Probing Dark Matter with Gravitational-Wave Interferometers in Space

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第五届粒子物理前沿研讨会

深圳, 2024.4.12-16

Contents



Motivation

- Standard model is not complete
- Dark Matter and Dark Energy
- Neutrino mass
- Matter-Antimatter asymmetry
- Theoretical Problems
 - ➢ Strong CP problem
 - Hierarchy problem
 - ➢ Fermion mass hierarchy
 - Unification of forces



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Evidence for Dark Matter



Dark Matter Candidates

- Primordial black holes
- Super heavy particles
- Asymmetric DM

.

Hidden sector DM

- Weakly-interacting (WIMP)
- Strongly-interacting (SIMP)
- Axion (ALP), Ultralight DM
- Sterile neutrino



Dark Matter Candidates

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WIMP

- ➤ Mass ~ 100 GeV
- Coupling constant ~ 0.5
- > Relic abundance $\Omega \sim 0.3$
- Thermal history
 - 1. Equilibrium XX <> ff
 - 2. Equilibrium XX > ff
 - 3. Freeze out
- Cold dark matter
- The&Exp attractive



WIMP

- > Theories: supersymmetry, extra dimensions,
- Direct, indirect and collider searches
- Impressive progresses



Ultralight Dark Matter

- Mass < 1 eV, QCD axion, axion-like-particles, bosonic</p>
- > Number density is very large, behaves as classical wave

$$\Phi(x) = \sum_{\mathbf{v}} \frac{\sqrt{2\rho/N}}{m} e^{i(\omega t - \mathbf{k} \cdot \mathbf{x} + \theta_{\mathbf{v}})},$$





Ultralight Dark Matter

 \succ Vector field A_{μ}

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_A^2 A^\nu A_\nu - \epsilon_D e J_D^\nu A_\nu, \quad \vec{A}(t, \vec{x}) = |\vec{A}|\hat{e}_A e^{i(\omega t - \vec{k} \cdot \vec{x})},$$
$$\delta x^i(t, \vec{x}) = \mathcal{M}_v \hat{e}_A^i e^{im_A(t - v\hat{k} \cdot \vec{x})}, \mathcal{M}_v \propto \epsilon_D e q_{D,j} |\vec{A}| / m_A M_j$$

DM property

$$\phi_{\vec{k}} = \frac{\sqrt{2\rho_{\rm DM}}}{m_{\phi}}, \qquad |\vec{A}| = \frac{\sqrt{2\rho_{\rm DM}}}{m_A}, \quad v \sim 10^{-3}, \quad \vec{k} \approx m_{\phi}\vec{v} \text{ and } \omega \approx m_{\phi}$$

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Physical Effects

Atomic physics

- Arvanitaki, Huang & Tilburg (2014), Graham, Kaplan, Mardon, Rajendran & Terrano (2015), Safronova, Budker, DeMille, Kimball, Derevianko & Clark (2018),
- ≻ Stadnik (2022), ……
- Astrophysical physics
 - Pierce, Riles & Zhao (2018), Morisaki & Suyama (2019), Guo, Riles, Yang & Zhao 2019, Grote & Stadnik (2019),
 - An, Huang, Liu & Xue (2021), Chen, Shu, Xue, Yuan & Zhao (2019), Xia, Xu & Zhou (2020), Sun, Yang & Zhang (2021), Wu, Chen, & Huang (2023),
 - ≻ Liu, Lou & Ren (2021), Luu, Liu, Ren, Broadhurst, Yang, Wang & Xie (2023), ……
- Underground searches
 - ➢ Dark Matter Experiments, PandaX, XENONnT, …

Space-based GW Interferometers

\succ LISA, Taiji, TianQin, DECIGO, BBO, LISAMax, ASTROD-GW, μ Ares



Signal Response

Gravitational wave can change the structure of spacetime, and the physical distance between objects

