中国科学院大学
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International Centre
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国际理论物理中心-亚太地区

耗散效应作为宇宙相变的新观测量

郭怀珂

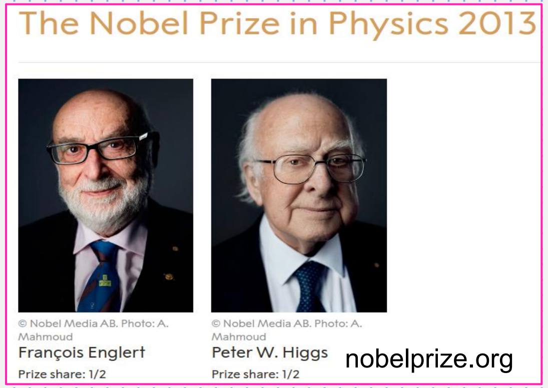
2024年4月13日

The Fifth Workshop on
Frontiers of Particle Physics
第五届
粒子物理前沿
研讨会

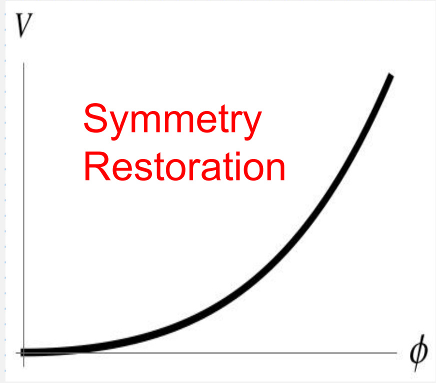
1924-2024
中山大学 世纪华诞

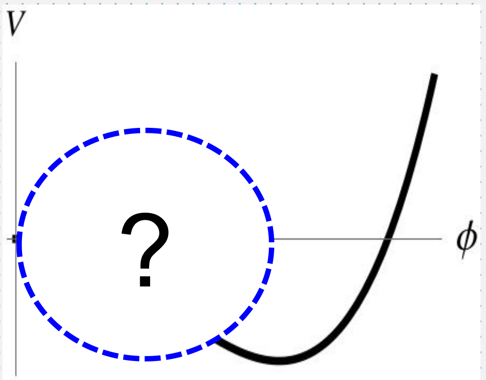
Based on HG [2310.10927]

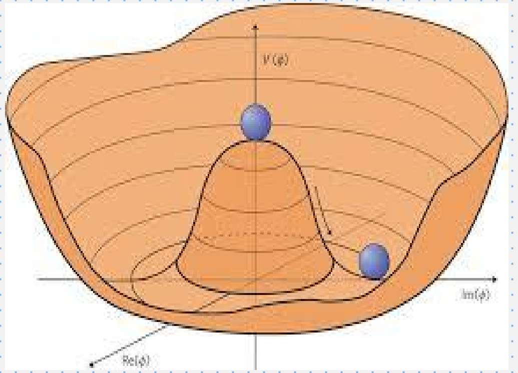
Electroweak Phase Transition



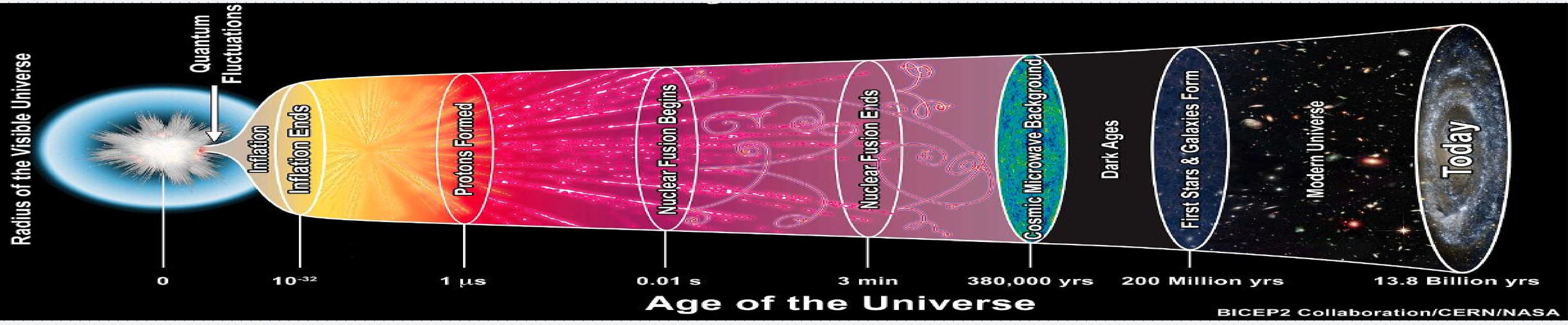
$$SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}$$

