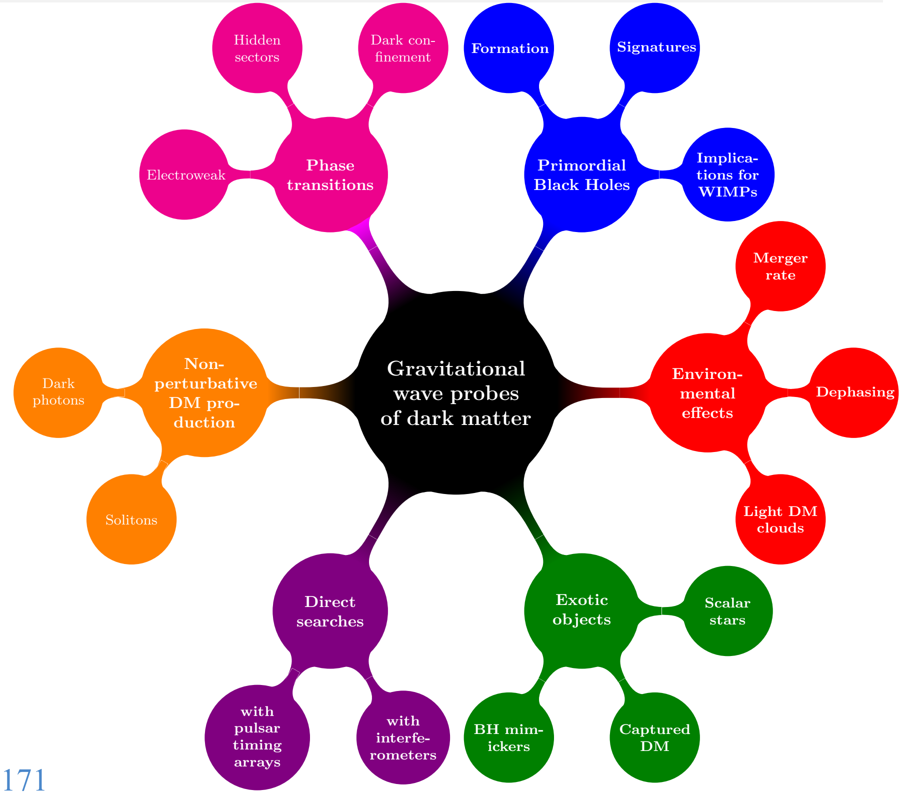
$$\eta_B = \frac{405\Gamma_{\rm sph}}{4\pi^2\gamma_w v_w g_\star T}\int dz\,\mu_{B_L} f_{\rm sph}\, e^{-45\Gamma_{\rm sph}|z|/4\gamma_w v_w}$$

D. E. Morrissey and M. J. Ramsey-Musolf, New J. Phys. 14, 125003 (2012).

Credit: T. Cohen

SFOPT and new DM mechanism/signal

- ➢ The observation of GW@LIGO initiates a new era of exploring DM by GW.
- ➢ SFOPT could provide a new approach for DM production.



Credit: Gianfranco Bertone et. al.

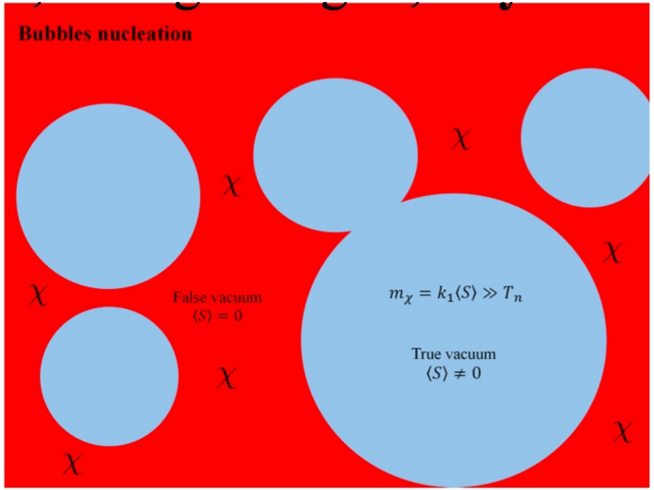
J.Jaeckel, V. V. Khoze, M. Spannowsky, Phys.Rev. D94 (2016) no.10, 103519
Zhaofeng Kang,et.al. arXiv:2101.03795, arXiv:2003.02465
Yan Wang, Chong Sheng Li, and **FPH**, arXiv:2012.03920
FPH, Eibun Senaha Phys.Rev. D100 (2019) no.3, 03501
FPH PoS ICHEP2018 (2019) 397
FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028
FPH, Jiang-Hao Yu, Phys.Rev. D98 (2018) no.9, 095022
FPH, Xinmin Zhang, Phys.Lett. B788 (2019) 288-
Haipeng An, et.al, arXiv: 2208.14857, arXiv:2009.12381, arXiv:2201.05171

SFOPT and new DM mechanism/signal

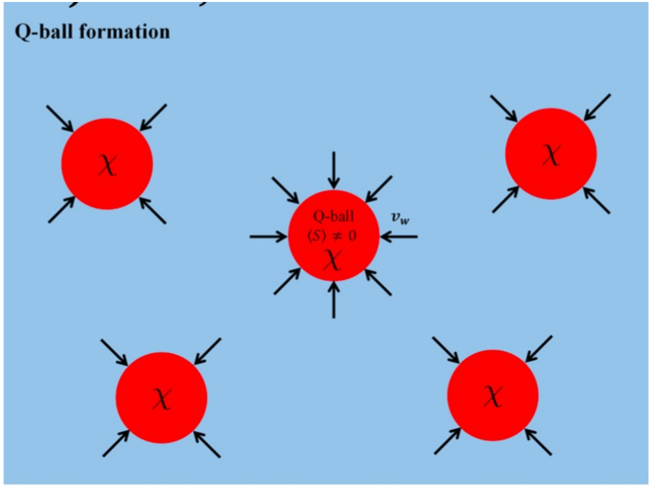
Case I: anti-filter case

The cosmic phase transition with Q-balls production can explain baryogenesis and DM simultaneously

$$\rho_{\rm DM}^4 v_b^{3/4} = 73.5(2\eta_B s_0)^3 \lambda_S \sigma^4 \Gamma^{3/4}$$



