When scalar-induced gravitational waves meet pulsar timing arrays

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Based on: *2307.01102, 2307.14911, 2401.09818*

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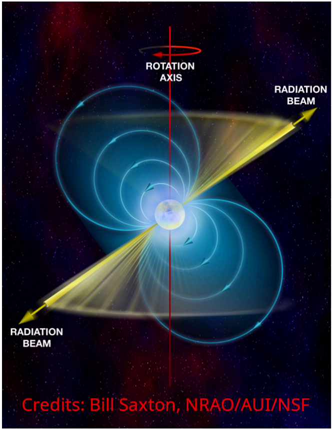
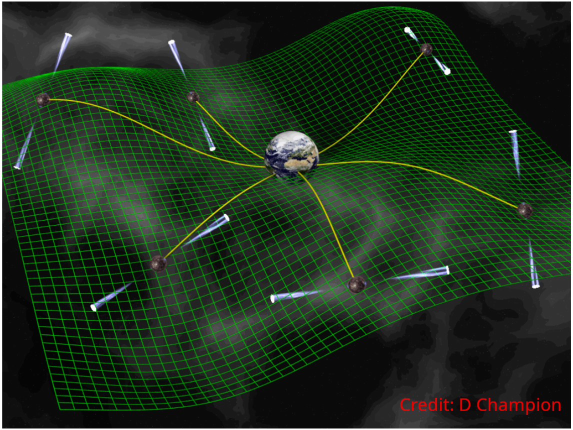
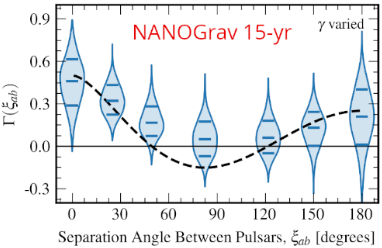
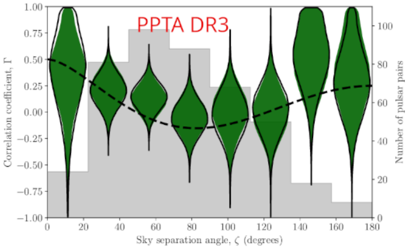
Pulsar and PTA

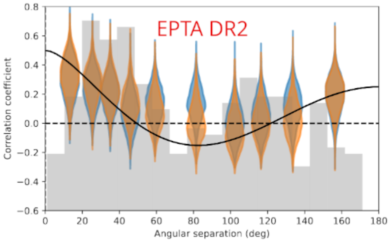


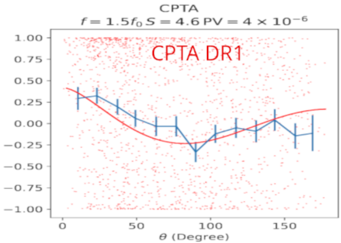


- Pulsars are highly magnetized, rotating neutron stars that emit regular pulses of electromagnetic radiation.
- GWs can cause tiny distortion in spacetime inducing variations in the time of arrivals (ToAs).
- A pulsar timing array (PTA) pursues to detect nHz GWs by regularly monitoring ToAs from an array of the ultra rotational stable millisecond pulsars.









Gabriella Agazie, et al., ApJL (2023); Daniel Reardon, et al. ApJL (2023)

J. Antoniadis, et al., A&A (2023); Heng Xu, et al., RAA (2023)

SIGWs can explain the PTA signal.

