中国科学院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

Primordial Black Holes and Non-Gaussianity in the Gravitational-Wave Era

Sai Wang

Institute of High Energy Physics, Chinese Academy of Sciences

- ✓ Phys.Rev.Res. 6 (2024) 1, L012060: **SW**, Zhi-Chao Zhao, Jun-Peng Li, Qing-Hua Zhu
- ✓ JCAP (2024), 2309.07792: Jun-Peng Li, **SW**, Zhi-Chao Zhao, Kazunori Kohri
- ✓ JCAP (2024), 2403.00238: Jun-Peng Li, **SW**, Zhi-Chao Zhao, Kazunori Kohri
- ✓ JCAP 10 (2023) 056: Jun-Peng Li, **SW**, Zhi-Chao Zhao, Kazunori Kohri
- ✓ Phys.Rev.D 106 (2022) 12, 123511: **SW**, Valeri Vardanyan, Kazunori Kohri
- ✓ Phys.Rev.D 99 (2019) 10, 103531: **SW**, Takahiro Terada, Kazunori Kohri
- ✓ Phys.Rev.Lett. 120 (2018) 19, 191102: **SW**, Yi-Fan Wang, Qing-Guo Huang, Tjonnie Li

Outline

※ **Introduction**

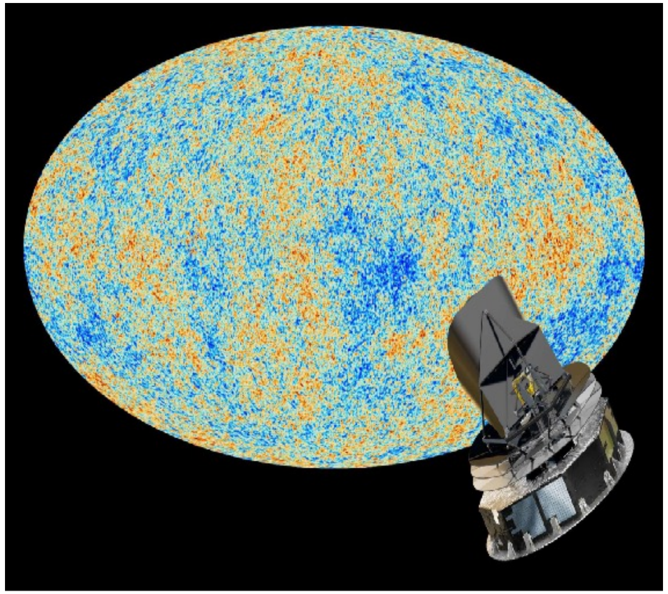
※ **SIGW background due to PNG**

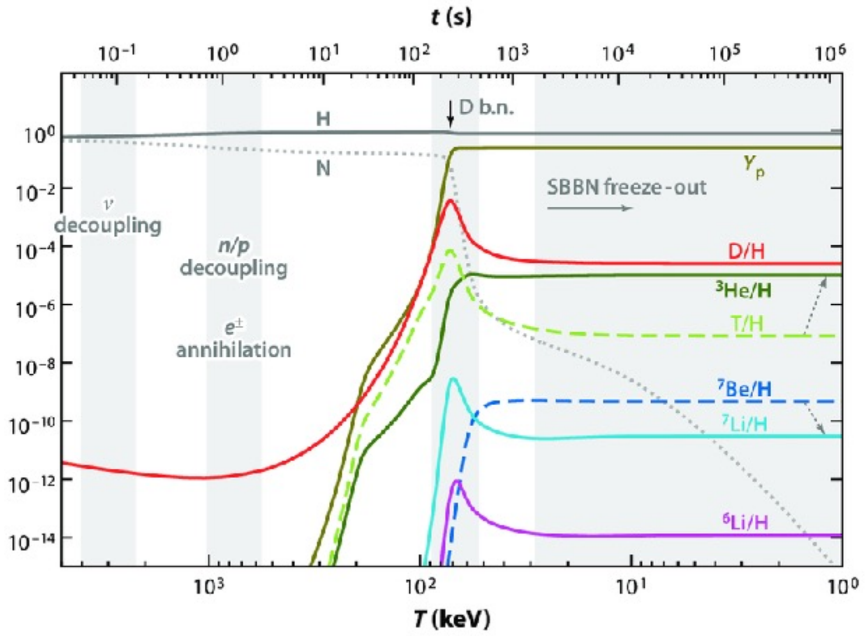
※ **SIGW inhomogeneities and anisotropies**

