Axion Bose-Einstein Condensate

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Axion Dark Matter

- The axion (axion-like particle) is a good candidate of DM.

See talks by Xiaoping Wang and Yong Tang

- Different from heavy cold dark matter.

Take $m_a = 10^{-4}$ eV , de Broglie wave length ~ 2 mm

Use local density 0.4 GeV/cm^3, $n \times \lambda_{dB}^3 \sim 3 \times 10^{10}$

Huge occupation number!

- In coherence $\Rightarrow$ the axions are in BEC!

wave-like, can be considered as a classical field.

Outline

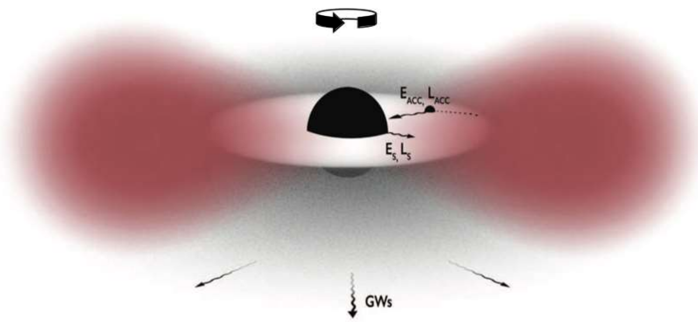
- ✧ **Dilute and Dense Axion Stars**

Properties & Radio Signals

Braaten, Mahapatra, **HZ,** *PRL* 117 (2016) 12, 121801
PRD 98 (2018) 9, 096012

Braaten, **HZ** *Rev.Mod.Phys.* 91 (2019) 4, 041002

- ✧ **Black Hole Superradiance**



Properties & GW Signals

Bao, Xu, **HZ,** *PRD* 106 (2022) 6, 064016
PRD 107 (2023) 6, 064037

Guo, Bao, **HZ** *PRD* 107 (2023) 7, 075009

- ✧ **Summary**

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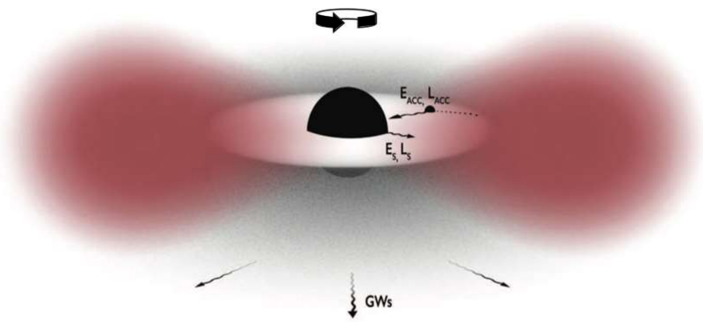

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✧ Summary

Non-relativistic EFT for Axions

- The BEC evolves with t, eventually to the lowest energy state.

  What is the lowest energy axion BEC?

- Better to use non-relativistic EFT   Non-trivial for real scalar field

  - NR: Schroedinger equation for ψ

    time evolution is in phase $\propto e^{-iEt}$, **complex** field

  - Rel: Klein-Gordon equation for **real** scalar ϕ

Non-relativistic EFT for Axions

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    time evolution is in phase $\propto e^{-iEt}$, **complex** field

  - Rel: Klein-Gordon equation for **real** scalar ϕ

  - Key pt.: NR field ψ is a mix of ϕ and its momentum π

$$\psi = e^{imt}\left[\sqrt{\frac{m}{2}}\left(1-\frac{\nabla^2}{m^2}\right)^{1/4}\phi + \frac{i}{\sqrt{2m}}\left(1-\frac{\nabla^2}{m^2}\right)^{-1/4}\pi\right]$$

Braaten, Mahapatra, **HZ** PRD 2016, PRD 2018; Namjoo, Guth PRD 2018

QCD Axion Star

- Take QCD axion as an example
- Non-relativistic effective Lagrangian

$$\mathcal{L}_{\rm eff} = \tfrac{1}{2} i \left(\psi^* \dot{\psi} - \dot{\psi}^* \psi \right) - \frac{1}{2m_a} \nabla \psi^* \cdot \nabla \psi - \mathcal{V}_{\rm eff}$$

$$\mathcal{V}_{\rm eff} = m_a \psi^* \psi - \frac{1}{16} \frac{(\psi^* \psi)^2}{f_a^2} + \frac{1}{288} \frac{(\psi^* \psi)^3}{m_a f_a^4} + \ldots$$

Attractive interaction!

Expand by $\dfrac{\psi^* \psi}{m_a f_a^2}$

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Dilute limit

Attractive interaction!

Expand by $\dfrac{\psi^* \psi}{m_a f_a^2}$

- EOM with Newtonian gravity

$$-\frac{\nabla^2}{2m_a} \psi + \left[(\mathcal{V}'_{\rm eff}(\psi^* \psi) + m_a \Phi \right] \psi = (\mu - m_a) \psi,$$

$$\nabla^2 \Phi = 4\pi G m_a \psi^* \psi.$$

Dilute QCD Axion Stars

Assume:
- Truncated potential, dilute axion limit
- Newtonian gravity
- Spherically symmetric (BEC cannot take global angular mom.)

$$-\frac{\nabla^2}{2m_a}\psi + \left[(\mathcal{V}'_{\rm eff}(\psi^*\psi) + m_a\Phi\right]\psi = (\mu - m_a)\psi,$$