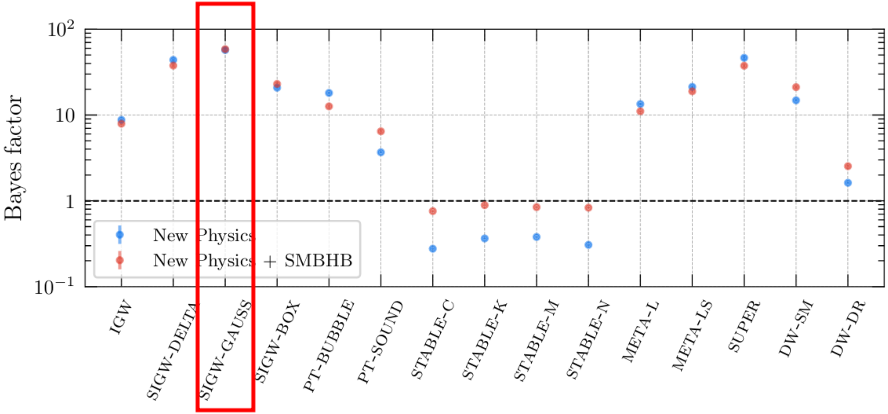


Figure 2. Bayes factors for the model comparisons between the new-physics interpretations of the signal considered in this work and the interpretation in terms of SMBHBs alone. Blue points are for the new physics alone, and red points are for the new physics in combination with the SMBHB signal. We also plot the error bars of all Bayes factors, which we obtain following the bootstrapping method outlined in Section 3.2. In most cases, however, these error bars are small and not visible.

Adeela Afzal, ApJL (2023)

Scalar-Induced Gravitational Waves (SIGWs)

- Primordial perturbations can be generated by quantum fluctuations during inflation.

- Metric perturbation in Newtonian gauge

$$\mathrm{d}s^2 = a^2 \left\{ -(1+2\phi)\mathrm{d}\eta^2 + \left[(1-2\phi)\delta_{ij} + \frac{h_{ij}}{2} \right] \mathrm{d}x^i \mathrm{d}x^j \right\}, \tag{1}$$

where $\phi \equiv \phi^{(1)}$ and $h_{ij} \equiv h_{ij}^{(2)}$ are the scalar and tensor perturbations, respectively.

- Primordial scalar perturbation can be the source of SIGWs, as well as primordial black holes (PBHs).

What is primordial black hole?

What is primordial black hole?

The concept was first proposed in 1971 by Stephen Hawking, who introduced the idea that black holes may exist that are smaller than stellar mass, and are thus not formed by stellar gravitational collapse. A primordial black hole is a hypothetical type of black hole formed during the high-density, in-homogeneous phase of the early Universe.

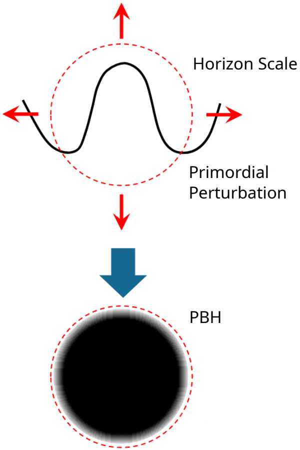
Motivations

- a perfect candidate for dark matter
- to provide seeds for super-massive BHs
- to provide seeds for cosmic structures
- to account for LVK events

PBH formation

- PBHs can be formed in the early universe by gravitational collapse of primordial density perturbations.
- PBH mass can span many orders

$$M_{\mathrm{PBH}} \sim \frac{t}{G} \sim \left(\frac{t}{10^{-5}s}\right) M_{\odot} \tag{2}$$



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Non-Gaussianity

- The local-type non-Gaussian curvature perturbations:

$$\mathcal{R}(\vec{x}) = \mathcal{R}_{\mathrm{G}}(\vec{x}) + F_{\mathrm{NL}} \left(\mathcal{R}_{\mathrm{G}}^2(\vec{x}) - \langle \mathcal{R}_{\mathrm{G}}^2(\vec{x}) \rangle\right). \tag{3}$$

- The effective curvature power spectrum

$$P_{\mathcal{R}}^{\mathrm{NG}} = P_{\mathcal{R}}(k) + F_{\mathrm{NL}}^2 \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \, \frac{P_{\mathcal{R}}(uk) P_{\mathcal{R}}(vk)}{2u^2 v^2}. \tag{4}$$

