第五届粒子物理前沿研讨会

The Fifth Workshop on Frontiers of Particle Physics

Axion-pion scattering at finite temperature and its influence in axion thermalization

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Introduction

Strong CP problem and QCD axion

- QCD Lagrangian could contain a P- and CP- odd term due to instantons effect [Belavin et al., 1975, Callan et al., 1976, Jackiw and Rebbi, 1976]

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi^2} \sum_{c=1}^{8} G^c_{\mu\nu} \widetilde{G}^{\mu\nu}_c \,, \quad \widetilde{G}^{\mu\nu}_c \equiv \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} G^c_{\rho\sigma} \,,$$

  which is called QCD θ-term. θ is an angular parameter of QCD θ-vacuum with $\theta \in [0, 2\pi)$.
- The measurable CP violation parameter is not θ but the combination of it and the phase angle of quark mass matrix

$$\bar{\theta} = \theta + \mathrm{ArgDet} Y_U Y_D \,.$$

- $\bar{\theta}$ can be determined from the measurement of neutron electric dipole moment (nEDM)

$$H = -d_n \vec{E} \cdot \hat{S} \,.$$

  Experimental upper limit on d_n [Abel et al., 2020]

$$|d_n|_{\mathrm{exp}} < 1.8 \times 10^{-26} \, e\,\mathrm{cm} \,.$$

  nEDM contributed from $\bar{\theta}$ [Pospelov and Ritz, 2000]

$$d_n = 2.4(1.0) \times 10^{-16} \bar{\theta} \, e\,\mathrm{cm} \,.$$

  Thus $|\bar{\theta}| \lesssim 10^{-10}$.
- **The strong CP problem is to understand why $\bar{\theta}$ is so small.**

Introduction

Strong CP problem and QCD axion

PQ solution of the strong CP problem

A global $U(1)_{\rm PQ}$ symmetry was introduced by Peccie and Quinn in order to preserve P and CP invariance of strong interactions despite the effects of instantions.
[Peccei and Quinn, 1977a, Peccei and Quinn, 1977b]
The spontaneous breakdown of the $U(1)_{\rm PQ}$ symmetry leads to a pseudoscalar pseudo-Goldstone boson, the "axion". [Weinberg, 1978, Wilczek, 1978]

- Modern perspective of PQ mechanism:
  - ✍ Introduce pseudoscalar axion field

    $$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a + \mathcal{L}(\partial_\mu a,\, \psi) + \frac{a}{f_a}\frac{g_s^2}{32\pi^2} G^c_{\mu\nu}\widetilde{G}^{\mu\nu}_c\,,$$

    with quasi shift symmetry: $a \to a + \kappa$.
  - ✍ By taking $\frac{\kappa}{f_a} + \bar{\theta} = 0$, $\bar{\theta}$ is effectively replaced with dynamical field $\frac{a}{f_a}$.
  - ✍ QCD vacuum energy density takes its minimum value when $\langle a \rangle = 0$,
    [Vafa and Witten, 1984]
    thus solve the strong CP problem.

- The $aG\widetilde{G}$ operator is commonly regarded as the model independent axion coupling in the construction of various axion models.
- There are UV model dependent axion coupling, like $aF\widetilde{F}$, $\partial_\mu a \bar{f} c_f^0 \gamma^\mu \gamma_5 f$. For rev, see [Di Luzio et al., 2020]
- Following we will stick to the model independent axion coupling, i.e., axion couples to SM particles entirely through the $aG\widetilde{G}$ operator.

Introduction

Thermal axion and its HDM bound

- Weak coupling of axion to SM particles ($f_a \gtrsim 10^7$ GeV) ➜ Dark matter candidate
- Axion could keep in thermal equilibrium with SM thermal bath in the early stage of thermal history, as long as the axion coupling is not too faint.
  (There is thermal axion production when $f_a \lesssim 10^{12}$ GeV [Masso et al., 2002])

  Thermal axion ➜ Hot dark matter (HDM) component ➜ Extra dark radiation

  Contribute to the effective number of extra relativistic degrees of freedom

$$\Delta N_{\text{eff}} \simeq \frac{4}{7}\left(\frac{43}{4g_{\star s}(T_D)}\right)^{4/3}, \quad \text{with } T_D \text{ the decoupling temperature of axion.}$$

- The value of ΔN_{eff} is constrained by cosmic microwave background (CMB) experiments

$$\Delta N_{\text{eff}} \lesssim 0.28\,, \quad \text{given by Planck'18 [Aghanim et al., 2020].}$$

