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The transverse spin dependent asymmetry in EicC and EIC

——phenomenology prediction

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Collaborated with Shi-Chen Xue, De-Min Li (Zhengzhou U.), Yongliang Yang (Qingdao U.) , Hui Li, and Zhun Lu (Southeast U.)
Based on Y. Yang, **X. Wang** and Z. Lu, Phys. Rev. D 103, 114011 (2021)

OUTLINE

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$\sin\phi_S$ asymmetry

Beam spin asymmetry?

Transversity distribution function

Transversity distribution function

- Fundamental distribution to encode the nucleon structure
- Chiral-odd
- Hard to access compared to helicity and unpolarized distribution function

How to access transversity

- (TMD) factorization frame in SIDIS ——Collins function
  J. C. Collins, NPB 396, 161 (1993)
- Collinear factorization in SIDIS——twist-3 fragmentation function
  X.Wang, Z. Lu PRD 93, 074009 (2016)
- Collinear factorization in SIDIS——dihadron fragmentation function
  A. Bacchetta et al., PRL107, 012001(2011)
- Drell-Yan process——the antiquark transversity
  V. Barone et al., Phys.Rept. 359 (2002) 1-168

Why twist-3 fragmentation?

- An alternative approach to access transversity in SIDIS——the twist-3 chiral-odd fragmentation function $\widetilde{H}(z)$ serves as a "spin analyzer"
- The advantage of this approach is that the transverse momentum of the final state hadron is not necessarily to be measured.
- The final-state hadron Lambda makes the flavor dependence fragmentation funtion as well as the sea quark contribution to transversity possible.

Xiaoyu Wang and Zhun Lu, PRD 93.074009(2016)
Y. Yang, X. Wang and Z. Lu, Phys. Rev. D 103, 114011 (2021)

What have we done?

sinϕ_S asymmetry

- twist-3 T-odd quark-gluon-quark fragmentation function of Λ
- include the leading-order evolution effects for the fragmentation functions and the distribution functions
- estimate the SSA in SIDIS process at the kinematics of EIC, EicC and COMPASS.

Reference frame and invariants

- SIDIS with unpolarized electron beam and transversely polarized target with a Lambda hyperon produced in the final state

$$l(\ell)+N^{\uparrow}(P)\longrightarrow l(\ell')+\Lambda(P_\Lambda)+X(P_X),$$

$$\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2}=\\
&\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h}\\
&\quad+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\\
&\quad+S_\parallel\Big[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\Big]\\
&\quad+S_\parallel\lambda_e\Big[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\Big]\\
&\quad+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\\
&\qquad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}\\
&\qquad+\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]\\
&\quad+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\\
&\qquad+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\},
\end{aligned}\tag{3.4}$$

- After integrating over P_{hT}, only several terms survive

$$F_{UU,T}=x\sum_a e_a^2\,f_1^a(x)\,D_1^a(z),$$

$$F_{UU,L}=0,$$

$$F_{LL}=x\sum_a e_a^2\,g_1^a(x)\,D_1^a(z),$$

$$F_{UT}^{\sin\phi_S}=-x\sum_a e_a^2\,\frac{2M_h}{Q}\,h_1^a(x)\,\frac{\tilde{H}^a(z)}{z},$$

$$F_{LT}^{\cos\phi_S}=-x\sum_a e_a^2\,\frac{2M}{Q}\left(xg_T^a(x)D_1^a(z)+\frac{M_h}{M}\,h_1^a(x)\,\frac{\tilde{E}^a(z)}{z}\right).$$

Definition of the asymmetry

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- For the single transversely polarized SIDIS process, the differential cross section becomes

$$\frac{d^4\sigma}{dx\,dy\,dz\,d\phi_S} = \frac{2\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)$$
$$\times\left\{F_{UU,L}(x,z) + |S_\perp|\sqrt{2\varepsilon(1+\varepsilon)}\sin\phi_S\, F_{UT}^{\sin\phi_S}(x,z) + \cdots\right\}.$$

$$F_{UU}(x,z) = x\sum_q e_q^2 f_1^q(x) D_1^q(z)\,,$$

$$F_{UT}^{\sin\phi_S}(x,z) = -x\sum_q e_q^2 \frac{2M_\Lambda}{Q} \boxed{h_1^q(x)} \frac{\tilde{H}^q(z)}{z}\,,$$

