Bootstrapping the Chiral Anomaly at Large N_c

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Based on:

Arxiv: JHEP 11 (2023) 176

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Bootstrapping EFT

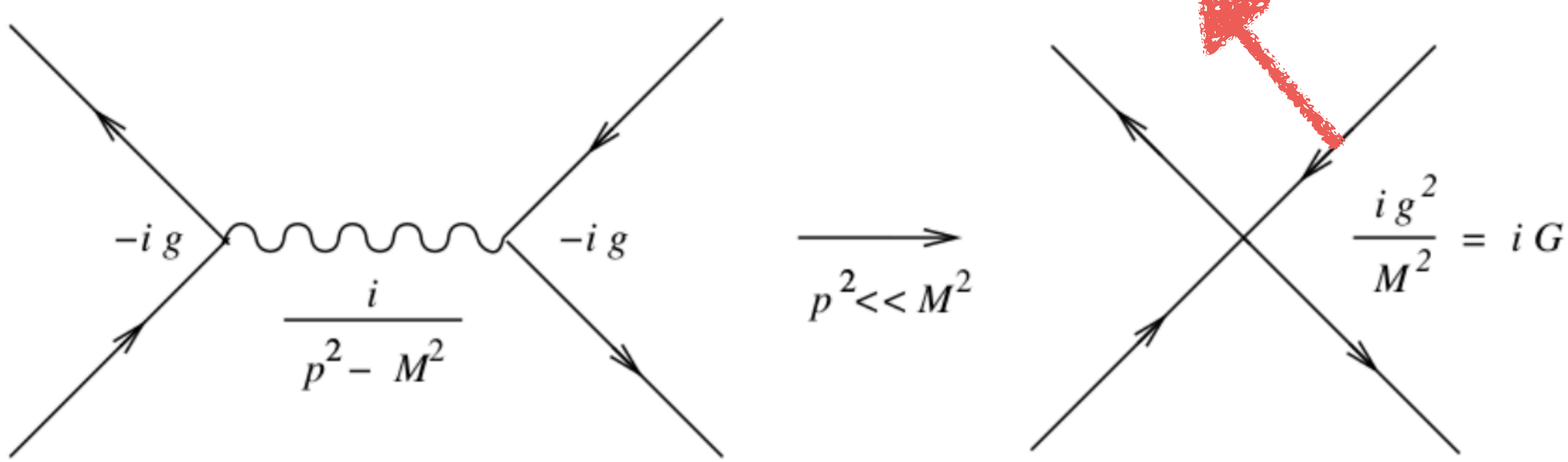
Bootstrapping EFT

- Effective field theory

Wilson coefficients

$$\mathscr{L}_{EFT} = \mathscr{L}_{renormalizable} + \sum \frac{c_i}{\Lambda^{d-4}} \mathcal{O}_i^d$$

EFT bases



UV Scale

IR Scale

- The Wilson coefficients can choose any value if UV physics is unknown

$$\frac{c_i}{\Lambda^{d-4}} \quad ?$$

Bootstrapping EFT

- The EFT is bounded if UV amplitudes follows consistency conditions

(1). Causality → Amplitude is analytical

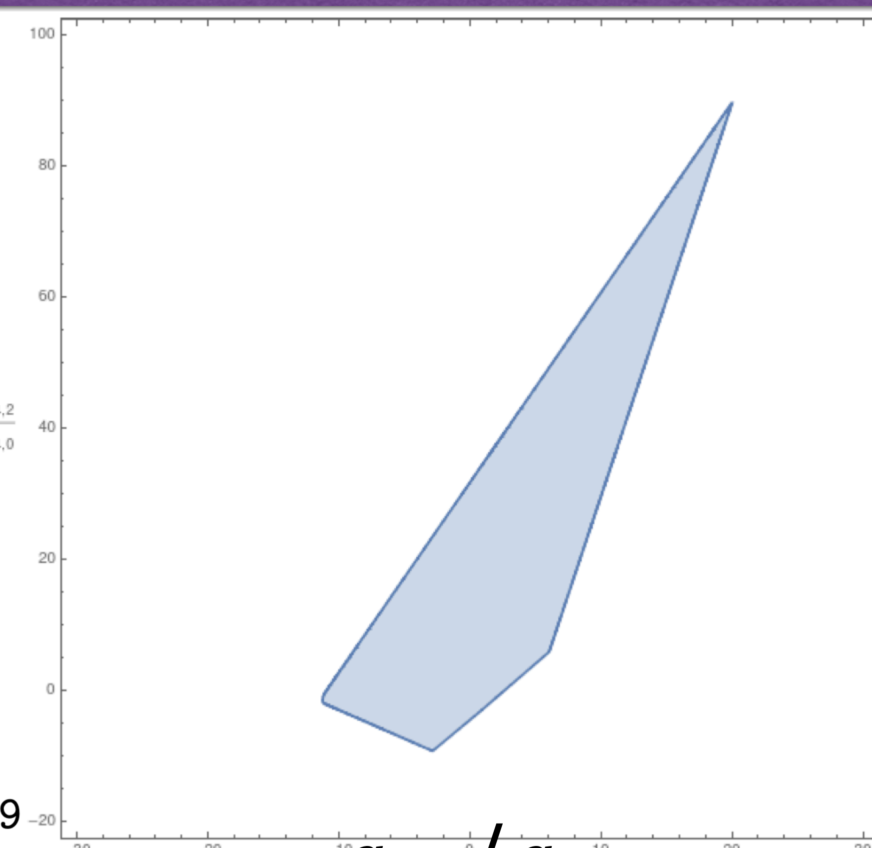
(2). Unitarity → Elastic amplitude imaginary part is positive

