Twist-3 Contributions for SIDIS in the Target Fragmentation Region

陈开宝

山东建筑大学

2023.11.18
近代物理所惠州研究部

Based on:

***K.B. Chen**, J.P. Ma and X.B. Tong, PRD 108, 094015 (2023)*

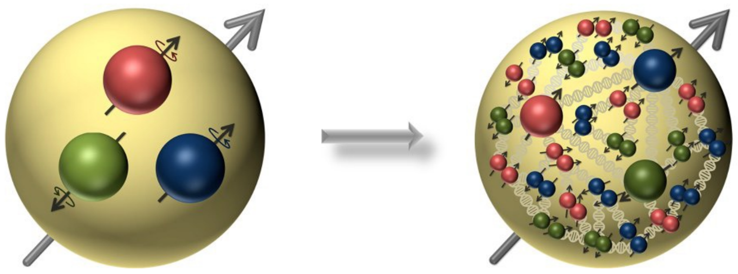
***K.B. Chen**, J.P. Ma and X.B. Tong, JHEP 11 (2021) 038*

Contents

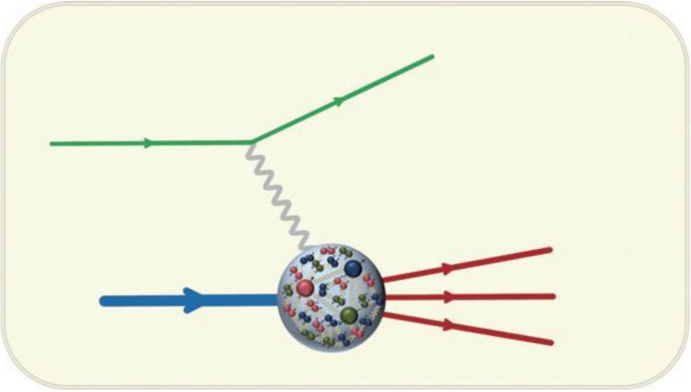
Contents

Introduction

■ Probing nucleon structure



Nucleon structure



Lepton-nucleon deep inelastic scattering (DIS)

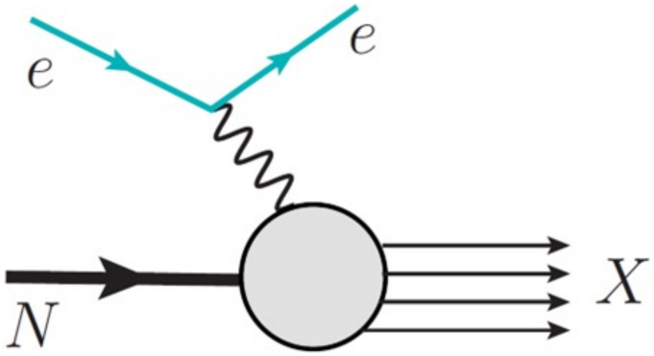
Images taken from Front. Phys. 16(6), 64701

Why nucleon structure important?

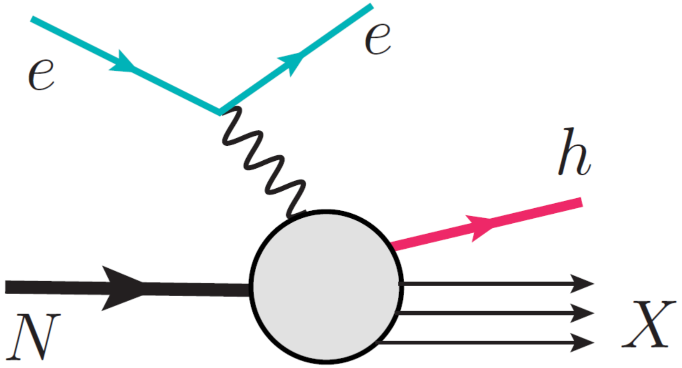
- Fundamental components of the structure of matter
- Insights into the properties of strong interaction
- Inputs for describing high energy reactions
- Prerequisite for doing new physics researches
- …… ……

Introduction

■ Semi-inclusive DIS (SIDIS)



Inclusive DIS



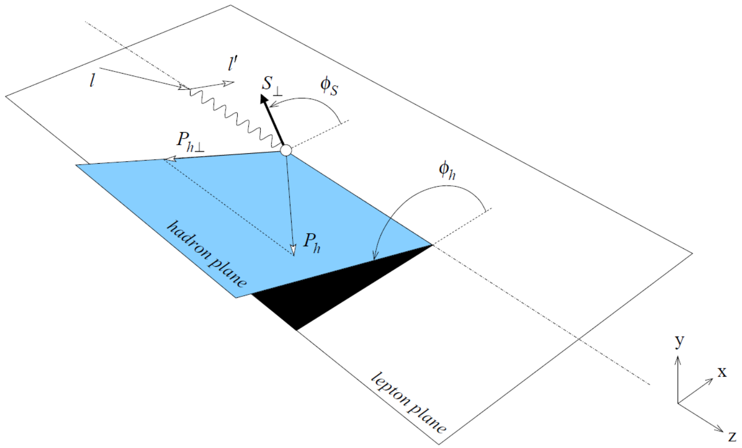
SIDIS

- One-dimensional imaging of the nucleon
- Without fragmentation functions (FFs)
- Target spin asymmetries

- Three-dimensional imaging of the nucleon
- Accessing fragmentation functions
- Azimuthal and/or spin asymmetries

More abundant physical contents!

- **Semi-inclusive DIS (SIDIS)**

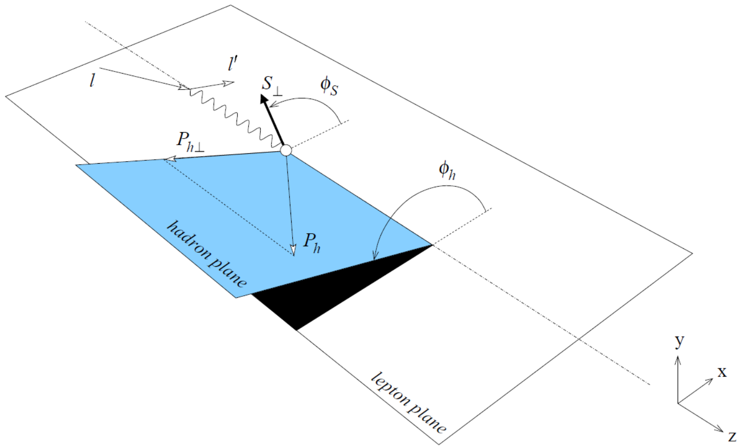


A. Bacchetta et al., JHEP 02, 093 (2007)

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} &= \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\\
&\Big\{F_{UU,T}+\varepsilon\,F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}\\
&+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}\\
&+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]\\
&+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]\\
&+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\\
&\quad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}\\
&\quad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big]\\
&+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}\\
&\quad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Big\}
\end{aligned}
$$

18 structure functions for SIDIS
with polarized lepton and nucleon

■ Semi-inclusive DIS (SIDIS)



A. Bacchetta et al., JHEP 02, 093 (2007)

$$P_{h\perp} \sim M \ll Q$$

TMD factorization applies.