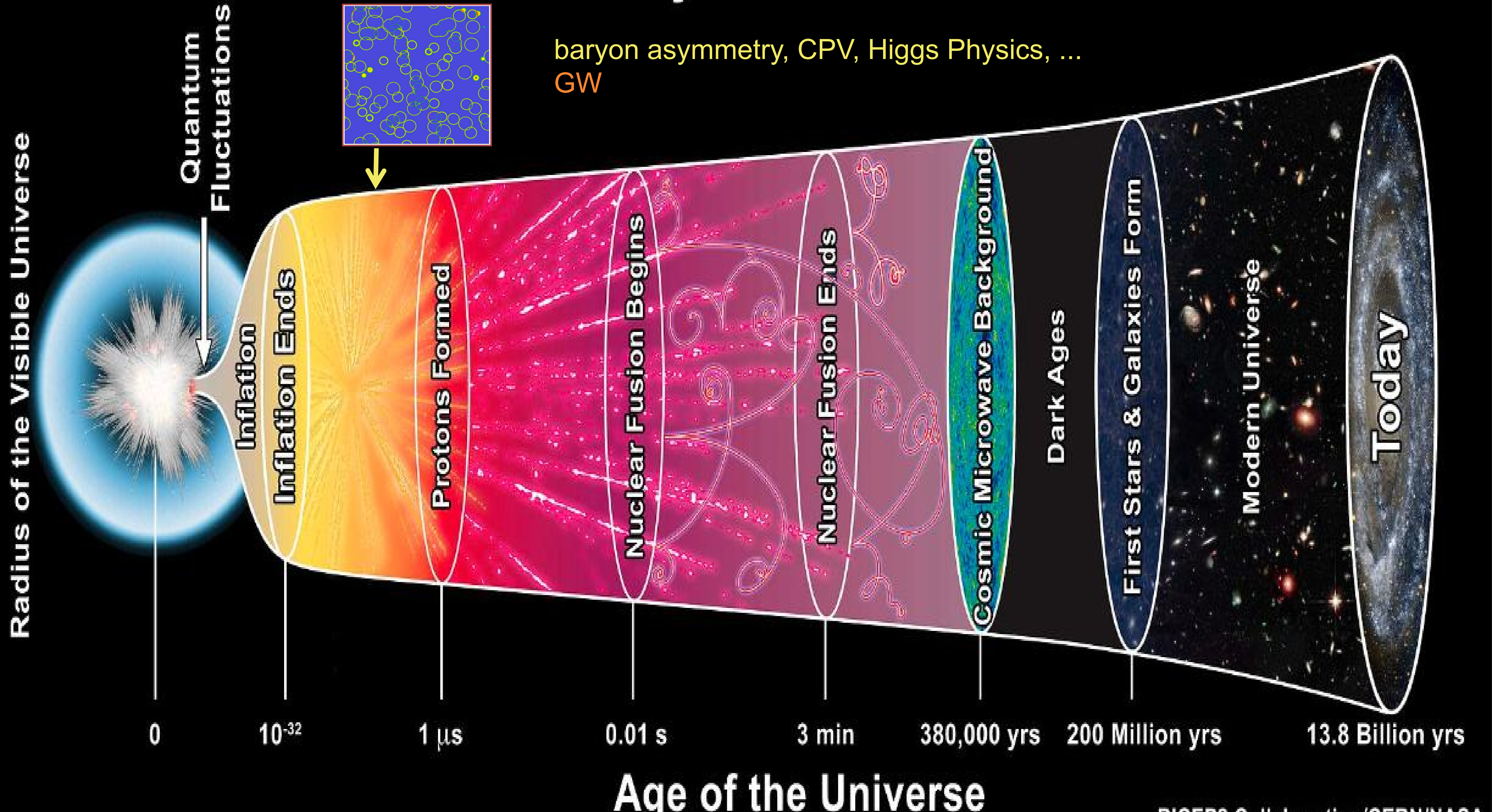




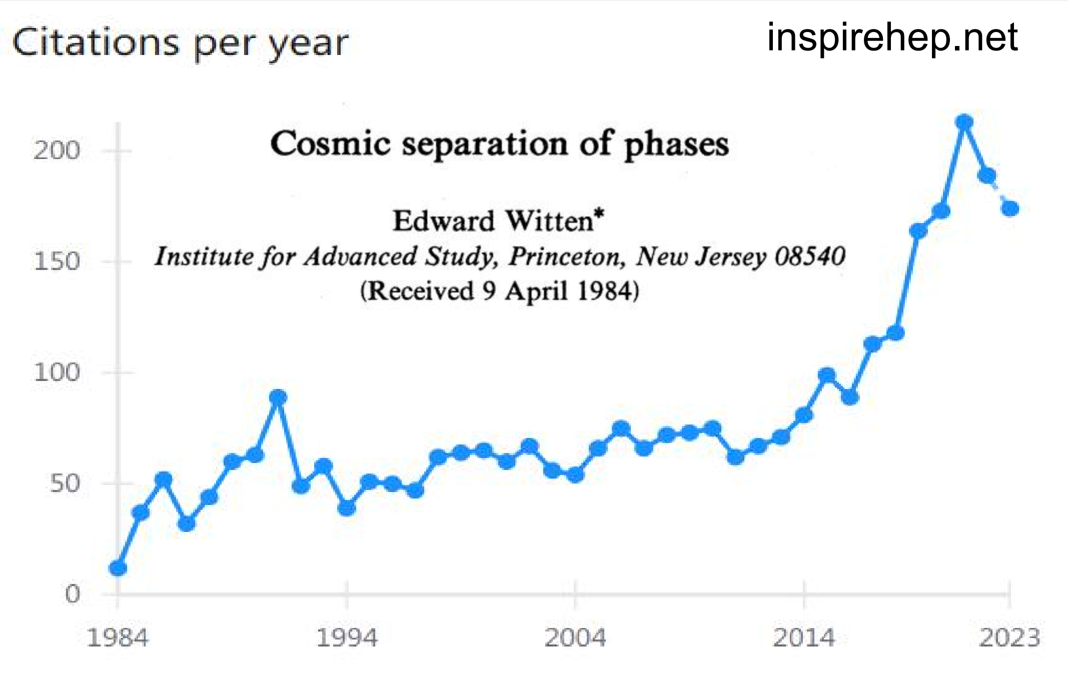


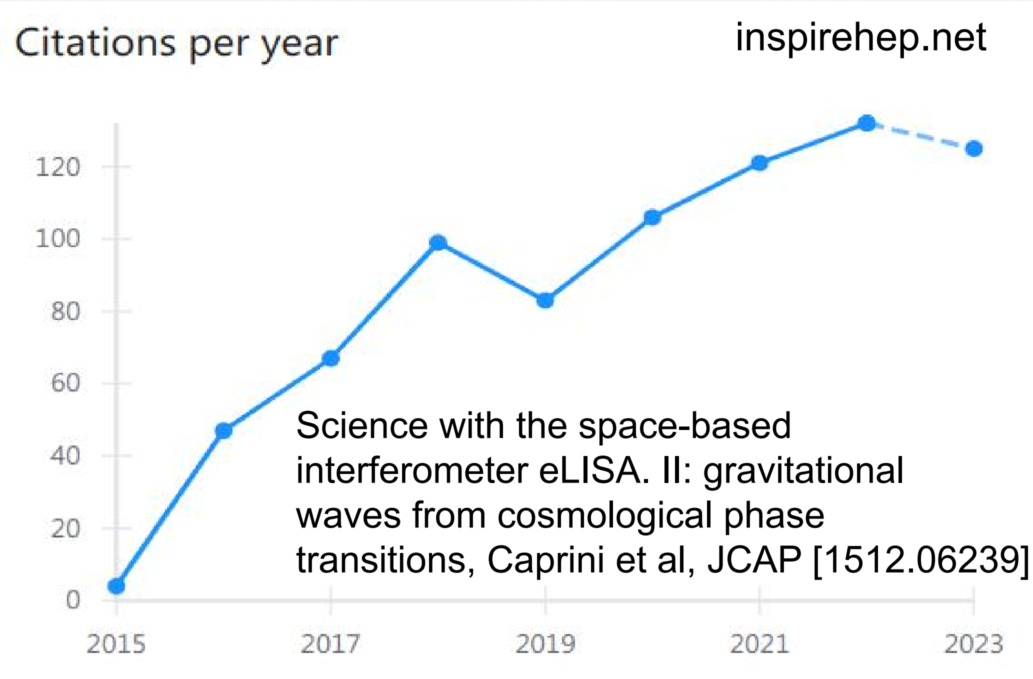
Temperature drops



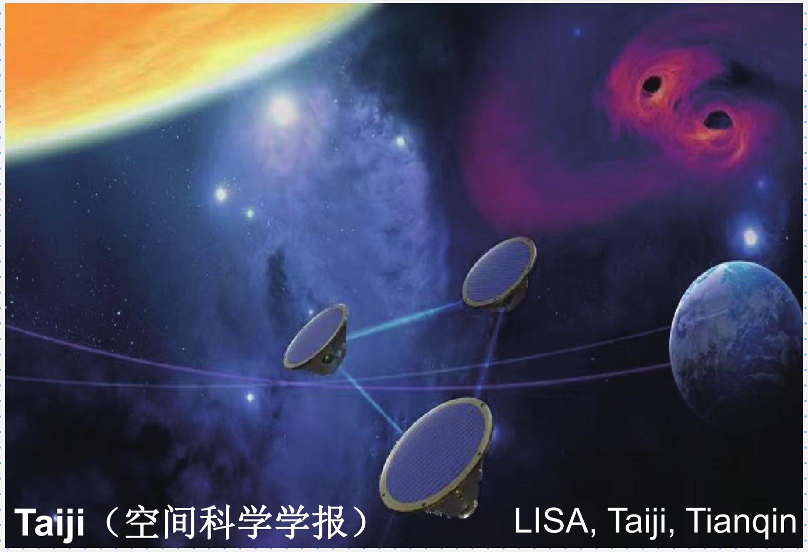


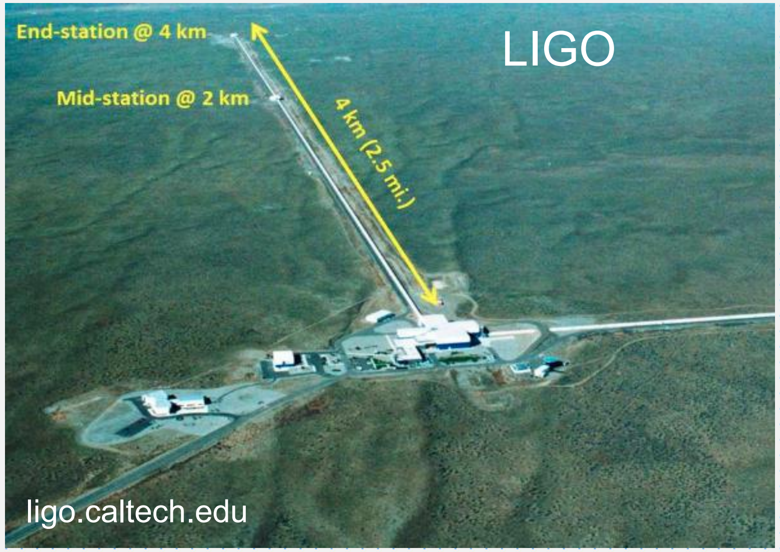
BICEP2 Collaboration/CERN/NASA





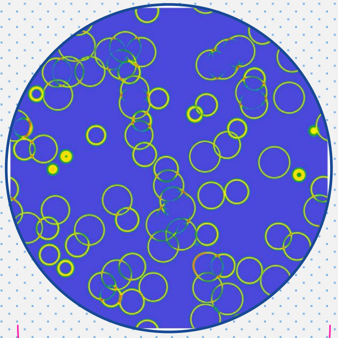
中国脉冲星测时阵列（CPTA）





See also Haipeng, Kepan, Zhenmin, Fapeng, Yun and Renhui’s talks

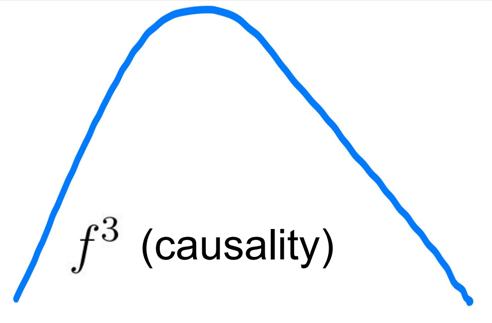
Properties



Horizon size: 1/H*