(3). Locality → Analyticity in t, the high energy bound of amplitudes $< s^2$

- Such as four photons EFT

$$\langle 12\rangle^{2h}[34]^{2h}(a_{4,0}s^4+a_{4,1}s^3t+a_{4,2}s^2t^2\cdots)$$

$$\frac{a_{4,1}}{a_{4,0}}$$



$a_{4,2}/a_{4,0}$

N. Arkani-Hamed, T.-C. Huang, and Y.-T. Huang, JHEP **05**, 259 (2021), arXiv: 2012.15849

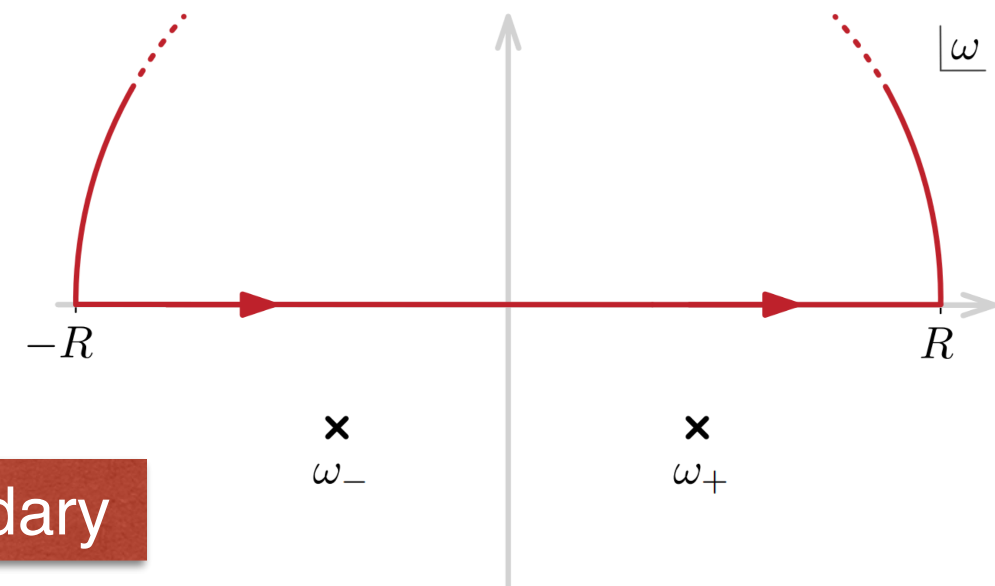
Causality vs Analyticity

- The electron motion under the electric field $E(x, t)$

$$\mathbf{x}(t) = \int_{-\infty}^{\infty} G(t-t')\mathbf{E}(t')\,\mathrm{d}t' \qquad \text{with} \qquad G(t-t') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{G}(\omega)\,\mathrm{e}^{-i\omega(t-t')}\mathrm{d}\omega$$

- Evaluation $G(t-t')$

For $t < t'$ $\quad \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{G}(\omega)\,\mathrm{e}^{-i\omega(t-t')}\mathrm{d}\omega$ →