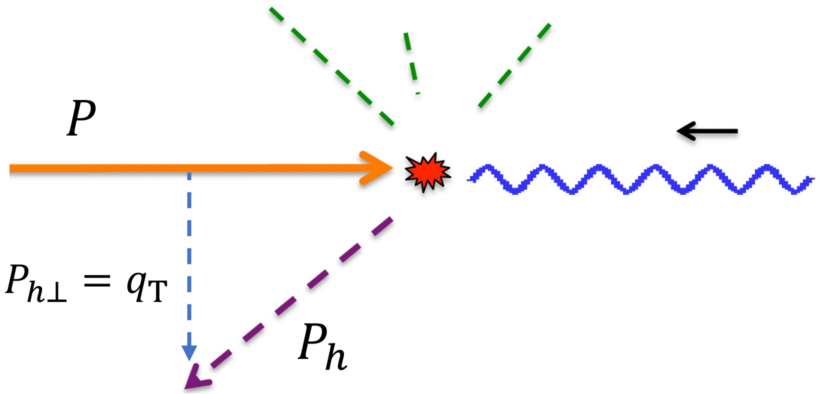
Structure functions are expressed by the convolution of TMD PDFs and FFs.

$F_{UU,T} \sim f_1 \otimes D_1$	Number density
$F_{LL} \sim g_{1L} \otimes D_1$	Helicity
$F_{UT,T}^{\sin(\phi_h-\phi_S)} \sim f_{1T}^{\perp} \otimes D_1$	Sivers
$F_{LT}^{\cos(\phi_h-\phi_S)} \sim g_{1T} \otimes D_1$	Worm-gear L-T
$F_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1 \otimes H_1^{\perp}$	Transversity
$F_{UU}^{\cos 2\phi_h} \sim h_1^{\perp} \otimes H_1^{\perp}$	Boer-Mulders
$F_{UL}^{\sin 2\phi_h} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$	Worm-gear T-L
$F_{UT}^{\sin(3\phi_h-\phi_S)} \sim h_{1T}^{\perp} \otimes H_1^{\perp}$	Pretzelosity

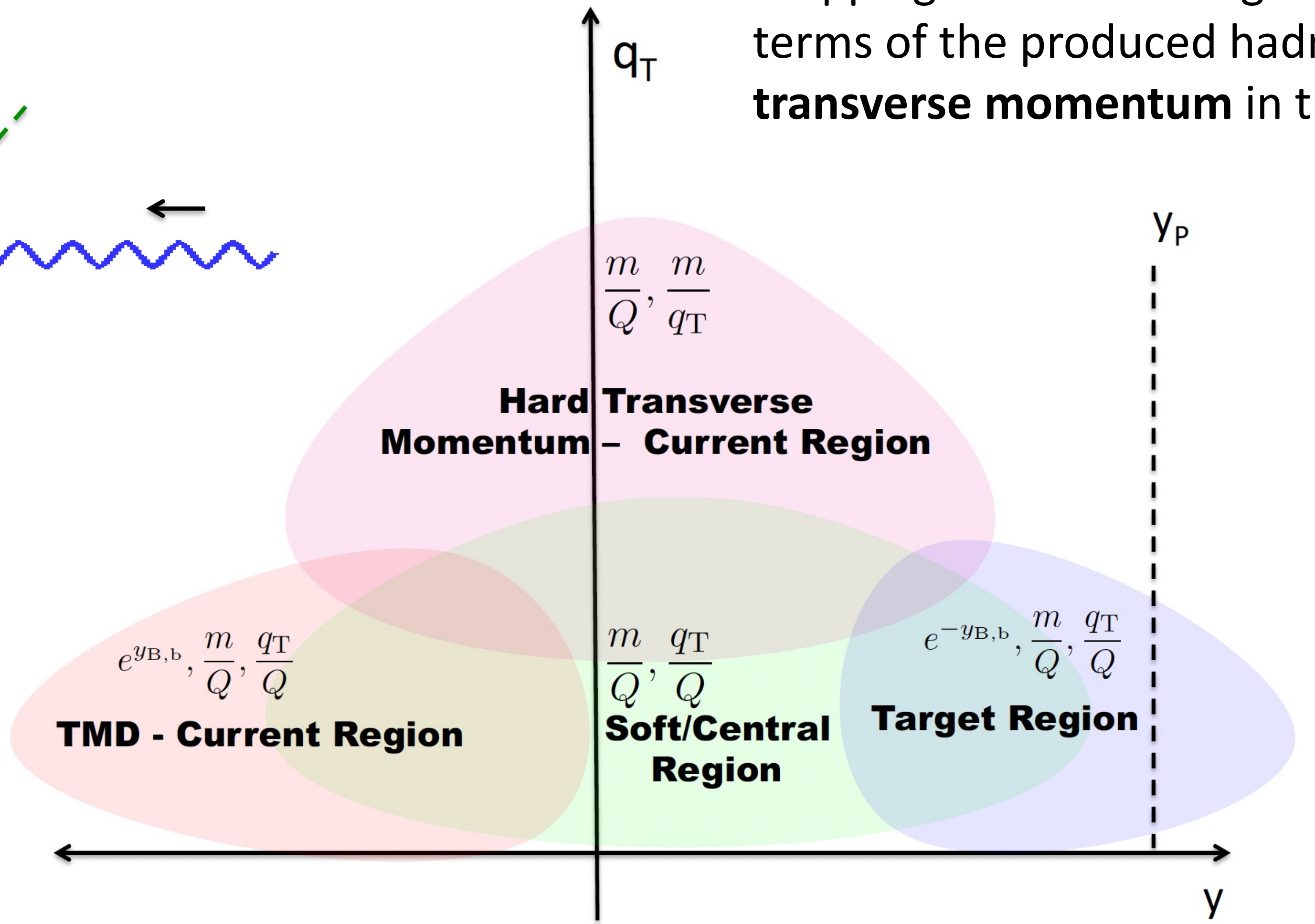
$\vdots$

Introduction

■ Kinematic regions of SIDIS



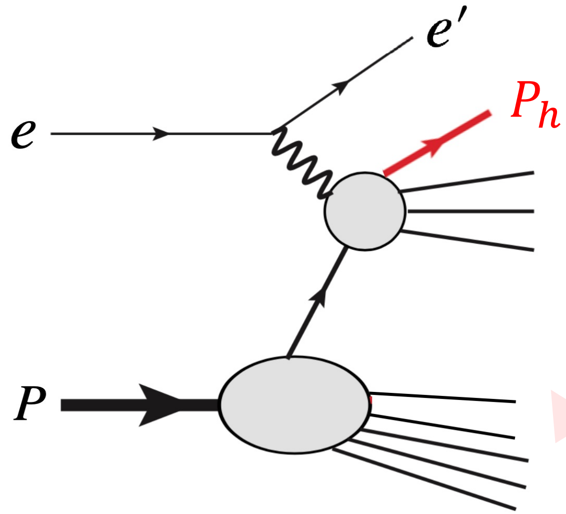
Mapping kinematical regions of SIDIS in terms of the produced hadron's **rapidity** and **transverse momentum** in the Breit frame