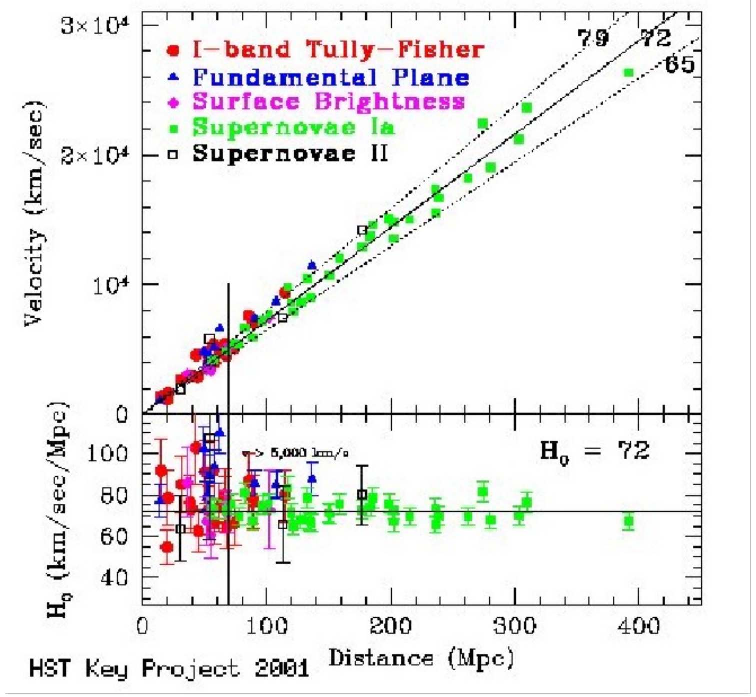
※ **Implications for PTA data and PBH**

※ **Summary**

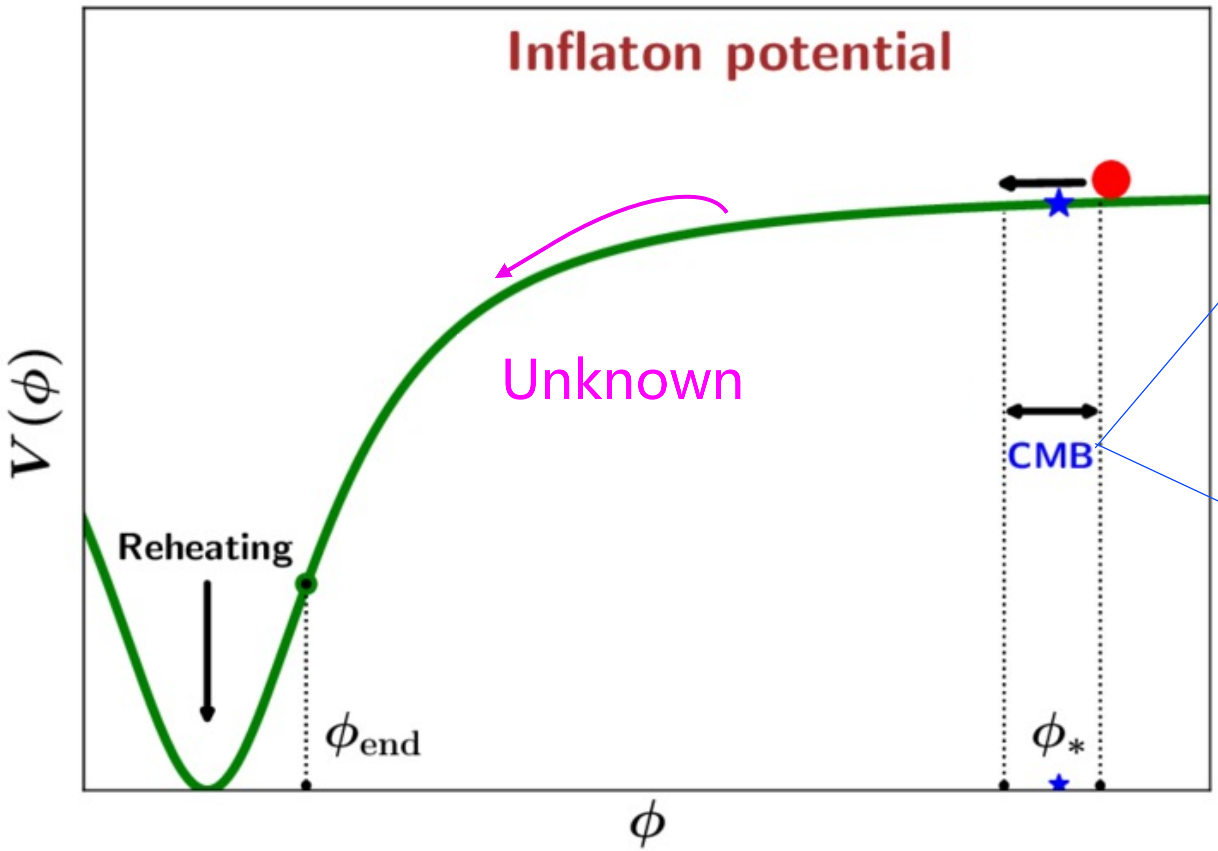
Standard model of cosmology

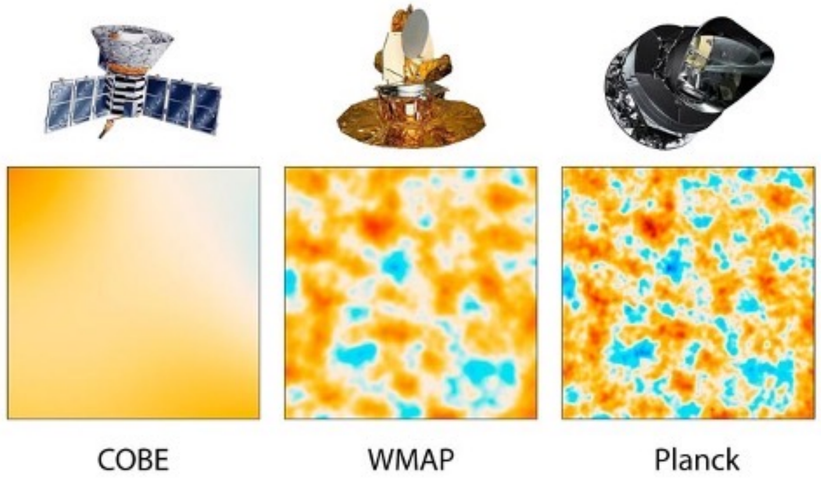




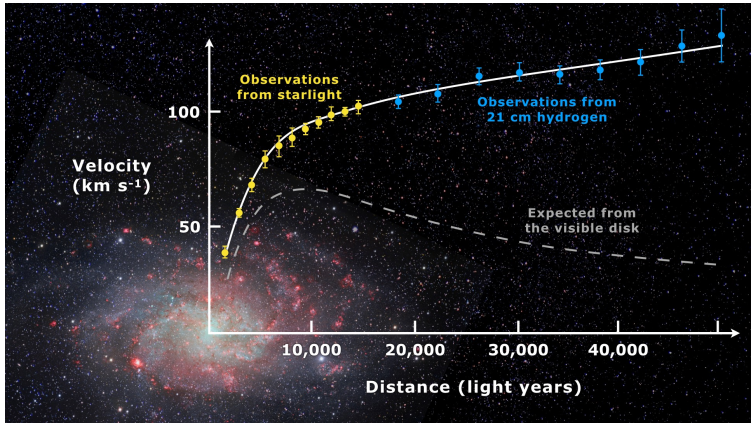


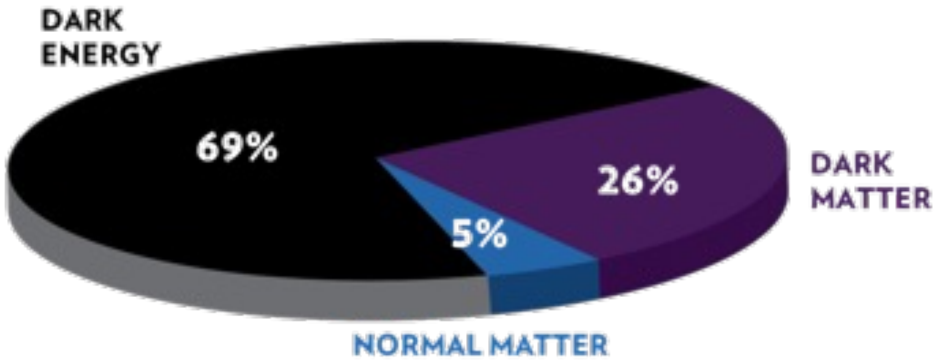
Open question: cosmic origin

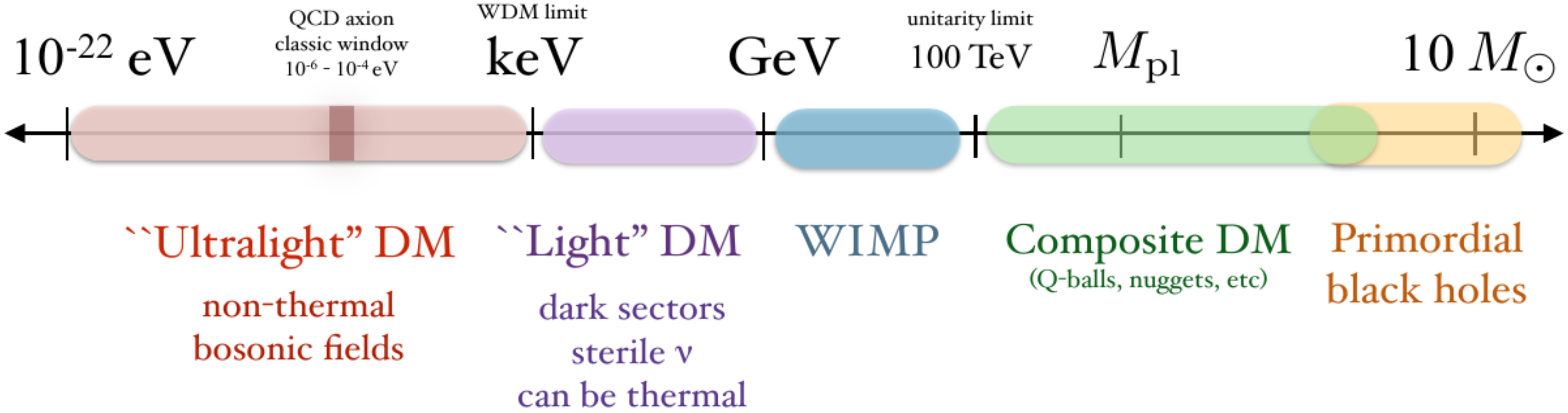


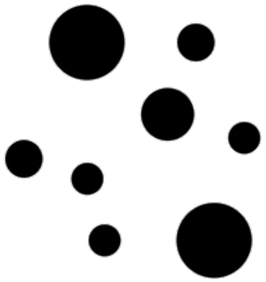


Open question: dark matter

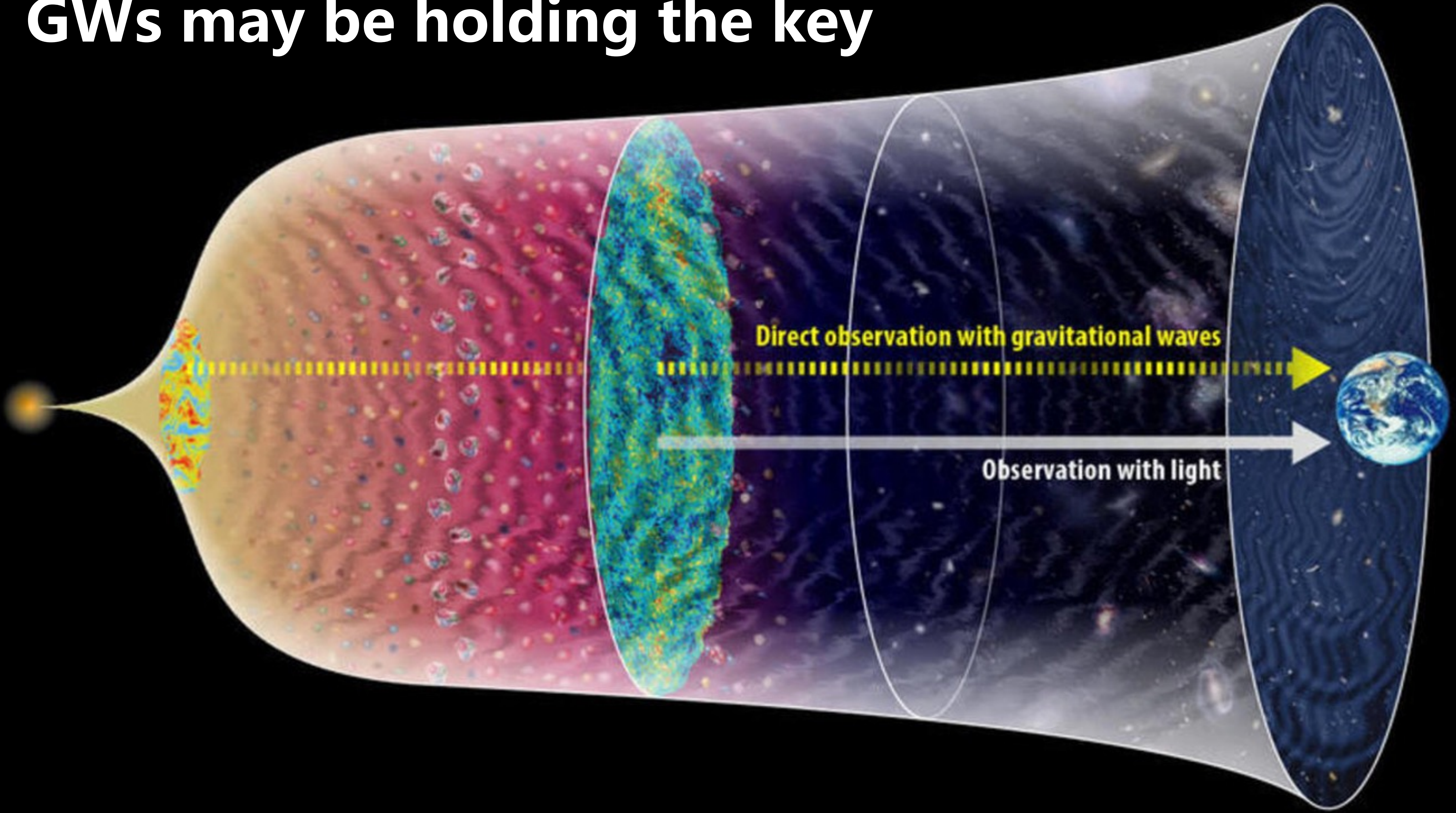






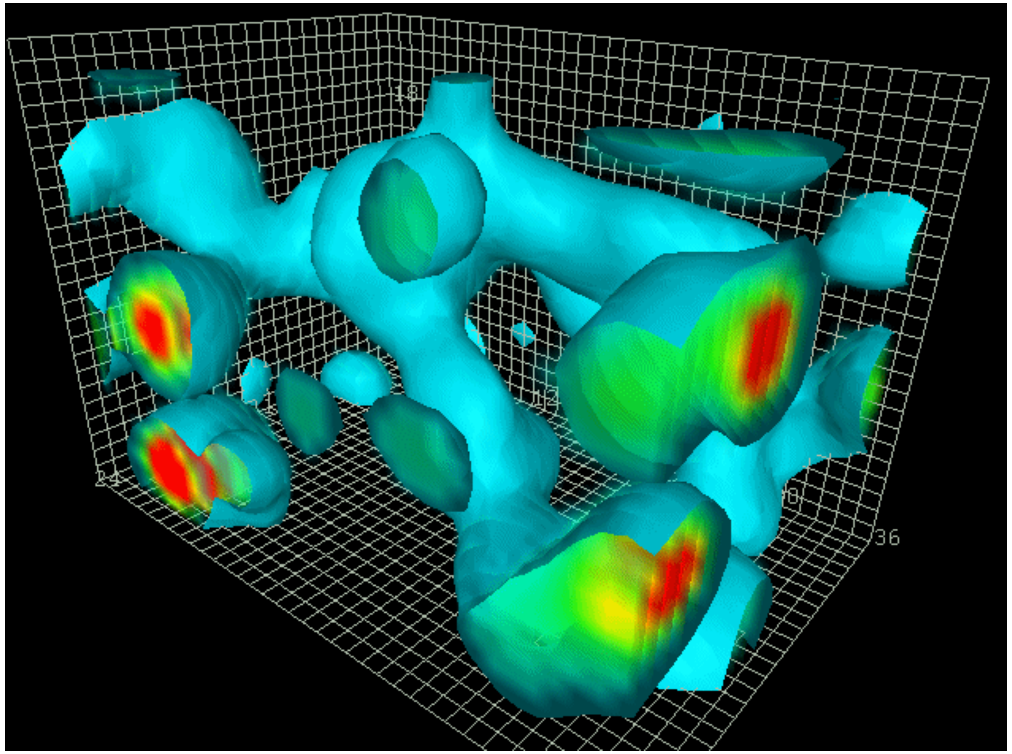


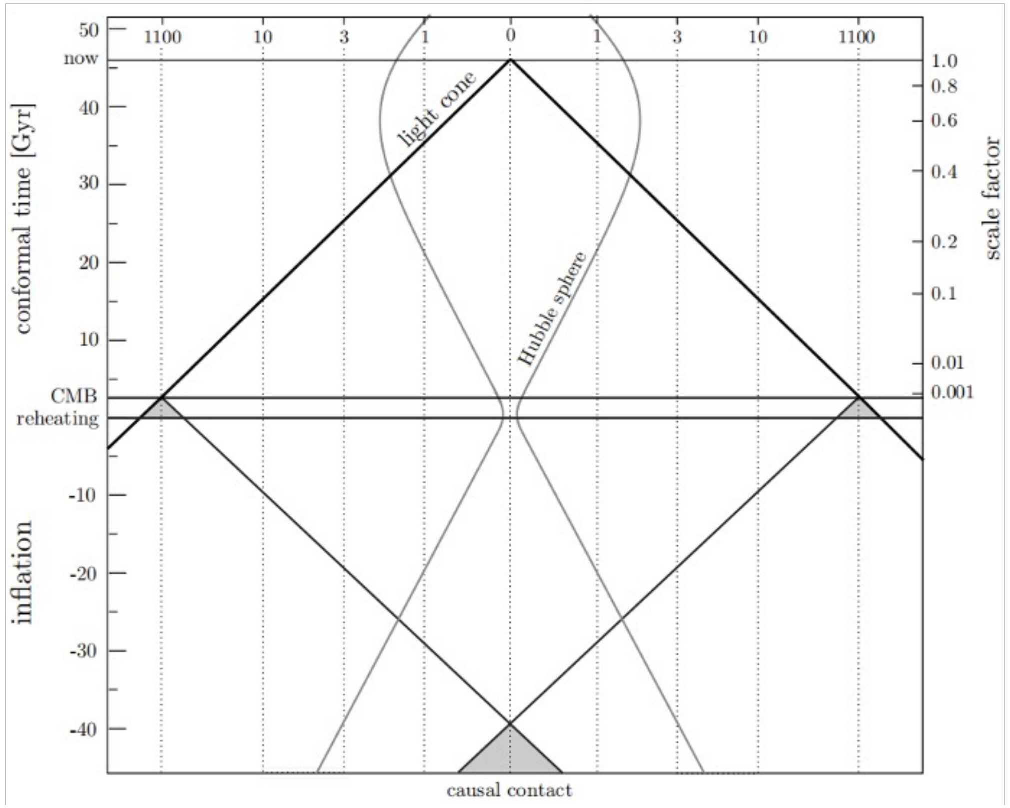
GWs may be holding the key



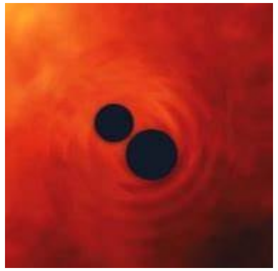
Perturbations during inflation

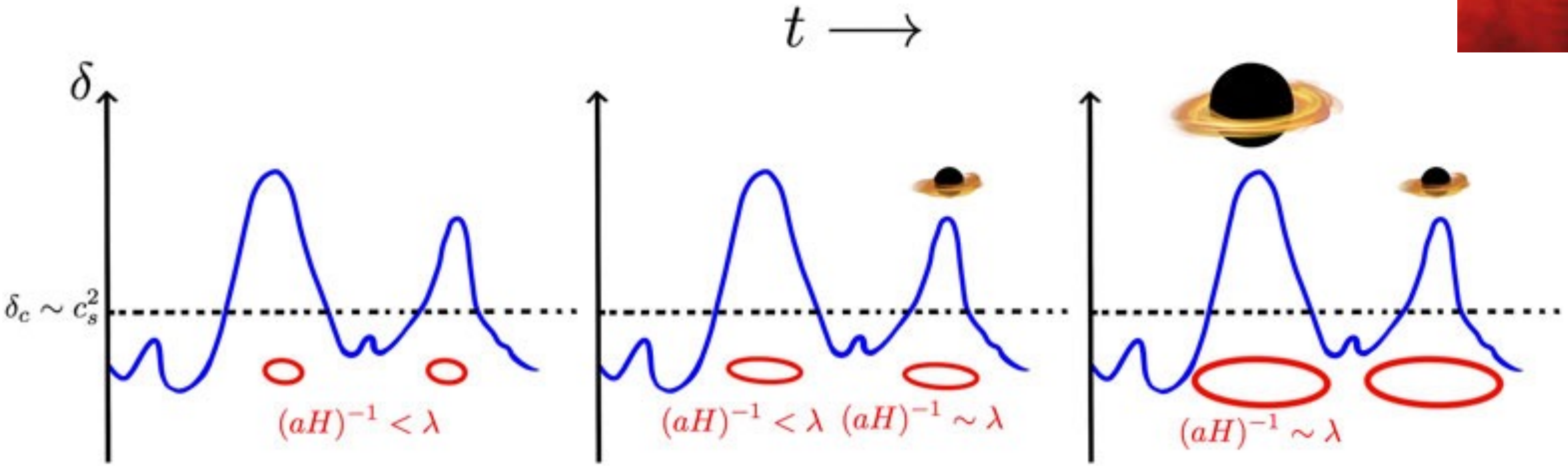
※ **quantum fluctuations**



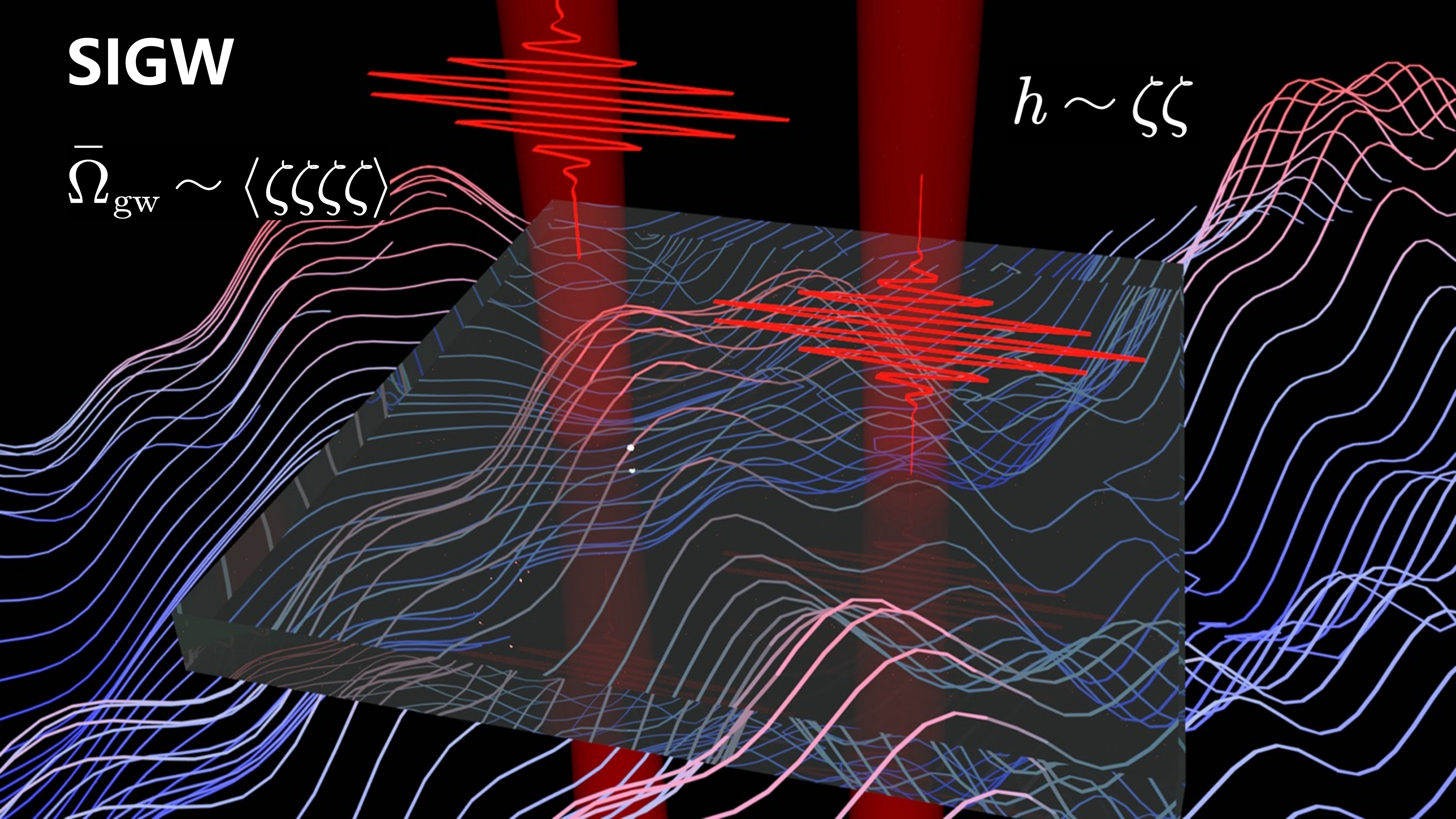


Primordial black holes

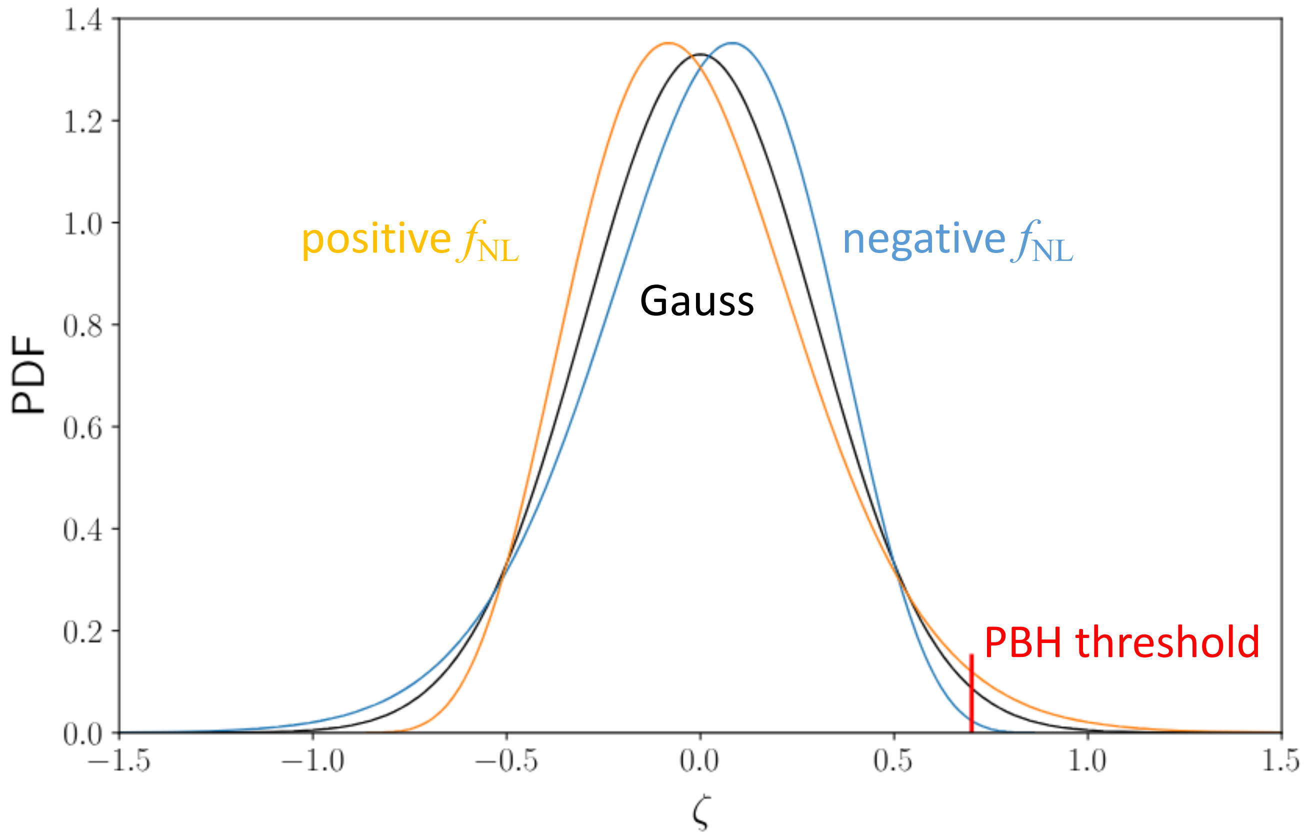




$$M_{PBH} \sim \frac{t}{G} \sim 10^5 \left(\frac{t}{1s}\right) M_\odot$$


SIGW
$\bar{\Omega}_{\rm gw} \sim \langle \zeta\zeta\zeta\zeta \rangle$
$h \sim \zeta\zeta$

Primordial non-Gaussianity



$$\zeta = \zeta_g + \frac{3}{5} f_{\rm NL} \left(\zeta_g^2 - \langle \zeta_g^2 \rangle\right) + \frac{9}{25} g_{\rm NL} \zeta_g^3$$

$$\langle \zeta\zeta\zeta \rangle \sim \mathcal{O}(f_{\rm NL})$$

$$\langle \zeta\zeta\zeta\zeta \rangle \sim \mathcal{O}(f_{\rm NL}^2) + \mathcal{O}(g_{\rm NL})$$

SIGW due to PNG

$$\mathrm{d}s^2 = a(\eta)^2\left[-e^{2\Phi}\mathrm{d}\eta^2 + \left(e^{-2\Psi}\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j\right]$$

$$h \sim \zeta\zeta$$

Scalar-induced GWs

Primordial non-Gaussianity

$$\Omega_{\rm gw}(\eta, k) \sim \langle hh \rangle$$

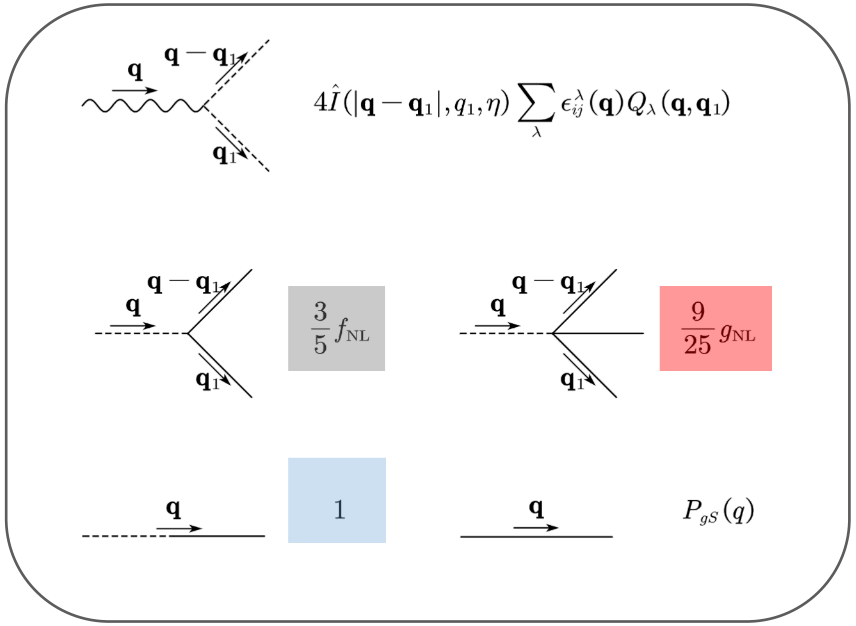
$$\zeta = \zeta_g + \frac{3}{5}f_{\rm NL}(\zeta_g^2 - \langle\zeta_g^2\rangle) + \frac{9}{25}g_{\rm NL}\zeta_g^3$$

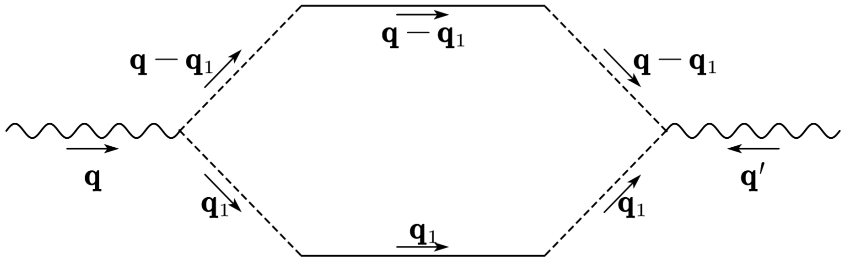
imprint

$$\sim \int\int (\text{Projection})^2 \times (\text{Kernel})^2 \times \langle\zeta\zeta\zeta\zeta\rangle$$

