

Consequences of phase transitions occurred during inflation

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2009.12381, 2201.05171 w/ Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou

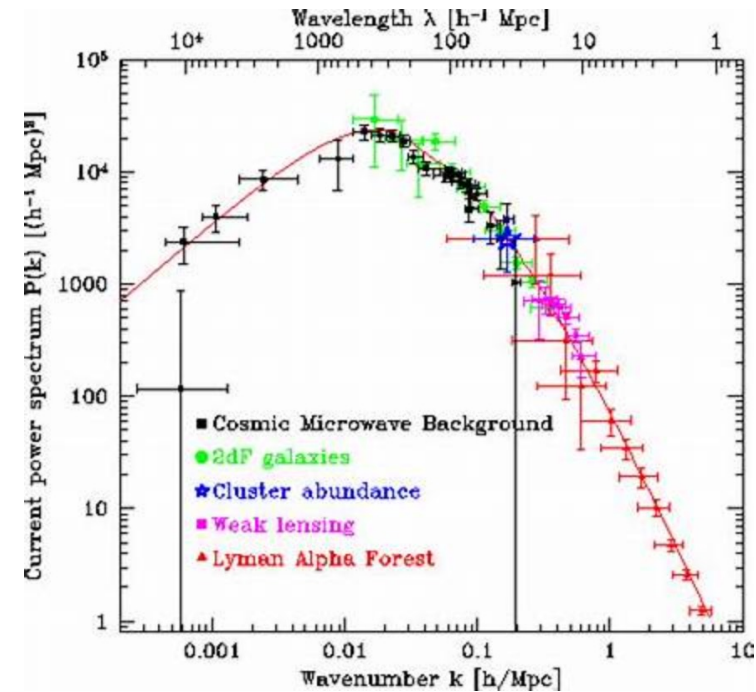
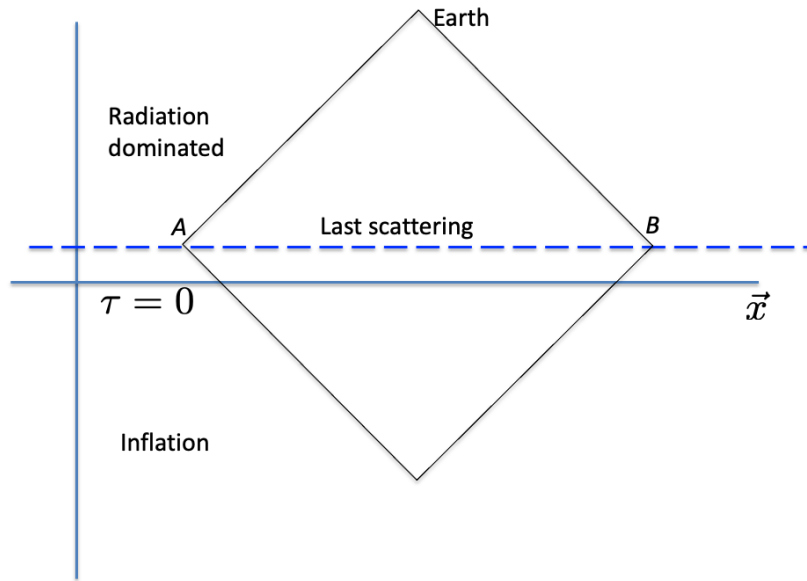
2208.14857 w/ Xi Tong and Siyi Zhou

2304.02361 w/ Chen Yang

2308.00070 w/ Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang

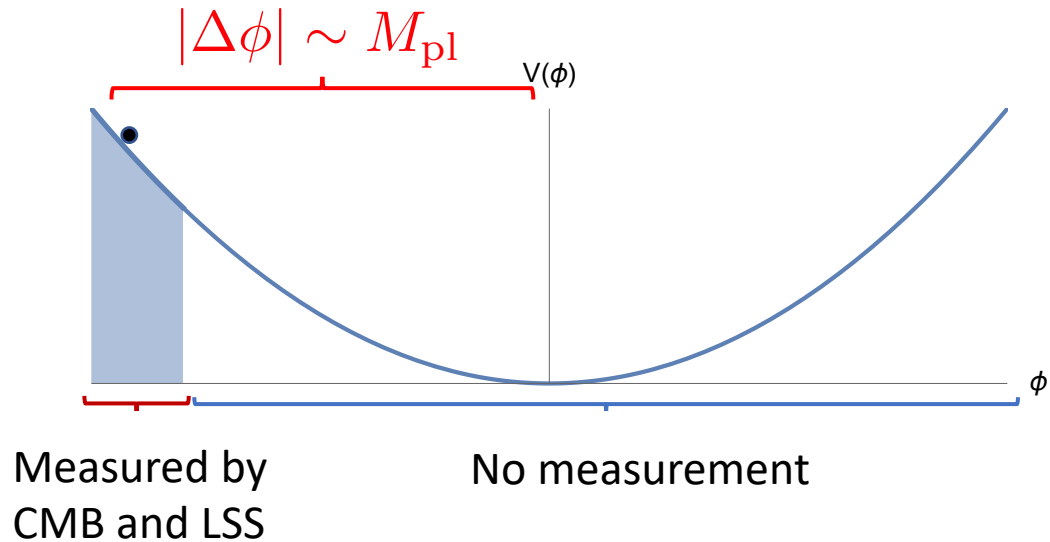
2404.xxxxxx w/ Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang

Very brief introduction of inflation



- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

Slow roll models



- The inflaton must couple to some spectator field.
- The masses or couplings in the spectator sector can be changed drastically due to the evolution of the inflaton field.

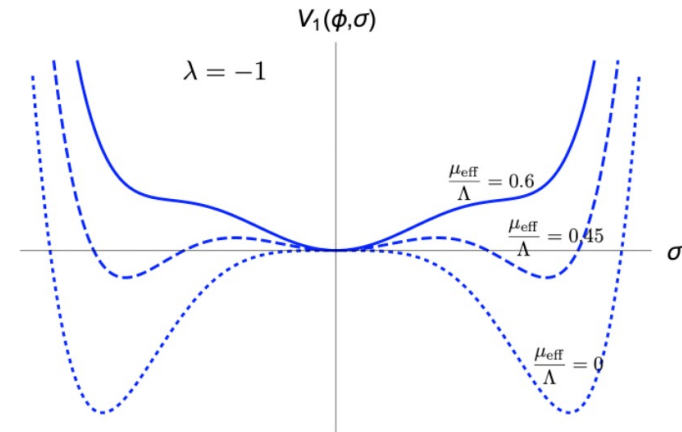
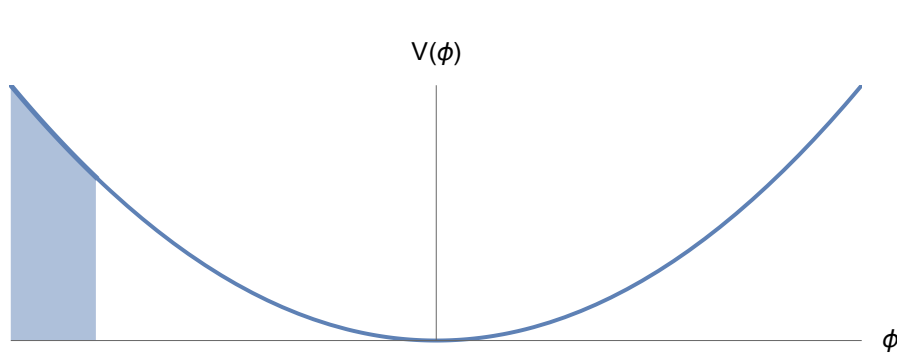
Induced phase transition in spectator sectors

- ϕ : inflaton field

σ : spectator field

Example 1:

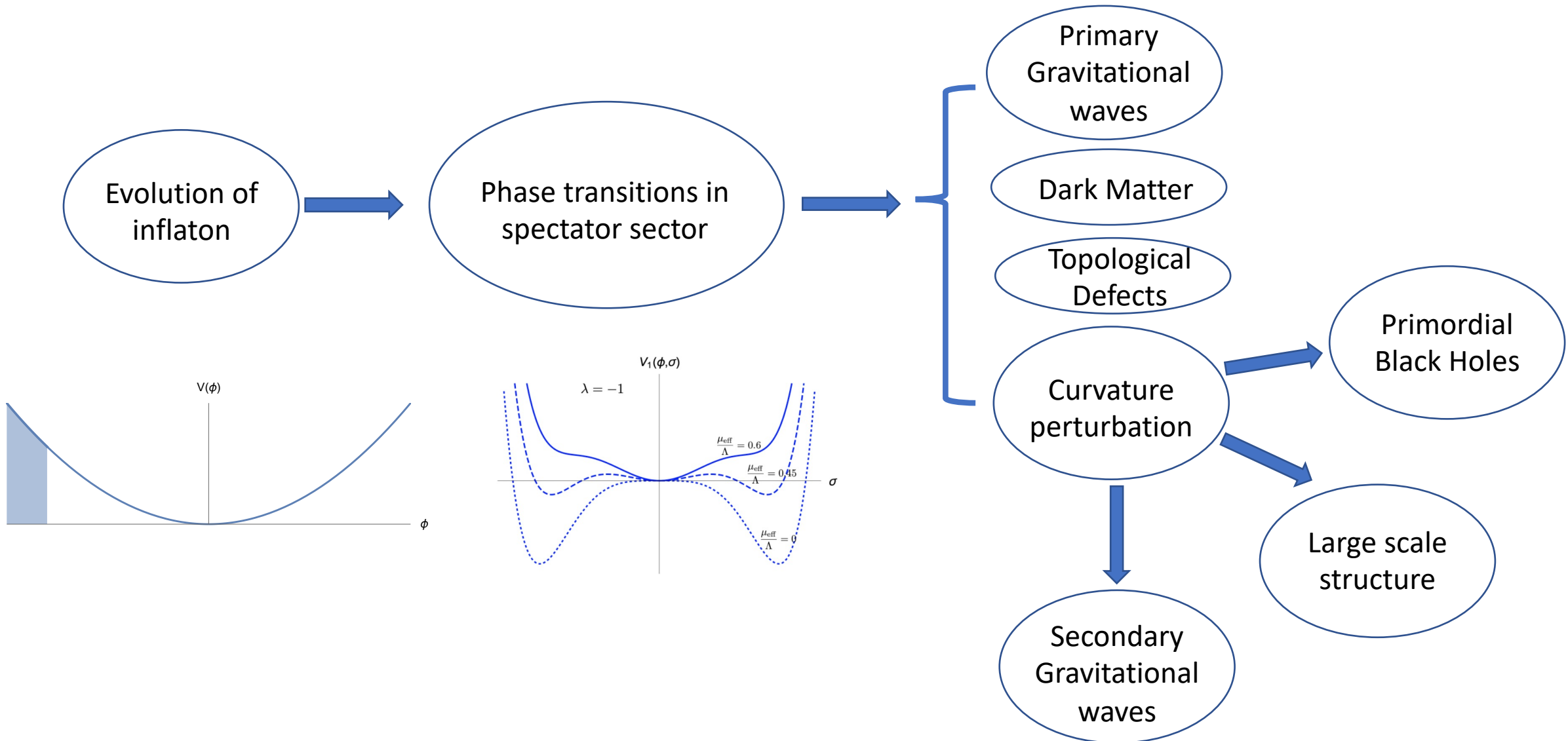
$$V_1(\phi, \sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$



Example 2:

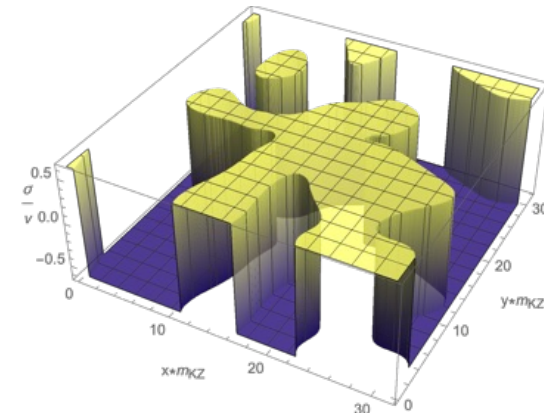
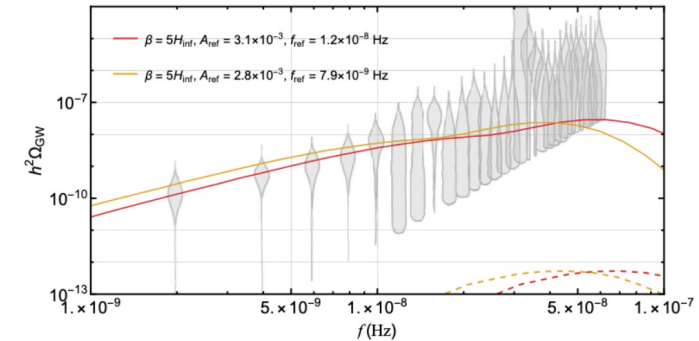
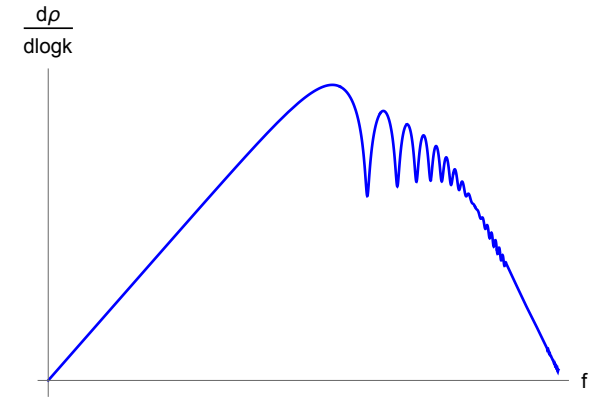
$$\mathcal{L}_\sigma = -\left(1 - \frac{c^2\phi^2}{\Lambda^2}\right)\frac{1}{4g^2}G_{\mu\nu}^a G^{a\mu\nu}$$

Induced phase transition in spectator sectors

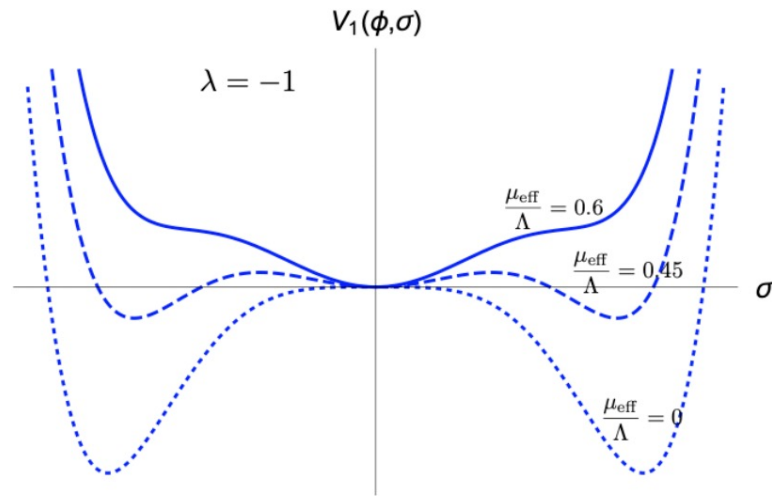


Outline

- First-order phase transitions during inflation.
 - Primary GWs
 - Dark Matter
 - Curvature perturbation
 - Secondary GWs
 - Primordial Black Holes
- GWs from second-order phase transitions (domain walls) during inflation.
- Summary and outlook

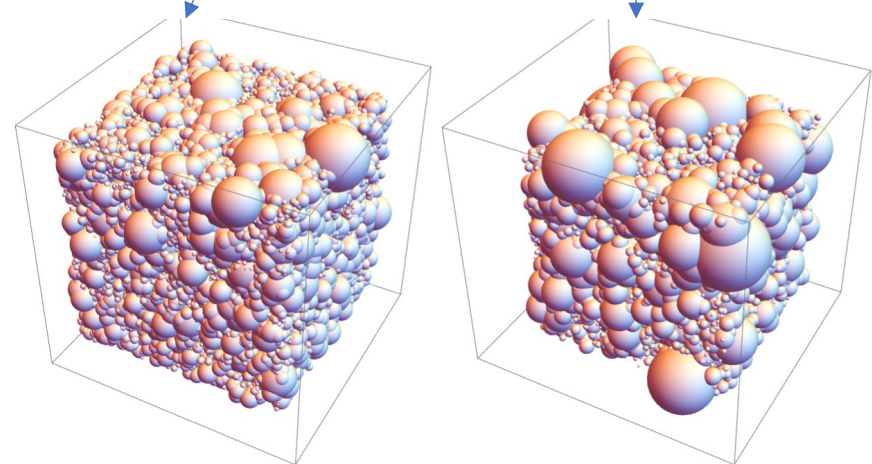
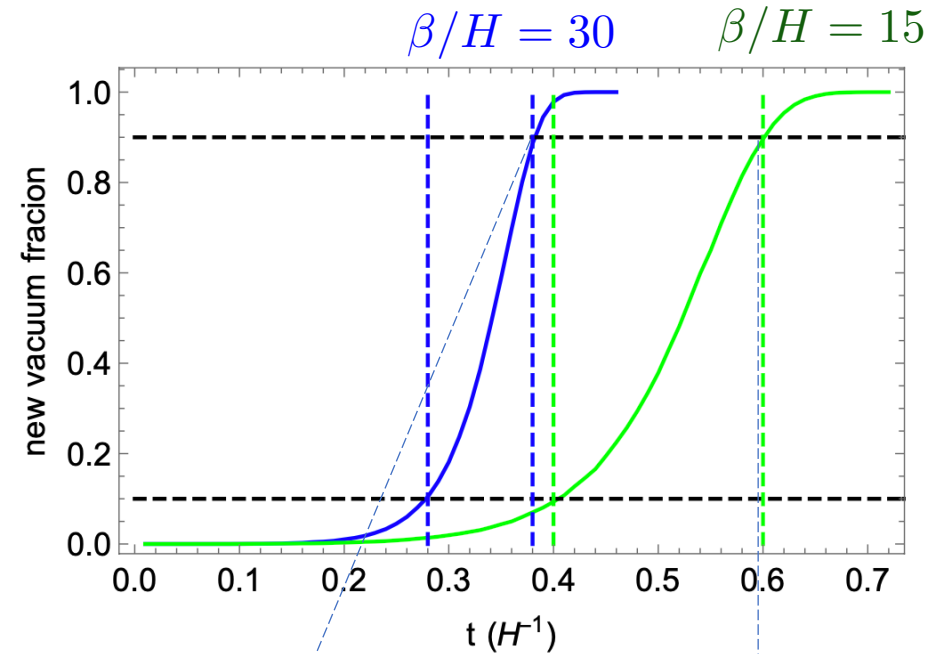


First-order phase transition during inflation



S_4 becomes smaller during

- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.