Twist-3 fragmentation function

Transversity

Definition of the asymmetry

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- The x-dependent and z-dependent asymmetry are defined as

$$A_{UT}^{\sin\phi_S}(x) = \frac{\int dy \int dz \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} (1+\frac{\gamma^2}{2x}) \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin\phi_s}(x,z)}{\int dy \int dz \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} (1+\frac{\gamma^2}{2x}) F_{UU}(x,z)},$$

$$A_{UT}^{\sin\phi_s}(z) = \frac{\int dx \int dy \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} (1+\frac{\gamma^2}{2x}) \sqrt{2\epsilon(1+\epsilon)} F_{UT}^{\sin\phi_s}(x,z)}{\int dx \int dy \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} (1+\frac{\gamma^2}{2x}) F_{UU}(x,z)}.$$

The solution for the Pol. PDFs

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- At the initial scale $Q^2 = 2.41\ \mathrm{GeV}^2$, we adopt the standard parametrization for the transversity

$$h_1^q(x) = \frac{1}{2}\mathcal{N}_q^T(x)[f_1^q(x) + g_1^q(x)],$$

$$\mathcal{N}_q^T(x) = N_q^T\, x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{\alpha+\beta}}{\alpha^\alpha \beta^\beta}.$$

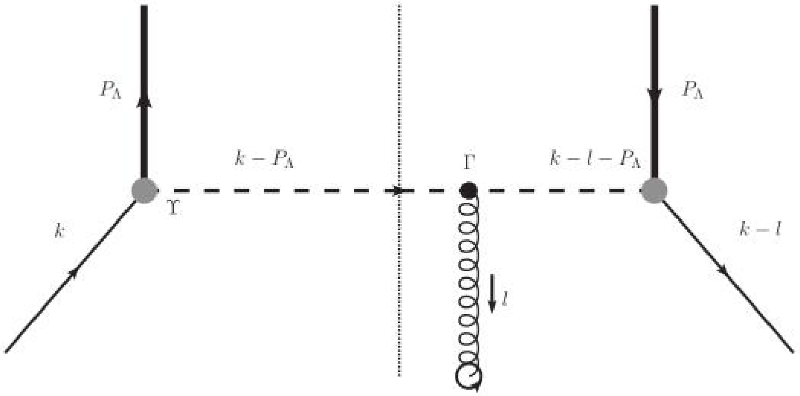
M. Anselmino *et al.* Phys. Rev. D 87, 094019 (2013)

- How to deal with the twist-3 fragmentation function for Lambda?

Diquark spectator model

- $\tilde{H}$ can be obtained from the Trace of the correlator as follows

$$\frac{z}{4M_\Lambda}\mathrm{Tr}[(\tilde{\Delta}_{A\alpha}(z,k_T;S_\Lambda)+\tilde{\Delta}_{A\alpha}(z,k_T;-S_\Lambda))\sigma^{\alpha-}]=\tilde{H}(z,k_T)+i\tilde{E}(z,k_T)$$