- The interaction rate between axions and SM particles is depicted by axion thermalization rate

$$\Gamma_a(T) = \frac{1}{n_a^{\text{eq}}}\int \mathrm{d}\widetilde{\Gamma}\sum|\mathcal{M}|^2\, n_B(E_1)n_B(E_2)[1+n_B(E_3)][1+n_B(E_4)]\,.$$

- The decoupling condition under instantaneous decoupling approximation

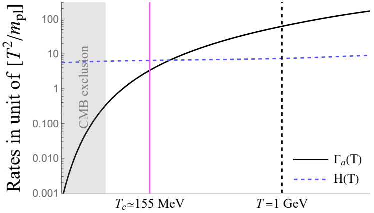
$$\Gamma_a(T_D) \simeq H(T_D)\,.$$

Introduction

Thermal axion and its HDM bound

Temperature regions and main thermal channels

- ☞ $T_D \gtrsim 1$ GeV: $ag \leftrightarrow gg$.
  [Masso et al., 2002, Graf and Steffen, 2011]
- ☞ $T_D \lesssim 1$ GeV: Hadrons need to be included.
- ☞ $T_D \lesssim 200$ MeV: $a\pi \leftrightarrow \pi\pi$.
  [Chang and Choi, 1993, Hannestad et al., 2005, Giusarma et al., 2014, D'Eramo et al., 2022]



- The constrain of $\Delta N_{\rm eff}$ gives a lower limit of f_a, or equivalently, upper limit of m_a by

$$m_a \simeq 5.7 \times (10^6\ \text{GeV}/f_a)\ \text{eV}\,.$$

  The highest attainable axion mass from the constrain of $\Delta N_{\rm eff}$ is called HDM bound.

- Reliable axion-pion interactions are needed to determine Γ_a for $T_D < T_c$.
  - ☞ The calculation of Γ_a up to NLO in ChPT show that the boundary of validity of chiral expansion for Γ_a is $T_\chi \simeq 70$ MeV. [Di Luzio et al., 2021]
  - ☞ Unitarization procedure is implemented to the chiral perturbative amplitudes in order to extend the applicable range of energy and temperature. [Di Luzio et al., 2023]
  - ☞ Other important works which are not listed...

To our knowledge, the thermal correction in $a\pi$ scattering amplitudes was ignored in all previous works.
In this talk, we consider all thermal correction in scattering amplitudes up to one loop level, and investigate its effect on axion thermalization rate.

Contents

Axion-pion $SU(2)$ chiral perturbation theory

- QCD-axion effective Lagrangian
  We stick to the model independent axion coupling

$$\mathcal{L}_{a\mathrm{QCD}} = \mathcal{L}_{\mathrm{QCD}} + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{g_s^2}{32\pi^2} G^c_{\mu\nu}\widetilde{G}^{\mu\nu}_c \,.$$

Eliminate $aG\widetilde{G}$ operator by a chiral transformation of quark field

$$q \to \exp\left(i\frac{a}{2f_a}Q_a\gamma_5\right) q\,, \quad Q_a^\dagger = Q_a\,, \quad \mathrm{Tr}Q_a = 1\,.$$

Arrive an other basis

$$\mathcal{L}'_{a\mathrm{QCD}} = \frac{1}{2}\partial_\mu a \partial^\mu a + \mathcal{L}^0_{\mathrm{QCD}} - \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q - \bar{q}_R M_a q_L - \bar{q}_L M_a^\dagger q_R\,,$$

$$M_a \equiv \exp\left(-i\frac{a}{2f_a}Q_a\right) M \exp\left(-i\frac{a}{2f_a}Q_a\right)\,, \quad M = \mathrm{diag}(m_u,\, m_d)\,,$$

where $\mathcal{L}^0_{\mathrm{QCD}}$ denotes QCD Lagrangian under χ-limit ($m_q \to 0$).

✍ There is arbitrariness for the choice of Q_a.

To get axion-pion interactions, we match $\mathcal{L}'_{a\mathrm{QCD}}$ to $SU(2)$ ChPT.

Axion-pion $SU(2)$ chiral perturbation theory

Chiral effective Lagrangian can be constructed order by order
[Gasser and Leutwyler, 1984, Gasser and Leutwyler, 1985]

$$\mathcal{L}_{a\chi\mathrm{PT}} = \frac{1}{2}\partial_\mu a \partial^\mu a + \mathcal{L}_2 + \mathcal{L}_4 + \cdots .$$

- Power counting rule
  Expansion by soft external momenta, light quarks masses are counted as $m_q = \mathcal{O}(p^2)$.
- The LO effective Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger\rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu\big|_{\mathrm{LO}} ,$$

$$J_A^\mu\big|_{\mathrm{LO}} = -i\frac{F^2}{2}\langle Q_a\{\partial^\mu U, U^\dagger\}\rangle .$$

$$U = \exp\left(\frac{i\phi}{F}\right) , \quad \phi = \sum_{i=1}^{3}\tau^i\phi_i = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} .$$

$$\chi_a = 2B_0 M_a .$$

Two parameters: F the decay constant of pion in χ-limit, B_0 is related to $\langle\bar{q}q\rangle|_{\chi\text{-limit}}$.