GW from instantaneous and local sources (qualitative study)

- E.O.M. of GW

$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$$

$$ds^2 = a^2(\tau) [-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$



Traceless and transverse

- For an instantaneous and local source,

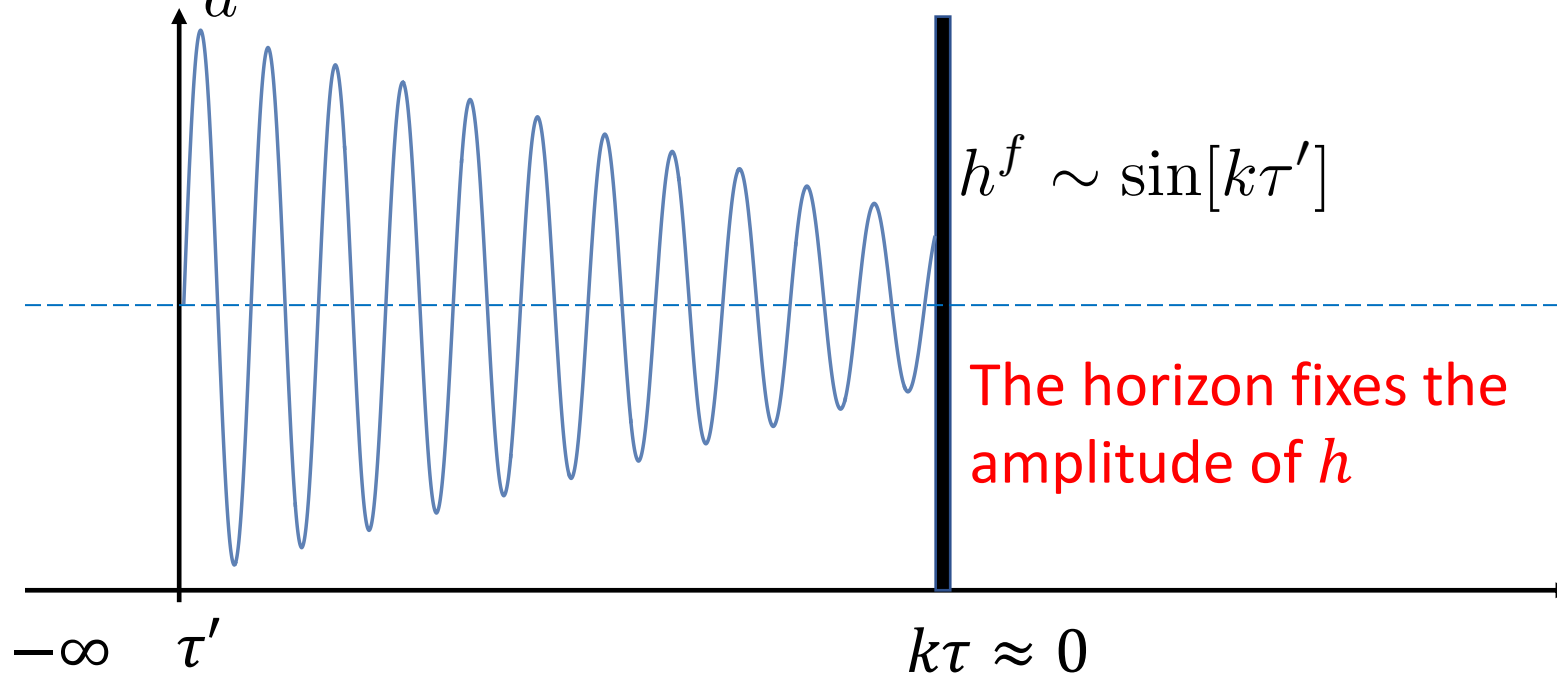
$$\sigma_{ij} \sim \delta(\mathbf{x})\delta(\tau - \tau')$$

- E.O.M. in Fourier space

$$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2 h(\tau, \mathbf{k}) = 16\pi G_N a^{-1} T \delta(\tau - \tau')$$

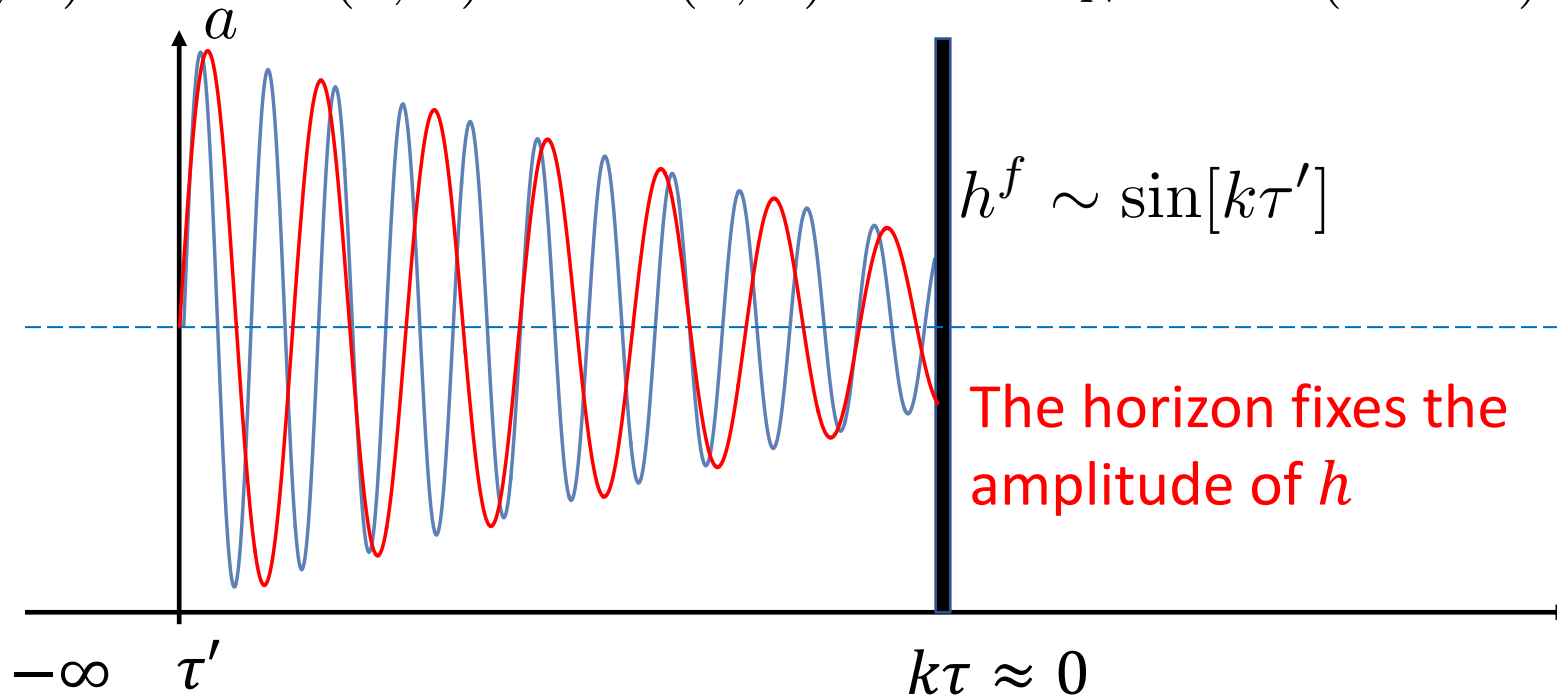
GW from instantaneous and local sources (qualitative study)

- $$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2h(\tau, \mathbf{k}) = 16\pi G_N a^{-1}T\delta(\tau - \tau')$$



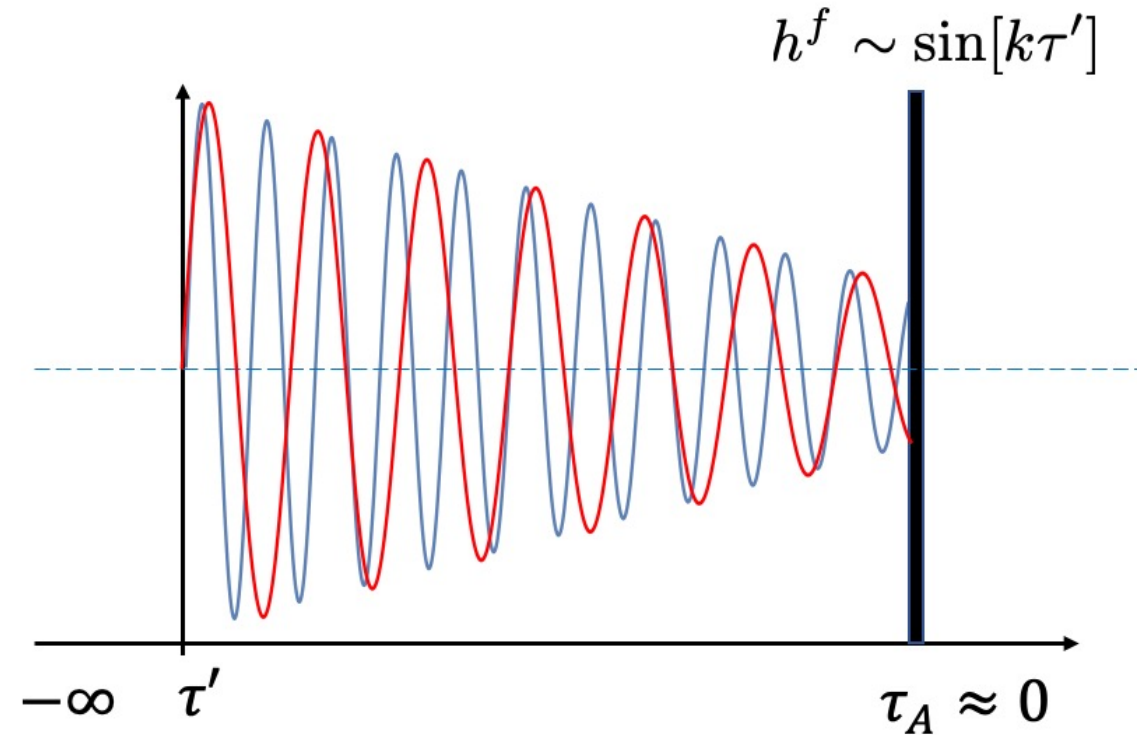
GW from instantaneous and local sources (qualitative study)

- $$h''(\tau, \mathbf{k}) + \frac{2a'}{a}h'(\tau, \mathbf{k}) + k^2h(\tau, \mathbf{k}) = 16\pi G_N a^{-1}T\delta(\tau - \tau')$$



GW from instantaneous and local sources (qualitative study)

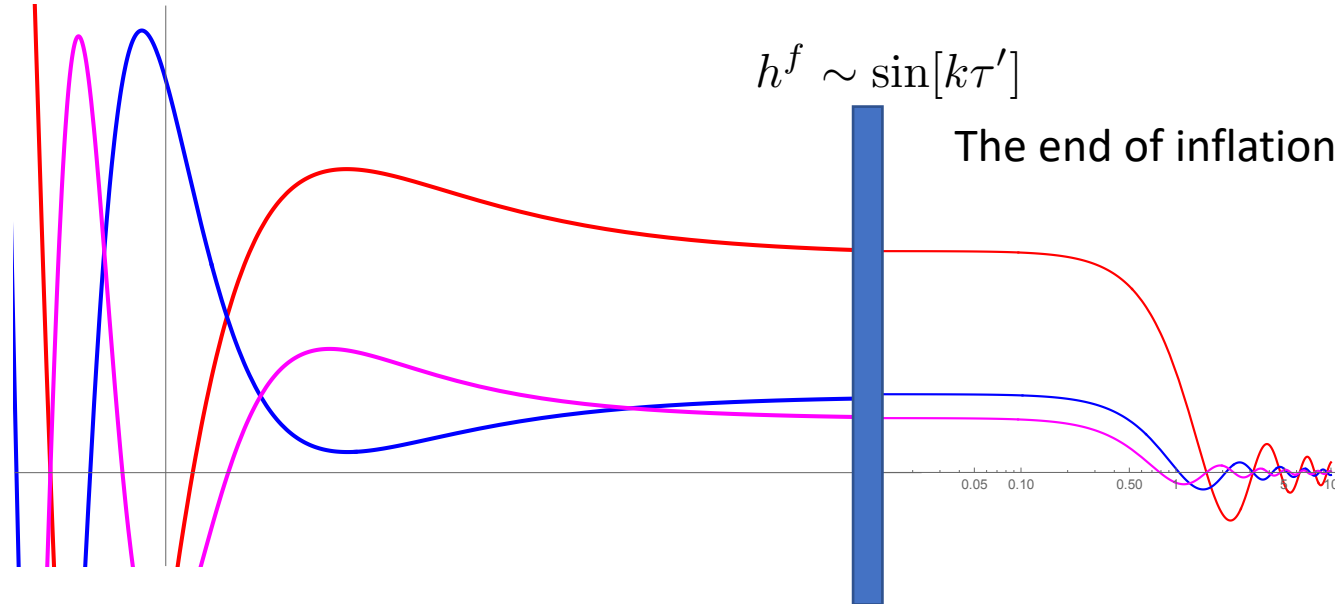
- The conformal time between the source and the horizon is fixed.
- The phase of h at the source is fixed.
- The value of h^f at the horizon **oscillates** with k .
- h^f is the **initial condition** for later evolution.



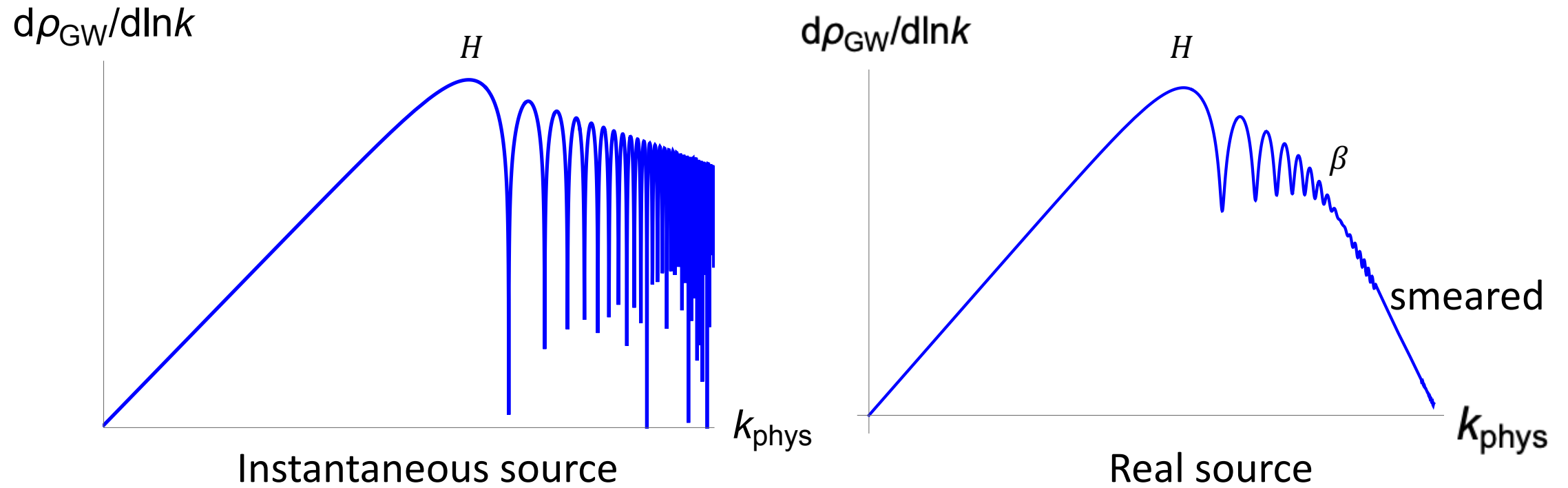
$$k\tau_A \approx 0$$

After inflation

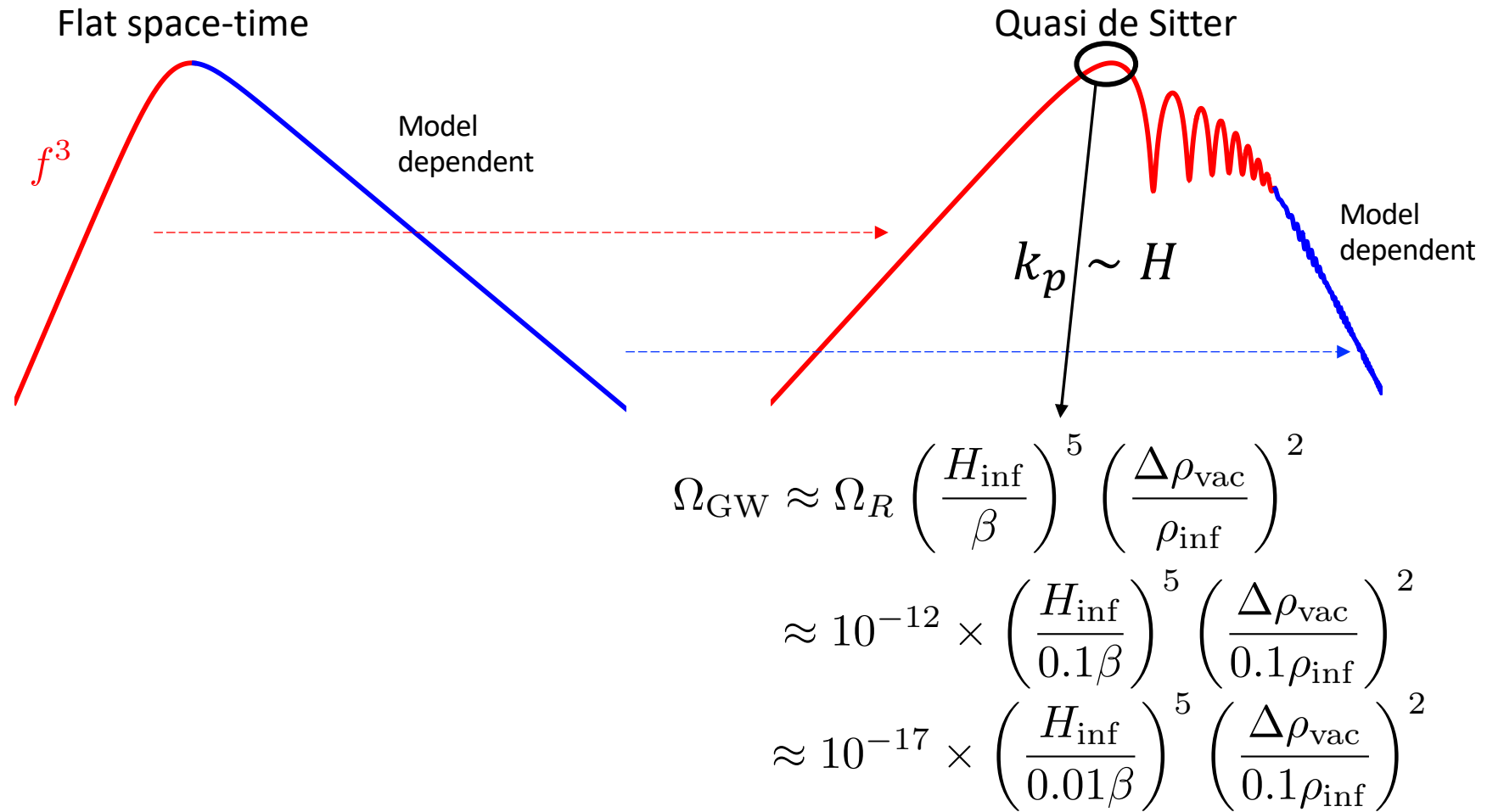
- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



Spectrum of GW from a real source



Spectrum distortion by inflation



First-order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering, and fast reheating

$$\Omega_{\text{GW}}(k_{\text{today}}) = \Omega_R \frac{H_{\text{inf}}^4}{k_p^4} \left[\frac{1}{2} + \mathcal{S}(k_p \beta^{-1}) \cos \left(\frac{2k_p}{H_{\text{inf}}} \right) \right] \left(\frac{\Delta \rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^2 \frac{d\rho_{\text{GW}}^{\text{flat}}}{\Delta \rho_{\text{vac}} d \log k_p}$$

Dilution factor

Smearing

Suppressed by
the energy
fraction

Redshift

$$\frac{f_{\text{today}}}{f_{\star}} = \frac{a(\tau_{\star})}{a_1} \left(\frac{g_{\star S}^{(0)}}{g_{\star S}^{(R)}} \right)^{1/3} \frac{T_{\text{CMB}}}{\left[\left(\frac{30}{g_{\star}^{(R)}} \pi^2 \right) \left(\frac{3H_{\text{inf}}^2}{8\pi G_N} \right) \right]^{1/4}}$$

e^{-N_e}

N_e : e-folds before the end of inflation

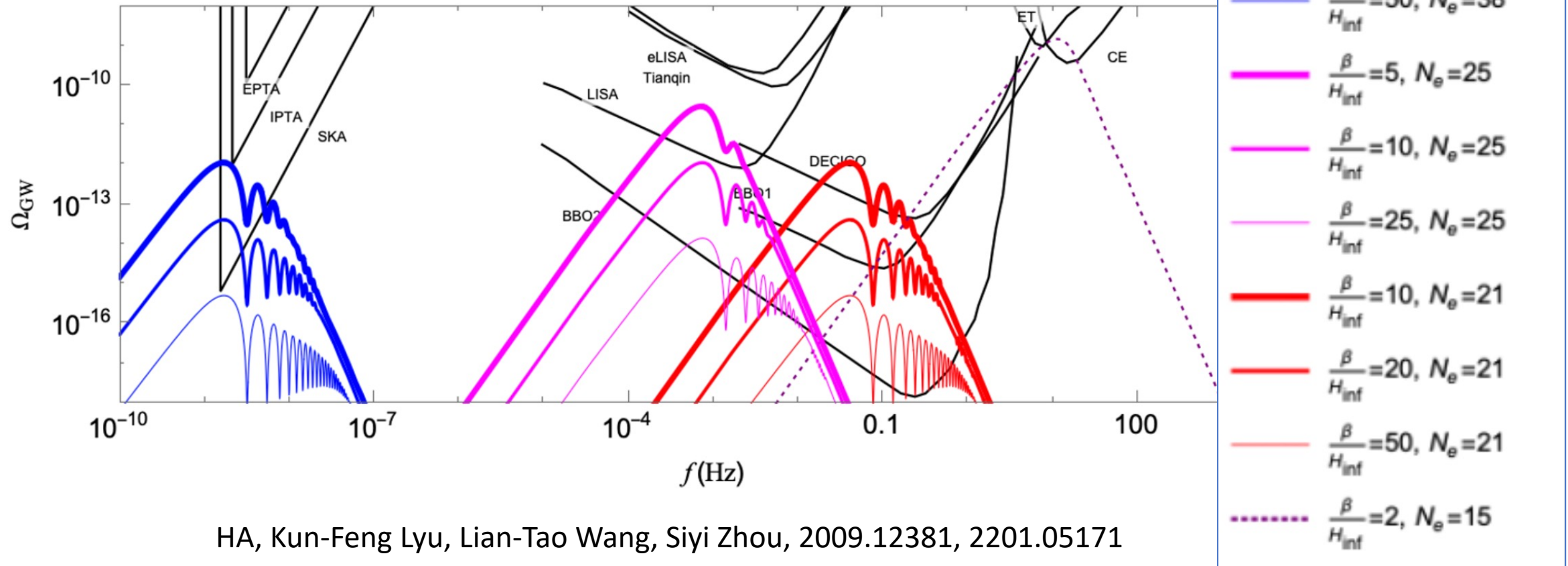
First-order phase transition during inflation