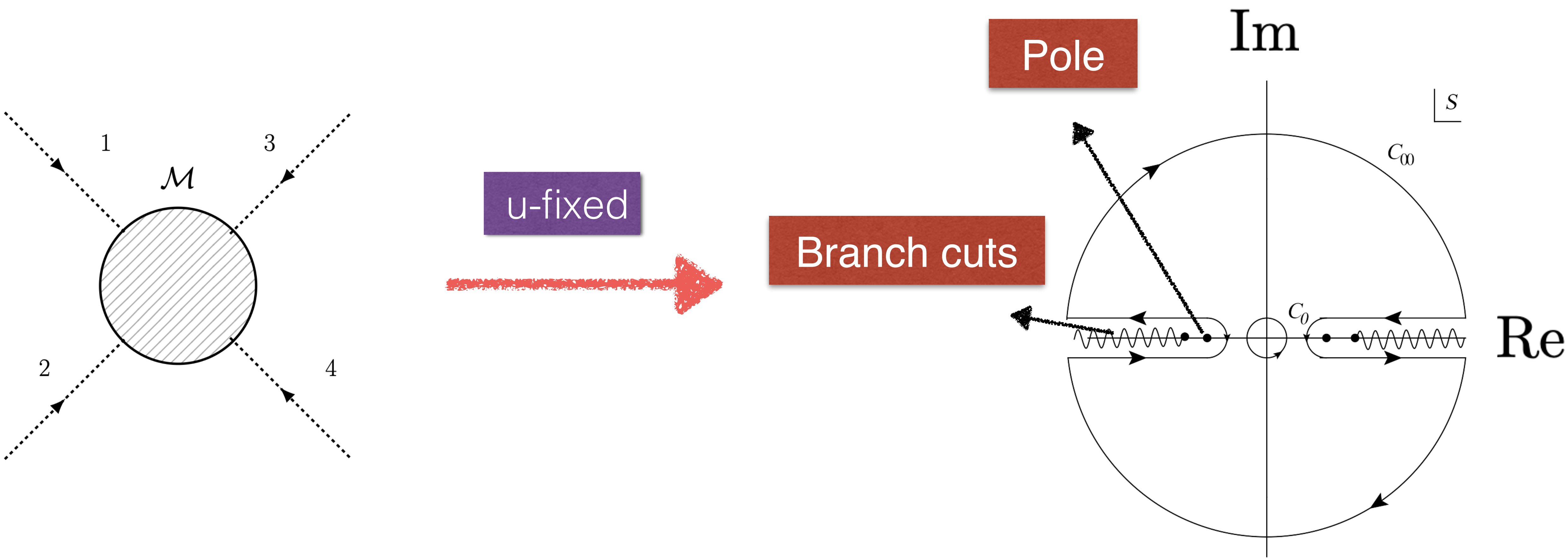
Vanish at the circle boundary

- Causality requires $G(\omega)$ is analytical in the upper half plane of ω

For $t < t'$ $\quad G(t-t') = 0$ → $G(\omega)$ no pole for $\mathrm{Im}[\omega] > 0$]

Causality vs Analyticity

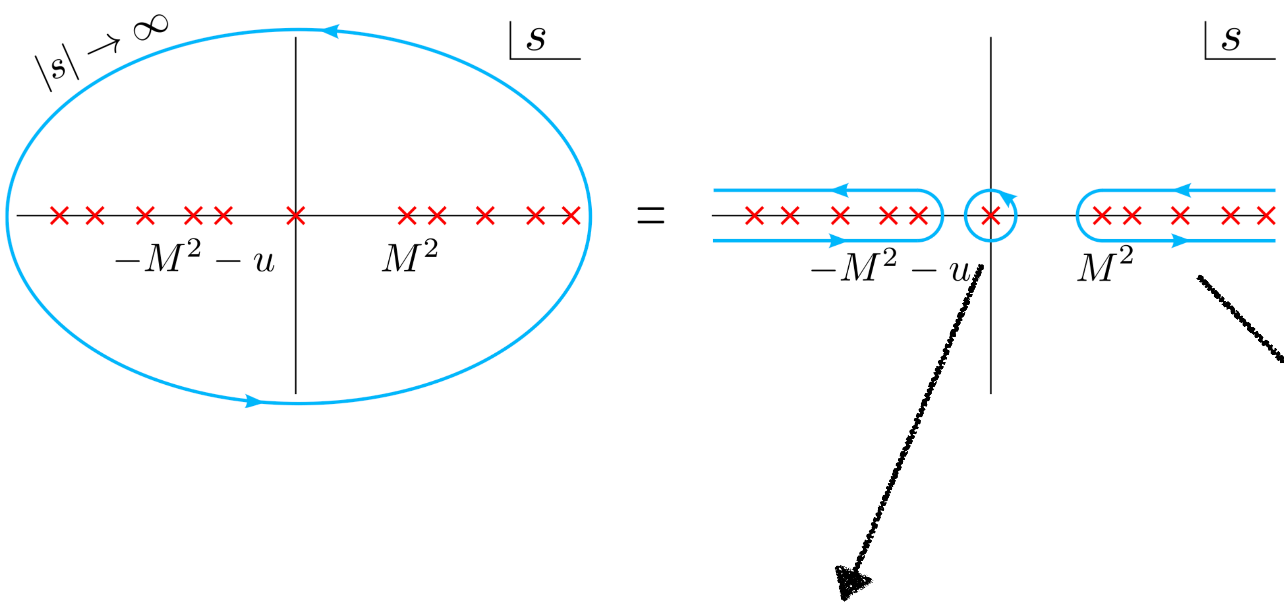
- Causality: four-point amplitude is analytic in whole s plane except its real axis



Dispersion Relation

- Analytic property allows to equate EFT amplitude to UV singularities

u-fixed



$$\frac{1}{2\pi i}\oint_{\infty} ds' \frac{M(s',u)}{s'^{k+1}} = 0 \longrightarrow \frac{1}{2\pi i}\oint_{0} ds' \frac{M_{\text{low}}(s',u)}{s'^{k+1}} = \frac{1}{\pi}\int_{M^2}^{\infty} ds' \frac{\text{Im}\, M_{\text{high}}(s',u)}{s'^{k+1}} + t - channel$$

IR Scale — UV Scale

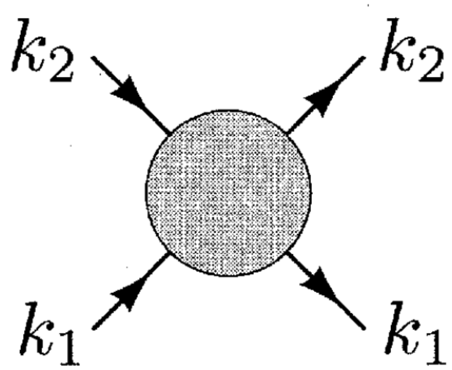
- Dispersion relation is the tool to constrain EFT