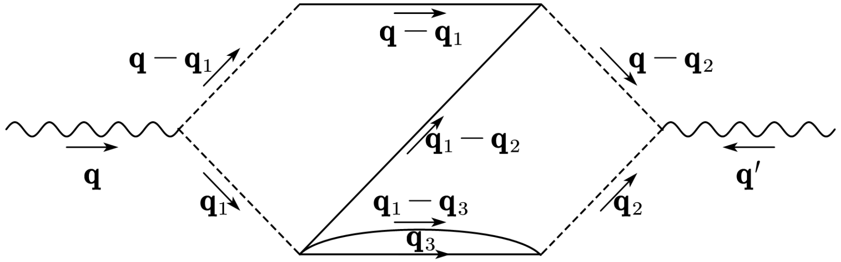
probe

Diagrammatic Approach



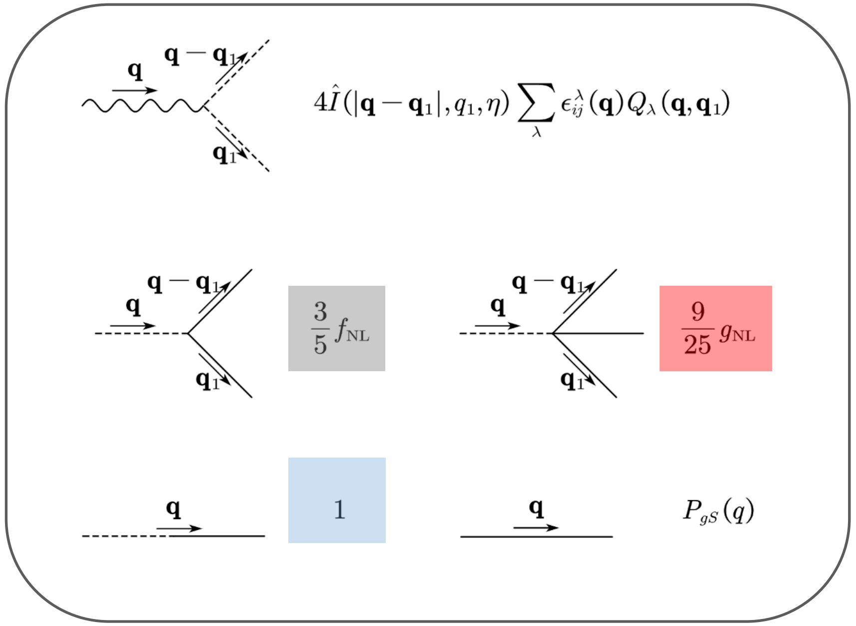


Gaussian

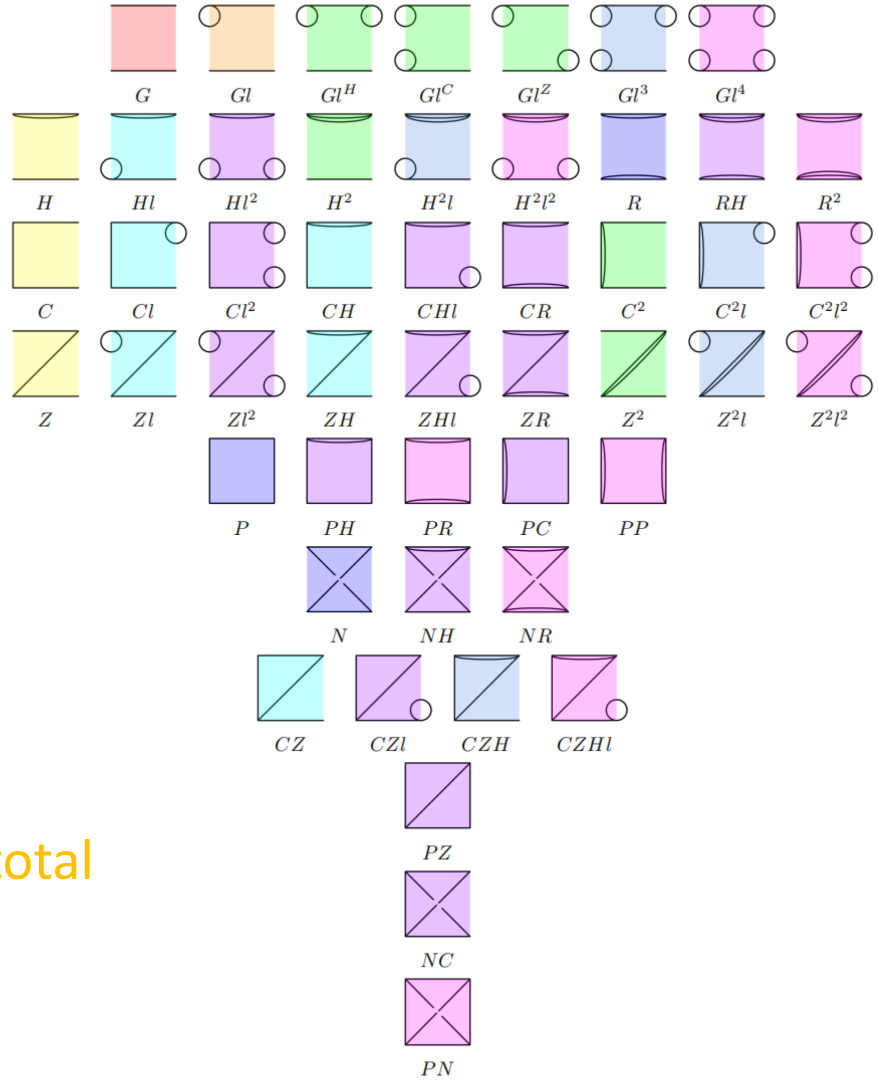


non-Gaussian

Diagrammatic Approach

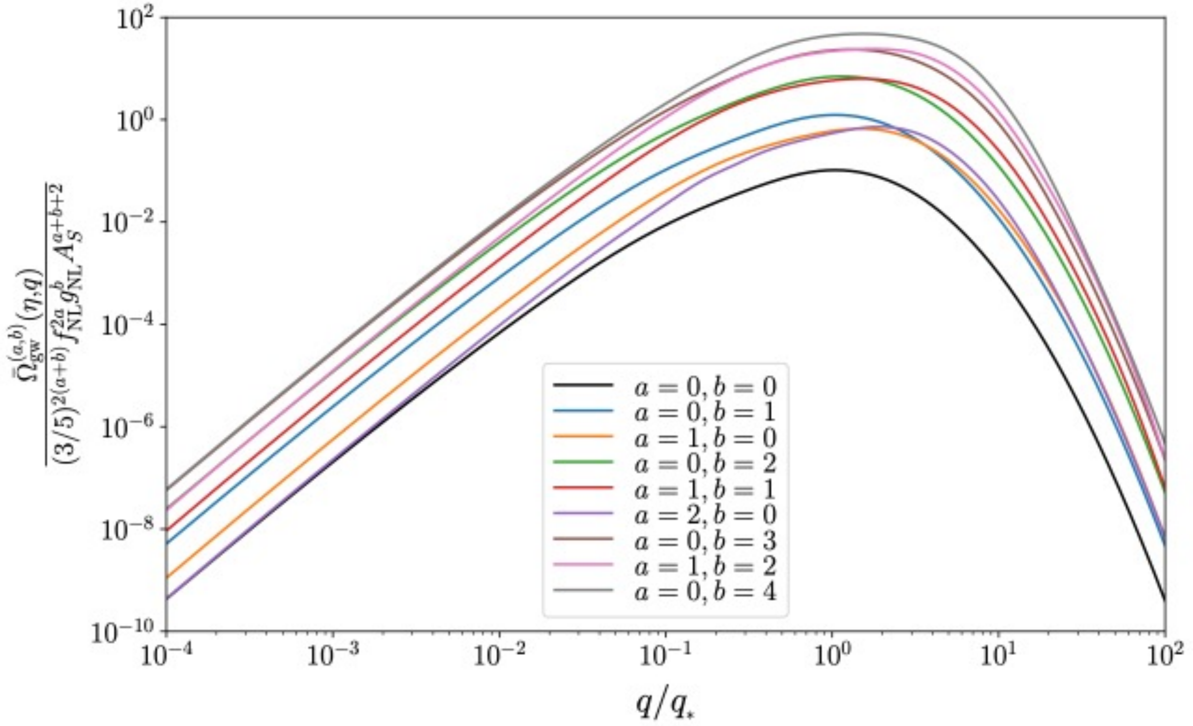


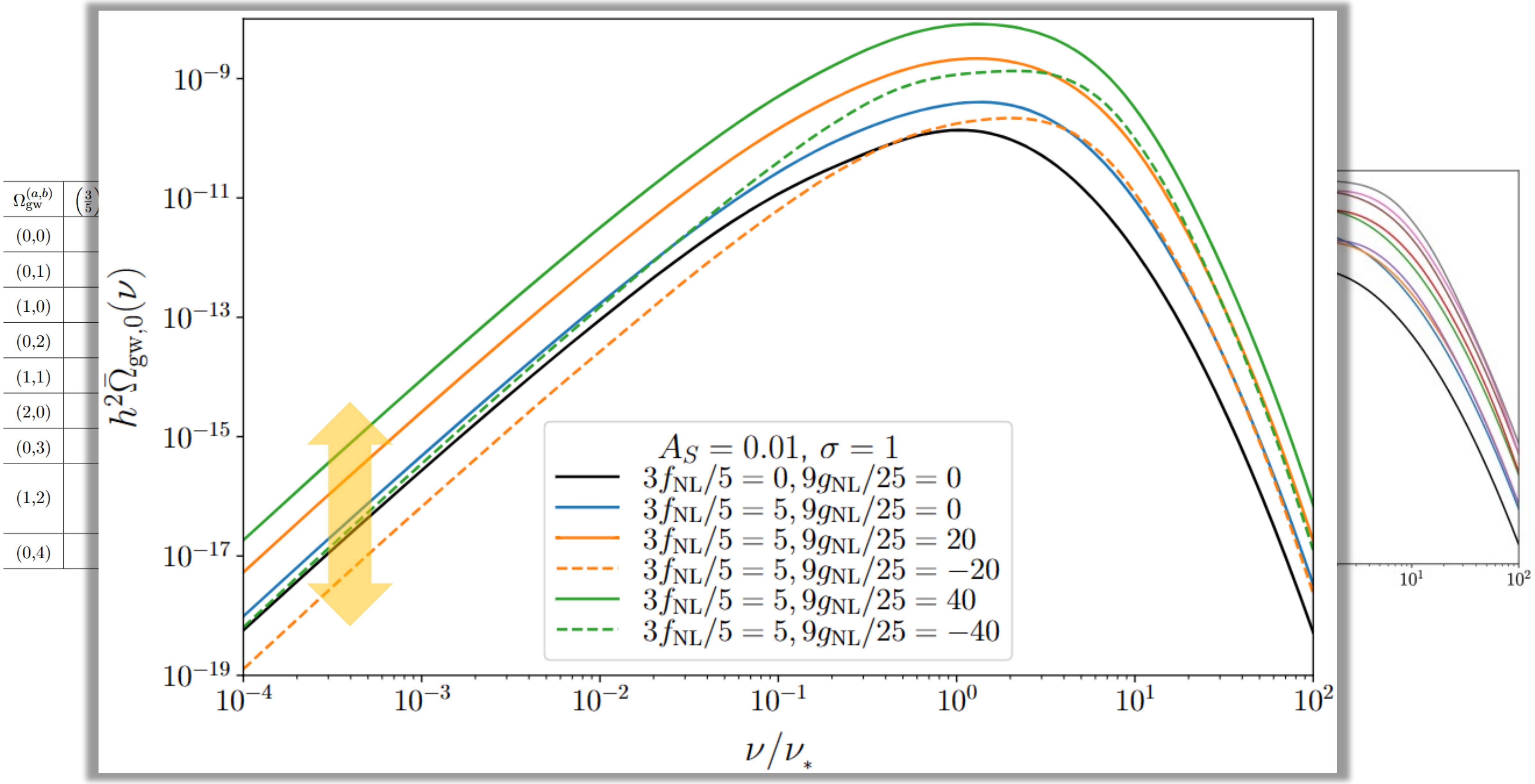
49 diagrams in total



Energy-density fraction spectrum

$\Omega_{\rm gw}^{(a,b)}$	$\left(\frac{3}{5}\right)^{2(a+b)} f_{\rm NL}^{2a} g_{\rm NL}^{b} A_S^{a+b+2}$	Diagram-X
(0,0)	A_S^2	G
(0,1)	$\left(\frac{3}{5}\right)^2 g_{\rm NL} A_S^3$	Gl
(1,0)	$\left(\frac{3}{5}\right)^2 f_{\rm NL}^2 A_S^3$	H, C, Z
(0,2)	$\left(\frac{3}{5}\right)^4 g_{\rm NL}^2 A_S^4$	Gl^H, Gl^C, Gl^Z, H^2, C^2, Z^2
(1,1)	$\left(\frac{3}{5}\right)^4 f_{\rm NL}^2 g_{\rm NL} A_S^4$	Hl, Cl, Zl, CH, ZH, CZ
(2,0)	$\left(\frac{3}{5}\right)^4 f_{\rm NL}^4 A_S^4$	R, P, N
(0,3)	$\left(\frac{3}{5}\right)^6 g_{\rm NL}^3 A_S^5$	Gl^3, H^2l, C^2l, Z^2l, CZH
(1,2)	$\left(\frac{3}{5}\right)^6 f_{\rm NL}^2 g_{\rm NL}^2 A_S^5$	Hl^2, Cl^2, Zl^2, CHl, ZHl, CZl, RH, CR, ZR, PH, PC, PZ, NH, NC
(0,4)	$\left(\frac{3}{5}\right)^8 g_{\rm NL}^4 A_S^6$	Gl^4, H^2l^2, R^2, C^2l^2, Z^2l^2, PR, PP, NR, $CZHl$, PN





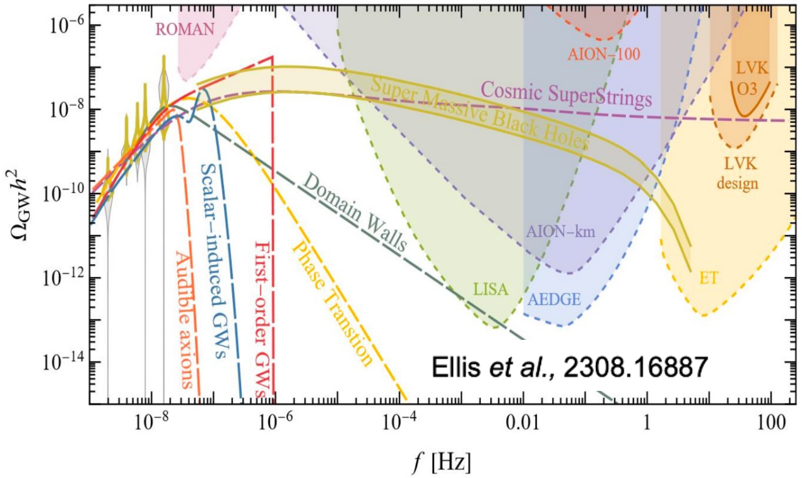
$\Omega_{\rm gw}^{(a,b)}$
(0,0)
(0,1)
(1,0)
(0,2)
(1,1)
(2,0)
(0,3)
(1,2)
(0,4)
$h^2\bar{\Omega}_{\rm gw,0}(\nu)$
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-19}
$A_S = 0.01,\ \sigma = 1$
$3f_{\rm NL}/5 = 0, 9g_{\rm NL}/25 = 0$
$3f_{\rm NL}/5 = 5, 9g_{\rm NL}/25 = 0$
$3f_{\rm NL}/5 = 5, 9g_{\rm NL}/25 = 20$
$3f_{\rm NL}/5 = 5, 9g_{\rm NL}/25 = -20$
$3f_{\rm NL}/5 = 5, 9g_{\rm NL}/25 = 40$
$3f_{\rm NL}/5 = 5, 9g_{\rm NL}/25 = -40$
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
ν/ν_*
10^{1}
10^{2}

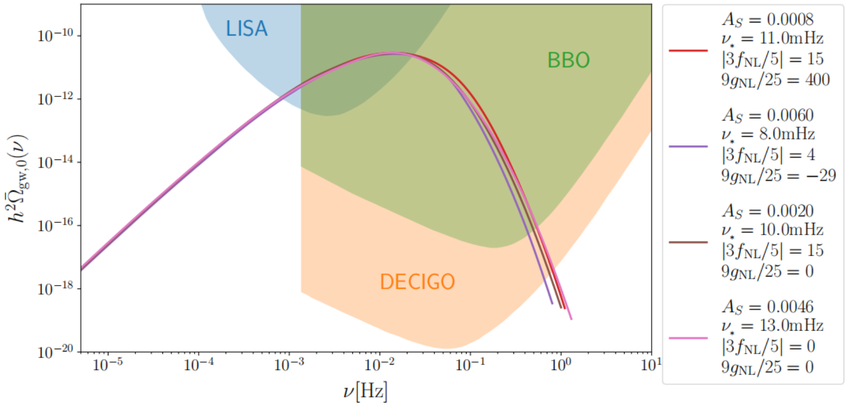
From background to fluctuations

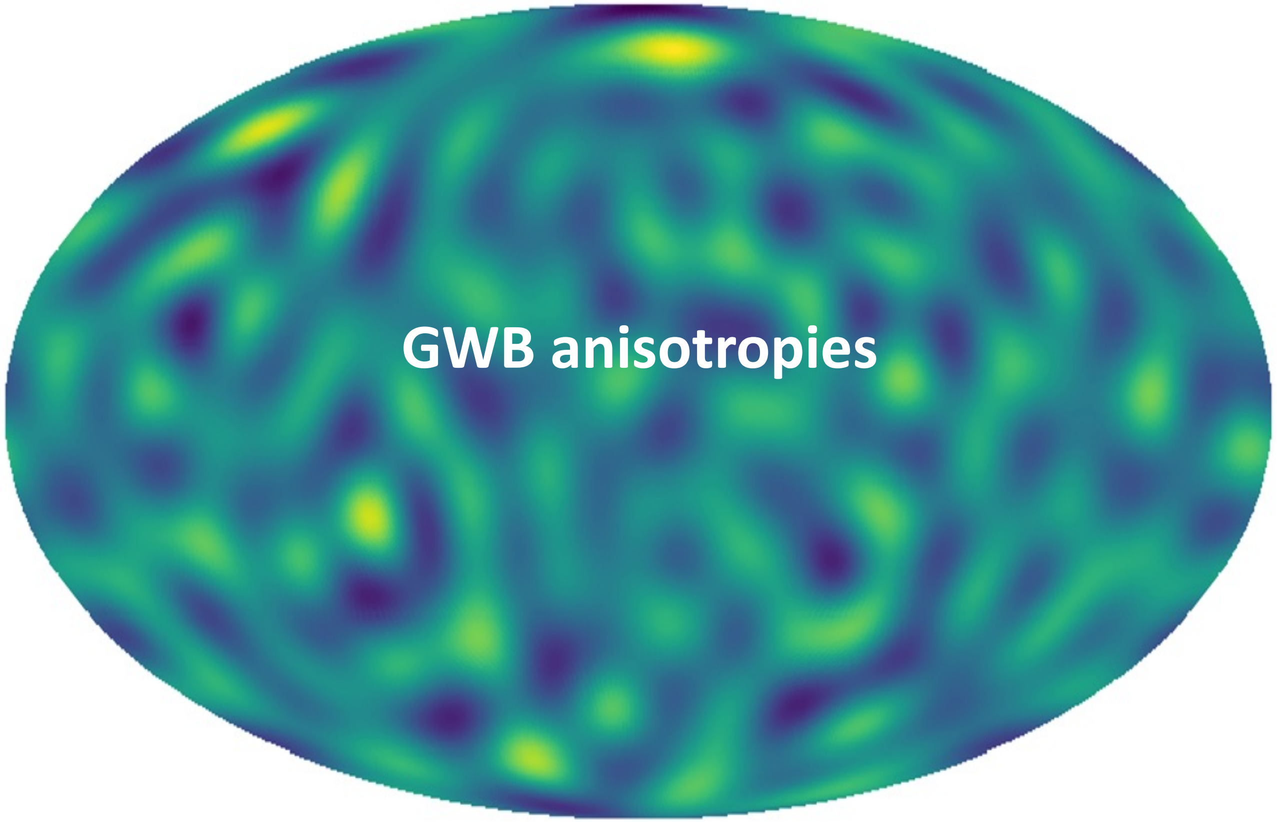
$$\rho_{\rm gw} = \bar{\rho}_{\rm gw} + \delta\rho_{\rm gw}$$

$\Omega_{\rm gw}(f)$

$C_\ell(f)$



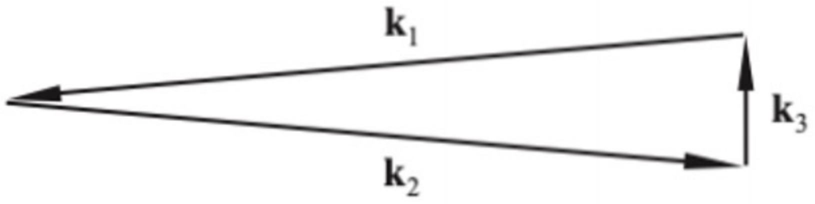


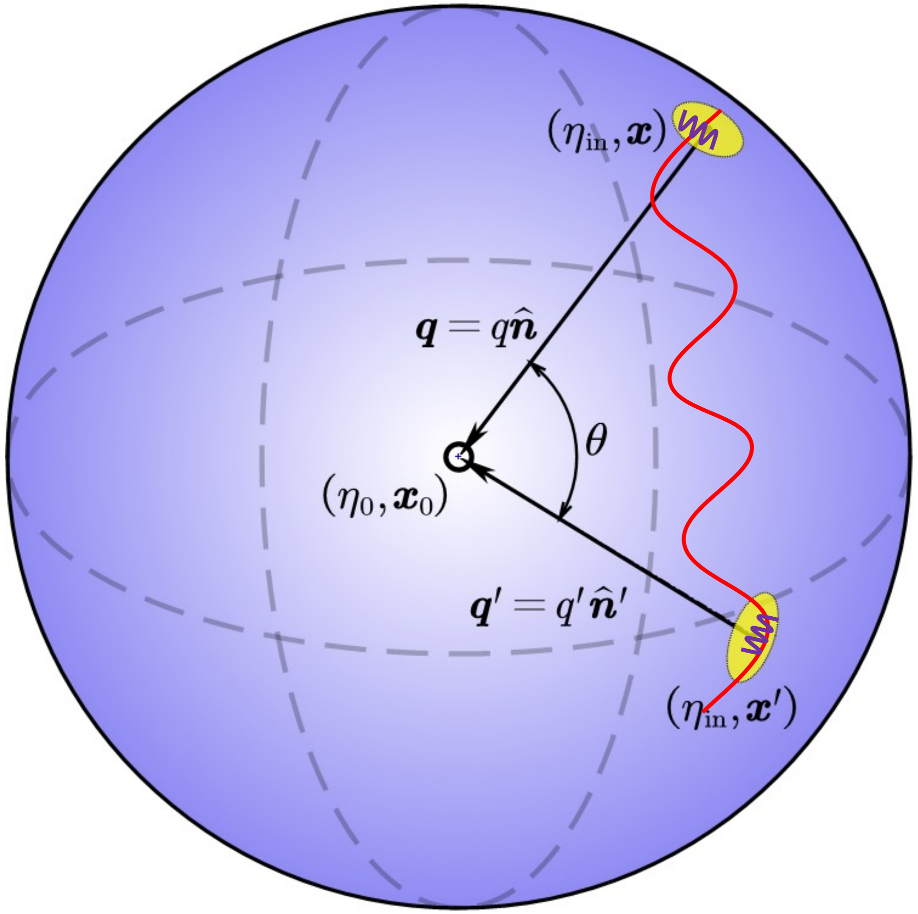


Inhomogeneities and Anisotropies

$$\delta_{\rm gw}(\eta, \mathbf{x}, \mathbf{q}) = \frac{\omega_{\rm gw}(\eta,\mathbf{x},\mathbf{q}) - \bar{\Omega}_{\rm gw}(\eta,q)}{\bar{\Omega}_{\rm gw}(\eta,q)}$$

- small-scale inhomogeneities: unobservable
- large-scale inhomogeneities: observabable!