$$f_{\rm now} = 1.65 \times 10^{-5} \left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz}$$

~100-1000



Cai, Pi, Sasak, PRD [1909.13728]

nHz (~100MeV) QCD scale

EPTA

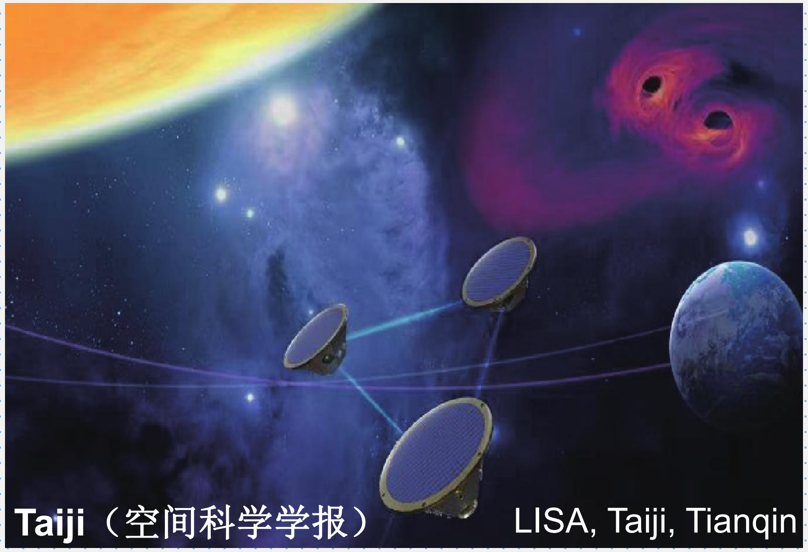
NANOGrav Physics Frontiers Center

PPTA

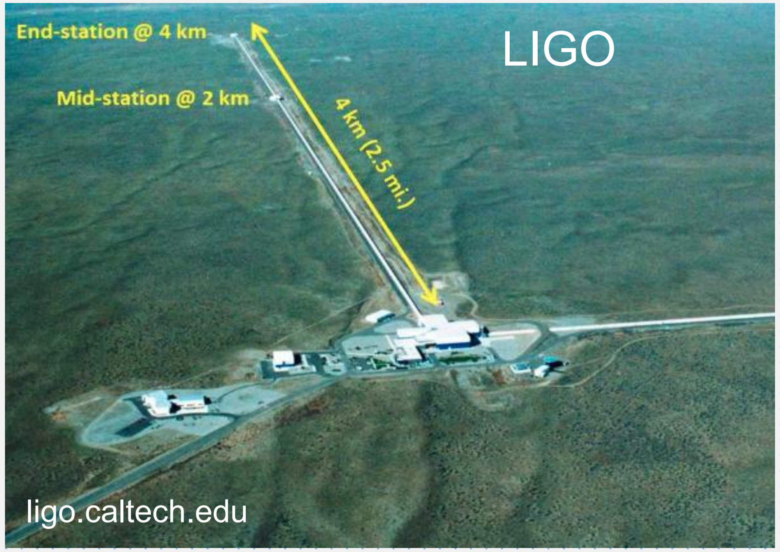
InPTA Indian Pulsar Timing Array

中国脉冲星测时阵列（CPTA）

~mHz : (~100GeV) weak scale



~100Hz (~PeV - EeV) high scale



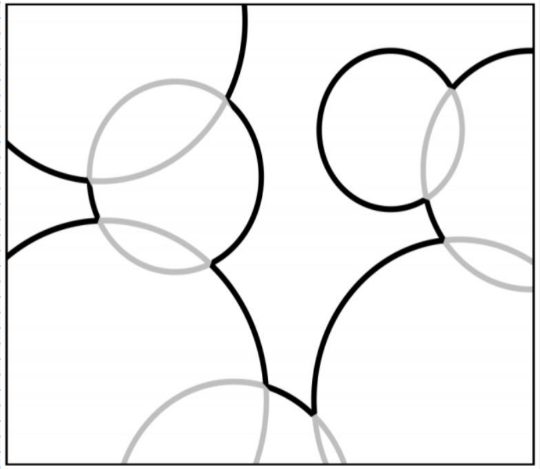
See also Haipeng, Kepan, Zhenmin, Fapeng, Yun and Renhui's talks