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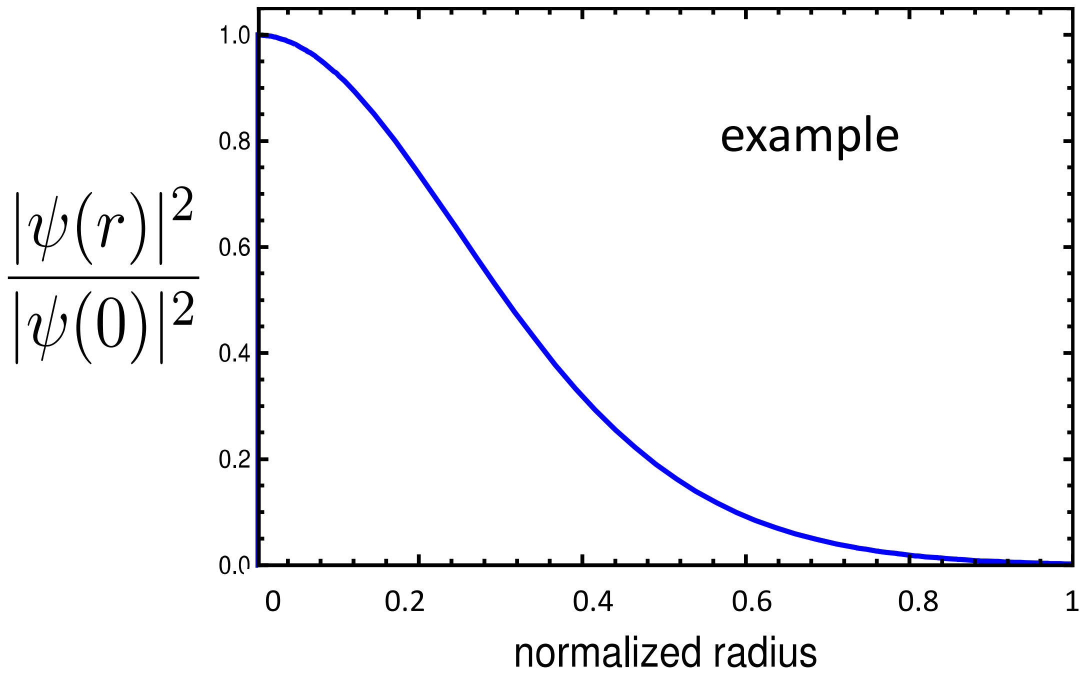
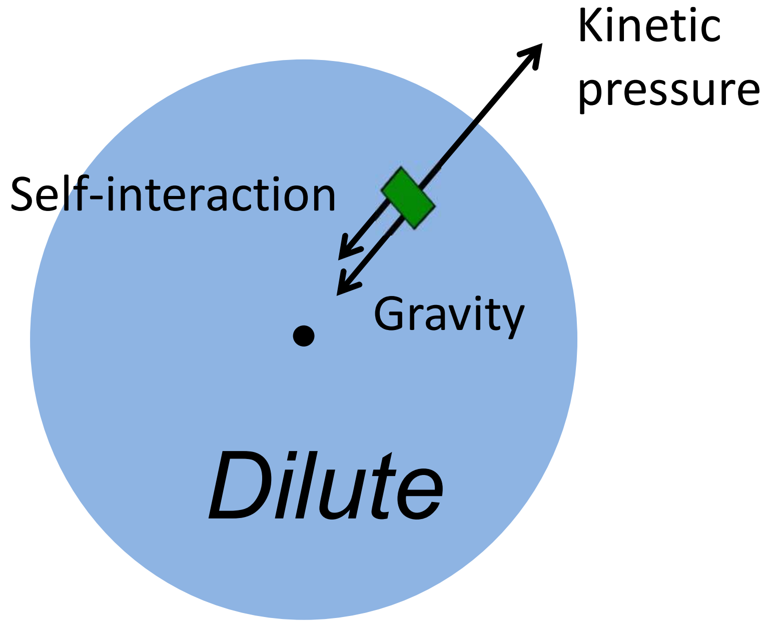
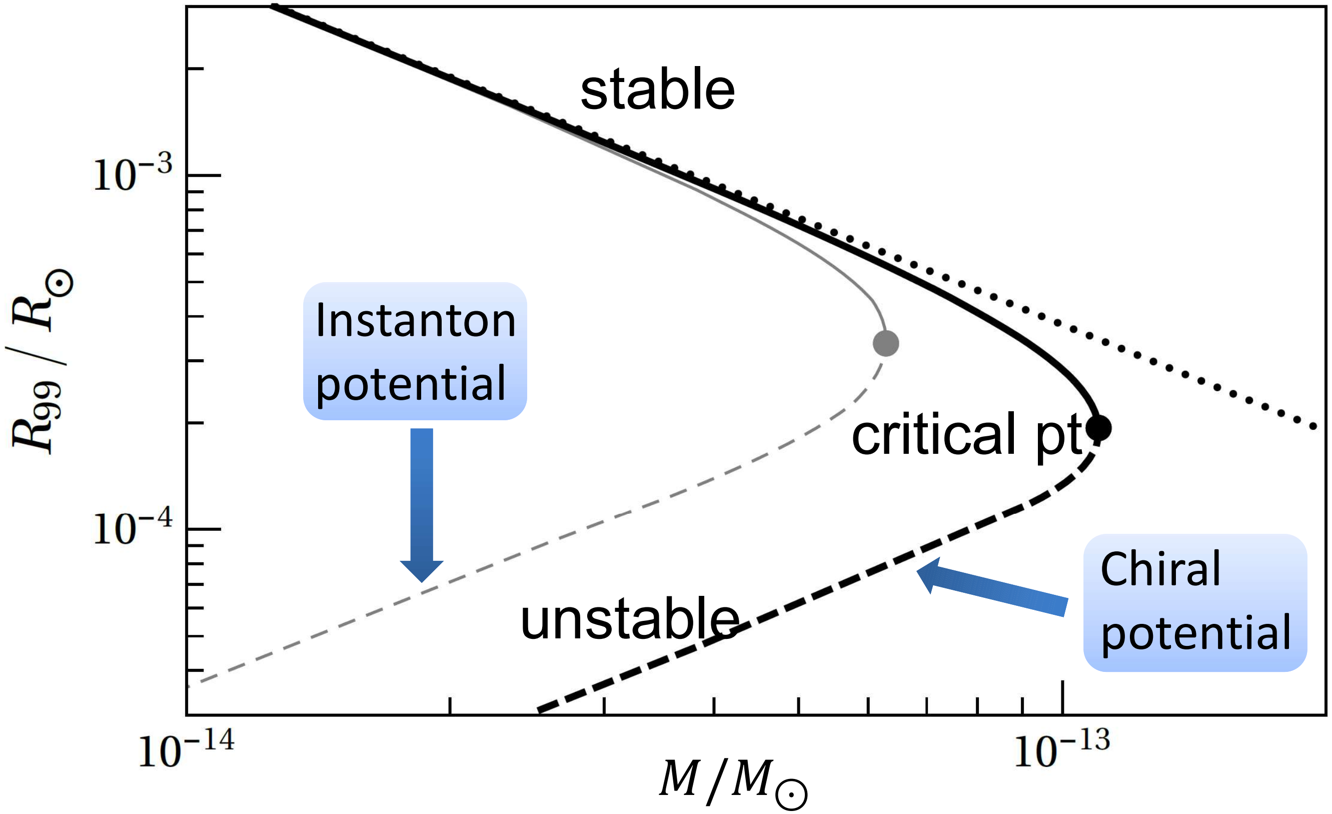




Dilute Axion Star: M vs R

- Heavier dilute axion stars have smaller radii.
- Grow by attracting surrounding axions.
- Critical mass: beyond which the kinetic pressure cannot balance the attractive self-interaction and gravity
- Collapse when heavier than the critical mass.



$$M_* \sim 10^{-13} M_\odot$$

$$R_* \sim 10^{-4} R_\odot \sim R_☽$$

Braaten, **HZ**
Rev.Mod.Phys. (2019)

Keep Potential to All Orders

$$\mathcal{L}_{\text{eff}} = \tfrac{1}{2} i \left(\psi^* \dot{\psi} - \dot{\psi}^* \psi \right) - \frac{1}{2m_a} \nabla \psi^* \cdot \nabla \psi - \mathcal{V}_{\text{eff}}$$

- Dilute axion field

$$\mathcal{V}_{\text{eff}} = m_a \psi^* \psi - \frac{1}{16} \frac{(\psi^* \psi)^2}{f_a^2} + \boxed{\frac{1}{288} \frac{(\psi^* \psi)^3}{m_a f_a^4} + \dots}$$

Dilute limit

- In dense axion field $(\psi^* \psi) \sim m_a f_a^2$, must keep all orders

Both instanton and chiral potential can be summed to all orders

e.g. Instanton potential:

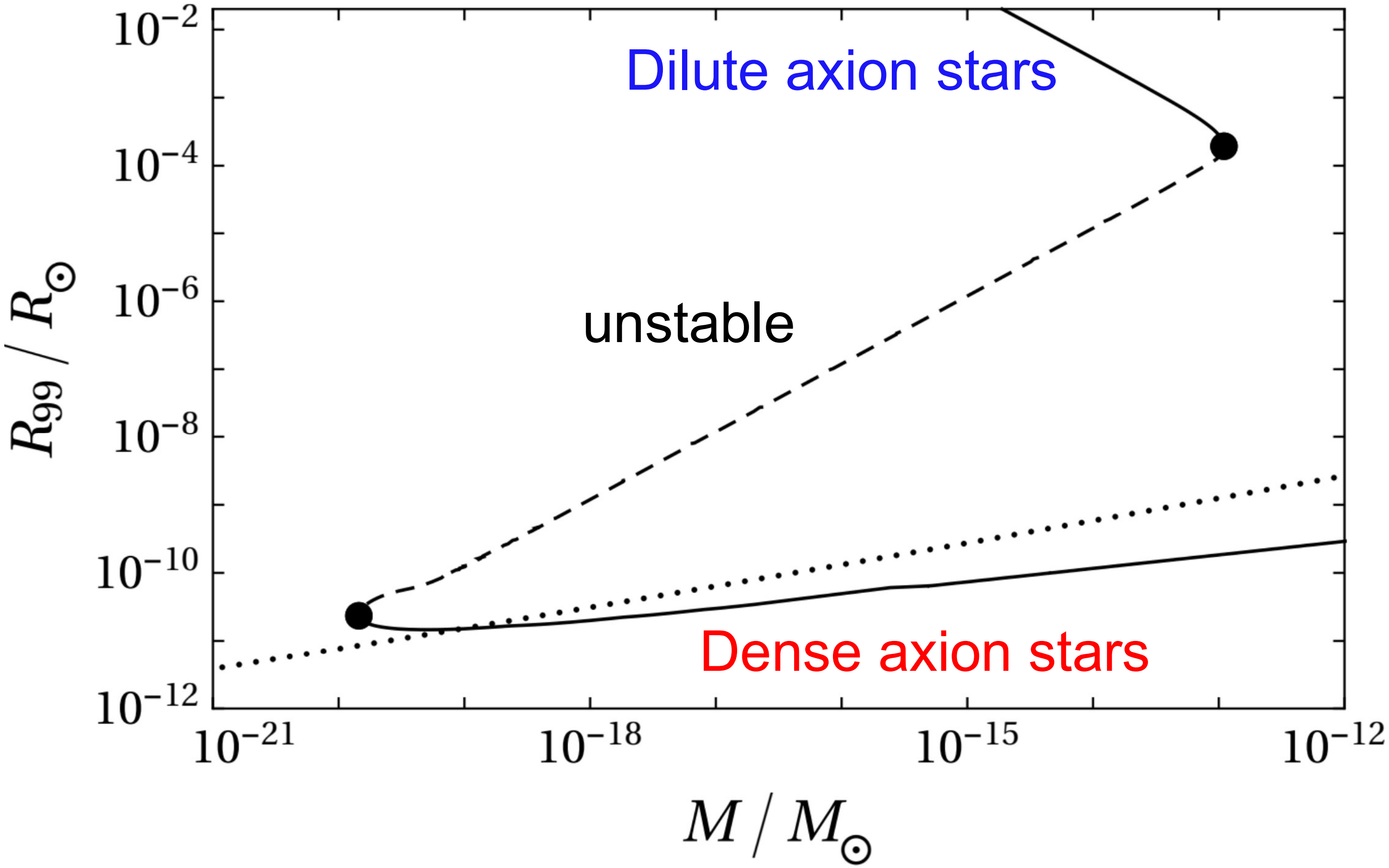
$$\mathcal{V}_{\text{eff}}(\psi^* \psi) = \tfrac{1}{2} m_a \psi^* \psi + m_a^2 f_a^2 \left[1 - J_0(2\psi^* \psi / m_a f_a^2) \right]$$

Braaten, Mahapatra, **HZ**, PRD (2016), PRD (2018)

Dense Branch

With complete potential, a **new dense branch** is found.

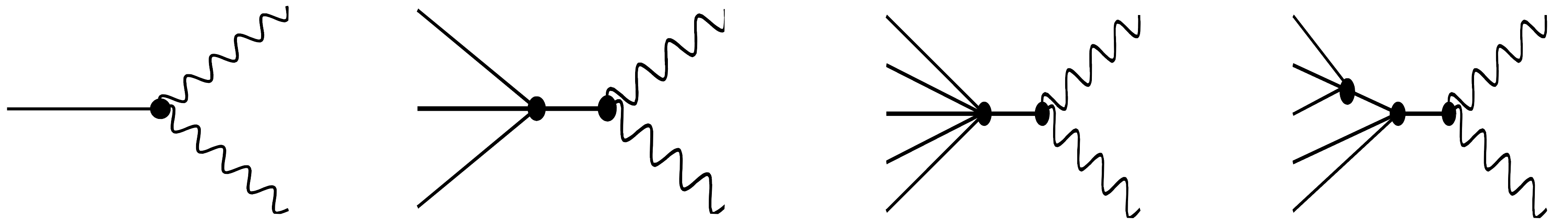
Assume: • NREFT • Newtonian gravity • Isotropic



Braaten, Mahapatra, **HZ,** PRL (2016),
Braaten, **HZ,** Rev.Mod.Phys. (2019)

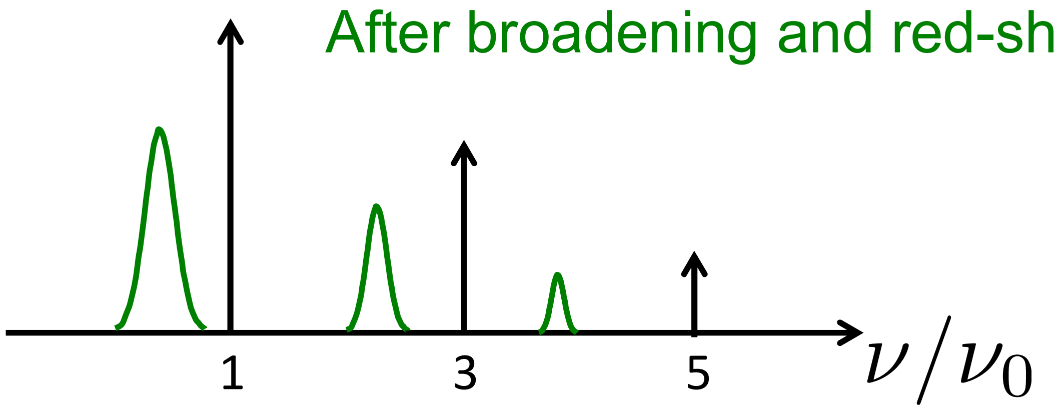
Odd-integer Harmonics

- If assume axion-photon coupling



$$\nu_0 = \frac{m_a}{2} \qquad \nu = 3\frac{m_a}{2} \qquad \nu = 5\frac{m_a}{2} \qquad \ldots$$

- Odd-integer harmonics of the fundamental radio frequency.



Unique feature of axions !!

Braaten, Mohapatra, **HZ**, PRD (2016)

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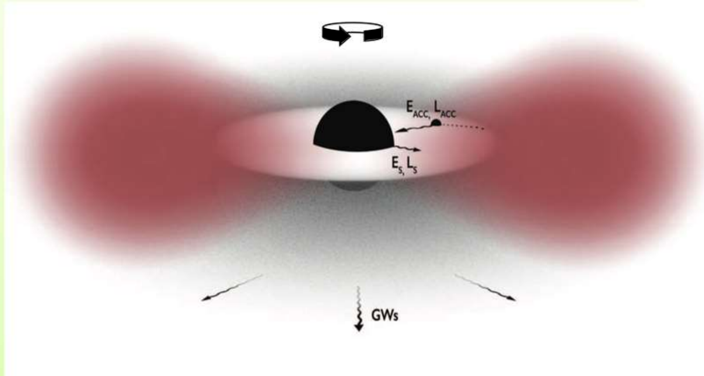

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✧ Summary

Massive Scalar in Kerr Metric

- **BEC can take global OAM if spacetime is rotating**

  Free scalar field with mass μ: $(\nabla^{\nu}\nabla_{\nu} + \mu^2)\Phi = 0$

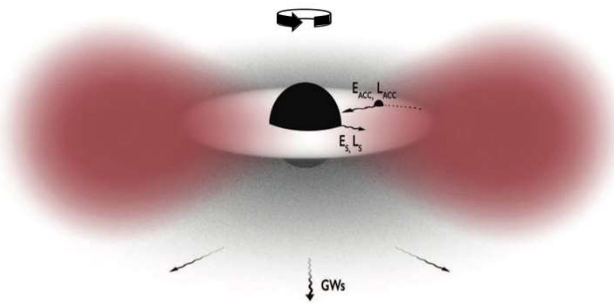
- The eigen-energy is **complex** $\omega_{n\ell m} = \omega_{n\ell m}^{(R)} + i\omega_{n\ell m}^{(I)}$