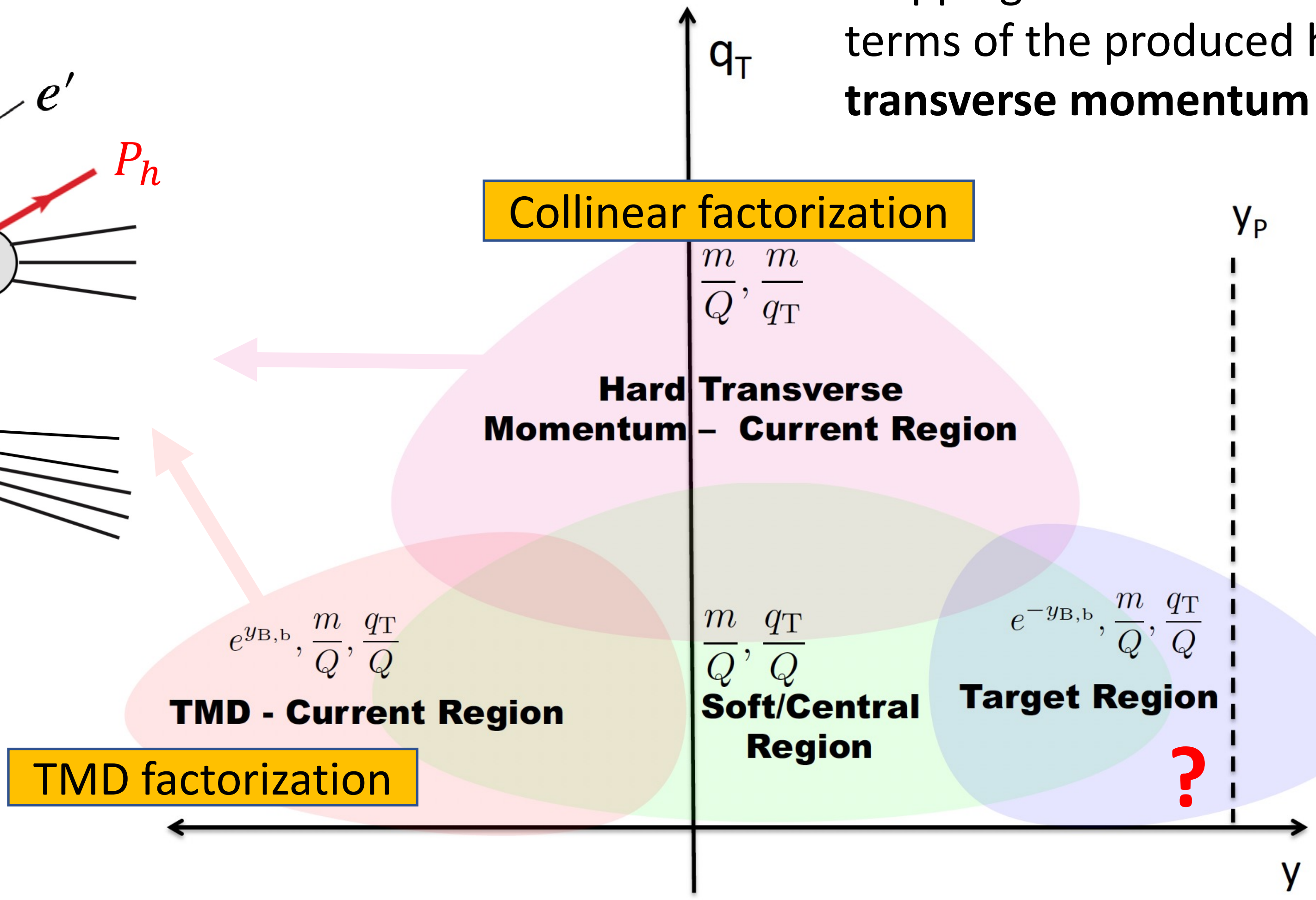


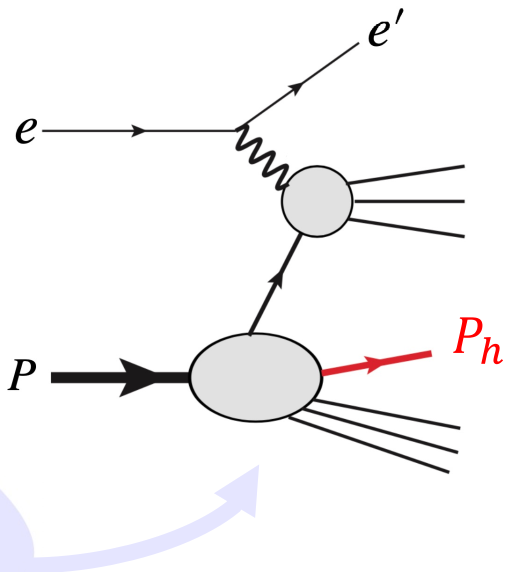
M. Boglione et al. JHEP 10 (2019) 122

Kinematic regions of SIDIS

Mapping kinematical regions of SIDIS in terms of the produced hadron's **rapidity** and **transverse momentum** in the Breit frame







M. Boglione et al. JHEP 10 (2019) 122

Introduction

■ SIDIS in the target fragmentation region (TFR)

- Surprisingly high number of events in the TFR were found at HERA.

M. Derrick et al. (ZEUS Collaboration), Phys. Lett. B 346, 399 (1995).

T. Ahmed et al. (H1 Collaboration), Phys. Lett. B 348, 681 (1995).

SIDIS in the target fragmentation region (TFR)

- Surprisingly high number of events in the TFR were found at HERA.

M. Derrick et al. (ZEUS Collaboration), Phys. Lett. B 346, 399 (1995).

T. Ahmed et al. (H1 Collaboration), Phys. Lett. B 348, 681 (1995).

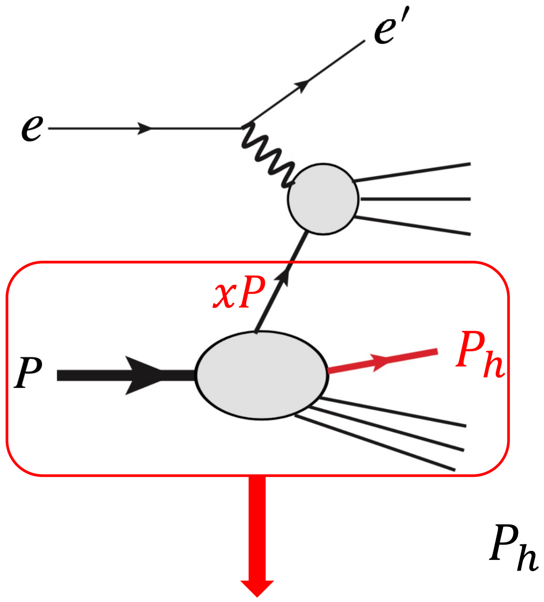
- Theoretical description: the concept of fracture function

$F(x, \xi_h)$: Fracture function

Parton distributions in the presence of an almost collinear particle observed in the final state.

(conditional probability, hybrid of PDF and fragmentation function)

L. Trentadue and G. Veneziano, Phys. Lett. B 323, 201, 1994



$P_h \approx \xi_h P$

$$d\sigma \sim F(x, \xi_h) \otimes H$$

Introduction

■ SIDIS in the target fragmentation region (TFR)

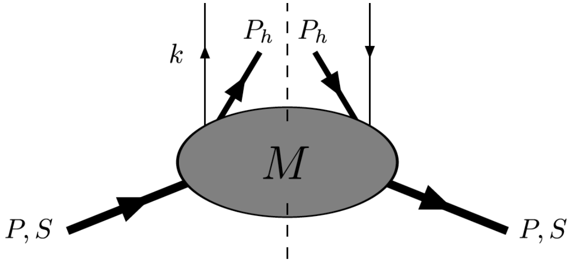
- Major theoretical developments
  - ✓ NLO calculation: *Graudenz, NPB 432, 351 (1994).*
  - ✓ Operator definition and evolution: *Berera and Soper, PRD 53, 6162 (1996).*
  - ✓ All-order proof of factorization (twist-2): *Collins, PRD 57, 3051 (1998).*
  - ✓ Application to hadron collisions:
    Ceccopieri and Trentadue, PLB 668, 319 (2008); Ceccopieri, PLB 703, 491 (2011).
  - ✓ Classification of quark fracture functions at twist-2 for a polarized nucleon target:
    Anselmino, Barone, and Kotzinian, PLB 699, 108 (2011); PLB 706, 46 (2011).

