香港科技大學
THE HONG KONG
UNIVERSITY OF SCIENCE
AND TECHNOLOGY

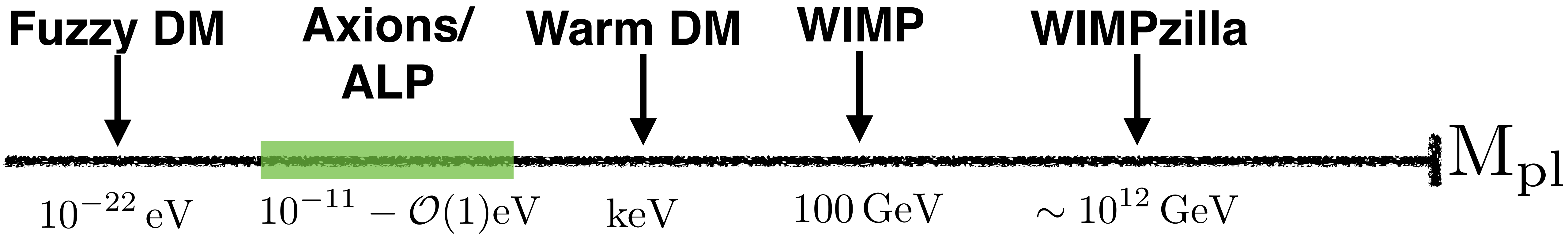
Macroscopic States as Dark Matter Candidates

Sida Lu (卢思达)

@SYSU Shenzhen, 04/13/2024

based on:
1810.04360, w/ Yang Bai and Andrew J. Long
2111.10360, 2208.12290, 2312.13378, w/ Yang Bai and Nicholas Orlofsky

DM Zoo



Fuzzy DM

$10^{-22}\,\mathrm{eV}$

Axions/ ALP

$10^{-11} - \mathcal{O}(1)\mathrm{eV}$

Warm DM

keV

WIMP

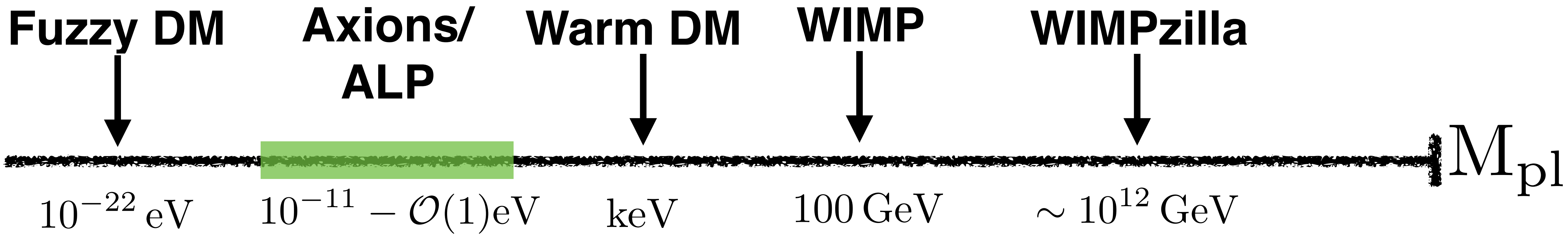
$100\,\mathrm{GeV}$

WIMPzilla

$\sim 10^{12}\,\mathrm{GeV}$

$\mathrm{M_{pl}}$

DM Zoo



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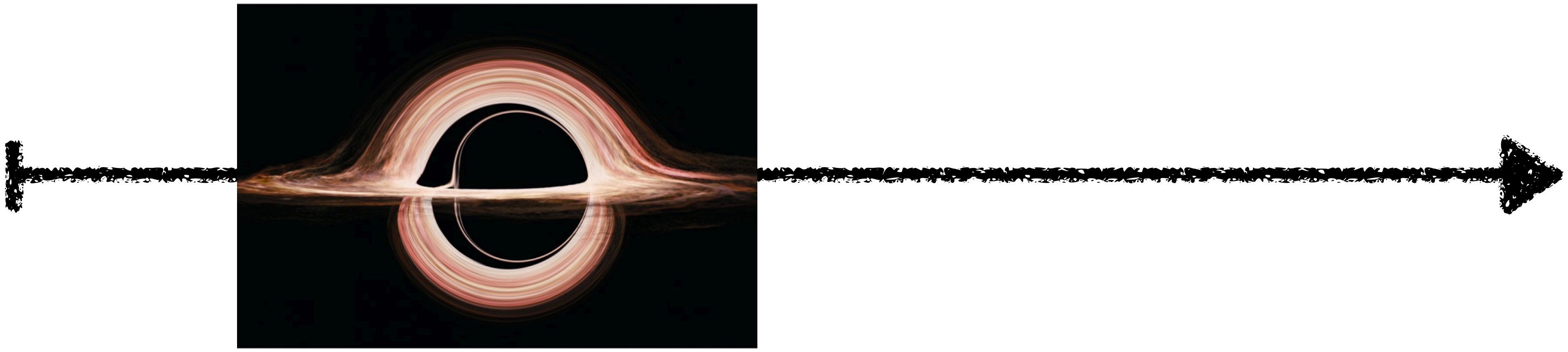
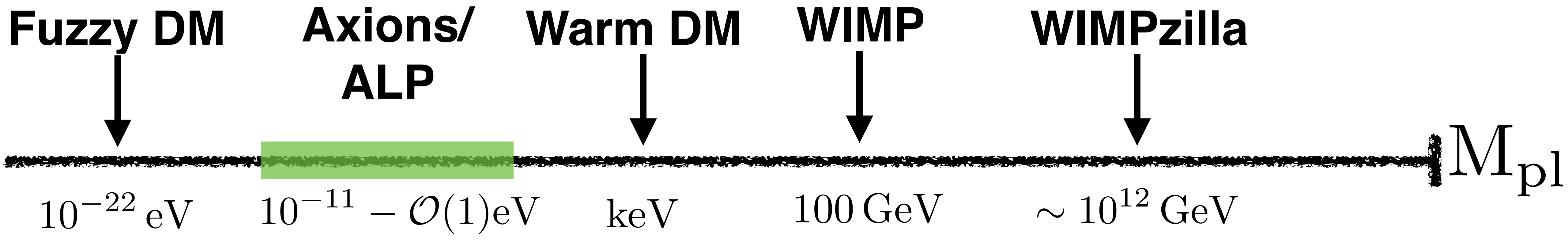
PBHs



See Prof. Sai Wang's and Prof. Ke-Pan Xie's talks this morning and Zeng Zhenmin's talk later for discussions on PBHs

DM Zoo



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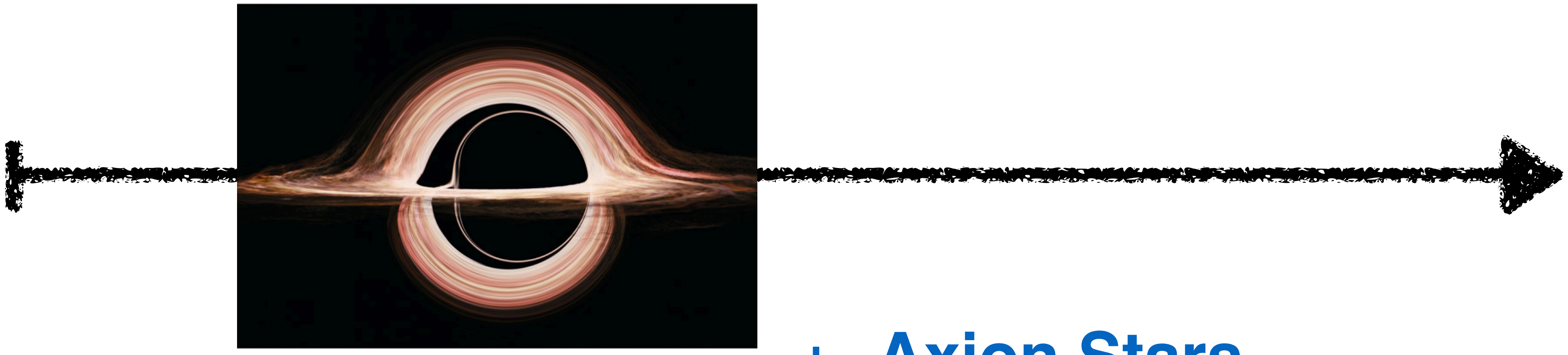
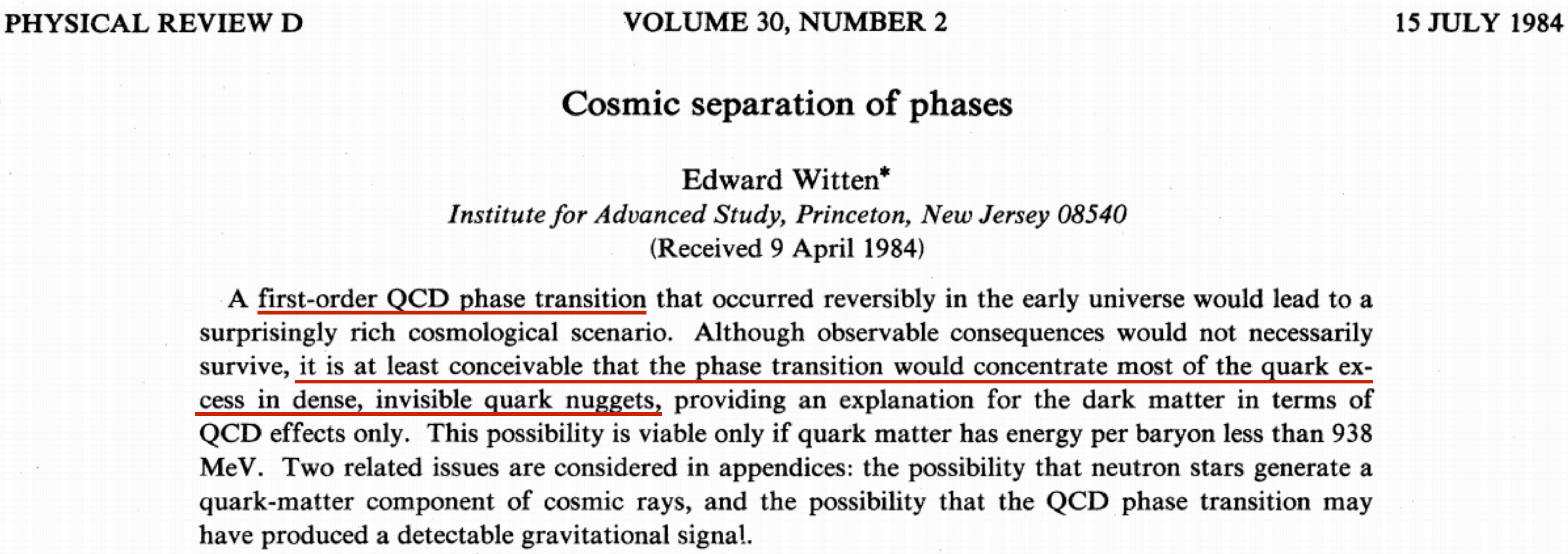


- **+ Axion Stars**
- **+ Dark Quark Nuggets**
- **+ Q-balls**
- **+ …**

See Prof. Hong Zhang's talk tomorrow for a discussion (possibly) on axion stars

MDMs from Phase Transitions



PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

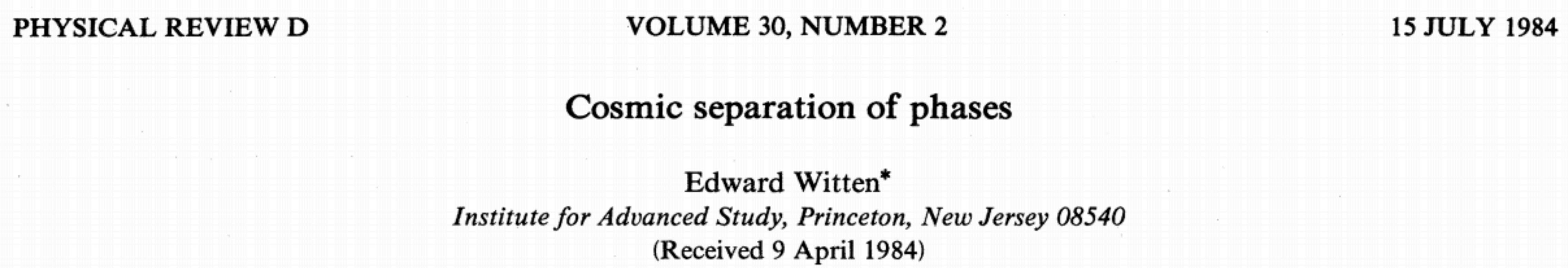
Cosmic separation of phases

Edward Witten*
Institute for Advanced Study, Princeton, New Jersey 08540

(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

MDMs from Phase Transitions



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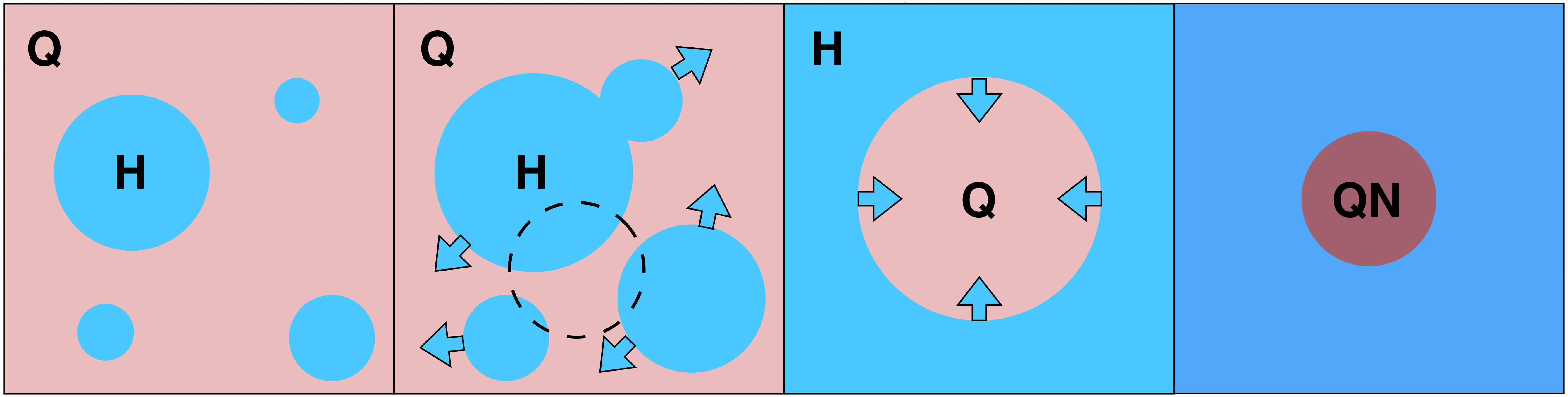
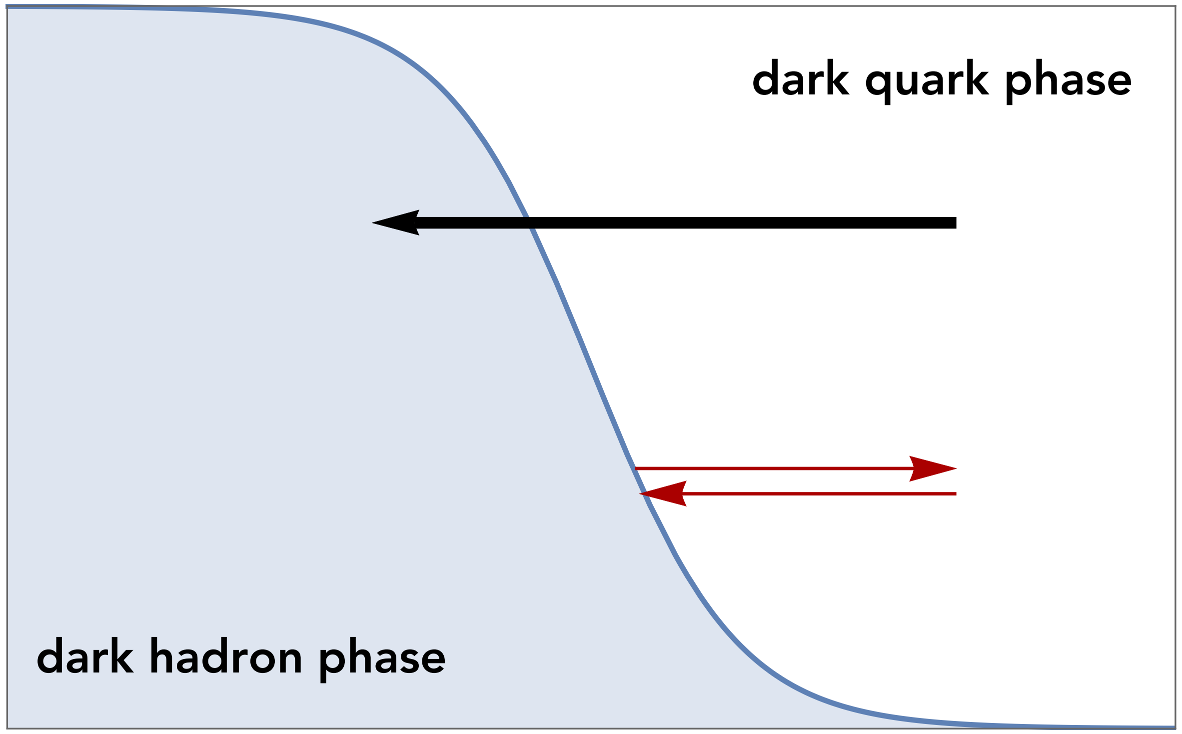
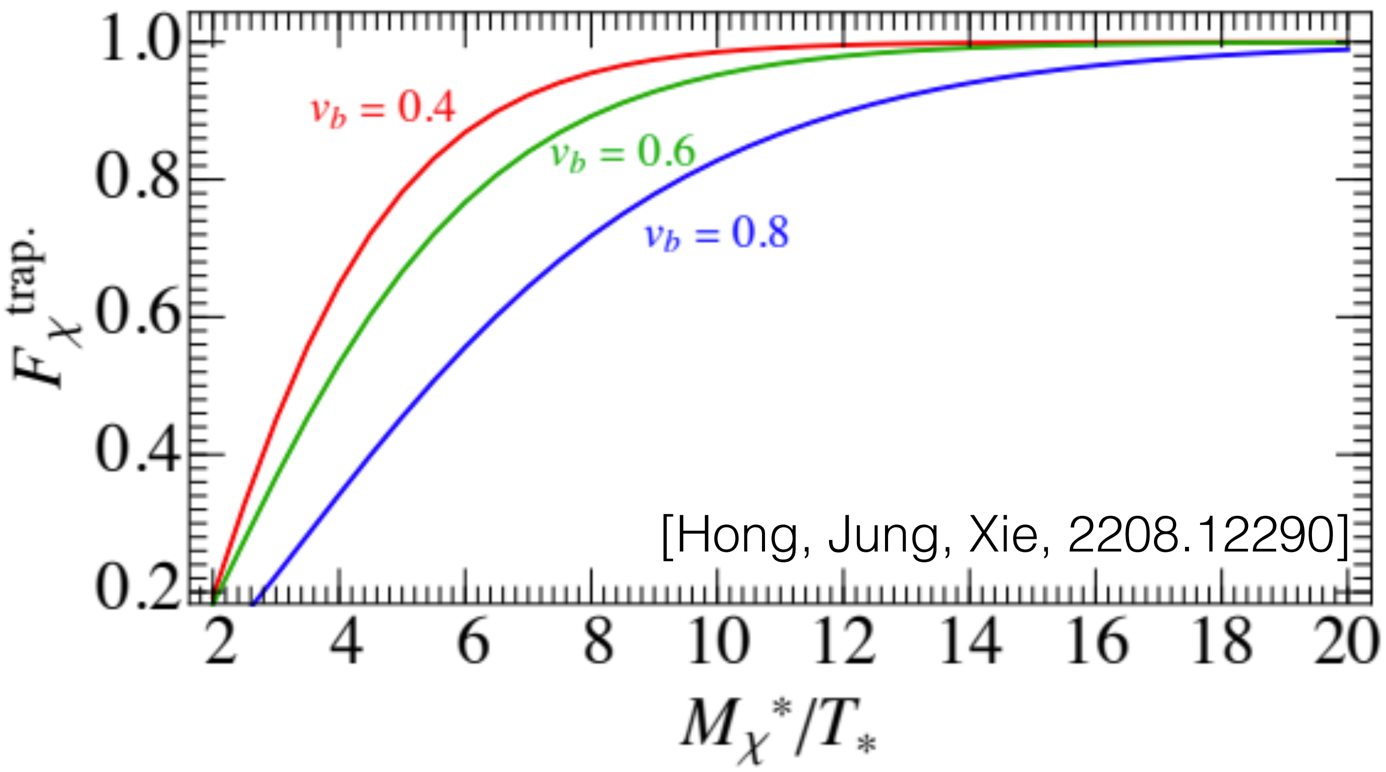