SIGW propogation

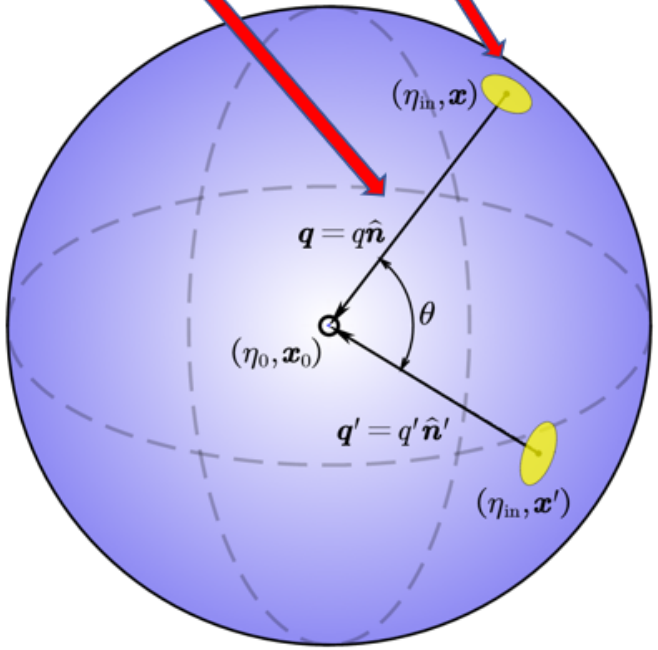
$$\delta_{\rm gw}(\eta_0, \mathbf{x}_0, \mathbf{q}) = \delta_{\rm gw}(\eta_{\rm in}, \mathbf{x}_{\rm in}, \mathbf{q}) + [4 - n_{\rm gw,0}(\nu)]\, \Phi(\eta_{\rm in}, \mathbf{x}_{\rm in})$$

$$+2\,[4 - n_{\rm gw,0}(\nu)] \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}_0} \int_{\eta_{\rm in}}^{\eta_0} \mathrm{d}\eta'\, e^{ik\mu(\eta'-\eta_0)} \partial_{\eta'} \Phi(\eta', \mathbf{k})$$

SW effect

ISW effect

- initial inhomogeneities
- Sachs-Wolfe effect:
  gravitational redshift / blueshift at initial time
- integrated Sachs-Wolfe effect:
  gravitational redshift / blueshift during the propogation of GWs



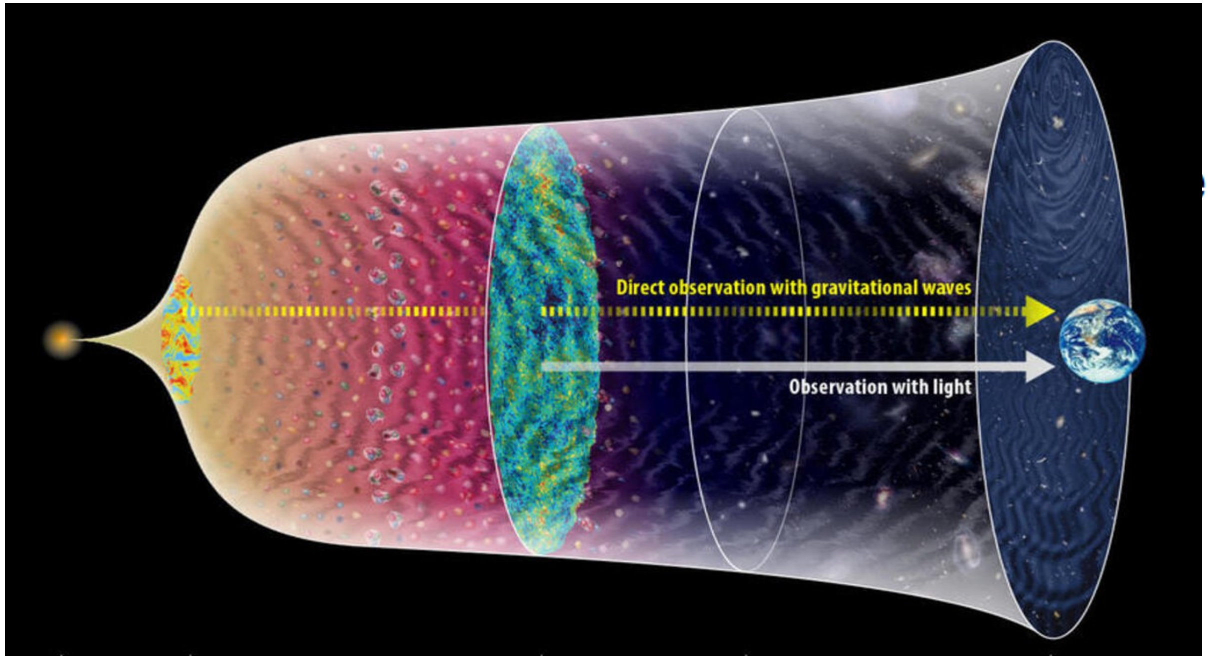
SIGW propogation

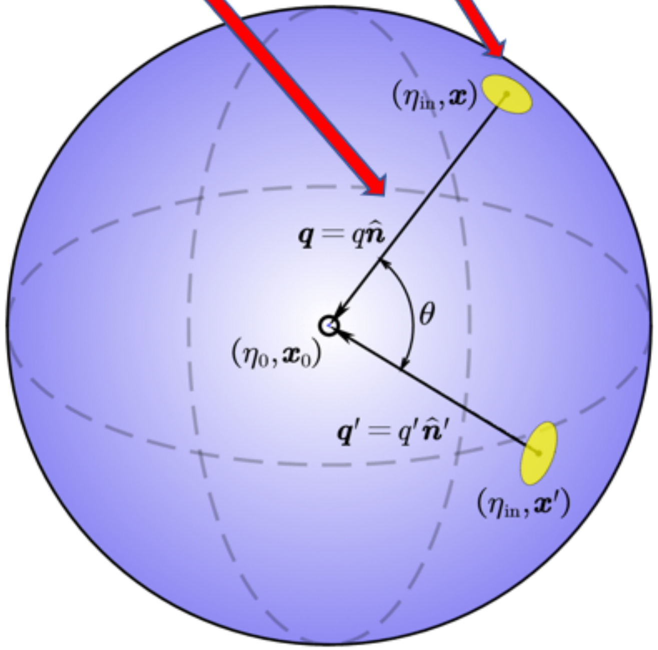
$$\delta_{\rm gw}(\eta_0,\mathbf{x}_0,\mathbf{q}) = \delta_{\rm gw}(\eta_{\rm in},\mathbf{x}_{\rm in},\mathbf{q}) + [4 - n_{\rm gw,0}(\nu)]\,\Phi(\eta_{\rm in},\mathbf{x}_{\rm in})$$
$$+2\,[4 - n_{\rm gw,0}(\nu)]\int \frac{{\rm d}^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}_0}\int_{\eta_{\rm in}}^{\eta_0} {\rm d}\eta'\, e^{ik\mu(\eta'-\eta_0)}\partial_{\eta'}\Phi(\eta',\mathbf{k})$$

SW effect

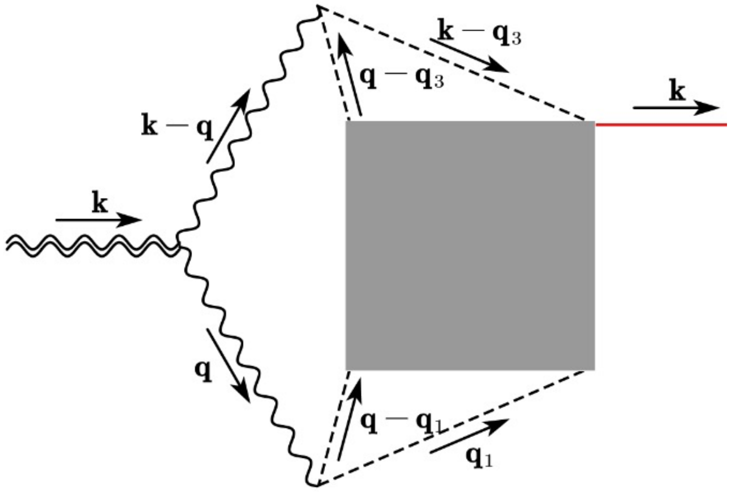
ISW effect

- initial inhomogeneities



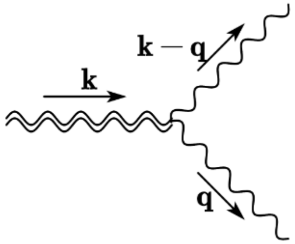


Initial inhomogeneities

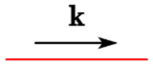


$$\omega_{\rm gw}(\eta,\mathbf{x},\mathbf{q}) \sim \langle\zeta^4\rangle_{\mathbf{x}} \sim \langle\zeta_S^4\rangle_{\mathbf{x}} + \mathcal{O}(\zeta_{gL}) f_{\rm NL}\langle\zeta_{gS}\zeta_S^3\rangle_{\mathbf{x}} + \mathcal{O}(\zeta_{gL}) g_{\rm NL}\langle\zeta_{gS}^2\zeta_S^3\rangle_{\mathbf{x}}$$

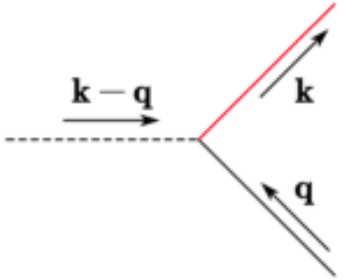
$$\zeta_g = \zeta_{gS} + \zeta_{gL}$$

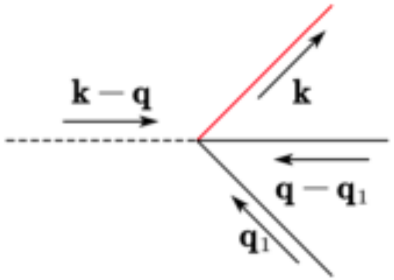


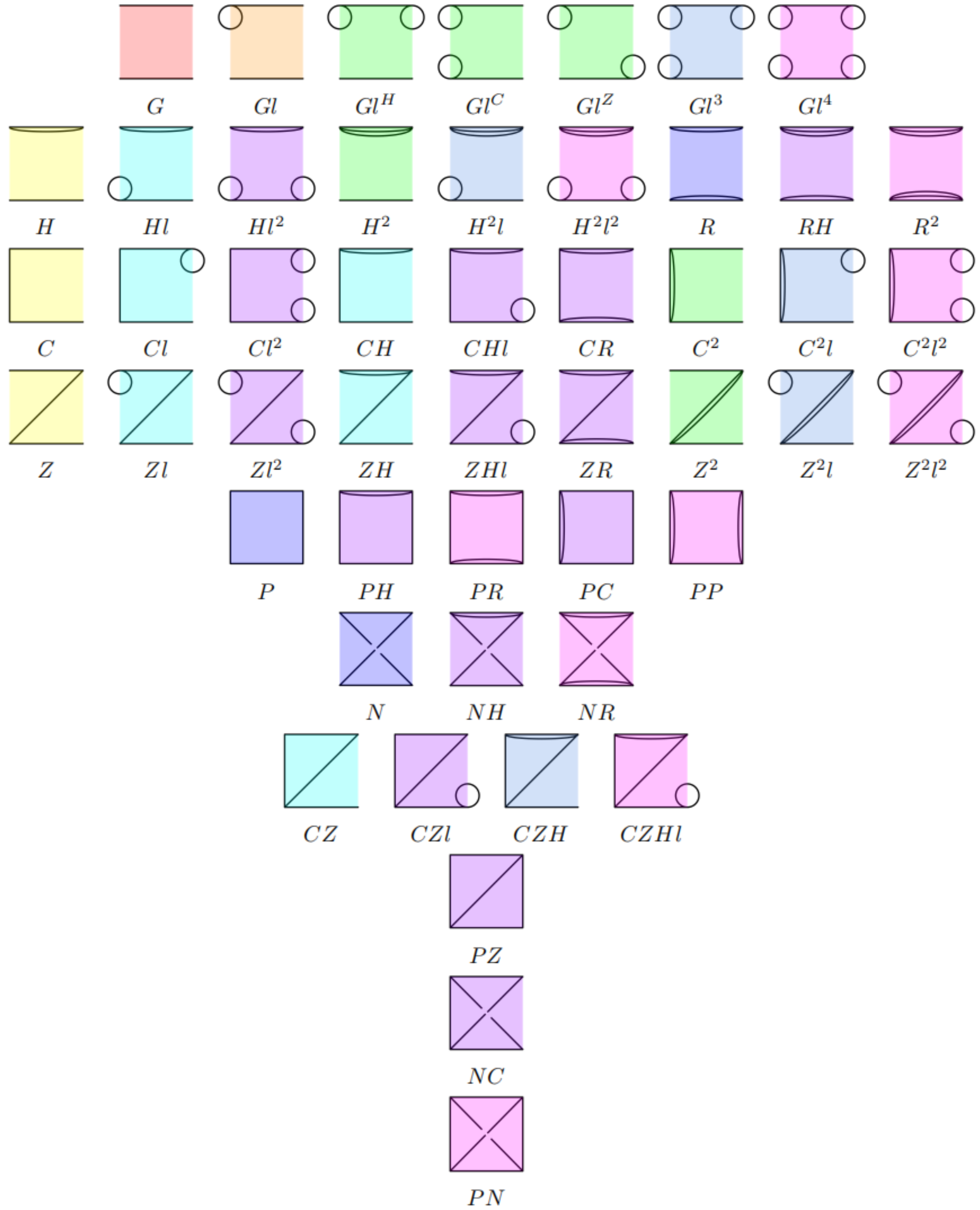
$$-\frac{1}{48\,(2\pi)^{3/2}}\frac{q^3}{\mathcal{H}^2}(\mathbf{k}-\mathbf{q})\cdot\mathbf{q}$$

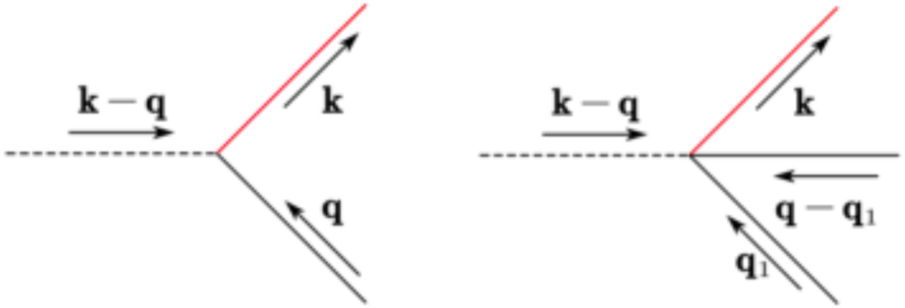


$P_{gL}(k)$



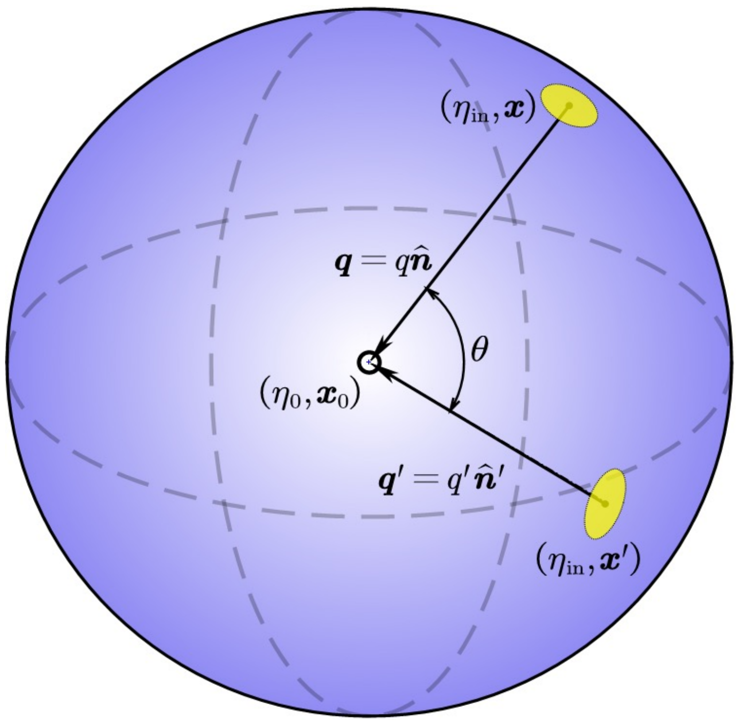


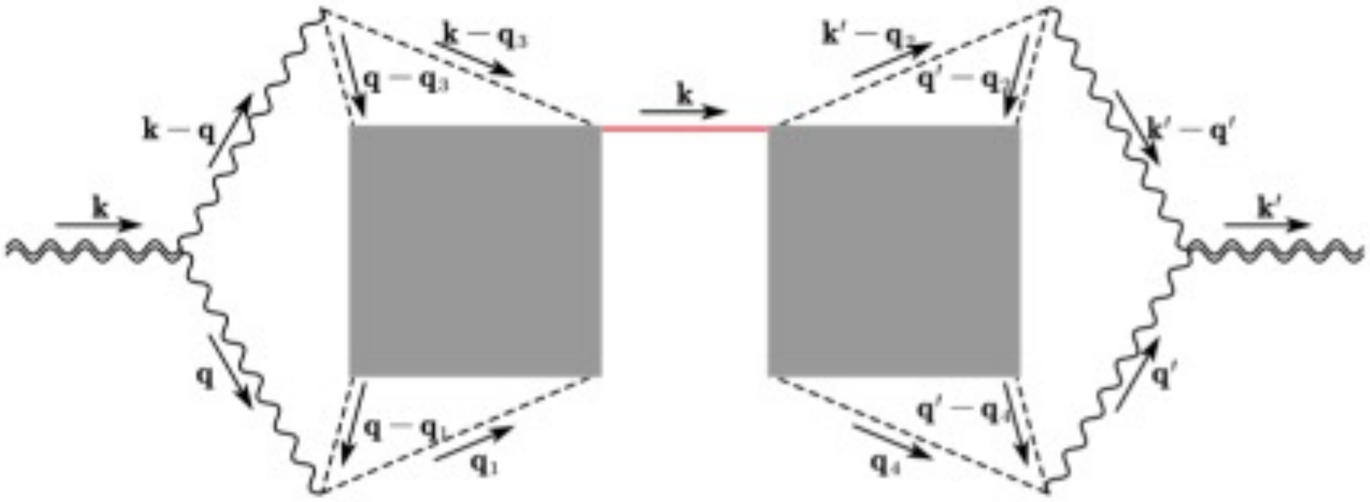




$\Omega_{\rm gw}^{(a,b)}$	$\left(\frac{3}{5}\right)^{2(a+b)} f_{\rm NL}^{2a} g_{\rm NL}^{b} A_S^{a+b+2}$	Diagram-X
(0,0)	A_S^2	G
(0,1)	$\left(\frac{3}{5}\right)^2 g_{\rm NL} A_S^3$	Gl
(1,0)	$\left(\frac{3}{5}\right)^2 f_{\rm NL}^2 A_S^3$	H, C, Z
(0,2)	$\left(\frac{3}{5}\right)^4 g_{\rm NL}^2 A_S^4$	Gl^H, Gl^C, Gl^Z, H^2, C^2, Z^2
(1,1)	$\left(\frac{3}{5}\right)^4 f_{\rm NL}^2 g_{\rm NL} A_S^4$	Hl, Cl, Zl, CH, ZH, CZ
(2,0)	$\left(\frac{3}{5}\right)^4 f_{\rm NL}^4 A_S^4$	R, P, N
(0,3)	$\left(\frac{3}{5}\right)^6 g_{\rm NL}^3 A_S^5$	Gl^3, H^2l, C^2l, Z^2l, CZH
(1,2)	$\left(\frac{3}{5}\right)^6 f_{\rm NL}^2 g_{\rm NL}^2 A_S^5$	Hl^2, Cl^2, Zl^2, CHl, ZHl, CZl, RH, CR, ZR, PH, PC, PZ, NH, NC
(0,4)	$\left(\frac{3}{5}\right)^8 g_{\rm NL}^4 A_S^6$	Gl^4, H^2l^2, R^2, C^2l^2, Z^2l^2, PR, PP, NR, $CZHl$, PN