Axion-pion $SU(2)$ chiral perturbation theory

- We take a specific Q_a in practical calculation

$$Q_a = \frac{M^{-1}}{\langle M^{-1}\rangle}\,.$$

The mass mixing of axion and π^0 field will be absent at LO, by such choice of Q_a.

- The the a-π^0 kinetic mixing still appears at LO, contributed by axial current coupling

$$\frac{\partial_\mu a}{2f_a} J^\mu_A\big|_{\mathrm{LO}} \supset \delta_{a\pi}\partial_\mu a \partial^\mu \pi^0\,, \quad \delta_{a\pi} = \frac{1}{2}\delta_I \frac{F}{f_a}\,, \quad \delta_I = \frac{m_d - m_u}{m_d + m_u}\,,$$

and this can be eliminated by the field redefination up to $\mathcal{O}(1/f_a^2)$

$$a \to a - \delta_{a\pi}\pi^0 + \mathcal{O}(1/f_a^3)\,, \quad \pi^0 \to \left(1 + \frac{1}{2}\delta_{a\pi}^2\right)\pi^0 + \mathcal{O}(1/f_a^3)\,.$$

Axion-pion $SU(2)$ chiral perturbation theory

- The effective Lagrangian at $\mathcal{O}(p^4)$

$$
\begin{aligned}
\mathcal{L}_4 \supset & \frac{l_3+l_4}{16}\left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle + \frac{l_4}{8}\left\langle \partial_\mu U \partial^\mu U^\dagger \right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle \\
& - \frac{l_7}{16}\left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle \left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle + \frac{h_1-h_3-l_4}{16}\left[\left(\left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle\right)^2 \right. \\
& \left. + \left(\left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle\right)^2 - 2\left\langle \chi_a U^\dagger \chi_a U^\dagger + U\chi_a^\dagger U\chi_a^\dagger \right\rangle\right] + \frac{\partial_\mu a}{2f_a} J_A^\mu\big|_{\mathrm{NLO}}\,, \\
J_A^\mu\big|_{\mathrm{NLO}} \supset & -il_1\left\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\right\rangle \left\langle \partial_\nu U \partial^\nu U^\dagger \right\rangle \\
& - i\frac{l_2}{2}\left\langle Q_a\left\{\partial_\nu U, U^\dagger\right\}\right\rangle \left\langle \partial^\mu U \partial^\nu U^\dagger + \partial^\nu U \partial^\mu U^\dagger \right\rangle \\
& - i\frac{l_4}{4}\left\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle\,.
\end{aligned}
$$

Only the operators relevant to this work are shown.

Contents

$a\pi \to \pi\pi$ scattering at finite temperature

- Finite-temperature effects are included by imaginary time formalism (ITF), where [Kapusta and Gale, 2011, Bellac, 2011, Laine and Vuorinen, 2016]

$$p^0 \to i\omega_n , \quad \text{with } \omega_n = 2\pi n T \,, n \in \mathbb{Z} \,,$$
$$-i \int \frac{\mathrm{d}^d q}{(2\pi)^d} \to -i \int_\beta \frac{\mathrm{d}^d q}{(2\pi)^d} \equiv T \sum_n \int \frac{\mathrm{d}^{d-1} q}{(2\pi)^{d-1}} \,.$$

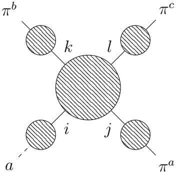
  The zeroth components of external momenta can be extrapolated to real and continuous energies

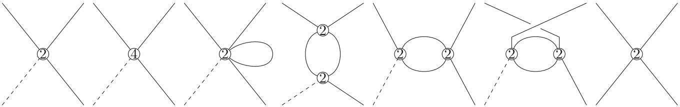
$$i\omega_n \to E + i0^+ \,.$$

  after completing the Matsubara sums.

- Compute the thermal Green functions in ITF

$$G^T_{a\pi^a;\pi^b\pi^c}(p_1, p_2; p_3, p_4) = \sum_{i,j,k,l} G_{ai}(p_1^2) G_{\pi^a j}(p_2^2) G_{k\pi^b}(p_3^2) G_{l\pi^c}(p_4^2) A_{ij;kl}(p_1, p_2; p_3, p_4) \,.$$





Feynman diagrams for amputated functions up to NLO.