(a) Bubble nucleation: χ particles trapped in the false vacuum due to Boltzmann suppression



(b) Q-ball formation: After the formation of Q-balls, they should be squeezed by the true vacuum

The global Q-ball model proposed by T.D. Lee

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028;

SFOPT and new DM mechanism/signal

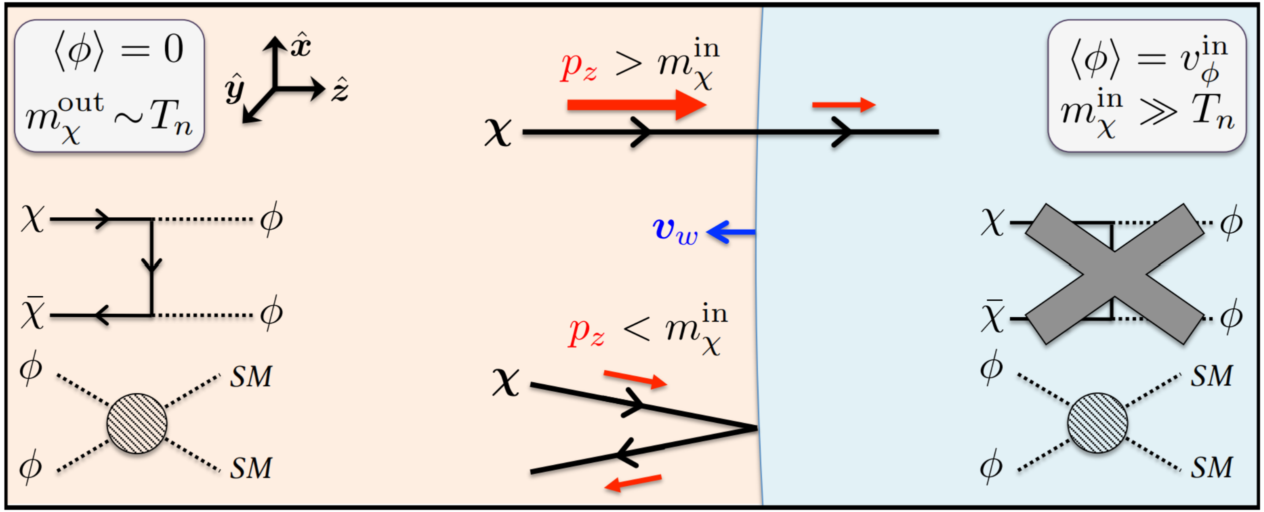
DM Case II: filter case

In the recent two years, this dynamical DM formed by phase transition has became a new idea and attracted more and more attentions.
Namely, bubbles in SFOPT can be the “filters” to packet your needed heavy DM.

FPH, Chong Sheng Li, Phys.Rev. D96 (2017) no.9, 095028
arXiv:1912.04238, Dongjin Chway, Tae Hyun Jung, Chang Sub Shin
arXiv:1912.02830, Michael J. Baker, Joachim Kopp,and Andrew J. Long
arXiv:2012.15113, Wei Chao, Xiu-Fei Li, Lei Wang
arXiv:2101.05721, Aleksandr Azatov, Miguel Vanvlasselaer, Wen Yin
arXiv:2103.09827, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2103.09822, Pouya Asadi , Eric D. Kramer, Eric Kuflik, Gregory W. Ridgway, Tracy R. Slatyer, J. Smirnov
arXiv:2008.04430 Jeong-Pyong Hong， Sunghoon Jung, Ke-pan Xie
Haipeng An, et.al, arXiv: 2208.14857
Siyu Jiang, FPH, Chong Sheng Li, arXiv:2305.02218
more and more new works…

SFOPT and new DM mechanism/signal

Michael J. Baker, Joachim Kopp, Andrew J. Long,Phys.Rev.Lett. 125 (2020) 15, 151102