Q: quarks H: hadrons QN: quark nuggets

Trapped in the False Vacuum

- **Some particles in the unbroken phase may not be energetic enough to tunnel through**
  - ➡ Bubble wall velocity
  - ➡ Mass of the corresponding state in the broken phase

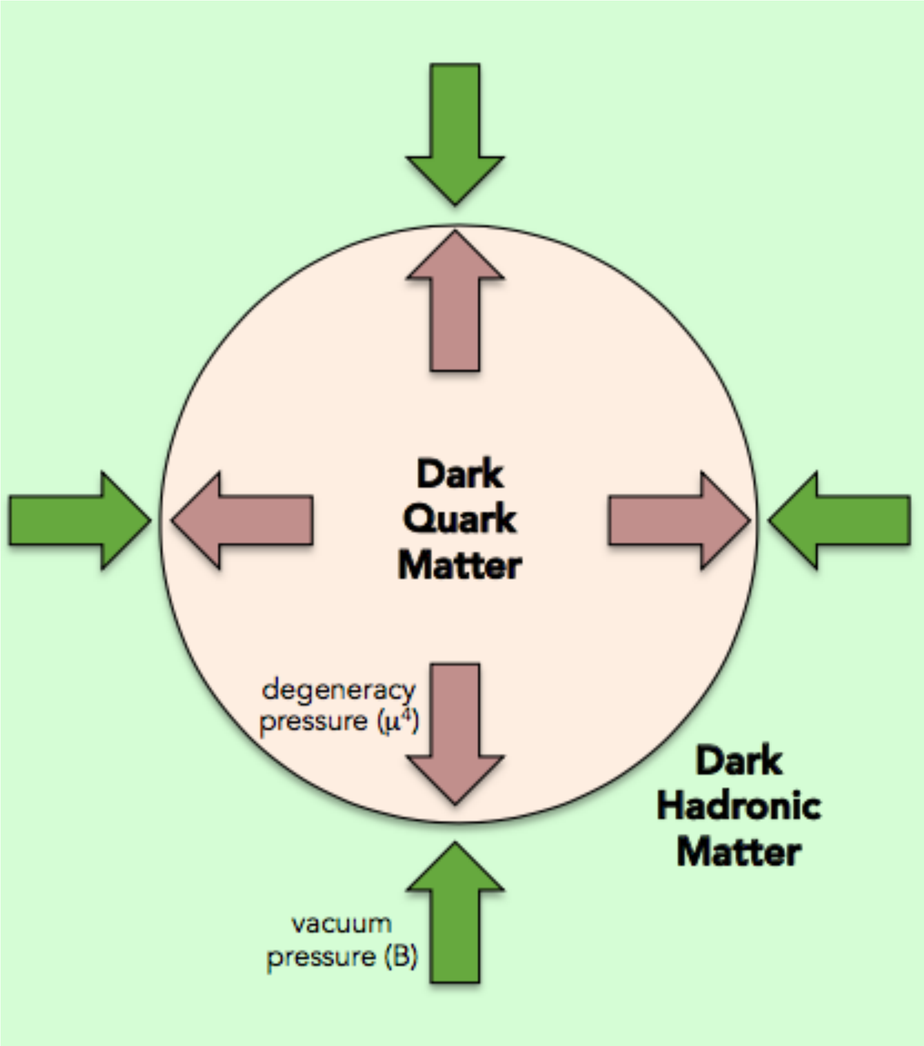




[Hong, Jung, Xie, 2208.12290]

Balancing the Vacuum Pressure

- **Pressure from the true vacuum is countered by the Fermi degeneracy pressure**



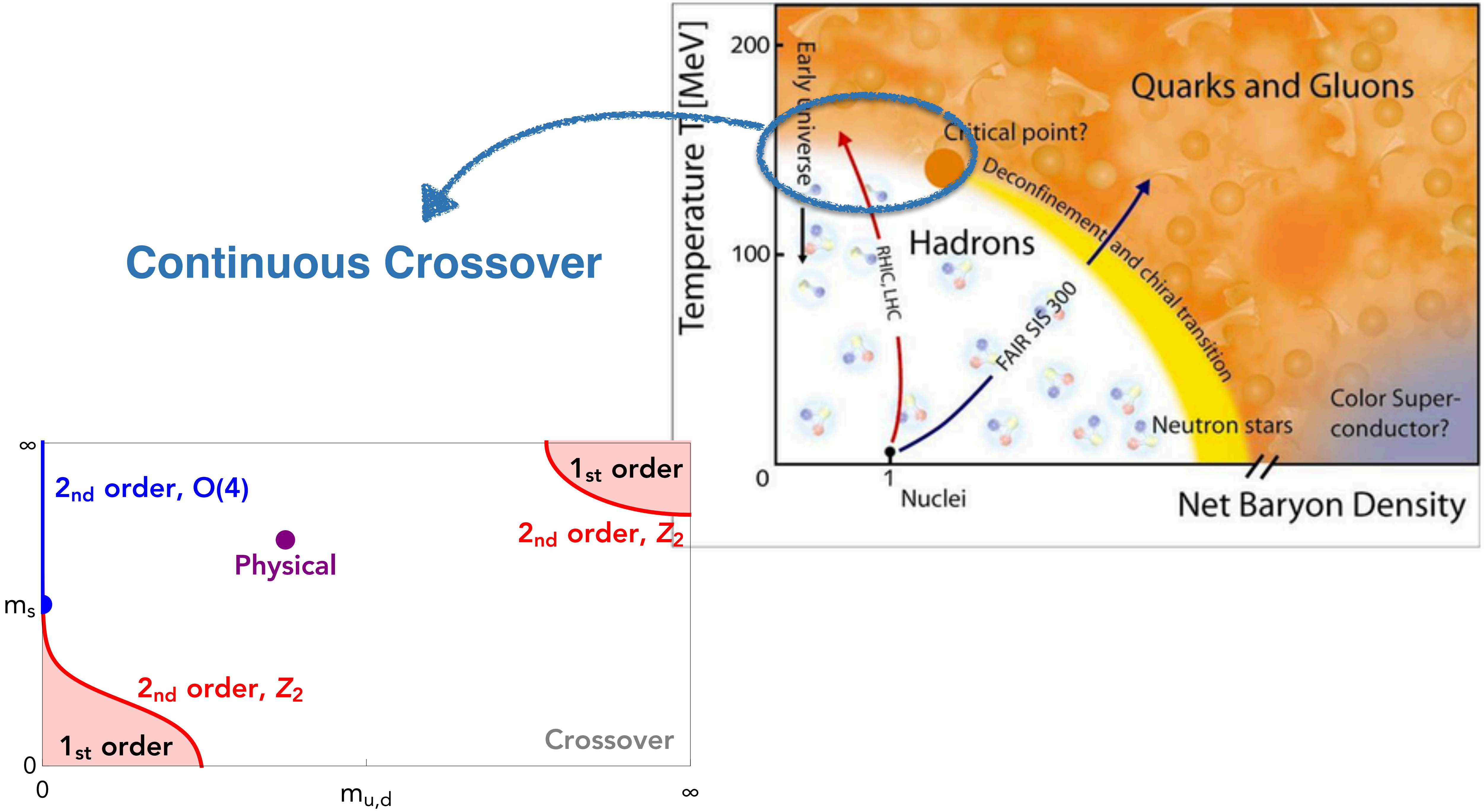
- **From thermal dynamics:**

$$n = g\frac{\mu^3}{6\pi^2} \quad \rho = g\frac{\mu^4}{8\pi^2} + B \quad P = g\frac{\mu^4}{24\pi^2} - B$$

$$g = 2N_d N_f \qquad n_{\mathsf{B}_{\mathrm{d}},\mathrm{nug}} = \frac{1}{N_d} n = N_f \frac{\mu^3}{3\pi^2}$$

- **The bag parameter:** $B \sim \Lambda_d^4$

QCD is Upsetting…



Continuous Crossover

…While the Dark Sector is Still Fine

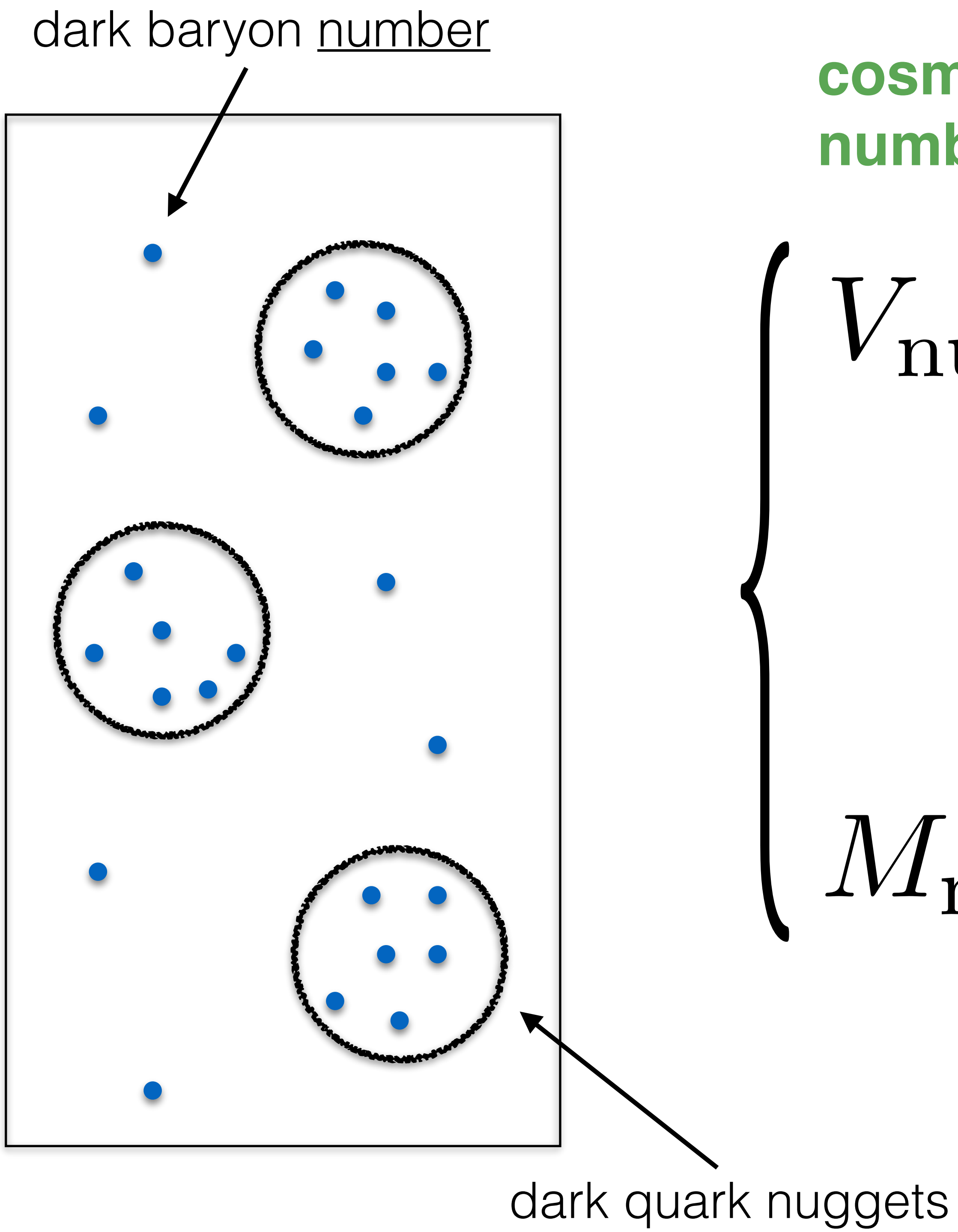
- **A direct FOPT from a scalar potential**
- **Or being lazy and just copy the QCD into the dark sector (but have light dark quarks s.t. FOPT can happen)**

$$\mathcal{L}_{\rm dQCD} = \sum_{i=1}^{N_f} \left[\bar{\psi}_i i\gamma^\mu D_\mu \psi_i - m_{\psi_i}\, \bar{\psi}_i \psi_i\right] \,-\, \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a}$$

  - ➡ The “dark quark nuggets”

[Bai, Long, *SL*, 1810.04360]

Size and Mass



cosmic dark baryon number density

fraction of dark baryon number as DQN

$$\begin{cases} V_{\mathrm{nug}} = \dfrac{n_{\mathrm{B}_d} \, f_{\mathrm{B}_d,\mathrm{nug}}}{n_{\mathrm{nug}} \, n_{\mathrm{B}_d,\mathrm{nug}}} \\[2em] M_{\mathrm{nug}} = \rho \, V_{\mathrm{nug}} \end{cases}$$

cosmic DQN number density

dark baryon number density in DQN

energy density inside DQN

$$\Rightarrow \begin{cases} \rho = 4B \\ n_{\mathrm{B_d},\mathrm{nug}} = \left(\dfrac{64N_f}{3\pi^2 N_d^3}\right)^{1/4} B^{3/4} \end{cases}$$