$a\pi \to \pi\pi$ scattering at finite temperature

- Thermal scattering amplitudes are extracted from thermal Green functions by LSZ reduction formula

$$i\mathcal{M}^T_{a\pi^a;\pi^b\pi^c}(p_1,p_2;p_3,p_4) = \frac{1}{Z_\pi^{3/2}} \lim_{p_i^0 \to E_{p_i}} \left[p_1^2 \prod_{i=2}^{4} \left(p_i^2 - m_\pi^2\right) \right] G^T_{a\pi^a;\pi^b\pi^c}(p_1,p_2;p_3,p_4)\,.$$

In the calculation of the $a\pi \to \pi\pi$ scattering amplitudes, we keep the terms up to $\mathcal{O}(1/f_a)$, and ignore the isospin breaking in m_π.

- ☞ $\mathcal{M}^T_{a\pi^a;\pi^b\pi^c}$ have well-defined zero temperature limitation $\mathcal{M}^{T=0^+}_{a\pi^a;\pi^b\pi^c}$;
- ☞ T-dependence of $\mathcal{M}^T_{a\pi^a;\pi^b\pi^c}$ entirely introduced by loop diagrams;
- ➥ We can make pure predictions at finite temperature, once the values of LECs are fixed at zero temperature.

For comparison with [Di Luzio et al., 2023], we take the same set of values for the parameters.
For perturbative calculation

$$m_\pi = 137\ \text{MeV}\,, \quad F_\pi = 92.1\ \text{MeV}\,, \quad m_u/m_d = 0.5\,,$$
$$\bar{l}_1 = -0.36(59)\,, \quad \bar{l}_2 = 4.31(11)\,, \quad \bar{l}_3 = 3.53(26)\,, \quad \bar{l}_4 = 4.73(10)\,, \quad l_7 = 0.007(4)\,.$$

$a\pi \to \pi\pi$ scattering at finite temperature

- Partial waves (PWs) and s-channel thermal unitarity relations

$$\mathcal{M}_{a\pi;IJ}(E_{cm}) = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\theta \, \mathcal{M}_{a\pi;I}(E_{cm},\cos\theta)P_J(\cos\theta)\,.$$

$$\mathrm{Im}\mathcal{M}_{a\pi;IJ}(E_{cm}) \overset{s}{=} \frac{1}{2}\rho^T_{\pi\pi}(E_{cm})\mathcal{M}^{IJ^*}_{\pi\pi;\pi\pi}\mathcal{M}_{a\pi;IJ}\,, \quad (E_{cm} > 2m_\pi)\,,$$

$$\rho^T_{\pi\pi}(E_{cm}) = \frac{\sigma_\pi(E^2_{cm})}{16\pi}\left[1 + 2n_B(\frac{E_{cm}}{2})\right]\,,$$

$$\sigma_\pi(s) = \sqrt{1 - \frac{4m_\pi^2}{s}}\,, \quad n_B(E) = \frac{1}{e^{E/T} - 1}\,.$$

  I denotes the total isospin of final $\pi\pi$ state, E_{cm} the center of mass (CM) energy and θ the scattering angle in CM collision.

- Perturbative PW amplitudes satisfy the thermal unitarity relations perturbatively

$$\mathrm{Im}\mathcal{M}^{(4)}_{a\pi;IJ}(E_{cm}) \overset{s}{=} \frac{1}{2}\rho^T_{\pi\pi}(E_{cm})\mathcal{M}^{IJ,(2)}_{\pi\pi;\pi\pi}\mathcal{M}^{(2)}_{a\pi;IJ}\,, \quad (E_{cm} > 2m_\pi)\,.$$

- There are additional pure thermal contributions from Landau cuts of cross channels above $\pi\pi$ threshold on right hand.
  A similiar case is thermal $K\pi \to K\pi$ scattering, where the thermal Landau cut generated in the u channel show up in the physical energy region above the πK threshold. [Gómez Nicola et al., 2023].