Gravitational Wave Sources

The current understanding:

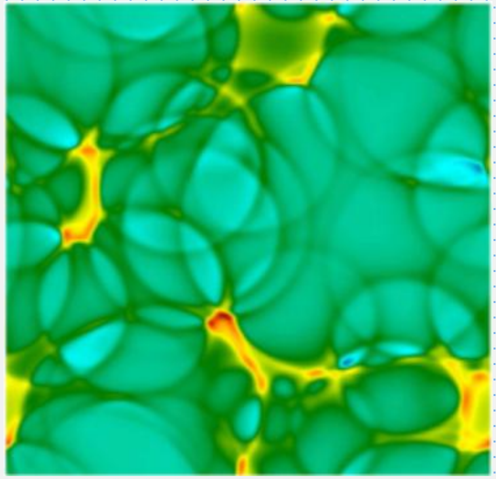
$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

energy near the wall



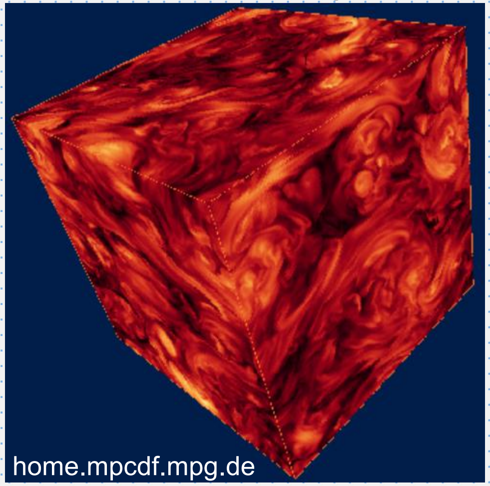
Bubble Collisions

fluid kinetic energy



Sound Waves

turbulent fluid + magnetic field



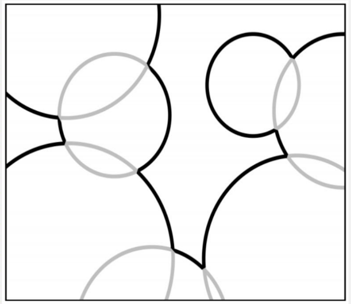
Magnetohydrodynamic Turbulence

GW Spectra

Energy density Spectrum

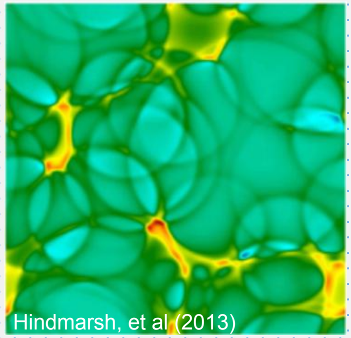
$$\Omega_{\rm GW}(f) = \frac{d\rho_{\rm GW}}{\rho_c d\log f}$$

bubble collision



$$\Omega_{\rm coll}(f)h^2 = 1.67\times 10^{-5}\Delta\left(\frac{H_{\rm pt}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2 \times\left(\frac{100}{g_*}\right)^{1/3} S_{\rm env}(f),$$

sound waves

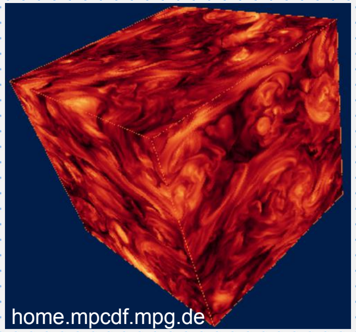


$$\Omega_{\rm sw}(f)h^2 = 2.65\times 10^{-6}\left(\frac{H_{\rm pt}}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} \times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}\Upsilon(\tau_{\rm sw}),$$

$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2}$$ (RD)

HG, Sinha, Vagie, White, JCAP [2007.08537]

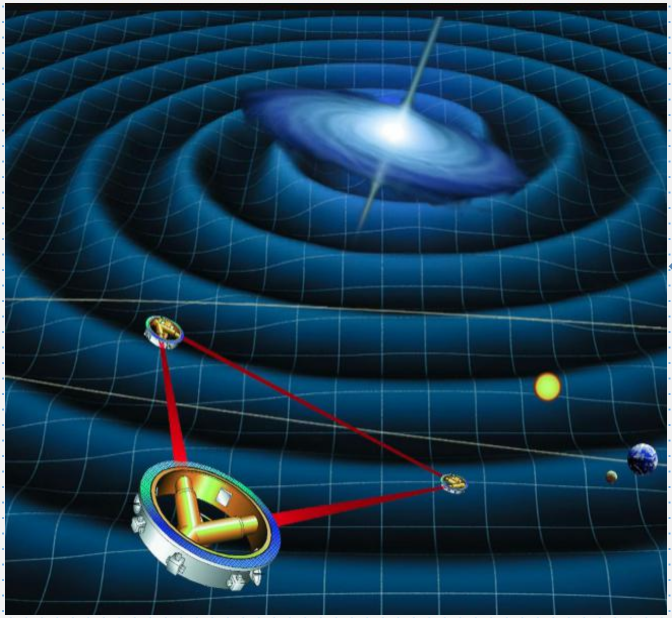
MHD



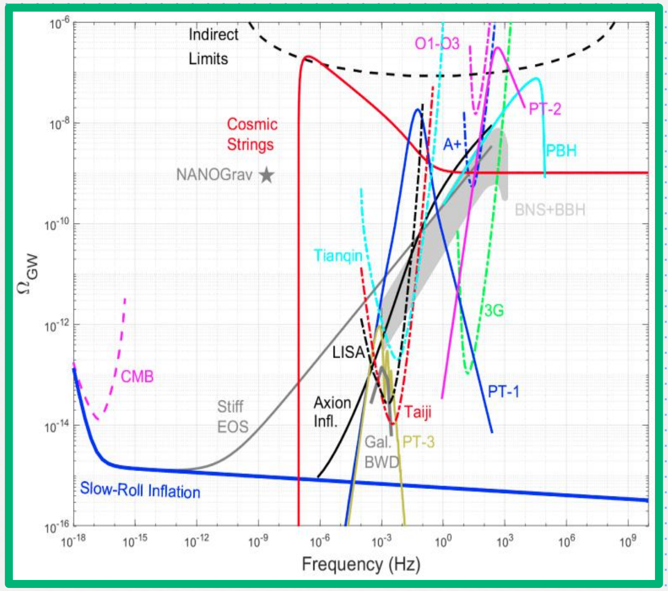
$$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w\, S_{\rm turb}(f)$$

Chiara Caprini et al, JCAP [1512.06239]

From Theory to Experiment



LIGO, LISA/Taiji/Tianqin, PTA, ...