  Three indices: $(n \geq 0, l, m)$, similar to hydrogen atom

- The BEC mass and angular momentum rise exponentially

$$M_{BEC} \propto \exp(\omega_{nlm}^{(I)}\, t)$$

superradiance rate



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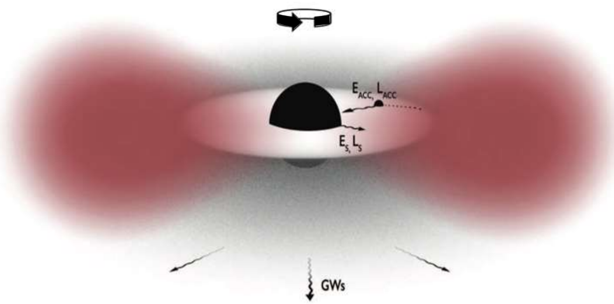
$$M_{BEC} \propto \exp(\omega_{nlm}^{(I)}\, t)$$ superradiance rate

➢ Do not depend on couplings between the axion and SM

➢ The rotating BEC has optical effects and emits gravitational waves



Superradiance Rate

$$M_{BEC} \propto \exp(\omega_{nlm}^{(I)}\, t)$$

- Analytic approximation at LO of $\alpha = M\mu$

M: BH mass
μ: axion mass

KLEIN-GORDON EQUATION AND ROTATING BLACK HOLES #3

Steven L. Detweiler (Yale U.) (1980)

Published in: *Phys.Rev.D* 22 (1980) 2323-2326

DOI cite claim reference search 540 citations

- Complicated numerical solution

Instability of the massive Klein-Gordon field on the Kerr spacetime #4

Sam R. Dolan (University Coll., Dublin) (May, 2007)

Published in: *Phys.Rev.D* 76 (2007) 084001 • e-Print: 0705.2880 [gr-qc]

pdf DOI cite claim reference search 425 citations

NLO Calculation

- We confirm a mistake in the LO calculation of $\omega_{nlm}^{(I)}$, and calculate the NLO contribution for the first time.

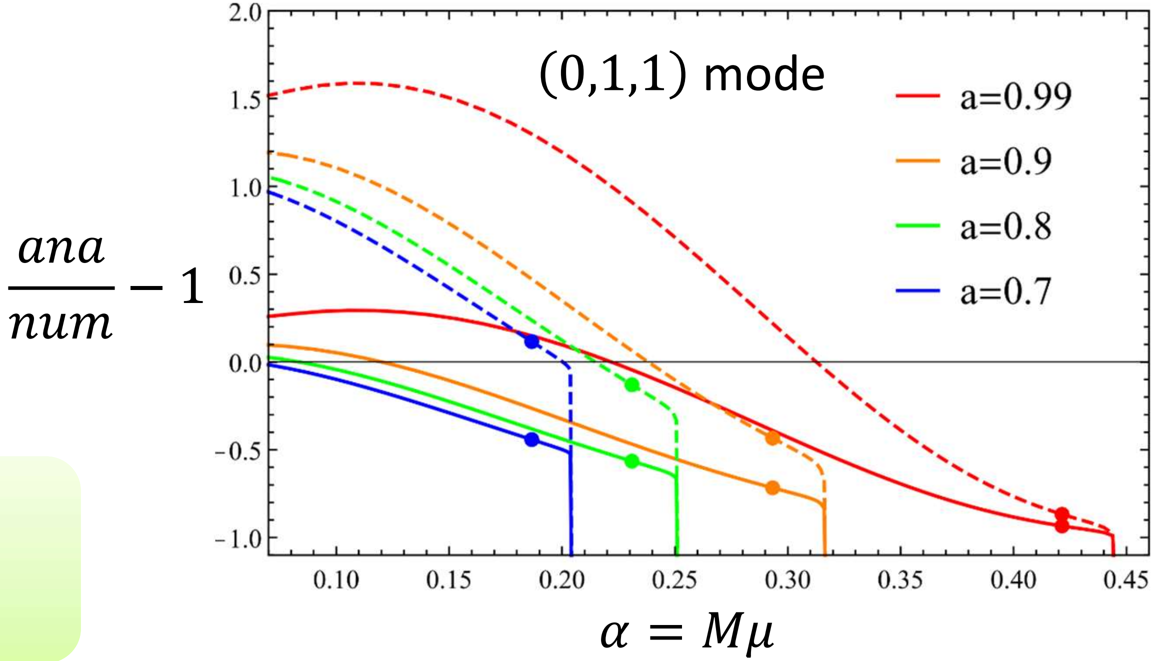
- Leading order

  At small α with $a = 0.99$,

  Err. of original result $\sim$ 150%

  Err. of corrected result $\sim$ 30%

Analytic and numerical results do not converge at $\alpha \to 0$!



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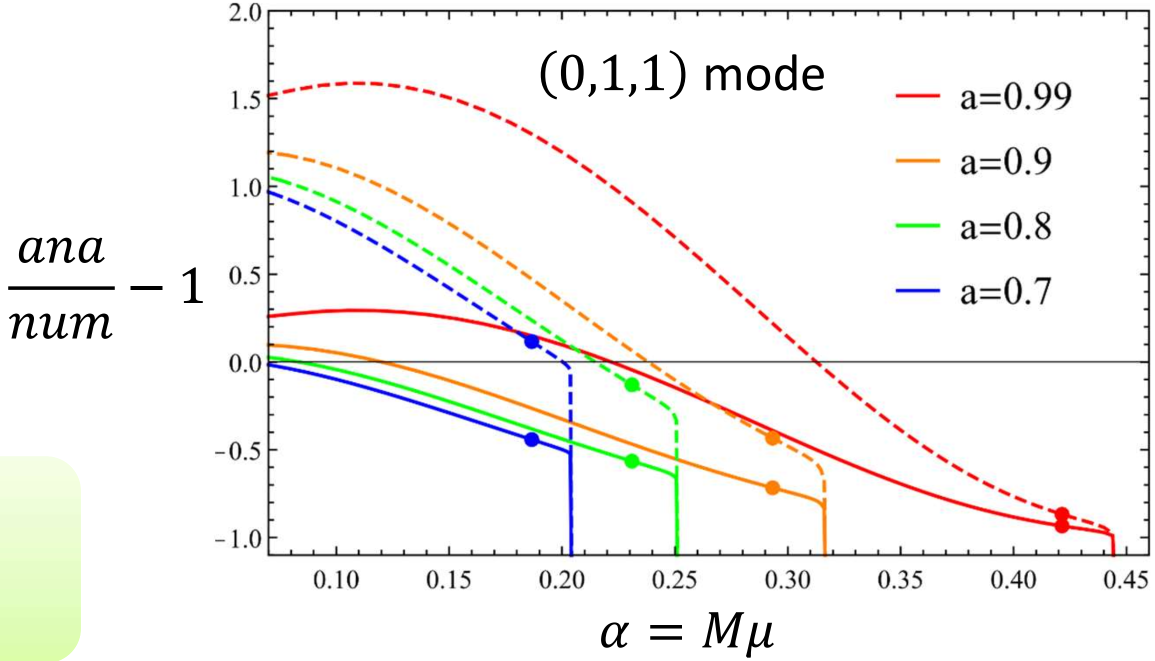
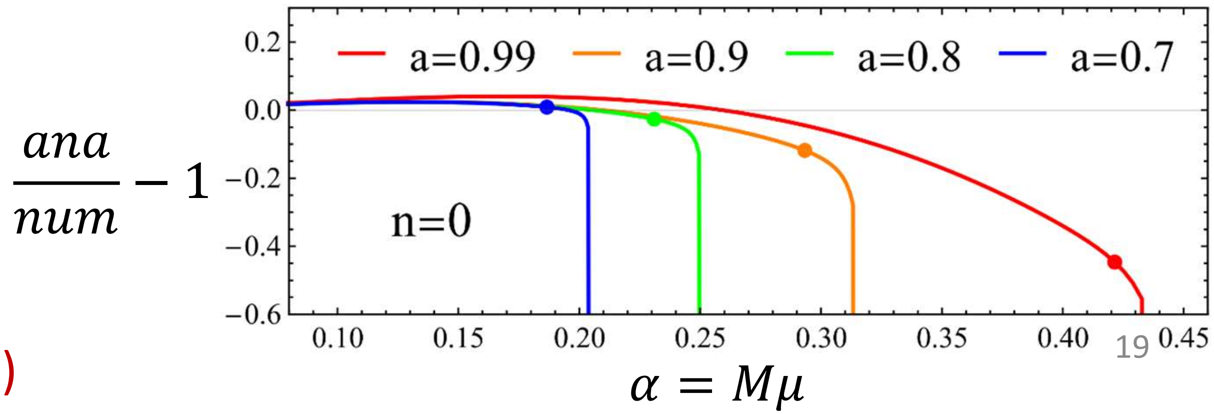