$a\pi \to \pi\pi$ scattering at finite temperature

- The unitarized PW $a\pi \to \pi\pi$ amplitudes in inverse amplitude method (IAM) take the form [Salas-Bernárdez et al., 2021]

$$\mathcal{M}^{\rm IAM}_{a\pi;IJ} = \frac{\left(\mathcal{M}^{(2)}_{a\pi;IJ}\right)^2}{\mathcal{M}^{(2)}_{a\pi;IJ} - \mathcal{M}^{(4)}_{a\pi;IJ}} .$$

We use two values of LECs in the calculation by IAM amplitudes

$$\bar{l}_1 - \bar{l}_2 = -5.95(2) , \quad \bar{l}_1 + \bar{l}_2 = 4.9(6) .$$

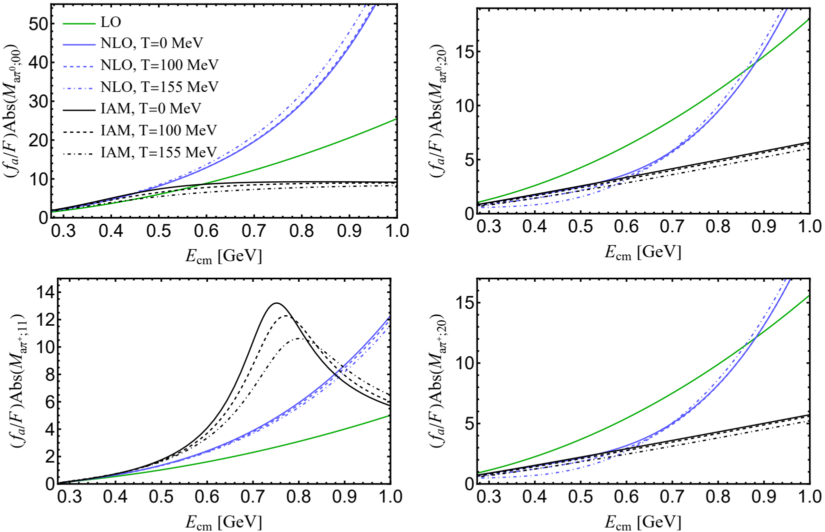
which were determined from $\pi\pi$ scattering to fit the pole position and width of the ρ resonance precisely [Dobado and Pelaez, 1997]. Such values are also same with that used in [Di Luzio et al., 2023].

- Resonances poles on the second Riemann sheet

	$f_0(500)/\sigma$		$\rho(770)$	
	$M_\sigma \pm i\frac{\Gamma_\sigma}{2}$	$\lvert f_a g_{\sigma a\pi}\rvert$	$M_\rho \pm i\frac{\Gamma_\rho}{2}$	$\lvert f_a g_{\rho a\pi}\rvert$
$T = 0$ MeV	$422 \pm i240$ MeV	0.032 GeV^2	$739 \pm i72$ MeV	0.035 GeV^2
$T = 100$ MeV*	$368 \pm i310$ MeV	0.037 GeV^2	$744 \pm i77$ MeV	0.036 GeV^2

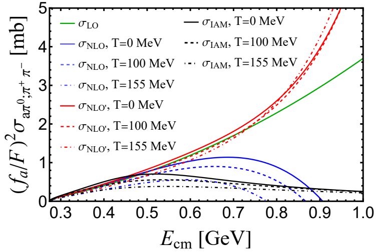
*Only include s-channel unitary thermal correction.

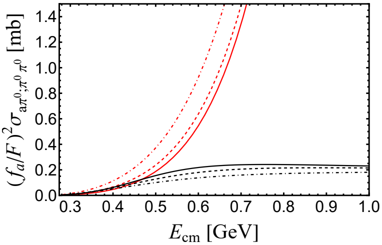
$a\pi \to \pi\pi$ scattering at finite temperature



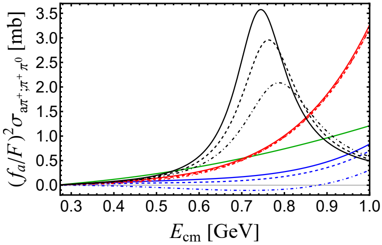
$a\pi \to \pi\pi$ scattering at finite temperature

$$|\mathcal{M}^{(2)} + \mathcal{M}^{(4)}|^2 = \left(\mathcal{M}^{(2)}\right)^2 + 2\mathcal{M}^{(2)}\mathrm{Re}\left(\mathcal{M}^{(4)}\right) + |\mathcal{M}^{(4)}|^2 .$$





- The chiral perturbative amplitudes up to NLO begin to be unreliable around $E \simeq 500$ MeV.