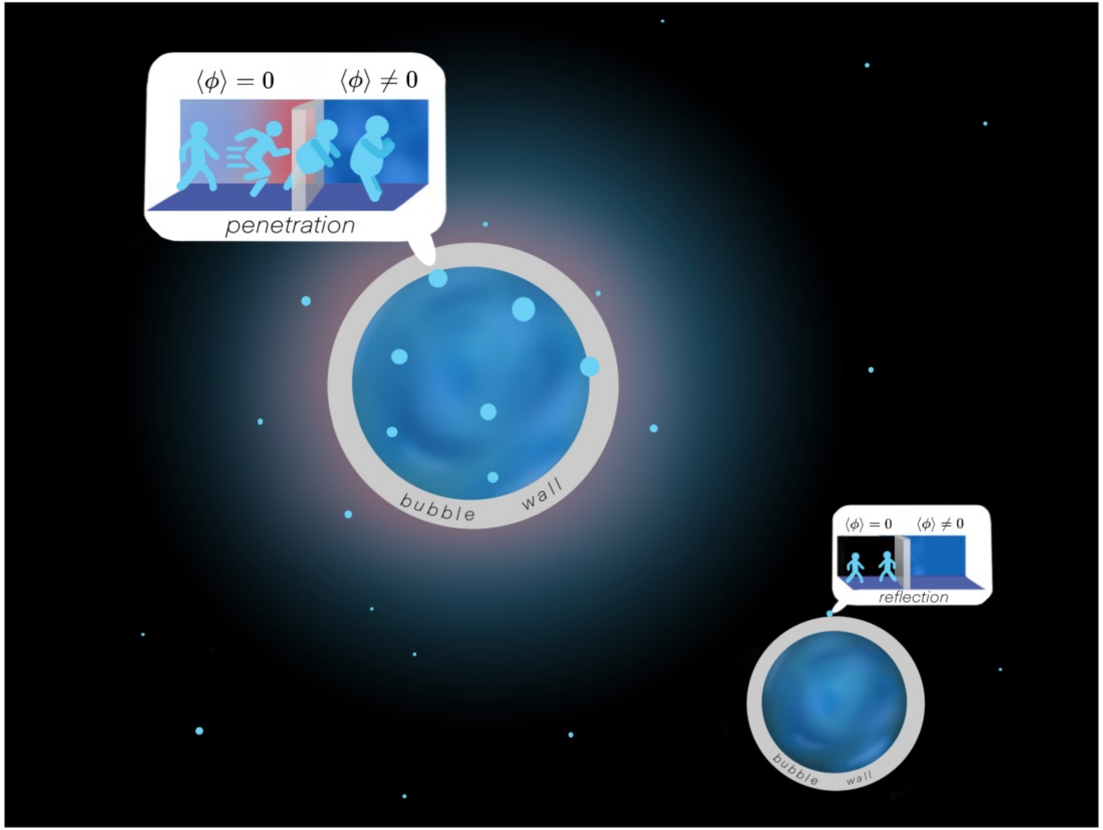


$$\Omega_{DM}h^2 \approx 0.17\left(\frac{T_n}{\mathrm{TeV}}\right)\left(\frac{m_\chi^\infty}{30T_n}\right)^{-\frac{5}{2}}\exp\left(-\frac{m_\chi^\infty}{30T_n}\right)$$

SFOPT and new DM mechanism/signal

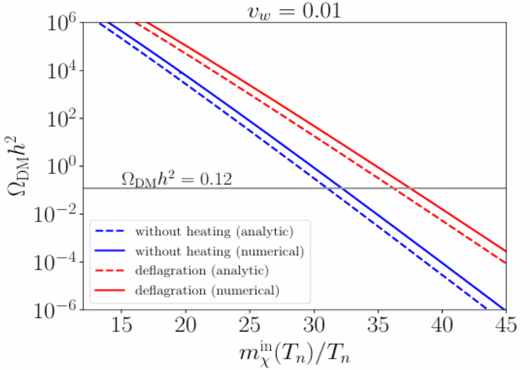
Bubble wall dynamics plays an essential role in the filtered DM mechanism.

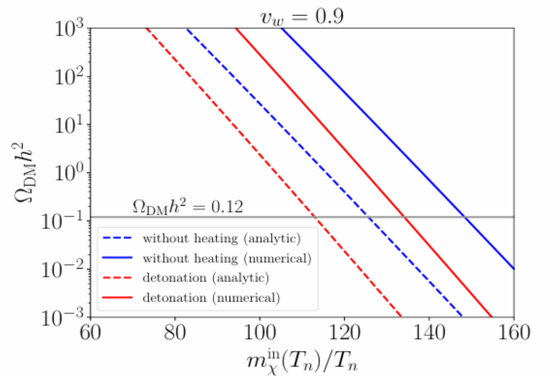
Siyu Jiang, FPH, Chong Sheng Li, Phys.Rev.D 108 (2023) 6, 063508



SFOPT and new DM mechanism/signal

$$n_\chi^{\rm in} = \frac{T_+}{\gamma_w \tilde{\gamma}_+} \int_0^\infty \frac{dp_z}{(2\pi)^2} \mathcal{A}(z \gg L_w, p_z) \exp\left[\tilde{\gamma}_+ \left(\tilde{v}_+ p_z - \sqrt{p_z^2 + (m_\chi^{\rm in})^2}\right) / T_+\right] \left(\sqrt{p_z^2 + (m_\chi^{\rm in})^2} + \frac{T_+}{\tilde{\gamma}_+}\right)$$





$v_w = 0.01$

	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)} h^2 / \Omega_{\rm DM}^{(0)} h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)} h^2 / \Omega_{\rm DM}^{(0)} h^2$
BP_1	31	66	32	71
BP_2	31.1	7.9	32.2	8.1
BP_3	30.8	778.8	31.9	858.5
BP_4	$\star$	$\star$	$\star$	$\star$