One can measure the phase by laser







 $\succ \text{ Response } \frac{\delta v(t)}{v_0} \equiv y_{BA} = -\frac{1}{2} \frac{n_i n_j}{1 + \vec{k} \cdot \vec{n}} \left[h_{ij} \left(t - \frac{\vec{k} \cdot \vec{x}_B}{c} \right) - h_{ij} \left(t - \frac{\vec{k} \cdot \vec{x}_A + L}{c} \right) \right]$

DM spikes

- > WIMP DM particles accretion around BH \rightarrow DM spike
- > NFW profile → spiky density profile Gondolo & Silk (1999)

$$\rho(r) \propto r^{-\gamma}, 0 \leq \gamma \leq 2 \quad \Longrightarrow \quad \rho_{\text{spike}}(r) = \rho_{\text{sp}} \left(\frac{r_{\text{sp}}}{r}\right)^{\alpha}, \quad \alpha = \frac{9 - 2\gamma}{4 - \gamma}$$

 \succ Dynamical friction \rightarrow Gravitational wave

Extreme-mass-ratio Inspiral (EMRI)





Eda, Itoh, Kuroyanagi & Silk (13', 15'), Yue & Cao(19'), Yue & Han (18'), Yue, Han & Chen(19')

DM spikes

- > WIMP DM particles accretion around BH \rightarrow DM spike
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$$\rho(r) \propto r^{-\gamma}, 0 \leq \gamma \leq 2 \implies \rho_{\text{spike}}(r) = \rho_{\text{sp}}\left(\frac{r_{\text{sp}}}{r}\right)^{\alpha}, \quad \alpha = \frac{9-2\gamma}{4-\gamma}$$

➤ Dynamical friction → Gravitational wave Extreme-mass-ratio Inspiral (EMRI)

Li, Tang, Wu, arXiv:2112.14041



$$m \frac{d^2 \mathbf{x}}{dt^2} = \mathbf{F}_{\rm G} + \mathbf{F}_{\rm DF}$$
$$h_{ij} \sim \frac{G}{d} \frac{d^2 Q_{ij}}{dt^2},$$
$$Q_{ij} \sim m \left(x_i x_j - \frac{1}{3} x^2 \delta_{ij} \right)$$



Eda, Itoh, Kuroyanagi & Silk (13', 15'), Yue & Cao(19'), Yue & Han (18'), Yue, Han & Chen(19')

Density Distribution

Relativistic treatment (Will et al) = \searrow

20 25 30 35 40

$$\rho(r, \alpha = 1) = \frac{\kappa}{(r/GM)^{\omega}} \left(1 - \frac{4GM}{r}\right)^{\eta} \cdot \frac{\text{Halo parameters}}{(1 - \frac{4GM}{r})^{\eta}} \cdot \frac{(1 - \frac{4GM}{r})^{\eta}}{(1 - \frac{4GM}{r})^{\eta}} \cdot \frac{(1 - \frac{4GM}{r})^{\eta}} \cdot \frac{(1 - \frac{4GM}{r})^{\eta}}{(1 - \frac{4GM}{r})^{\eta}} \cdot \frac{(1 - \frac{4$$

v/v_{max}

r/GM

16

5

10

15

r/GM

ρ(r)/10¹⁸GeV·cm⁻³

Velocity Distribution

Fitted with Gaussian distribution

$$f_r(v) = rac{1}{\sqrt{2\pi}\sigma(r)} \exp\left(-rac{(v-ar v(r))^2}{2\sigma^2(r)}
ight),$$





Effects on EMRI

Phase shift of the waveform of GW



Ultralight DM - Signal Response

- DM couples to SM particles, inducing oscillations of test mass, effectively changing the length $A \xrightarrow{L \to L + \Delta L}$
- One-way Doppler shift

$$\delta t_{rs} = -\hat{n}_{rs} \cdot \left[\delta \vec{x}(t, \vec{x}_r) - \delta \vec{x}(t - L, \vec{x}_s)\right]$$

$$\frac{\delta\nu_{rs}}{\nu_0} = \frac{\nu_{rs} - \nu_0}{\nu_0} = -\frac{d\,\delta t_{rs}}{dt}$$