- Primordial stochastic GW signals

Instantaneous reheating

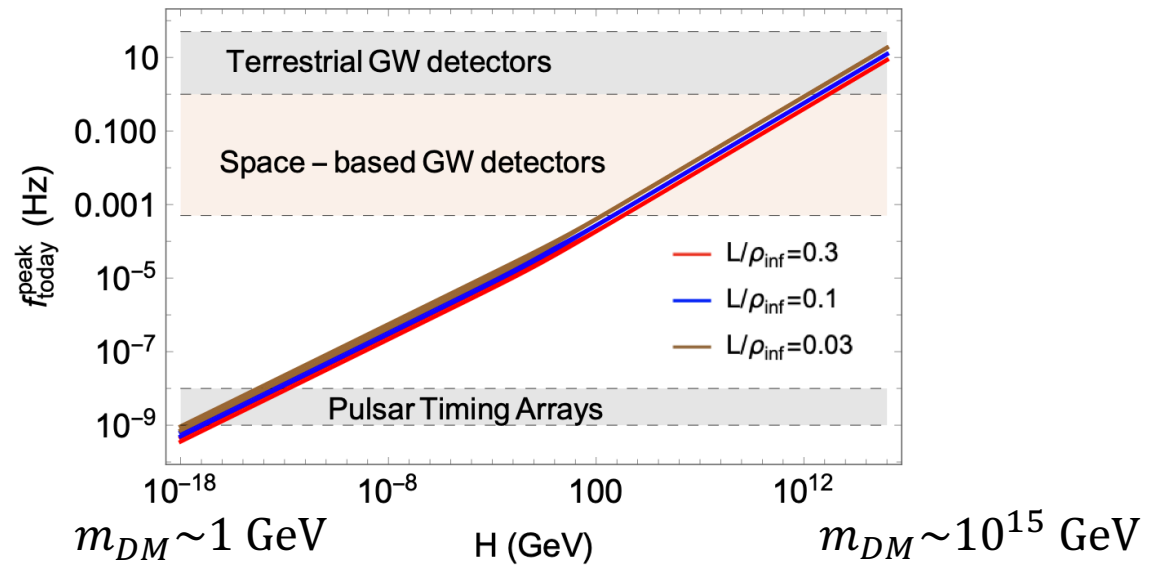
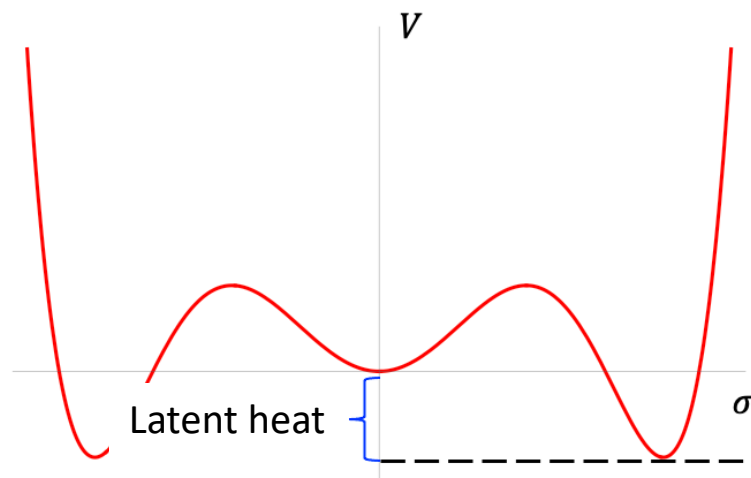
$$H_{\text{inf}} = 10^{12} \text{ GeV}$$

$$\Delta\rho_{\text{vac}}/\rho_{\text{inf}} = 0.3$$

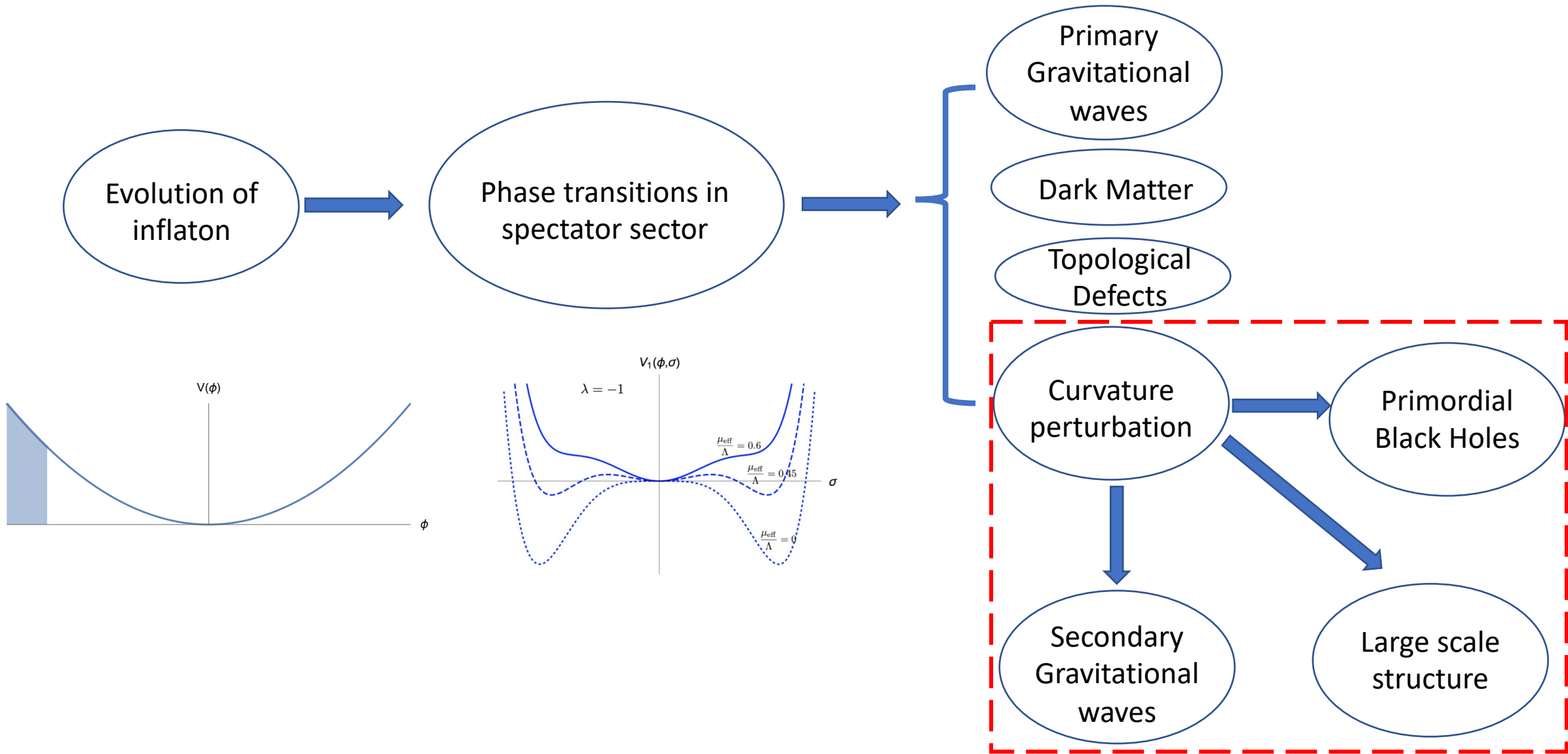


Producing super heavy DM

- Where does the latent heat go?
- σ particles produced during bubble collision and thermalization.
- If the phase transition is ***symmetry-restoration***, σ particles can be DM.



Induced phase transition in spectator sectors



Induced scalar perturbation $\delta\phi$

- Interactions

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi, \sigma)$$

$$V(\phi, \sigma) = V_0(\phi) + V_1(\phi, \sigma) \quad \xrightarrow{\phi = \phi_0 + \delta\phi} \quad \frac{\partial V_1}{\partial \phi_0} \delta\phi \quad \text{Source term for } \delta\phi$$

$$\delta\tilde{\phi}_{\mathbf{q}}'' - \frac{2}{\tau}\delta\tilde{\phi}_{\mathbf{q}}' + \left(q^2 + \frac{1}{H^2\tau^2} \frac{\partial^2 V_0}{\partial \phi_0^2} \right) \delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} ,$$

$$\mathcal{S}_{\mathbf{q}} = -\frac{1}{H^2\tau^2} \left[\frac{\partial V_1}{\partial \phi} \right]_{\mathbf{q}} - \underbrace{\left\{ \frac{2\Phi_{\mathbf{q}}}{H^2\tau^2} \left(\frac{\partial V_0}{\partial \phi_0} + \left[\frac{\partial V_1}{\partial \phi} \right]_0 \right) + \frac{\dot{\phi}_0}{H\tau} (3\Psi'_{\mathbf{q}} + \Phi'_{\mathbf{q}}) \right\}}_{\text{Pure gravitational, subdominant}}$$

Pure gravitational, subdominant

Induced curvature perturbation ζ

- We solve the following equations of motion numerically with a $1000 \times 1000 \times 1000$ lattice

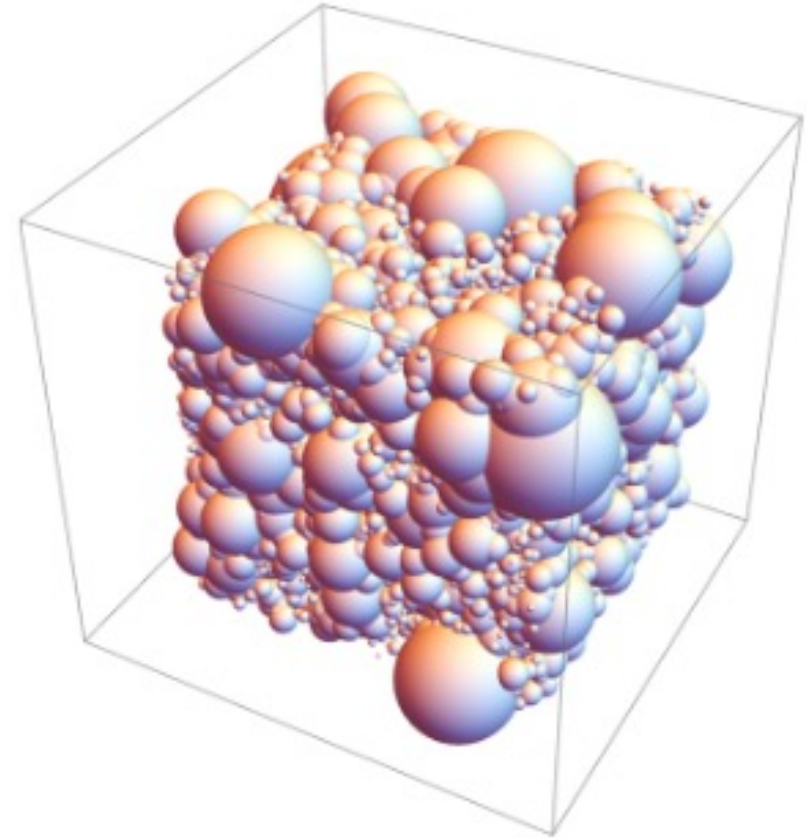
$$\delta\tilde{\phi}_{\mathbf{q}}'' - \frac{2}{\tau}\delta\tilde{\phi}_{\mathbf{q}}' + \left(q^2 + \frac{1}{H^2\tau^2} \frac{\partial^2 V_0}{\partial\phi_0^2}\right) \delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} ,$$

$$\tilde{\Psi}'_{\mathbf{q}} - \frac{\tilde{\Phi}_{\mathbf{q}}}{\tau} = -4\pi G_N \left(\frac{\dot{\phi}_0 \delta\tilde{\phi}_{\mathbf{q}}}{H_{\text{inf}}\tau} + \left[\frac{\partial_i}{\partial^2} (\sigma' \partial_i \sigma) \right]_{\mathbf{q}} \right)$$

$$\tilde{\pi}_{\mathbf{q}}^S = -\frac{3}{2} H_{\text{inf}}^2 \tau^2 q_i q_j q^{-4} [(\partial_i \sigma \partial_j \sigma)^{\text{TL}}]_{\mathbf{q}}$$

- Conserved quantity after the phase transition

$$\zeta_{\mathbf{q}} = -\tilde{\Psi}_{\mathbf{q}} - \frac{H_{\text{inf}} \delta\tilde{\phi}_{\mathbf{q}}}{\dot{\phi}_0}$$



Power spectrum of ζ

- After the collision of the bubbles, σ field oscillates and keeps producing ζ .
- The production of ζ lasts about H^{-1} , longer than β^{-1} .

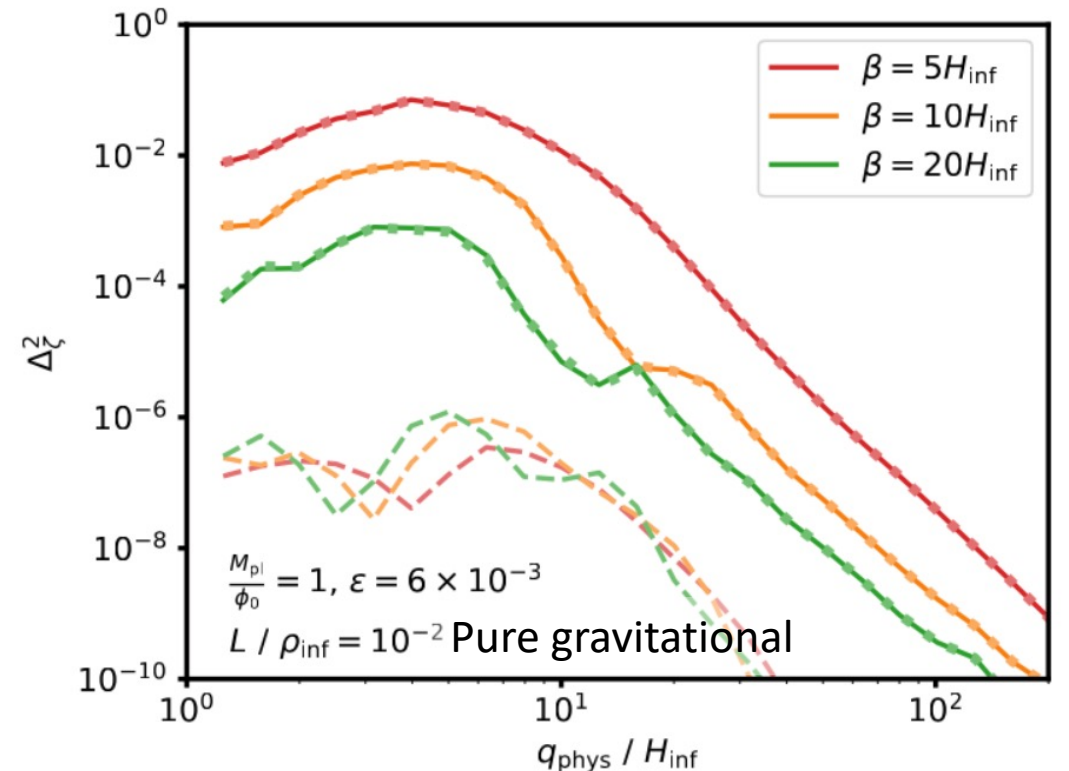
$$\zeta_{\mathbf{q}} \approx \frac{H_{\text{inf}}}{\dot{\phi}_0 q^2} \int \frac{d\tau'}{\tau'} \left(\cos q\tau' - \frac{\sin q\tau'}{q\tau'} \right) \frac{c_m \phi_0 [\sigma^2(\tau')]_{\mathbf{q}}}{H_{\text{inf}}^2 \tau'^2}$$

$$\Delta_{\zeta}^{2(\text{emp})}(q) = A_{\text{ref}} \mathcal{F} \left(\frac{q_{\text{phys}}}{H_{\text{inf}}} \right)$$

$$\mathcal{F}(x) = \frac{x^3}{1 + (\alpha_1 x)^4 + (\alpha_2 x)^9}$$

$$A_{\text{ref}} = \frac{\mathcal{A}}{\epsilon} \left(\frac{M_{\text{pl}}}{\phi_0} \right)^2 \left(\frac{H_{\text{inf}}}{\beta} \right)^3 \left(\frac{L}{\rho_{\text{inf}}} \right)^2$$