- The energy density of GWs:

$$\Omega_{\mathrm{GW}}(k) = \int_0^\infty \mathrm{d}v \int_{|1-v|}^{|1+v|} \mathrm{d}u \mathcal{T} \, P_{\mathcal{R}}^{\mathrm{NG}}(vk) P_{\mathcal{R}}^{\mathrm{NG}}(uk), \tag{5}$$

where the transfer function $\mathcal{T} = \mathcal{T}(u, v)$ is given by

$$\begin{aligned}\mathcal{T}(u, v) =& \frac{3}{1024 v^8 u^8} \left[4v^2 - \left(v^2 - u^2 + 1\right)^2\right]^2 \left(v^2 + u^2 - 3\right)^2 \\ & \times \left\{ \left[\left(v^2 + u^2 - 3\right) \ln\left(\left|\frac{3 - (v+u)^2}{3 - (v-u)^2}\right|\right) - 4vu\right]^2 \right. \\ & \left. + \pi^2 \left(v^2 + u^2 - 3\right)^2 \Theta(v + u - \sqrt{3}) \right\}.\end{aligned} \tag{6}$$

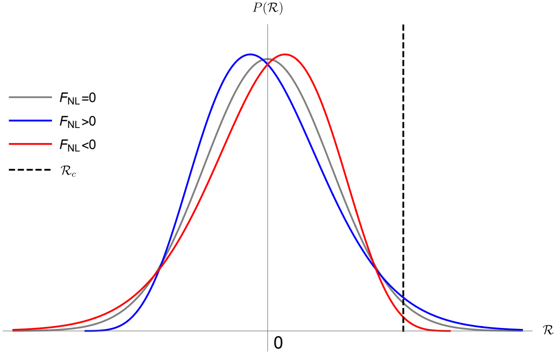

Figure: An illustration of the impact of the non-Gaussianity parameter $F_{\rm NL}$ on the probability distribution of the curvature perturbation $\mathcal{R}$. Notably, non-Gaussianity induces a skew in the distribution. A positive $F_{\rm NL}$ extends the tail of the probability distribution for $\mathcal{R} > 0$, thereby elevating the likelihood of $\mathcal{R} > \mathcal{R}_c$ and consequently leading to increased production of PBHs. Conversely, a negative $F_{\rm NL}$ diminishes the PBH production.

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- Power spectrum:

$$P_{\mathcal{R}}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(k/k_*)}{2\Delta^2}\right). \tag{7}$$

- The PBH mass fraction at formation time

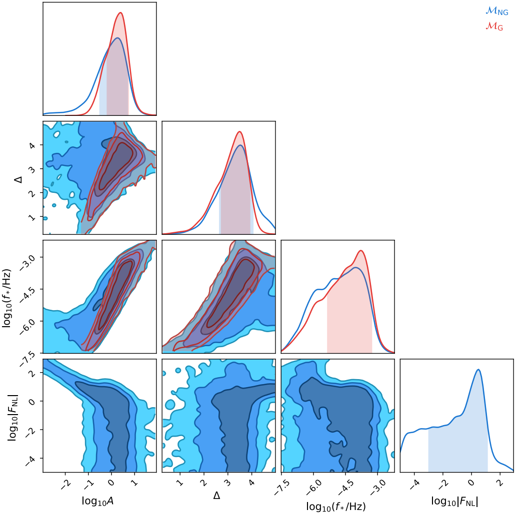
$$\beta(M) \simeq \frac{1}{2}\begin{cases} \operatorname{erfc}\left(\frac{\mathcal{R}_{\mathrm{G}}^{+}(\mathcal{R}_{\mathrm{c}})}{\sqrt{2\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}}\right) + \operatorname{erfc}\left(-\frac{\mathcal{R}_{\mathrm{G}}^{-}(\mathcal{R}_{\mathrm{c}})}{\sqrt{2\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}}\right); & F_{\mathrm{NL}} > 0, \\ \operatorname{erf}\left(\frac{\mathcal{R}_{\mathrm{G}}^{+}(\mathcal{R}_{\mathrm{c}})}{\sqrt{2\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}}\right) - \operatorname{erf}\left(\frac{\mathcal{R}_{\mathrm{G}}^{-}(\mathcal{R}_{\mathrm{c}})}{\sqrt{2\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}}\right); & F_{\mathrm{NL}} < 0, \end{cases} \tag{8}$$

with

$$\mathcal{R}_{\mathrm{G}}^{\pm}(\mathcal{R}) = \frac{1}{2F_{\mathrm{NL}}}\left(-1 \pm \sqrt{1 + 4F_{\mathrm{NL}}\mathcal{R} + 4F_{\mathrm{NL}}^{2}\langle\mathcal{R}_{\mathrm{G}}^{2}\rangle}\right). \tag{9}$$