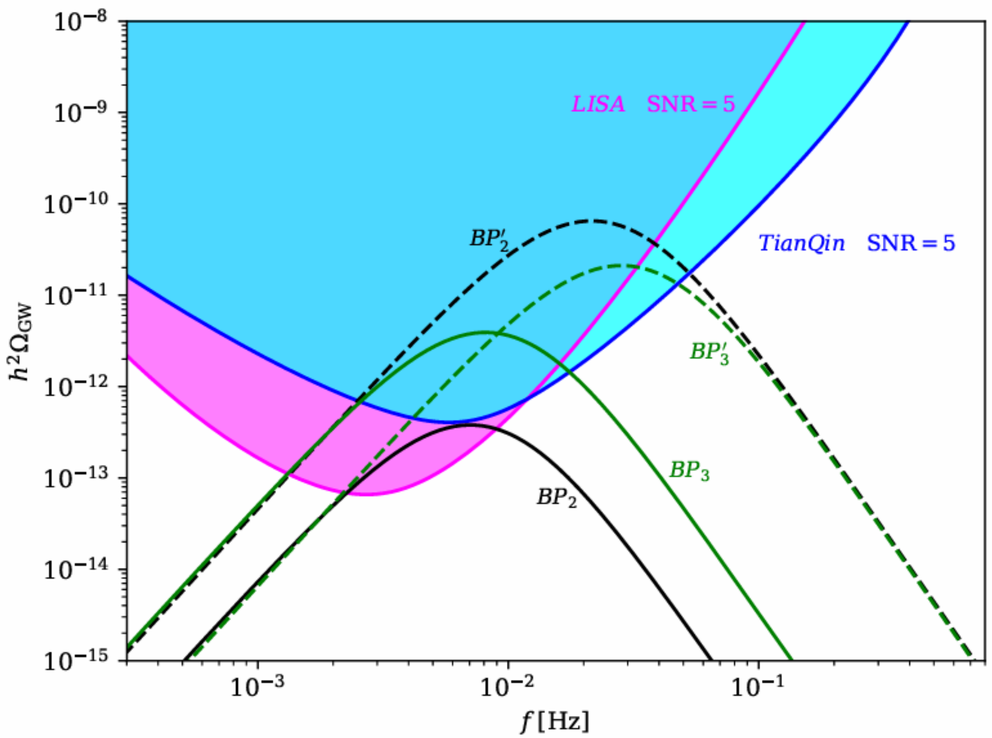
$v_w = 0.9$

	analytic		numerical	
	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)} h^2 / \Omega_{\rm DM}^{(0)} h^2$	$m_\chi^{\rm in}(T_n)/T_n$	$\Omega_{\rm DM}^{\rm (hy)} h^2 / \Omega_{\rm DM}^{(0)} h^2$
BP_1	125.3	1/19	147.8	1/27
BP_2	125.9	1/7	148.7	1/9
BP_3	124.6	1/10	147.3	1/12
BP_4	123.8	$1/(1.2 \times 10^{13})$	146.5	$1/(2.2 \times 10^{15})$

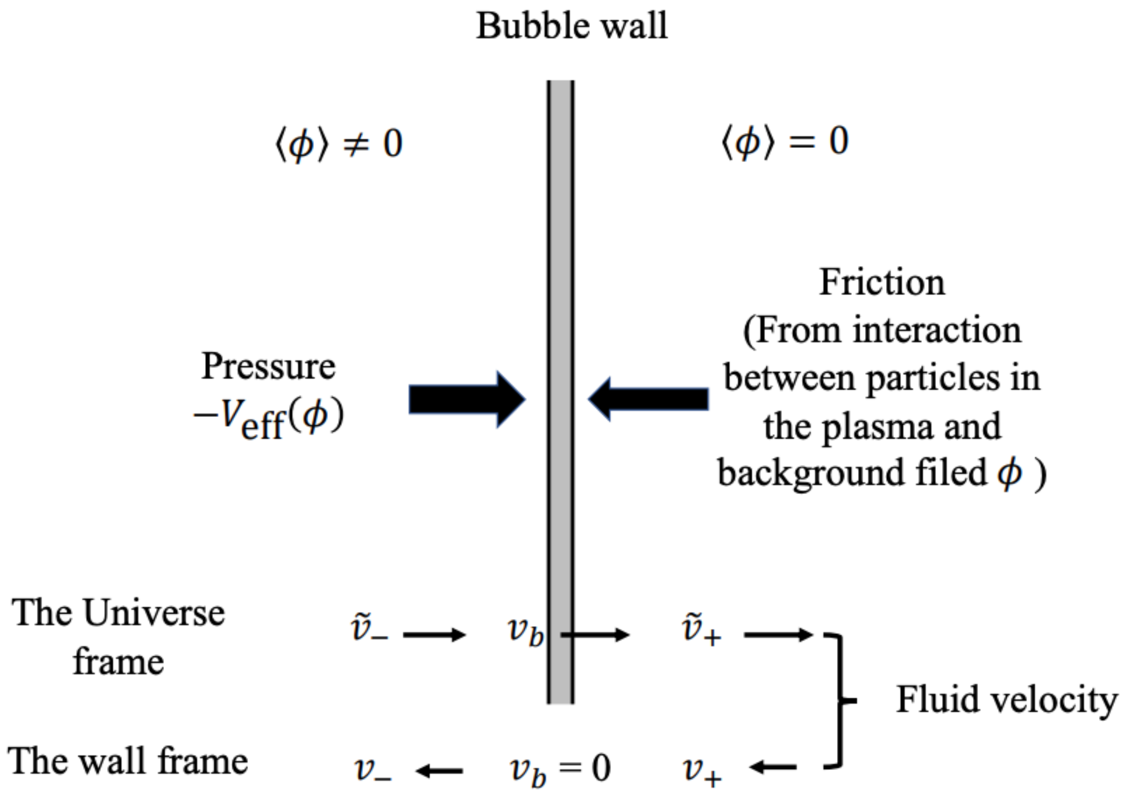
SFOPT and new DM mechanism/signal

The hydrodynamic effects play essential roles in the filtered DM mechanism. For the deflagration mode with low bubble wall velocity, the hydrodynamic effects significantly enhance the relic density. In contrast, for the detonation mode, the relic density is obviously reduced. For the hybrid mode, the hydrodynamic correction is extremely large.

Precise calculation of filtered DM relic density can help to decide the phase-transition parameters precisely. This gives more accurate phase-transition GW spectra.



Phase transition dynamics



泡壁速度计算流程

基于有效势，通过能动张量守恒得到早期宇宙plasma中的希格斯场运动方程

$$(1-v_w^2)h'' + \sum_i \frac{dm_i^2}{dh}\int \frac{d^3p}{(2\pi)^3 2E_i}\delta f_i(x,p) + \frac{\partial V_{\rm eff}(h,T)}{\partial h} = 0$$

求解玻尔兹曼方程得到粒子偏离热平衡的扰动分布

$$\frac{d}{dt}(f_i^{\rm eq}+\delta f_i) = \left(\left(v_w + \frac{p_z}{E}\right)\frac{\partial}{\partial z} - \frac{(m_i^2)'}{2E}\frac{\partial}{\partial p_z}\right)(f_i^{\rm eq}+\delta f_i) = -C[(f_i^{\rm eq}+\delta f_i)]$$