Fractional frequency change

$$y_{rs}(t) \equiv \frac{\delta \nu_{rs}}{\nu_0} = \mu_{rs} \left[h(t, \vec{x_r}) - h(t - L, \vec{x_s}) \right], \ h(t, \vec{x}) \propto e^{im(t - v\hat{k} \cdot \vec{x})}$$
$$\mu_{rs} = \begin{cases} \hat{k} \cdot \hat{n}_{rs} & \text{for scalar field,} \\ \hat{e}_A \cdot \hat{n}_{rs} & \text{for vector field,} \\ \frac{\hat{n}_{rs}^i \hat{n}_{rs}^j e_{ij}(\hat{k}, \psi)}{2(1 + \hat{n}_{rs} \cdot \hat{k})} & \text{for gravitational wave,} \end{cases}$$



Yu, Yao, Tang, Wu, arXiv: 2307.09197

Time-Delay Interferometry

- > The arm lengths are not equal
- Laser frequency noises dominate

$$\begin{aligned} X(t) &\equiv \left[\Delta y_{PD1}(t) - \Delta y_{PD2}(t) \right] - \left[\Delta y_{PD1}(t - T_2) - \Delta y_{PD2}(t - T_1) \right] \\ &= \left[H_1(t) - H_2(t) + p(t - T_1) - p(t - T_2) \right] \\ &- \left[H_1(t - T_2) - H_2(t - T_1) + p(t - T_1) - p(t - T_2) \right] \\ &= H_1(t) - H_2(t) - H_1(t - T_2) + H_2(t - T_1) \,, \end{aligned}$$

Michelson interferometry

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- $X(t) \equiv [\Delta y_{PD2} (t T_1) + \Delta y_{PD1}(t)]$ $-[\Delta y_{PD1} (t - T_2) + \Delta y_{PD2}(t)]$
- TDI-virtual equal-arm interference





 $\Delta y_{PD1}(t) = H_1(t) + p(t - T_1) - p(t),$ $\Delta y_{PD2}(t) = H_2(t) + p(t - T_2) - p(t),$

 $T_1 = 2L_1 \qquad T_2 = 2L_2$

Time-Delay Interferometry





➢ Input for TDI

$$\eta_{i'} \equiv s_{i'} + \frac{\varepsilon_{i'} - \tau_{i'}}{2} + D_{i+1'} \frac{\varepsilon_{i-1} - \tau_{i-1}}{2} + \frac{\tau_i - \tau_{i'}}{2}$$
$$\eta_i \equiv s_i + \frac{\varepsilon_i - \tau_i}{2} + D_{i-1} \frac{\varepsilon_{i+1'} - \tau_{i+1'}}{2} - D_{i-1} \frac{\tau_{i+1} - \tau_{i+1'}}{2}$$

$$\begin{split} \eta_{1'} &\sim D_{2'} p_3 - p_1, \, \eta_1 \sim D_3 p_2 - p_1, \\ \eta_{2'} &\sim D_{3'} p_1 - p_2, \, \eta_2 \sim D_1 p_3 - p_2, \\ \eta_{3'} &\sim D_{1'} p_2 - p_3, \, \eta_3 \sim D_2 p_1 - p_3. \end{split}$$