Contents

Effect of thermal correction in the scattering amplitudes on HDM bound

- We calculate the axion rate by the temperature dependent $a\pi \to \pi\pi$ scattering amplitudes [Chang and Choi, 1993, Hannestad et al., 2005]

$$\Gamma_a(T) = \frac{1}{n_a^{\mathrm{eq}}} \int \mathrm{d}\widetilde{\Gamma} \sum |\mathcal{M}_{a\pi;\pi\pi}|^2 \, n_B(E_1) n_B(E_2)[1 + n_B(E_3)][1 + n_B(E_4)] \,,$$

where the phase space integral

$$\int \mathrm{d}\widetilde{\Gamma} = \int \left(\prod_{i=1}^{4} \frac{\mathrm{d}^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \right) (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \,.$$

- Decoupling temperature of axion T_D is extracted from the decoupling condition under instantaneous decoupling approximation

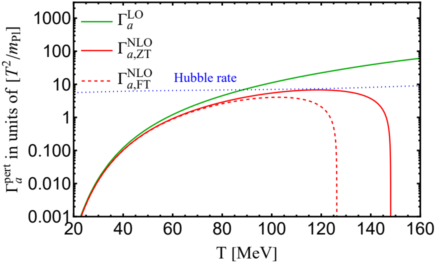
$$\Gamma_a(T_D) \simeq H(T_D) \,.$$

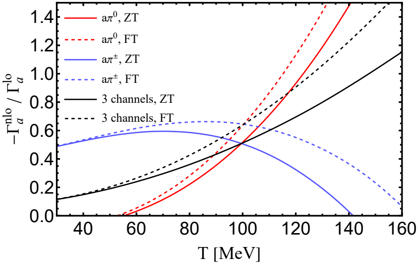
And T_D is constrained by the limitation of ΔN_{eff} from CMB experiments.

☞ We follow the procedure of cosmological discussion as in [Di Luzio et al., 2023].

Effect of thermal correction in the scattering amplitudes on HDM bound

By perturbative amplitudes up to NLO



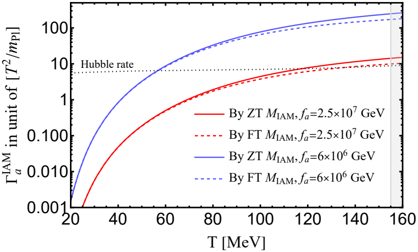


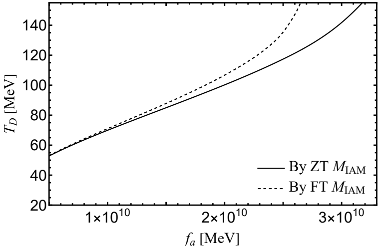
- We confirm that the breakdown temperature of NLO chiral perturbative description for $\Gamma_a(T)$, when using zero temperature $a\pi \to \pi\pi$ scattering amplitudes, is around 70 MeV.
- The convergence of the chiral expansion for $\Gamma_a(T)$ gets worse after taking the thermal corrections into the $a\pi$ scattering amplitudes.

➥ It is still unlikely to extract reliable bounds from the axion thermalization rate by using the chiral perturbative amplitudes, after including the thermal corrections to the $a\pi$ amplitudes.

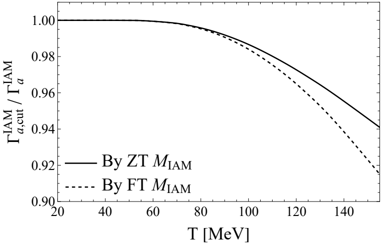
Effect of thermal correction in the scattering amplitudes on HDM bound

By IAM amplitudes



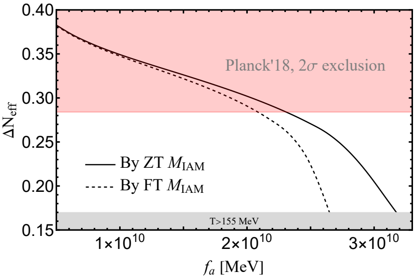


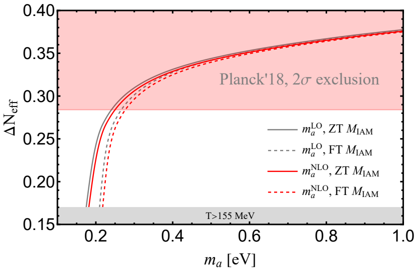
- $\Gamma^{\rm IAM}_{a,cut}$ is obtained by cutting off the contributions at $E^{\rm cut}_{cm} = 1$ GeV.
- The significant contributions of $\Gamma^{\rm IAM}_{a}$ are from the region of $E_{cm} < 1$ GeV.