用量子场论的方法计算碰撞项C

计算粒子在泡壁处的散射

采用合理的flow ansatz以及truncation scheme，求解扰动的演化方程组

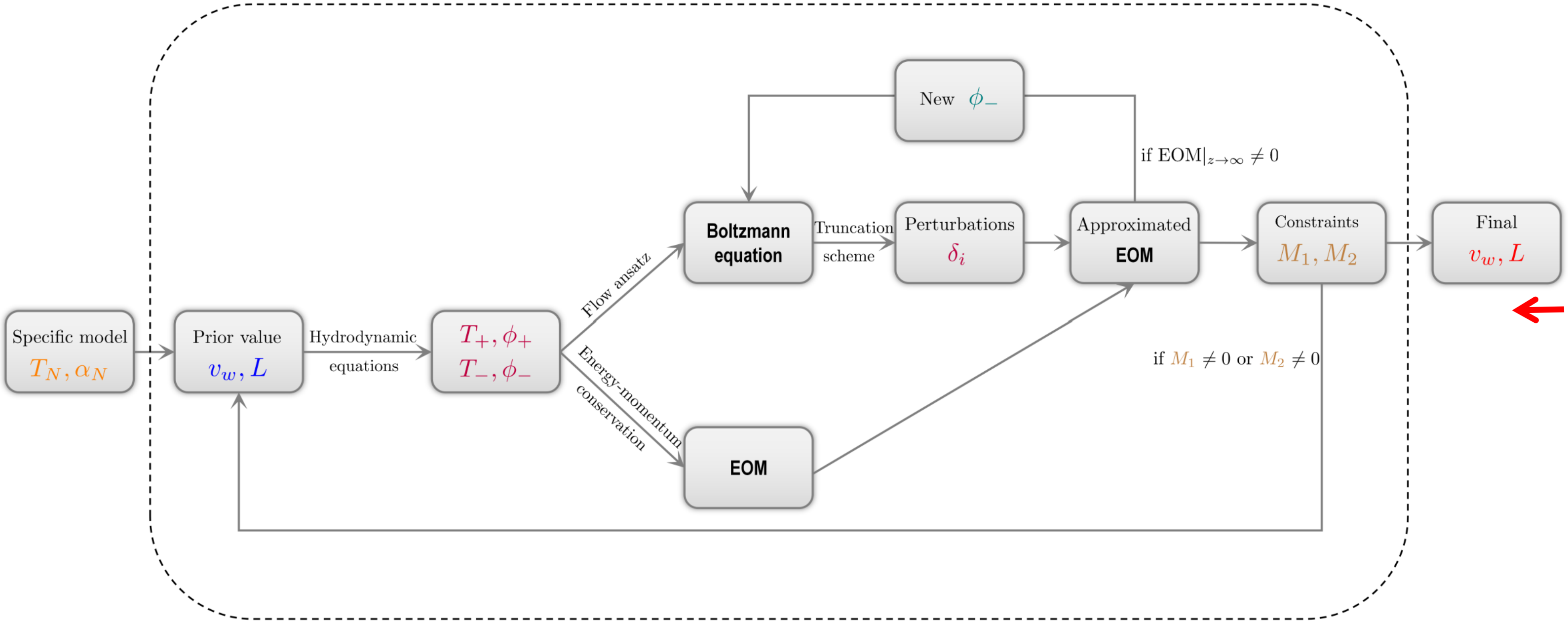
δf_i

将扰动代入运动方程，数值求解提取出泡壁速度和泡壁厚度 v_w, L_w

进一步可讨论相变动力学暗物质、电弱重子生成和相变引力波信号

Phase transition dynamics

A simple DM Model: **Bubble wall velocity in inert doublet model**



More accurate processes

Siyu Jiang, **FPH**, Xiao Wang, Phys.Rev.D 107 (2023) no.9, 095005

早期宇宙plasma中希格斯的运动方程

The energy-momentum tensor of the scalar field

$$T^{\mu\nu}_{\phi} = \partial^{\mu}\phi\partial^{\nu}\phi - g^{\mu\nu}\left(\frac{1}{2}\partial_{\alpha}\phi\partial^{\alpha}\phi - V_{T=0}(\phi)\right)$$

The energy-momentum tensor of the plasma

$$T^{\mu\nu}_{\rm pl} = \sum_i \int \frac{d^3p}{(2\pi)^3}\frac{p^{\mu}p^{\nu}}{E_i} f_i(x,p)$$

Using energy-momentum conservation $\nabla_{\mu}(T^{\mu\nu}_{\phi} + T^{\mu\nu}_{\rm pl}) = 0$, $V_{\rm eff}(\phi, T) = V_{T=0}(\phi) + V_T(\phi)$

$$\rightarrow \Box\phi + \frac{\partial V_{\rm eff}(\phi, T)}{\partial\phi} + \underbrace{\sum_i \frac{dm_i^2}{d\phi}\int \frac{d^3p}{(2\pi)^3 2E_i}\delta f_i(x,p)}_{\text{friction term}} = 0 \quad \text{(EOM)}$$

早期宇宙plasma中希格斯的运动方程

Higgs EOM in the plasma frame

$$\left(1-v_w^2\right)\phi'' + \underbrace{\sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3p}{(2\pi)^3 2E_i}\delta f_i(x,p)}_{friction\ \ term} + \frac{\partial V_{\text{eff}}\left(\phi,T\right)}{\partial\phi} = 0$$

Key point to calculate bubble wall velocity is to obtain the distribution function for the massive particles or the friction term.

How to calculate the distribution function or the friction term?
Boltzmann equation !

摩擦项/偏离热平衡分布的计算

The distribution function for each particle can be described by Boltzmann equation under WKB approximation

$$\frac{d}{dt}f = \left(\frac{\partial}{\partial t} + \dot{z}\frac{\partial}{\partial z} + \dot{p}_z\frac{\partial}{\partial p_z}\right) f = -C[f]$$

$\dot{z} = p_z/E$ and $\dot{p}_z = -\partial_z E = -(m^2)'/(2E)$ (external force from bubble wall)

Parameterization of the distribution function

$$f = \frac{1}{e^{(E+\delta)/T} \pm 1}$$

采用合理的flow ansatz

$$\delta = -\mu - \mu_{bg} - \frac{E}{T}(\delta T + \delta T_{bg}) - p_z(\delta v + \delta v_{bg})$$

The light particles contributes to the background.