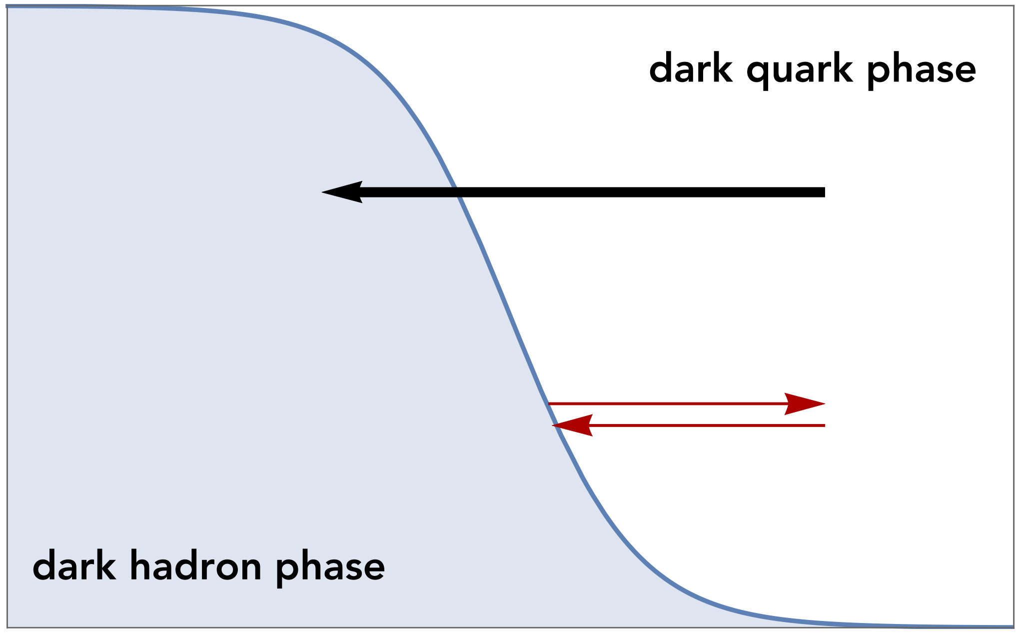
$$V_{\mathrm{nug}} = \frac{\underline{n_{\mathrm{B}_d}}\,\underline{f_{\mathrm{B}_d,\mathrm{nug}}}}{\underline{n_{\mathrm{nug}}}\,n_{\mathrm{B}_d,\mathrm{nug}}}$$

✤ Cosmic dark baryon number density

$$n_{\mathrm{B}_d}(t_c) = \underset{\text{dark baryon asymmetry}}{\underline{Y_{\mathrm{B}_d}}}\, s(t_c), \qquad s = (2\pi^2/45)\, g_{*S}\, T^3_{\gamma,c}$$

✤ Fraction of dark baryon number carried by DQN

$$f_{B_d} = \frac{N_{B_d} N_d}{N_f} \frac{\sqrt{2\pi}}{3\zeta(3)} \left(\frac{m_{B_d}}{T_c}\right)^{3/2} e^{-m_{B_d}/T_c}, \qquad f_{\mathrm{B_d},\mathrm{nug}} \approx 1 - f_{B_d}$$



$$V_{\rm nug} = \frac{\underline{n_{\mathrm{B}_d}}\, \underline{f_{\mathrm{B}_d,\rm nug}}}{\underline{n_{\rm nug}}\, n_{\mathrm{B}_d,\rm nug}}$$

✤ Cosmic DQN number density

$$n_{\rm nug}(t_c) \approx n_{\rm nucleation}(t_c) = \left(2.1\times 10^{14}\right)\left(\frac{\tilde\sigma}{0.1}\right)^{-9/2} H(t_c)^3$$

nugget number comparable to nucleation sites

$$\tilde\sigma \equiv \sigma/(B^{2/3}T_c^{1/3})$$

- ➡ bubble nucleation rate

$$\gamma \approx \zeta\, T^4 \left(\frac{S_3}{2\pi\, T}\right)^{3/2} e^{-S_3/T}$$

- ➡ fraction of space in the unbroken phase

$$f_{\rm unbroken}(t) = \exp\left[-\frac{4\pi}{3}\int_{t_c}^{t} dt'\, \beta_{\rm w}^3\, (t-t')^3\, \gamma(t')\right]$$

- ➡ number density of nucleation sites

$$n_{\rm nuclear tion} = \int_{t_c}^{\infty} dt' \gamma(t') f_{\rm unbroken}(t')$$

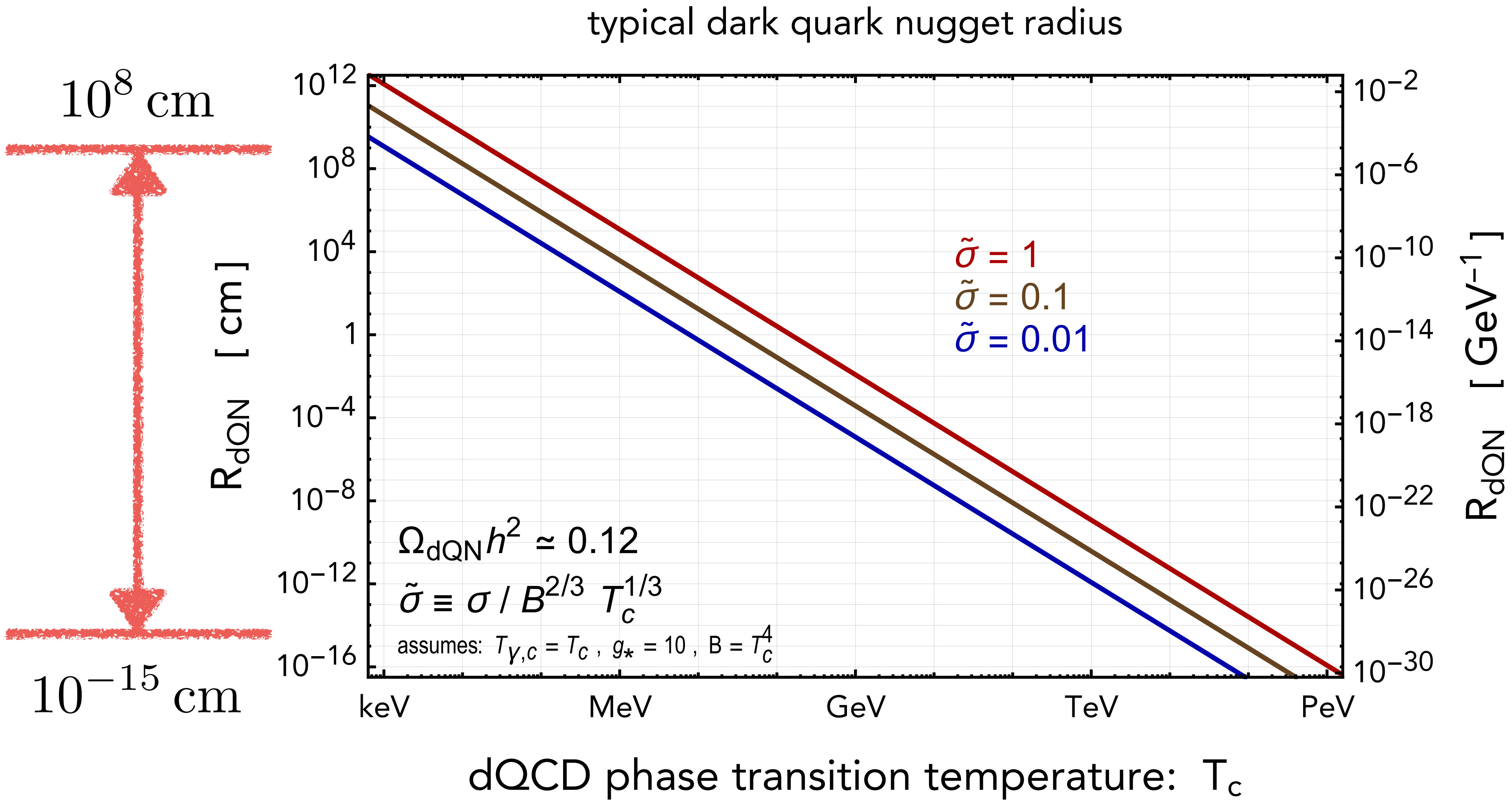
See Kawana, Lu, Xie 2206.09923
for an *ab initio* calculation on $n_{\rm nug}$

[Fuller, Mathews, Alcock, Phys.Rev.D 37 (1988) 1380]

R and M at the Correct Abundance

- **Combining the ingredients before**

$$R_{\rm nug} \simeq (0.081\ {\rm cm}) \left[\frac{B}{(0.1\ {\rm GeV})^4}\right]^{-1/3} \left(\frac{T_c}{0.1\ {\rm GeV}}\right)^{-1} \left(\frac{\tilde{\sigma}}{0.1}\right)^{3/2}$$

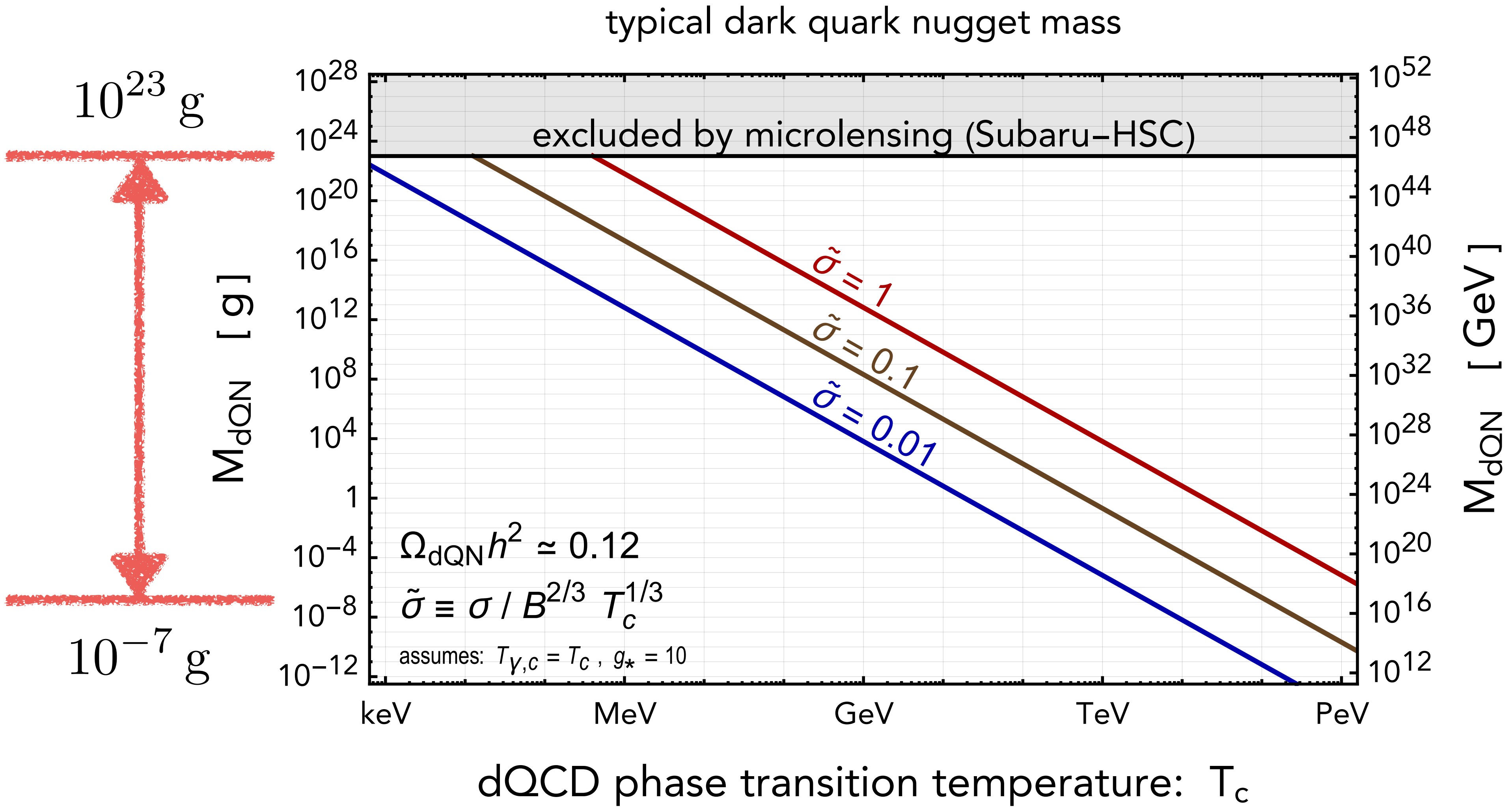


(for $N_f = N_d = 3$)

R and M at the Correct Abundance

- **Combining the ingredients before**

$$M_{\rm nug} \simeq (2.1 \times 10^{11}\ {\rm g}) \left(\frac{T_c}{0.1\ {\rm GeV}}\right)^{-3} \left(\frac{\tilde{\sigma}}{0.1}\right)^{9/2}$$



(for $N_f = N_d = 3$)

Non-topological Solitons

- **Stable macroscopic states may also exist in a scalar theory**
  - A (global) symmetry to protect the stability
  - A scalar potential providing an attractive force

[See Lee and Pang, Phys. Rept. 221 (1992) 251-350 for a review]

- **Let's use Q-ball with a *global* U(1) as an example**
  - Let $\Phi = \phi(r)\, e^{-i\omega t}/\sqrt{2}$
  - From the Lagrangian we immediately have

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} \underline{+ \omega^2\phi - \frac{dU'}{d\phi}} = 0$$

effective potential

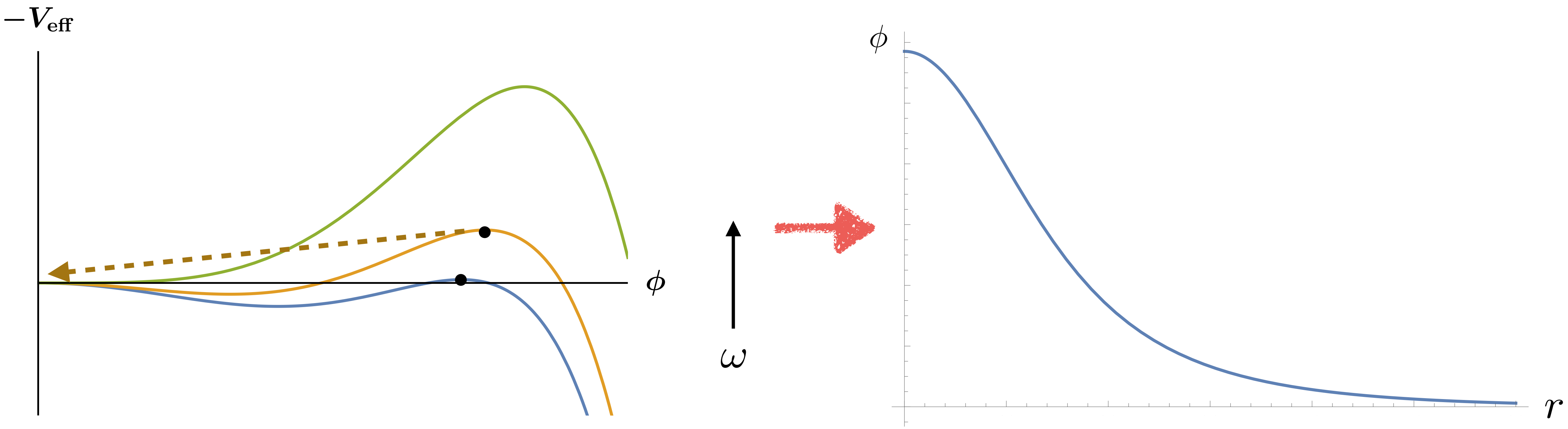
$$V_{\rm eff} = U - \frac{1}{2}\omega^2\phi^2$$

  - By defining an effective potential, the EOM has a Newtonian interpretation if we take $r \to t,\ \phi \to x$

Non-topological Solitons

- **A particle moving along -V**

$$\phi(r=0)=\phi_0,\ \phi(r=\infty)=0 \to x(t=0)=x_0,\ x(t=\infty)=0$$

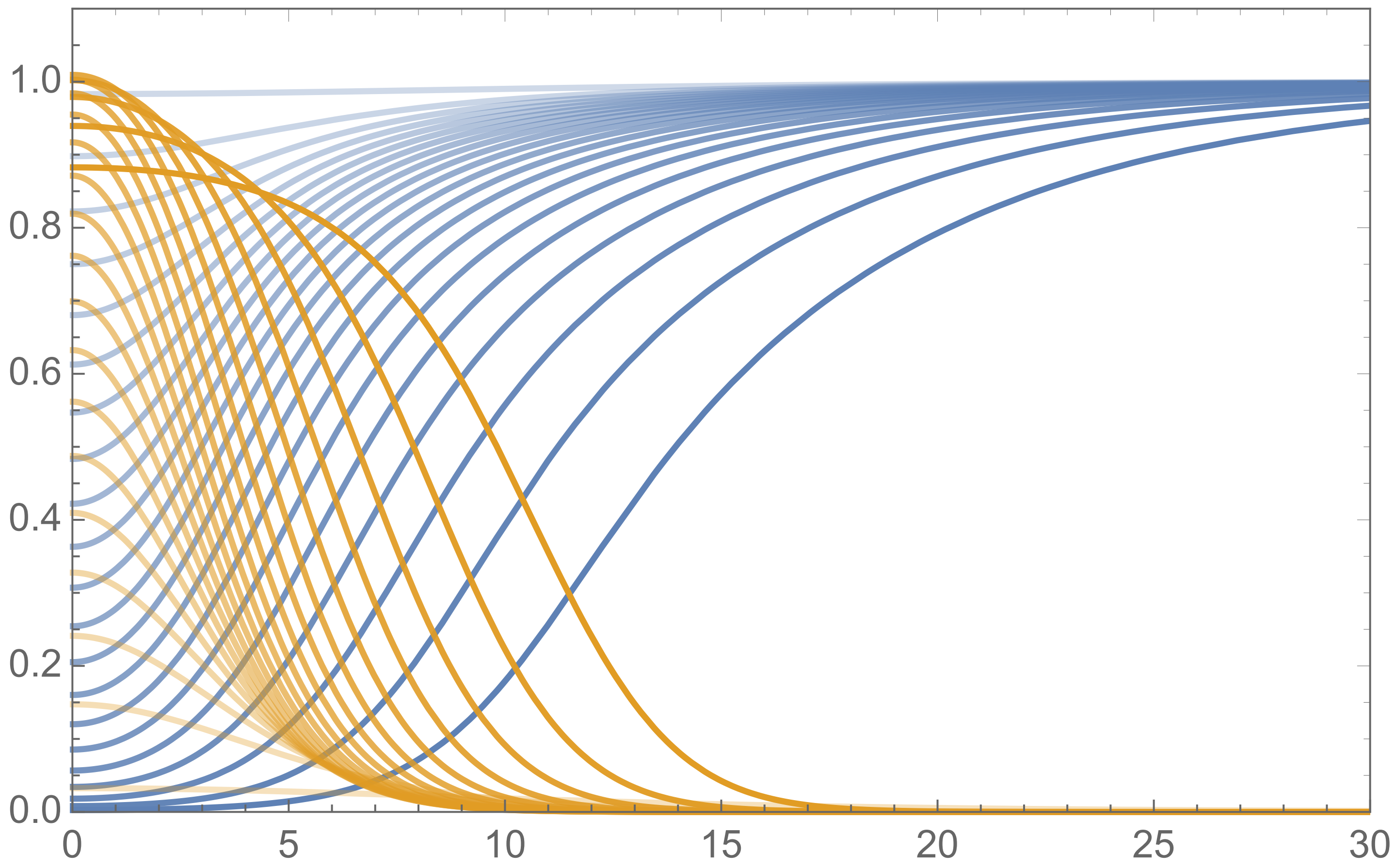


- ➡ There must be a local minimum and a local maxima
- ➡ The local maxima must be greater than zero
- ➡ The larger ω, the smaller charge for the Q-ball (and *vice versa*)