  ⋮

Introduction

SIDIS in the target fragmentation region (TFR)

- Quark fracture functions at twist-2



$$\mathcal{M}^{[\Gamma]}(x_B, \boldsymbol{k}_\perp, \zeta, \boldsymbol{P}_{h\perp}) \equiv \frac{1}{4\zeta}\int \frac{\mathrm{d}k^+\,\mathrm{d}k^-}{(2\pi)^3}\delta(k^- - x_B P^-)\,\mathrm{Tr}(\mathcal{M}\Gamma)$$

$$= \frac{1}{4\zeta}\int \frac{\mathrm{d}\xi^+\,\mathrm{d}^2\boldsymbol{\xi}_\perp}{(2\pi)^6} e^{\mathrm{i}(x_B P^-\xi^+ - \boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp)} \sum_X \int \frac{\mathrm{d}^3\boldsymbol{P}_X}{(2\pi)^3 2E_X} \langle P, S|\overline{\psi}(0)\Gamma|P_h, S_h; X\rangle\langle P_h, S_h; X|\psi(\xi^+, 0, \boldsymbol{\xi}_\perp)|P, S\rangle$$

16 TMD fracture functions at twist-2

$$\mathcal{M}^{[\gamma^-]} = \hat{M} + \frac{\boldsymbol{P}_{h\perp}\times\boldsymbol{S}_\perp}{m_h}\hat{M}_T^h + \frac{\boldsymbol{k}_\perp\times\boldsymbol{S}_\perp}{m_N}\hat{M}_T^\perp + \frac{S_\parallel(\boldsymbol{k}_\perp\times\boldsymbol{P}_{h\perp})}{m_N m_h}\hat{M}_L^{\perp h},$$

$$\mathcal{M}^{[\gamma^-\gamma_5]} = S_\parallel\Delta\hat{M}_L + \frac{\boldsymbol{P}_{h\perp}\cdot\boldsymbol{S}_\perp}{m_h}\Delta\hat{M}_T^h + \frac{\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp}{m_N}\Delta\hat{M}_T^\perp + \frac{\boldsymbol{k}_\perp\times\boldsymbol{P}_{h\perp}}{m_N m_h}\Delta\hat{M}^{\perp h},$$

$$\mathcal{M}^{[\mathrm{i}\sigma^{i-}\gamma_5]} = S_\perp^i\Delta_T\hat{M}_T + \frac{S_\parallel P_{h\perp}^i}{m_h}\Delta_T\hat{M}_L^h + \frac{S_\parallel k_\perp^i}{m_N}\Delta_T\hat{M}_L^\perp + \frac{(\boldsymbol{P}_{h\perp}\cdot\boldsymbol{S}_\perp)P_{h\perp}^i}{m_h^2}\Delta_T\hat{M}_T^{hh} + \frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp)k_\perp^i}{m_N^2}\Delta_T\hat{M}_T^{\perp\perp}$$

$$+ \frac{(\boldsymbol{k}_\perp\cdot\boldsymbol{S}_\perp)P_{h\perp}^i - (\boldsymbol{P}_{h\perp}\cdot\boldsymbol{S}_\perp)k_\perp^i}{m_N m_h}\Delta_T\hat{M}_T^{\perp h} + \frac{\epsilon_\perp^{ij}P_{h\perp j}}{m_h}\Delta_T\hat{M}^h + \frac{\epsilon_\perp^{ij}k_{\perp j}}{m_N}\Delta_T\hat{M}^\perp,$$

Anselmino, Barone, and Kotzinian, Phys. Lett. B699, 108 (2011); Phys. Lett. B706, 46 (2011).

Introduction

■ SIDIS in the target fragmentation region (TFR)

Only four structure functions at twist-2

- Structure functions in the TFR

$F_{UU,T}$ $\quad F_{UT,T}^{\sin(\phi_h-\phi_S)}$

$$\frac{d\sigma^{\mathrm{TFR}}}{dx_B\, dy\, d\zeta\, d^2\boldsymbol{P}_{h\perp}\, d\phi_S} = \frac{2\alpha_{\mathrm{em}}^2}{Q^2 y}\left\{\left(1-y+\frac{y^2}{2}\right)\sum_a e_a^2\left[M(x_B,\zeta,\boldsymbol{P}_{h\perp}^2) - |\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\sin(\phi_h-\phi_S)\right]\right.$$
$$\left. + \lambda_l y\left(1-\frac{y}{2}\right)\sum_a e_a^2\left[S_\parallel \Delta M_L(x_B,\zeta,\boldsymbol{P}_{h\perp}^2) + |\boldsymbol{S}_\perp|\frac{|\boldsymbol{P}_{h\perp}|}{m_h}\Delta M_T^h(x_B,\zeta,\boldsymbol{P}_{h\perp}^2)\cos(\phi_h-\phi_S)\right]\right\}.$$

F_{LL} $\quad F_{LT}^{\cos(\phi_h-\phi_S)}$

Can we go beyond the twist-2? For:

1. More accurate description of the process

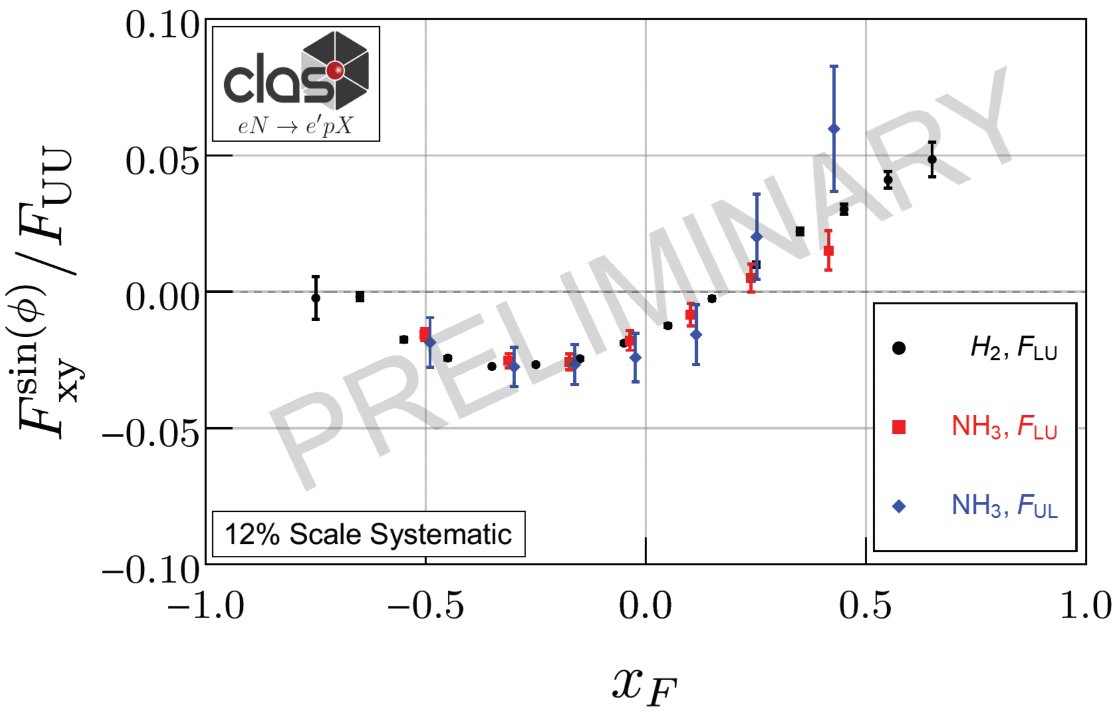
2. Novel azimuthal/spin asymmetries

Anselmino, Barone, and Kotzinian, Phys. Lett. B699, 108 (2011); Phys. Lett. B706, 46 (2011).