- Next-to-leading order

  At small α, **err.** $<$ **5%**

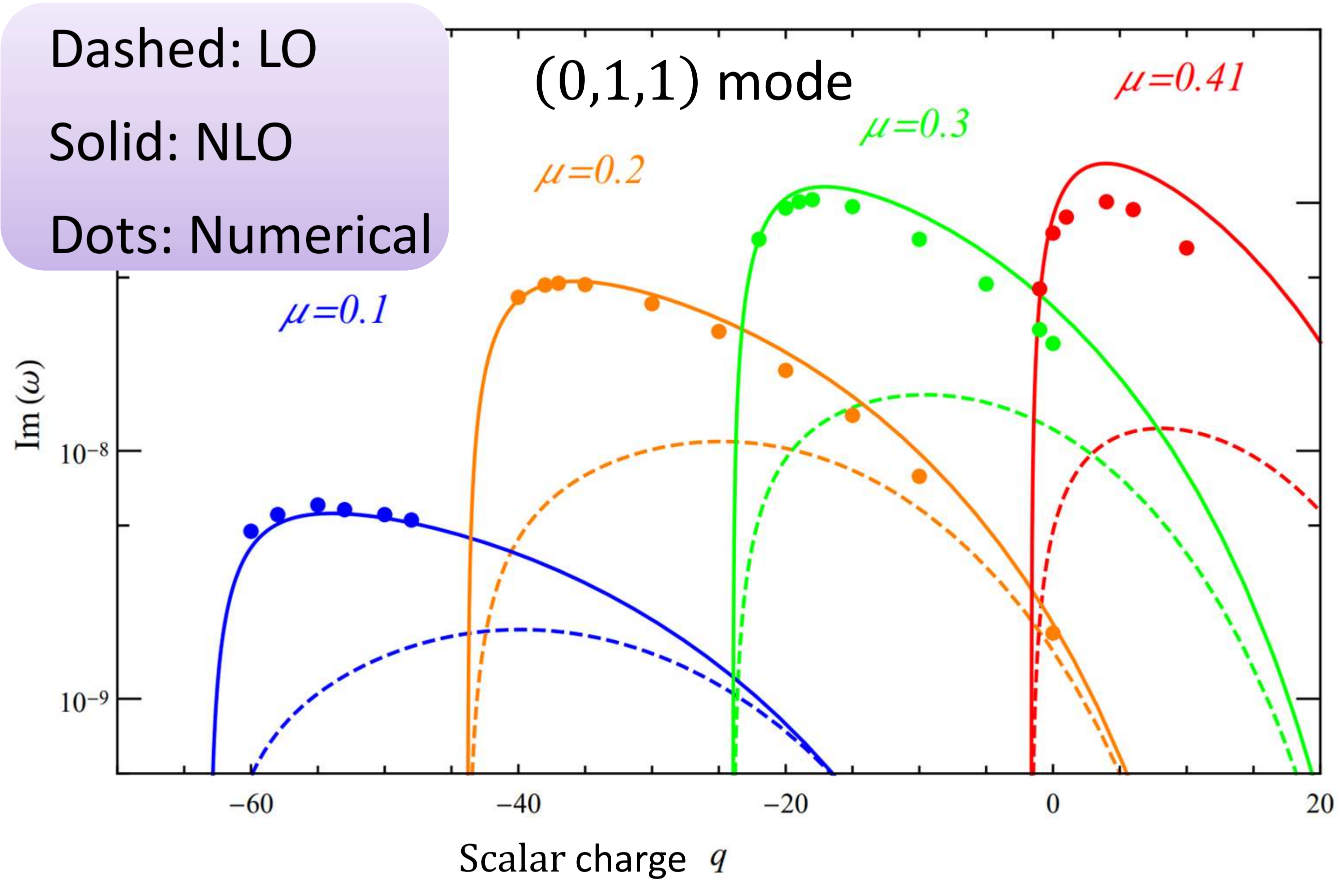


Bao, Xu and **HZ,** PRD (2022)

NLO ω_{nlm} of Kerr-Newman BH

- NLO solution greatly improves the precision

  BH mass is normalized to 1, BH charge $Q = 0.02$

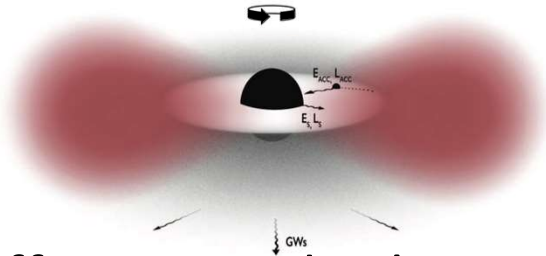


Bao, Xu and **HZ,** PRD (2023)

Numerical results from H. Furuhashi and Y. Nambu, Prog. Theor. Phys. (2004)

GW Emission

- Previous calculation only consider the $(n = 0, l = 1, m = 1)$ mode



Monochromatic, constant energy flux,
Cannot distinguish from neutron stars

- Different modes have slightly different angular speeds

$$\phi(t,\vec{r}) = \sum_{nlm} \int d\omega \left[e^{i(m\varphi - \omega t)} R_{nlm}(r) S_{lm}(\theta) + \text{c.c.} \right]$$

$$\omega_R^{nlm} \approx \mu \left[1 - \frac{\alpha^2}{2(n+l+1)^2} \right] + O(\alpha^4)$$

e.g.

$$\cos[(\omega + \Delta\omega)t] + \cos[(\omega - \Delta\omega)t] = 2\boxed{\cos(\Delta\omega\, t)}\cos(\omega t)$$

Modulation of amp. and energy flux. **Beat!**

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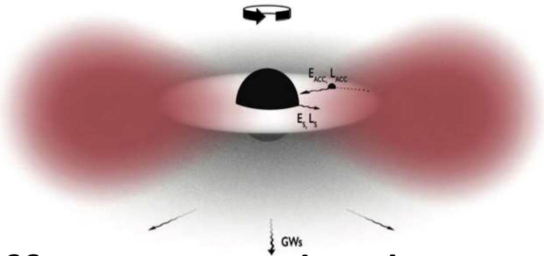

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e.g $\quad \cos[(\omega + \Delta\omega)t] + \cos[(\omega - \Delta\omega)t] = 2\cos(\Delta\omega\, t)\cos(\omega t)$

- Strength of the beat signal.