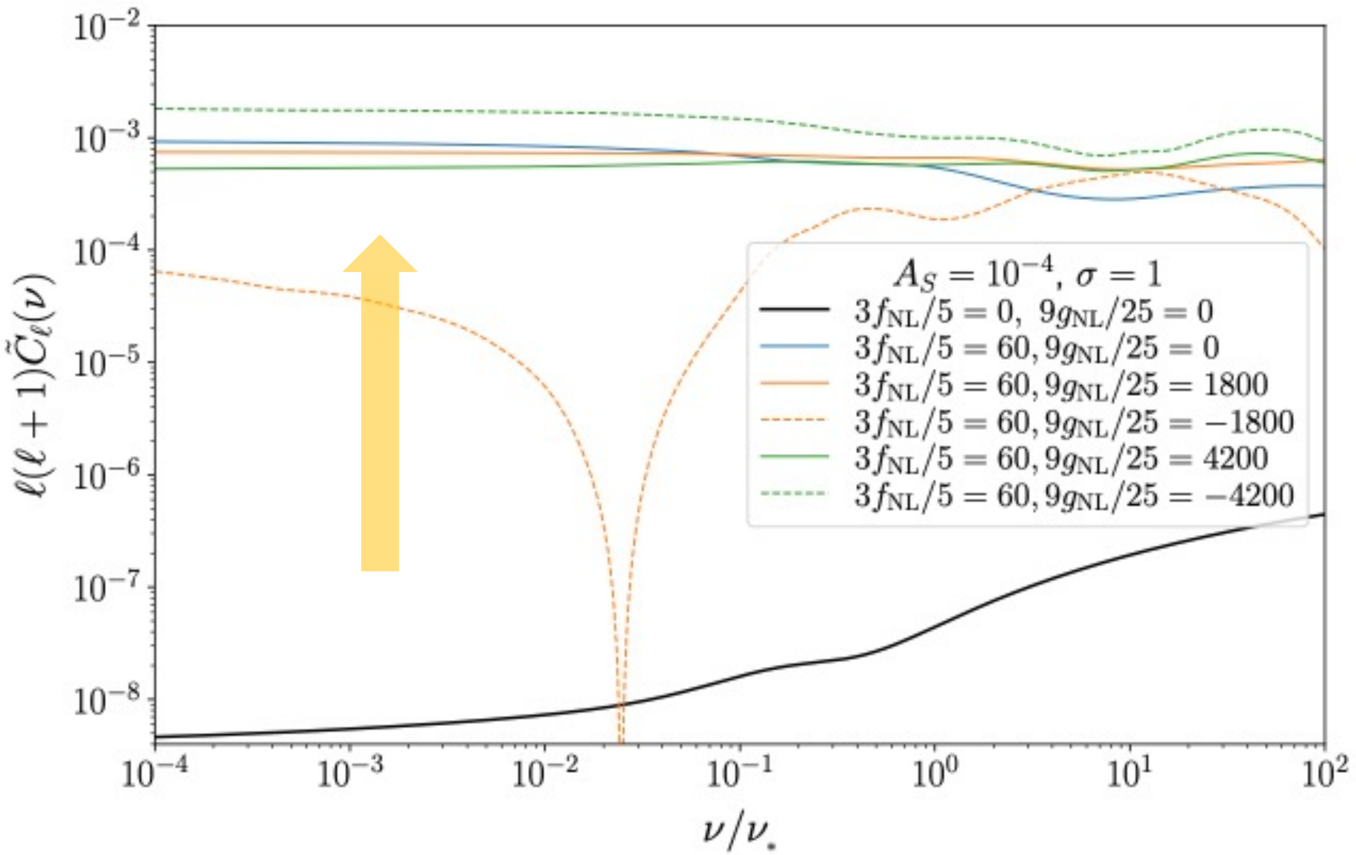
Angular 2-point correlation

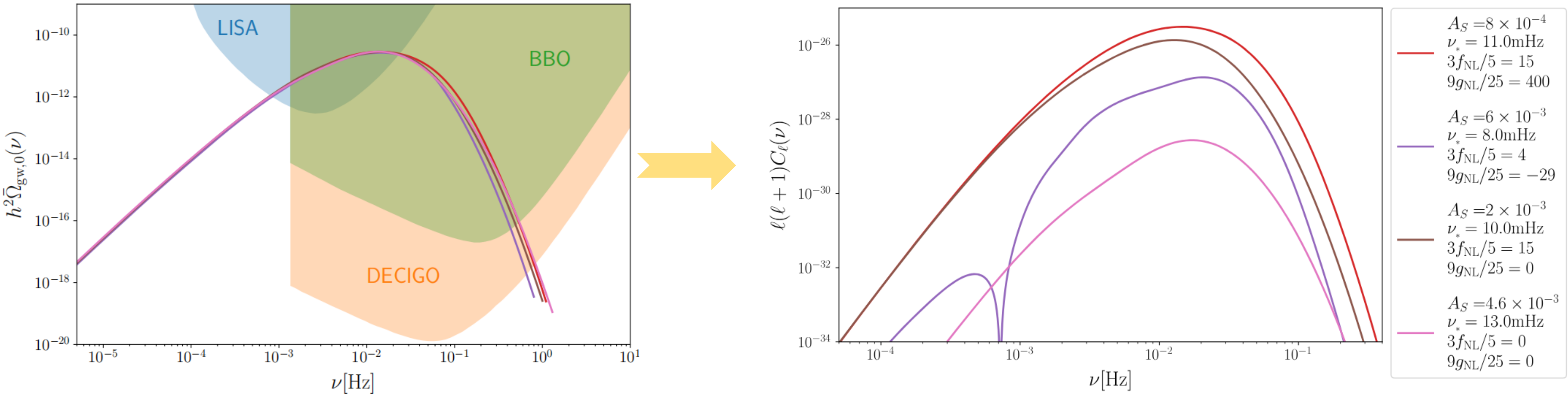




~$O(10^3)$ diagrams in total

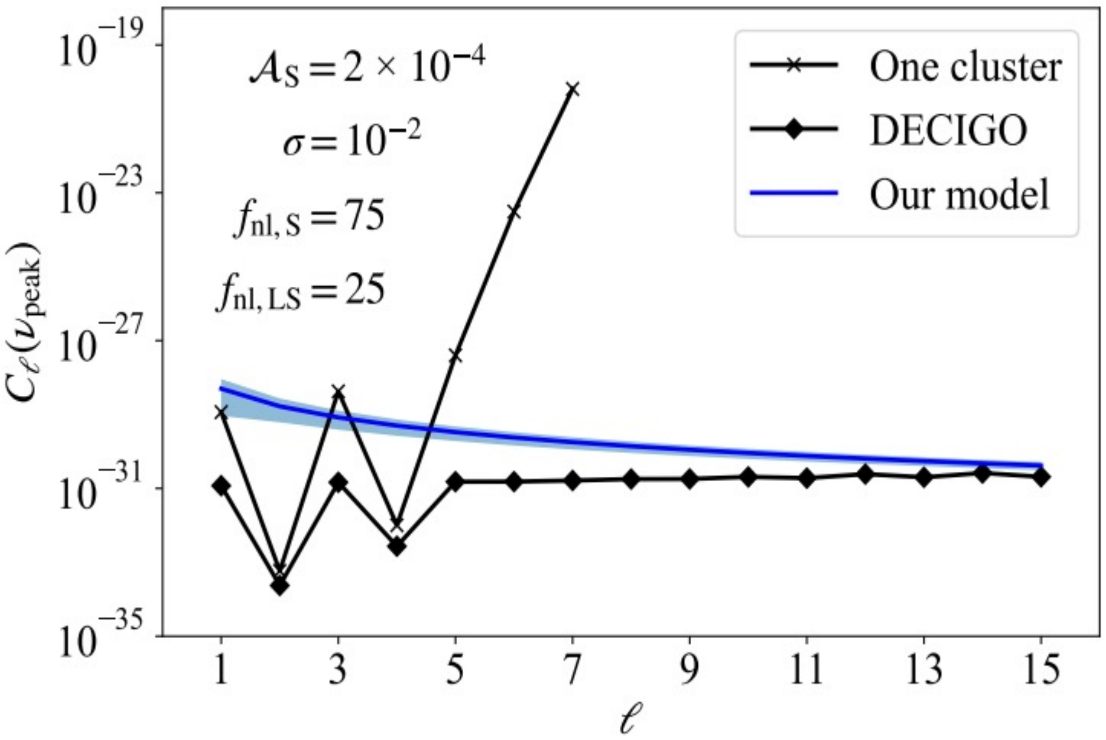
Angular power spectrum

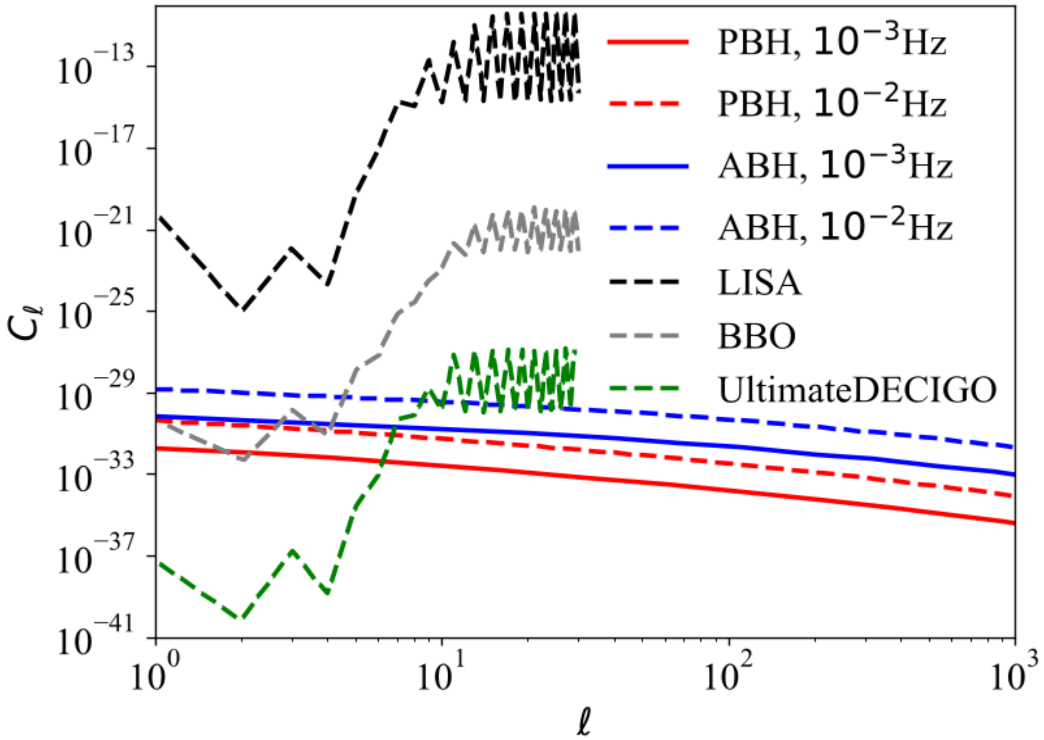




Parameter degeneracy

Foreground



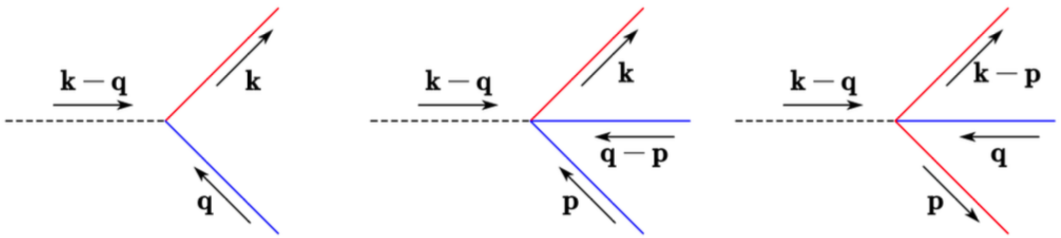


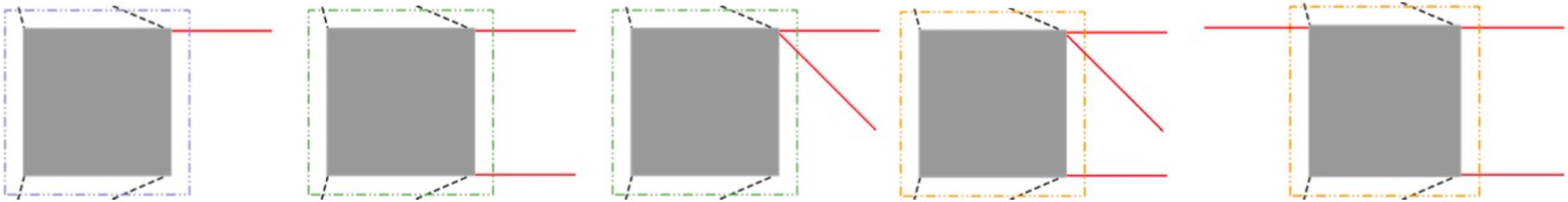
$C_\ell \propto [\ell(\ell+1)]^{-1}$

$C_\ell \propto (\ell+1/2)^{-1}$

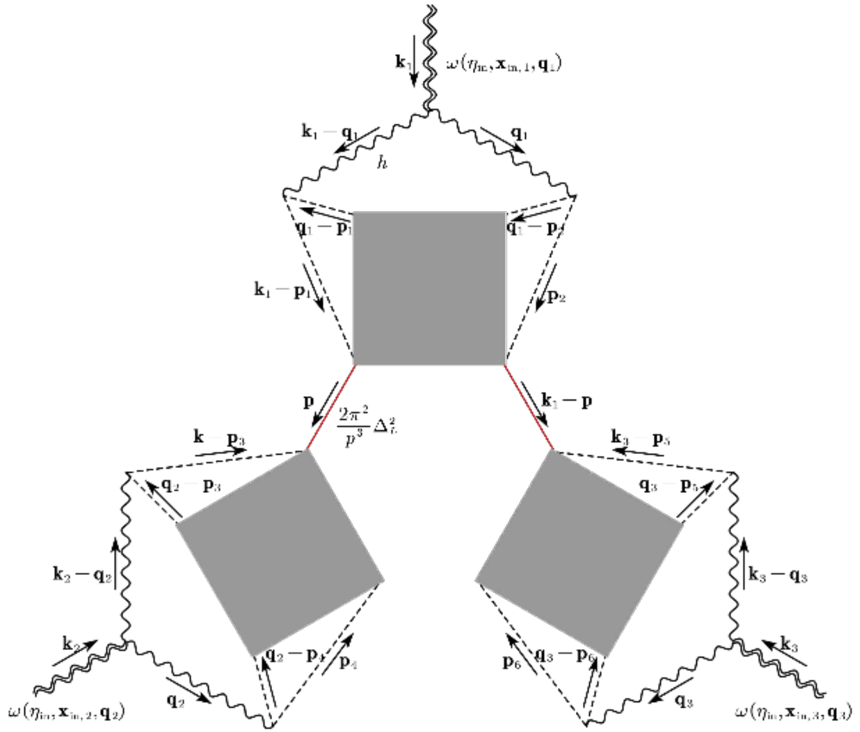
Parameter degeneracy
Foreground

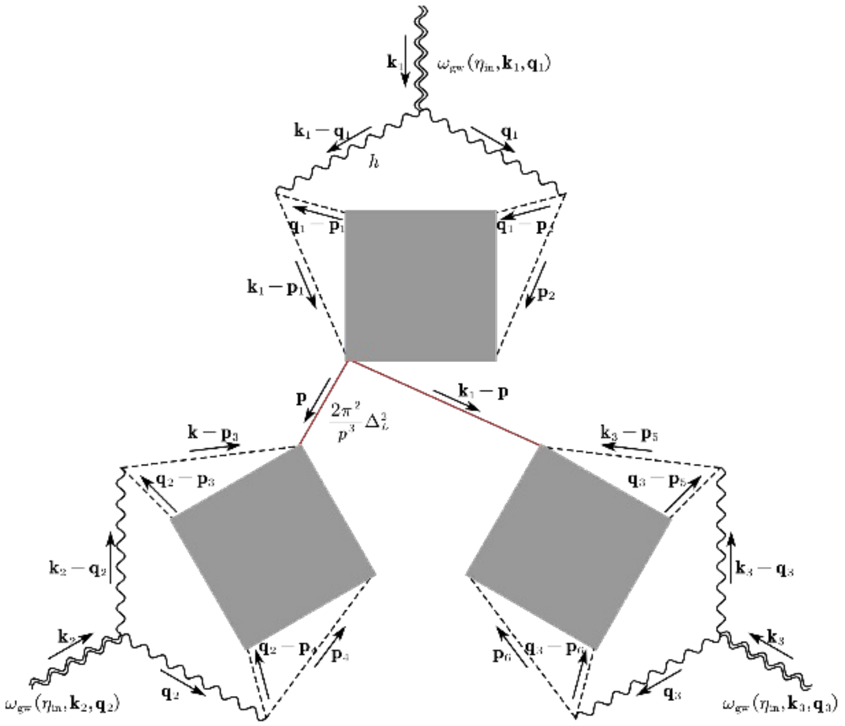
$$\omega_{\rm gw}(\eta_{\rm in},\mathbf{x}_i,\mathbf{q}_i)\sim\langle\zeta^4\rangle_{\mathbf{x}_i}\ \sim\ \langle\zeta_S^4\rangle_{\mathbf{x}_i}+\mathcal{O}(\zeta_{gL})\left[f_{\rm NL}\langle\zeta_{gS}\zeta_S^3\rangle_{\mathbf{x}_i}+g_{\rm NL}\langle\zeta_{gS}^2\zeta_S^3\rangle_{\mathbf{x}_i}\right]$$
$$+\mathcal{O}(\zeta_{gL}^2)\left[f_{\rm NL}^2\langle\zeta_{gS}^2\zeta_S^2\rangle_{\mathbf{x}_i}+f_{\rm NL}g_{\rm NL}\langle\zeta_{gS}^3\zeta_S^2\rangle_{\mathbf{x}_i}+g_{\rm NL}^2\langle\zeta_{gS}^4\zeta_S^2\rangle_{\mathbf{x}_i}+g_{\rm NL}\langle\zeta_{gS}\zeta_S^3\rangle_{\mathbf{x}_i}\right]$$
$$+\mathcal{O}(\zeta_{gL}^3)\left[f_{\rm NL}^3\langle\zeta_{gS}^3\zeta_S\rangle_{\mathbf{x}_i}+f_{\rm NL}^2g_{\rm NL}\langle\zeta_{gS}^4\zeta_S\rangle_{\mathbf{x}_i}+f_{\rm NL}g_{\rm NL}^2\langle\zeta_{gS}^5\zeta_S\rangle_{\mathbf{x}_i}\right.$$
$$\left.+g_{\rm NL}^3\langle\zeta_{gS}^6\zeta_S\rangle_{\mathbf{x}_i}+f_{\rm NL}g_{\rm NL}\langle\zeta_{gS}^2\zeta_S^2\rangle_{\mathbf{x}_i}+g_{\rm NL}^2\langle\zeta_{gS}^3\zeta_S^2\rangle_{\mathbf{x}_i}\right],$$