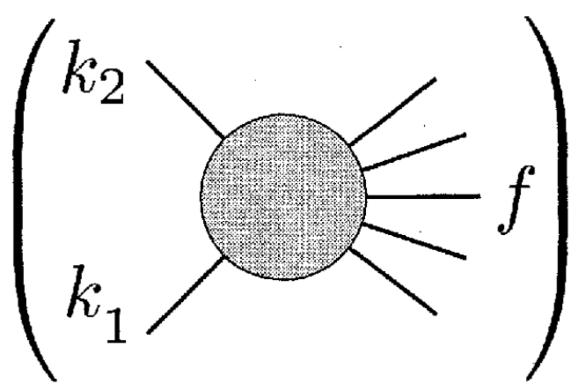
Dispersion Relation

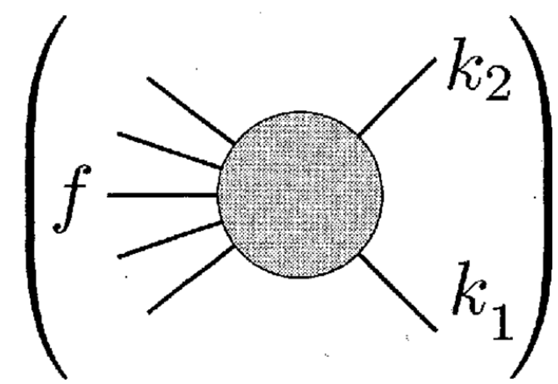
- Dispersion relation relates the EFT with UV singularities

u-fixed
$$\frac{1}{2\pi i}\oint_0 ds' \frac{M_{\text{low}}(s',u)}{s'^{k+1}} = \frac{1}{\pi}\int_{M^2}^{\infty} ds' \frac{\text{Im}\, M_{\text{high}}(s',u)}{s'^{k+1}} \quad + t - channel$$

- Unitarity: discontinuity is positive for elastic amplitudes

2Im 

$= \sum_f \int d\Pi_f$ 

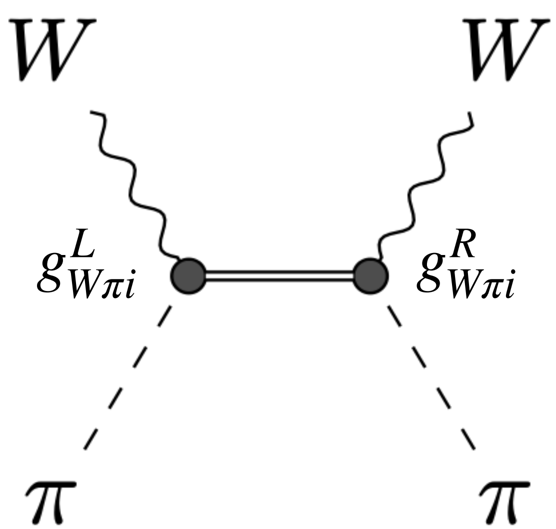


$$-i(T - T^\dagger) \qquad = \qquad T^\dagger T$$

Optical theorem
$$\text{Im}\, M_{\text{high}}(s',u) = 2E_{\text{cm}} p_{\text{cm}} \sigma_{\text{tot}}(k_1, k_2 \to \text{anything})$$

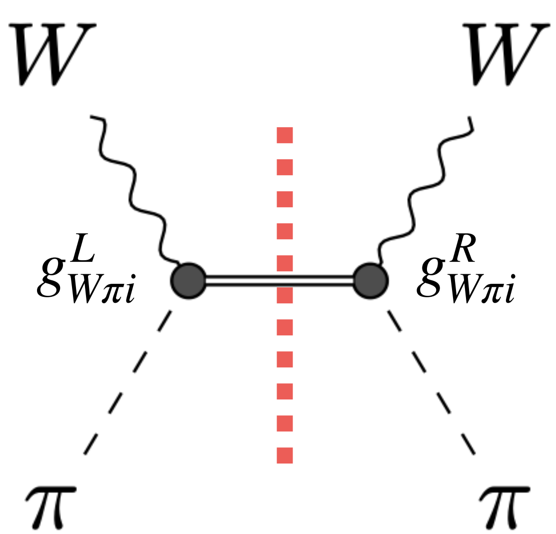
Dispersion Relation

- Unitarity at tree level



$$M(s,t) = \frac{\mathrm{res}[M(s=m_i^2)]}{s-m_i^2+i\epsilon} + t\ \&\ u - channel$$

- Amplitude imaginary at tree level



$$\mathrm{Im} M = -\pi\delta(s-m_i^2)\mathrm{res}[M(s=m_i^2)]$$

$$\sim g^L_{W\pi i}(g^R_{W\pi i})^*$$

Is determined by 3-point couplings

Application of Dispersion Relation

- Constrain generic EFT

A. Adams, N. Arkani-Hamed, S. Dubovsky, A. Nicolis and R. Rattazzi, *Causality, analyticity and an IR obstruction to UV completion*, *JHEP* **10** (2006) 014, [hep-th/0602178].

N. Arkani-Hamed, T.-C. Huang and Y.-T. Huang, *The EFT-Hedron*, 2012.15849.

X. Li, H. Xu, C. Yang, C. Zhang and S.-Y. Zhou, *Positivity in Multifield Effective Field Theories*, *Phys. Rev. Lett.* **127** (2021) 121601, [2101.01191].

- Gravity EFT

S. Caron-Huot, D. Mazac, L. Rastelli and D. Simmons-Duffin, *Sharp Boundaries for the Swampland*, *JHEP* **07** (2021) 110, [2102.08951].