$$\tilde{\Delta}_A^\alpha(z,k_T;S_\Lambda)=\sum\!\!\!\!\!\!\!\int_X \frac{1}{2z}\int\frac{d\xi^+d^2\boldsymbol{\xi}_T}{(2\pi)^3}\int e^{ik\cdot\xi}\langle 0|\int_{\pm\infty^+}^{\xi^+}d\eta^+\mathcal{U}^{\boldsymbol{\xi}_T}_{(\infty^+,\eta^+)}$$
$$\times\, gF_\perp^{-\alpha}(\eta)\mathcal{U}^{\boldsymbol{\xi}_T}_{(\eta^+,\xi^+)}\psi(\xi)|P_\Lambda,S_\Lambda;X\rangle\langle P_\Lambda,S_\Lambda;X|\bar{\psi}(0)\mathcal{U}^{\mathbf{0}_T}_{(0^+,\infty^+)}\mathcal{U}^{\infty^+}_{(\mathbf{0}_T,\boldsymbol{\xi}_T)}|0\rangle\Big|_{\substack{\eta^+=\xi^+=0\\ \eta_T=\xi_T}}\,.$$

Diquark spectator model

- The diquark spectator can be scalar or axial-vector

$$\langle P_\Lambda, S_\Lambda; X|\bar{\psi}(0)|0\rangle = \begin{cases} \bar{U}(P_\Lambda, S_\Lambda)\,\Upsilon_s\,\dfrac{i}{\not{k} - m_q} & \text{scalar diquark,} \\ \bar{U}(P_\Lambda, S_\Lambda)\Upsilon_v^\mu\,\dfrac{i}{\not{k} - m_q}\,\varepsilon_\mu & \text{axial-vector diquark.} \end{cases}$$

$$\Upsilon_s = g_s(k^2) \qquad \Upsilon_v^\mu = \frac{g_v(k^2)}{\sqrt{3}}\gamma_5(\gamma^\mu + \frac{P_\Lambda^\mu}{M_\Lambda}) \qquad g_s = g_v = g_{qh}$$

$$g_{qh}(k^2) \mapsto \frac{g_D}{z}\,e^{-\frac{k^2}{\Lambda^2}},$$

Diquark spectator model

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- The calculator can be derived as

$$\tilde{\Delta}^{\alpha}_{A\,s}(z,k_T,S_\Lambda) = -i\frac{C_F\alpha_S}{2(2\pi)^2(1-z)P_\Lambda^-}\frac{1}{k^2-m^2}\int\frac{d^4l}{(2\pi)^4}$$
$$\times\frac{(l^- g_T^{\alpha\rho} - l^\alpha n_+^\rho)(\not{k}-\not{l}+m)\bar{\Upsilon}_s U(P_\Lambda,S_\Lambda)\bar{U}(P_\Lambda,S_\Lambda)\Upsilon_s(\not{k}+m)}{((k-l)^2-m^2)(l^2-i\epsilon)((k-l-P_\Lambda)^2-m_s^2)(-l^- - i\epsilon)}\bar{\Gamma}_\rho$$

$$\tilde{\Delta}^{\alpha}_{A\,v}(z,k_T,S_\Lambda) = i\frac{C_F\alpha_S}{2(2\pi)^2(1-z)P_\Lambda^-}\frac{1}{k^2-m^2}\int\frac{d^4l}{(2\pi)^4}$$
$$\times\frac{(l^- g_T^{\alpha\rho} - l^\alpha n_+^\rho)(\not{k}-\not{l}+m)\bar{\Upsilon}_v^\nu U(P_\Lambda,S_\Lambda)\bar{U}(P_\Lambda,S_\Lambda)\Upsilon_v^\mu(\not{k}+m)}{((k-l)^2-m^2)(l^2-i\epsilon)((k-l-P_\Lambda)^2-m_s^2)(-l^- - i\epsilon)}d^{i\mu}d^{j\nu}\bar{\Gamma}_{i\,j\rho}$$

$$\Gamma_s^{\rho,a} = i\,T^a(2k-2P-l)^\rho$$
$$\Gamma_v^{\rho\mu\nu,a} = -\,i\,T^a[(2k-2P-l)^\rho g^{\mu\nu} - (k-P-l)^\mu g^{\nu\rho} - (k-P)^\nu g^{\rho\mu}];$$