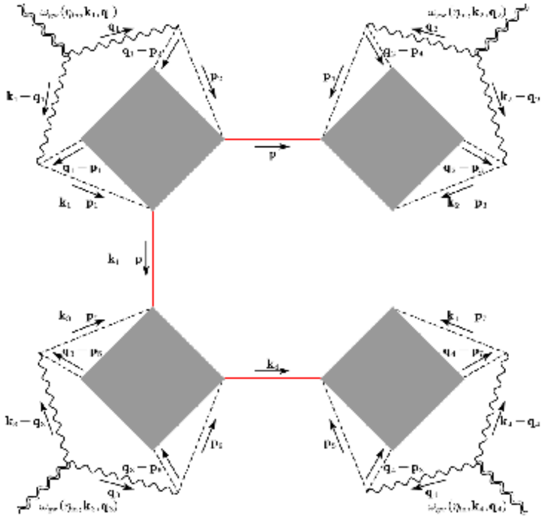
Angular bispectrum

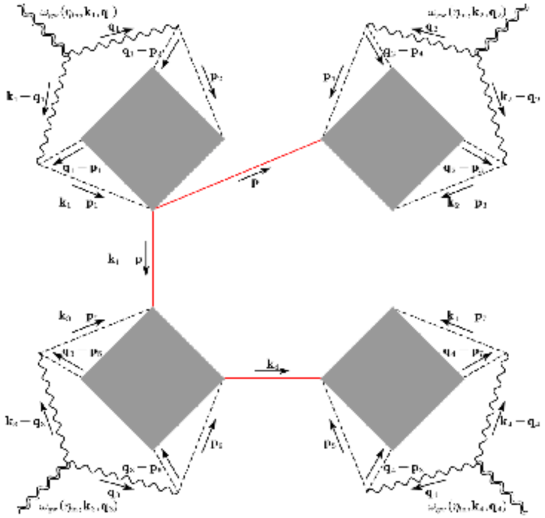


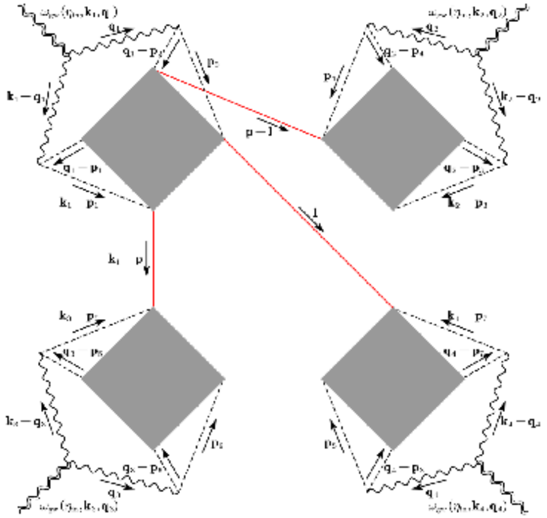


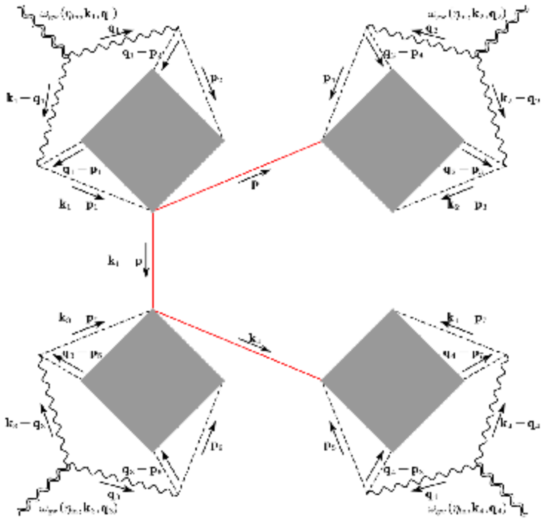
~$O(10^5)$ diagrams in total

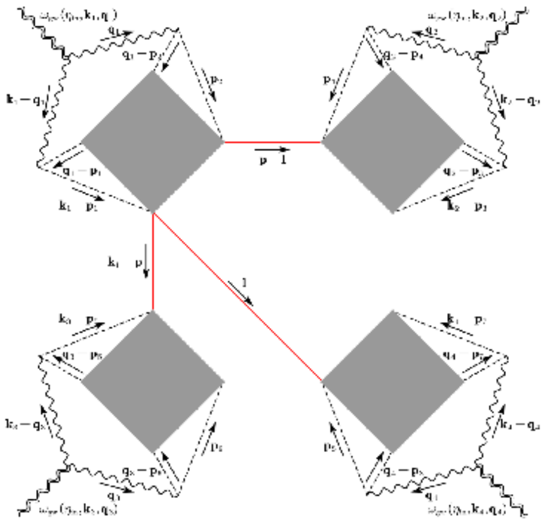
Angular trispectrum





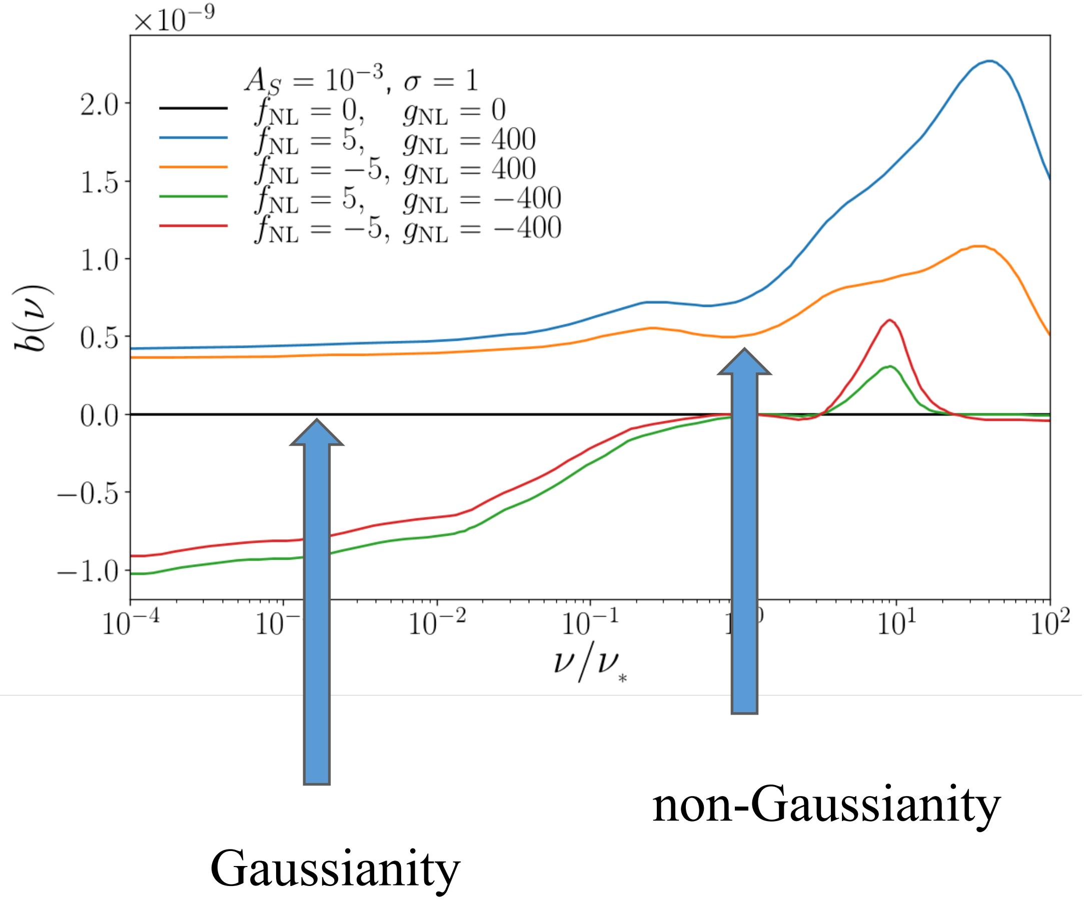


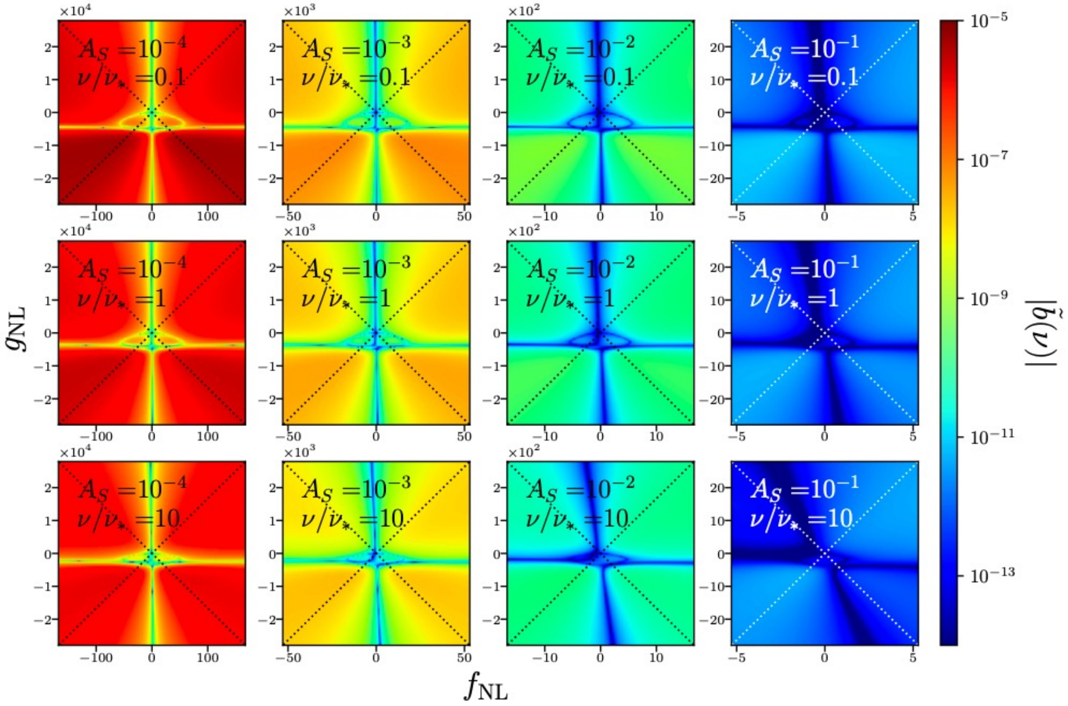




~$O(10^6)$ diagrams in total

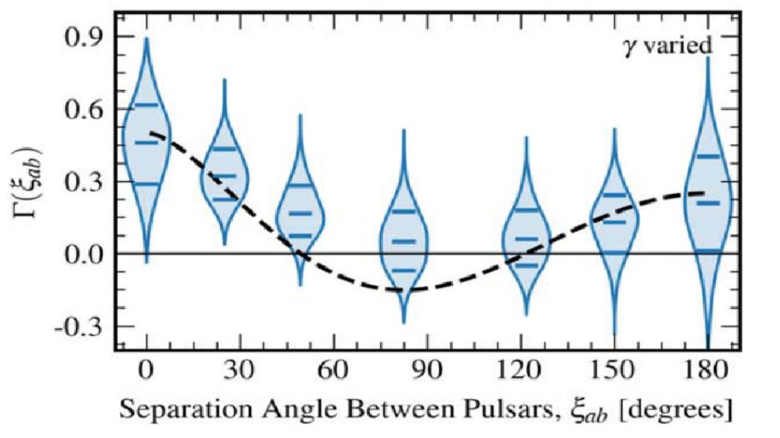
Gaussianity or not



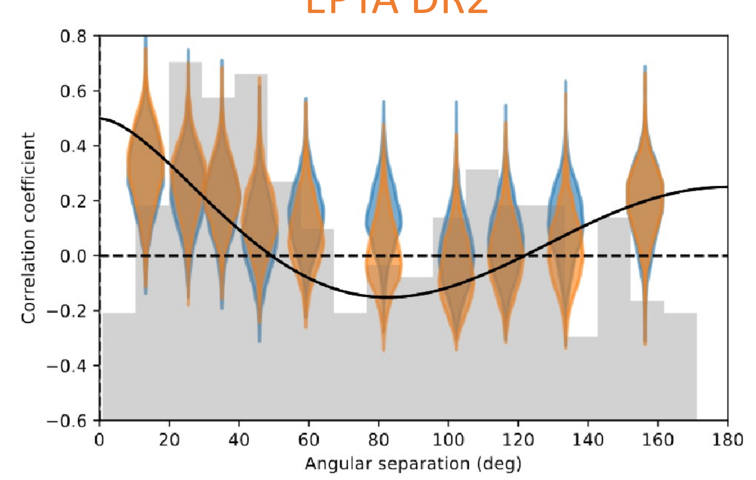


Evidence for nanohertz GWB

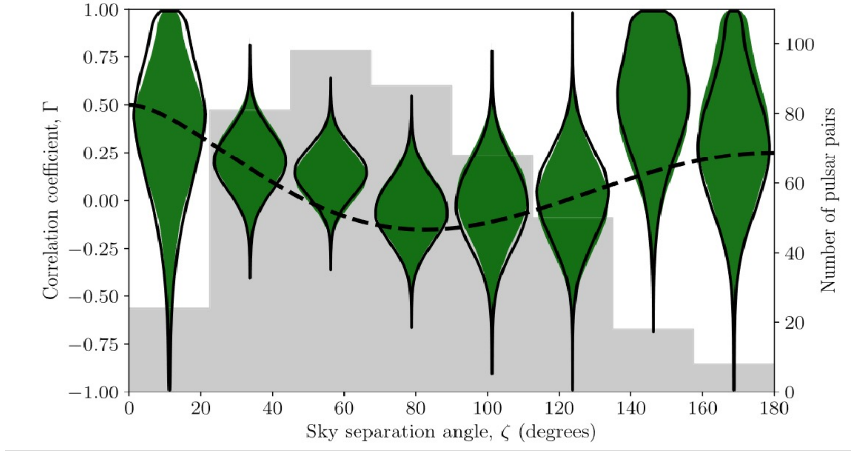
NANOGrav 15-year



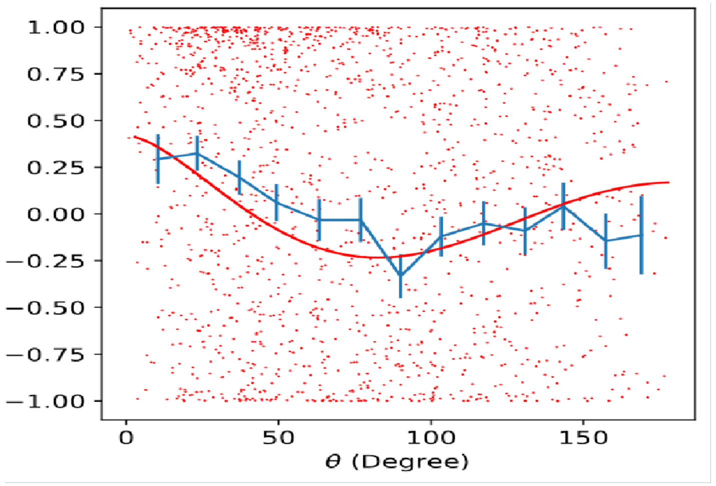
EPTA DR2



PPTA DR3

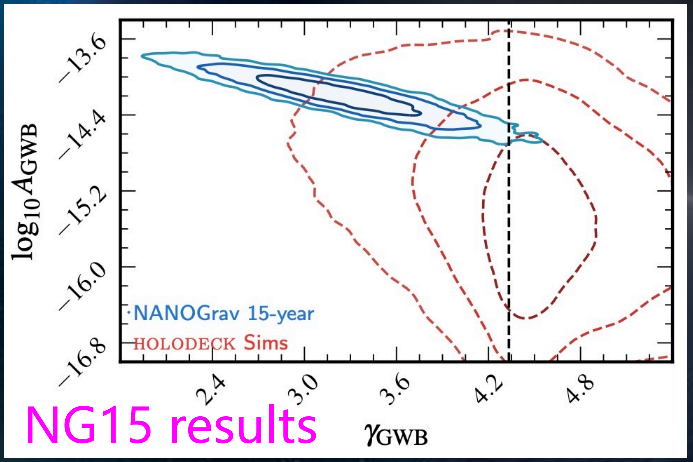


CPTA DR1

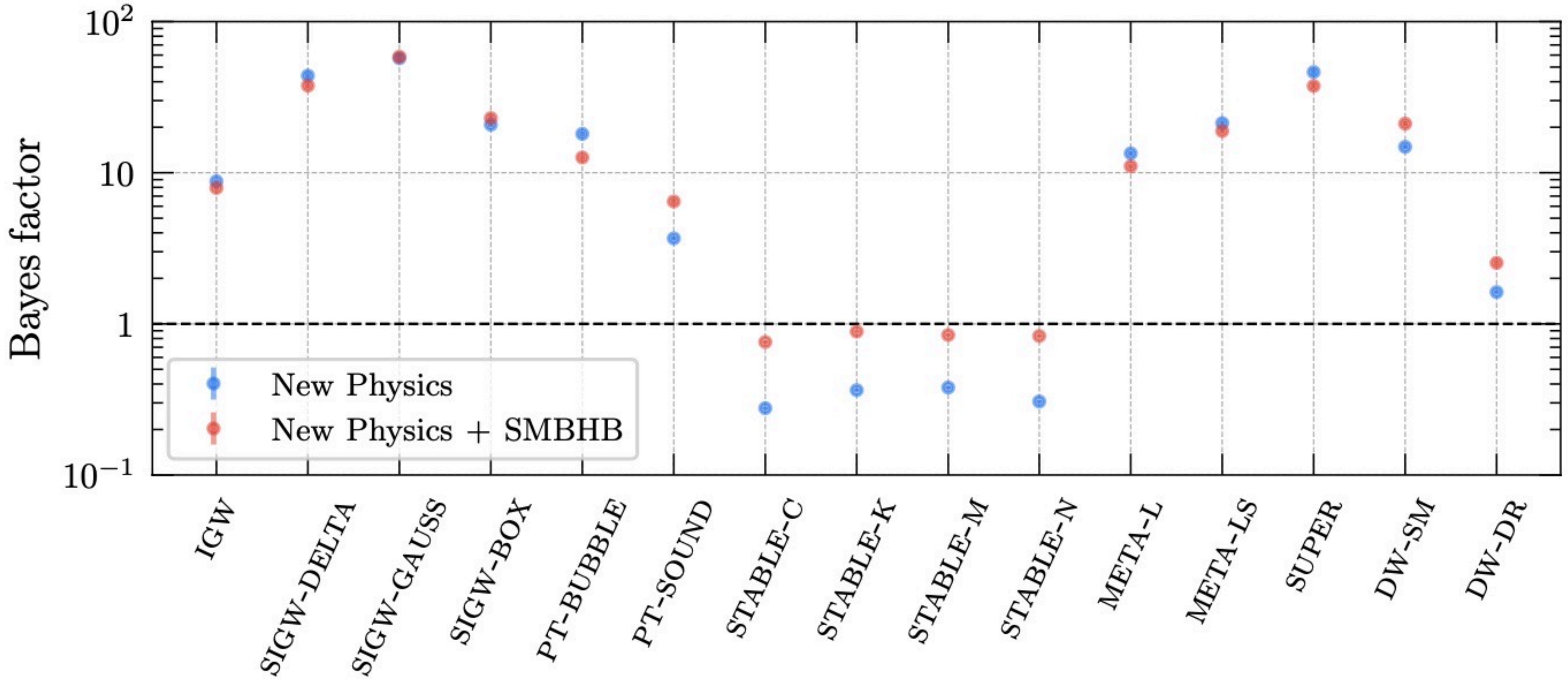


HD curve: ~3.5-4sigma (NG15); 4.6sigma (CPTA)