摩擦项/偏离热平衡分布的计算

求解玻尔兹曼方程得到粒子偏离热平衡的扰动分布

$$\frac{d}{dt}(f_i^{\rm eq}+\delta f_i) = \left(\left(v_w+\frac{p_z}{E}\right)\frac{\partial}{\partial z} - \frac{(m_i^2)'}{2E}\frac{\partial}{\partial p_z}\right)(f_i^{\rm eq}+\delta f_i) = -C[(f_i^{\rm eq}+\delta f_i)]$$

Boltzmann equation for each particle species:

$$(-f_0')\left(\frac{p_z}{E}\left[\partial_z\mu+\frac{E}{T}\partial_z\left(\delta T+\delta T_{bg}\right)+p_z\partial_z\left(\delta v+\delta v_{bg}\right)\right]+\partial_t\mu\right.$$
$$\left.+\frac{E}{T}\partial_t\left(\delta T+\delta T_{bg}\right)+p_z\partial_t\left(\delta v+\delta v_{bg}\right)\right)+TC[\mu,\delta T,\delta v]=(-f_0')\frac{\partial_t\left(m^2\right)}{2E}$$

How to solve this equation?

摩擦项/偏离热平衡分布的计算

采用truncation scheme，求解扰动的演化方程组

After integration by $\int d^3p/(2\pi)^3$, $\int Ed^3p/(2\pi)^2$, and $\int p_z d^3p/(2\pi)^3$, (truncation scheme)

$$v_w c_2^i\left(\mu_i' + \mu_{bg}'\right) + v_w c_3^i\left(\delta T_i' + \delta T_{bg}'\right) + \frac{c_3^i T}{3}\left(\delta v_i' + \delta v_{bg}'\right) + \mu_i \Gamma_{\mu 1,i} + \delta T_i \Gamma_{T1,i} = \frac{v_w c_1^i}{2T}(m_i^2)' \,,$$

$$v_w c_3^i\left(\mu_i' + \mu_{bg}'\right) + v_w c_4^i\left(\delta T_i' + \delta T_{bg}'\right) + \frac{c_4^i T}{3}\left(\delta v_i' + \delta v_{bg}'\right) + \mu_i \Gamma_{\mu 2,i} + \delta T_i \Gamma_{T2,i} = \frac{v_w c_2^i}{2T}(m_i^2)' \,,$$

$$\frac{c_3^i}{3}\left(\mu_i' + \mu_{bg}'\right) + \frac{c_4^i}{3}\left(\delta T_i' + \delta T_{bg}'\right) + \frac{v_w c_4^i T}{3}\left(\delta v_i' + \delta v_{bg}'\right) + \delta v_i T \Gamma_{v,i} = 0$$

Phase transition dynamics

$$V_0 = \mu_1^2|\Phi|^2 + \mu_2^2|\eta|^2 + \frac{1}{2}\lambda_1|\Phi|^4 + \frac{1}{2}\lambda_2|\eta|^4 + \lambda_3|\Phi|^2|\eta|^2 + \lambda_4|\Phi^\dagger\eta|^2 + \frac{1}{2}\{\lambda_5(\Phi^\dagger\eta)^2 + \text{H.c.}\} ,$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h + v + iG^0) \end{pmatrix} , \ \eta = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + iA) \end{pmatrix} ,$$

$$V_{\text{CW}}(\phi, T=0) = \sum_i \frac{n_i}{64\pi^2}\left[m_i^4(\phi)\left(\ln\frac{m_i^2(\phi)}{\overline{m}_i^2} - \frac{3}{2}\right) + 2\overline{m}_i^2 \overline{m}_i^2(\phi)\right] ,$$

$$V_{\text{T}}(\phi, T>0) = \sum n_i \frac{T^4}{2\pi^2} I_b\left(\frac{M_i^2}{T^2}\right) ,$$

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + V_{\text{CW}}(\phi) + V_{\text{T}}(\phi, T) .$$

Phase transition dynamics

$$f = \frac{1}{e^{(E+\delta)/T} \pm 1}$$

$$(-f_0')\left(\frac{p_z}{E}\left[\partial_z\mu + \frac{E}{T}\partial_z\left(\delta T + \delta T_{bg}\right) + p_z\partial_z\left(\delta v + \delta v_{bg}\right)\right] + \partial_t\mu + \frac{E}{T}\partial_t\left(\delta T + \delta T_{bg}\right) + p_z\partial_t\left(\delta v + \delta v_{bg}\right)\right) + TC[\mu, \delta T, \delta v] = (-f_0')\frac{\partial_t\left(m^2\right)}{2E}$$

truncation scheme →

$$\hat{A}\delta' + \Gamma\delta = \Sigma,$$

source term (Σ)

collision term (Γ)

$$\delta = (\mu_t, \delta T_t, T\delta v_t, \mu_W, \delta T_W, T\delta v_W, \mu_A, \delta T_A, T\delta v_A),$$

$$\Sigma = \frac{v_w}{2T}\left(c_1^t(m_t^2)', c_2^t(m_t^2)', 0, c_1^W(m_W^2)', c_2^W(m_W^2)', 0, c_1^A(m_A^2)', c_2^A(m_A^2)', 0\right),$$

$$\hat{A} = \begin{pmatrix} \hat{A}_t & 0 & 0 \\ 0 & \hat{A}_W & 0 \\ 0 & 0 & \hat{A}_A \end{pmatrix}, \quad \text{where} \quad \hat{A}_i = \begin{pmatrix} v_w c_2^i & v_w c_3^i & \frac{1}{3}c_3^i \\ v_w c_3^i & v_w c_4^i & \frac{1}{3}c_4^i \\ \frac{1}{3}c_3^i & \frac{1}{3}c_4^i & \frac{1}{3}v_w c_4^i \end{pmatrix},$$