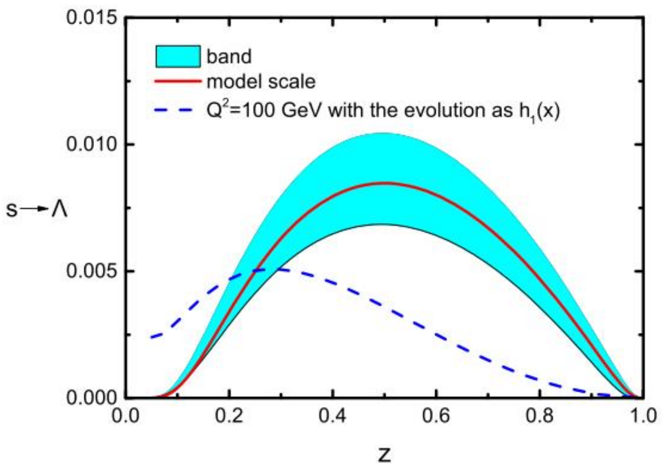
$$\frac{1}{l^2+i\varepsilon} \to -2\pi i\delta(l^2), \qquad \frac{1}{(k-l)^2-m^2+i\varepsilon} \to -2\pi i\delta((k-l)^2-m^2)\,.$$

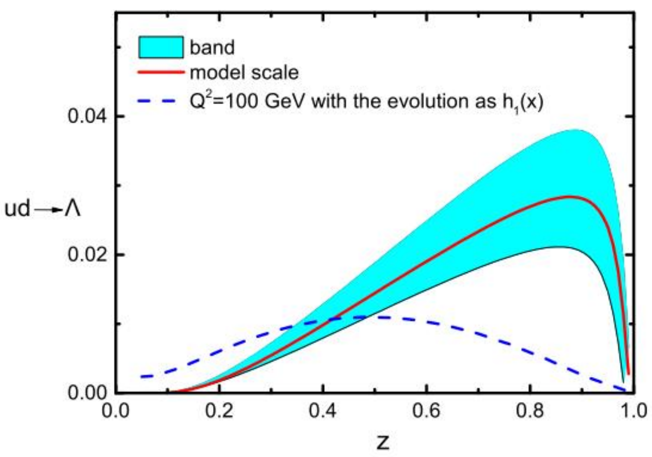
Diquark spectator model

- Assuming an SU(6) spin-flavor symmetry

$$\tilde{H}^{u\to\Lambda} = \tilde{H}^{d\to\Lambda} = \frac{1}{4}\tilde{H}^{(s)} + \frac{3}{4}\tilde{H}^{(v)}, \quad \tilde{H}^{s\to\Lambda} = \tilde{H}^{(s)},$$

- Up to now, we can obtain the results at a relatively low energy scale, to perform the prediction at higher energy scale, one should perform the evolution effect on the fragmentation function, the DGLAP evolution kernel has been chosen as the same as that for transversity.





Kinematical region

- For EIC

$$0.001 < x < 0.4, \quad 0.01 < y < 0.95, \quad 0.2 < z < 0.8,$$
$$Q^2 > 1\text{GeV}^2, \quad \sqrt{s} = 45 \text{ GeV}, \quad W > 5 \text{ GeV}.$$

- For EicC

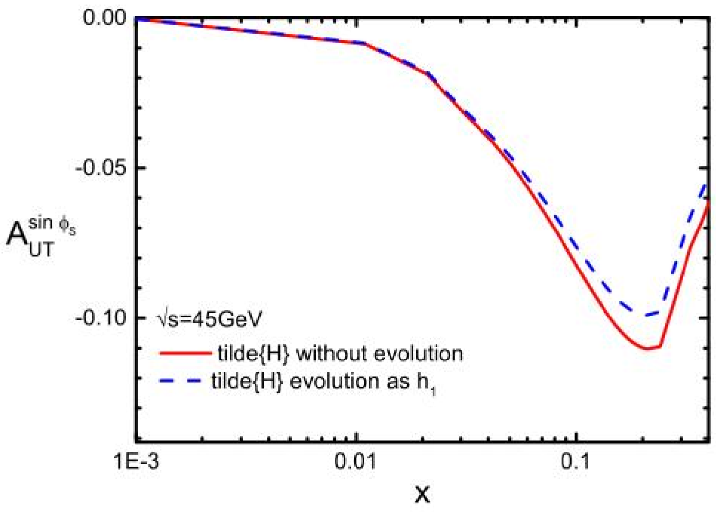
$$0.005 < x < 0.5, \quad 0.07 < y < 0.9, \quad 0.2 < z < 0.7,$$
$$Q^2 > 1\text{GeV}^2, \quad \sqrt{s} = 16.7 \text{ GeV}, \quad W > 2\text{GeV}.$$