Gravitational Wave Spectrum

Ω_{GW}

α

β

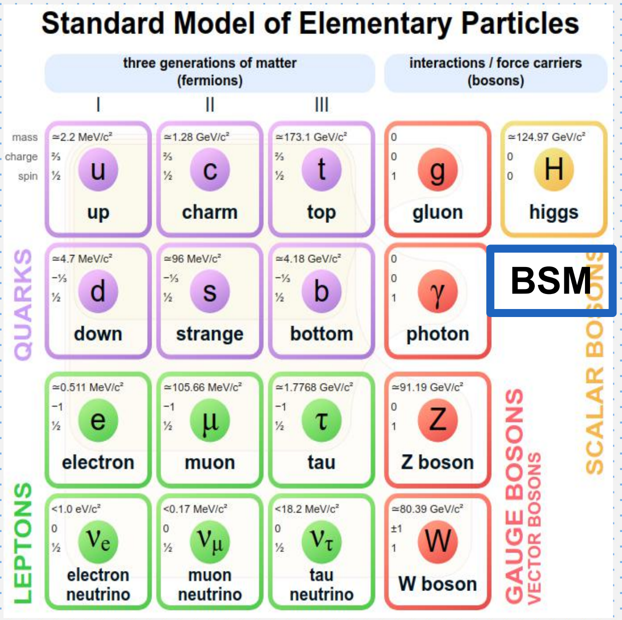
v_w

T_*

g_s

...

Phase Transition Parameters



Particle Physics Model

Problem: parameter degeneracy

Models	Strong 1^{st} order phase transition	GW signal	Cold DM	Dark Radiation and small scale structure
SM charged				
Triplet [20–22]	✓	✓	✓	✗
complex and real Triplet [23] (Georgi-Machacek model)	✓	✓	✓	✗
Multiplet [24]	✓	✓	✓	
2HDM [25–30]	✓	✓		✗
MLRSM [31]	✓	✓	✗	✗
NMSSM [32–36]	✓	✓	✓	✗
SM uncharged				
S_r (xSM) [37–49]	✓	✓	✗	✗
2 S_r's [50]	✓	✓	✓	✗
S_c (cxSM) [49, 51–54]	✓	✓	✓	✗
$U(1)_D$ (no interaction with SM) [55]	✓	✓	✓	✗
$U(1)_D$ (Higgs Portal) [56]	✓	✓	✓	
$U(1)_D$ (Kinetic Mixing) [57]	✓	✓	✓	
Composite SU(7)/SU(6) [58]	✓	✓	✓	
$U(1)_L$ [59]	✓	✓	✓	✗
$SU(2)_D \to$ global $SO(3)$ by a doublet [60–62]			✓	✗
$SU(2)_D \to U(1)_D$ by a triplet [63–65]			✓	✓
$SU(2)_D \to Z_2$ by two triplets [66]			✓	✗
$SU(2)_D \to Z_3$ by a quadruplet [67, 68]			✓	✗
$SU(2)_D \times U(1)_{B\text{-}L} \to Z_2 \times Z_2$ by a quintuplet and a S_c [69]			✓	✗
$SU(2)_D$ with two dark Higgs doublets [70]	✓	✓	✗	✗
$SU(3)_D \to Z_2 \times Z_2$ by two triplets [62, 71]			✓	✗
$SU(3)_D$ (dark QCD) (Higgs Portal) [72, 73]	✓	✓	✓	
$G_{SM} \times G_{D,SM} \times Z_2$ [74]	✓	✓	✓	
$G_{SM} \times G_{D,SM} \times G_{D,SM} \cdots$ [75]	✓	✓	✓	
Current work				
$SU(2)_D \to U(1)_D$ (see the text)	✓	✓	✓	✓

Ghosh, HG, Han, Liu, JHEP [2012.09758]

See also Kepan's talk

Many models can lead to the same PT parameter values

Solutions: New Observables

- Anisotropy

  Geller, Hook, Sundrum, Yuhsin Tsai, PRL [1803.10780]
  Li, Huang, Wang, Zhang, PRD [2112.01409]
  Li, Yan, Huang, PRD [2211.03368]

- Primordial magnetic field

  Di,Wang,Zhou,Bian,Cai, PRL [2012.15625]
  Yang,Bian,PRD [2102.01398], ...

- Primordial black holes and solitons

  Hong, Jung, Xie, PRD [2008.04430]
  Kawana,Xie,PLB [2106.00111]
  Liu,Bian,Cai,Guo,Wang, PRD [2106.05637]
  Lu,Kawana,Xie, PRD [2202.03439]

- Curvature perturbations

  Liu,Bian,Cai,Guo,Wang,PRL[2208.14086]
  Jiang,Liu,Sun,Wang, PLB [1512.07538]

Anything directly readable from the isotropic GW spectrum?

Dissipative Effects as New Observables

GW depends on (large) bulk velocity of the system

$$h \sim 10^{-22} \frac{M/M_\odot}{r/100\mathrm{Mpc}} \left(\frac{v}{c}\right)^2$$

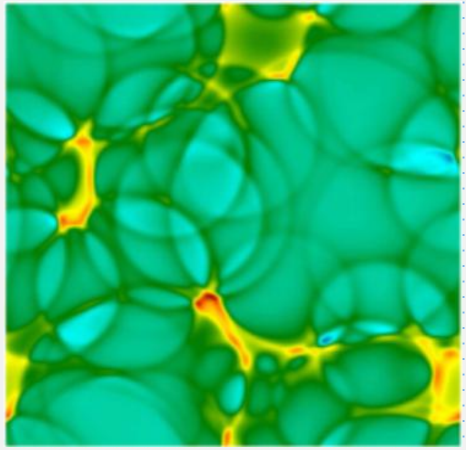
Dissipative effects dissipate away the bulk kinetic energy (leaves imprint)

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + (\zeta + \frac{1}{3}\mu)\nabla(\nabla\cdot\mathbf{v})$$

Navier–Stokes equations (Newtonian fluid mechanics)

GW calculation requires: relativistic (magneto-)hydrodynamics

Sound Waves



Usually the dominant source (Hindmarsh,Huber,Rummukainen,Weir, PRL [1304.2433])

$$T^{ij} \propto (p+e) v^i v^j$$

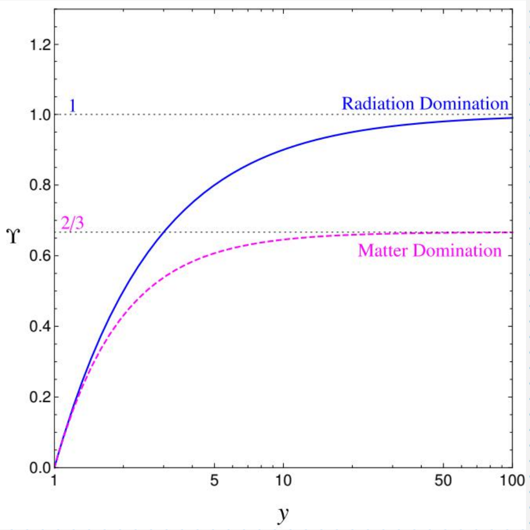
$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$

$$S_{\rm sw}(f) = \left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2} \qquad f_* = \frac{2\beta}{\sqrt{3}v_w} \approx \frac{3.4}{R_*}$$