B. Bellazzini, M. Lewandowski, and J. Serra, "Positivity of Amplitudes, Weak Gravity Conjecture, and Modified Gravity," *Phys. Rev. Lett.* **123** no. 25, (2019) 251103,

- Constrain QCD EFT (focus on π)

J. Albert and L. Rastelli, JHEP **08**, 151 (2022), arXiv: 2203.11950.

C. Fernandez, A. Pomarol, F. Riva, and F. Sciotti, JHEP **06**, 094 (2023), arXiv: 2211.12488.

Understanding Vector Meson Dominance and holographic QCD

QCD in large N_c

QCD at large N_c

- Understanding strong dynamics is still an open problem, such QCD

Chiral Lagrangian

$$\mathcal{L}_{\rm Ch} = -\frac{f_\pi^2}{4}\,\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right) + L_1'\left[\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)\right]^2$$

? ?

No connection with UV

- The anomaly is bridge to connect the IR physics with the UV physics

Chiral anomaly

$$A = \frac{ne^2}{48\pi^2 F_\pi}\pi^0 \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

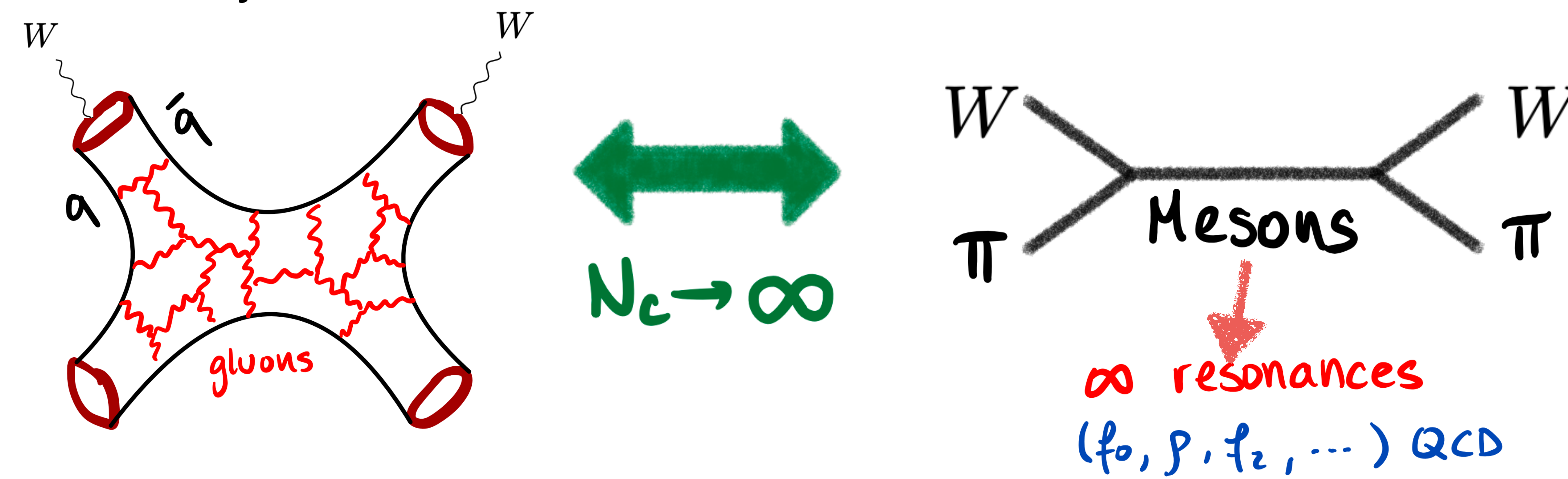
$$B = -\tfrac{2}{3} ie \frac{n}{\pi^2 F_\pi^3} \varepsilon^{\mu\nu\alpha\beta} A_\mu\, \partial_\nu \pi^+\, \partial_\alpha \pi^-\, \partial_\beta \pi^0$$

Determined by UV

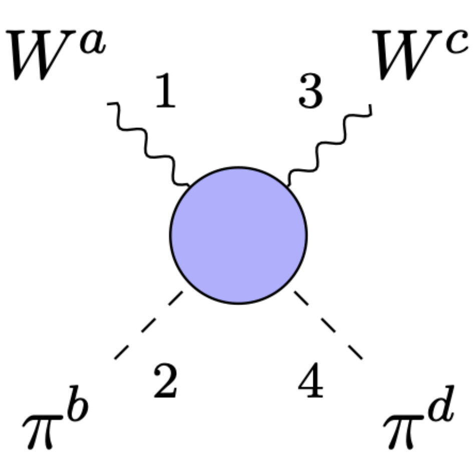
- Bound the anomaly can provide non-trivial connection between UV and IR and consistency conditions for phenomenological models for QCD

QCD at large N_c

- For large N_c, the strong $SU(N_c)$ gauge theory has a weakly coupled dual theory with infinite mesons

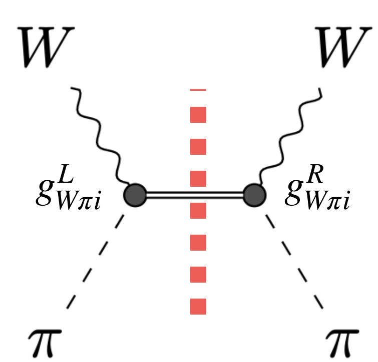


- For large N_c, the EFT can well bounded by the tree level amplitudes of weakly coupled theory



$$\frac{1}{2\pi i}\oint_0 ds' \frac{M_{\text{low}}(s',u)}{s'^{k+1}} = \frac{1}{\pi}\int_{M^2}^{\infty} ds' \frac{\text{Im}\, M_{\text{high}}(s',u)}{s'^{k+1}}$$