- The total abundance of PBHs in the dark matter at present

$$f_{\mathrm{PBH}} \equiv \frac{\Omega_{\mathrm{PBH}}}{\Omega_{\mathrm{CDM}}} = 2.7 \times 10^{8} \int_{-\infty}^{\infty} \mathrm{d}\ln M \left(\frac{g_{*,r}}{10.75}\right)^{3/4} \left(\frac{g_{*,s}}{10.75}\right)^{-1} \left(\frac{M}{M_{\odot}}\right)^{-1/2} \beta(M). \tag{10}$$



- $|F_{\rm NL}| \lesssim 13.9$
- $-13.9 \lesssim F_{\rm NL} \lesssim -0.1$ when further requiring $f_{\rm PBH} \lesssim 1$.

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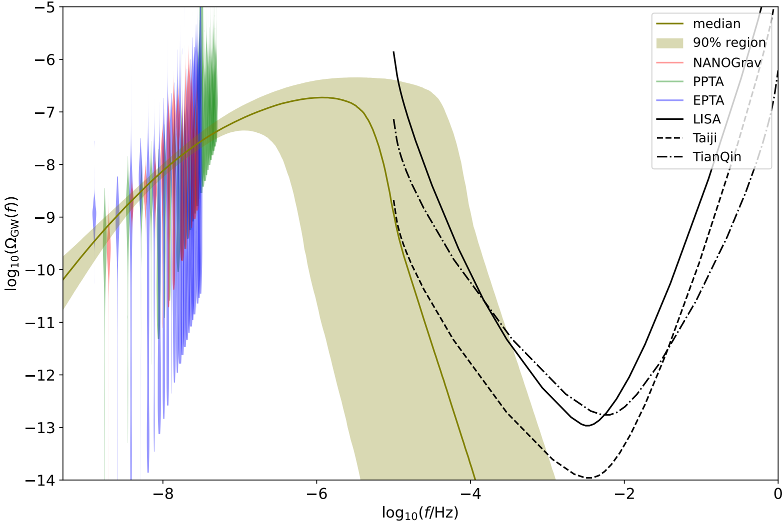

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Implications

- The constraints on $F_{\rm NL}$ have significant implications for Multi-field inflation models.
- For instance, adiabatic curvaton models predict that

$$f_{\rm NL} = \frac{5}{3}F_{\rm NL} = \frac{5}{4r_{\rm D}} - \frac{5r_{\rm D}}{6} - \frac{5}{3}, \tag{11}$$

  where $r_{\rm D} = 3\rho_{\rm curvaton}/(3\rho_{\rm curvaton} + 4\rho_{\rm radiation})$ represents the "curvaton decay fraction" at the time of curvaton decay.

- Our constraint $|F_{\rm NL}| \lesssim 13.9$ implies

$$r_{\rm D} \gtrsim 0.05 \quad (95\%), \tag{12}$$

  and the further constraint that $F_{\rm NL} \lesssim -0.1$ yields

$$r_{\rm D} \gtrsim 0.62 \quad (95\%), \tag{13}$$

  indicating that the curvaton field has a non-negligible energy density when it decays.

- Our findings, therefore, pave the way to constrain inflation models with PTAs.