Binary SMBHs



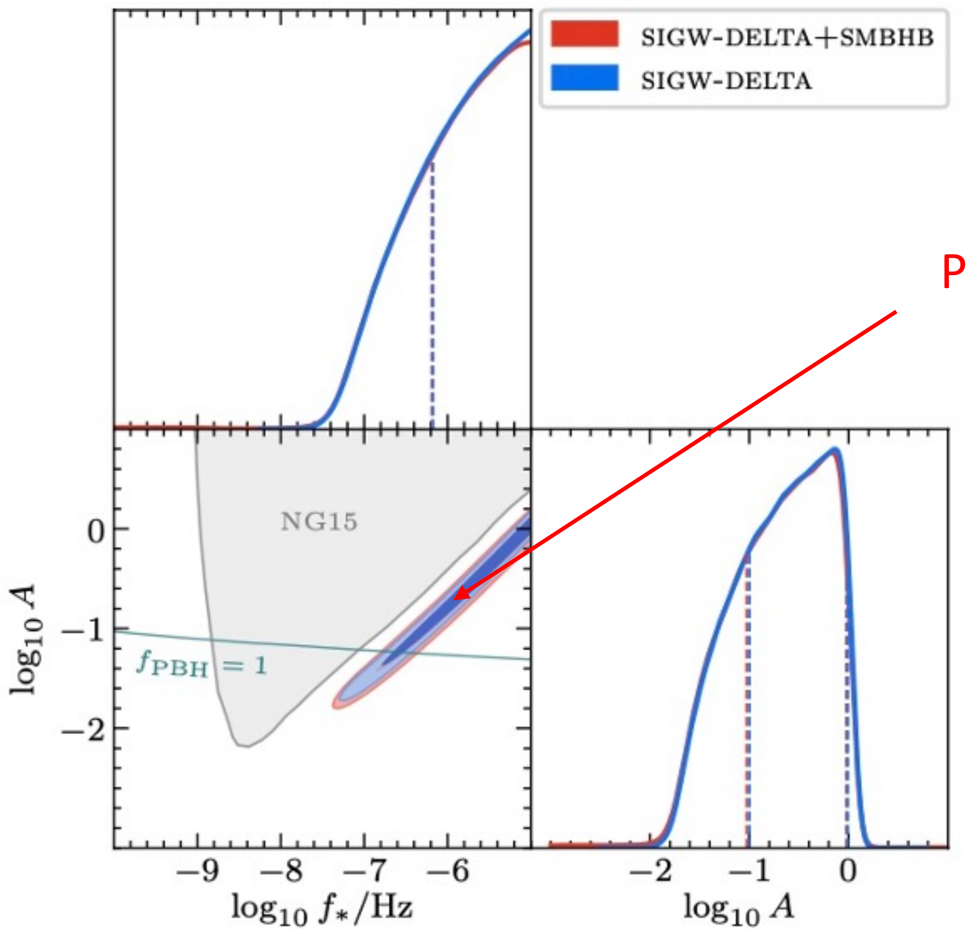
New physics



NG15 results

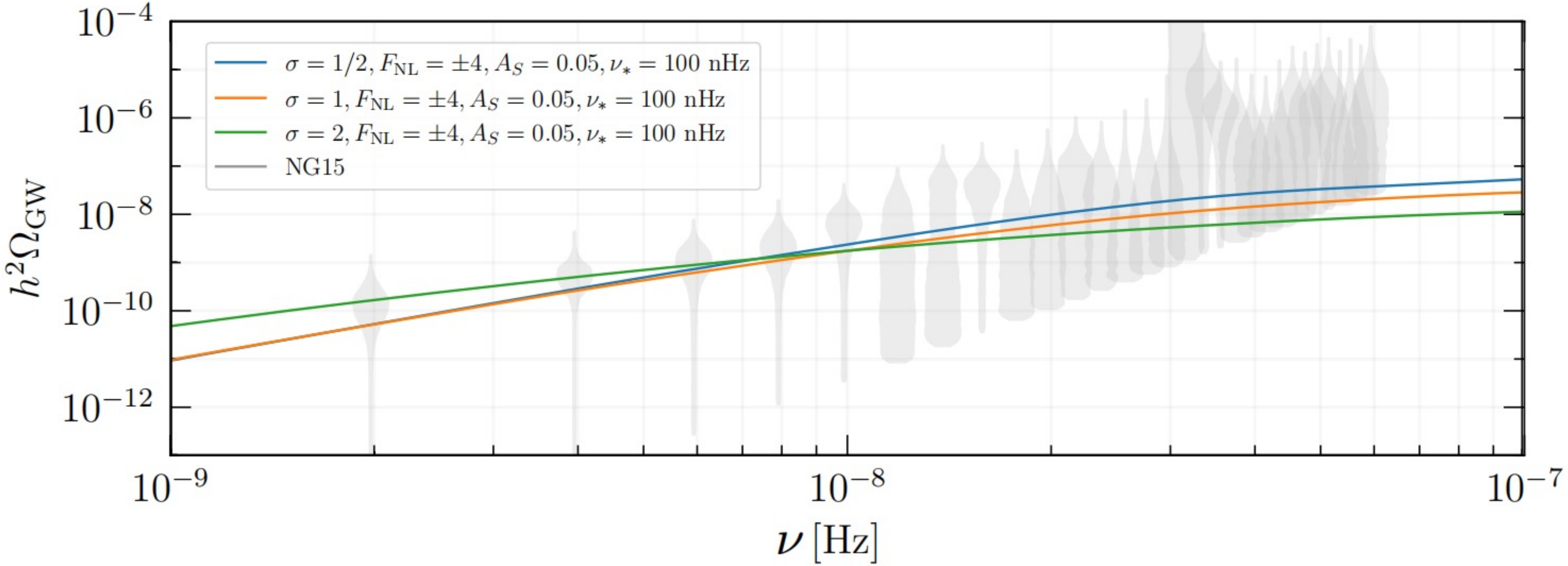
Gaussianity

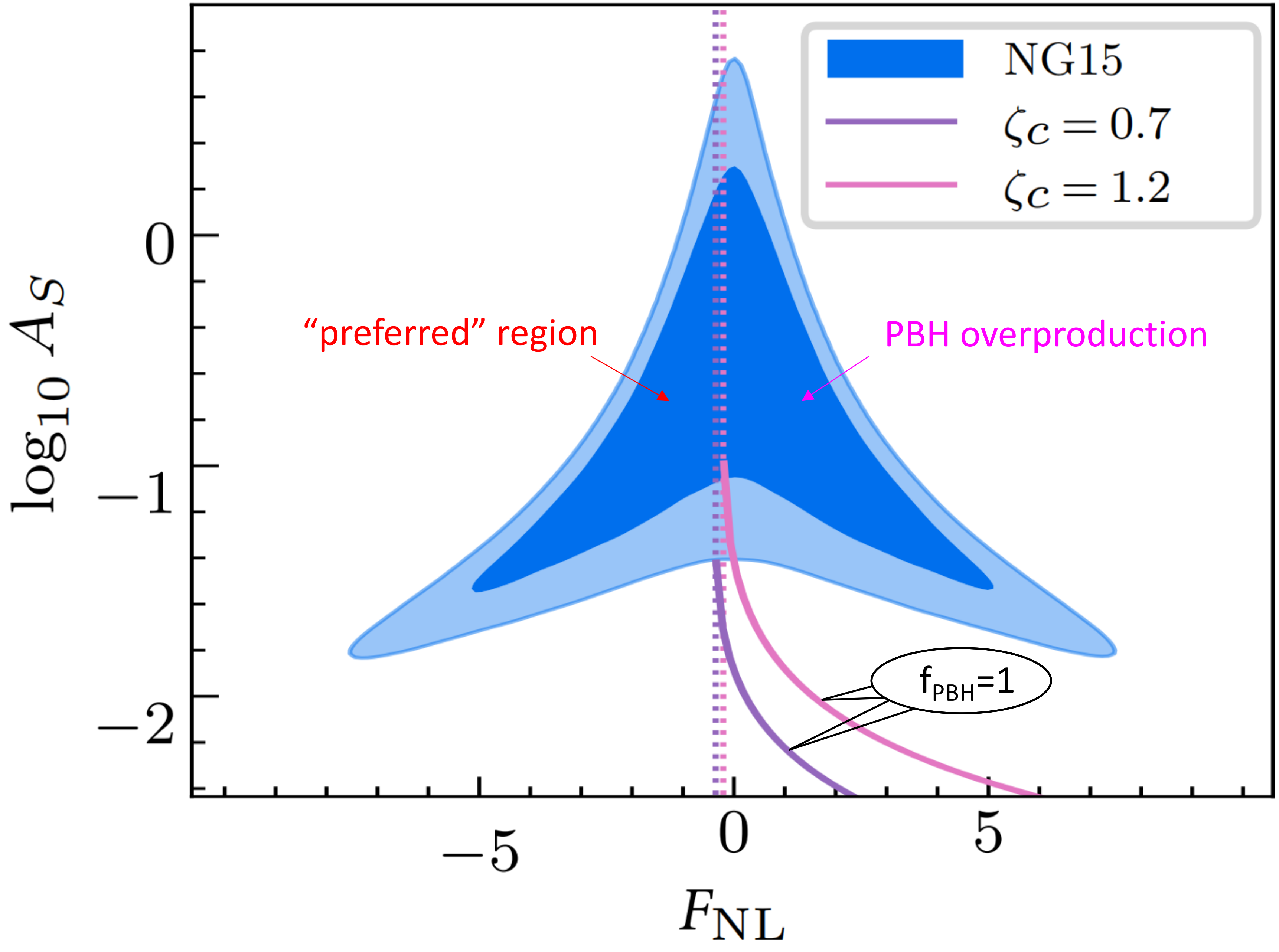
NANOGrav 15-year



PBH overproduction

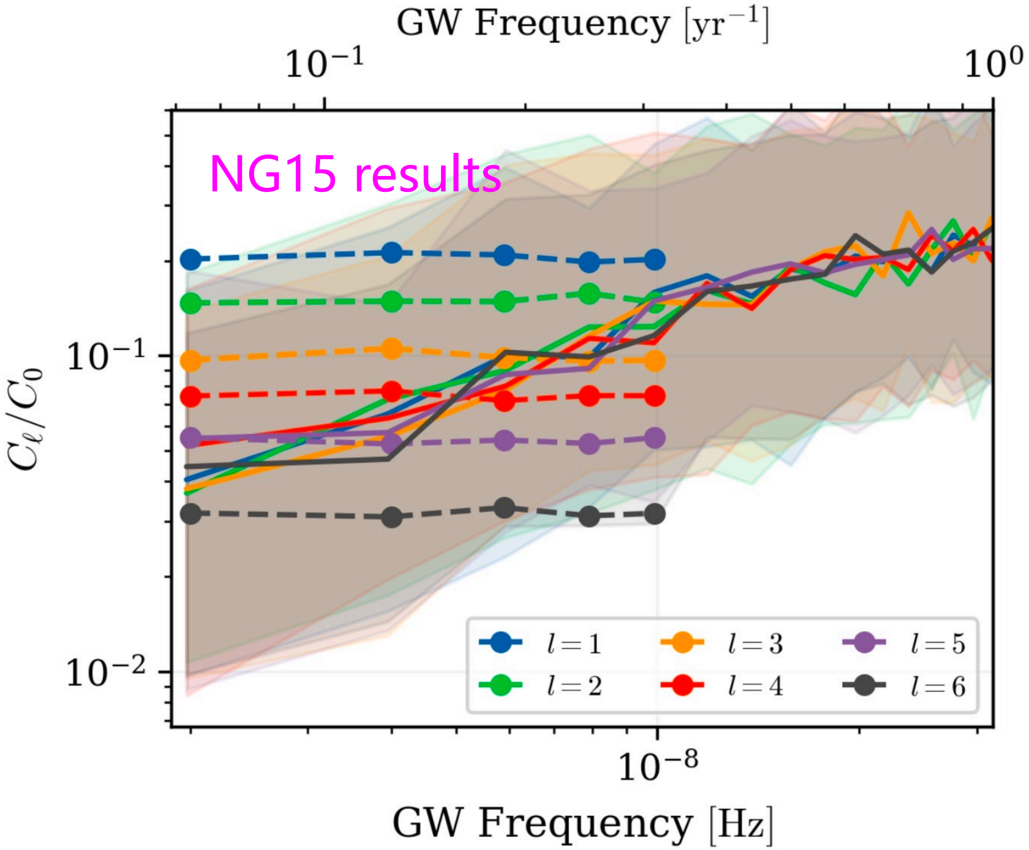
Non-Gaussianity

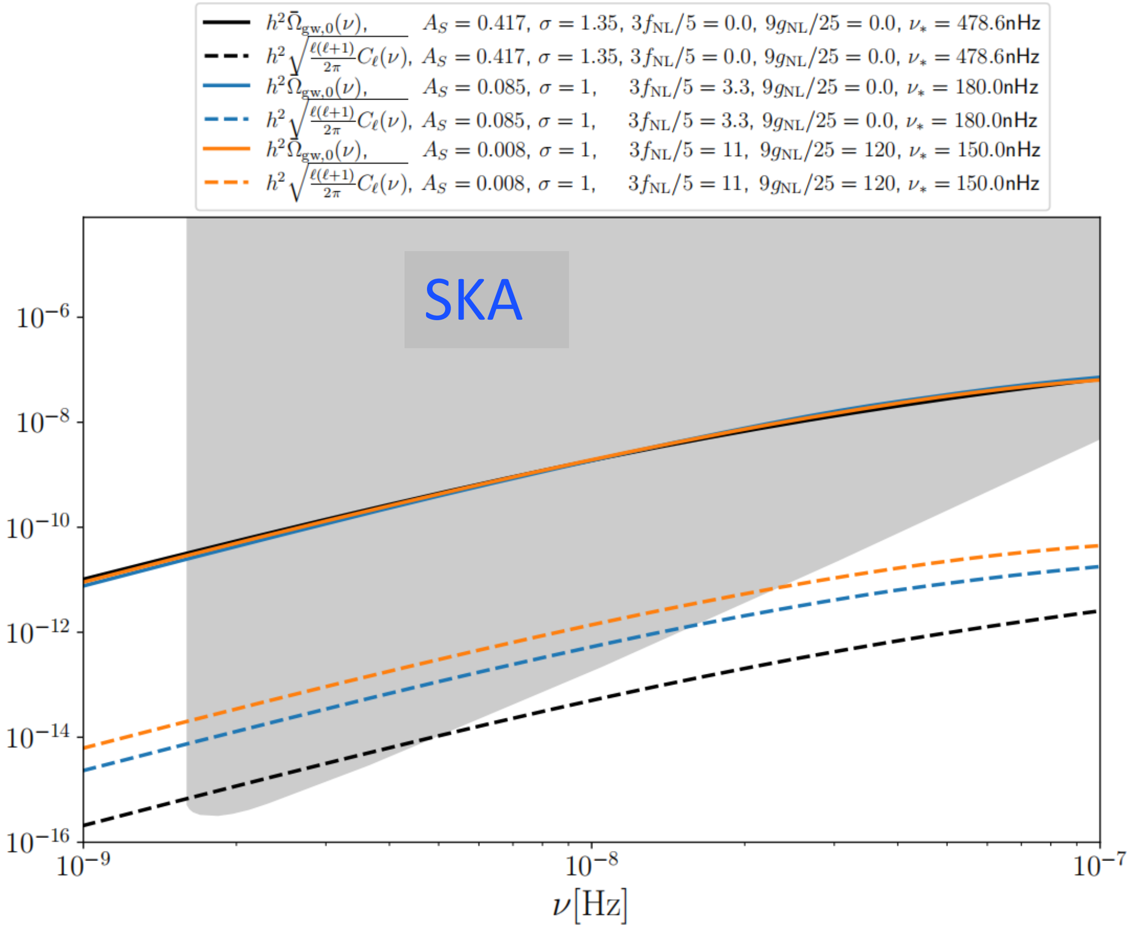




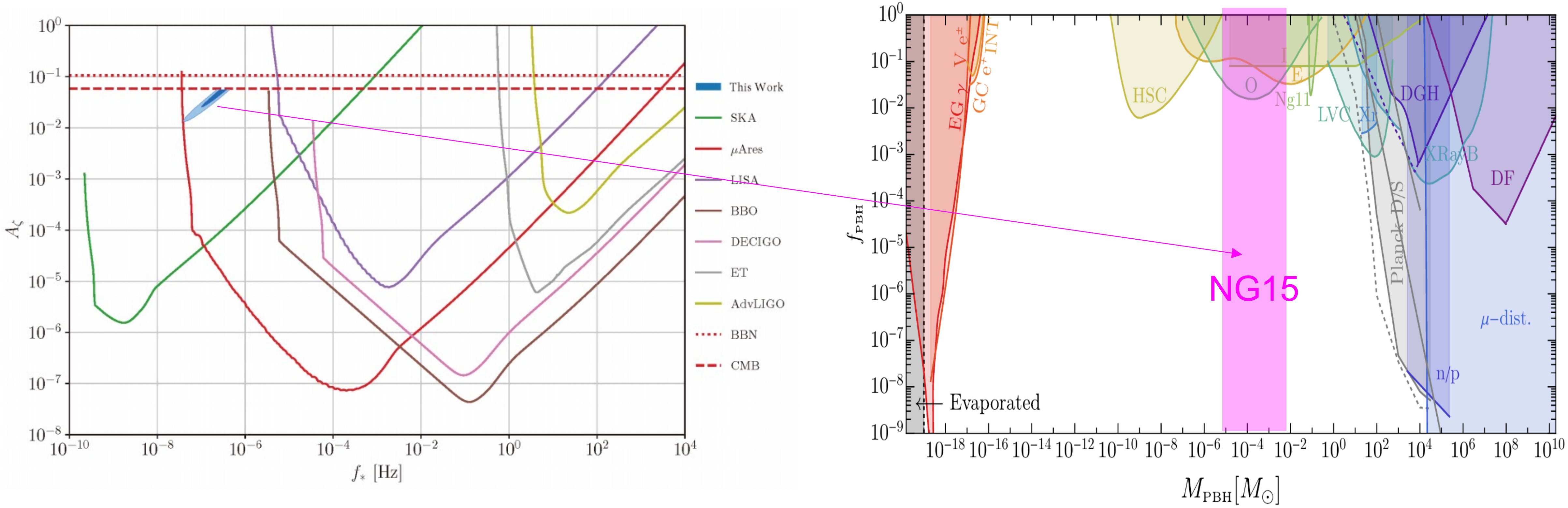
NG15
$\zeta_c = 0.7$
$\zeta_c = 1.2$
"preferred" region
PBH overproduction
$f_{PBH}=1$
$\log_{10} A_S$
F_{NL}
0
−1
−2
−5
0
5

Tests by anisotropies



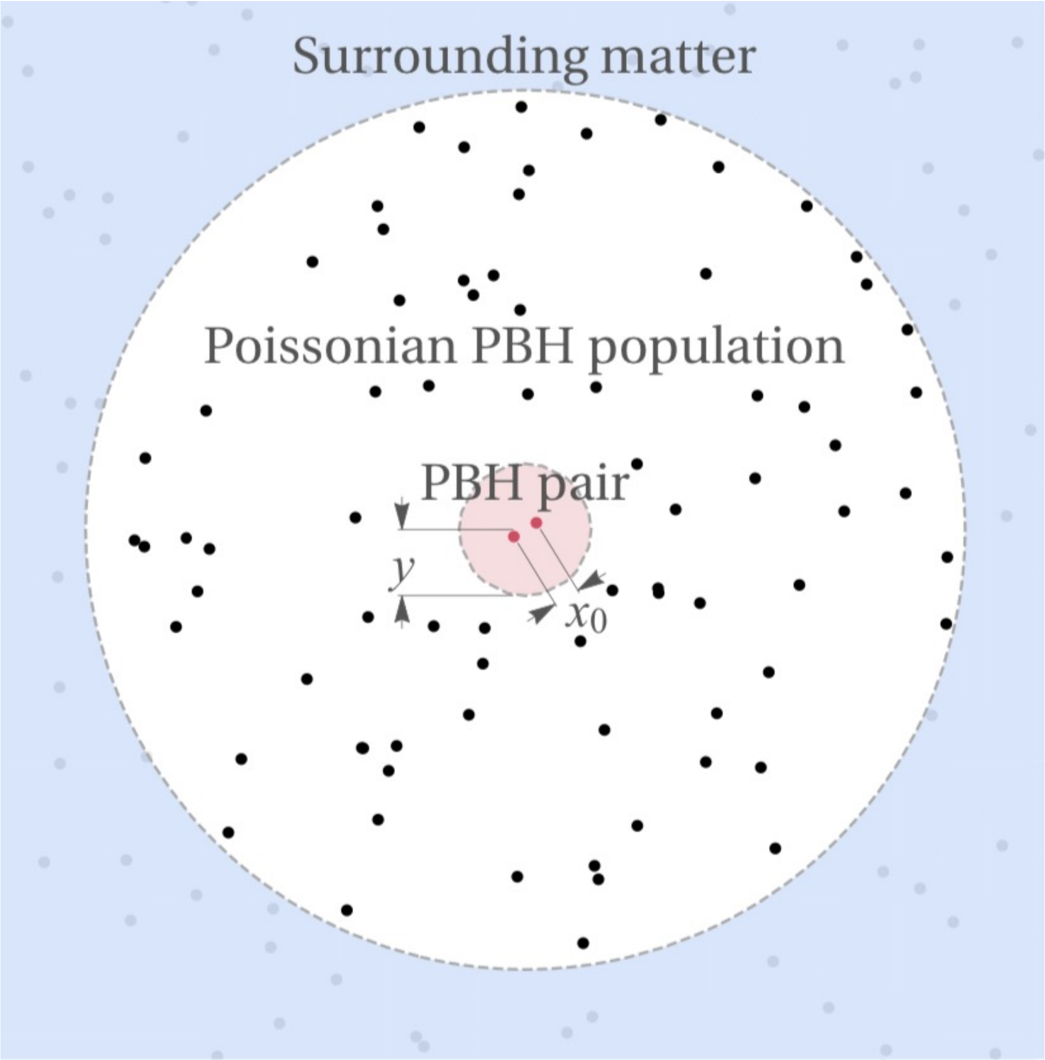


PBH mass regime

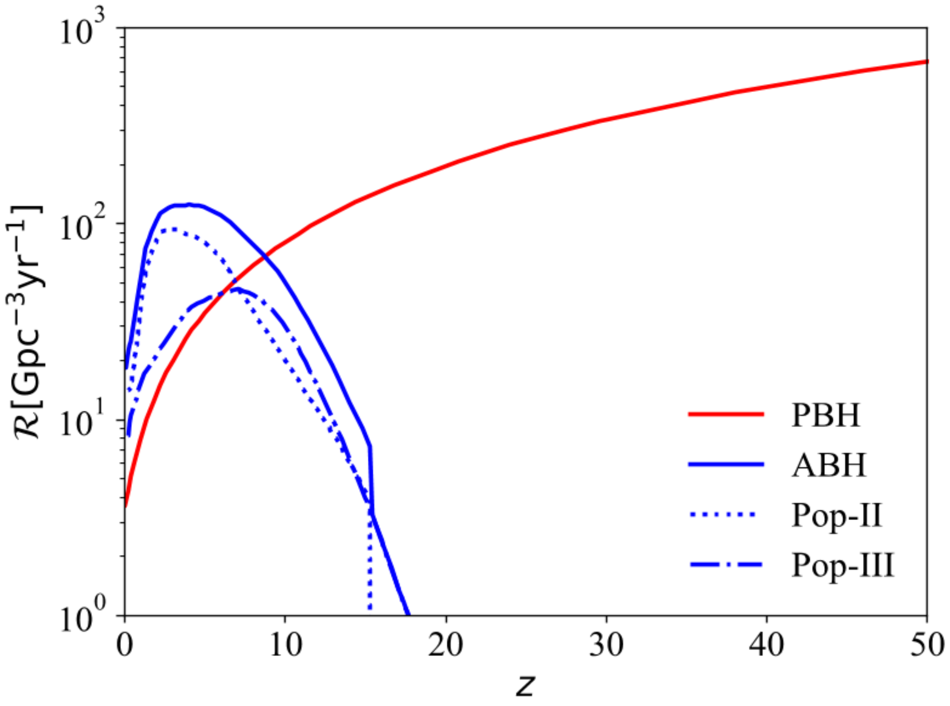


$$\frac{m_{\rm pbh}}{M_\odot} \simeq 8.5 \times 10^{-4} \left(\frac{g_{*,\rho}}{10.75}\right)^{-1/6} \left(\frac{100{\rm nHz}}{f_*}\right)^2$$