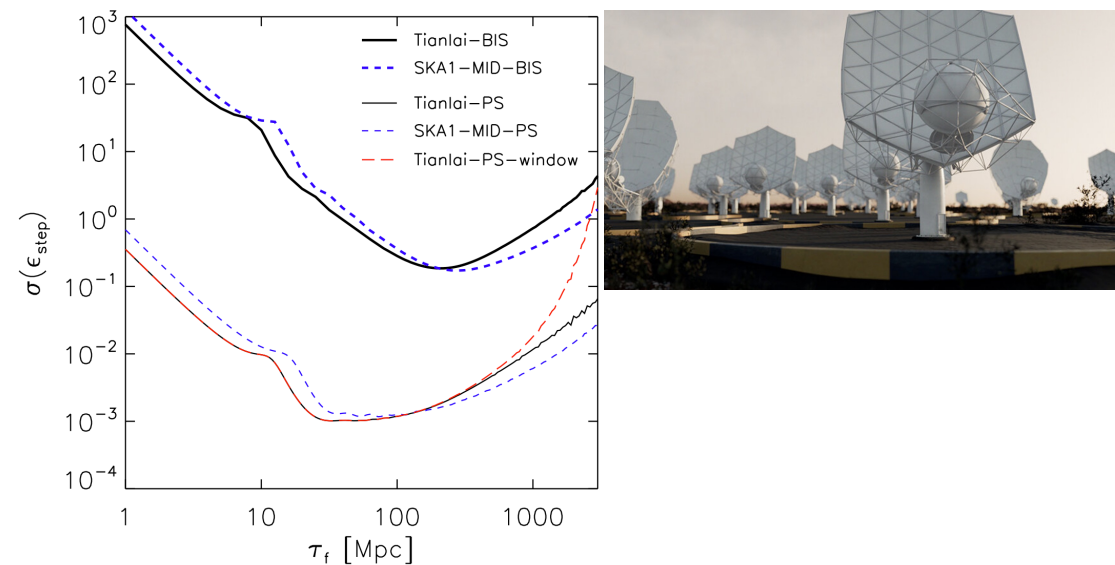
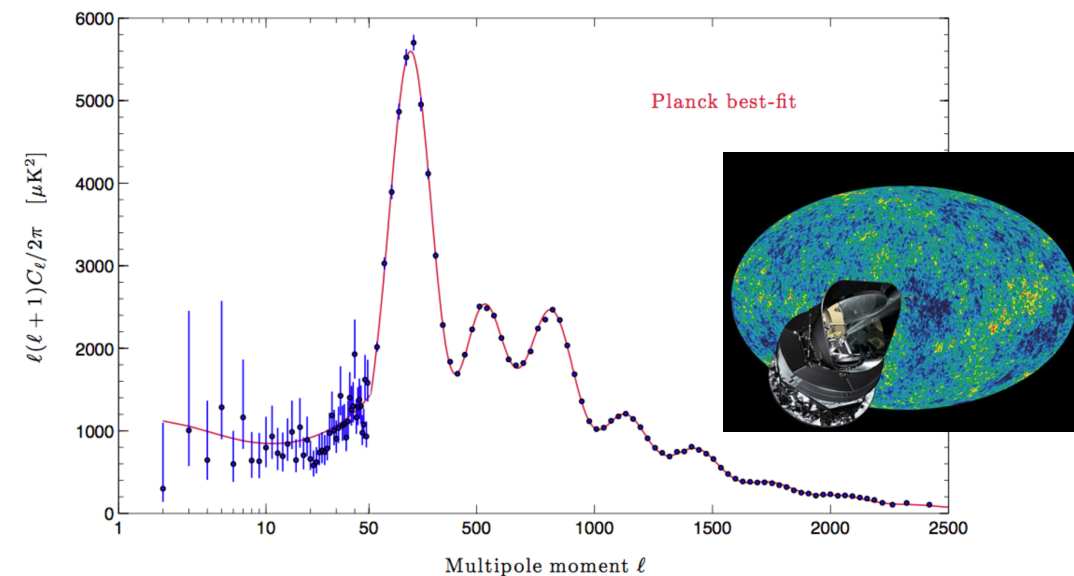
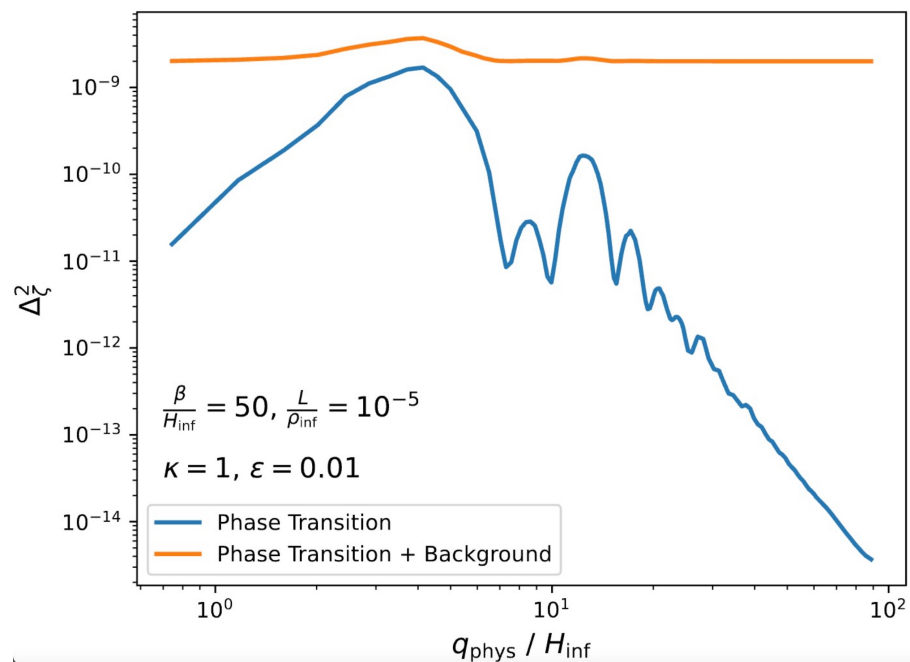
$$\mathcal{A} \approx 24 \quad L \equiv \Delta\rho \quad \alpha_1 \approx 0.31, \alpha_2 \approx 0.17$$



Power spectrum of ζ

$$k_{\text{today}} = (2000 \text{ Mpc})^{-1} \times e^{60-N_e} \times \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)$$

$$A_{\text{ref}} = \frac{\mathcal{A}}{\epsilon} \left(\frac{M_{\text{pl}}}{\phi_0} \right)^2 \left(\frac{H_{\text{inf}}}{\beta} \right)^3 \left(\frac{L}{\rho_{\text{inf}}} \right)^2$$



Secondary GWs

- After inflation $\zeta \rightarrow \Phi, \Psi$
- Expand the Einstein equation to second order:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm} \mathcal{S}_{lm} ,$$

$$\begin{aligned} \mathcal{S}_{ij} \equiv & 2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i(\Psi' + \mathcal{H}\Phi)\partial_j(\Psi' + \mathcal{H}\Phi) - \frac{2c_s^2}{3w\mathcal{H}^2}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi) . \end{aligned}$$

Scalar induced GWs

Matarrese, Mollerach, and Bruni, astro-hp/9707278

Mollerach, Harari, and Matarrese, astro-hp/0310711

Ananda, Clarkson, and Wands, gr-qc/0612013

Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290

Secondary GWs

$$\Omega_{\text{GW}}^{(2)}(f) = \Omega_R A_{\text{ref}}^2 \mathcal{F}_2 \left(\frac{q_{\text{phys}}}{H_{\text{inf}}} \right)$$

$$f = \frac{q}{2\pi a_0} = f_{\text{ref}} \times \frac{q_{\text{phys}}}{H_{\text{inf}}}$$

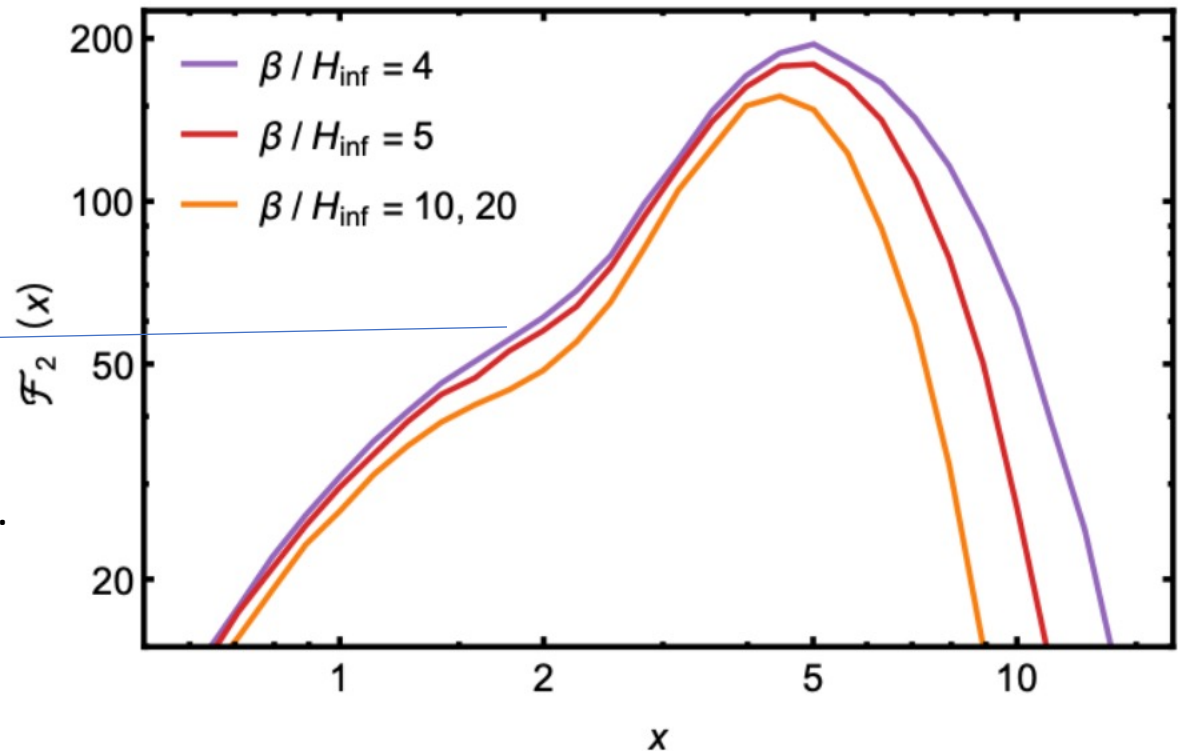
$$f_{\text{ref}} = 10^{-9} \text{ Hz} \times e^{40-N_e} \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^{1/2}$$

$$\mathcal{F}_2^{\text{IR}}(x) \approx x^3 \left(\frac{6}{5} \log^2 x + \frac{16}{25} \log x + \frac{28}{125} \right) \quad \leftarrow \mathcal{F}_2(x)$$

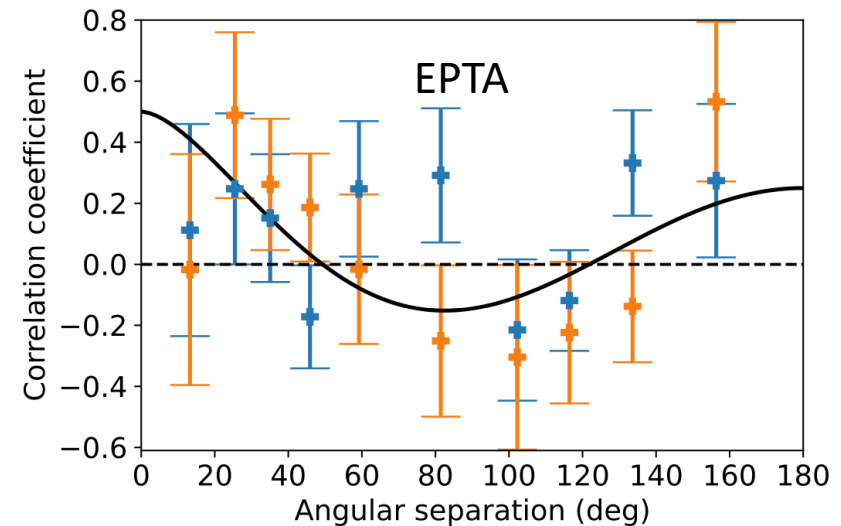
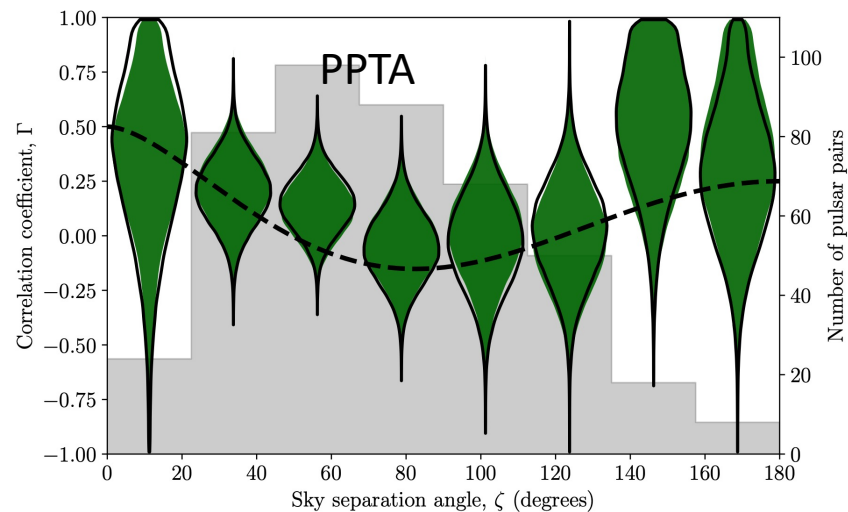
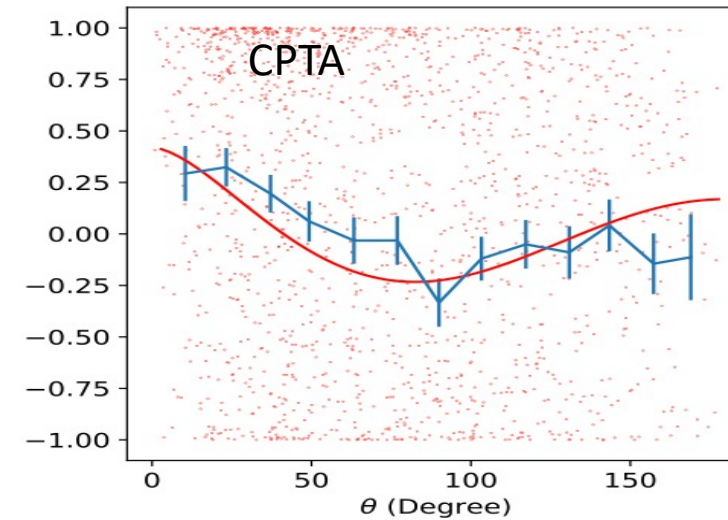
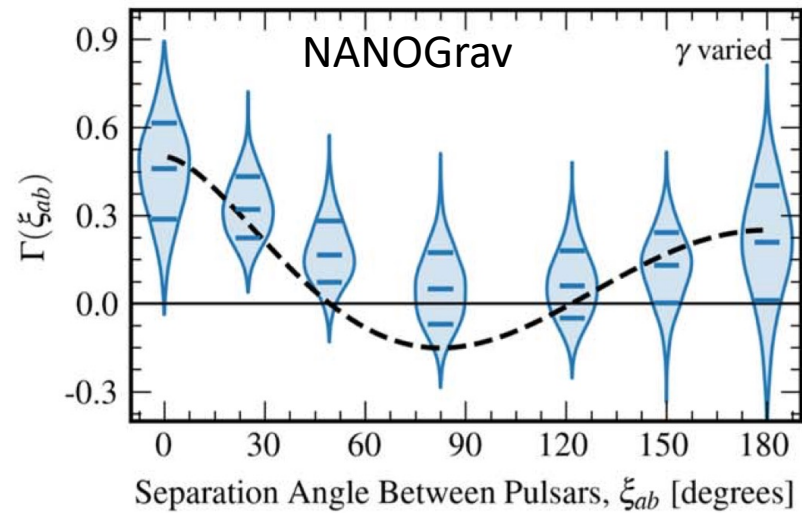
$\mathcal{F}_2$ Collects information of the transfer functions.

$$A_{\text{ref}} = \frac{\mathcal{A}}{\epsilon} \left(\frac{M_{\text{pl}}}{\phi_0} \right)^2 \left(\frac{H_{\text{inf}}}{\beta} \right)^3 \left(\frac{L}{\rho_{\text{inf}}} \right)^2$$

$$\mathcal{F}_2^{\text{max}} \approx 200$$

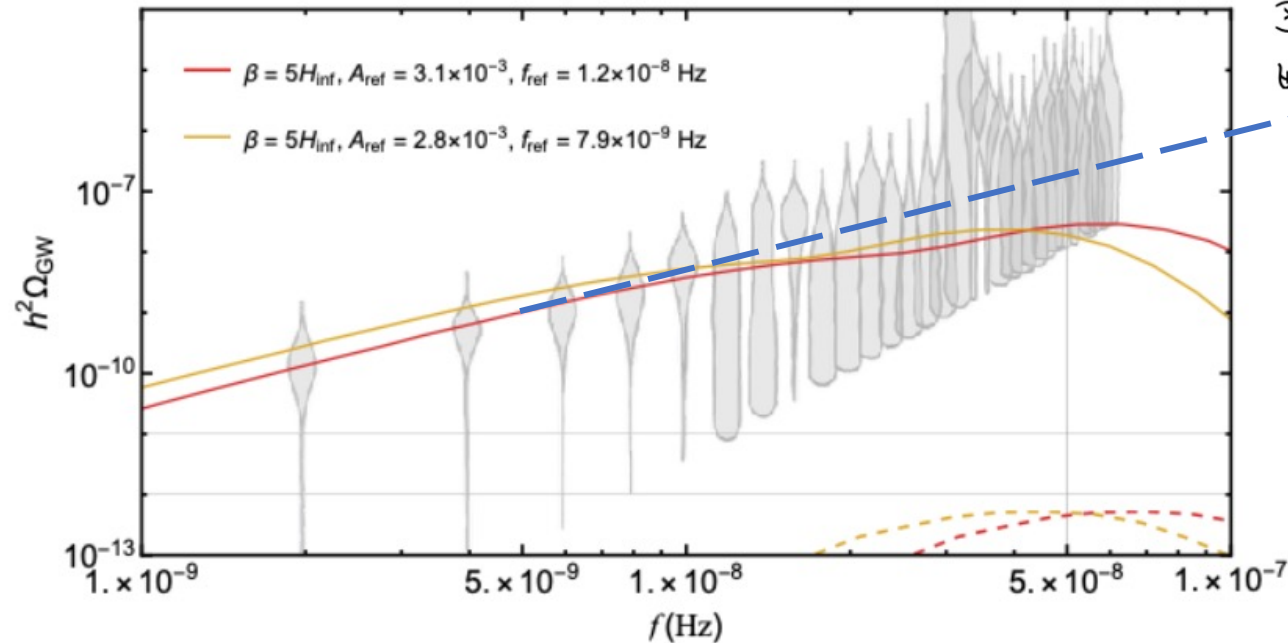


Observation from PTAs

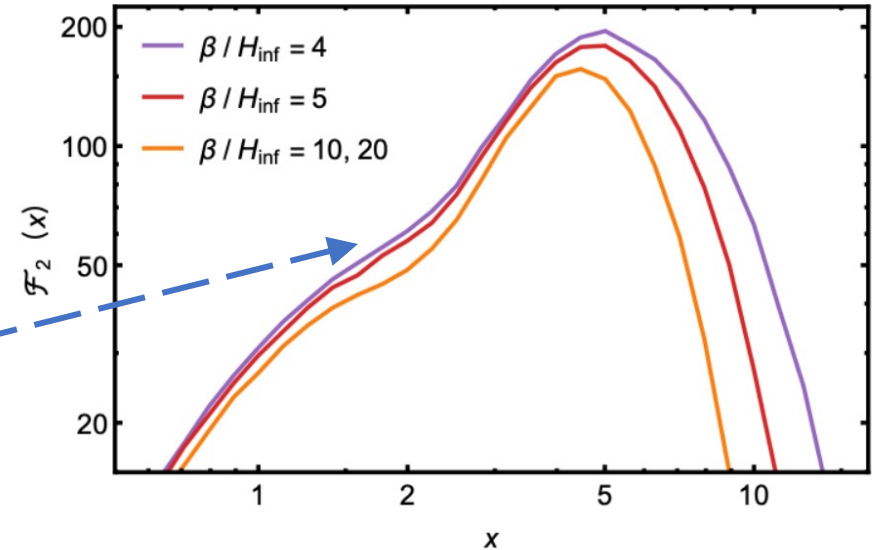


Observation from PTAs

- The slope is around 2 in the IR region



$$\mathcal{F}_2^{\text{IR}}(x) \approx x^3 \left(\frac{6}{5} \log^2 x + \frac{16}{25} \log x + \frac{28}{125} \right)$$



$$\Omega_{\text{GW}}^{(2)}(f) = \Omega_R A_{\text{ref}}^2 \mathcal{F}_2 \left(\frac{q_{\text{phys}}}{H_{\text{inf}}} \right)$$

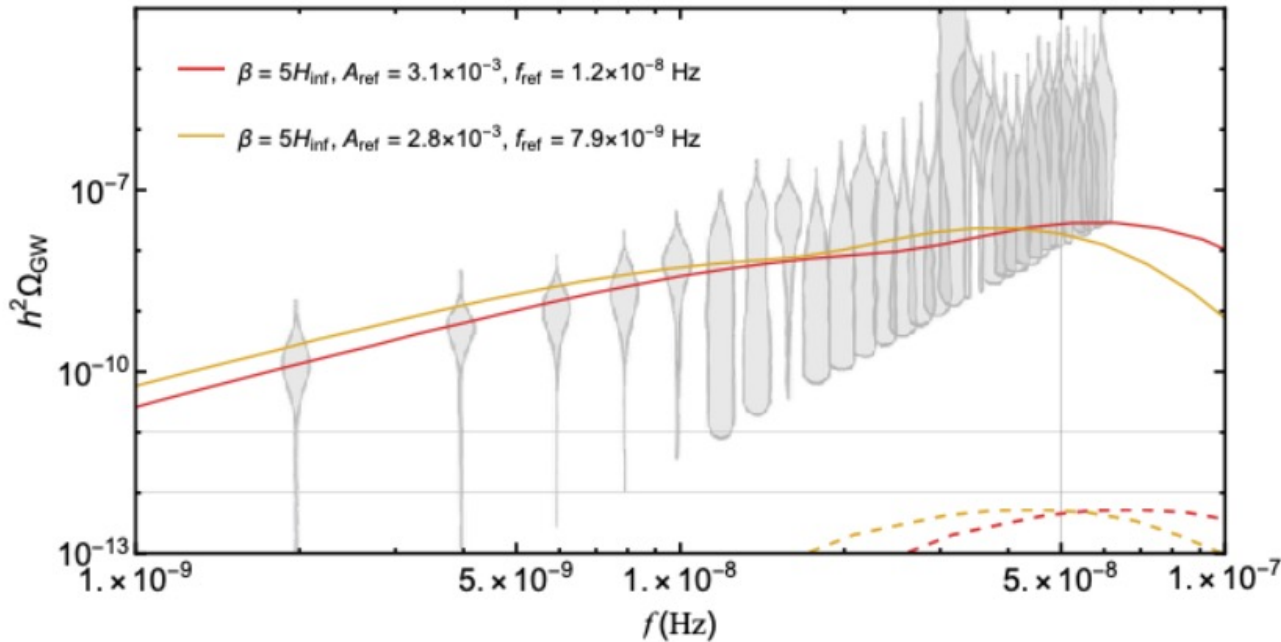
$$f = \frac{q}{2\pi a_0} = f_{\text{ref}} \times \frac{q_{\text{phys}}}{H_{\text{inf}}}$$

$$f_{\text{ref}} = 10^{-9} \text{ Hz} \times e^{40-N_e} \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^{1/2}$$