Effect of thermal correction in the scattering amplitudes on HDM bound

By IAM amplitudes





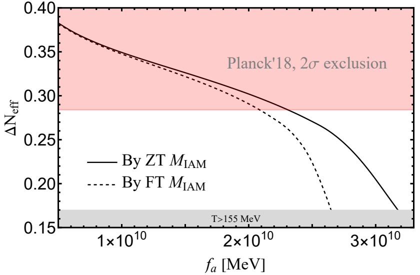
- The QCD axion mass up to LO & NLO

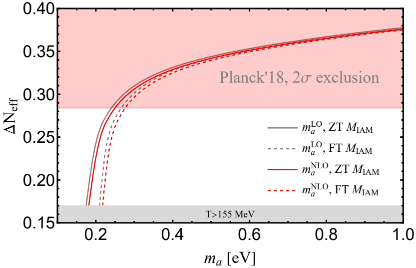
$$m_a^2|_{\rm LO} = \gamma_{ud}\, m_\pi^2 \frac{F^2}{f_a^2}\,, \qquad \text{where} \quad \gamma_{ud} = \frac{m_u m_d}{(m_u+m_d)^2}\,,$$

$$m_a^2|_{\rm NLO} = \gamma_{ud}\, m_\pi^2 \frac{F^2}{f_a^2} \left\{ 1 - 2\frac{m_\pi^2}{(4\pi F)^2} \log\frac{m_\pi^2}{\mu^2} + 2\left[h_1^r(\mu^2) - h_3\right]\frac{m_\pi^2}{F^2} - 8 l_7 \gamma_{ud} \frac{m_\pi^2}{F^2} \right\}.$$

Effect of thermal correction in the scattering amplitudes on HDM bound

By IAM amplitudes





- The constrians

	lower limit of f_a	upper limit of m_a by m_a^{LO}	upper limit of m_a by m_a^{NLO}
ZT	2.3×10^7 GeV	0.24 eV	0.25 eV
FT	2.1×10^7 GeV	0.27 eV	0.28 eV

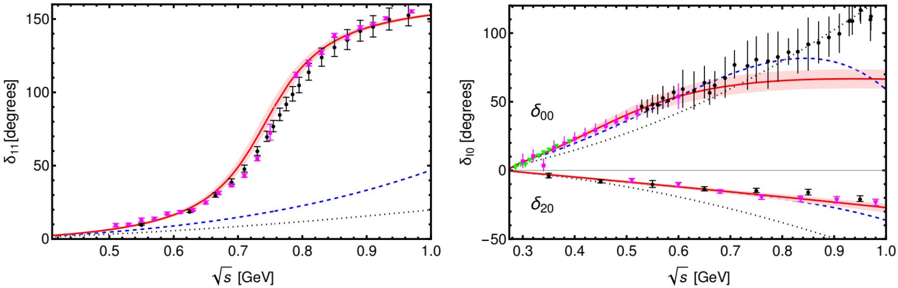
The shift of the bounds introduced by the thermal corrections in the $a\pi$ amplitudes is around 10%.

Contents

Summary and conclusions

- The thermal $a\pi \to \pi\pi$ scattering amplitudes, up to one loop level, are calculated in SU(2) ChPT.
- To include the effects of the relevant resonances (ρ and σ), unitarized amplitudes are constructed by IAM.
- Taking the thermal IAM amplitudes as reliable input below $\sqrt{s} \lesssim 1\ \mathrm{GeV}$, we calculate the axion thermalization rate at the temperatures below $155\ \mathrm{MeV}$.
- We find that the thermal correction in the axion-pion scattering amplitudes can cause around 10% shift to the bounds of axion decay constant and mass constrained by Planck$'$18 data.

Thank you for listening



[Di Luzio et al., 2023]

References I

[Abel et al., 2020] Abel, C. et al. (2020).
Measurement of the Permanent Electric Dipole Moment of the Neutron.
Phys. Rev. Lett., 124(8):081803.

[Aghanim et al., 2020] Aghanim, N. et al. (2020).
Planck 2018 results. VI. Cosmological parameters.
Astron. Astrophys., 641:A6.
[Erratum: Astron.Astrophys. 652, C4 (2021)].

[Belavin et al., 1975] Belavin, A. A., Polyakov, A. M., Schwartz, A. S., and Tyupkin, Y. S. (1975).
Pseudoparticle Solutions of the Yang-Mills Equations.
Phys. Lett. B, 59:85–87.

[Bellac, 2011] Bellac, M. L. (2011).
Thermal Field Theory.
Cambridge Monographs on Mathematical Physics. Cambridge University Press.

[Callan et al., 1976] Callan, Jr., C. G., Dashen, R. F., and Gross, D. J. (1976).
The Structure of the Gauge Theory Vacuum.
Phys. Lett. B, 63:334–340.

References II

[Chang and Choi, 1993] Chang, S. and Choi, K. (1993).
Hadronic axion window and the big bang nucleosynthesis.
Phys. Lett. B, 316:51–56.