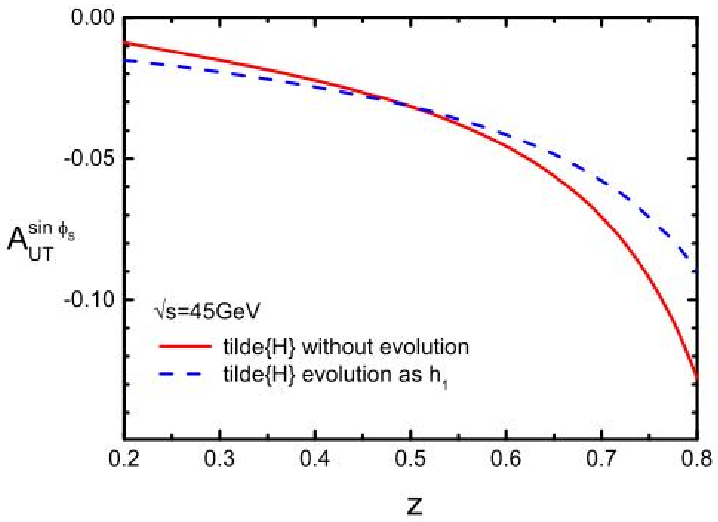
- For COMPASS

$$Q^2 > 1\text{GeV}^2, \quad 0.004 < x < 0.7, \quad 0.1 < y < 0.9,$$
$$z > 0.2, \quad W > 5 \text{ GeV}, \quad E_h > 1.5 \text{ GeV}.$$