Observation from PTAs

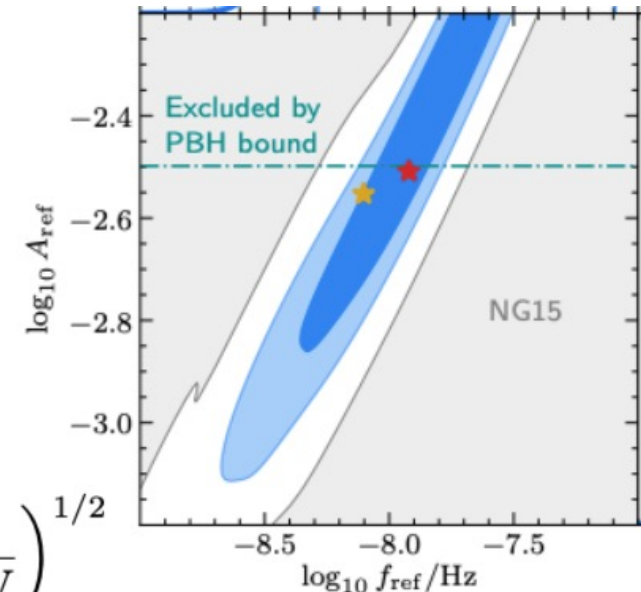
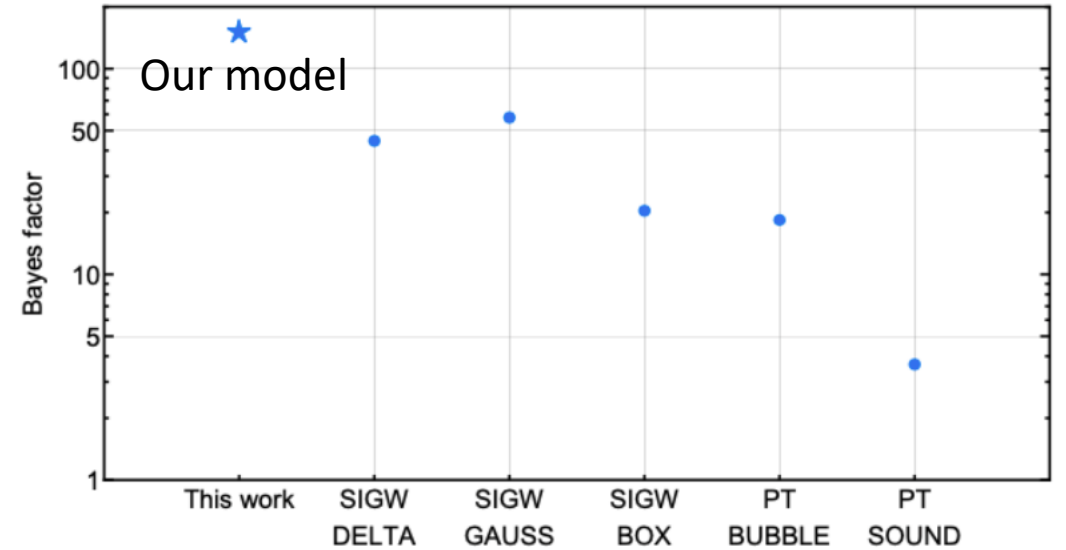
HA, Boye Su, Hanwen Tai, Lian-Tao Wang, 2308.00070

- Bayes factor against SMBHB



$$\Omega_{\text{GW}}^{(2)}(f) = \Omega_R \underline{A_{\text{ref}}^2} \mathcal{F}_2 \left(\frac{q_{\text{phys}}}{\underline{H_{\text{inf}}}} \right)$$

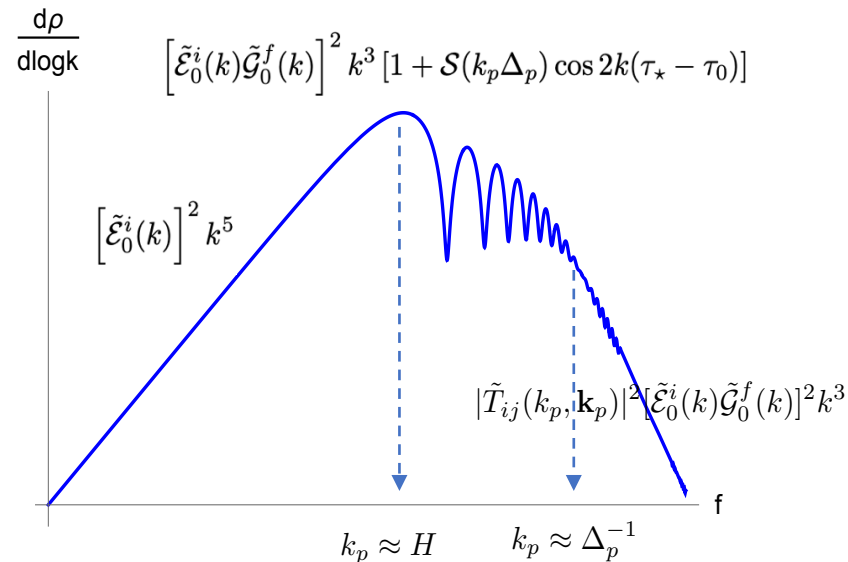
$$f = \frac{q}{2\pi a_0} = f_{\text{ref}} \times \frac{q_{\text{phys}}}{H_{\text{inf}}} \quad f_{\text{ref}} = 10^{-9} \text{ Hz} \times e^{40-N_e} \left(\frac{H_{\text{inf}}}{10^{14} \text{ GeV}} \right)^{1/2}$$



Comparison between primary GW and secondary GW

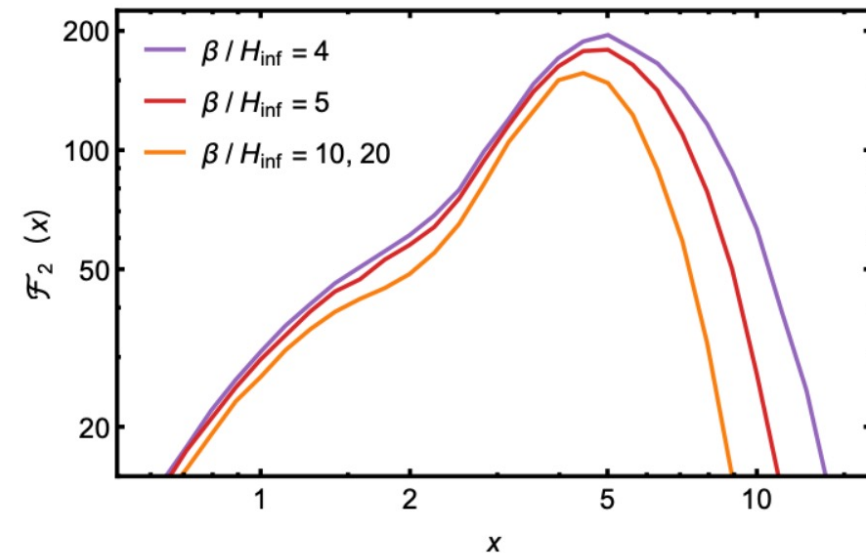
- Primary

$$\Omega_{\text{GW}} \approx \Omega_R \left(\frac{H_{\text{inf}}}{\beta} \right)^5 \left(\frac{\Delta \rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^2$$



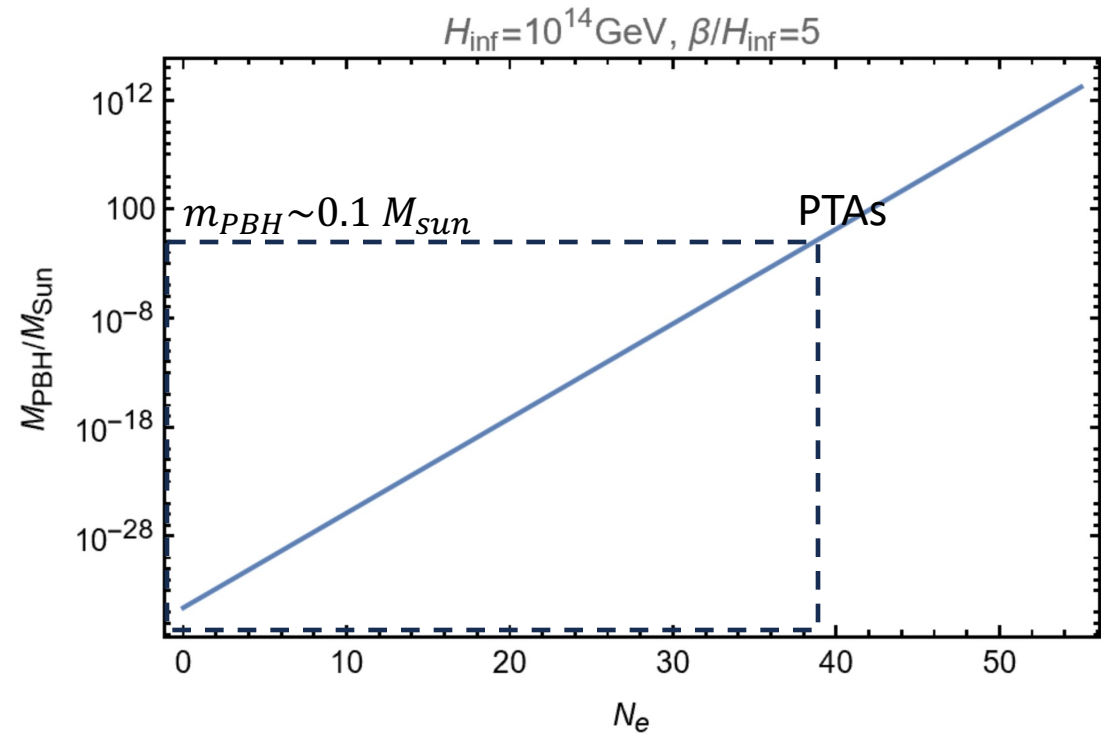
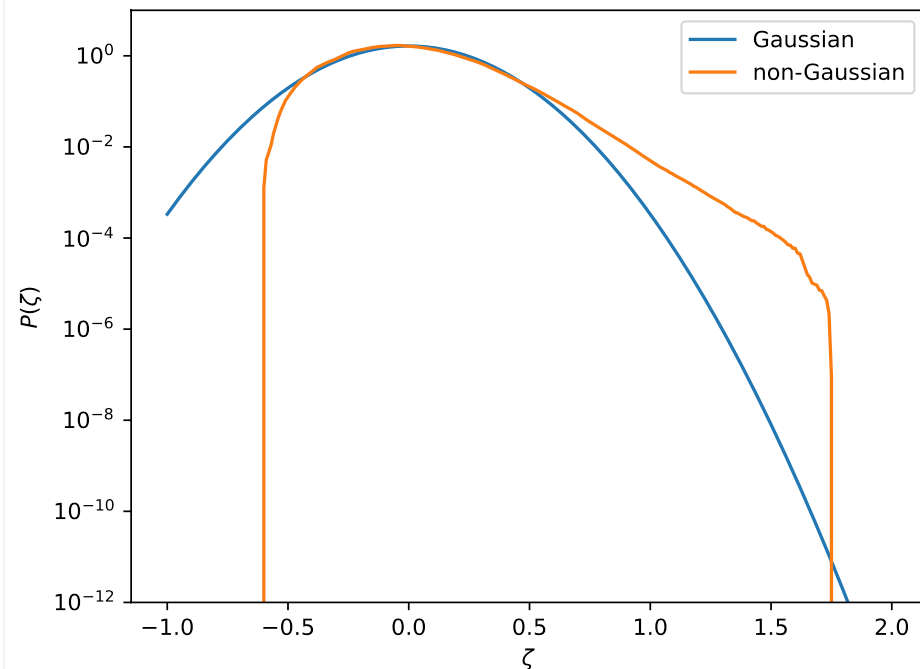
- Secondary

$$\Omega_{\text{GW}} \sim \Omega_R \left(\frac{\mathcal{A}}{\epsilon} \right)^2 \left(\frac{M_{\text{pl}}}{\phi_0} \right)^4 \left(\frac{H_{\text{inf}}}{\beta} \right)^6 \left(\frac{L}{\rho_{\text{inf}}} \right)^4$$



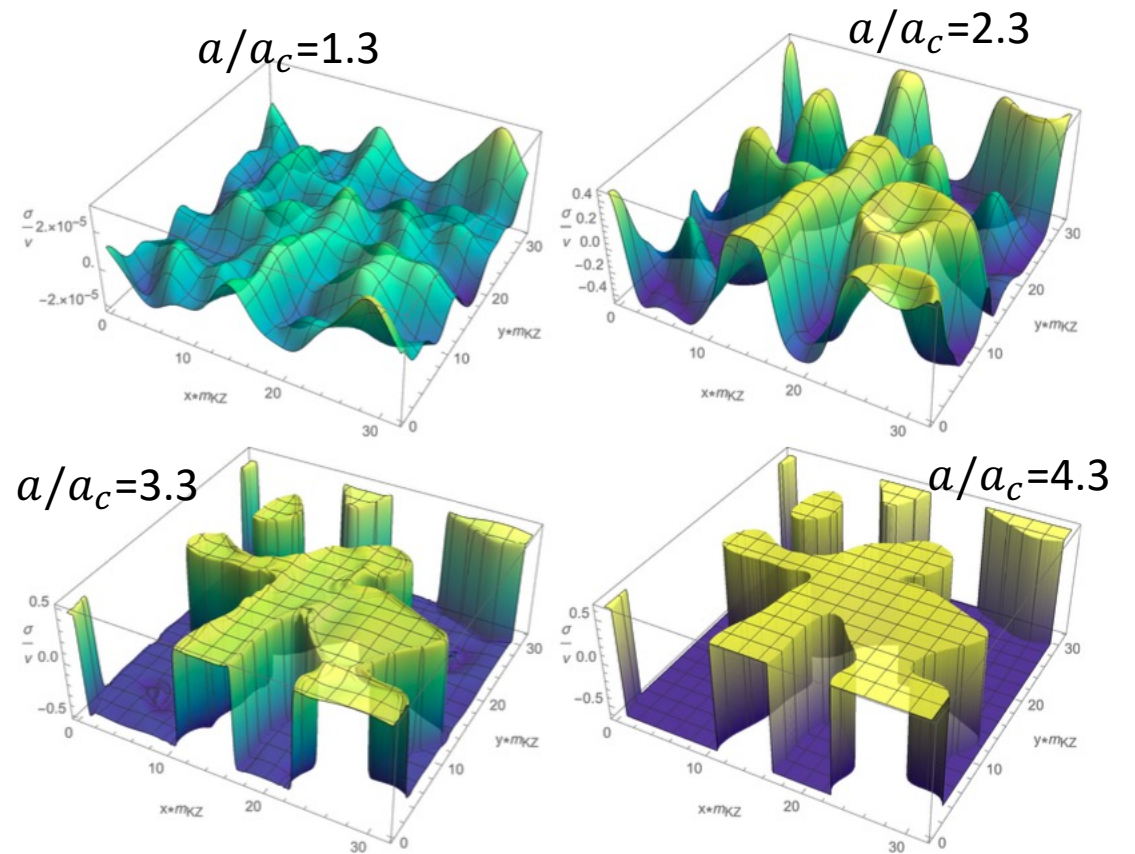
Primordial Black Holes

- PBHs will form if $\Delta_{\zeta}^2 \sim 0.01$
- The power spectrum is highly non-Gaussian



Formation of domain walls

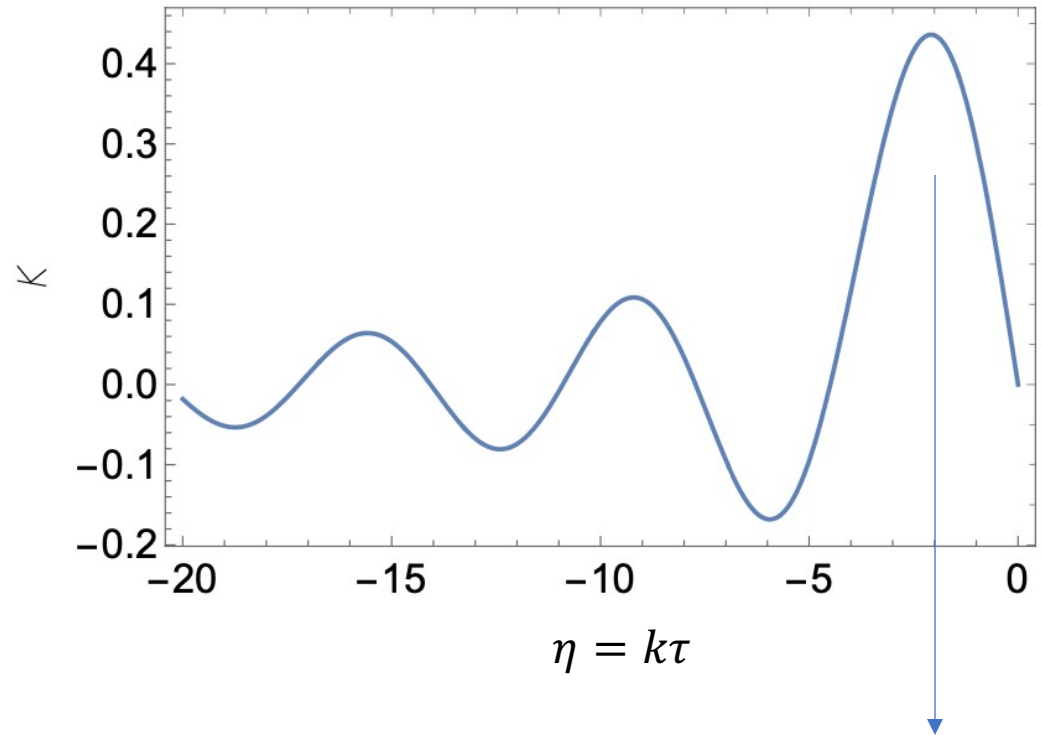
- Symmetry breaking via a **second order phase transition**.
- We numerically solve the nonlinear evolution of σ field with $1000 \times 1000 \times 1000$ lattice.
- At the beginning there are fluctuations, dying out after a few e-folds.
- The configuration becomes comovingly static after a few e-folds.