$$c^k_{EFT} \sim \sum_{i=1}^{\infty}$$



$$\sim \sum_i^{\infty} g^L_{W\pi i}(g^R_{W\pi i})^*/M^{2k}$$

QCD at large N_c

- UV set up: $N_f = 2$ massless quarks in the fundamental Rep of $SU(N_c)$

Chiral symmetry breaking pattern

$$U(N_f)_L \times U(N_f)_R \rightarrow SU(N_f) \times U(1)$$

- Generate a iso-spin triplet π^a and η singlet under $SU(N_f = 2)$

$$U = \mathrm{Exp}(i\eta/F_\pi)\mathrm{Exp}(2i\pi^a\tau^a/F_\pi)$$

- Gauge the unbroken global $SU(N_f = 2)$: W^a

Anomaly WZW

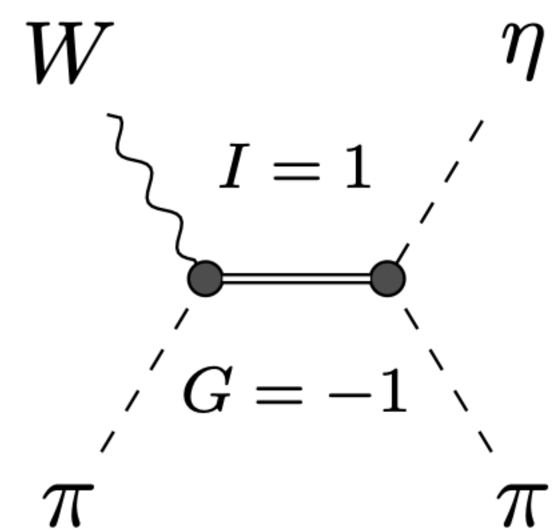
$$\kappa \; \varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left[A_{\mu L}U_{\nu L}U_{\alpha L}U_{\beta L} + \mathrm{L} \rightarrow \mathrm{R}\right]$$

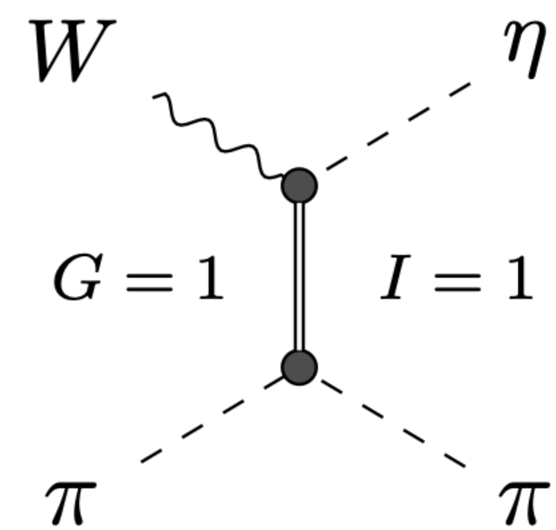
$$\rightarrow \quad \mathcal{M}(W^{a+}, \pi^b, \eta, \pi^c) \propto [12]\langle 24\rangle[41] \propto \sqrt{stu}$$

QCD at large N_c

- UV scattering amplitude associated with chiral anomaly

$$\mathcal{M}(W^{a+}, \pi^b, \eta, \pi^c) =$$





$$\mathcal{J}^2 = J(J+1)$$

dispersion relation of Anomaly

$$\frac{\kappa}{2\sqrt{2}} = \sum_i^{J^-_{\rm odd}} \frac{g_{W\eta i}\, g_{\pi\pi i}\, \mathcal{J}}{m_i^3} - \sum_i^{J^+_{\rm even}} \frac{g_{W\pi i}\, g_{\pi\eta i}\, \mathcal{J}}{m_i^3}$$

- $U(2)$ symmetry relates coupling $g_{W\eta i}$ with $g_{W\pi i}$ ($g_{\pi\eta i}$ with $g_{\pi\pi i}$)

- To bound anomaly, should constrain $g_{W\pi i}$ ($g_{\pi\pi i}$) and spectrum through $\mathscr{M}(W\pi W\pi)$ ($\mathscr{M}(\pi\pi\pi\pi)$)

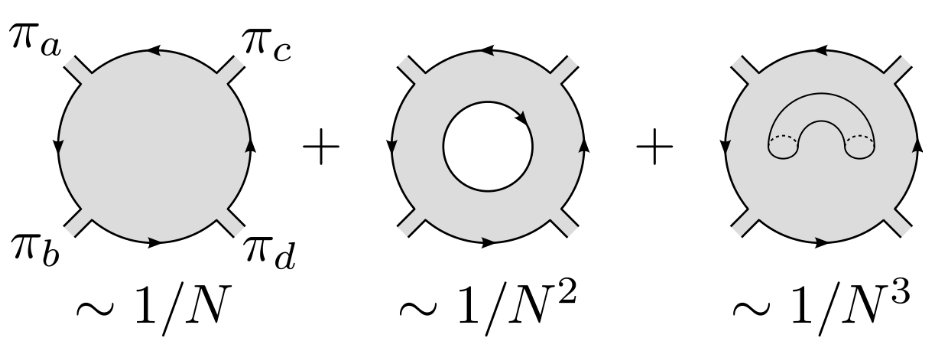
$$W\pi \rightarrow W\pi$$

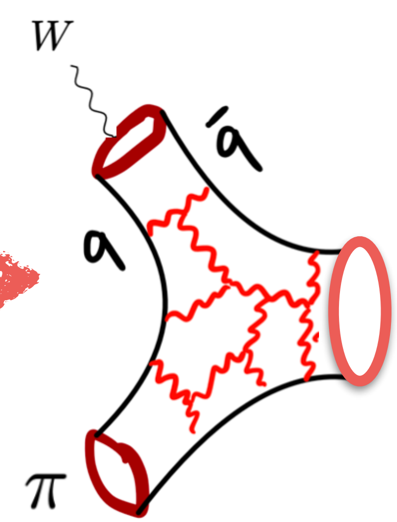
$W\pi \to W\pi$

- $W\pi \to W\pi$: the quantum number of the $W^a\pi^b$ state under Isospin $SU(N_f = 2)$