TDI cancels laser frequency noise

Time-Delay Interferometry

- There are multiple combinations
- Michelson channels
- $$\begin{split} X(t) &= (\eta_{2':\mathbf{322'}} + \eta_{1:\mathbf{22'}} + \eta_{3:\mathbf{2'}} + \eta_{1'}) (\eta_{3:\mathbf{2'3'3}} + \eta_{1':\mathbf{3'3}} + \eta_{2':\mathbf{3}} + \eta_{1}), \\ Y(t) &= (\eta_{3':\mathbf{133'}} + \eta_{2:\mathbf{33'}} + \eta_{1:\mathbf{3'}} + \eta_{2'}) (\eta_{1:\mathbf{3'1'1}} + \eta_{2':\mathbf{1'1}} + \eta_{3':\mathbf{1}} + \eta_{2}), \\ Z(t) &= (\eta_{1':\mathbf{211'}} + \eta_{3:\mathbf{11'}} + \eta_{2:\mathbf{1'}} + \eta_{3'}) (\eta_{2:\mathbf{1'2'2}} + \eta_{3':\mathbf{2'2}} + \eta_{1':\mathbf{2}} + \eta_{3}). \end{split}$$

Sagnac channels

$$\begin{aligned} \alpha(t) &= (\eta_{2':\mathbf{1'2'}} + \eta_{3':\mathbf{2'}} + \eta_{1'}) - (\eta_{3:\mathbf{13}} + \eta_{2:\mathbf{3}} + \eta_{1}), \\ \beta(t) &= (\eta_{3':\mathbf{2'3'}} + \eta_{1':\mathbf{3'}} + \eta_{2'}) - (\eta_{1:\mathbf{21}} + \eta_{3:\mathbf{1}} + \eta_{2}), \\ \gamma(t) &= (\eta_{1':\mathbf{3'1'}} + \eta_{2':\mathbf{1'}} + \eta_{3'}) - (\eta_{2:\mathbf{32}} + \eta_{1:\mathbf{2}} + \eta_{3}). \end{aligned}$$

$\succ \zeta$ channel

$$\zeta(t) = (\eta_{1':1'} + \eta_{2':2'} + \eta_{3':3'}) - (\eta_{1:1} + \eta_{2:2} + \eta_{3:3}).$$



Transfer Functions

- Fourier transform
- > One-way single link _y

$$h(t) = \frac{\sqrt{T}}{2\pi} \int_0^\infty \tilde{h}(\omega) e^{i\omega t} d\omega$$

$$y_{rs}(t) = \mu_{rs} \frac{\sqrt{T}}{2\pi} \int_0^\infty d\omega \ \tilde{h}(\omega) e^{i\omega t} \left[e^{-i\vec{k}\cdot\vec{x}_r} - e^{-i(\tau + \vec{k}\cdot\vec{x}_s)} \right],$$

$$\tilde{y}_{rs}(\omega) = \mu_{rs} \ \tilde{h}(\omega) \left[e^{-i(\vec{k}\cdot\vec{x}_r)} - e^{-i(\tau + \vec{k}\cdot\vec{x}_s)} \right].$$

Transfer function, sky and polarization averaged



Transfer Functions

- Different channels have different transfer functions
- > DM is also different from gravitational wave, velocity effect, ...



Sensitivity

$$be fined by S_{O}(f) = \frac{N_{O}(f)}{R_{O}(f)}, \quad N_{X} = 16 \sin^{2}(\tau) \left\{ [3 + \cos(2\tau)] S_{acc} + S_{oms} \right\}, \quad \tau = 2\pi f L \\ S_{oms}(f) = \left(s_{oms} \frac{2\pi f}{c} \right)^{2} \left[1 + \left(\frac{2 \times 10^{-3} \text{ Hz}}{f} \right)^{4} \right] \frac{1}{\text{Hz}}, \qquad \text{LISA: } s_{oms} = 15 \times 10^{-12} \text{ m}, s_{acc} = 3 \times 10^{-15} \text{ m/s}^{2}, \\ S_{acc}(f) = \left(\frac{s_{acc}}{2\pi fc} \right)^{2} \left[1 + \left(\frac{0.4 \times 10^{-3} \text{ Hz}}{f} \right)^{2} \right] \left[1 + \left(\frac{f}{8 \times 10^{-3} \text{ Hz}} \right)^{4} \right] \frac{1}{\text{Hz}}, \qquad \text{TianQin: } s_{oms} = 1 \times 10^{-12} \text{ m}, s_{acc} = 1 \times 10^{-15} \text{ m/s}^{2}.$$