Some Q-ball Examples

- **At the renormalizable level, Q-balls should involve at least two fields**

$$V(S,\phi) = \frac{1}{4}\lambda_\phi(|\phi|^2 - v^2)^2 + \frac{1}{4}\lambda_{\phi S}|S|^2|\phi|^2 + \lambda_S|S|^4 + m_{S,0}^2|S|^2$$



symmetry "restoration"

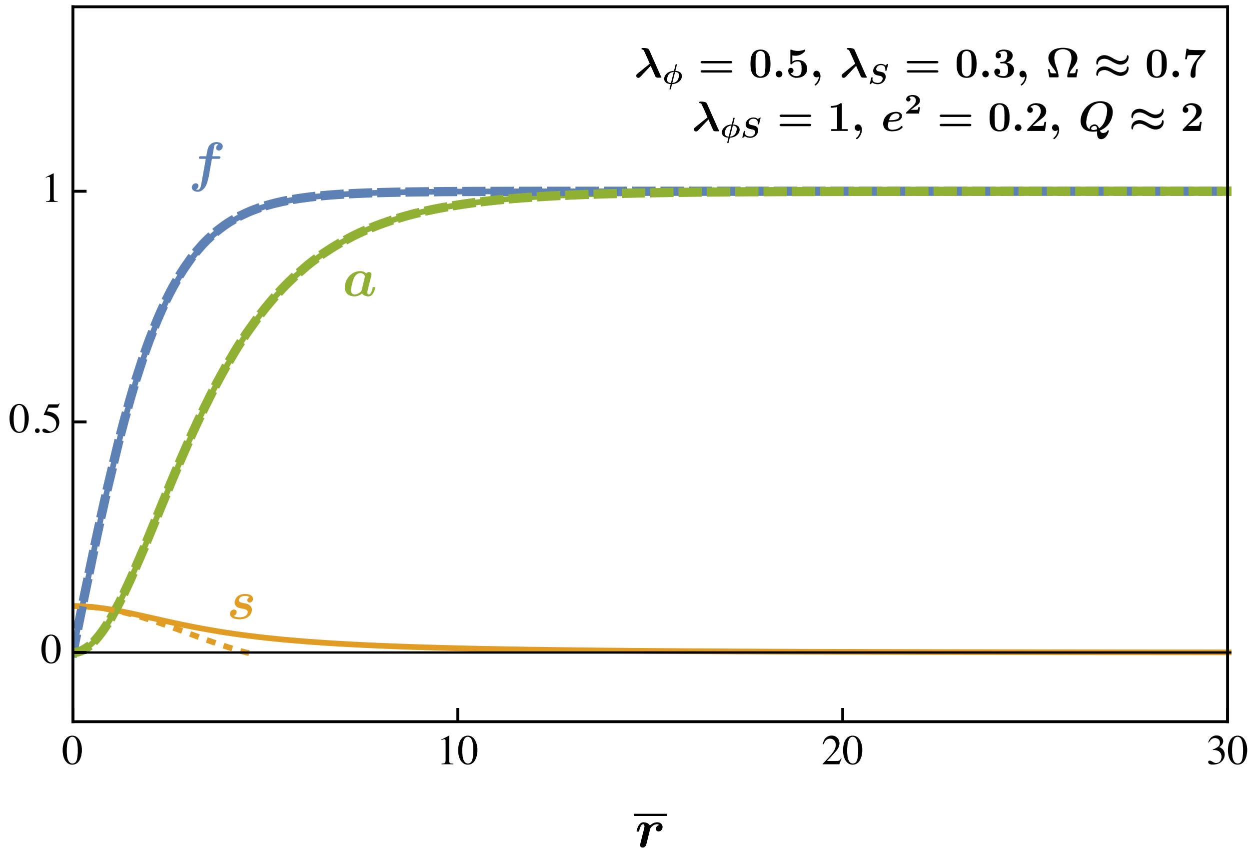
[Bai, *SL*, Orlofsky, 2208.12290]

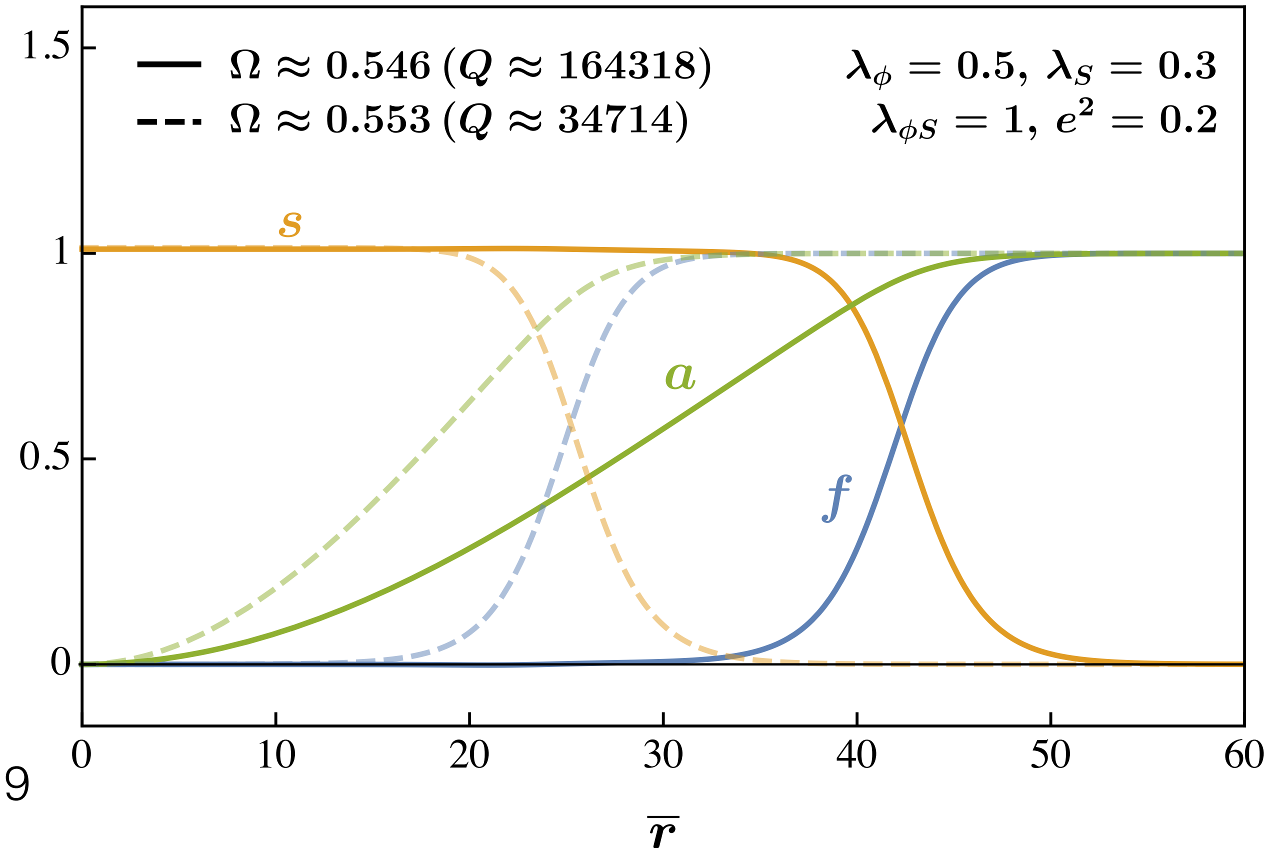
Some Q-ball Examples

- **At the renormalizable level, Q-balls should involve at least two fields**
  - ➡ sth fancy: a *non-topological* soliton w/ a *topological* charge
    - the “Q-monopole-ball”

[Bai, *SL*, Orlofsky, 2111.10360]

  - ➡ Consider *gauged* SU(2) × global U(1), SU(2)->U(1)





Solitosynthesis

- So far we are assuming that the relic abundance to be completely determined by the phase transition

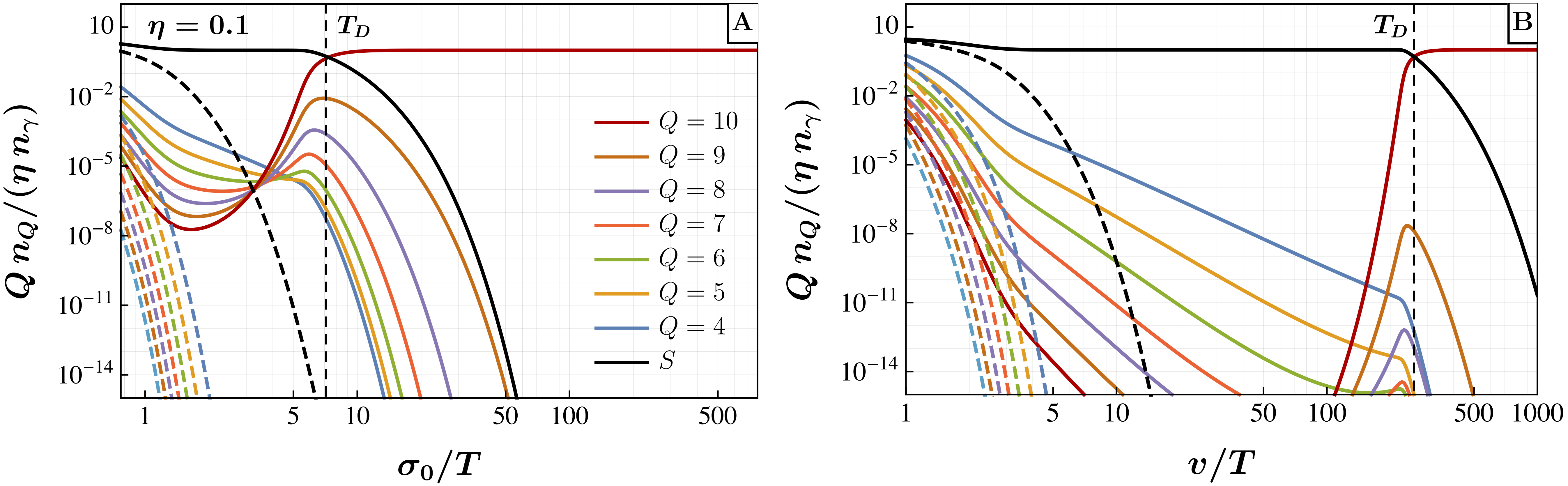
- Late universe evolution could change the story

$$
\begin{aligned}
S + S^\dagger \quad &\leftrightarrow \quad \phi + \phi^\dagger \ , \\
(Q) + S \quad &\leftrightarrow \quad (Q+1) + X \ , \\
(Q) + S^\dagger \quad &\leftrightarrow \quad (Q-1) + X \ , \\
(Q_{\rm min}) + S^\dagger \quad &\leftrightarrow \quad \underbrace{S + S + \cdots + S}_{Q_{\rm min}-1} + X \ . \\
(Q_1) + (Q_2) \quad &\leftrightarrow \quad (Q_1 + Q_2) + X \ , \\
(Q_1) + (-Q_2) \quad &\leftrightarrow \quad \begin{cases} (Q_1 - Q_2) + X & \text{for } Q_1 - Q_2 \geq Q_{\rm min} \ , \\ \underbrace{S + S + \cdots + S}_{Q_1 - Q_2} + X & \text{for } Q_{\rm min} > Q_1 - Q_2 \geq 0 \ . \end{cases}
\end{aligned}
$$

[K. Griest, E. Kolb, *Phys.Rev.D* 40 (1989) 3231]

Q-ball Charge Domination

- **Assuming certain amount of asymmetry within the dark sector**
  - ➡ In equilibrium and with a reasonable* M(Q) vs. Q, the binding energy will push the Q charges into larger Q-balls



*Say $M \propto Q^p$, “reasonable” means p<1

The Freeze-out of the System

- Write down all the Boltzmann equations

$$\begin{aligned}
\dot{n}_Q + 3Hn_Q = &-\delta_{Q,Q_{\text{min}}}(\sigma v_{\text{rel}})_{Q_{\text{min}}}\left(n_{Q_{\text{min}}}n_{S^\dagger} - n^{\text{eq}}_{Q_{\text{min}}}n^{\text{eq}}_{S^\dagger}\left(\frac{n_S}{n^{\text{eq}}_S}\right)^{Q_{\text{min}}-1}\right)\\
&-(1-\delta_{Q,Q_{\text{max}}})(\sigma v_{\text{rel}})_Q\left(n_Q n_S - n^{\text{eq}}_Q n^{\text{eq}}_S\left(\frac{n_{Q+1}}{n^{\text{eq}}_{Q+1}}\right)\right)\\
&+(1-\delta_{Q,Q_{\text{min}}})(\sigma v_{\text{rel}})_{Q-1}\left(n_{Q-1}n_S - n^{\text{eq}}_{Q-1}n^{\text{eq}}_S\left(\frac{n_Q}{n^{\text{eq}}_Q}\right)\right)\\
&-(1-\delta_{Q,Q_{\text{min}}})(\sigma v_{\text{rel}})_Q\left(n_Q n_{S^\dagger} - n^{\text{eq}}_Q n^{\text{eq}}_{S^\dagger}\left(\frac{n_{Q-1}}{n^{\text{eq}}_{Q-1}}\right)\right)\\
&+(1-\delta_{Q,Q_{\text{max}}})(\sigma v_{\text{rel}})_{Q+1}\left(n_{Q+1}n_{S^\dagger} - n^{\text{eq}}_{Q+1}n^{\text{eq}}_{S^\dagger}\left(\frac{n_Q}{n^{\text{eq}}_Q}\right)\right)
\end{aligned}$$