Calculation of GWs

- In Minkowski spacetime, static source cannot radiate due to energy-momentum conservation.
- During inflation, energy conservation is badly broken, so the even static source can produce GWs.

$$\tilde{h}_{ij}^f(\mathbf{k}) = \frac{16\pi G_N}{k} \int_{-\infty}^0 d\tau' \mathcal{K}(k\tau') \tilde{T}_{ij}^{TT}(\tau', \mathbf{k})$$



The dominant contribution

Numerical results for GWs

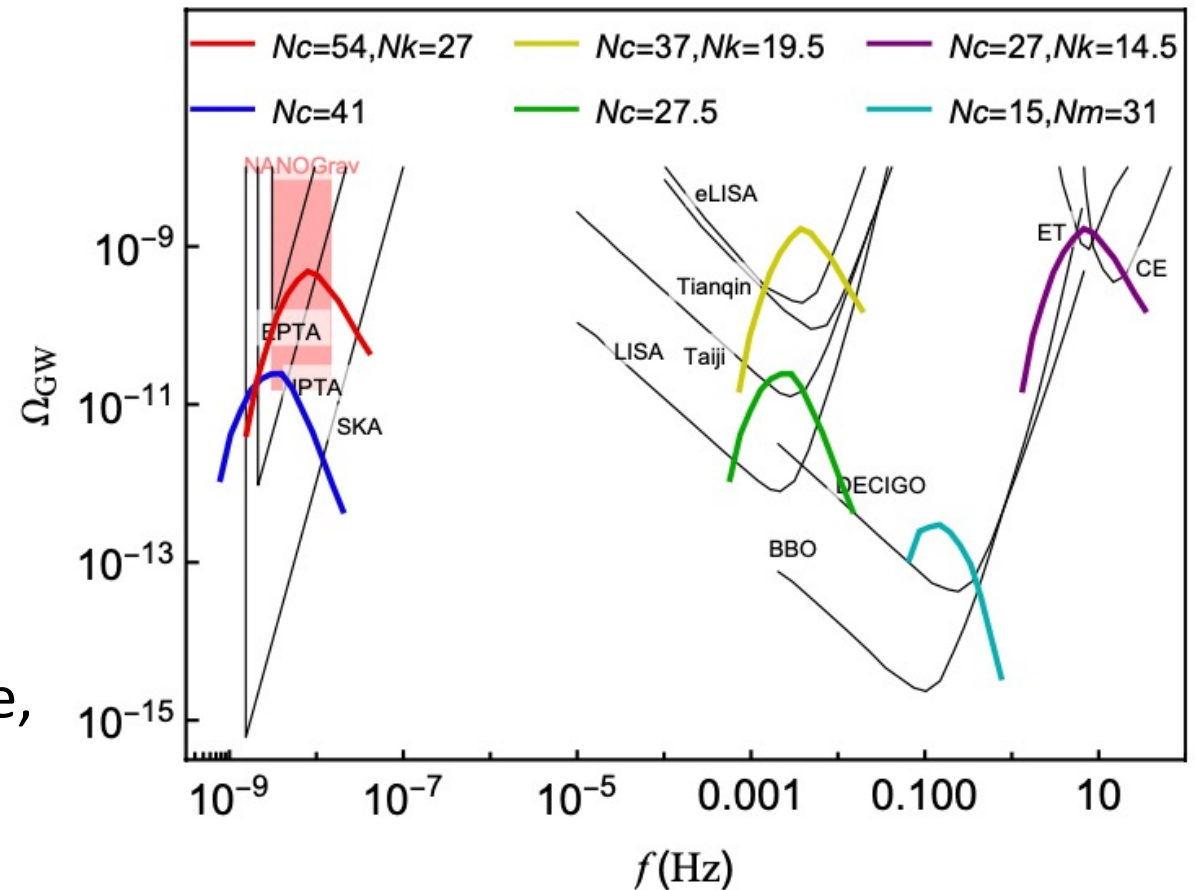
$$\Omega_{\text{GW}}(f) = \Omega_R \times \rho_R^{-1} \left. \frac{d\rho_{\text{GW}}}{d \ln f} \right|_{\text{today}}$$

$$\frac{f_{\text{today}}}{f_{\star}} = \frac{a(\tau_{\star})}{a_1} \left(\frac{g_{\star S}^{(0)}}{g_{\star S}^{(R)}} \right)^{1/3} \frac{T_{\text{CMB}}}{\left[\left(\frac{30}{g_{\star}^{(R)} \pi^2} \right) \left(\frac{3H_{\text{inf}}^2}{8\pi G_N} \right) \right]^{1/4}}$$

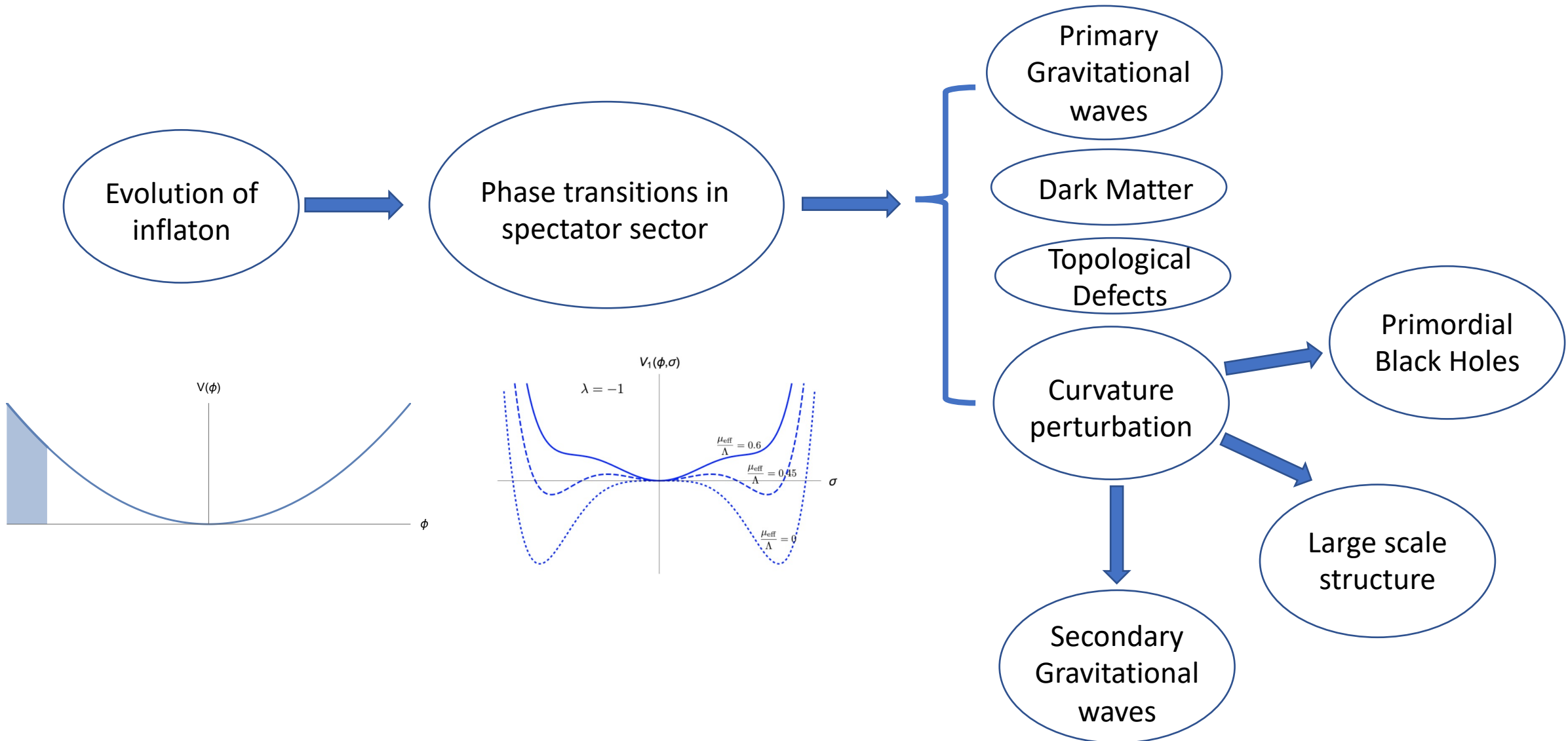
The detailed shape and strength also depends on the evolution of the universe.

- Instantaneous reheating,
- Matter dominated intermediate stage,
- Kination dominated intermediate stage.

HA, Chen Yang, 2304.02361

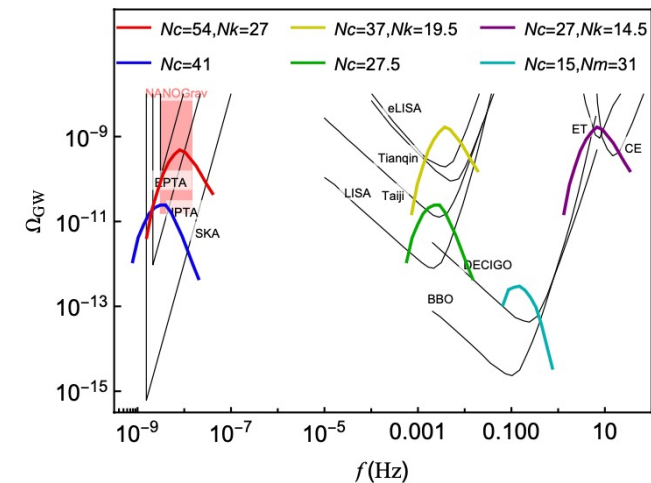
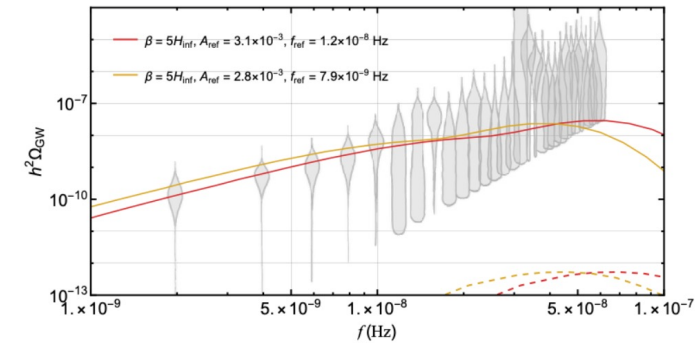
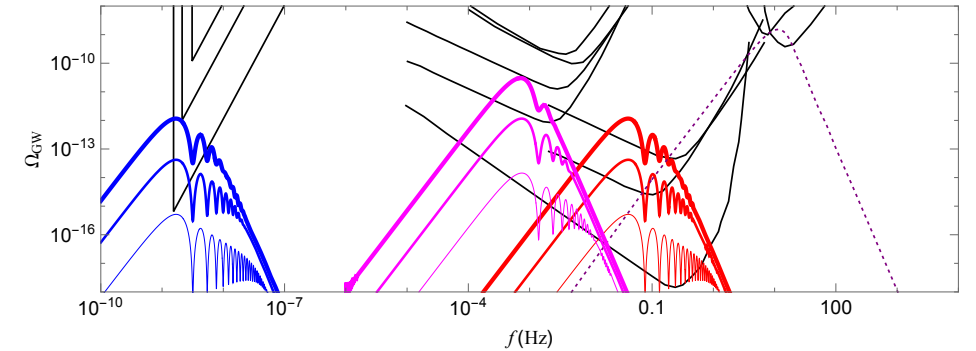


Summary



Summary

- Phase transitions can happen in a spectator sector during inflation.
- We show that there is an oscillatory feature in the GW spectrum if it is produced by first-order phase transition during inflation.
- We show that the secondary GW can be strong enough to explain the signals observed by PTAs
- Static topological defects can produce GWs during inflation.



Outlook

- The fate of the domain walls.
- Other topological defects.
- Application to high scale particle physics models.

Backups

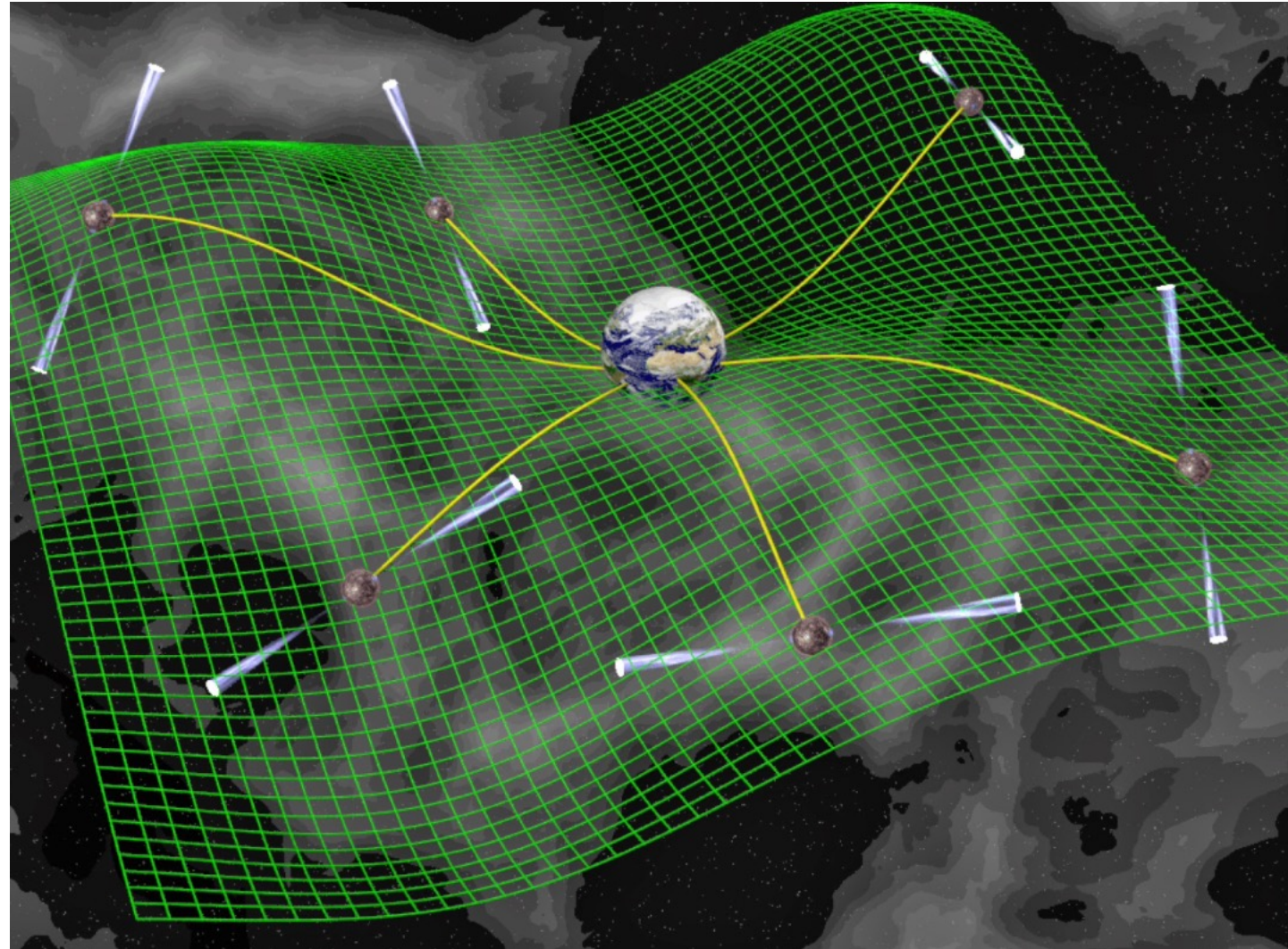
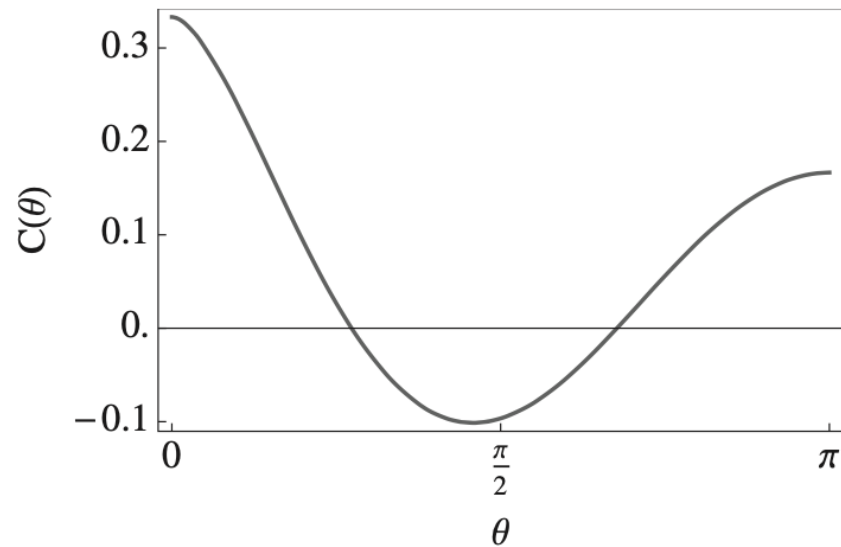
Observation from PTAs

- Hellings-Downs curve

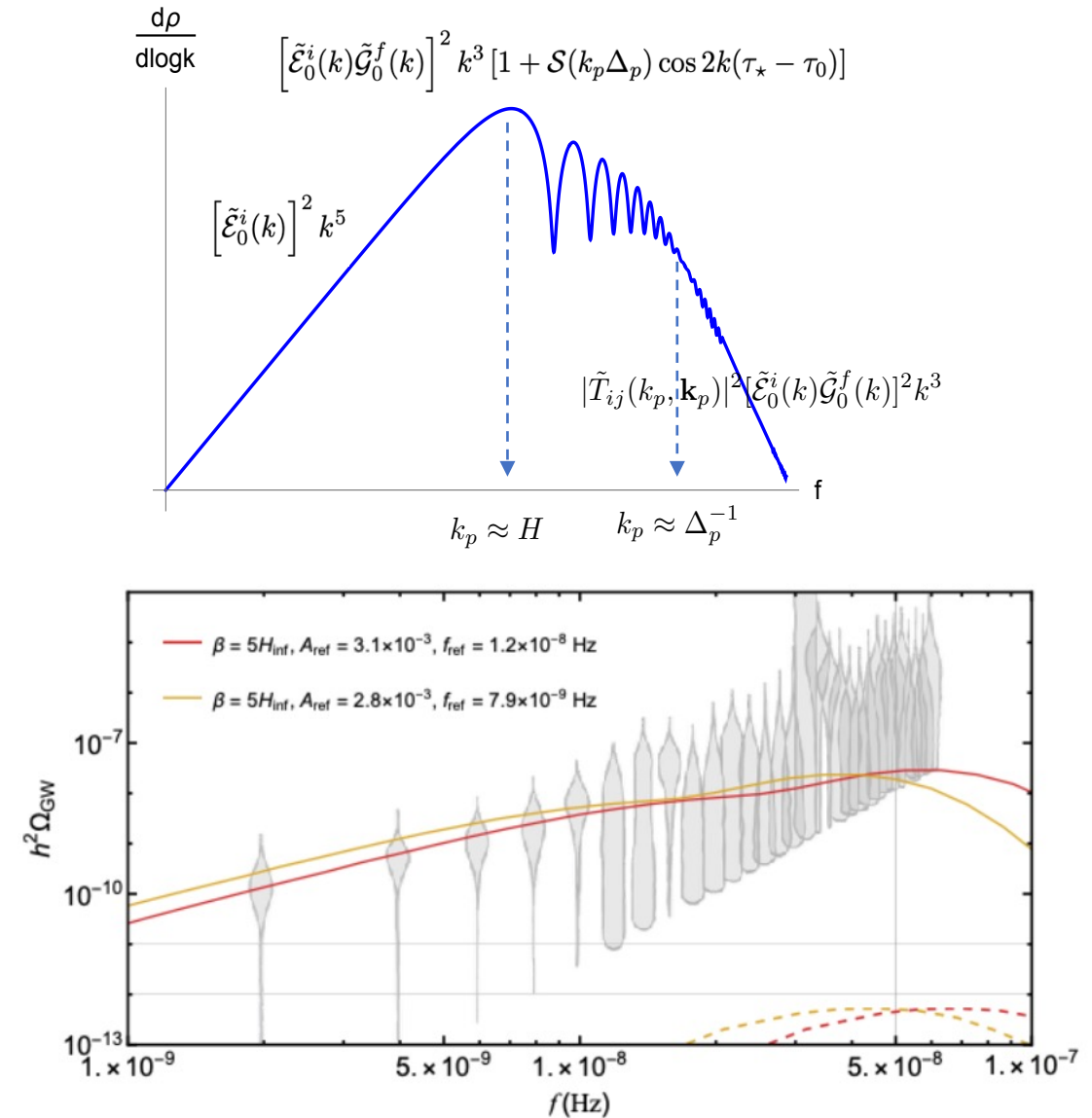
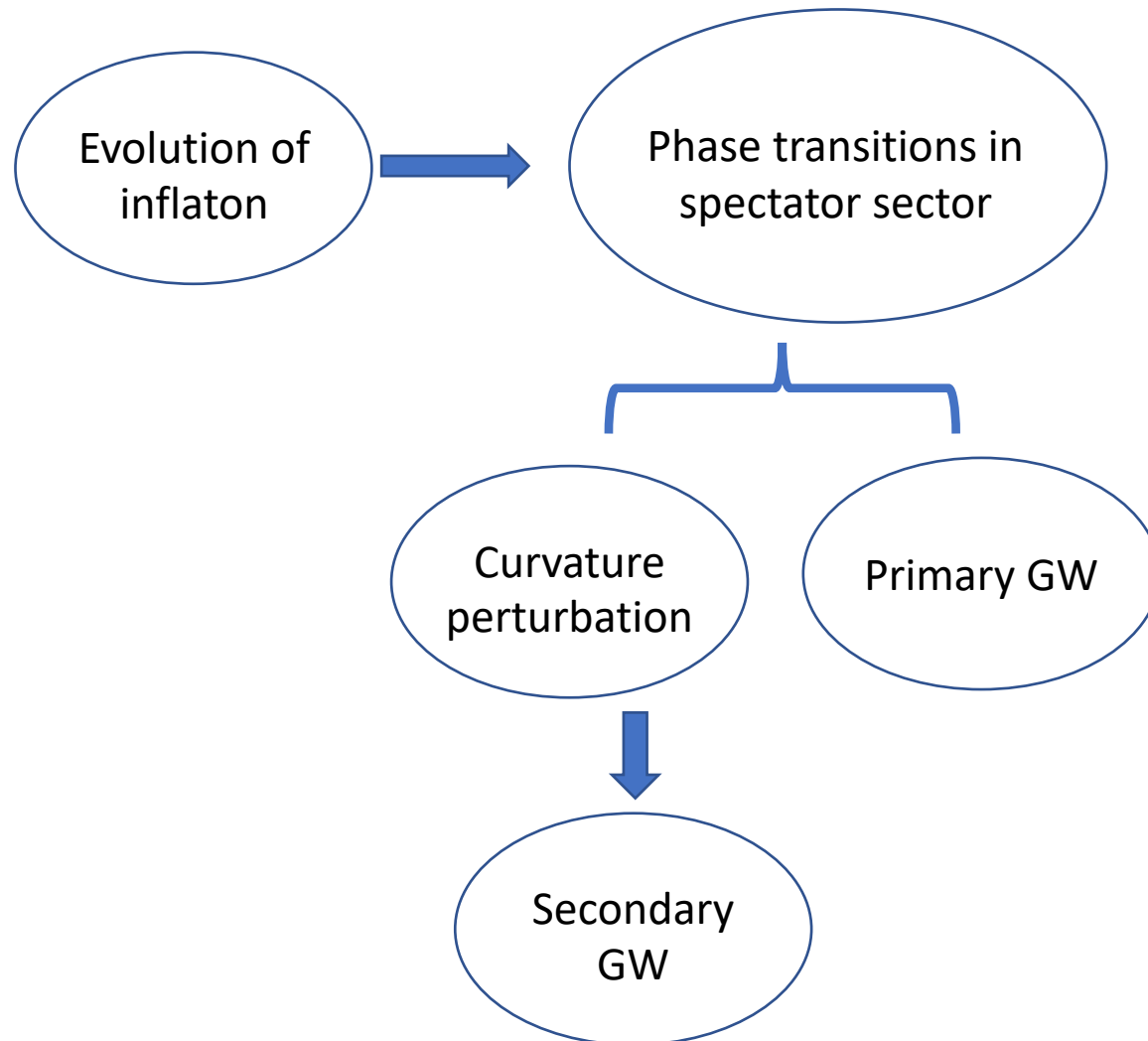
$$\langle z_a(t) z_b(t) \rangle = C(\theta_{ab}) \int_0^\infty df S_h(f)$$