Two (0,1,1) axions $\Longrightarrow$ graviton: Amp.$\propto N_{011}$, freq. $= 2\omega^{011}$

(0,1,1) + (1,1,1) $\Longrightarrow$ graviton: Amp.$\propto \sqrt{N_{011} N_{111}}$, freq. $= \omega^{011} + \omega^{111}$

Energy flux $\propto Amp^2$, so beat Amp. $\propto \sqrt{\frac{N_{111}}{N_{011}}}$, with freq. $\omega^{111} - \omega^{011}$

Guo, Bao and **HZ,** PRD (2023)

GW Emission

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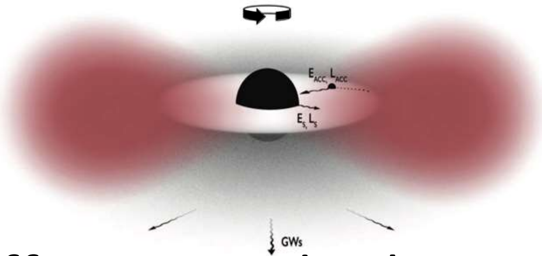

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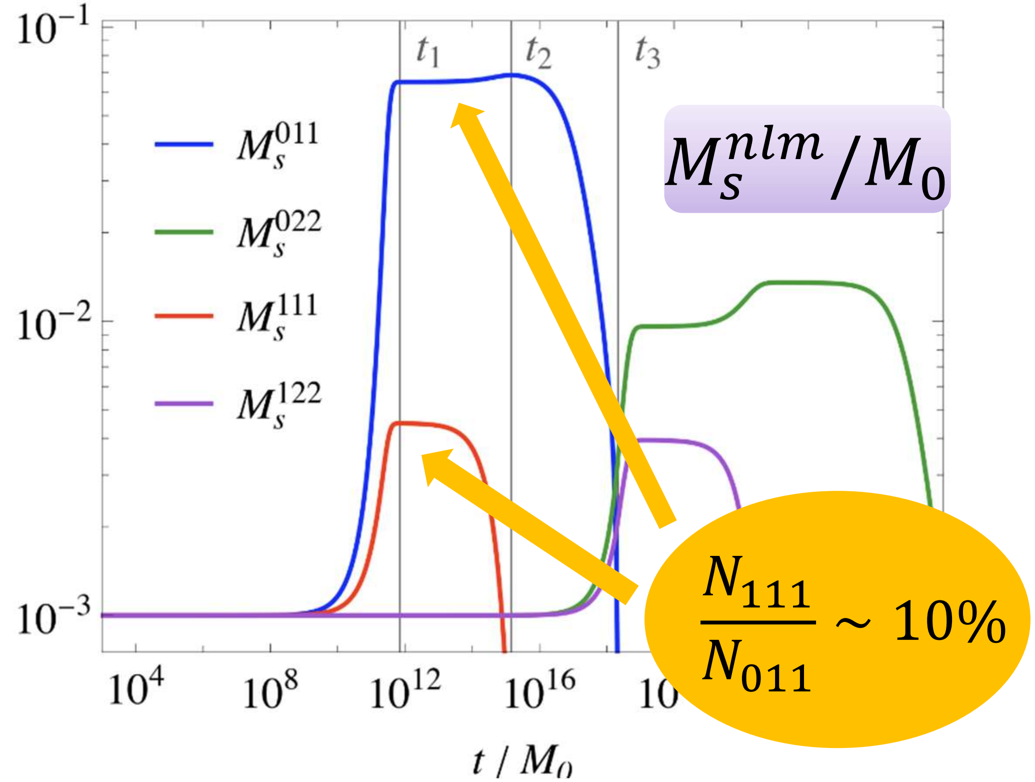
e.g $\cos[(\omega+\Delta\omega)t] + \cos[($



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Guo, Bao and **HZ,** PRD (2023)

GW Emission

- Use Teukolsky formalism to calculate the beat signal

LO

NLO

$\sqrt{N_{111}/N_{011}}$

suppressed

$$
\begin{aligned}
\frac{dE_{\rm GW}}{dt} = \frac{1}{8\pi}\sum_{\tilde{l}}\Bigg\{ & \frac{N_{011}^2}{{\omega^{(011)}}^2}\frac{\left|U_{l2}^{(\tilde{\omega}_1)}\right|^2}{\tilde{\omega}_1^2} + \frac{N_{111}^2}{{\omega^{(111)}}^2}\frac{\left|U_{l2}^{(\tilde{\omega}_2)}\right|^2}{\tilde{\omega}_2^2} + 4\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{l2}^{\tilde{\omega}_3}\right|^2}{\tilde{\omega}_3^2} \\
& + 4\sqrt{\frac{N_{011}^3 N_{111}}{{\omega^{(011)}}^3\omega^{(111)}}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_1)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_3)}\right|}{\tilde{\omega}_1\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde{\omega}_3)} + \phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right] \\
& + 2\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_1)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_2)}\right|}{\tilde{\omega}_1\tilde{\omega}_2}\cdot\cos\left[2\tilde{\omega}_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde{\omega}_2)} + \phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right] \\
& + 4\sqrt{\frac{N_{011}N_{111}^3}{\omega^{(011)}{\omega^{(111)}}^3}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_2)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_3)}\right|}{\tilde{\omega}_2\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde{\omega}_2)} + \phi_{\tilde{l}2}^{(\tilde{\omega}_3)}\right]\Bigg\}.
\end{aligned}
$$

$$
\tilde{\omega}_1 \equiv 2\omega^{(011)},\ \tilde{\omega}_2 \equiv 2\omega^{(111)},
$$
$$
\tilde{\omega}_3 \equiv \omega^{(011)} + \omega^{(111)},\ \tilde{\omega}_4 \equiv \omega^{(111)} - \omega^{(011)}
$$

Guo, Bao and **HZ,** PRD (2023)

GW Emission

- Use Teukolsky formalism to calculate the beat signal

LO

$$\frac{dE_{\rm GW}}{dt} = \frac{1}{8\pi}\sum_{\tilde{l}}\left\{ \boxed{\frac{N_{011}^2}{\omega^{(011)\,2}}\frac{\left|U_{l2}^{(\tilde{\omega}_1)}\right|^2}{\tilde{\omega}_1^2}} + \frac{N_{111}^2}{\omega^{(111)\,2}}\frac{\left|U_{l2}^{(\tilde{\omega}_2)}\right|^2}{\tilde{\omega}_2^2} + 4\frac{N_{011}N_{111}}{\omega^{(011)}\omega^{(111)}}\frac{\left|U_{l2}^{\tilde{\omega}_3}\right|^2}{\tilde{\omega}_3^2} \right.$$