Hindmarsh,Huber,Rummukainen,Weir, PRD [1504.03291]

$$\Omega_{\rm SW}(f \gtrsim f_{\rm peak}) \propto f^{-4}$$

$$\Omega_{\rm SW}(f \lesssim f_{\rm peak}) \propto f^{3}$$



Slight different fit obtained by the same group, PRD [1704.05871]

$$\Upsilon = 1 - (1 + 2\tau_{\rm sw} H_{\rm pt})^{-1/2}$$ (radiation domination)

HG, Sinha, Vagie, White, JCAP [2007.08537]

Sound Waves: Recent Development

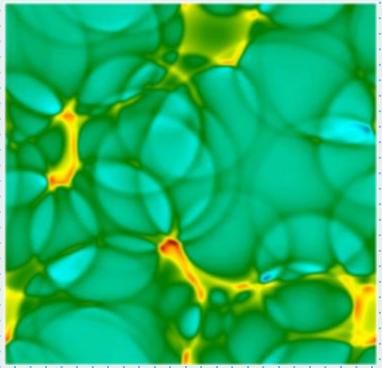
Analytical Modelling

- Refine the sound shell model
- Synergy with simulations

Sound Shell Model

Hindmarsh, PRL [1608.04735]
Hindmarsh, Hijazi, JCAP [1909.10040]
HG, Sinha, Vagie, White, JCAP [2007.08537]
Cai, Wang, Yuwen, PRD Letter [2305.00074]
Pol, Procacci, Caprini [2308.12943]

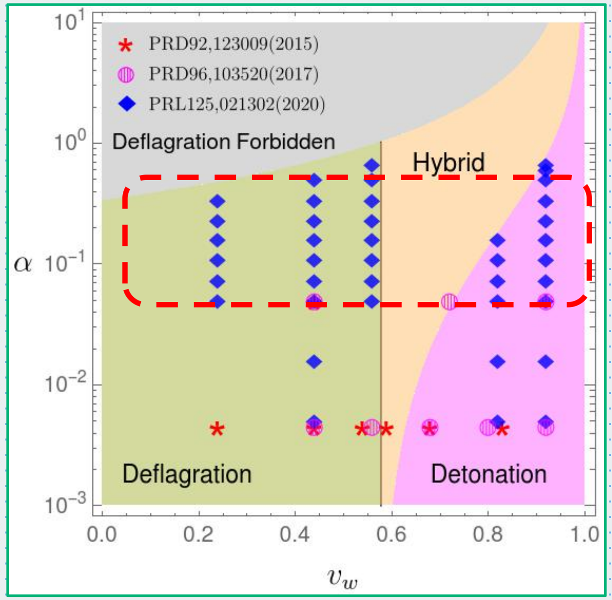
$$v_{\mathbf{q}}^{i} = \sum_{n=1}^{N_b} v_{\mathbf{q}}^{i(n)}$$

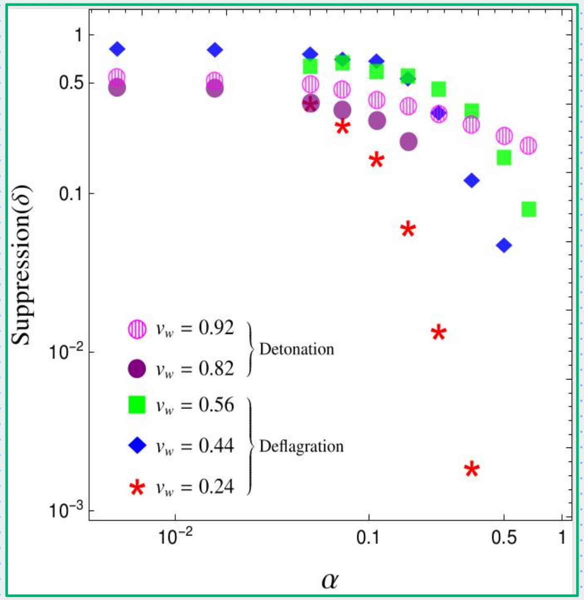


Numerical Simulation

- Suppression found for strong transitions with small vw
- Need to cover more parameter space (very strong PT)

$$h^2\Omega_{\rm sw}(f) = 2.65\times10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$





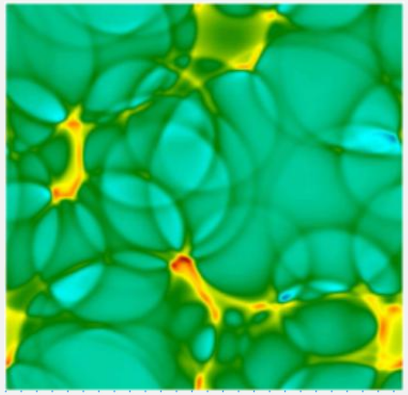
Cutting, Hindmarsh, Weir, PRL [1906.00480]

All based on perfect fluid approximation

Effects of Dissipation

- Disturbed fluid comes into rest eventually

$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3} \left[v^i_{\mathbf{q}} e^{-i\omega\eta + i\mathbf{q}\cdot\mathbf{x}} + c.c.\right]$$



$$v^i_{\mathbf{q}}(\eta) \propto \exp\left[-\int \Gamma(\mu, \zeta, \xi) d\eta\right]$$

$$\Gamma \propto q^2$$

shear viscosity

bulk viscosity

$$\Delta T^{ij} = -\mu\left(\frac{\partial U_i}{\partial x^j} + \frac{\partial U_j}{\partial x^i} - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{U}\right) - \zeta\delta_{ij}\nabla\cdot\mathbf{U},$$

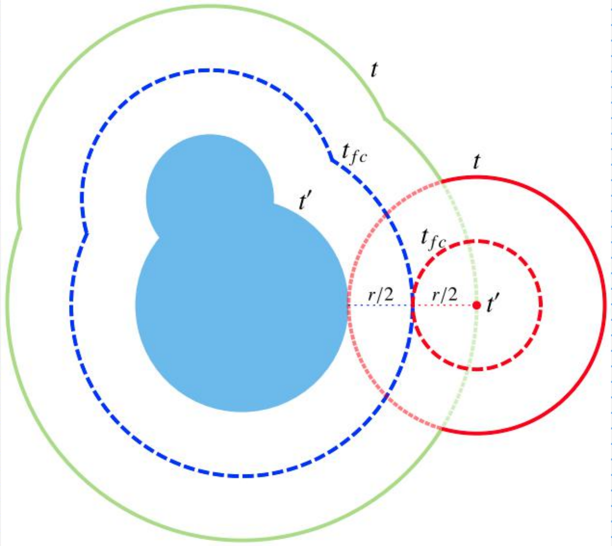
$$\Delta T^{i0} = -\chi\left(\frac{\partial T}{\partial x^i} + T\dot{U}_i\right). \qquad (1)$$