Angular correlation

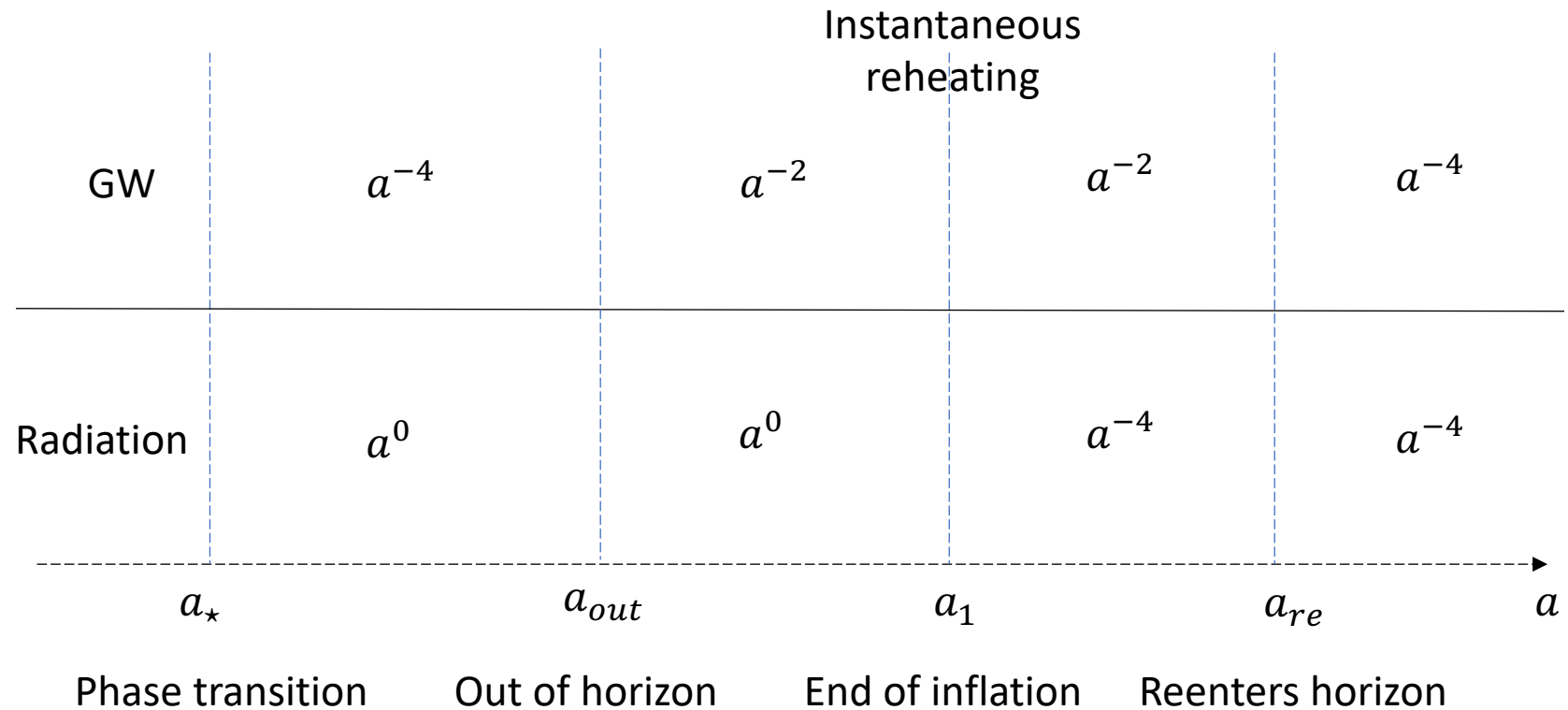
$$z_a(t) = -(\Delta\nu_a/\nu_a)(t) = \Delta T_a/T_a$$



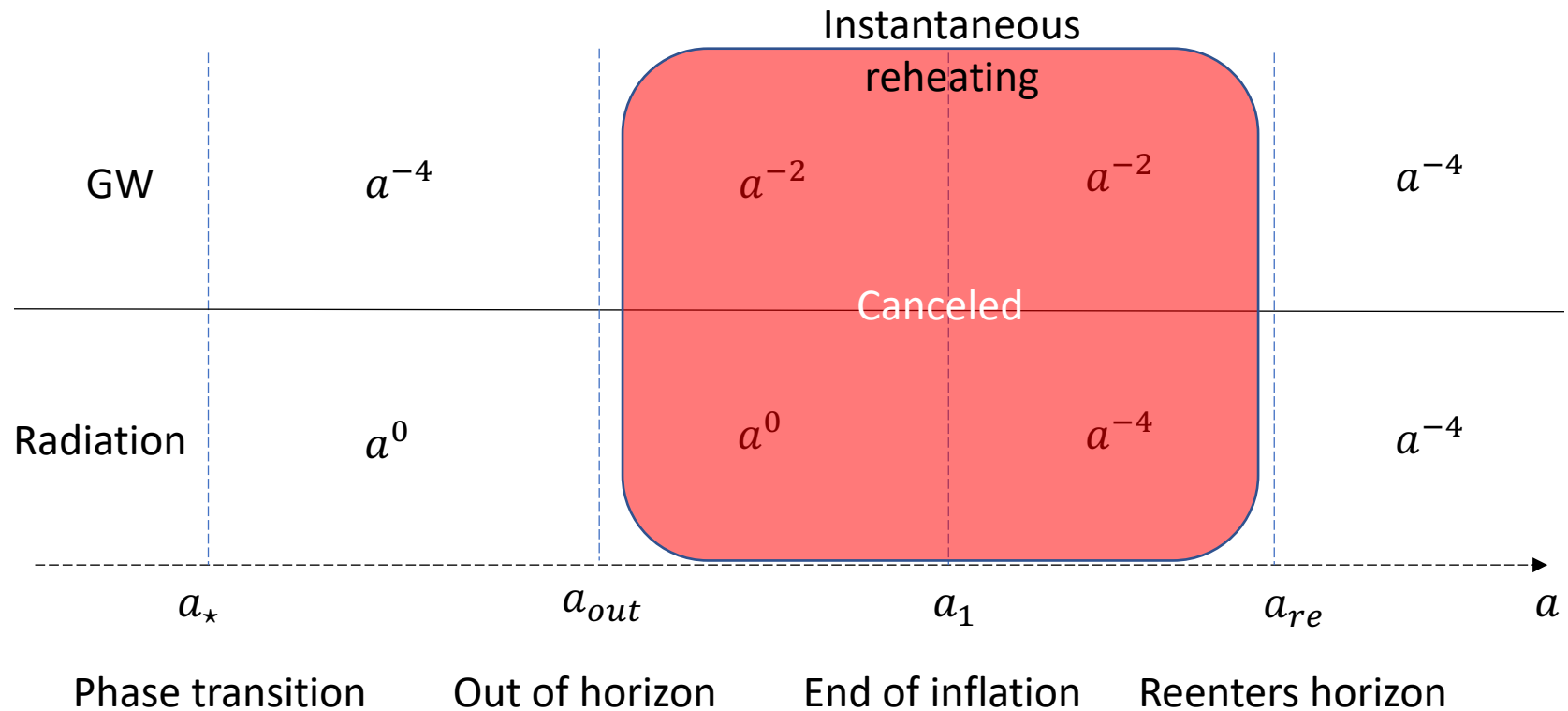
Summary for FOPT



Redshifts of the GW signal

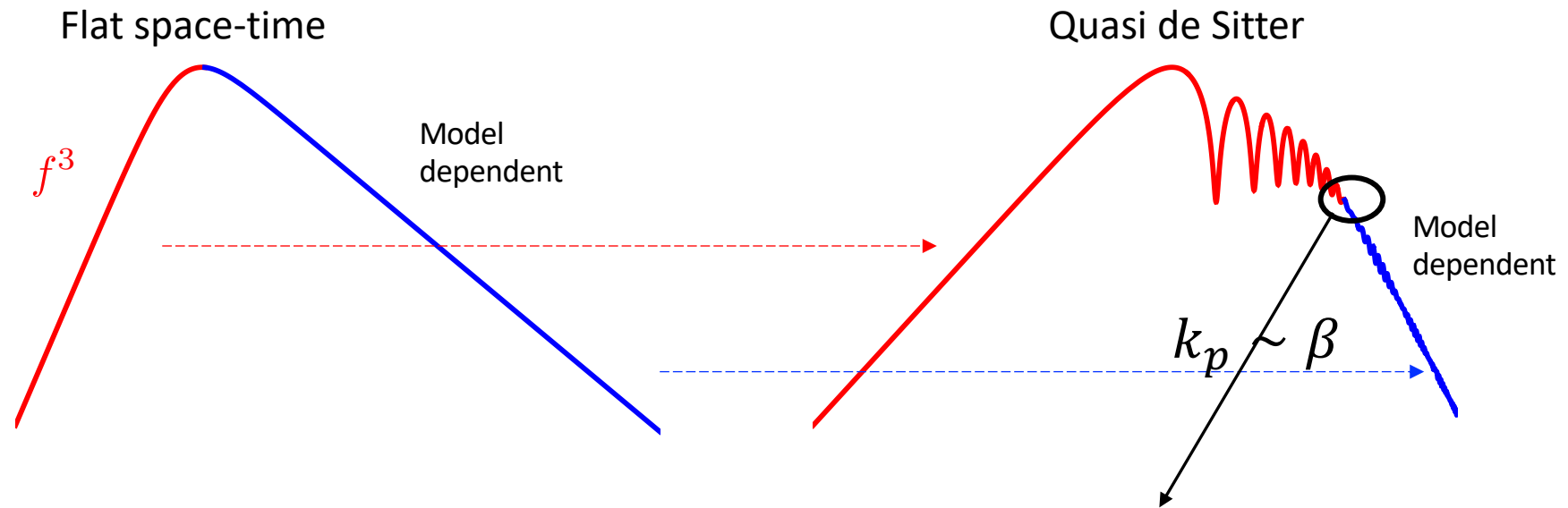


Redshifts of the GW signal



$$\frac{\Omega_{\text{GW}}}{\Omega_{\gamma}} \sim \left(\frac{a_{\star}}{a_{\text{out}}} \right)^4 \sim \left(\frac{H}{\beta} \right)^4$$

Spectrum distortion by inflation

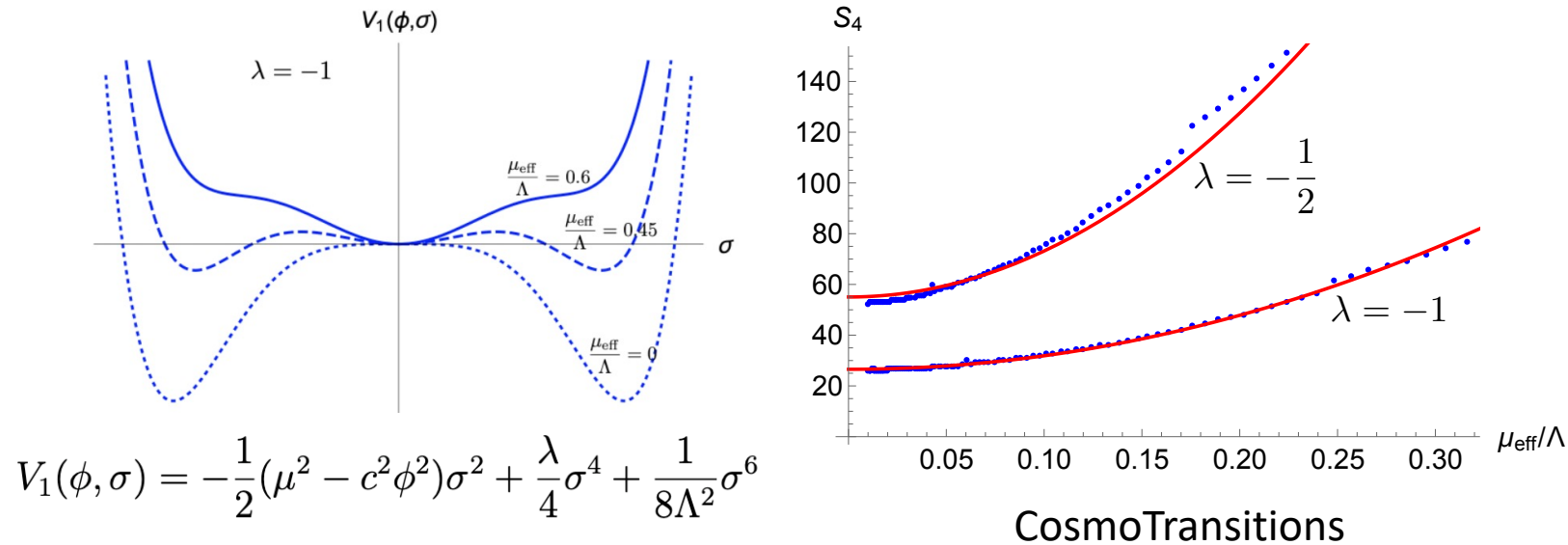


$$\Omega_{\text{GW}} \approx \Omega_R \left(\frac{H_{\text{inf}}}{\beta} \right)^6 \left(\frac{\Delta \rho_{\text{vac}}}{\rho_{\text{inf}}} \right)^2$$

First order phase transition during inflation

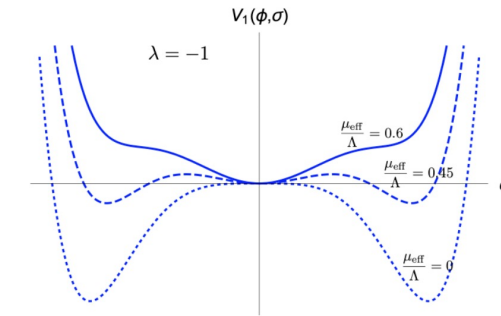
- $$\beta = \left| \frac{dS_4}{dt} \right| = \frac{dS_4}{d \log \mu_{\text{eff}}^2} \times \left| \frac{2\dot{\phi}}{\phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right)} \right| \quad \mu_{\text{eff}}^2 = -(\mu^2 - c^2 \phi^2)$$

$$\longrightarrow \frac{\beta}{H} = \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| (2\epsilon)^{1/2} \times \frac{M_{\text{pl}}}{\left| \phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right) \right|}$$



First order phase transition during inflation

- $$\beta = \left| \frac{dS_4}{dt} \right| = \frac{dS_4}{d \log \mu_{\text{eff}}^2} \times \left| \frac{2\dot{\phi}}{\phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right)} \right|$$



$$\Rightarrow \frac{\beta}{H} = \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| (2\epsilon)^{1/2} \times \frac{M_{\text{pl}}}{\left| \phi \left(1 - \frac{\mu^2}{c^2 \phi^2} \right) \right|}$$

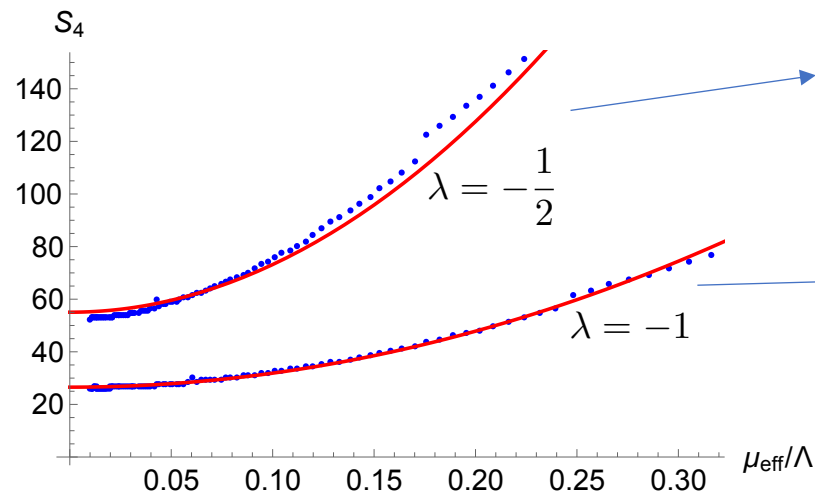
$\sim \mu_{\text{eff}}^2 / \Lambda^2$

$$\int_{\phi_{\text{end}}}^{\phi_{\text{PT}}} \frac{d\phi}{\sqrt{2\epsilon} M_{\text{pl}}} = N_e$$

$$\frac{\beta}{H} \sim \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$

First order phase transition during inflation

- $$\frac{\beta}{H} \sim \left| \frac{dS_4}{d \log \mu_{\text{eff}}^2} \right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$