[D'Eramo et al., 2022] D'Eramo, F., Hajkarim, F., and Yun, S. (2022).
Thermal Axion Production at Low Temperatures: A Smooth Treatment of the QCD Phase Transition.
Phys. Rev. Lett., 128(15):152001.

[Di Luzio et al., 2020] Di Luzio, L., Giannotti, M., Nardi, E., and Visinelli, L. (2020).
The landscape of QCD axion models.
Phys. Rept., 870:1–117.

[Di Luzio et al., 2023] Di Luzio, L., Martin Camalich, J., Martinelli, G., Oller, J. A., and Piazza, G. (2023).
Axion-pion thermalization rate in unitarized NLO chiral perturbation theory.
Phys. Rev. D, 108(3):035025.

[Di Luzio et al., 2021] Di Luzio, L., Martinelli, G., and Piazza, G. (2021).
Breakdown of chiral perturbation theory for the axion hot dark matter bound.
Phys. Rev. Lett., 126(24):241801.

[Dobado and Pelaez, 1997] Dobado, A. and Pelaez, J. R. (1997).
The Inverse amplitude method in chiral perturbation theory.
Phys. Rev. D, 56:3057–3073.

References III

[Gasser and Leutwyler, 1984] Gasser, J. and Leutwyler, H. (1984).
Chiral Perturbation Theory to One Loop.
Annals Phys., 158:142.

[Gasser and Leutwyler, 1985] Gasser, J. and Leutwyler, H. (1985).
Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark.
Nucl. Phys. B, 250:465–516.

[Giusarma et al., 2014] Giusarma, E., Di Valentino, E., Lattanzi, M., Melchiorri, A., and Mena, O. (2014).
Relic Neutrinos, thermal axions and cosmology in early 2014.
Phys. Rev. D, 90(4):043507.

[Gómez Nicola et al., 2023] Gómez Nicola, A., de Elvira, J. R., and Vioque-Rodríguez, A. (2023).
The pion-kaon scattering amplitude and the $K_0^*(700)$ and K*(892) resonances at finite temperature.
JHEP, 08:148.

[Graf and Steffen, 2011] Graf, P. and Steffen, F. D. (2011).
Thermal axion production in the primordial quark-gluon plasma.
Phys. Rev. D, 83:075011.

[Hannestad et al., 2005] Hannestad, S., Mirizzi, A., and Raffelt, G. (2005).
New cosmological mass limit on thermal relic axions.
JCAP, 07:002.

References IV

[Jackiw and Rebbi, 1976] Jackiw, R. and Rebbi, C. (1976).
Vacuum Periodicity in a Yang-Mills Quantum Theory.
Phys. Rev. Lett., 37:172–175.

[Kapusta and Gale, 2011] Kapusta, J. I. and Gale, C. (2011).
Finite-temperature field theory: Principles and applications.
Cambridge Monographs on Mathematical Physics. Cambridge University Press.

[Laine and Vuorinen, 2016] Laine, M. and Vuorinen, A. (2016).
Basics of Thermal Field Theory, volume 925.
Springer.

[Masso et al., 2002] Masso, E., Rota, F., and Zsembinszki, G. (2002).
On axion thermalization in the early universe.
Phys. Rev. D, 66:023004.

[Peccei and Quinn, 1977a] Peccei, R. D. and Quinn, H. R. (1977a).
Constraints Imposed by CP Conservation in the Presence of Instantons.
Phys. Rev. D, 16:1791–1797.

[Peccei and Quinn, 1977b] Peccei, R. D. and Quinn, H. R. (1977b).
CP Conservation in the Presence of Instantons.
Phys. Rev. Lett., 38:1440–1443.

References V

[Pospelov and Ritz, 2000] Pospelov, M. and Ritz, A. (2000).
Theta vacua, QCD sum rules, and the neutron electric dipole moment.
Nucl. Phys. B, 573:177–200.

[Salas-Bernárdez et al., 2021] Salas-Bernárdez, A., Llanes-Estrada, F. J., Escudero-Pedrosa, J., and Oller, J. A. (2021).
Systematizing and addressing theory uncertainties of unitarization with the Inverse Amplitude Method.
SciPost Phys., 11(2):020.

[Vafa and Witten, 1984] Vafa, C. and Witten, E. (1984).
Parity Conservation in QCD.
Phys. Rev. Lett., 53:535.

[Weinberg, 1978] Weinberg, S. (1978).
A New Light Boson?
Phys. Rev. Lett., 40:223–226.

[Wilczek, 1978] Wilczek, F. (1978).
Problem of Strong P and T Invariance in the Presence of Instantons.
Phys. Rev. Lett., 40:279–282.