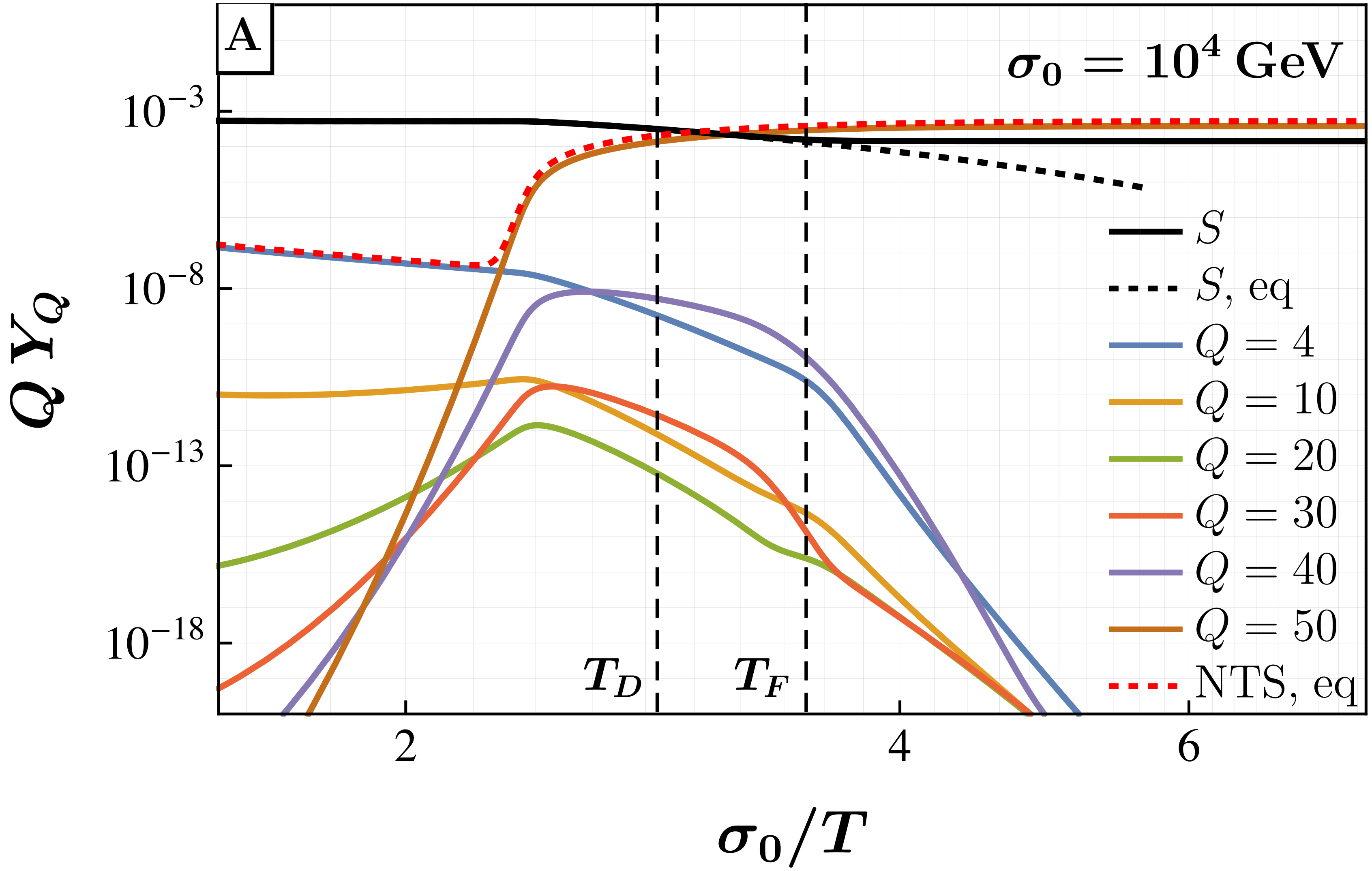
- Summing over all Qs to determine T_F

$$\dot{n}_{\text{NTS}} + 3Hn_{\text{NTS}} = -\sigma v(Q_{\text{min}})\left(n_{Q_{\text{min}}}n_{\bar{\phi}} - n^{\text{eq}}_{Q_{\text{min}}}n^{\text{eq}}_{\bar{\phi}}\left(\frac{n_\phi}{n^{\text{eq}}_\phi}\right)^{Q_{\text{min}}-1}\right)$$

$$\Rightarrow Hn_{\text{NTS}} \sim \sigma v(Q_{\text{min}})n_{Q_{\text{min}}}n_{\bar{\phi}}|_{T=T_F}$$

The Freeze-out of the System

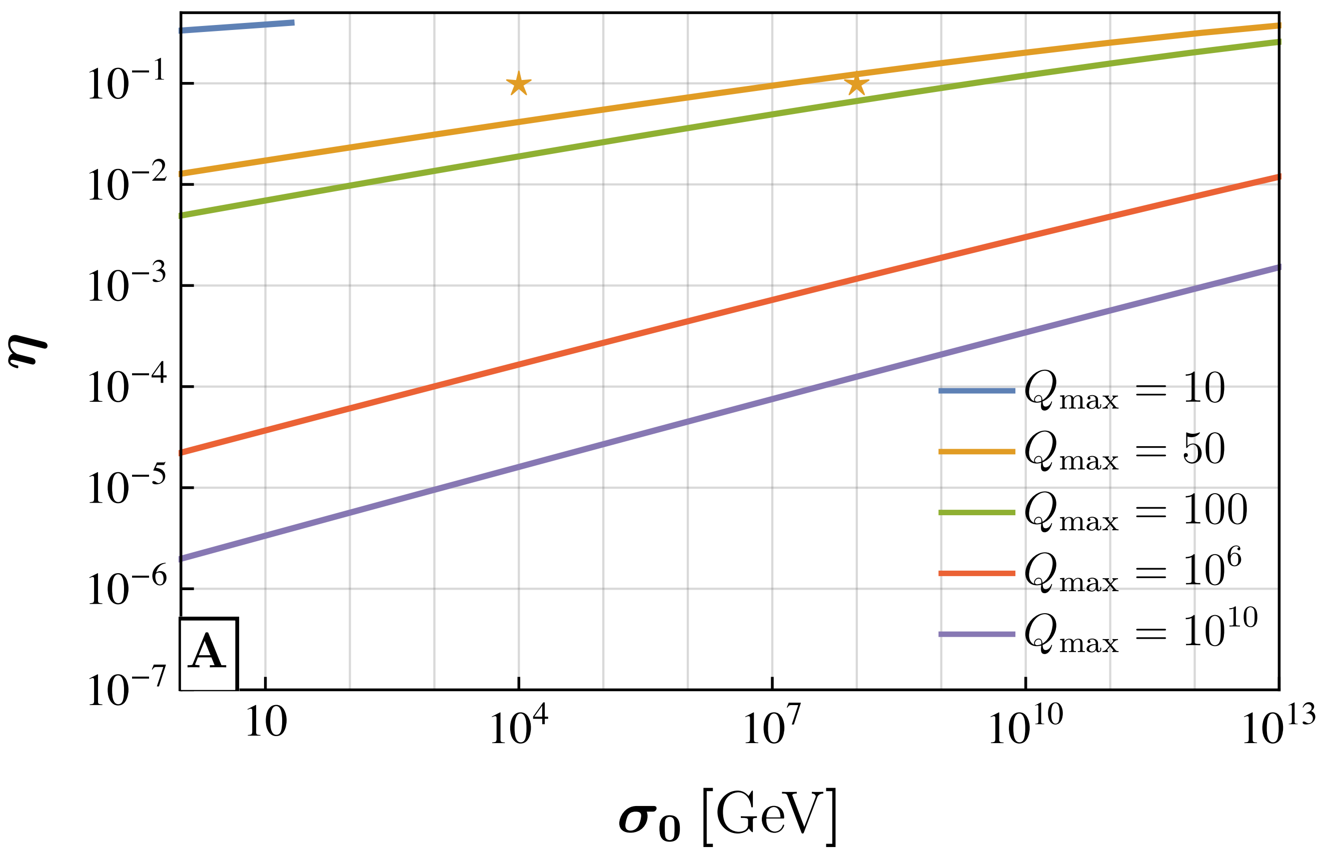
- **Compare with numerics**



The Parameter Space

- **For solitons to be relevant, we require** $T_D > T_F$
  - Equating the two temperature gives the boundary of solitosynthesis

$$\log\eta = \frac{m_S + m_{Q_{\min}}}{m_{Q_{\min}}}\log\left[\frac{2}{c_\gamma}\left(\frac{m_S}{2\pi T_F}\right)^{\frac{3}{2}}\right] + \frac{m_S}{m_{Q_{\min}}}\log\left[\frac{\pi g_*^{1/2} c_\gamma\, T_F^{1/2}}{\sqrt{90}Q_{\max}M_{\rm pl}\left(\sigma v_{\rm rel}\right)_{Q_{\min}}}\left(\frac{4\pi^2 T_F}{m_S\, m_{Q_{\min}}}\right)^{\frac{3}{2}}\right]$$

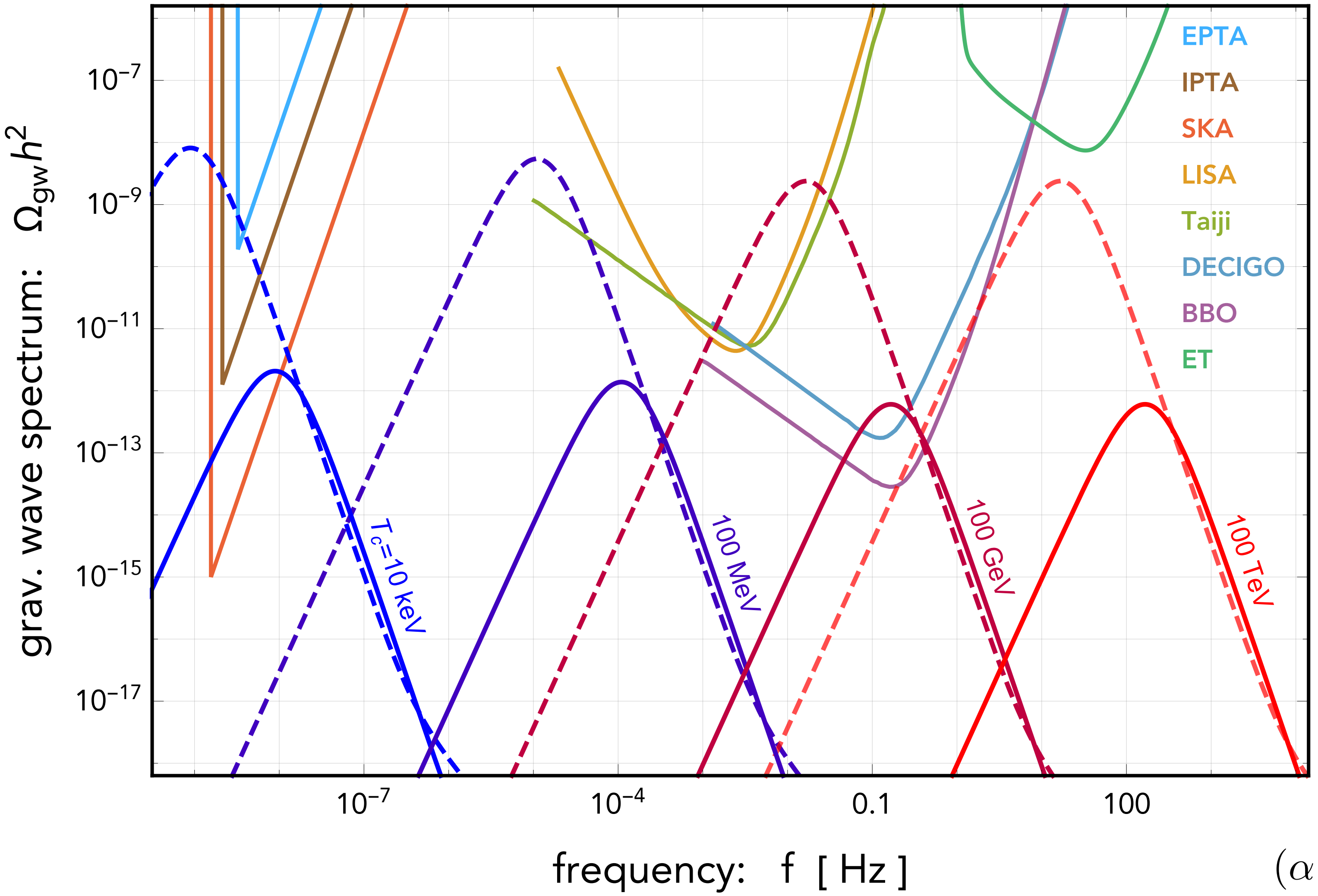


Gravitational Waves at Formation

- **GW spectrum:** $\Omega_{\rm gw}\, h^2 = \Omega_\phi\, h^2 + \Omega_{\rm sw}\, h^2 + \Omega_{\rm turb}\, h^2$

[Caprini+, 1512.06239]

$\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm rad}}$ energy density ratio $\frac{\beta}{H} \equiv T\,\frac{{\rm d}S}{{\rm d}T}$ strength of the phase transition



$(\alpha, \beta/H) = (0.1, 10^4)$ for solid
$= (1, 10^3)$ for dashed

MDMs Being More Attractive

- **Using the dark quark nuggets as an example**

$$\mathcal{L}_{\mathrm{dQCD}} = \sum_{i=1}^{N_f} \left[\bar{\psi}_i i\gamma^\mu D_\mu \psi_i - m_{\psi_i}\,\bar{\psi}_i\psi_i\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}$$

- **Adding an additional attraction between the MDMs**

$$\mathcal{L}_{\mathrm{Yukawa}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \sum_i (m_{\psi_i} + y_i\,\phi)\,\bar{\psi}_i\,\psi_i - V_0(\phi)\,, \quad V_0(\phi) = \frac{1}{2}m^2_{\mathrm{med}}\phi^2$$

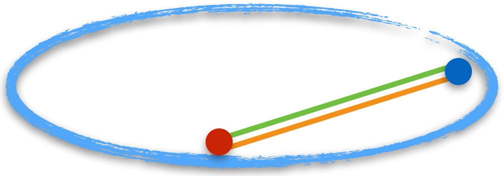
  - ➡ In the massless mediator limit

$$\alpha = y^2 q_1 q_2/(4\pi G m_1 m_2)$$

$$F = -G' m_1 m_2/r^2, \quad G' = (1-\alpha)G \equiv \underline{\beta}\,G$$

$$\omega^2 = \frac{G'm}{a^3}\,, \quad E = -\frac{G'm^2\eta}{2a} \quad e^2 = 1 + \frac{2EL^2}{G'^2m^5\eta^3}$$

- **MDMs may form binaries in the early universe**



[Bai, *SL*, Orlofsky, 2312.13378]

Energy Spectrum from the Binary

- **Energy emission from the binary is different now**
  - Energy emission through GW portal

$$\langle \dot{E}_{\rm GW} \rangle = \frac{32 G G'^3 \eta^2 m^5}{5 a^5 (1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

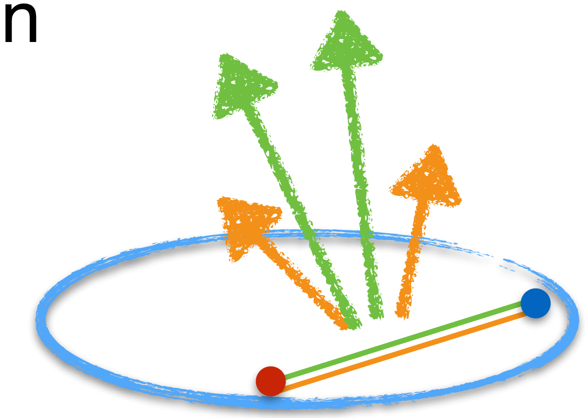
  - Energy emission through the dark force portal via dipole

$$\langle \dot{E}_{\rm DF} \rangle = \frac{G G'^2}{12\pi} \eta^2 m^4 \left(\frac{g q_1}{\sqrt{G} m_1} - \frac{g q_2}{\sqrt{G} m_2} \right)^2 \frac{1}{a^4} \frac{2+e^2}{(1-e^2)^{5/2}}$$

    - charge separation from a parity violating bubble

[Kharzeev, Zhitnitsky, 0706.1026]

    - (possibly large) fermion flavor fluctuation at formation



$$m = m_1 + m_2 \,, \quad \eta = \frac{m_1 m_2}{m^2}$$

Energy Spectrum from the Binary

- **The GW energy spectrum of the binary**
  - GW frequency is related to orbital frequency: $f_{\mathrm{GW},s} = \omega/\pi$

$$\frac{dE_{\mathrm{GW}}}{df_{\mathrm{GW},s}} = \frac{\dot{E}_{\mathrm{GW}}}{\dot{f}_{\mathrm{GW,s}}} = \frac{\pi \dot{E}_{\mathrm{GW}}}{\dot{\omega}} = \frac{\pi \dot{E}_{\mathrm{GW}}}{-\frac{3\sqrt{2}}{G' m^{5/2} \eta^{3/2}} \sqrt{-E}\dot{E}}$$

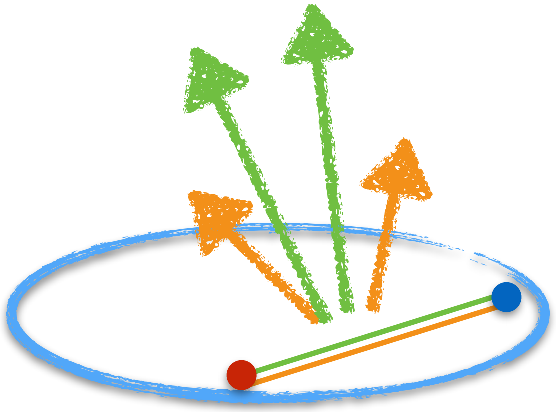
$$\dot{E} = \dot{E}_{\mathrm{GW}} + \dot{E}_{\mathrm{DF}}$$

same mass, opposite charge

$$\frac{dE_{\mathrm{GW}}}{df_{\mathrm{GW},s}} = \frac{\pi\sqrt{a}\left(37e^4 + 292e^2 + 96\right) G'^{3/2} M_{\mathrm{obj}}^{5/2}}{3\sqrt{2}\,\left(\underbrace{10\,a(1-e^2)(2+e^2)(\beta-1)}_{\text{DF portal}}\right) + \left(\underbrace{\left(37e^4 + 292e^2 + 96\right) G' M_{\mathrm{obj}}}_{\text{GW portal}}\right)}$$