Introduction

■ SIDIS in the target fragmentation region (TFR)

- Preliminary results from JLab



CLAS12 beam-spin asymmetry results

show different behavior in CFR and TFR !

sensitive to $F_{LU}^{\sin\phi}$, $F_{UL}^{\sin\phi}$.

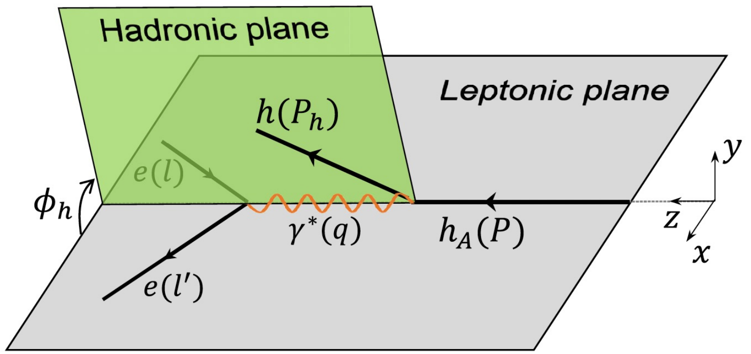
Twist-3 structure functions!

A. Accardi et al., arXiv:2306.09360. *See also the talk given by T. Hayward at **SPIN2023***

Contents

Kinematics and structure functions

$$e(l,\lambda_e)+h_A(P,S)\to e(l')+h(P_h)+X$$



$$\frac{d\sigma}{dx_B dy d\xi_h d\psi d^2 P_{h\perp}}=\frac{\alpha^2 y}{4\xi_h Q^4}L_{\mu\nu}(l,\lambda_e,l')W^{\mu\nu}(q,P,S,P_h)$$

$$L^{\mu\nu}(l,\lambda_e,l')=2(l^\mu l'^\nu+l^\nu l'^\mu-l\cdot l' g^{\mu\nu})+2i\lambda_e\epsilon^{\mu\nu\rho\sigma}l_\rho l'_\sigma$$

$$W^{\mu\nu}(q,P,S,P_h)=\sum_X\int\frac{d^4x}{(2\pi)^4}e^{iq\cdot x}\langle S;h_A|J^\mu(x)|hX\rangle\langle Xh|J^\nu(0)|h_A;S\rangle$$

$$P^\mu\approx(P^+,0,0,0) \qquad l^\mu=(\frac{1-y}{y}x_BP^+,\frac{Q^2}{2x_ByP^+},\frac{Q\sqrt{1-y}}{y},0)$$

$$P_h^\mu=(P_h^+,P_h^-,\vec{P}_{h\perp}) \qquad q^\mu=(-x_BP^+,\frac{Q^2}{2x_BP^+},0,0)$$

$$S^\mu=(\frac{S_LP^+}{M},-\frac{S_LM}{2P^+},\vec{S}_\perp)$$

$$Q^2=-q^2,$$

$$x_B=\frac{Q^2}{2P\cdot q},$$

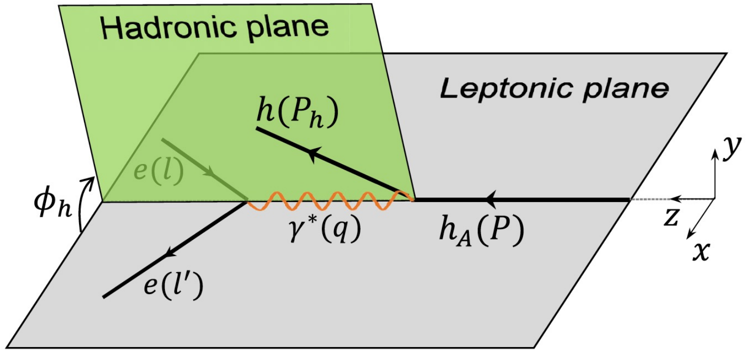
$$y=\frac{P\cdot q}{P\cdot l},$$

$$\xi_h=\frac{P_h\cdot q}{P\cdot q}\approx\frac{P_h^+}{P^+},$$

$$\cancel{z_h=\frac{P\cdot P_h}{P\cdot q}}\ll 1.$$

Twist-3 contributions for SIDIS in TFR

■ Kinematics and structure functions



$$
\begin{aligned}
\frac{d\sigma}{dx_B dy d\xi_h d\psi d^2 P_{h\perp}} = \\
\frac{\alpha^2}{x_B y Q^2}\Big\{ & A(y)F_{UU,T} + E(y)F_{UU,L} + B(y)F_{UU}^{\cos\phi_h}\cos\phi_h + E(y)F_{UU}^{\cos 2\phi_h}\cos 2\phi_h \\
& + \lambda_e D(y)F_{LU}^{\sin\phi_h}\sin\phi_h + \lambda_e S_L\Big[C(y)F_{LL} + D(y)F_{LL}^{\cos\phi_h}\cos\phi_h\Big] \\
& + S_L\Big[B(y)F_{UL}^{\sin\phi_h}\sin\phi_h + E(y)F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\Big] \\
& + |\vec{S}_\perp|\Big[\Big(A(y)F_{UT,T}^{\sin(\phi_h-\phi_S)} + E(y)F_{UT,L}^{\sin(\phi_h-\phi_S)}\Big)\sin(\phi_h-\phi_S) + E(y)F_{UT}^{\sin(\phi_h+\phi_S)}\sin(\phi_h+\phi_S) \\
& \quad + B(y)F_{UT}^{\sin\phi_S}\sin\phi_S + B(y)F_{UT}^{\sin(2\phi_h-\phi_S)}\sin(2\phi_h-\phi_S) + E(y)F_{UT}^{\sin(3\phi_h-\phi_S)}\sin(3\phi_h-\phi_S)\Big] \\
& + \lambda_e|\vec{S}_\perp|\Big[D(y)F_{LT}^{\cos\phi_S}\cos\phi_S + C(y)F_{LT}^{\cos(\phi_h-\phi_S)}\cos(\phi_h-\phi_S) + D(y)F_{LT}^{\cos(2\phi_h-\phi_S)}\cos(2\phi_h-\phi_S)\Big]\Big\}.
\end{aligned}
$$