$$I = 1 \otimes 1 = 0 + 1 + 2$$

- For large N_c, the mesons are only the bound state of two quark doublets, no mesons in $I = 2$ Rep

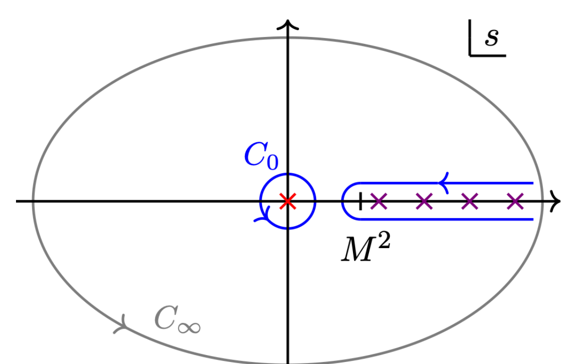




$$\langle q\bar{q}\rangle : \frac{1}{2} \otimes \frac{1}{2} = 0 + 1$$

- The amplitude $M_t^{I=2}(s,u)$ with $I = 2$ in t-channel do not have t-channel poles.

Pole structure for u-fixed



Impose most strong constraints on the EFT and mesons

$W\pi \to W\pi$

- There are six types of mesons can couple to $W\pi$

	Isospin $I=0$	Isospin $I=1$		n
$G=+1$	$J^+_{\rm odd}\ (f_J)$	$J^+_{\rm odd}\ (a_J)$	$G=-1$	1
	$J^+_{\rm even}\ (f_J)$	$J^+_{\rm even}\ (a_J)$		2
	$J^-_{\rm even}\ (\eta_J)$	$J^-_{\rm even}\ (\pi_J)$		3
$G=-1$	$J^+_{\rm odd}\ (h_J)$	$J^+_{\rm odd}\ (b_J)$	$G=+1$	4
	$J^-_{\rm odd}\ (\omega_J)$	$J^-_{\rm odd}\ (\rho_J)$		5
	$J^-_{\rm even}\ (\omega_J)$	$J^-_{\rm even}\ (\rho_J)$		6

Only meson-2,5 relevant to anomaly

$q\bar{q}$ states classified in terms of their Isospin, G−parity and J^P

- The sign of the $\mathscr{M}(W^{\lambda_1}\pi W^{\lambda_2}\pi)$ residues from mesons depends on the helicity λ of W and CP property of mesons

Unitarity

Elastic (λ_1, λ_2) = (+ -)

$$R^{+-}_{I=0} = +g_4^2 + g_5^2 + g_6^2 \qquad R^{+-}_{I=1} = +g_1^2 + g_2^2 + g_3^2$$

P and C unbroken

In-elastic (λ_1, λ_2) = (+ +)

$$R^{++}_{I=0} = -g_4^2 + g_5^2 - g_6^2 \qquad R^{++}_{I=1} = -g_1^2 + g_2^2 - g_3^2$$

$W\pi \to W\pi$

- The dispersion relations of $\mathscr{M}(W^{\lambda_1}\pi W^{\lambda_2}\pi)$ tell us (plus other conditions)

$$\left\langle \frac{\mathcal{J}^2}{m^4} \right\rangle_2 + \left\langle \frac{\mathcal{J}^2}{m^4} \right\rangle_5 = 3f_{1,0} - 2g_{0,0} \equiv \mathcal{P} \geq 0$$

$\langle \frac{\mathcal{J}^2}{m^4} \rangle_2 \sim \frac{g_2^2 J(J+1)}{m^4} > 0$ is the spectrum integration of meson-2

	Isospin $I=0$	Isospin $I=1$		n
$G=+1$	$J^+_{\text{odd}}\ (f_J)$	$J^+_{\text{odd}}\ (a_J)$	$G=-1$	1
	$J^+_{\text{even}}\ (f_J)$	$J^+_{\text{even}}\ (a_J)$		2
	$J^-_{\text{even}}\ (\eta_J)$	$J^-_{\text{even}}\ (\pi_J)$		3
$G=-1$	$J^+_{\text{odd}}\ (h_J)$	$J^+_{\text{odd}}\ (b_J)$	$G=+1$	4
	$J^-_{\text{odd}}\ (\omega_J)$	$J^-_{\text{odd}}\ (\rho_J)$		5
	$J^-_{\text{even}}\ (\omega_J)$	$J^-_{\text{even}}\ (\rho_J)$		6

Only meson-2,5 relevant to anomaly

$f_{1,0}, g_{0,0}$ are the d-8 Wilson coefficients relevant to polarizabilities of the pions