NLO

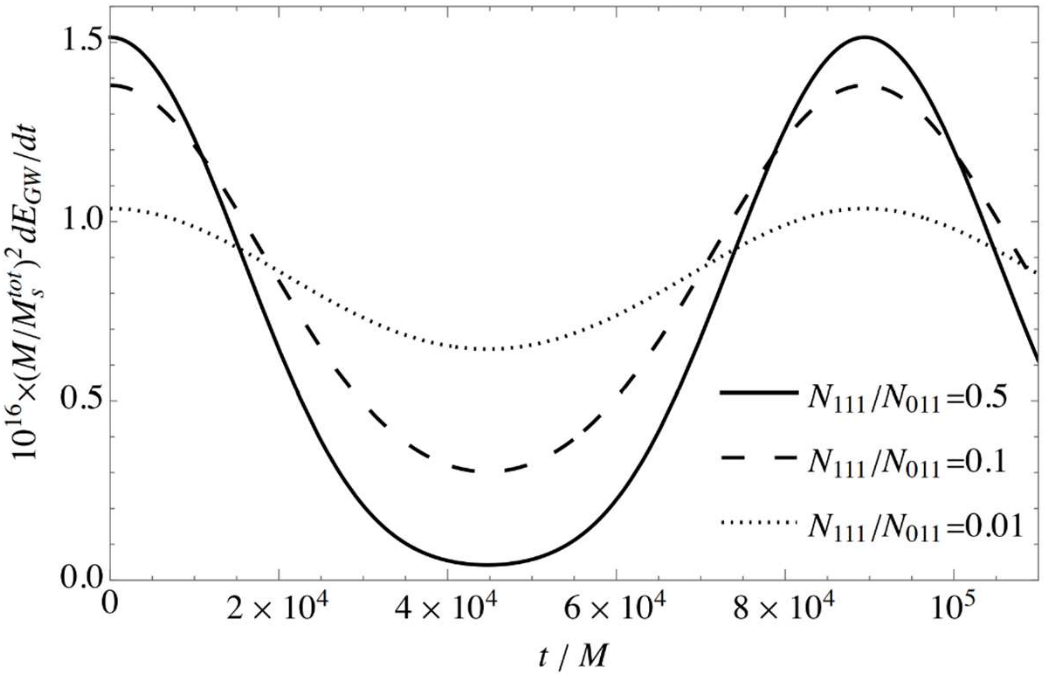
$\sqrt{N_{111}/N_{011}}$

suppressed

$$\boxed{+4\sqrt{\frac{N_{011}^3 N_{111}}{\omega^{(011)\,3}\omega^{(111)}}}\frac{\left|U_{\tilde{l}2}^{(\tilde{\omega}_1)}\right|\left|U_{\tilde{l}2}^{(\tilde{\omega}_3)}\right|}{\tilde{\omega}_1\tilde{\omega}_3}\cdot\cos\left[\tilde{\omega}_4(t-r_*) - \phi_{\tilde{l}2}^{(\tilde{\omega}_3)} + \phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\right]}$$

$+2$ … $\phi_{\tilde{l}2}^{(\tilde{\omega}_1)}\Big]$

$+4$ … $+\phi_{\tilde{l}2}^{(\tilde{\omega}_3)}\Big]\Big\}.$



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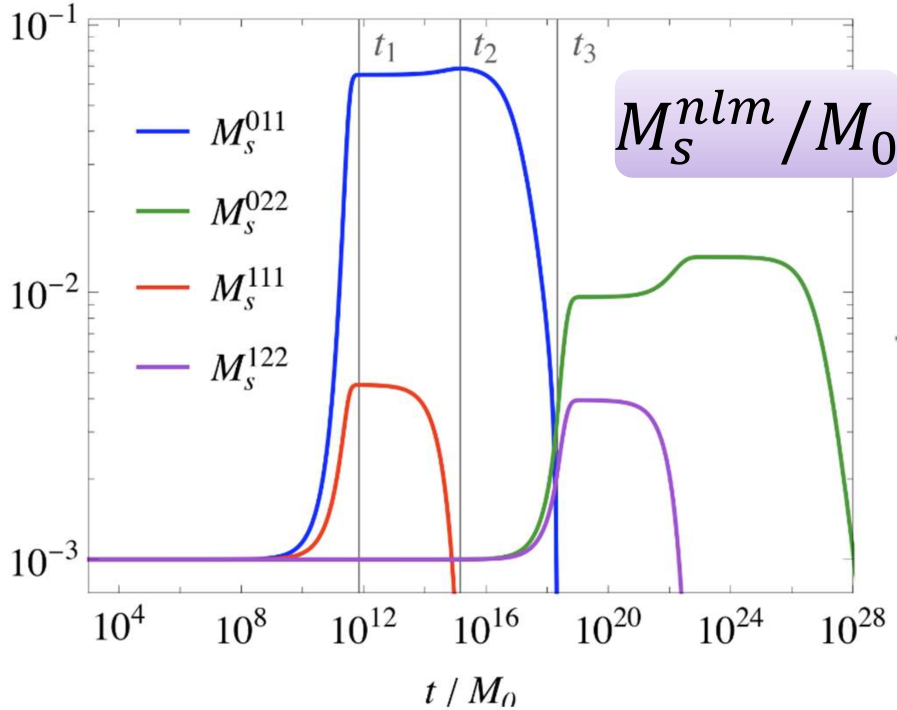
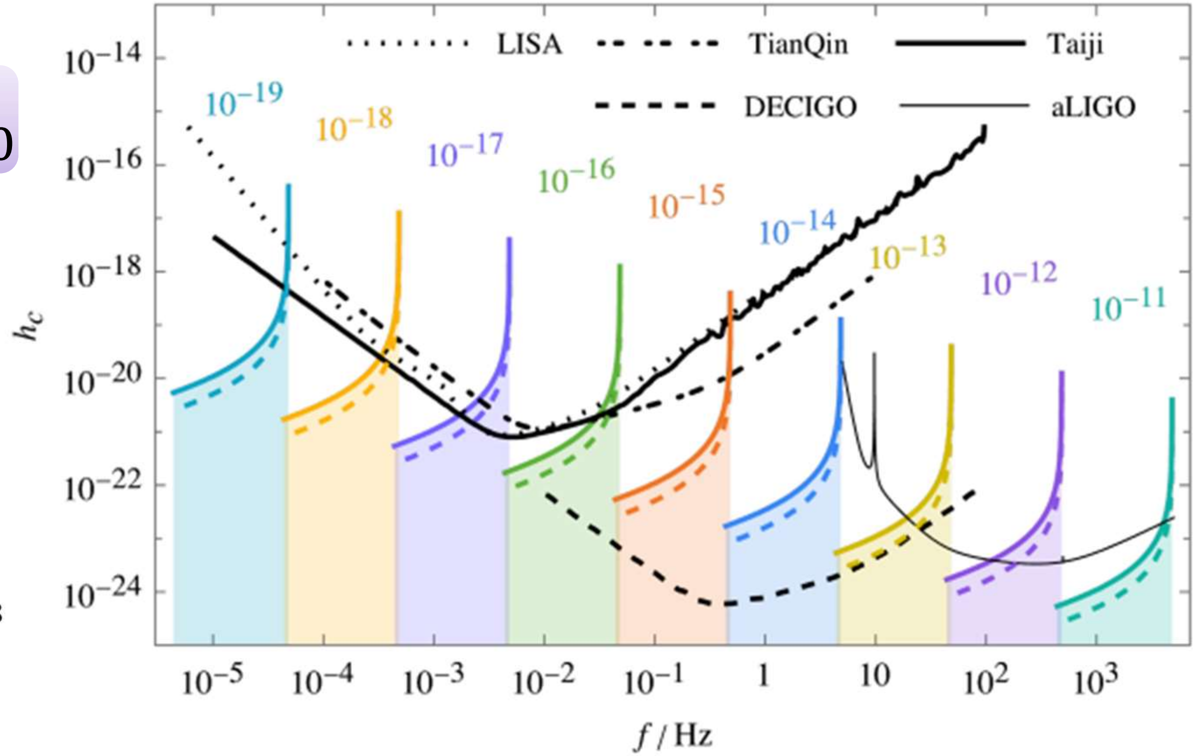
'RD (2023)

GW Beat: Observation



M_s^{nlm}/M_0



The BH spin here is determined by $M\mu$

- Parameters: $M\mu = 0.17$ (so $a_C = 0.6$), $M_s/M = 0.1$, $N_{111}/N_{011} = 0.1$
- The red shift ranges from 0.001 to 10
- The current and future GW telescope can cover a large range of axion mass.