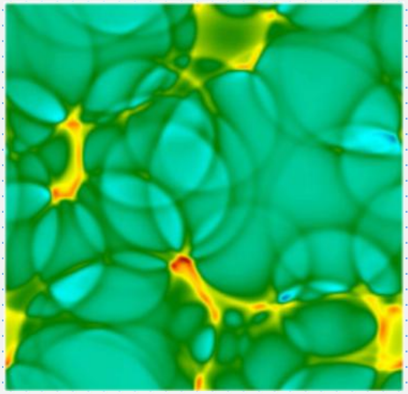
thermal conduction

Weinberg, ApJ, 1971

Euler equation -> Navier–Stokes equations
-> Relativistic hydrodynamics

Sound Shell Model with Dissipation





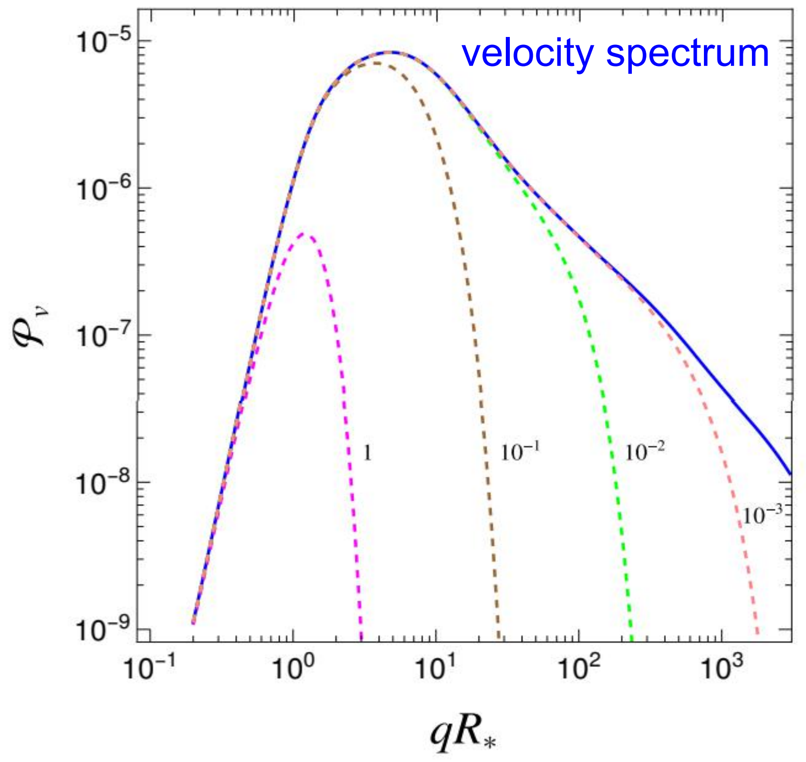
$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

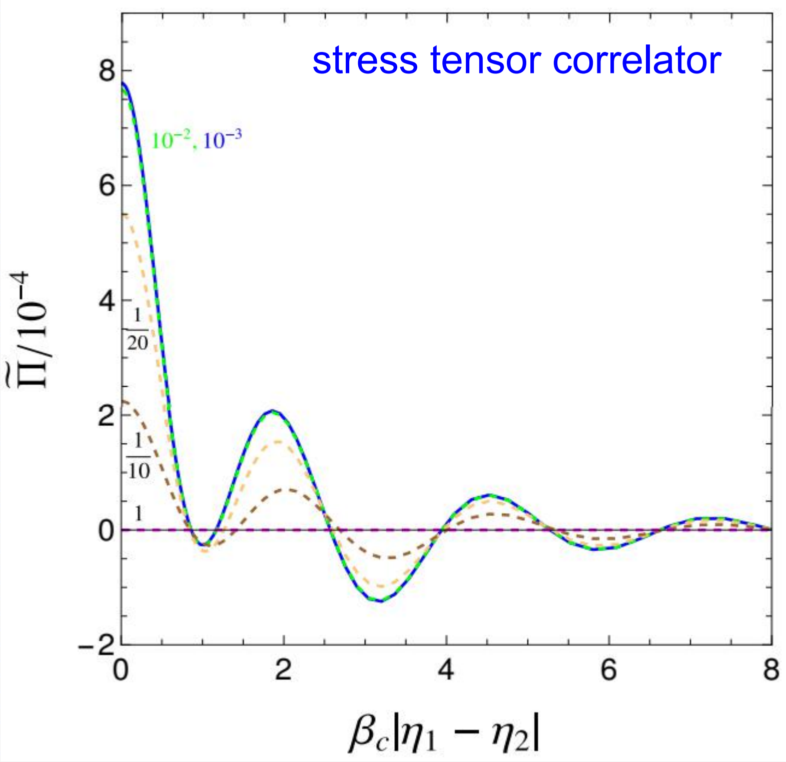
$$v^i_{\mathbf{q}}(\eta) = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}} \exp\left[-\int_{\eta_d^{(n)}}^{\eta} \Gamma d\bar{\eta}\right] \theta(\eta - \eta_d^{(n)})$$

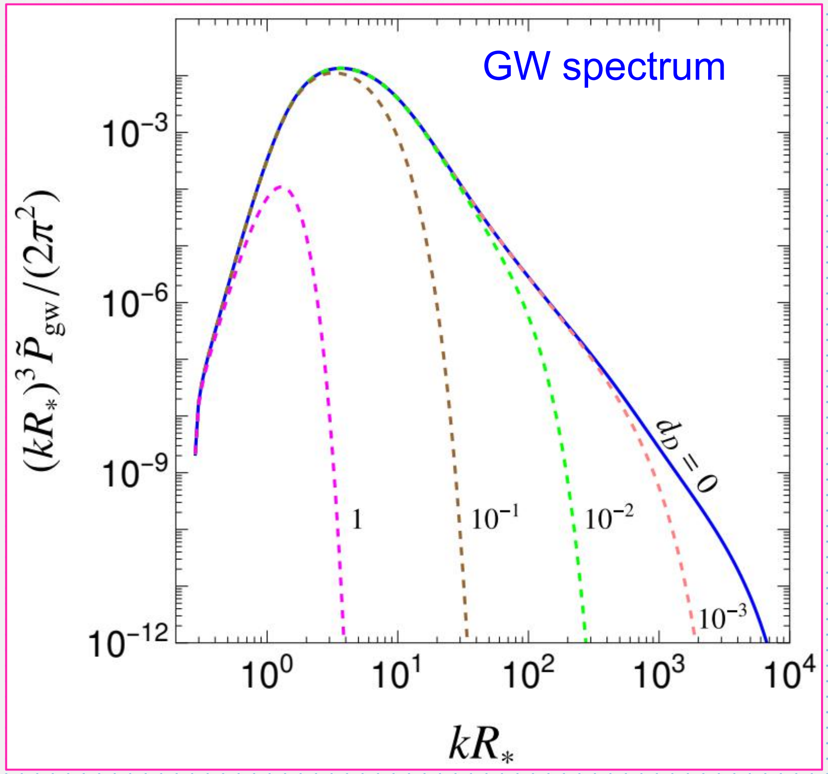
bubble destruction time
(when SW forms)