Sensitivity



Sensitivity on scalar DM

$$\succ \text{ Strong sector } \delta \mathcal{L} = \frac{\phi}{M_P} \left[-\frac{d_g \beta_3}{2g_3} F^A_{\mu\nu} F^{A\mu\nu} - \sum_{i=u,d} \left(d_{m_i} + \gamma_{m_i} d_g \right) m_i \bar{\psi}_i \psi_i \right] \text{ Damour \& Donoghue}$$



Sensitivity on vector DM

 \succ For example, vector fields couple to baryon number B, or B-L, effectively neutron number. Sensitivity on ratio $\epsilon_D = e_D/e$

Statistical Effects

Velocity distributions, likelihood analysis

Correlation

Network of detectors

$$\gamma_{ab}(f,v) := \int d\hat{k} \ \chi_{ab}(f,\hat{k},v), \ \ \lambda_{\min}^2 = \frac{N}{2\Gamma_X S_\Phi} \left(\frac{\ln lpha}{\ln \gamma} - 1\right)$$

- Correlation length is larger
- Sensitivity is enhanced by

$$\sqrt{n(n-1)/2}$$

- > Different from SGWB searches $\stackrel{>}{\gtrsim}$ which has \sqrt{T} enhancement.
- ➤ How to distinguish.

Yao, Tang, arXiv: 2404.01494

Detecting Ultralight Dark Matter Gravitationally

- > Metric perturbation in solar system $ds^{2} = -(1+2\Psi)dt^{2} + (1-2\Phi)\delta_{ij}dx^{i}dx^{j} + h_{ij}dx^{i}dx^{j}$
- $$\begin{split} \succ & \mathsf{Einstein equations} \\ \partial_i \partial^i \Phi = 4\pi G T_{00}, \\ & 3\ddot{\Phi} + \partial_i \partial^i (\Psi \Phi) = 4\pi G T_k^k, \\ & \ddot{h}_{ij} = 16\pi G \left(T_{ij} \frac{1}{3} \delta_{ij} T_k^k \right), \\ & \Psi^j \simeq \Phi^j \simeq \pi G \frac{\rho}{m^2} = \frac{7 \times 10^{-26} \rho}{0.4 \, \mathrm{GeV/cm}^3} \left(\frac{10^{-18} \mathrm{eV}}{m} \right)^2, \\ & h_{ij}^v \propto h_0 \simeq \frac{8}{3} \pi G \frac{\rho}{m^2} = \frac{2 \times 10^{-25} \rho}{0.4 \, \mathrm{GeV/cm}^3} \left(\frac{10^{-18} \mathrm{eV}}{m} \right)^2, \\ & h_{ij}^s \simeq h_0 v^2 / 2 \end{split}$$
- Tensor perturbation for scalar and tensor ULDM is suppressed.

Detecting Ultralight Dark Matter Gravitationally

> Metric perturbation in solar system $ds^{2} = -(1 + 2\Psi)dt^{2} + (1 - 2\Phi)\delta_{ij}dx^{i}dx^{j} + h_{ij}dx^{i}dx^{j}$

Detecting Ultralight Dark Matter Gravitationally

Metric perturbation in solar system

 $ds^2 = -(1+2\Psi)dt^2 + (1-2\Phi)\delta_{ij}dx^i dx^j + h_{ij}dx^i dx^j$

Different channels have different sensitivities

Summary

- We discuss how gravitational-wave detectors in space may help to understand the nature of dark matter
- > Weakly interacting massive particles
 - can form spikes around black holes and affect the orbiting compact objects, imprinting on the waveform of GW
 - \succ Relativistic effect, velocity distribution, ...
- Ultralight bosonic, wave like
 - can couple to normal matter and induce the oscillation of test masses, leading to signals in detectors and very good sensitivities.
 - > Metric perturbation by vector ULDM may be detectable in future.