Numerical results

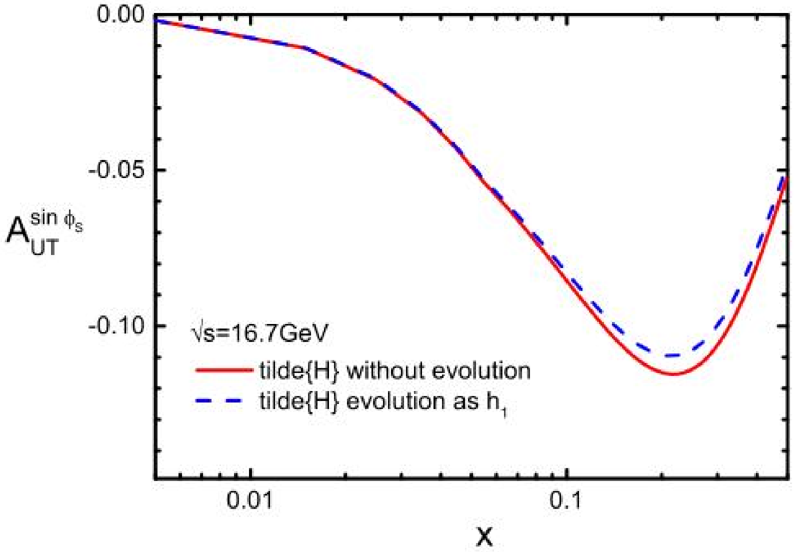
- For EIC

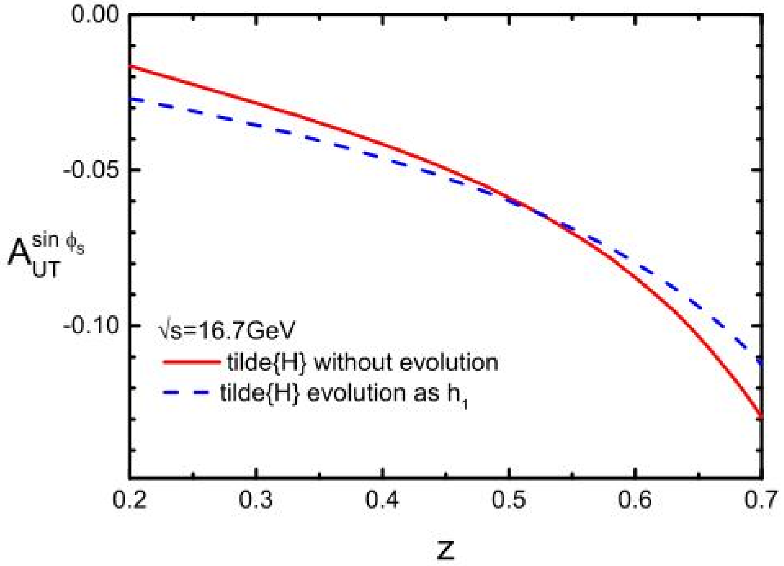




Numerical results

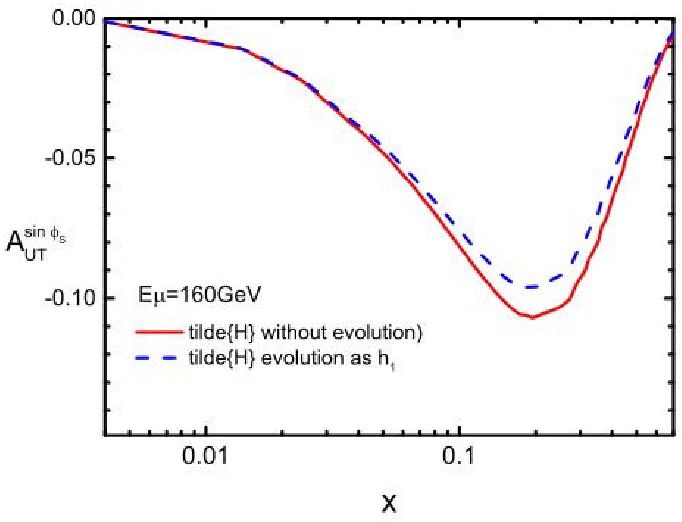
➤ For EicC

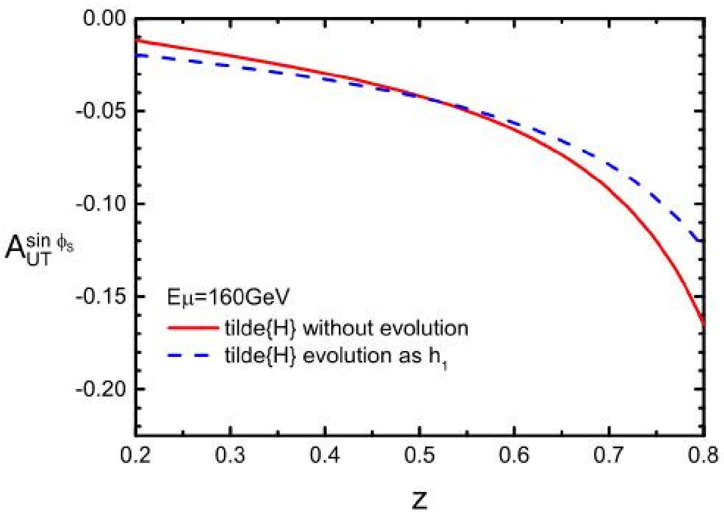




Numerical results

- For COMPASS





Discussion

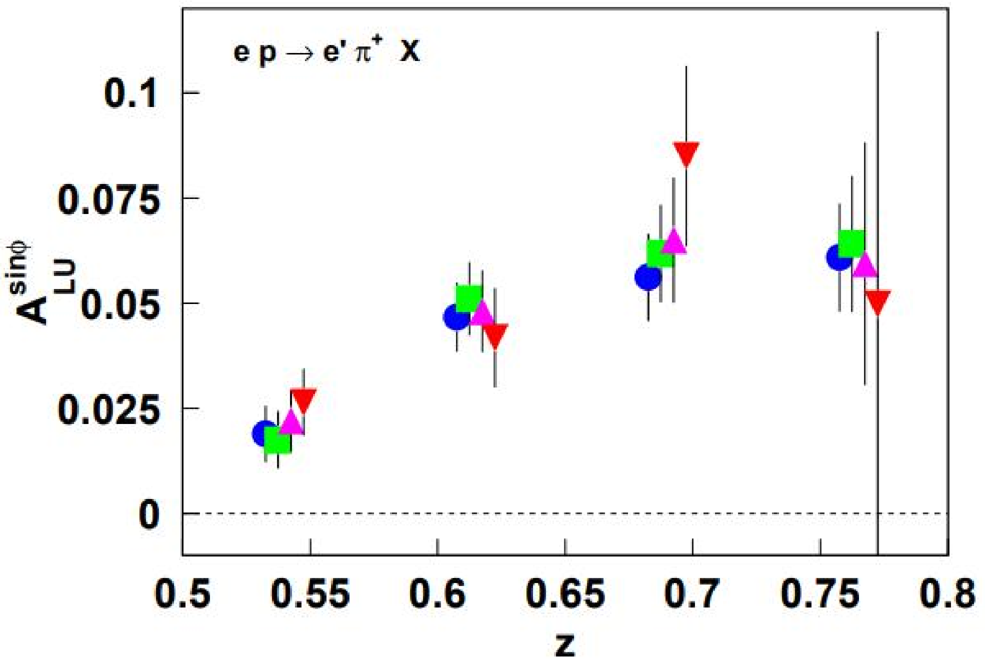
- In the phenomenological analysis, we only consider the contributions from the up and down quark because the lack of information for transversity of the sea quarks.
- We consider the Λ production in SIDIS process, which is normally more difficult to be measured than the meson production.
- The asymmetry is quite sizable at EIC, EicC and COPMASS due to the factor M_Λ/Q is much larger than M_π/Q.
- We expect that the $\sin\phi_S$ asymmetry of Λ production in SIDIS can be measured with the help of high statistic of the future ep facilities.