Anomaly $W\pi \to W\eta$

Anomaly $W\pi \to W\eta$

- $U(N_f = 2)$ symmetry can relate the anomaly with meson-2, 5 interactions

$$\frac{\kappa}{4\sqrt{2}} = \sum_i^{J^+_{\text{even}}} \frac{(g_2 g_{\pi\pi})_i \mathcal{J}}{m_i^3} = \sum_i^{J^-_{\text{odd}}} \frac{(g_5 g_{\pi\pi})_i \mathcal{J}}{3m_i^3}$$

- Non-trivial informations

Anomaly requires that QCD should contain at least one vector and one spin-two resonance

Holographic QCD describing anomaly only contain vector is not consistent

spin two particles heavier than spin one

Anomaly $W\pi \to W\eta$

- $U(N_f = 2)$ symmetry can relate the anomaly with meson-2, 5 interactions

$$\frac{\kappa}{4\sqrt{2}} = \sum_i^{J^+_{\text{even}}} \frac{(g_2 g_{\pi\pi})_i \mathcal{J}}{m_i^3} = \sum_i^{J^-_{\text{odd}}} \frac{(g_5 g_{\pi\pi})_i \mathcal{J}}{3m_i^3}$$

- Using Cauchy identities, anomaly can be bounded as

$$\frac{\kappa}{4\sqrt{2}} \leq \sqrt{\left(\sum_i^{J^+_{\text{even}}} \frac{g_{\pi\pi i}^2}{m_i^2}\right) \left\langle \frac{\mathcal{J}^2}{m^4} \right\rangle_2}$$

?

$$\frac{\kappa}{4\sqrt{2}} \leq \frac{1}{3}\sqrt{\left(\sum_i^{J^-_{\text{odd}}} \frac{g_{\pi\pi i}^2}{m_i^2}\right) \left\langle \frac{\mathcal{J}^2}{m^4} \right\rangle_5}$$

Bounded by $f_{1,0}, g_{0,0}$

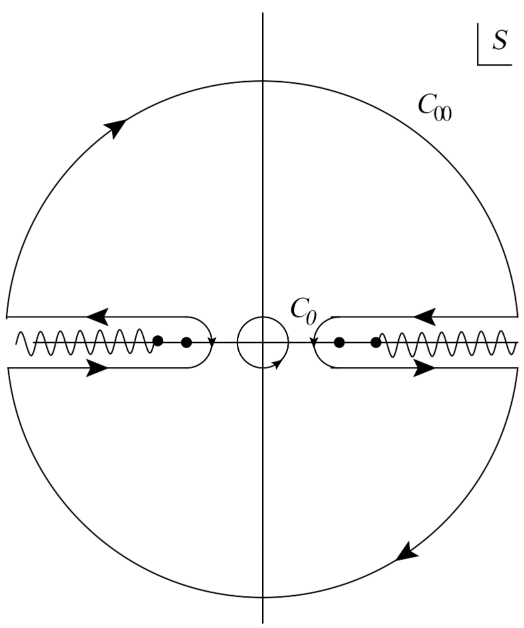
$\pi\pi \to \pi\pi$

- $\pi\pi \to \pi\pi$: the couplings of π-pair to the mesons $g_{\pi\pi i}$ are bounded by Goldstone decay constant

$$U = \mathrm{Exp}(i\eta/F_\pi)\mathrm{Exp}(2i\pi^a\tau^a/F_\pi)$$

The leading chiral operator

$$\mathcal{L}_{\mathrm{Ch}} = -\frac{f_\pi^2}{4}\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)$$



$$F_\pi^{-2} = 2\left(\sum_i^{J^+_{\mathrm{even}}} \frac{g^2_{\pi\pi i}}{m_i^2} + \sum_i^{J^-_{\mathrm{odd}}} \frac{g^2_{\pi\pi i}}{m_i^2}\right)$$

J. Albert and L. Rastelli, JHEP **08**, 151 (2022), arXiv: 2203.11950

C. Fernandez, A. Pomarol, F. Riva, and F. Sciotti, JHEP **06**, 094 (2023), arXiv: 2211.12488

Chiral Anomaly Upper bound

- Anomaly can be bounded as

$$\frac{\kappa}{4\sqrt{2}} \leq \sqrt{\left(\sum_i^{J^+_{\text{even}}} \frac{g^2_{\pi\pi i}}{m_i^2}\right)\left\langle \frac{\mathcal{J}^2}{m^4} \right\rangle_2}$$

Bounded by $f_{1,0}, g_{0,0}$

$$\frac{\kappa}{4\sqrt{2}} \leq \frac{1}{3}\sqrt{\left(\sum_i^{J^-_{\text{odd}}} \frac{g^2_{\pi\pi i}}{m_i^2}\right)\left\langle \frac{\mathcal{J}^2}{m^4} \right\rangle_5}$$

Bounded by F_π

- Optimizing this bound, the upper bound of anomaly is

$$\frac{\kappa}{\sqrt{\mathcal{P}/F_\pi^2}} \leq \frac{1}{\sqrt{2}}$$

$$3f_{1,0} - 2g_{0,0} \equiv \mathcal{P}$$

Polarizabilities of the pions

Summary

- Goldstone anomaly is determined by UV
- Bootstrap anomaly can provide non-trivial connection UV and IR

$$\frac{\kappa}{\sqrt{\mathcal{P}/F_\pi^2}} \leq \frac{1}{\sqrt{2}}$$

- Anomaly require the QCD should contain both vector and high spin resonance
- Phenomenological models for QCD should follow above conditions
- Graviton anomaly?

Thanks !!