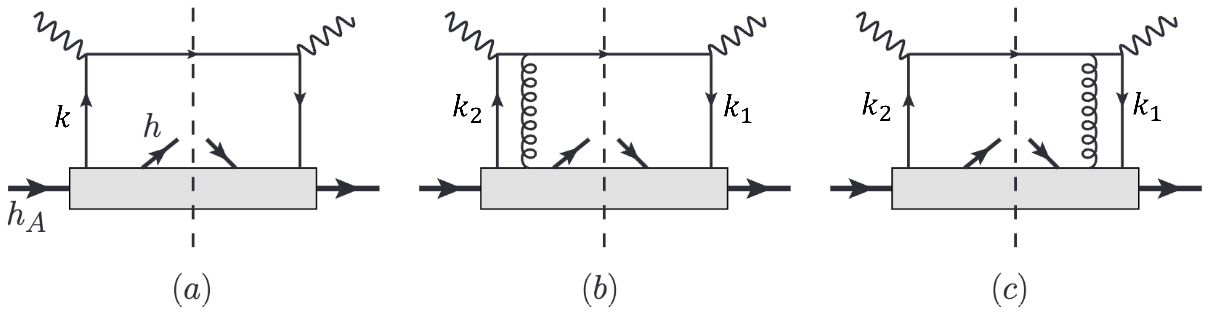
$$
\begin{aligned}
A(y) &= y^2 - 2y + 2, \\
B(y) &= 2(2-y)\sqrt{1-y}, \\
C(y) &= y(2-y), \\
D(y) &= 2y\sqrt{1-y}, \\
E(y) &= 2(1-y).
\end{aligned}
$$

18 structure functions for TFR SIDIS with polarized lepton and nucleon beam

Twist-3 contributions for SIDIS in TFR

■ Hadronic tensor

Hadronic tensor at the tree level



… …

Multiple gluon scattering in the final state.

$$W^{\mu\nu}\Big|_a = \int \frac{d^3k}{(2\pi)^3}\left[\left(\gamma^\mu(\not{k}+\not{q})\gamma^\nu\right)_{ij} 2\pi\delta\big((k+q)^2\big)\right]\sum_X \int \frac{d^3\eta}{(2\pi)^4} e^{-ik\cdot\eta}\langle h_A|\bar\psi_i(\eta)|hX\rangle\langle Xh|\psi_j(0)|h_A\rangle,$$

$$W^{\mu\nu}\Big|_b = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\left[\left(\gamma^\mu(\not{k}_1+\not{q})\gamma_\alpha \frac{i(\not{k}_2+\not{q})}{(k_2+q)^2+i\epsilon}\gamma^\nu\right)_{ij} 2\pi\delta\big((k_1+q)^2\big)\right]$$
$$\times(-ig_s)\sum_X \int \frac{d^3\eta d^3\eta_1}{(2\pi)^4} e^{-ik_1\cdot\eta}e^{i(k_1-k_2)\cdot\eta_1}\langle h_A|\bar\psi_i(\eta)|hX\rangle\langle Xh|G^\alpha(\eta_1)\psi_j(0)|h_A\rangle,$$

$k_i \sim Q(1,\lambda^2,\lambda)$, with $\lambda \sim \frac{\Lambda_{QCD}}{Q}$

$$W^{\mu\nu}\Big|_c = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6}\left[\left(\gamma^\mu \frac{i(\not{k}_1+\not{q})}{(k_1+q)^2-i\epsilon}\gamma_\alpha(\not{k}_2+\not{q})\gamma^\nu\right)_{ij} 2\pi\delta\big((k_2+q)^2\big)\right]$$
$$\times(-ig_s)\sum_X \int \frac{d^3\eta d^3\eta_1}{(2\pi)^4} e^{-ik_1\cdot\eta}e^{i(k_1-k_2)\cdot\eta_1}\langle h_A|\bar\psi_i(\eta)G^\alpha(\eta_1)|hX\rangle\langle Xh|\psi_j(0)|h_A\rangle,$$

Carrying out the collinear expansion procedures … …
(Prof. Liang's talk)

Twist-3 contributions for SIDIS in TFR

■ Hadronic tensor

twist-2

twist-3

$$W^{\mu\nu} = \xi_h(\gamma^\mu\gamma^+\gamma^\nu)_{ij}\mathcal{M}_{ji}(x_B) + \left[\frac{-i\xi_h}{2q^-}(\gamma^\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma^\nu)_{ij}\mathcal{M}^\alpha_{\partial,ji}(x_B) + (\mu\leftrightarrow\nu)^*\right]$$

$$+\left\{\frac{-i\xi_h}{2q^-}(\gamma^\mu\gamma^+\gamma_{\perp\alpha}\gamma^-\gamma^\nu)_{ij}\int dx_2\left[\mathrm{P}\frac{1}{x_2-x_B} - i\pi\delta(x_2-x_B)\right]\mathcal{M}^\alpha_{F,ji}(x_B,x_2) + (\mu\leftrightarrow\nu)^*\right\}$$

$$\mathcal{M}_{ij}(x) = \int\frac{d\eta^-}{2\xi_h(2\pi)^4}e^{-ixP^+\eta^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)\mathcal{L}_n^\dagger(\eta^-)|hX\rangle\langle Xh|\mathcal{L}_n(0)\psi_i(0)|h_A\rangle,$$

$$\mathcal{M}^\alpha_{\partial,ij}(x) = \int\frac{d\eta^-}{2\xi_h(2\pi)^4}e^{-ixP^+\eta^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)\mathcal{L}_n^\dagger(\eta^-)|hX\rangle\langle Xh|\partial_\perp^\alpha(\mathcal{L}_n\psi_i)(0)]|h_A\rangle,$$

$$\mathcal{M}^\alpha_{F,ij}(x_1,x_2) = \int\frac{d\eta^-d\eta_1^-}{4\pi\xi_h(2\pi)^4}e^{-ix_1P^+\eta^- - i(x_2-x_1)P^+\eta_1^-}\sum_X\langle h_A|\bar\psi_j(\eta^-)|hX\rangle\langle Xh|g_sF^{+\alpha}(\eta_1^-)\psi_i(0)|h_A\rangle.$$

$$\mathcal{L}_n(x) = \mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\, G^+(\lambda n+x)\right\}$$

Gauge invariant matrix elements!

$$g_sF^{+\alpha} = g_s[\partial^+G_\perp^\alpha - \partial_\perp^\alpha G^+] + \mathcal{O}(g_s^2)$$

Twist-3 contributions for SIDIS in TFR

Fracture functions defined via the correlation matrices

$$\mathcal{M}_{ij}(x) = \frac{(\gamma_\rho)_{ij}}{2N_c}\left[\bar{n}^\rho\left(u_1 - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^h\right) + \frac{1}{P^+}\left(P_{h\perp}^\rho u^h - M\tilde{S}_\perp^\rho u_T - S_L\tilde{P}_{h\perp}^\rho u_L^h - \frac{P_{h\perp}^{\langle\rho}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}u_T^h\right)\right]$$
$$- \frac{(\gamma_\rho\gamma_5)_{ij}}{2N_c}\left[\bar{n}^\rho\left(S_L l_{1L} - \frac{P_{h\perp}\cdot S_\perp}{M}l_{1T}^h\right) + \frac{1}{P^+}\left(\tilde{P}_{h\perp}^\rho l^h + MS_\perp^\rho l_T + S_L P_{h\perp}^\rho l_L^h - \frac{P_{h\perp}^{\langle\rho}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}l_T^h\right)\right] + \cdots,$$

$$\mathcal{M}_{\partial,ij}^\alpha(x) = \frac{(\gamma^-)_{ij}}{2N_c}i\left(-P_{h\perp}^\alpha u_\partial^h + M\tilde{S}_\perp^\alpha u_{\partial T} + S_L\tilde{P}_{h\perp}^\alpha u_{\partial L}^h + \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}u_{\partial T}^h\right)$$
$$+ \frac{(\gamma^-\gamma_5)_{ij}}{2N_c}i\left(\tilde{P}_{h\perp}^\alpha l_\partial^h + MS_\perp^\alpha l_{\partial T} + S_L P_{h\perp}^\alpha l_{\partial L}^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}l_{\partial T}^h\right) + \cdots,$$

$$\mathcal{M}_{F,ij}^\alpha(x_1,x_2) = \frac{(\gamma^-)_{ij}}{2N_c}\left(P_{h\perp}^\alpha w^h - M\tilde{S}_\perp^\alpha w_T - S_L\tilde{P}_{h\perp}^\alpha w_L^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}\tilde{S}_{\perp\beta}w_T^h\right)$$
$$- \frac{(\gamma^-\gamma_5)_{ij}}{2N_c}i\left(\tilde{P}_{h\perp}^\alpha v^h + MS_\perp^\alpha v_T + S_L P_{h\perp}^\alpha v_L^h - \frac{P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle}}{M}S_{\perp\beta}v_T^h\right) + \cdots,$$