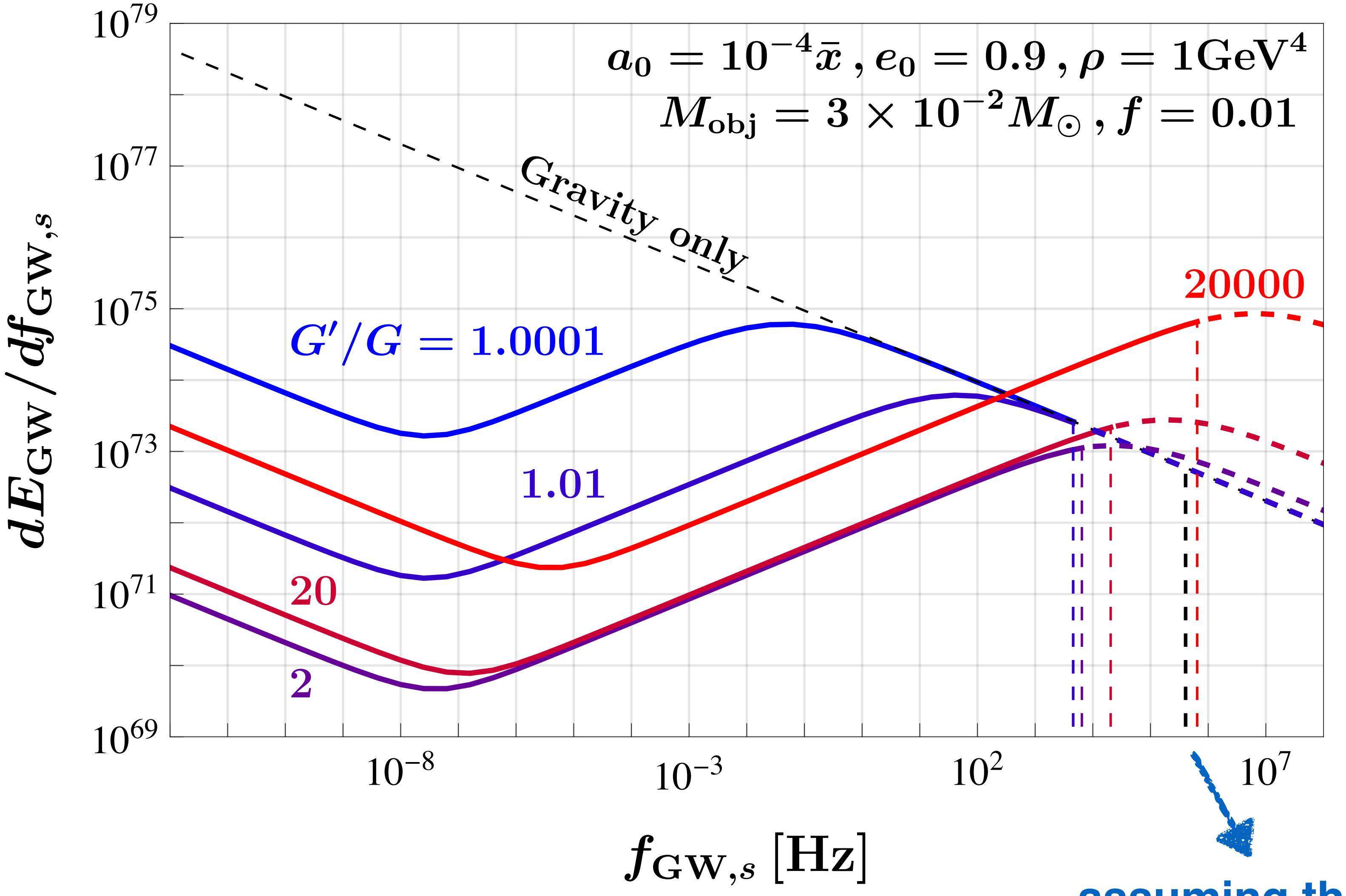
  - Without the dark force

$$\frac{dE_{\mathrm{GW}}}{df_{\mathrm{GW,s}}} \propto \sqrt{a} \propto f_{\mathrm{GW,s}}^{-1/3}$$



Energy Spectrum from the Binary

- **The GW emission spectrum of the binary is changed**



assuming the inspiral ends when the binary touches each other

SGWB from Dark Binaries

- **Convolution over cosmic history**
  - For primordial black holes (gravity only)

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c}\int_0^{z_{\rm sup}} dz \frac{R(z)}{(1+z)H(z)}\frac{dE_{\rm GW}}{df_{{\rm GW},s}}((1+z)f_{\rm GW})$$

  - With additional interactions, orbital geometry becomes extremely important

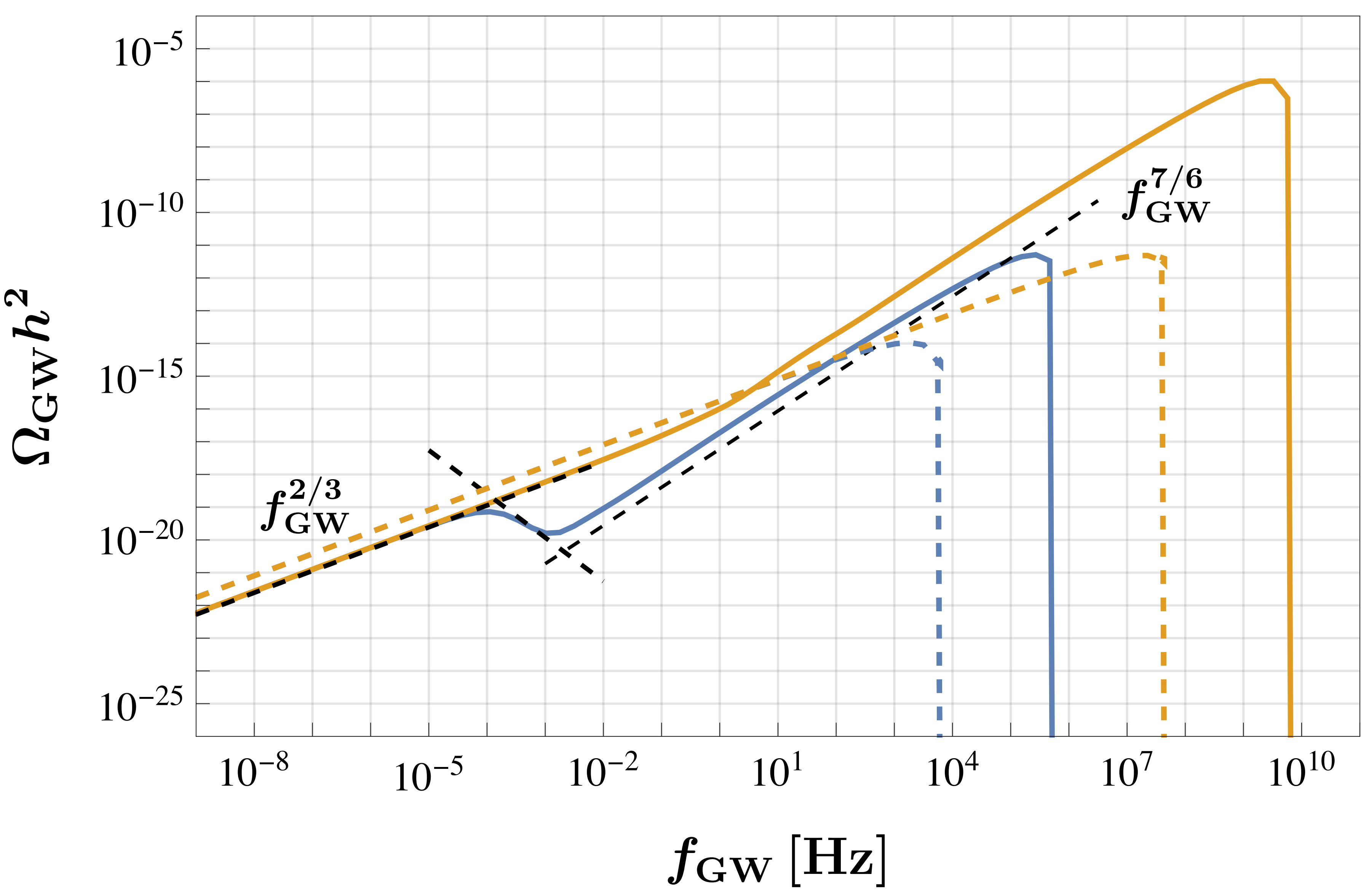
$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c}\int_{\sqrt{1-c_2^2}}^{e_{0,\rm max}} \underline{de_0} \int \underline{d\tau}\,\frac{n_{\rm obj}}{4}\mathcal{P}(\tau,e_0)\frac{dE_{\rm GW}}{df_{{\rm GW},s}}\,[(1+z(t))f_{\rm GW}]$$

orbital eccentricity

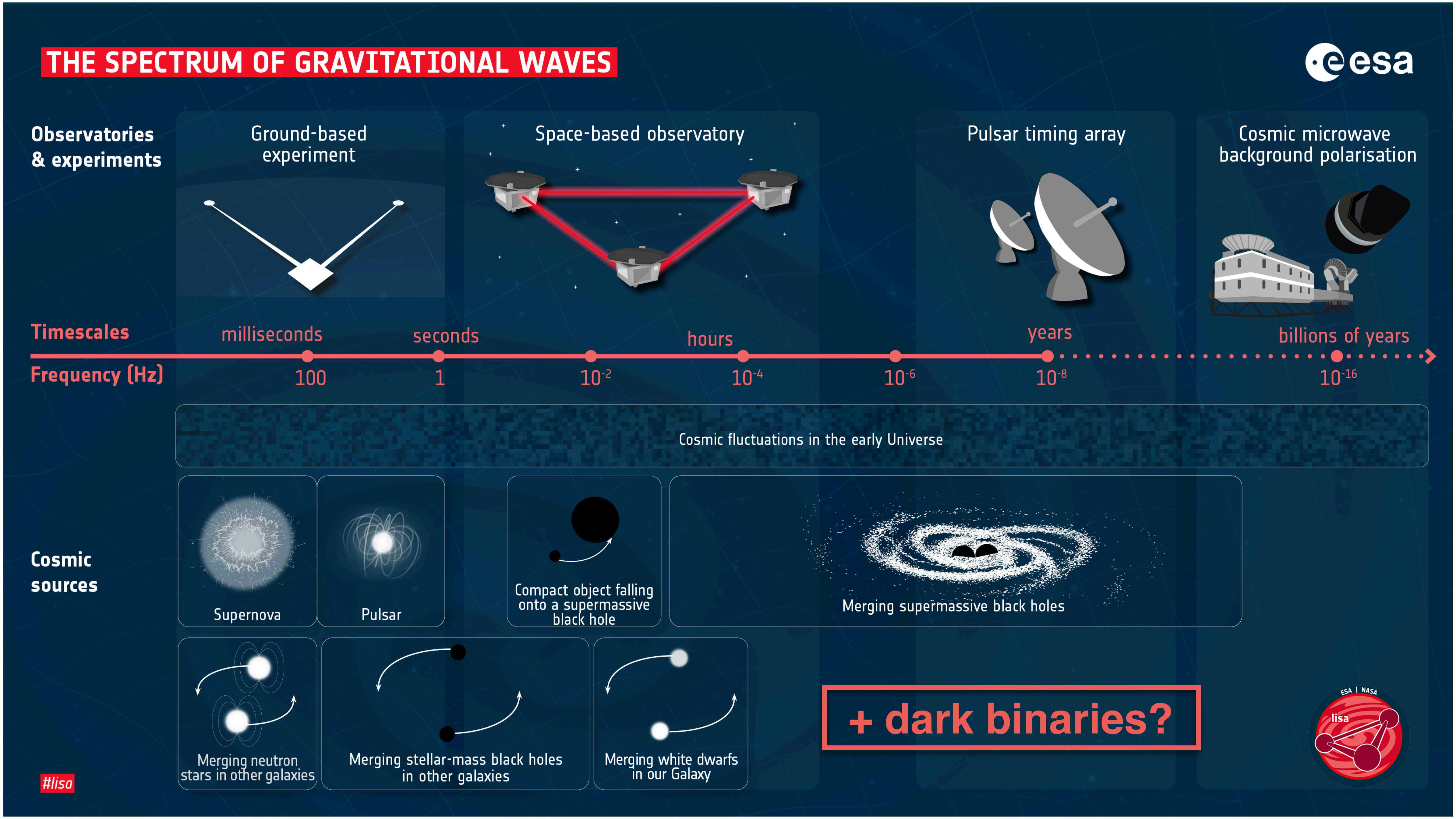
merger lifetime, related to semi-major axis

Spectral Shape

- A two- or three-stage power-law spectrum

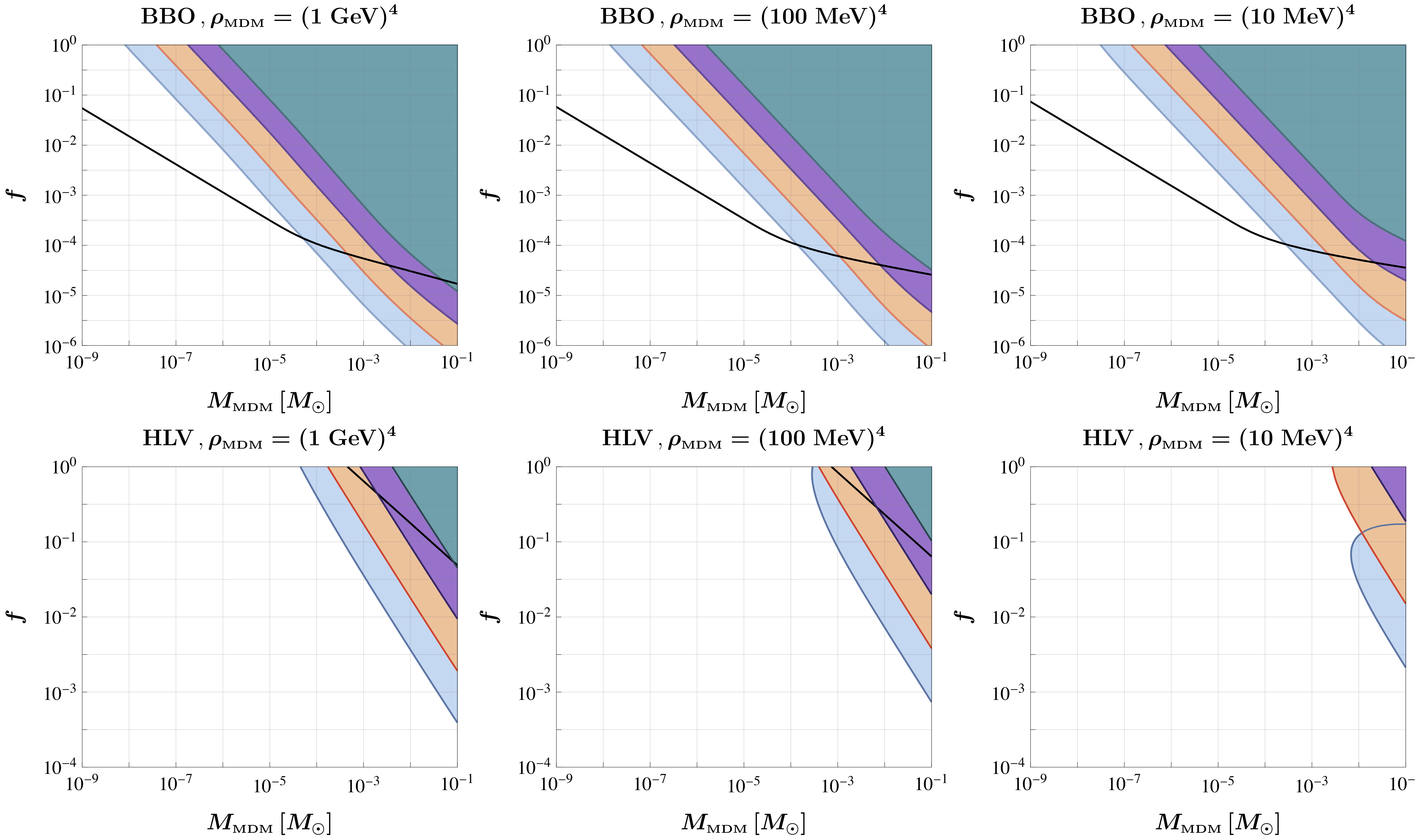


Sensitivity at Experiments



- ➡ Consider SKA, LISA, BBO and LIGO-Virgo (HLV)

Sensitivity at Experiments



$\varrho_{\rm th} = 1$, $T_{\rm obs} = 20$ yr (SKA)/1 yr (others)

🟦🟧🟪🟩 : $\beta \equiv G'/G = 10^4, 10^3, 10^2, 10$

Complications from the Model

- **The Yukawa sector**

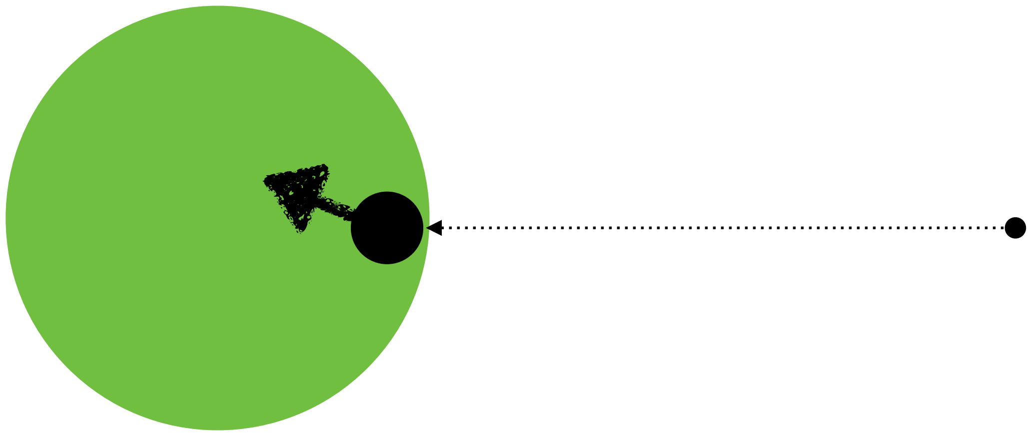
$$\mathcal{L}_{\rm dQCD} = \sum_{i=1}^{N_f} \left[\bar{\psi}_i i\gamma^\mu D_\mu \psi_i - m_{\psi_i}\,\bar{\psi}_i\psi_i\right] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a}$$

$$\mathcal{L}_{\rm Yukawa} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \sum_i (m_{\psi_i} + y_i\,\phi)\,\bar{\psi}_i\,\psi_i - V_0(\phi)\,, \quad V_0(\phi) = \frac{1}{2}m^2_{\rm med}\phi^2$$

- **Finite density effect**

$$V_1 = g\,\frac{1}{2\pi^2}\int_0^{k_F} dk\,k^2\sqrt{k^2 + (m_\psi + y\,\phi)^2} \approx \frac{g}{8\pi^2}\,\underline{\mu^2}\,\left[\mu^2 - (m_\psi + \underline{y\,\phi})^2\right]$$

➡ The mediator becomes heavy inside: a screening effect!