$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

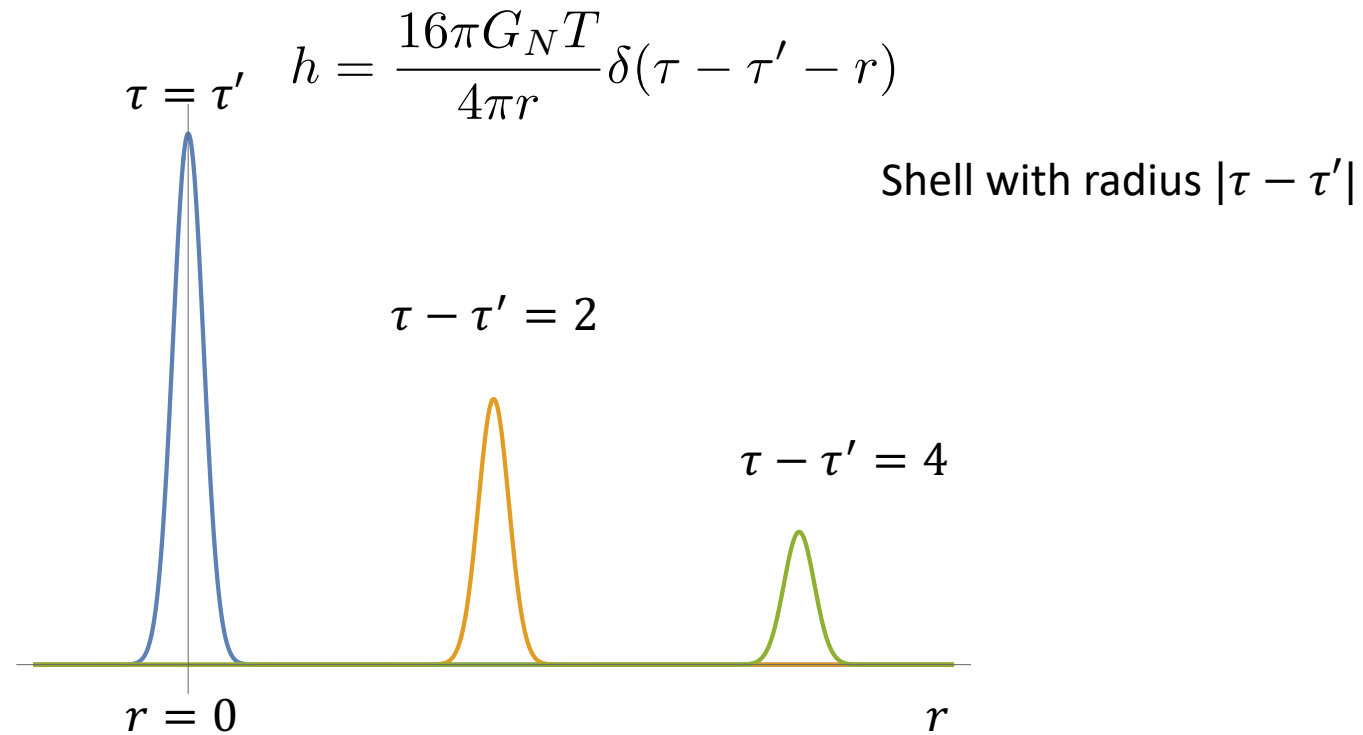
N_e : e-folds before the end of inflation

$$V_1(\phi, \sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij} ?
- In Minkovski space



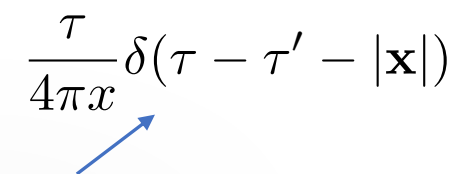
de Sitter inflation as an example

- What is the spatial configuration of h_{ij} ?
- In de Sitter space

$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} + \left(\frac{1}{k^2 \tau} - \frac{1}{k^2 \tau'} \right) \cos k(\tau - \tau') + \frac{1}{k^3 \tau \tau'} \sin k(\tau - \tau') \right]$$

de Sitter inflation as an example

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$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} \right. \\ \left. + \left(\frac{1}{k^2 \tau} - \frac{1}{k^2 \tau'} \right) \cos k(\tau - \tau') + \frac{1}{k^3 \tau \tau'} \sin k(\tau - \tau') \right] \\ \frac{1}{4\pi} \Theta(\tau - \tau' - |\mathbf{x}|)$$


de Sitter inflation as an example

- What is the spatial configuration of h_{ij} ?
- In de Sitter space

$$h(\tau, \mathbf{x}) \sim \frac{\tau}{4\pi x} \delta(\tau - \tau' - x) + \frac{1}{4\pi} \Theta(\tau - \tau' - x)$$

Similar to Minkovski

Intrinsic in de Sitter

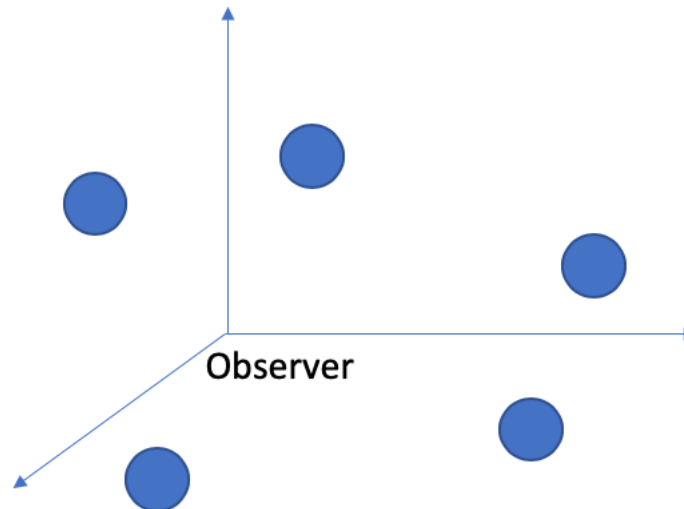
Decreases with both x and τ

constant

Vanishes out of horizon

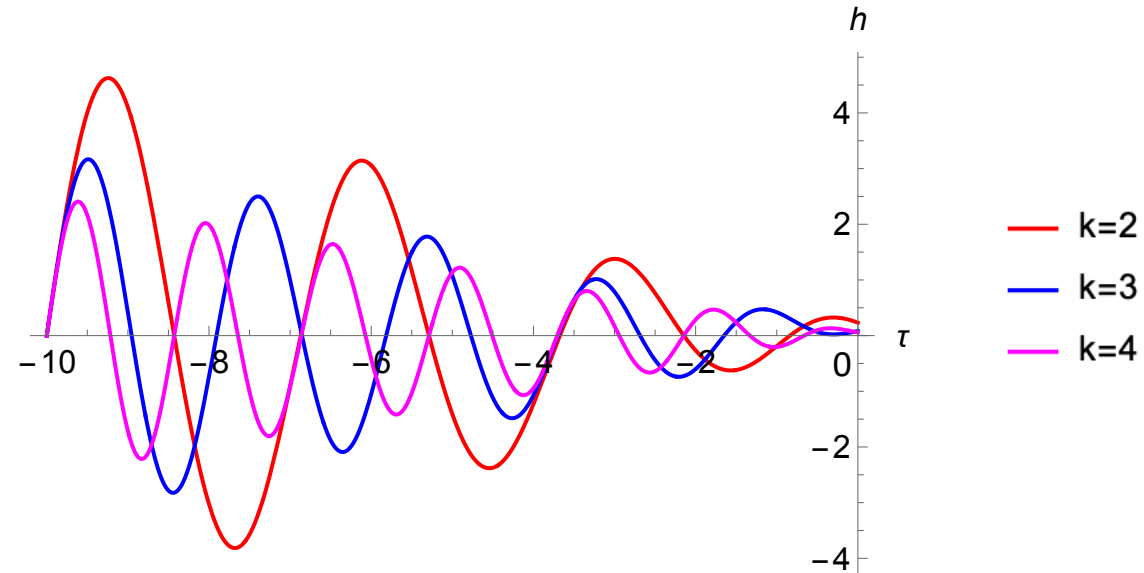
de Sitter inflation as an example

- At $\tau \rightarrow 0$ $h(\tau, \mathbf{x}) \sim \frac{1}{4\pi} \Theta(|\tau'| - x)$
- A ball of GW, with radius $|\tau'|$
- h uniformly distributed inside the GW balls.
- All the balls have the same radius.



Quasi-de Sitter inflation as an example

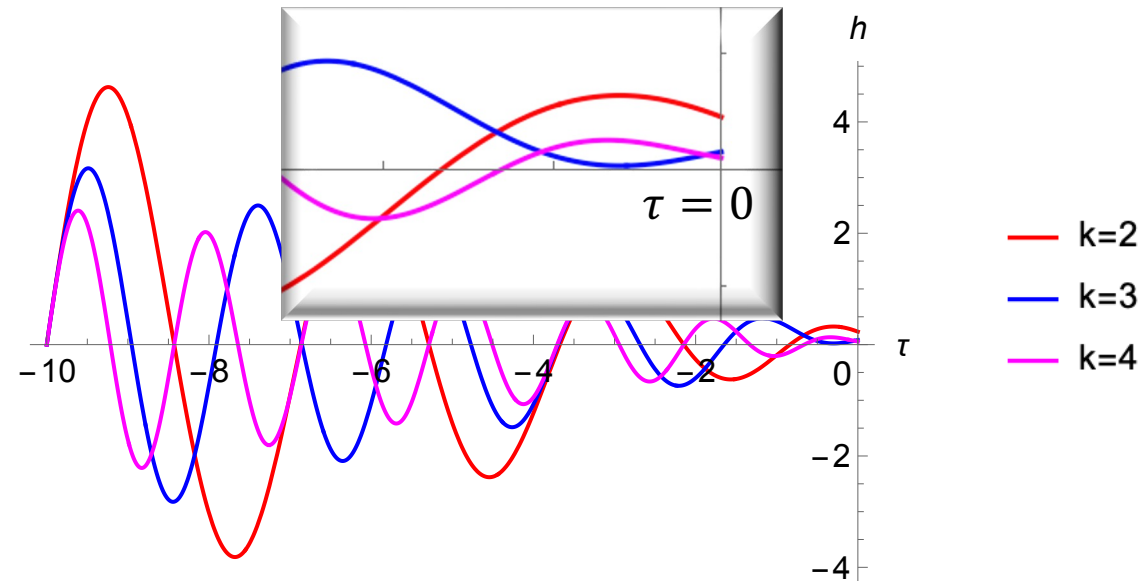
- $a = -\frac{1}{H\tau}$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij} \tau}{k} \left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'} \right) \cos k(\tau - \tau') + \left(1 + \frac{1}{k^2 \tau \tau'} \right) \sin k(\tau - \tau') \right]$$



De Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$

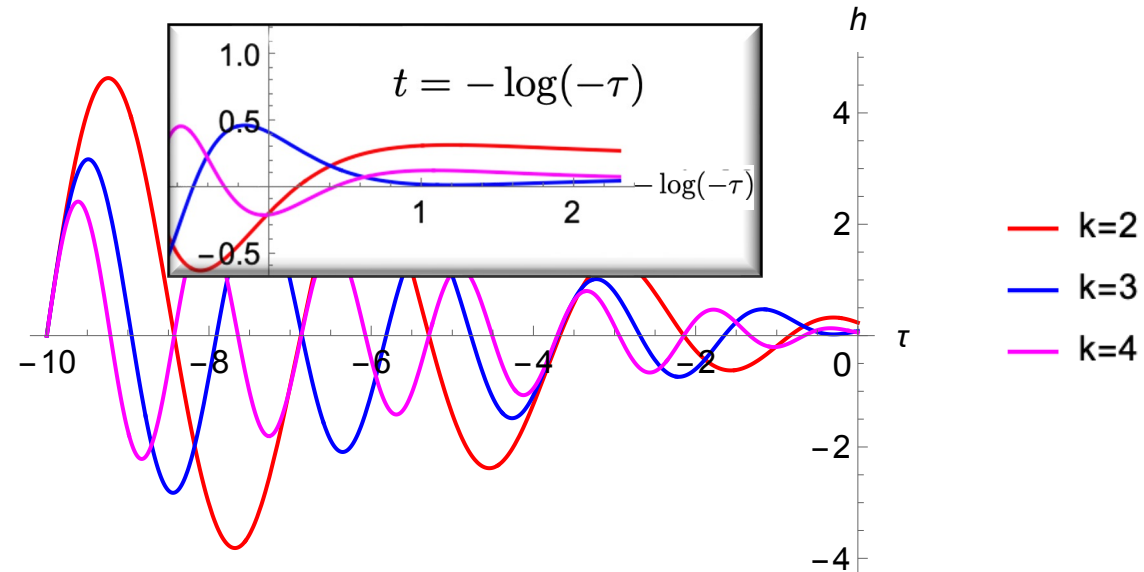
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij} \tau}{k} \left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'} \right) \cos k(\tau - \tau') + \left(1 + \frac{1}{k^2 \tau \tau'} \right) \sin k(\tau - \tau') \right]$$



De Sitter inflation as an example

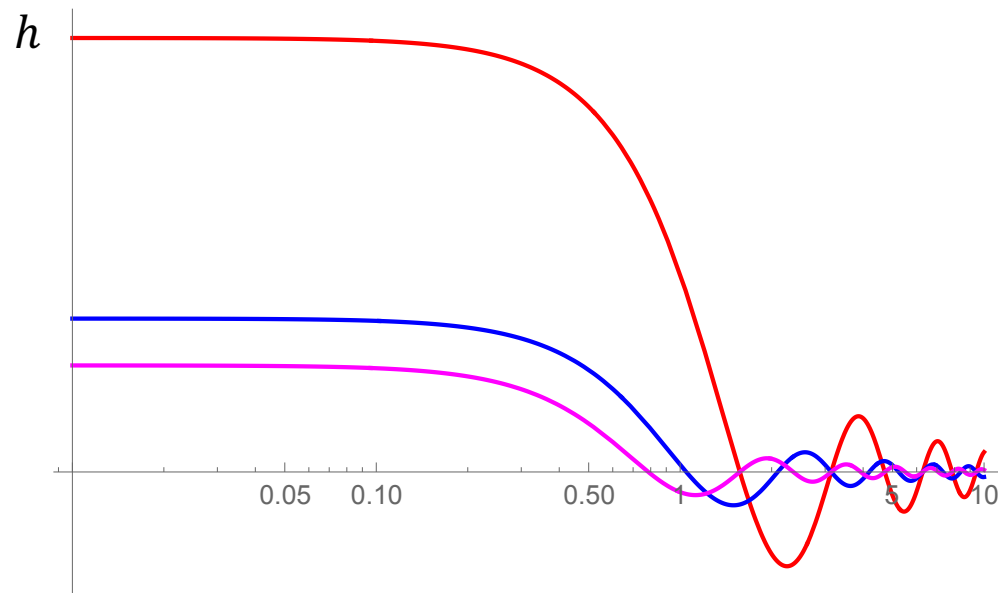
- $$a = -\frac{1}{H\tau}$$

- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij} \tau}{k} \left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'} \right) \cos k(\tau - \tau') + \left(1 + \frac{1}{k^2 \tau \tau'} \right) \sin k(\tau - \tau') \right]$$



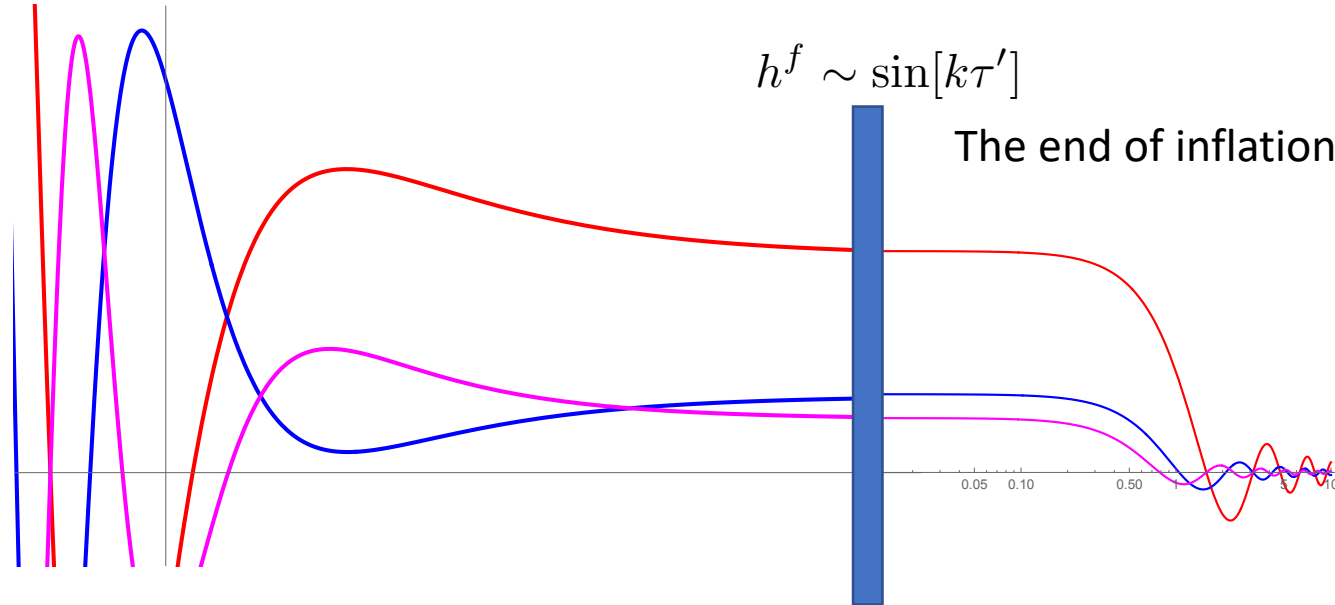
After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



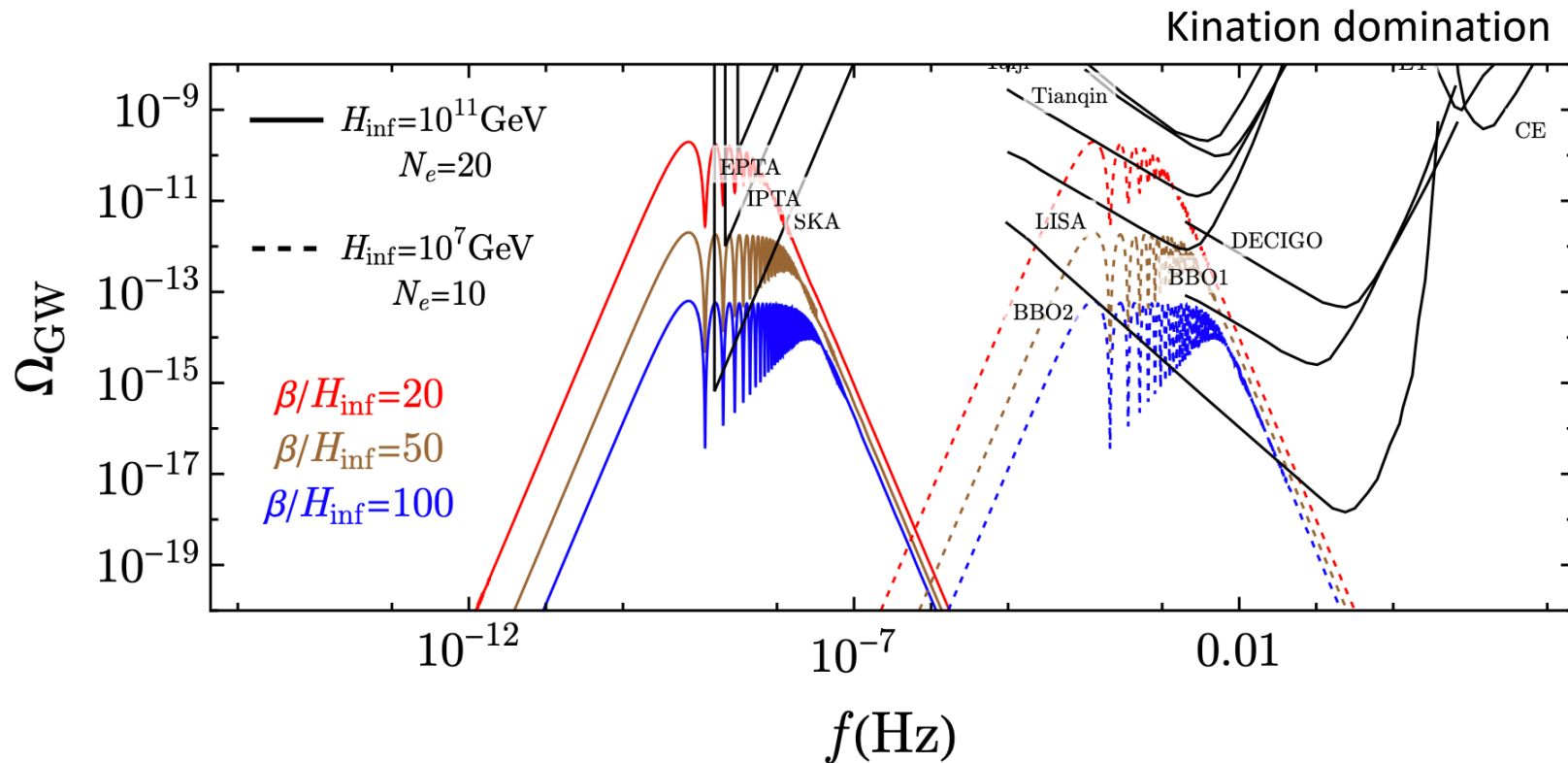
After inflation

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- All the modes start to oscillate with the same phase.
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First order phase transition during inflation

- Signal strength is also sensitive to intermediate stages



First order phase transition during inflation