Binary PBHs

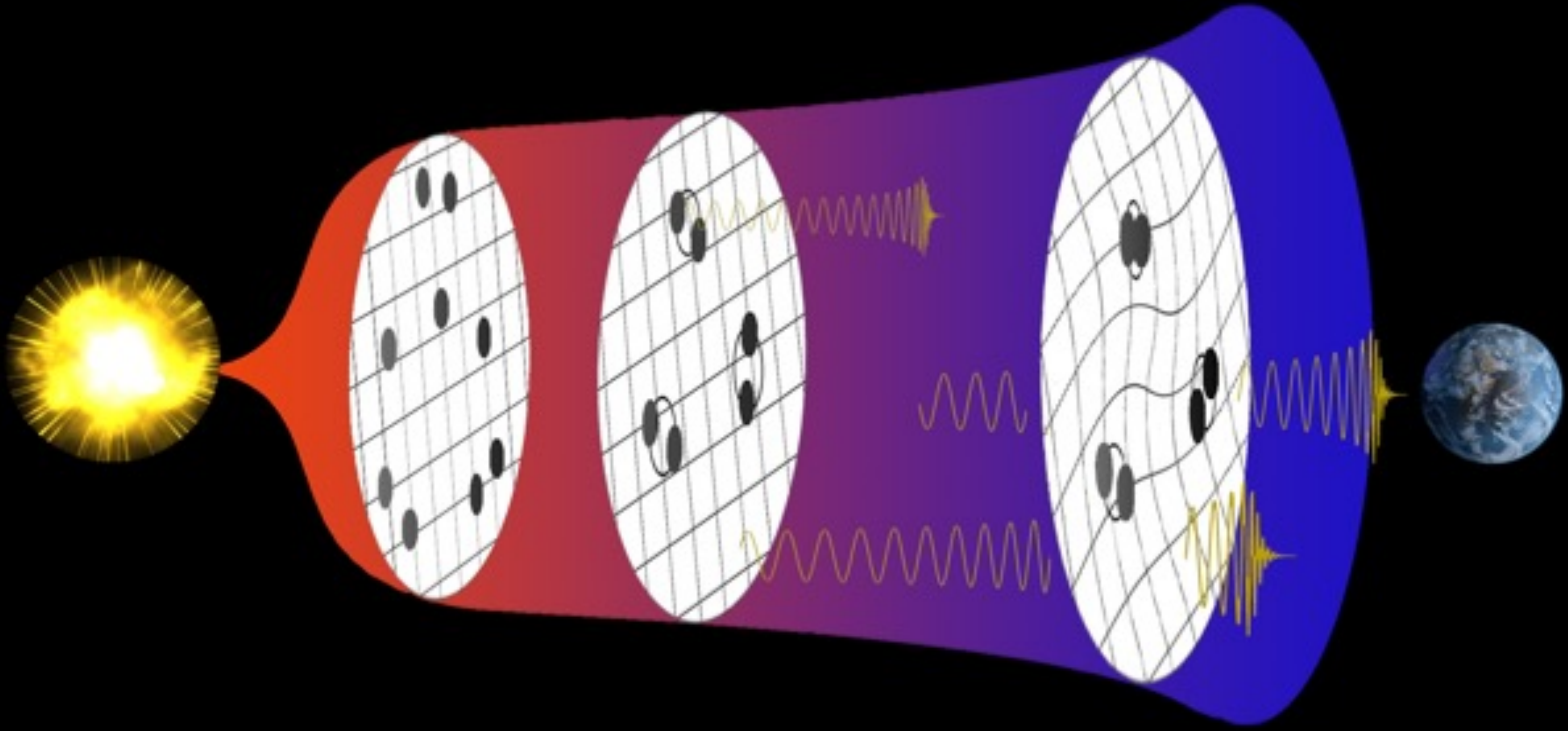


Surrounding matter
Poissonian PBH population
PBH pair
y
x_0

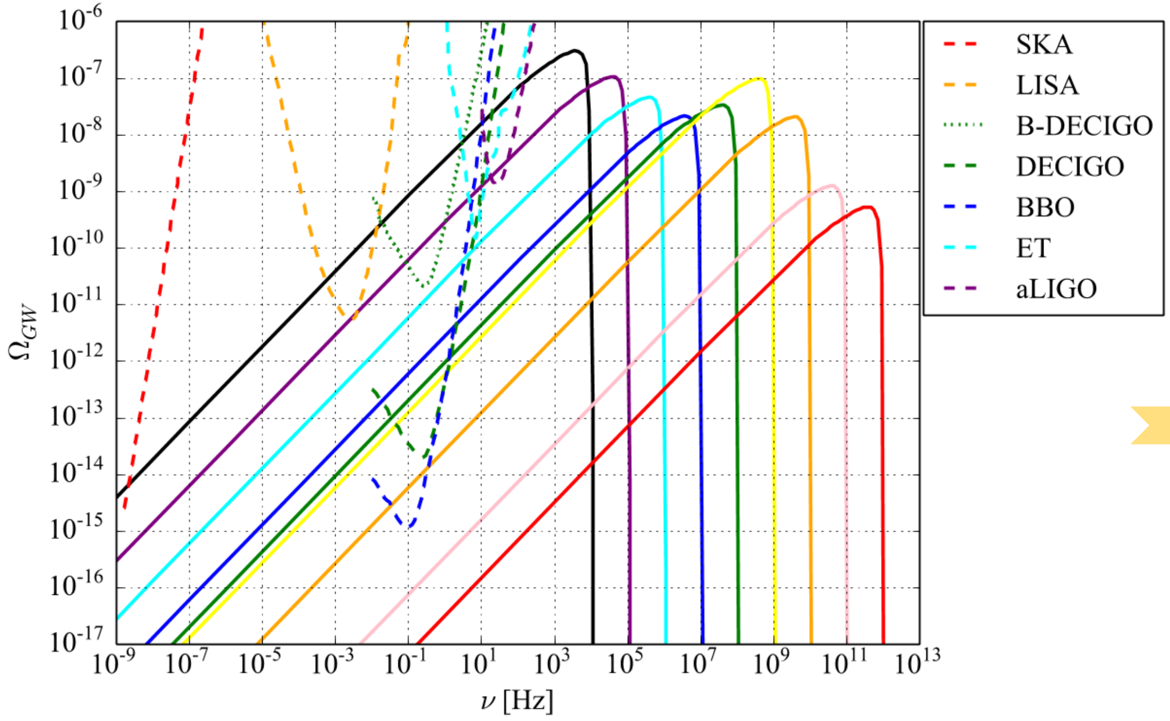


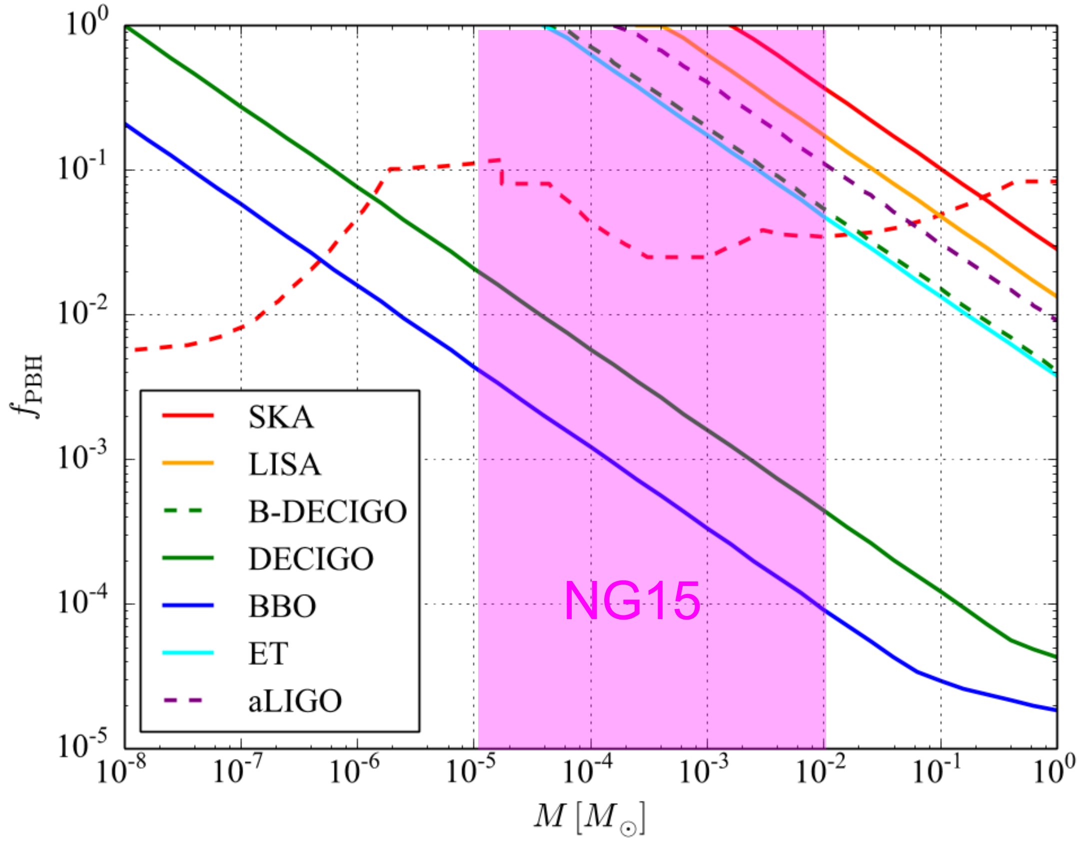
$\mathcal{R}[\mathrm{Gpc}^{-3}\mathrm{yr}^{-1}]$
10^3
10^2
10^1
10^0
0
10
20
30
40
50
z
PBH
ABH
Pop-II
Pop-III

SGWB



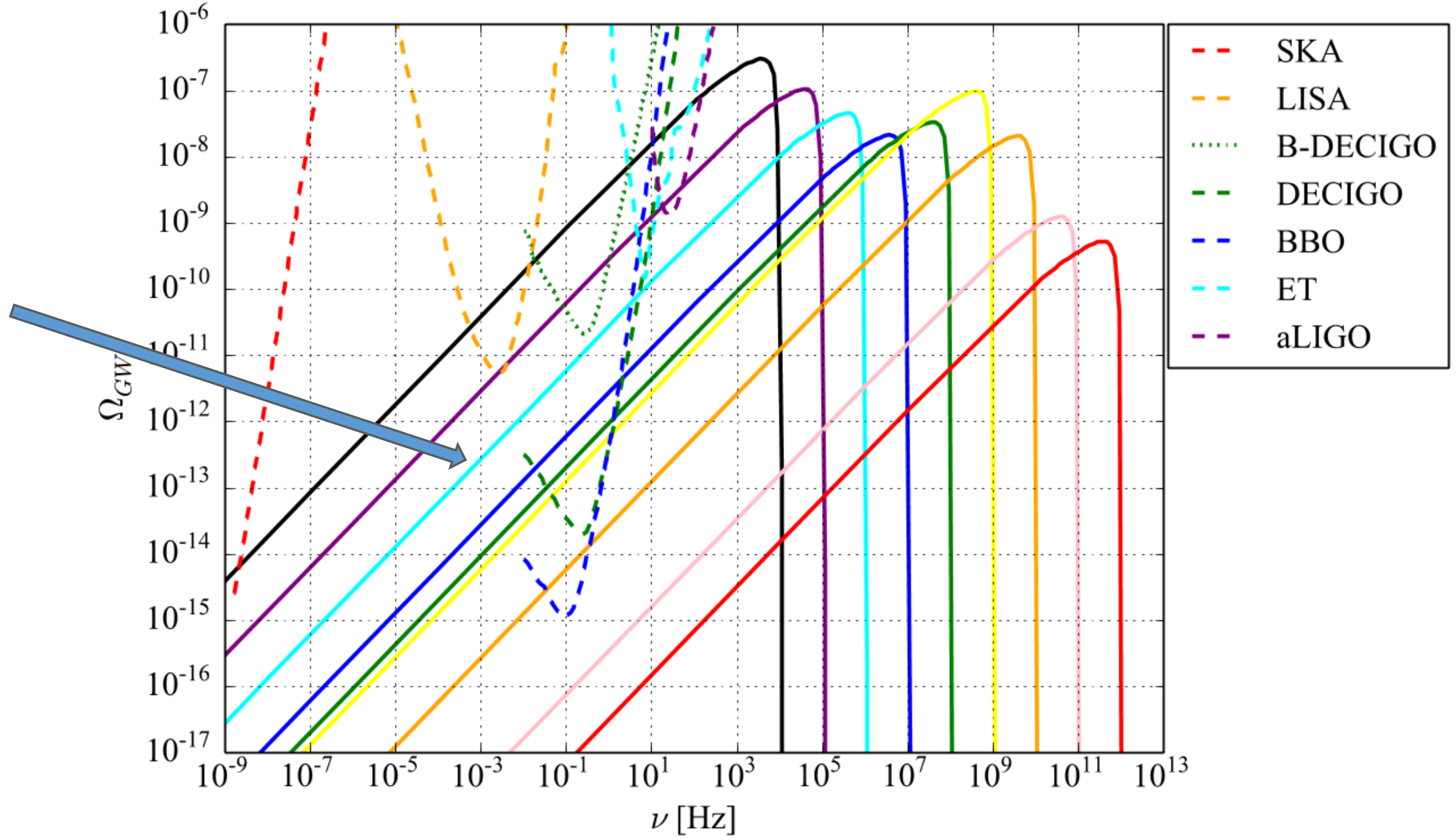
Energy-density fraction spectrum



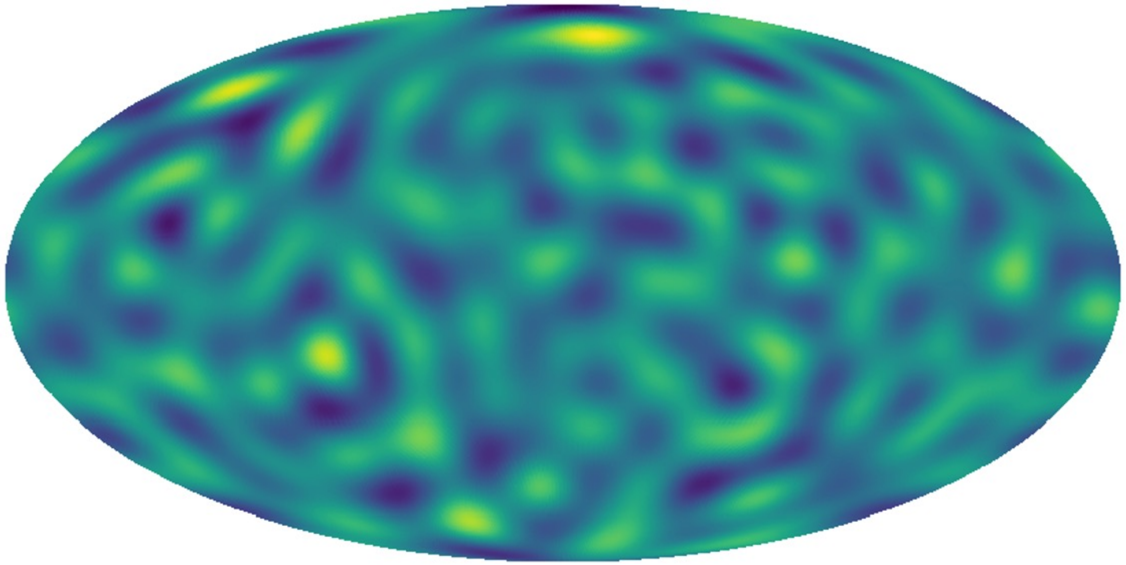


A challenge

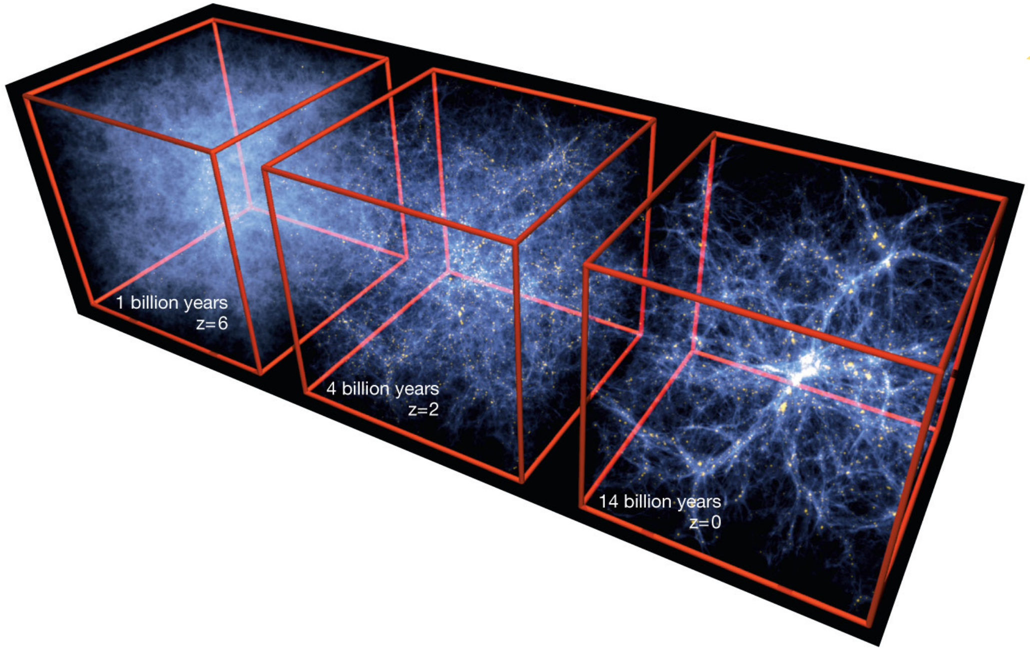
$\Omega_{GW}(f) \propto f^{2/3}$



SGWB anisotropies



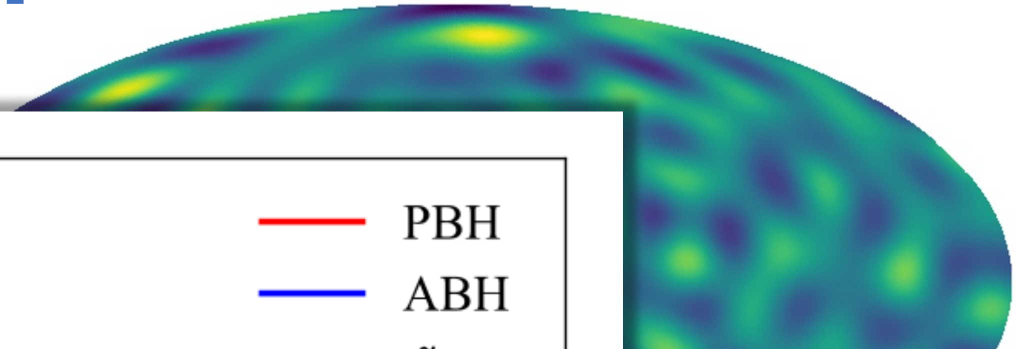
different clustering

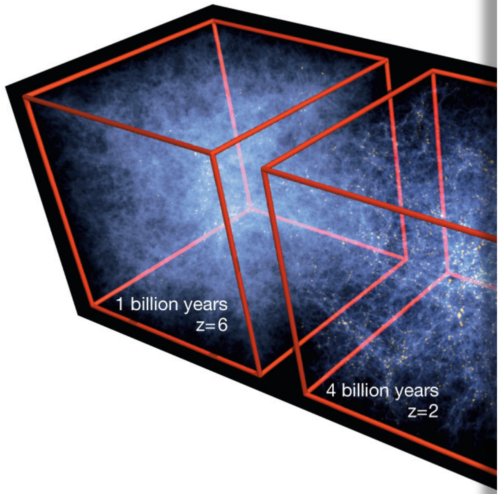


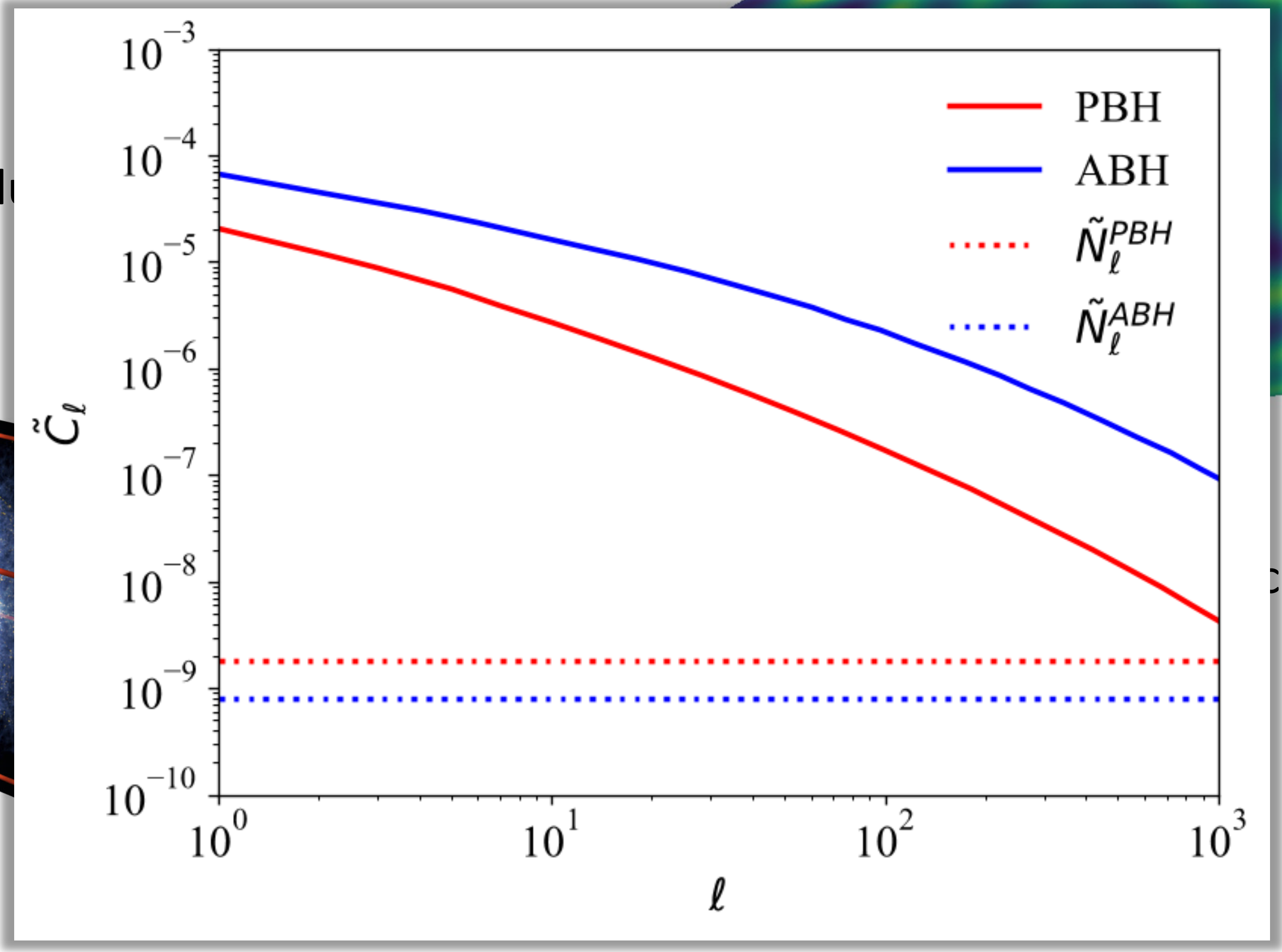
different anisotropic maps

Angular power spectrum

different cl







c maps

Summary

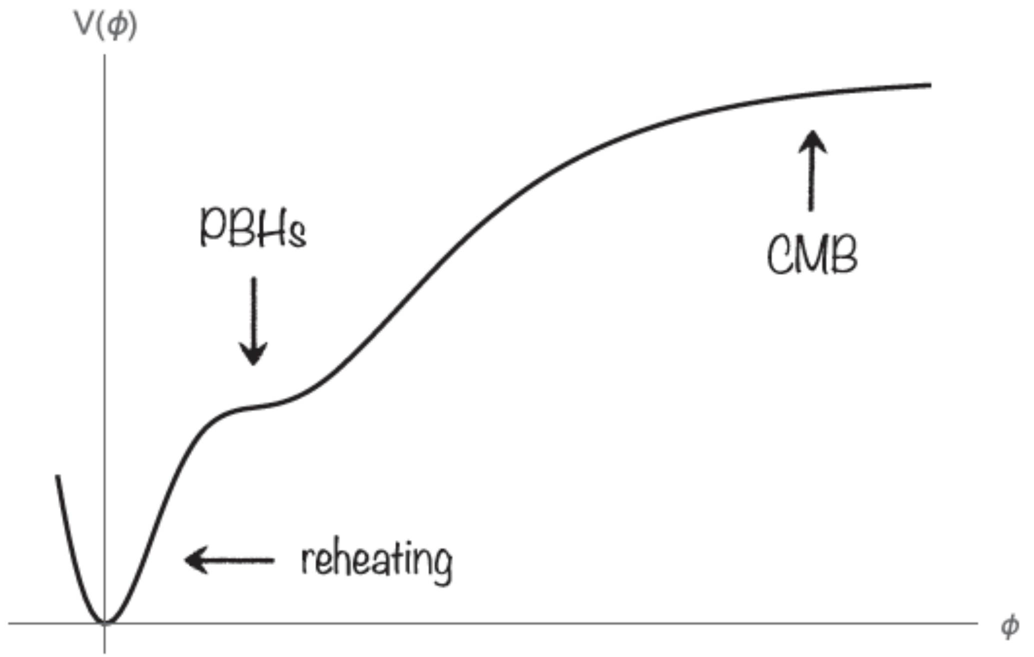
Take-home messages:

✓ SIGW

- Statistics of primordial perturbations
- Origin of the Universe

✓ SGWB

- Primordial black holes vs. Astro black holes
- Nature of dark matter



Thanks for your attention!