Complications from the Model

- **Requiring the dark force range to cover the MDM**

$$|m_{\rm in}|R < 1$$

$$\Rightarrow y < 2^{19/12} 3^{-7/12} \pi^{5/6} g^{-1/4} \Lambda_d^{1/3} M^{-1/3} = (2 \times 10^{-19}) \left(\frac{\Lambda_d}{1\,{\rm GeV}}\right)^{1/3} \underline{\left(\frac{M_\odot}{M}\right)^{1/3}}$$

$$\alpha = \frac{3\, g\, y^2\, m_\psi^2}{128\pi^3\, G\, \Lambda_d^4} \lesssim (0.02) \left(\frac{m_\psi/\Lambda_d}{0.5}\right)^2 \underline{\left(\frac{1\,{\rm GeV}}{\Lambda_d}\right)^{4/3}} \underline{\left(\frac{M_\odot}{M}\right)^{2/3}}$$

- ➡ DF may not be arbitrarily strong for MDM
- ➡ Model dependent

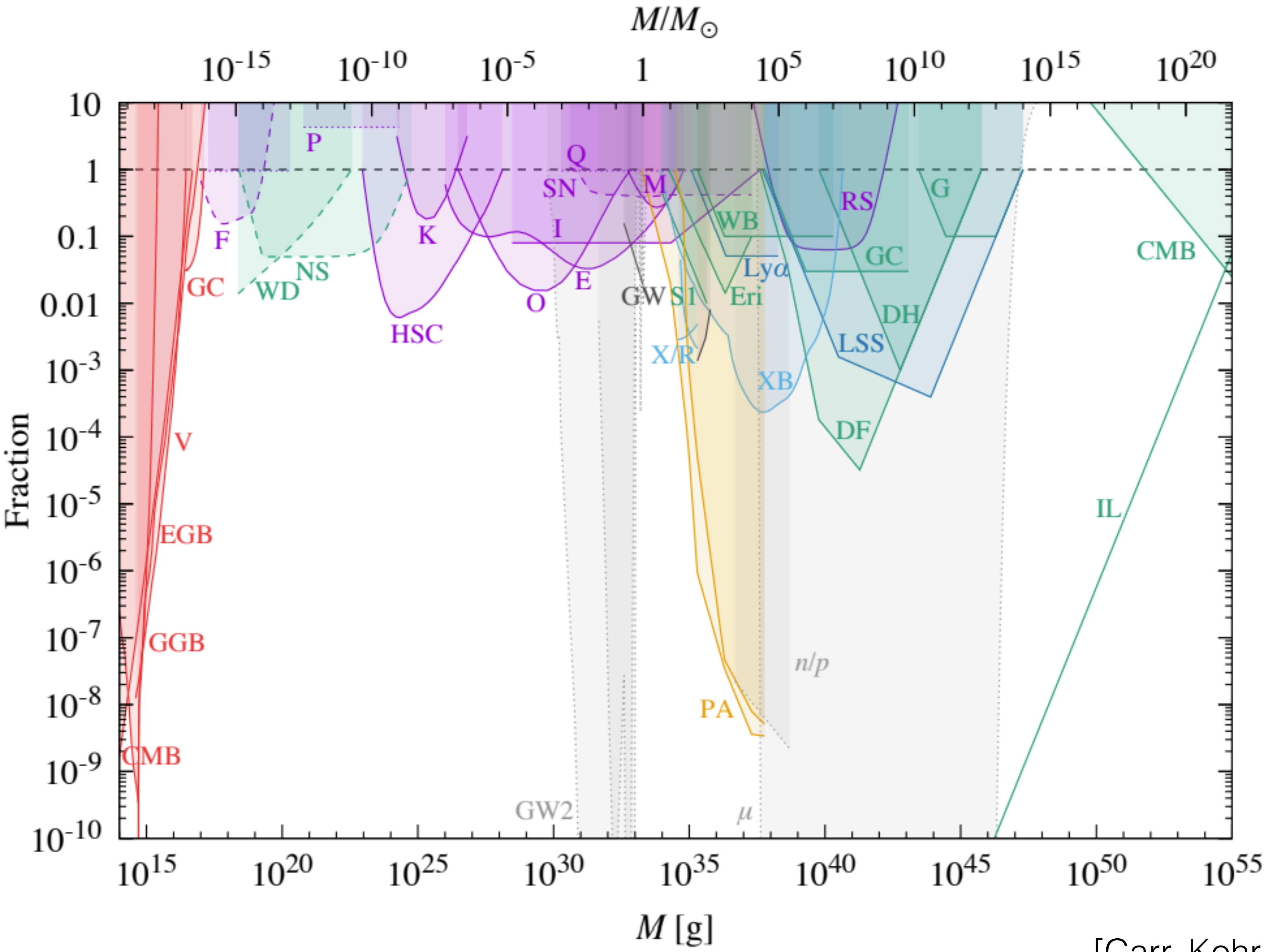
Conclusions

- MDMs can be naturally formed from cosmic phase transitions. Later evolution may also be important in determining their properties
- Candidates can come from either fermionic or bosonic theories
- Unique phenomenologies. E.g. with addition attractive interactions, the SGWB from binary MDMs is distinctive from the gravity-only case

Thank you!

Backup

Lensing



[Carr, Kohri, Sendouda, Yokoyama, 2002.12778]

More on the Screening

- **The scalar field with the effective potential**

$$\partial_r^2\phi + \frac{2}{r}\partial_r\phi = \frac{\partial V_{\rm eff}(\phi)}{\partial\phi}$$

$$V_{\rm eff}(\phi) = -\,a\,\phi\,\Theta(R-r) + \frac{1}{2}\,\left[m_{\rm in}^2\,\Theta(R-r) + m_{\rm med}^2\,\Theta(r-R)\right]\,\phi^2$$

- **The solution**

$$\phi_{\rm out} = c_1 e^{-m_{\rm med}\,r}\,, \qquad \phi_{\rm in} = a/m_{\rm in}^2 + c_2(e^{-m_{\rm in}\,r} - e^{m_{\rm in}\,r})/r$$

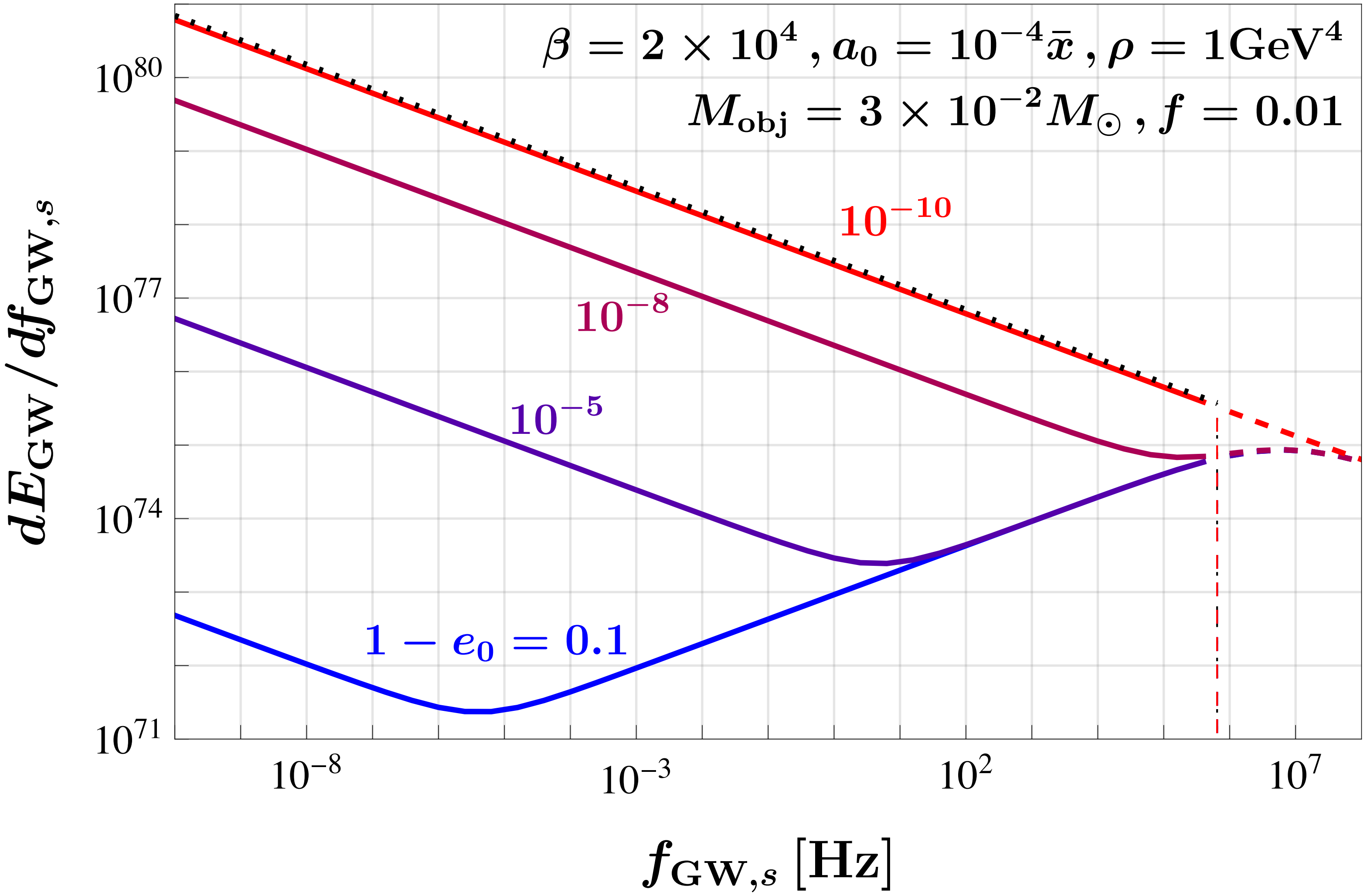
$$\frac{q_{\rm eff}\,y}{4\pi} = c_1 = a\,\frac{e^{m_{\rm med}\,R}[m_{\rm in}\,R\,\cosh\,(m_{\rm in}\,R) - \sinh\,(m_{\rm in}\,R)]}{m_{\rm in}^2[m_{\rm med}\,\sinh\,(m_{\rm in}\,R) + m_{\rm in}\,\cosh\,(m_{\rm in}\,R)]}$$

$$m_{\rm med}R \ll 1\,,\; m_{\rm in}R \ll 1 \Longrightarrow c_1 = a\,R^3/3$$

$$m_{\rm med}R \ll 1\,,\; m_{\rm in}R \gg 1 \Longrightarrow c_1 = a\,\frac{R^3}{(m_{\rm in}\,R)^2}$$

Energy Spectrum from the Binary

- **The GW energy spectrum of the binary**
  - ➡ Small a_0 or e_0



The Merger Rate

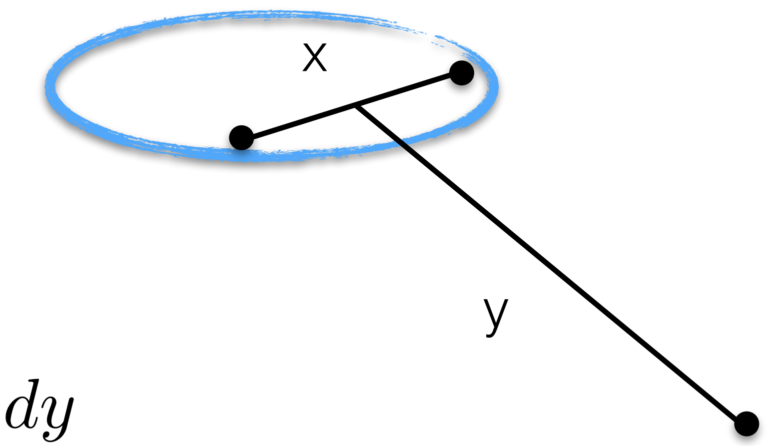
- **The merger rate depends on the geometry of the binary and its nearest neighbor**

$$R(x,y) = \frac{1}{2}\frac{n_{\rm obj}}{2}P = \frac{1}{2}\frac{3H_0^2}{8\pi G}\frac{f\,\Omega_{\rm DM}}{2M_{\rm obj}}P(x,y)$$

x: comoving distance between the binary

y: comoving distance to the nearest neighbor

f: fraction of the binaries among all DM



➡ Assuming random formation

$$P(x,y)\,dxdy = \frac{9x^2y^2}{\bar{x}^6}e^{-(y/\bar{x})^3}dxdy$$

➡ In terms of the orbital parameters

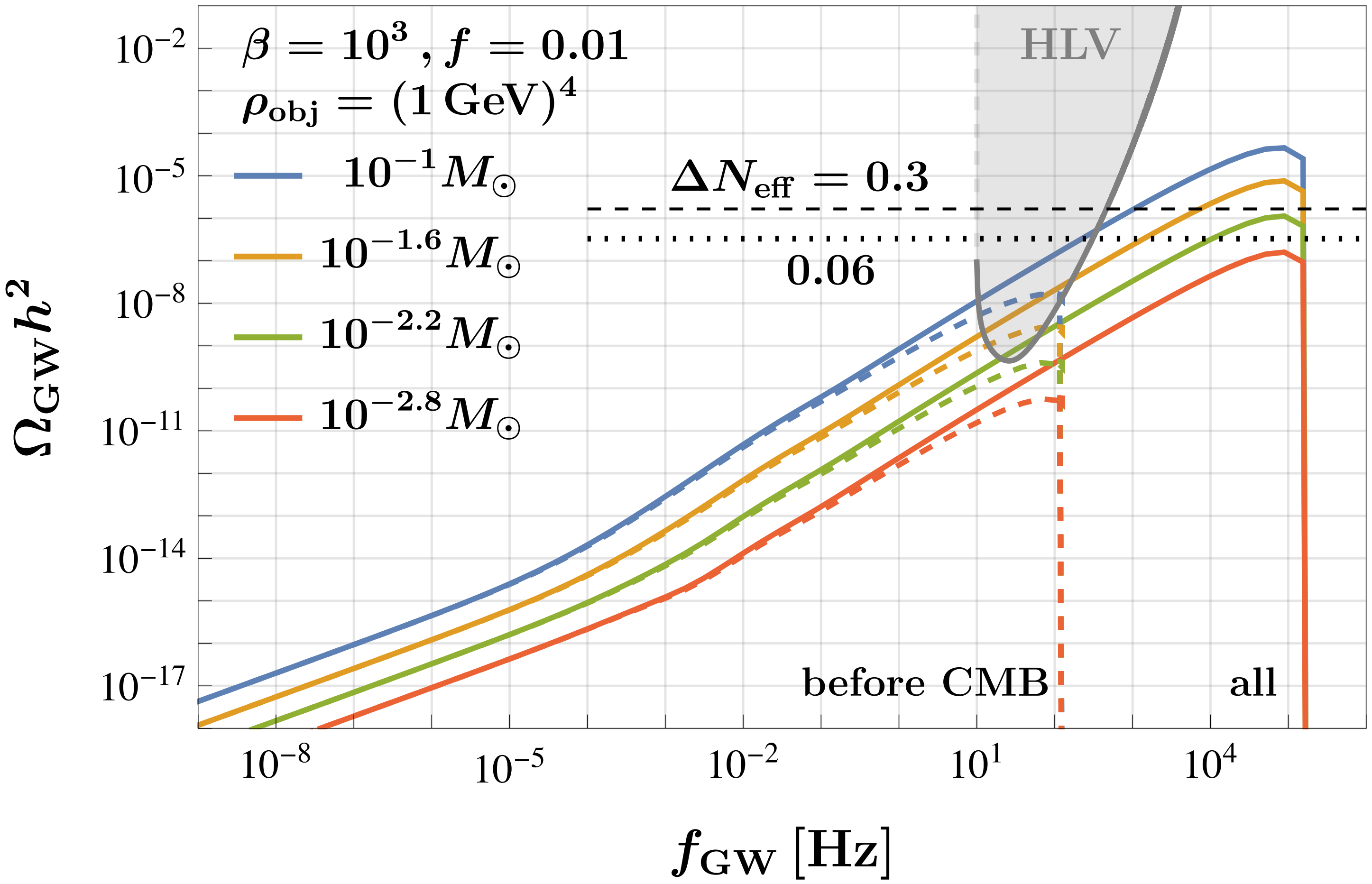
$$a_0 = \frac{c_1}{\beta}\frac{1}{f}\frac{x^4}{\bar{x}^3}\,, b_0 = c_2\left(\frac{x}{y}\right)^3 a_0\,, e_0 = \sqrt{1-\left(\frac{b_0}{a_0}\right)^2}\,.$$

$$\bar{x} = \frac{1}{1+z_{\rm eq}}\left(\frac{8\pi G M_{\rm obj}}{3H_0^2\,f\,\Omega_{\rm DM}}\right)^{1/3}$$