Conclusion

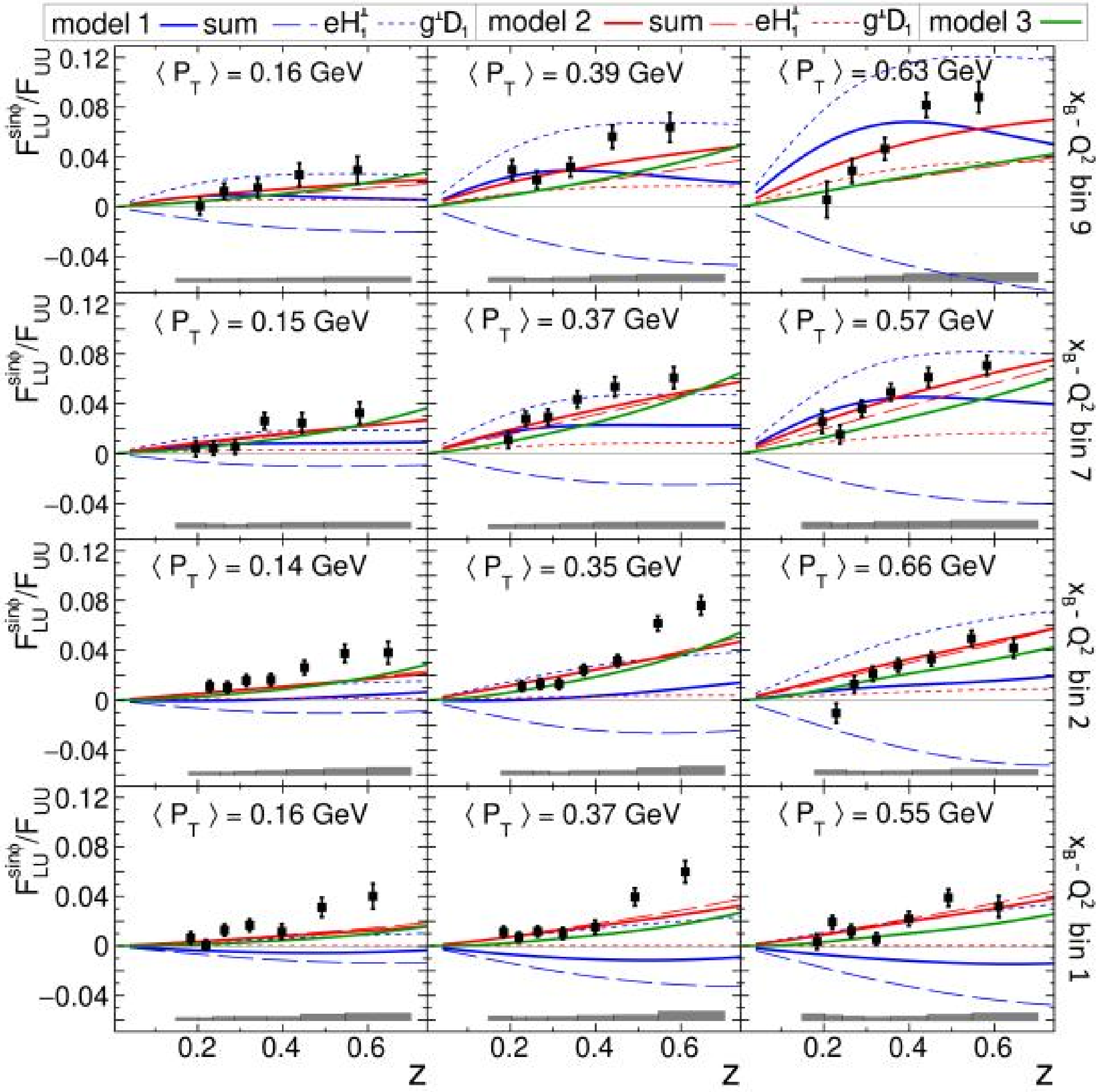
- The estimated $\sin\phi_S$ asymmetry for the Λ production in SIDIS is sizable, around 0.1. The sign of the asymmetry is negative in the entire x and z region.
- The inclusion of the evolution effects of $\widetilde{H}$ can change the shape and size of the asymmetry at the intermediate region of x and z.
- $\sin\phi_S$ asymmetries of Λ production in SIDIS may be measured at the kinematics of EIC, EicC and COMPASS, which provide a feasible way to access the twist-3 collinear fragmentation of the Λ as well as the sea quark transversity.

Beam spin asymmetry?

Phys.Rev.D 69 (2004) 112004
CLAS



Phys.Rev.Lett. 128 (2022) 6, 062005
CLAS 12



Beam spin asymmetry?

$$F_{LU}^{\sin\phi} = \frac{2M}{Q}\,\mathcal{C}\Bigg[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(x_B e H_1^{\perp} + \frac{M_h}{M} f_1 \frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(x_B g^{\perp} D_1 + \frac{M_h}{M} h_1^{\perp} \frac{\tilde{E}}{z}\right)\Bigg]$$

twist-3 observable

measured by CLAS、HERMES、COMPASS last two decades

still can not understand the contribution from each part to the total structure function consistently

Beam spin asymmetry?

1. What shall we do to calculate the qgq twist-3 FF?

2. What shall we do about the evolution of higher twist PDF/FF?