Dampings due to Dissipation

- Velocity power spectrum and stress tensor correlator are generally non-stationary
  (unequal time correlator depends not just on time difference)
- Damping at large frequencies (small scales)

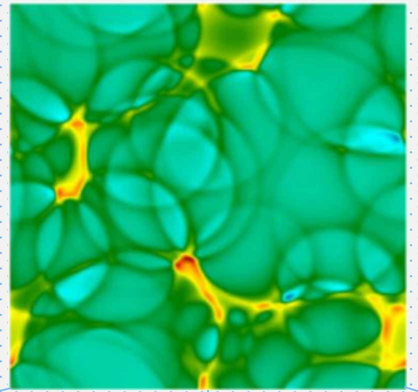






All plots assuming constant effective damping length for illustration (leads to stationary spectrum)

Lifetime of Sound Waves



expansion of the Universe
(dilution)

dissipation
(damping)

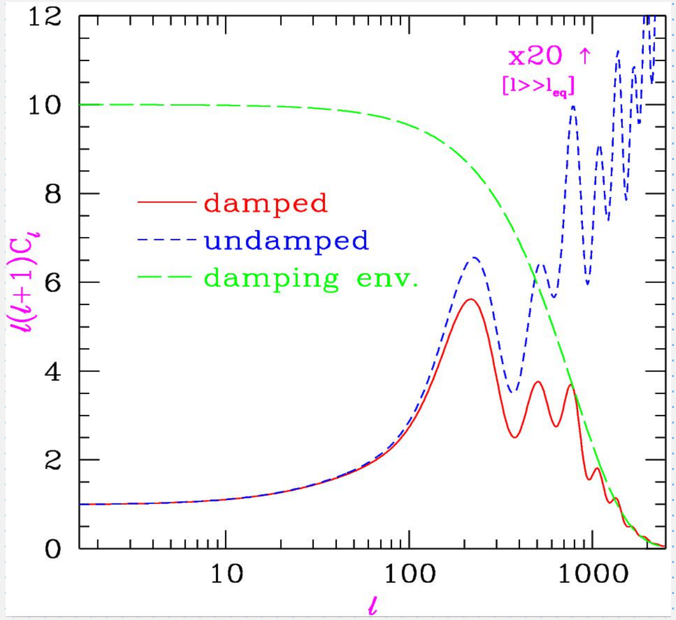
onset of MHD turbulence
(abrupt turnoff)

- Realistic cases: intertwining of these effects (makes GW spectrum model dependent)
- GW spectrum carries information about each model (break parameter degeneracy)
- Upsilon factor becomes frequency and model dependent

Microscopic Origin

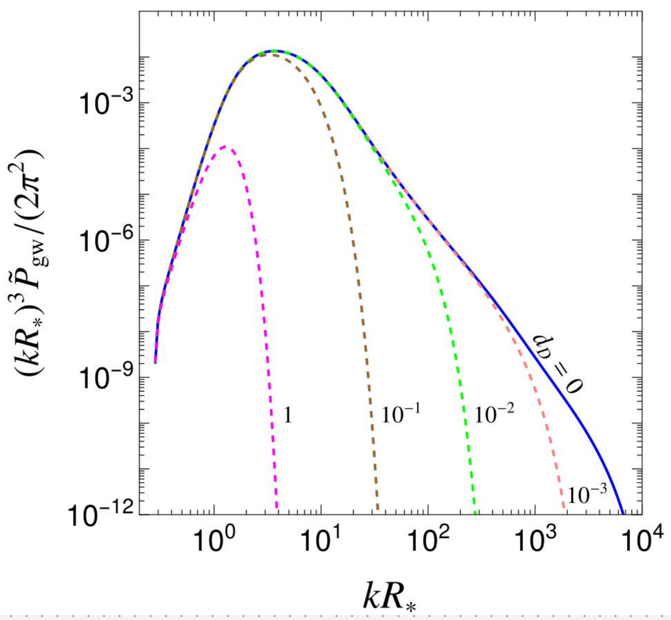
- Viscosity can be strong for phase transitions in the dark sector
- Can also be stronger when BSM physics are included
- Calculable from semi-classical kinetic theory or Green-Kubo relations

Analogy: Silk damping of CMB Anisotropy



Hu,White, ApJ [9609079]

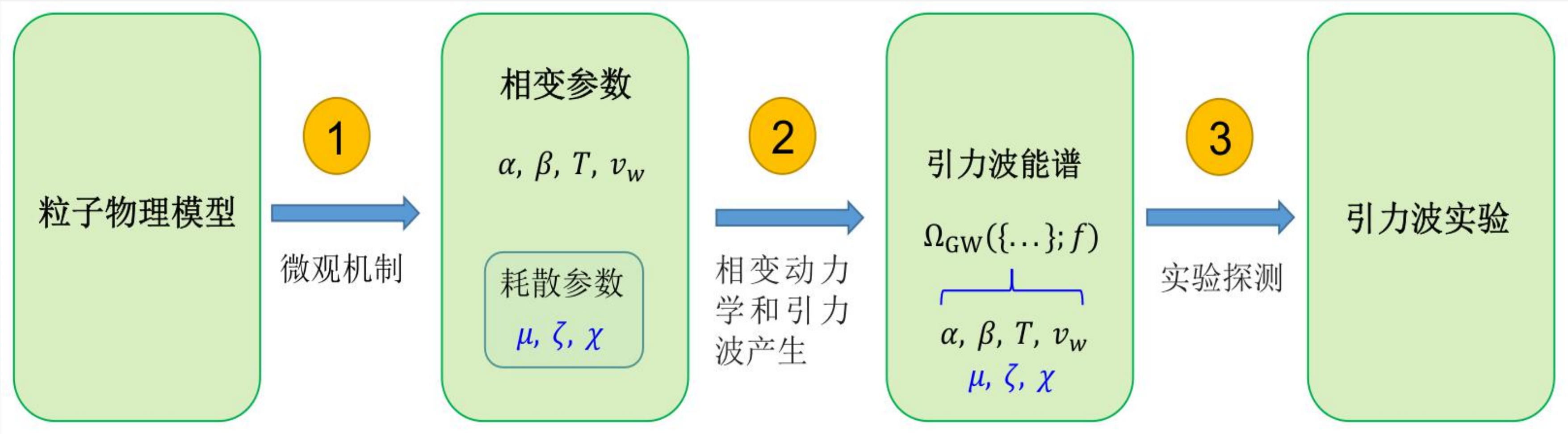
damping of GW



HG [2310.10927]

Future Directions

- Microscopic origin (classifications of BSM)
- Dynamics and spectra calculation (simulation and modelling)
- Experimental detection (LIGO, LISA/Taiji/Tianqin, PTA)



Summary

- Dissipative effects can serve as new obervables
- New portals to probe microscopic particle (very weak) interactions
- Experimental searches of new spectrum are desired

Thanks!