[Ioka, Chiba, Tanaka, Nakamura, astro-ph/9807018]

Constraints from N_{eff}

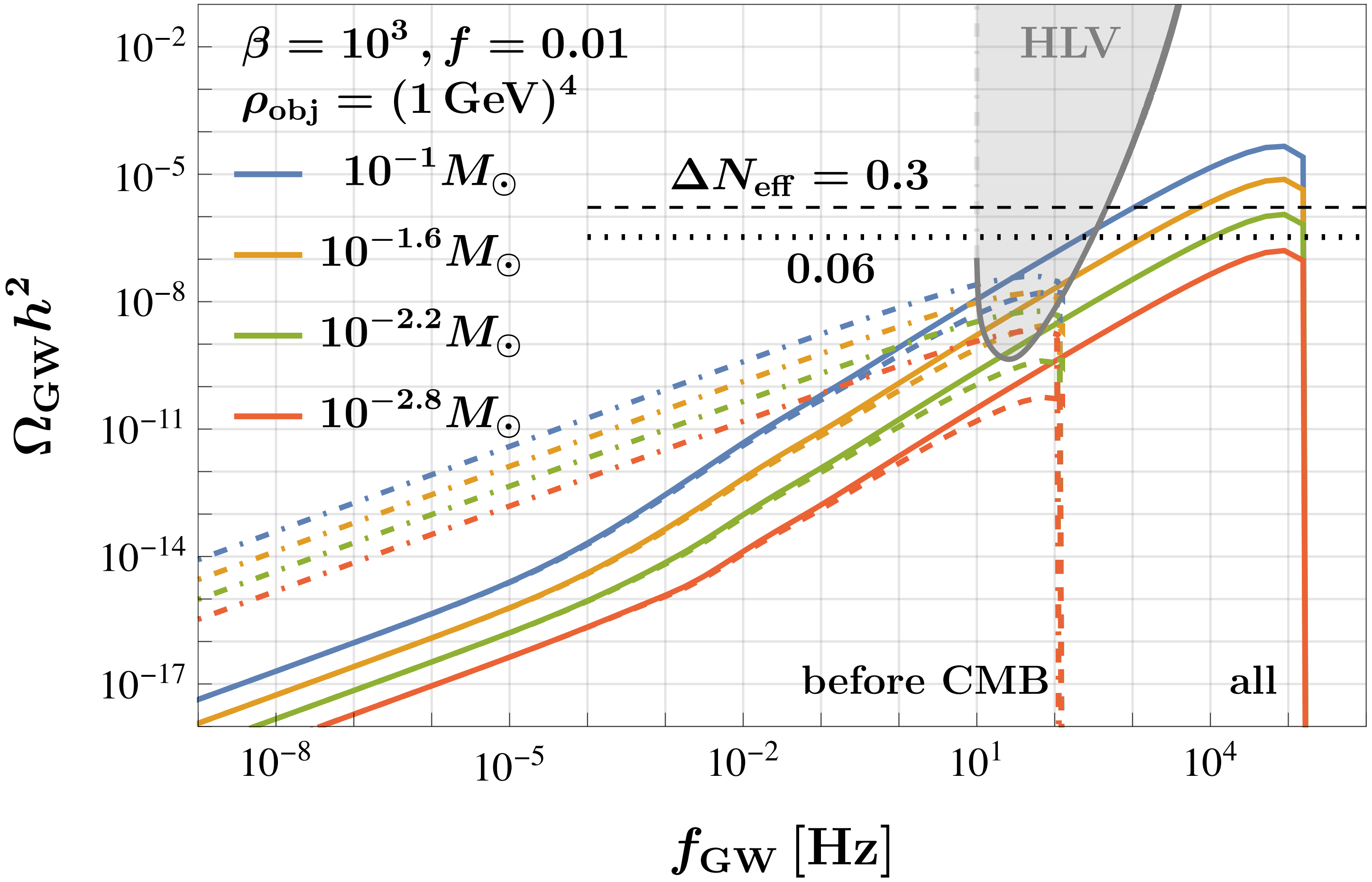
- ✤ **Counting contributions from only before the CMB**



➡ Amplitude of that part is ~$10^{3.5}$ smaller compared with the full spectrum for the GW

Constraints from N_{eff}

- Counting contributions from only before the CMB



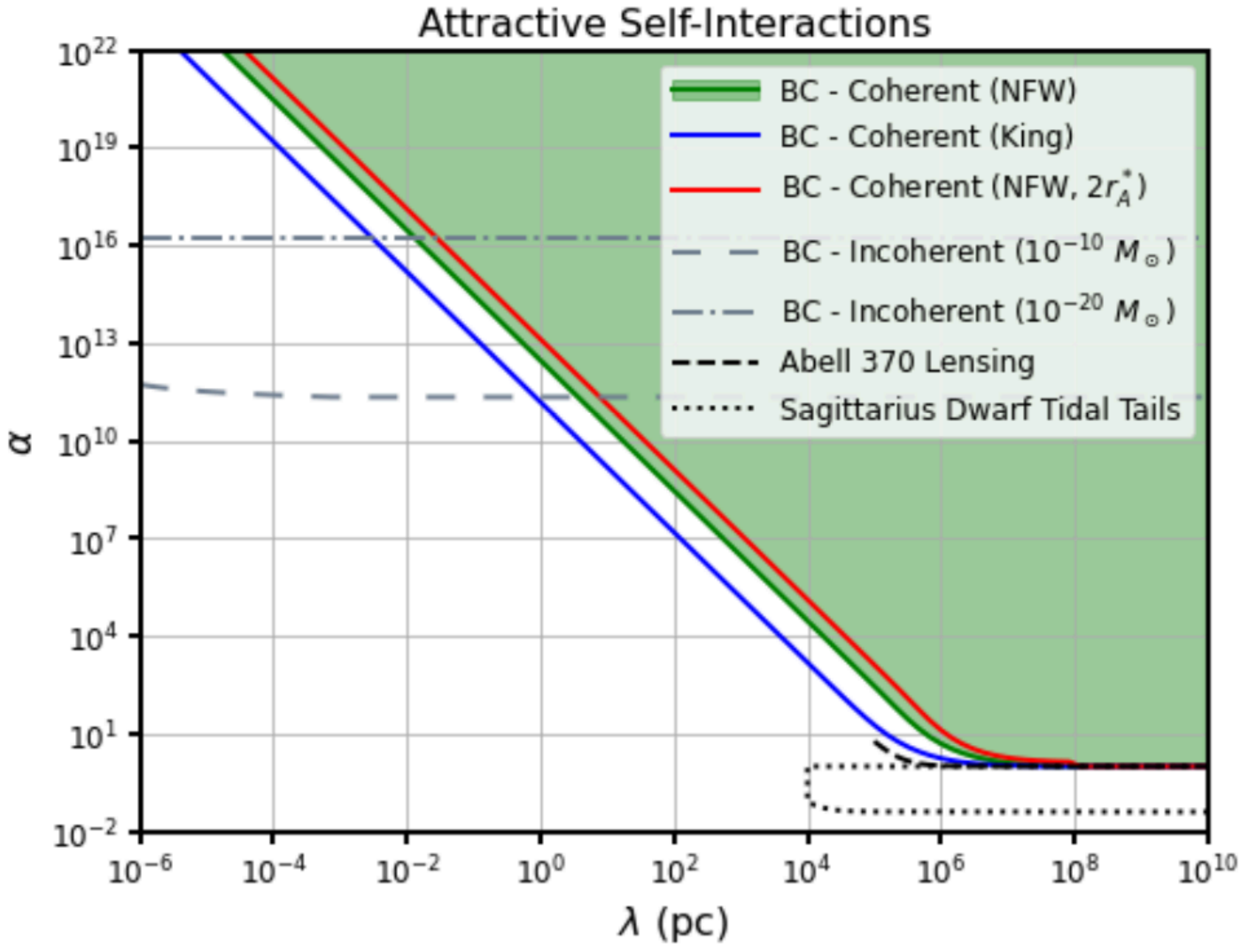
➡ DF emission is not a huge issue either

DF Mediator Mass

- **Typical distance**

$$\bar{x} = \frac{1}{1+z_{\rm eq}}\left(\frac{8\pi G M_{\rm obj}}{3H_0^2\, f\, \Omega_{\rm DM}}\right)^{1/3} \approx 0.1\,{\rm pc}\left(\frac{M_{\rm obj}}{M_\odot}\right)^{1/3}\left(\frac{1}{f}\right)^{1/3} \sim (6\times 10^{-23}\,{\rm eV})^{-1}\left(\frac{M_{\rm obj}}{M_\odot}\right)^{1/3}\left(\frac{1}{f}\right)^{1/3}$$

- **Cosmological constraints from bullet clusters**



[Bogorad, Graham, Ramani, 2311.07648]

Scalar and Vector Emission

- **Emission through vector or scalar takes the same form in the massless mediator limit**

$$\langle \dot{E}_S \rangle = \frac{1}{3}\eta^2 m^2 \omega^4 r^2 g_S(m_S, e)(\tilde{\mathrm{q}}_1 - \tilde{\mathrm{q}}_2)^2,$$

$$\langle \dot{E}_V \rangle = \frac{2}{3}\eta^2 m^2 \omega^4 r^2 g_V(m_V, e)(\tilde{\mathrm{q}}_1 - \tilde{\mathrm{q}}_2)^2,$$

$$g_S(m_S, e) = \sum_{n=1}^{\infty} 2n^2 \left[\mathcal{J}_n'^2(ne) + \left(\frac{1-e^2}{e^2}\right)\mathcal{J}_n^2(ne)\right] \times \left[1 - \left(\frac{m_S}{n\omega}\right)^2\right]^{3/2},$$

$$g_V(m_V, e) = \sum_{n=1}^{\infty} 2n^2 \left[\mathcal{J}_n'^2(ne) + \left(\frac{1-e^2}{e^2}\right)\mathcal{J}_n^2(ne)\right] \times \left[1 - \left(\frac{m_V}{n\omega}\right)^2\right]^{1/2} \left[1 + \frac{1}{2}\left(\frac{m_V}{n\omega}\right)^2\right],$$

[Krause, Kloor, Fischbach, 1994]

[Alexander, McDonough, Sims, Yunes, 1808.05286]

Higher Harmonics

- **With an eccentric orbit, the binary should emit GW at all harmonics of the orbital frequency**
  - ➡ We are effectively assuming all energy are emitted through the n=2 channel
  - ➡ To account for the other modes

$$\frac{dE_{\rm GW}}{dt} = \frac{32 G G'^3 \eta^2 m^5}{5a^5} \frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1-e^2)^{7/2}} = \frac{32 G G'^3 \eta^2 m^5}{5a^5} \sum_n g(n,e)\,,$$

$$g(n,e) = \frac{n^4}{32} \Bigg\{ \left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{2}{n} J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne) \right]^2 + (1-e^2) \left[J_{n-2}(ne) - 2eJ_n(ne) + J_{n+2}(ne) \right]^2 + \frac{4}{3n^2} J_n^2(ne) \Bigg\}\,,$$

[Peters and Mathews, Phys. Rev. 131 (1963) 435-439]

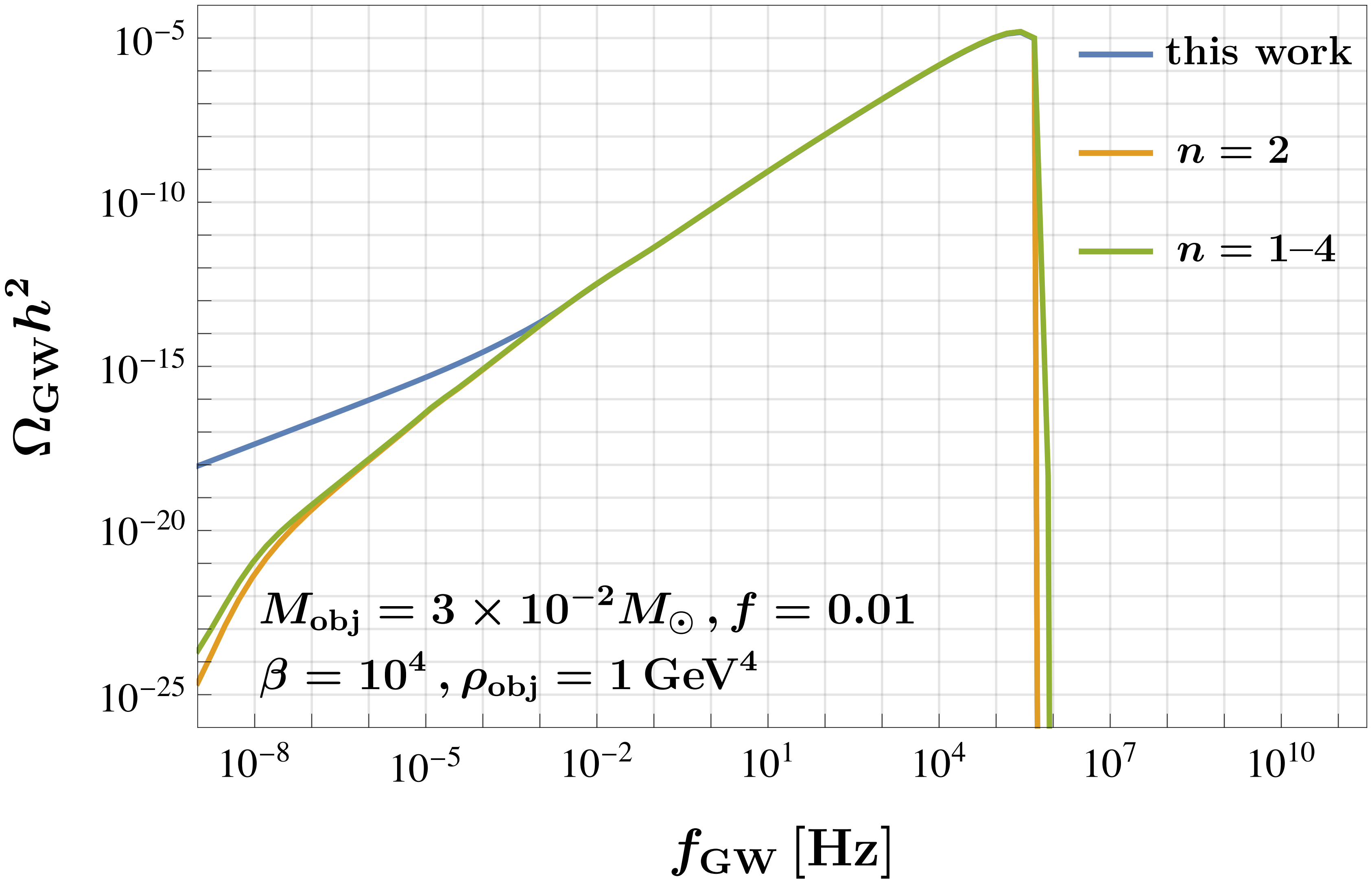
[Enoki, Nagashima, astro-ph/0609377]

Higher Harmonics

- **The following calculation is straight-forward**

$$
\begin{aligned}
\frac{d^2 E_{\rm GW}}{dt\, df_{{\rm GW},s}} &= \frac{32 G G'^3 \eta^2 m^5}{5a^5} \sum_n g(n,e)\,\delta(f_{{\rm GW},s} - n f_{\rm orb})\,, \\
\frac{dE_{\rm GW}}{df_{{\rm GW},s}} &= \sum_n \int dt \frac{32 G G'^3 \eta^2 m^5}{5a^5} g(n,e)\,\delta(f_{{\rm GW},s} - n f_{\rm orb}) \\
&= \sum_n \int \frac{de}{de/dt} \frac{32 G G'^3 \eta^2 m^5}{5a^5} g(n,e)\,\delta(f_{{\rm GW},s} - n f_{\rm orb}) \\
&= \sum_n \int de \frac{32 G G'^3 \eta^2 m^5}{5a^5} \frac{a^3 (1-e^2)^{3/2}}{4eG^2(\mathcal{G}-1)\mathcal{G} M_{\rm obj}^2} \frac{g(n,e)}{n} \delta(f_{\rm orb} - f_{{\rm GW},s}/n) \\
&= \sum_n \frac{32 G G'^3 \eta^2 m^5}{5a^5} \frac{a^3 (1-e^2)^{3/2}}{4eG^2(\mathcal{G}-1)\mathcal{G} M_{\rm obj}^2} \frac{g(n,e)}{n} \frac{1}{\left|\frac{df_{\rm orb}}{de}\right|_{e=e_n}} \\
&\equiv \sum_n \left(\frac{E_{\rm GW}}{df_{{\rm GW},s}} \right)_n \,,
\end{aligned}
$$

Higher Harmonics



Sensitivity at Experiments

- **Signal-to-noise ratio for multiple detectors where cross-correlation can be performed**

$$\varrho^2 = n_{\mathrm{det}} T_{\mathrm{obs}} \int df_{\mathrm{GW}} \left(\frac{\Omega_{\mathrm{GW}}}{\Omega_{\mathrm{noise}}} \right)^2$$

[Schmitz, 2002.04615]

  - ➡ n_det=2 for cross-correlation, and 1 for auto-correlation (if applicable)
  - ➡ Nontrivial noise subtraction is required for auto-correlation
    - TDI interfereometry?

[Smith and Caldwell, 1908.00546]