Outline

- The observed spectrum of SIGW per $\ln k$ today is

$$\Omega_{\mathrm{GW},0}h^2 \approx 1.62\times 10^{-5}\left(\frac{\Omega_{r,0}h^2}{4.18\times 10^{-5}}\right)\left(\frac{g_{*r}\left(T_{\mathrm{rh}}\right)}{106.75}\right)\left(\frac{g_{*s}\left(T_{\mathrm{rh}}\right)}{106.75}\right)^{-\frac{4}{3}}\Omega_{\mathrm{GW,rh}}. \quad (14)$$

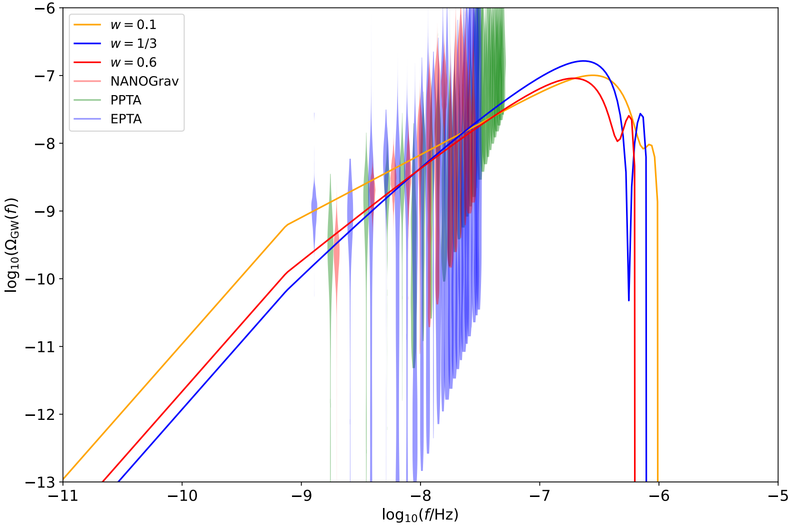
The SIGW spectrum for the scales $k \gtrsim k_{\mathrm{rh}}$ is

$$\Omega_{\mathrm{GW,rh}} = \left(\frac{k}{k_{\mathrm{rh}}}\right)^{-2b}\int_0^\infty dv\int_{|1-v|}^{1+v} du\, \mathcal{T}\,\mathcal{P}_{\mathcal{R}}(ku)\mathcal{P}_{\mathcal{R}}(kv), \quad (15)$$

where $b \equiv (1-3w)/(1+3w)$. And $\Omega_{\mathrm{GW,rh}} \propto (k/k_{\mathrm{rh}})^2$ when $k \lesssim k_{\mathrm{rh}}$.

- The primordial power spectrum:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{A}{\sqrt{2\pi}\Delta}\exp\left(-\frac{\ln^2(k/k_*)}{2\Delta^2}\right). \quad (16)$$



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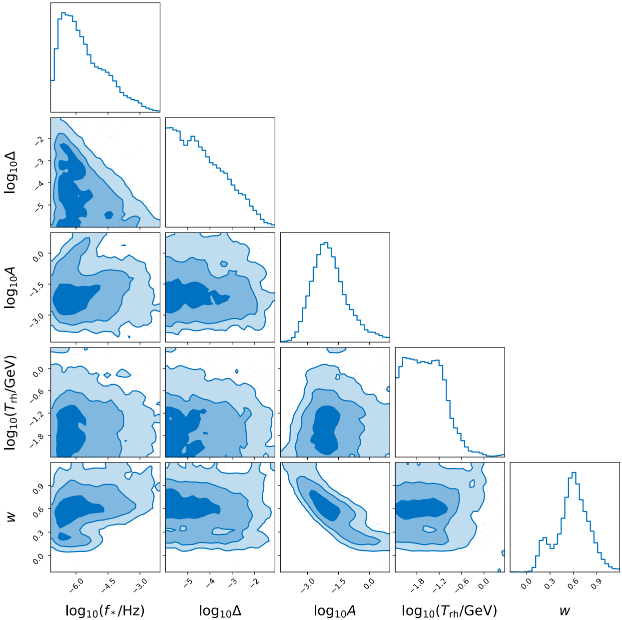