Fracture functions up to twist-3

$$\tilde{a}_\perp^\mu \equiv \varepsilon_\perp^{\mu\nu}a_{\perp\nu}$$

$$P_{h\perp}^{\langle\alpha}P_{h\perp}^{\beta\rangle} \equiv P_{h\perp}^\alpha P_{h\perp}^\beta + g_\perp^{\alpha\beta}\vec{P}_{h\perp}^2/2$$

- Red: twist-2.
- Green: twist-3
- Functions of $(x, \xi_h, P_{h\perp}^2)$

- Only chiral-even terms contribute
- Similar form ($k_\perp \to P_{h\perp}$) and naming rules as TMDs

See e.g., S.Y. Wei, Y.K. Song, KBC, Z.T. Liang, PRD 95, 074017

Twist-3 contributions for SIDIS in TFR

Fracture functions defined via the correlation matrices

The twist-3 fracture functions are not independent from each other!

Applying QCD equation of motion $i\gamma \cdot D\psi = 0$, gives:

$$x[u_S^K(x) + il_S^K(x)]$$
$$= u_{\partial S}^K(x) + il_{\partial S}^K(x) + i\int dy \left[\mathrm{P}\frac{1}{y-x} - i\pi\delta(y-x)\right][w_S^K(x,y) - v_S^K(x,y)]$$

The relations take a unified form!

S	K
null	h
L	h
T	null
T	h

Four sets of equations

Twist-3 contributions for SIDIS in TFR

Hadronic tensor in terms of fracture functions

$$
\begin{aligned}
W^{\mu\nu} =& -2g_\perp^{\mu\nu}\Big(u_1 - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^h\Big) + 2i\varepsilon_\perp^{\mu\nu}\Big(S_L l_{1L} - \frac{P_{h\perp}\cdot S_\perp}{M}l_{1T}^h\Big) \\
&+ \frac{2}{P\cdot q}P_{h\perp}^{\{\mu}\bar{q}^{\nu\}}\Big(u^h - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_T^h\Big) + \frac{2i}{P\cdot q}P_{h\perp}^{[\mu}\bar{q}^{\nu]}\Big(l^h - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}l_T^h\Big) - \frac{2M}{P\cdot q}\tilde{S}_\perp^{\{\mu}\bar{q}^{\nu\}}\Big(u_T - \frac{\vec{P}_{h\perp}^2}{2M^2}u_T^h\Big) \\
&- \frac{2iM}{P\cdot q}\tilde{S}_\perp^{[\mu}\bar{q}^{\nu]}\Big(l_T - \frac{\vec{P}_{h\perp}^2}{2M^2}l_T^h\Big) - \frac{2S_L}{P\cdot q}\tilde{P}_{h\perp}^{\{\mu}\bar{q}^{\nu\}}u_L^h - \frac{2iS_L}{P\cdot q}\tilde{P}_{h\perp}^{[\mu}\bar{q}^{\nu]}l_L^h,
\end{aligned}
$$

$$\bar{q}^\mu \equiv q^\mu + 2x_B P^+ \bar{n}^\mu$$

$$A^{\{\mu}B^{\nu\}} \equiv A^\mu B^\nu + A^\nu B^\mu$$

$$A^{[\mu}B^{\nu]} \equiv A^\mu B^\nu - A^\nu B^\mu$$

The hadronic tensor can be expressed by those fracture functions defined only from the quark-quark correlator.

Current conservation satisfied: $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$

Twist-3 contributions for SIDIS in TFR

Results for structure functions and azimuthal asymmetries

Structure functions

Azimuthal/spin asymmetries

$$F_{UU,T}=x_B u_1,\qquad F_{UT,T}^{\sin(\phi_h-\phi_S)}=\frac{|\vec{P}_{h\perp}|}{M}x_B u_{1T}^h,$$

$$F_{LL}=x_B l_{1L},\qquad F_{LT}^{\cos(\phi_h-\phi_S)}=\frac{|\vec{P}_{h\perp}|}{M}x_B l_{1T}^h.$$

Twist-2

$$\langle\sin(\phi_h-\phi_S)\rangle_{UT}=\frac{|\vec{P}_{h\perp}|}{2M}\frac{u_{1T}^h}{u_1},$$

$$\langle\cos(\phi_h-\phi_S)\rangle_{LT}=\frac{|\vec{P}_{h\perp}|C(y)}{2MA(y)}\frac{l_{1T}^h}{u_1}.$$

$$F_{UU}^{\cos\phi_h}=-\frac{2|\vec{P}_{h\perp}|}{Q}x_B^2u^h,\qquad F_{LU}^{\sin\phi_h}=\frac{2|\vec{P}_{h\perp}|}{Q}x_B^2l^h,$$

$$F_{UL}^{\sin\phi_h}=-\frac{2|\vec{P}_{h\perp}|}{Q}x_B^2u_L^h,\qquad F_{LL}^{\cos\phi_h}=-\frac{2|\vec{P}_{h\perp}|}{Q}x_B^2l_L^h,$$

$$F_{UT}^{\sin\phi_S}=-\frac{2M}{Q}x_B^2u_T,\qquad F_{LT}^{\cos\phi_S}=-\frac{2M}{Q}x_B^2l_T,$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)}=-\frac{\vec{P}_{h\perp}^2}{QM}x_B^2u_T^h,\qquad F_{LT}^{\cos(2\phi_h-\phi_S)}=-\frac{\vec{P}_{h\perp}^2}{QM}x_B^2l_T^h.$$

Twist-3

$$\langle\cos\phi_h\rangle_{UU}=-\frac{|\vec{P}_{h\perp}|}{Q}\frac{B(y)}{A(y)}\frac{x_Bu^h}{u_1},\qquad \langle\sin\phi_h\rangle_{LU}=\frac{|\vec{P}_{h\perp}|}{Q}\frac{D(y)}{A(y)}\frac{x_Bl^h}{u_1},$$