Phase transition dynamics

Collision terms (Monte Carlo integration)

$$\Gamma_{\mu1,t} \simeq \left(5.0\times10^{-4}g_s^4 + 5.8\times10^{-4}g_s^2y_t^2\right)T\ ,$$

$$\Gamma_{T1,t} \simeq \Gamma_{\mu2,t} \simeq \left(1.1\times10^{-3}g_s^4 + 1.3\times10^{-3}g_s^2y_t^2\right)T\ ,$$

$$\Gamma_{T2,t} \simeq \left(1.1\times10^{-2}g_s^4 + 4.0\times10^{-3}g_s^2y_t^2\right)T\ ,$$

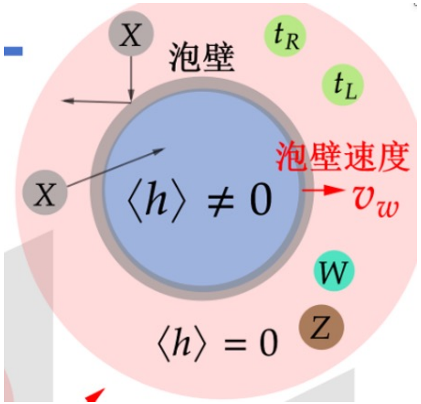
$$\Gamma_{v,t} \simeq \left(2.0\times10^{-2}g_s^4 + 1.8\times10^{-3}g_s^2y_t^2\right)T\ ,$$

$$\Gamma_{\mu1,W} \simeq \left(2.3\times10^{-3}g_s^2g_w^2 + 2.0\times10^{-3}g_w^4\right)T\ ,$$

$$\Gamma_{T1,W} \simeq \Gamma_{\mu2,W} \simeq \left(4.7\times10^{-3}g_s^2g_w^2 + 4.1\times10^{-3}g_w^4\right)T$$

$$\Gamma_{T2,W} \simeq \left(1.5\times10^{-2}g_s^2g_w^2 + 1.5\times10^{-2}g_w^4\right)T\ ,$$

$$\Gamma_{v,W} \simeq \left(5.7\times10^{-2}g_s^2g_w^2 + 1.5\times10^{-2}g_w^4\right)T\ ,$$



$$\Gamma_{\mu1,A} \simeq 1.0\times10^{-2}\lambda_3^4T\ ,$$

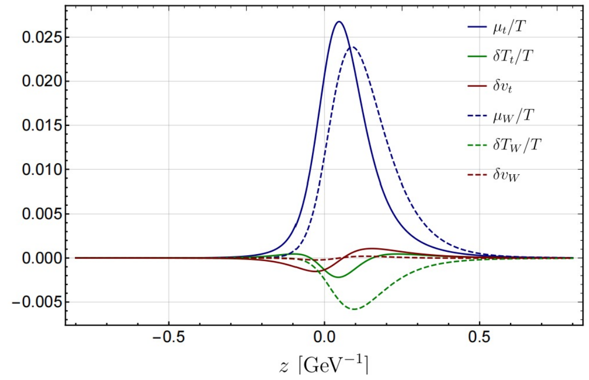
$$\Gamma_{T1,A} \simeq \Gamma_{\mu2,A} \simeq 4.9\times10^{-3}\lambda_3^4T\ ,$$

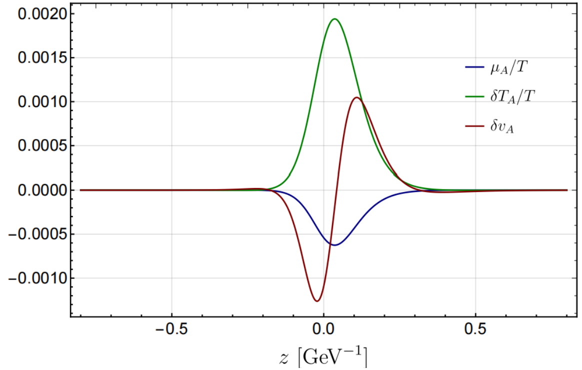
$$\Gamma_{T2,A} \simeq 5.1\times10^{-3}\lambda_3^4T\ ,$$

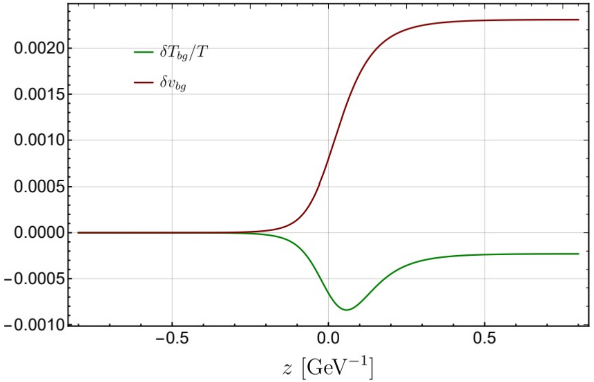
$$\Gamma_{v,A} \simeq 1.8\times10^{-3}\lambda_3^4T\ .$$