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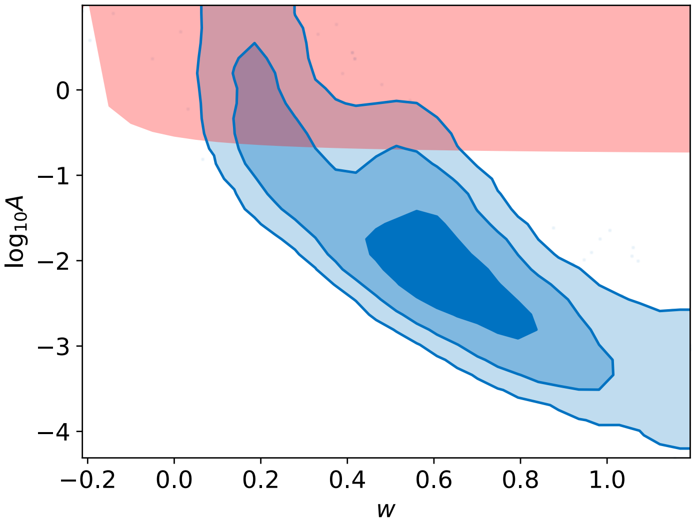
Implications

- Reheating temperature $T_{\rm rh} \lesssim 0.2\,\mathrm{GeV}$.
- $w < 0$ is excluded at 95% confidence level.
- $w = 1/3$ is consistent with the PTA data.
- w peaks at around 0.6.
- Since during the oscillation of inflaton, $w = \frac{p-2}{p+2}$ for an power-law potential $V(\phi) \propto \phi^p$, then, the constraint on w implies a ϕ^8 bottom of the inflationary potential.

Overcoming excessive PBH production



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Outline

- The SIGW spectrum:

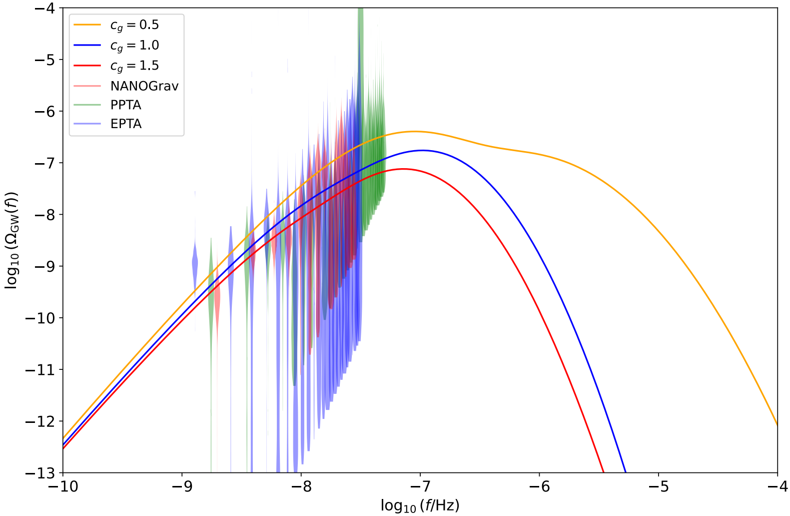
$$\Omega_{\mathrm{GW}}(k) = \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \mathcal{T}(u, v, c_g) P_\zeta(vk) P_\zeta(uk). \tag{17}$$

- The transfer function $\mathcal{T}$:

$$\begin{aligned}\mathcal{T}(u, v, c_g) = & \frac{3\left[4v^2 - \left(v^2 - u^2 + 1\right)^2\right]^2 \left(v^2 + u^2 - 3c_g^2\right)^2}{1024 v^8 u^8} \\ & \times \left\{ \left[\left(v^2 + u^2 - 3c_g^2\right) \ln\left(\left|\frac{3c_g^2 - (v+u)^2}{3c_g^2 - (v-u)^2}\right|\right) - 4vu\right]^2 \right. \\ & \left. + \pi^2 \left(v^2 + u^2 - 3c_g^2\right)^2 \Theta(v + u - \sqrt{3}c_g)\right\}.\end{aligned} \tag{18}$$

- The primordial power spectrum:

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{A}{\sqrt{2\pi}\Delta} \exp\left(-\frac{\ln^2(k/k_*)}{2\Delta^2}\right). \tag{19}$$



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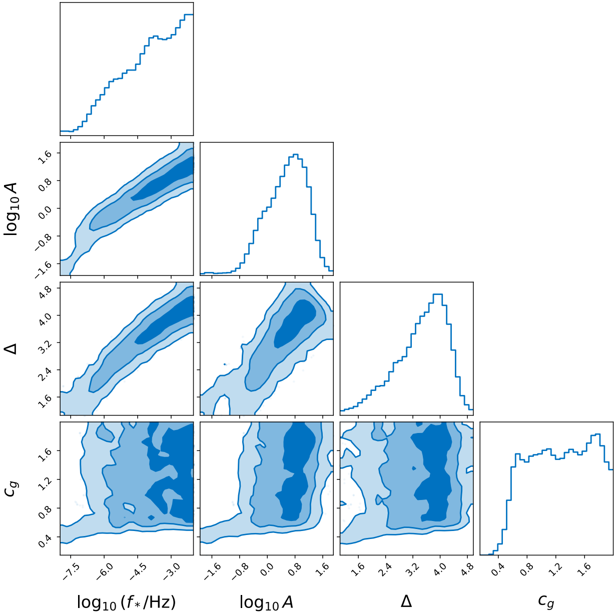

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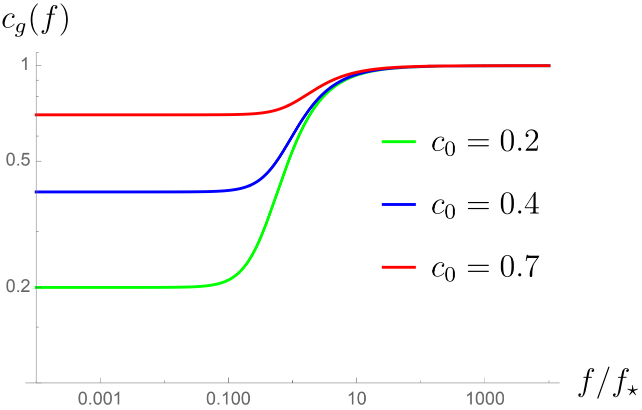
- $c_g \gtrsim 0.61$ at a 95% credible interval.

- $c_g = 1$ is consistent with the PTA data.

The frequency-dependent GW propagation speed

$$c_g(f) = \left[1 + \frac{f_\star^2}{f^2} - \frac{f_\star^2}{f^2}\sqrt{1 + 2\left(1 - c_0^2\right)\frac{f^2}{f_\star^2}}\right]^{1/2} . \tag{20}$$



Outline

Conclusions

- PTAs are opening a new window in the nHz band.
- SIGWs can explain recent PTA signal.
- PTA can explore the nature of the early Universe through SIGWs, including
  - the non-Gaussianity of curvature perturbation
    Lang Liu, *Zu-Cheng Chen*†, *Qing-Guo Huang*†, *2307.01102 PRDL*
  - the equation of state of the early Universe
    Lang Liu, *Zu-Cheng Chen*†, *Qing-Guo Huang*†, *2307.14911 JCAP*
  - the sound speed of the early Universe
    Lang Liu, *You Wu*†, *Zu-Cheng Chen*†, *2310.16500 JCAP*
  - sound speed resonance
    Jia-Heng Jin, Zu-Cheng Chen, Zhu Yi, Zhi-Qiang You, **Lang Liu**†, *You Wu*† *2307.08687 JACP*
  - parametric amplification in Higgs inflation
    Zhu Yi, Zhi-Qiang You, You Wu, Zu-Cheng Chen†, **Lang Liu**†, *2308.14688*
  - the speed of SIGW
    Zu-Cheng Chen, Jun Li, **Lang Liu**†, *Zhu Yi 2401.09818*
  - the initial condition of curvature perturbation
    Zu-Cheng Chen, **Lang Liu**† *2402.16781*
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Thank you for your attention!