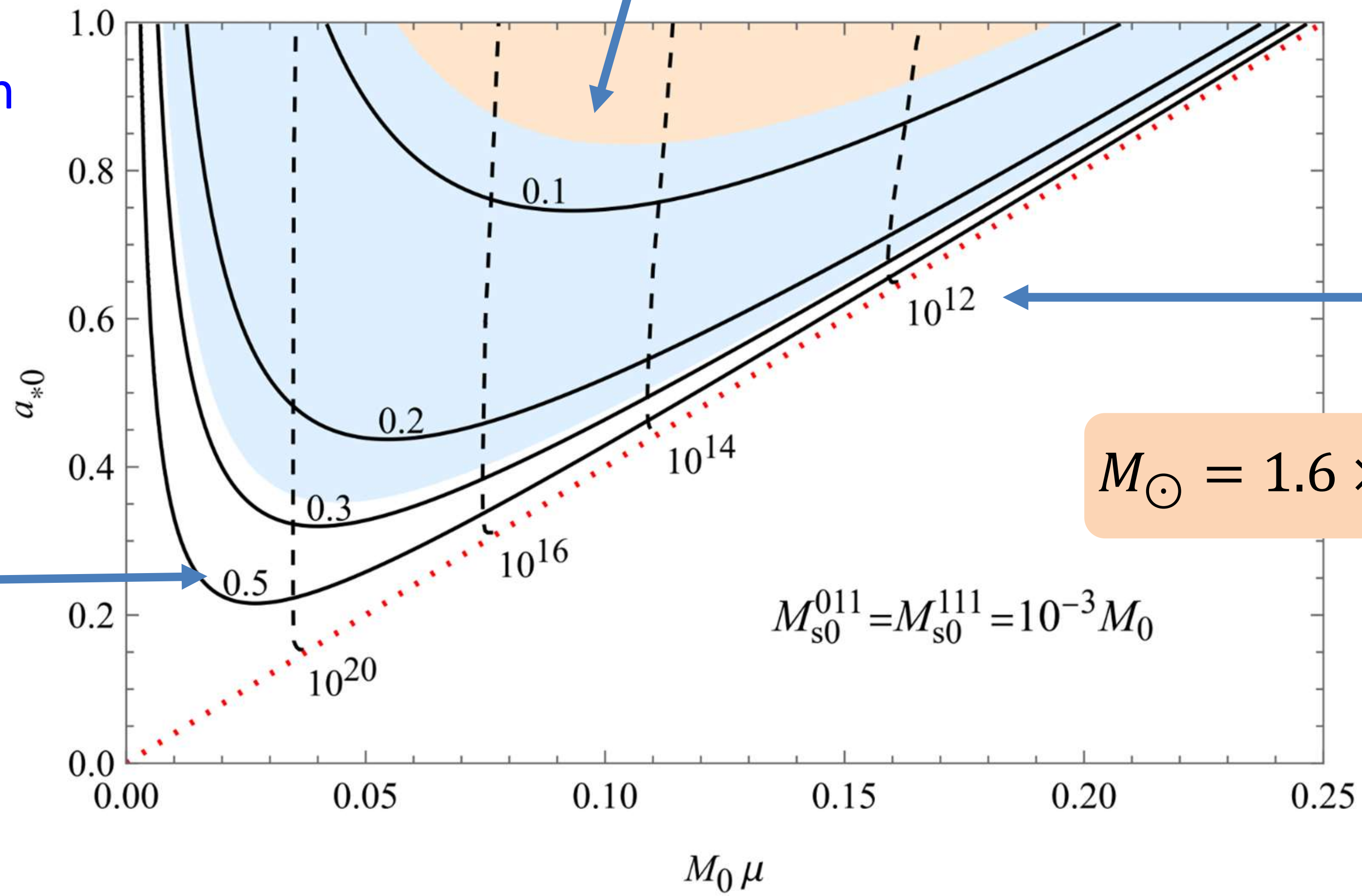
Guo, Bao and **HZ,** PRD (2023)

GW Beat: Observation

M_s/M_0 = 0.05 (orange) and 0.01 (blue)

No accretion

$\frac{N_{111}}{N_{011}}$



$\frac{\Delta t}{M_0}$

$M_\odot = 1.6 \times 10^{-13}$yr

- **Three factors** to consider for observation
  total signal strength, beat signal strength, beat duration.

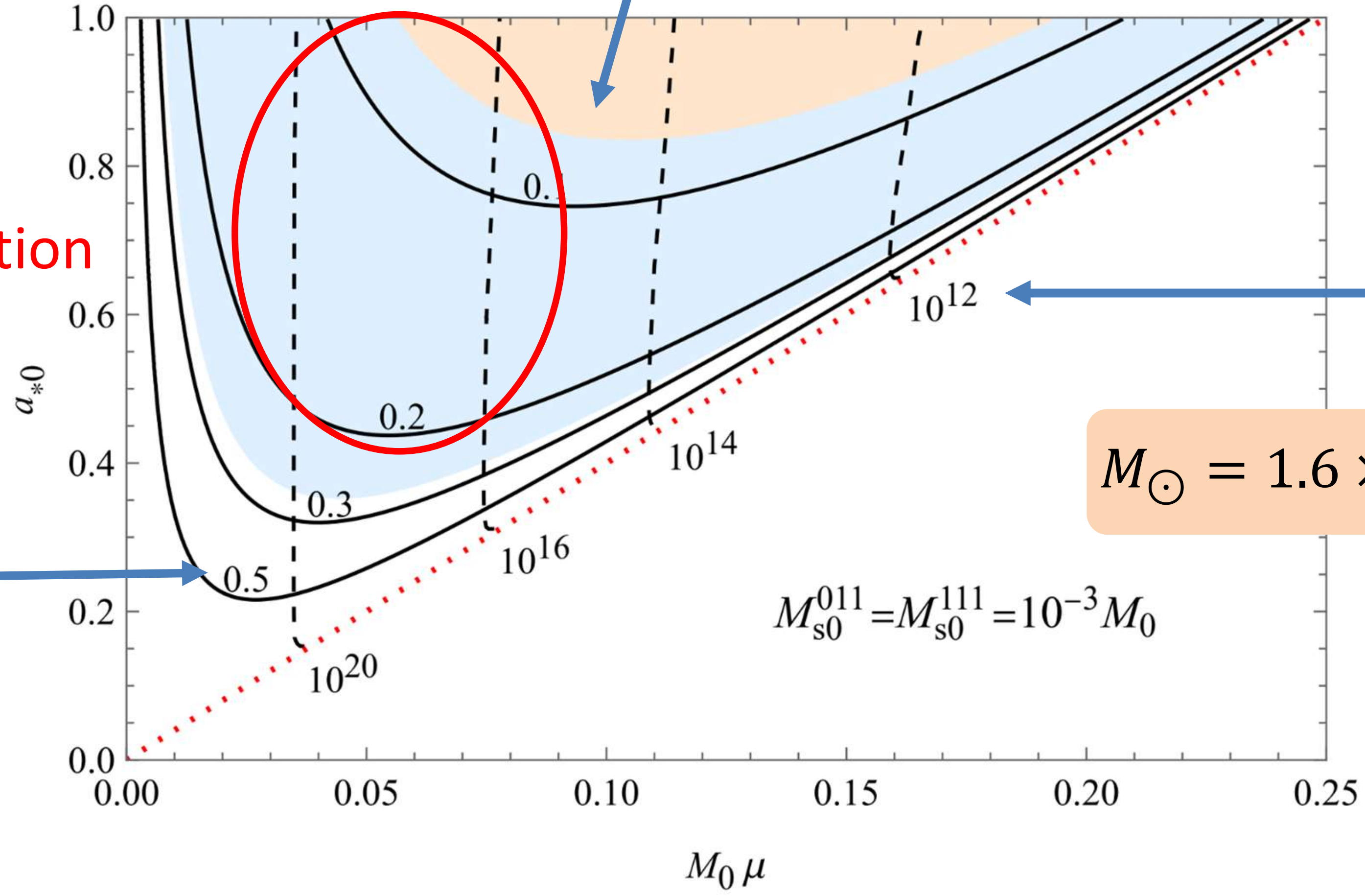
Guo, Bao and **HZ,** PRD (2023)

GW Beat: Observation

M_s/M_0 = 0.05 (orange) and 0.01 (blue)

Best region
for observation

$\frac{\Delta t}{M_0}$



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$\frac{N_{111}}{N_{011}}$

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Guo, Bao and **HZ,** PRD (2023)

Summary

- Dark matter axions would exist in BEC state in the universe, in forms of axion stars and superradiant axion clouds.
- We improve the widely-used analytic expression for superradiance rate, reducing error from 150% to $\leq 5\%$
- The photons in odd-integer harmonics of a fundamental frequency are a unique signature of dense axion BEC.
- The gravitational wave emitted by axion BEC around rotating BHs have unique “beat” signal.

Thank you!

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