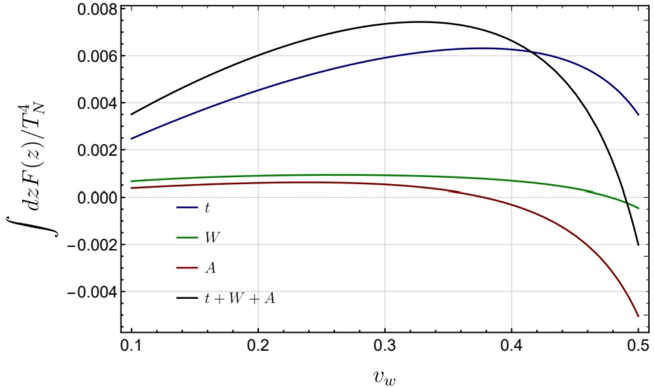
Phase transition dynamics

Solving perturbation equations:
Green Function Method







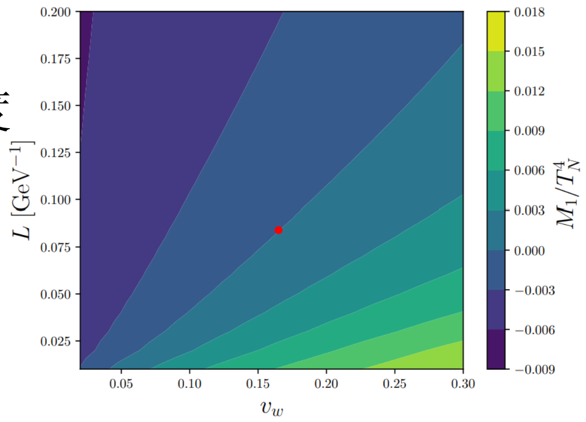


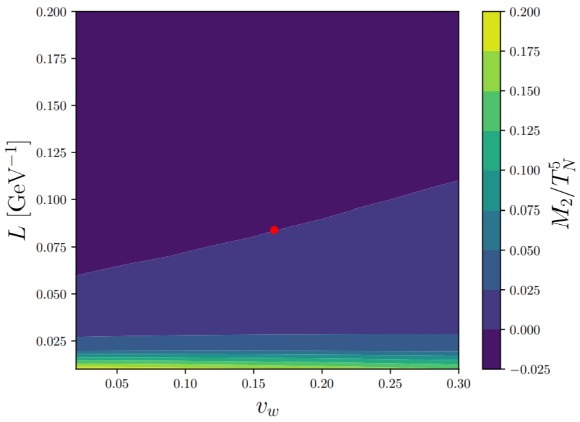
Phase transition dynamics

$$S_{\rm EOM} \equiv (1-v_w^2)\,\phi'' + \frac{\partial V_{\rm eff}(\phi,T_+)}{\partial\phi} + \frac{N_t T_+}{2}\frac{dm_t^2}{d\phi} \times \left(c_1^t \mu_t + c_2^t(\delta T_t + \delta T_{bg})\right)$$
$$+ \sum_b \frac{N_b T_+}{2}\frac{dm_b^2}{d\phi}\left(c_1^b \mu_b + c_2^b(\delta T_b + \delta T_{bg})\right) = 0\,,$$

Solving the EOM:
bubble wall 内外压强差为0;
bubble wall厚度不变

$$M_1 = \int S_{\rm EOM}\phi' dz = 0, \quad M_2 = \int S_{\rm EOM}(2\phi - \phi_-)\phi' dz = 0\,.$$





Phase transition dynamics

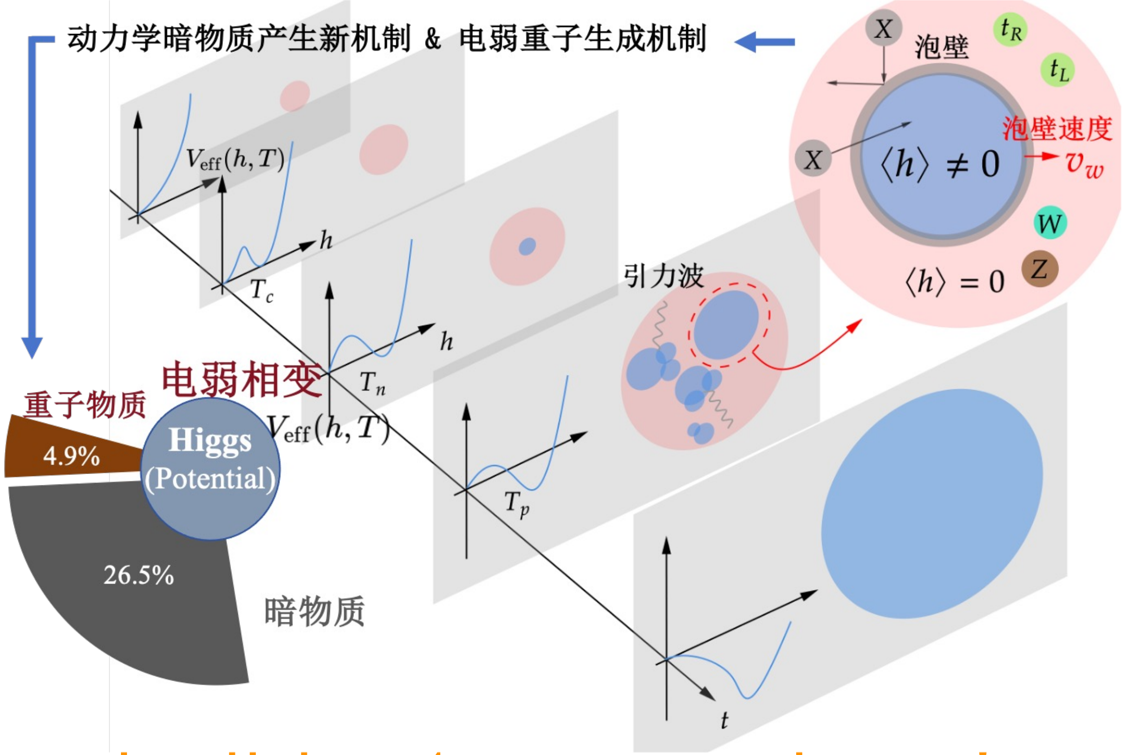
	T_c [GeV]	T_N [GeV]	v_w	L [GeV^{-1}]
Benchmark A	118.3	117.1	0.165	0.084
Benchmark B	118.6	117.5	0.164	0.085
Benchmark C	119.4	118.4	0.164	0.088

In the allowed parameter spaces, the bubble wall velocity varies slightly around 0.165.

The basic procedure in this work can also be used for any other SFOPT model.

Summary and outlook

精确计算泡壁速度等
相变动力学量对理解
电弱重子生成
和动力学暗物质产生新机制
至关重要。



Thanks! Comments and collaborations are welcome!

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