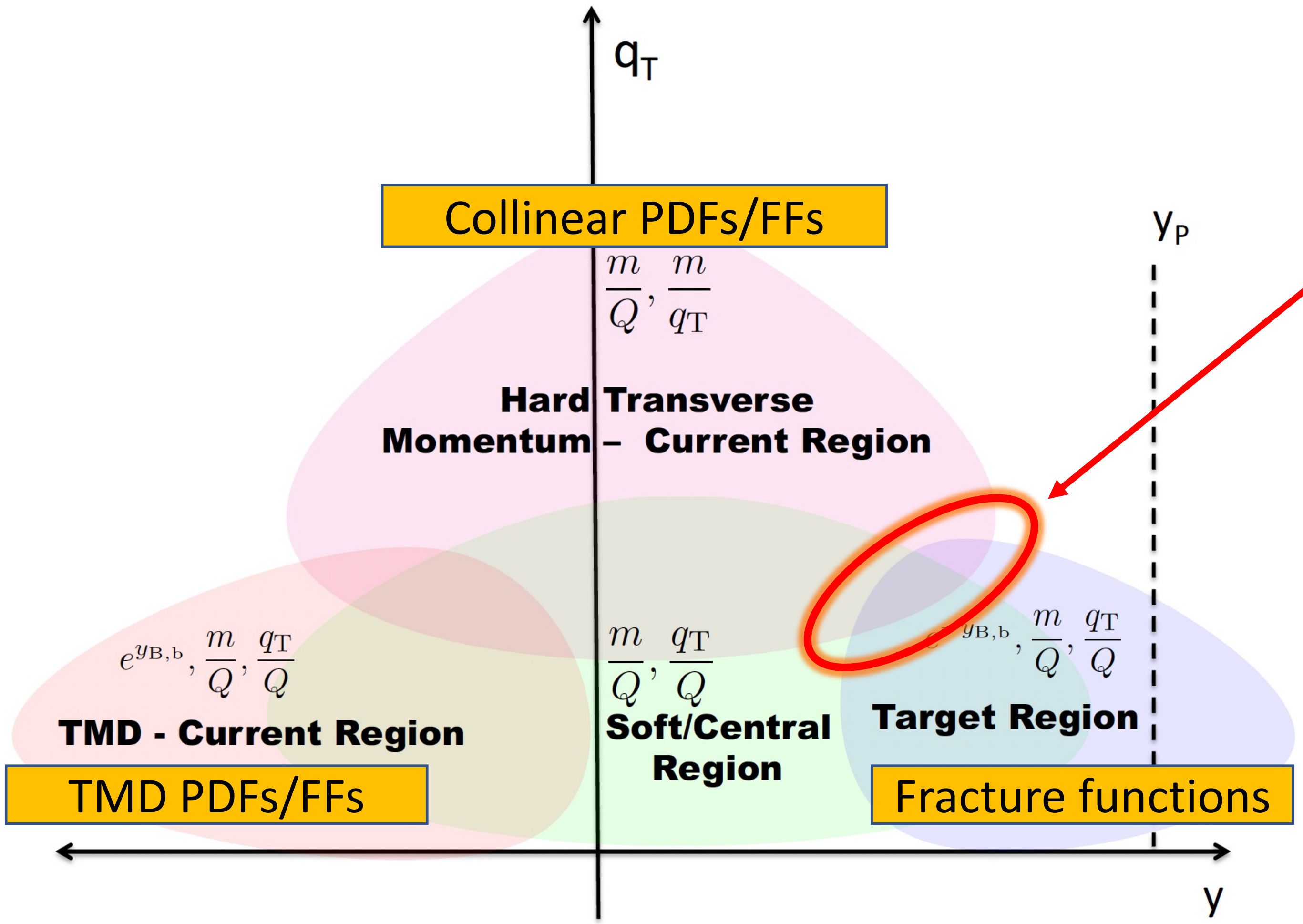
$$\langle\sin\phi_h\rangle_{UL}=-\frac{|\vec{P}_{h\perp}|}{Q}\frac{B(y)}{A(y)}\frac{x_Bu_L^h}{u_1},\qquad \langle\cos\phi_h\rangle_{LL}=-\frac{|\vec{P}_{h\perp}|}{Q}\frac{D(y)}{A(y)}\frac{x_Bl_L^h}{u_1},$$

$$\langle\sin\phi_S\rangle_{UT}=-\frac{M}{Q}\frac{B(y)}{A(y)}\frac{x_Bu_T}{u_1},\qquad \langle\cos\phi_S\rangle_{LT}=-\frac{M}{Q}\frac{D(y)}{A(y)}\frac{x_Bl_T}{u_1},$$

$$\langle\sin(2\phi_h-\phi_S)\rangle_{UT}=-\frac{\vec{P}_{h\perp}^2}{2MQ}\frac{B(y)}{A(y)}\frac{x_Bu_T^h}{u_1},\qquad \langle\cos(2\phi_h-\phi_S)\rangle_{LT}=-\frac{\vec{P}_{h\perp}^2}{2MQ}\frac{D(y)}{A(y)}\frac{x_Bl_T^h}{u_1}.$$

Contents

Collinear matching of fracture functions



Overlap region $\Lambda_{QCD} \ll P_{h\perp} \ll Q$

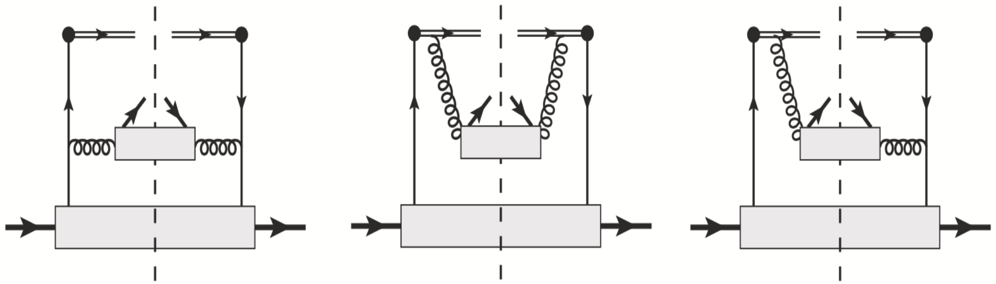
Large transverse momentum is generated perturbatively.

Fracture functions can be re-factorized and matched to collinear PDFs/FFs in the overlap region.

Collinear matching of fracture functions

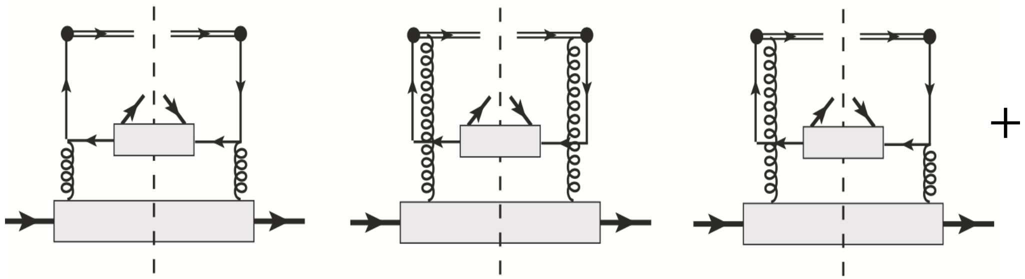
■ Matching of unpolarized/longitudinal polarized fracture function

$$\mathcal{M}_{ij} = \frac{(\gamma^-)_{ij}}{2N_c}\left[\boxed{u_1(x,\xi_h,P_{h\perp})} - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\right] + \frac{(\gamma_5\gamma^-)_{ij}}{2N_c}\left[S_L\boxed{l_{1L}(x,\xi_h,P_{h\perp})} - \frac{P_{h\perp}\cdot S_\perp}{M}l_{1T}^h(x,\xi_h,P_{h\perp})\right] + \cdots$$



$+h.c.$

quark PDF ⊗ gluon FF



$+h.c.$

gluon PDF ⊗ antiquark FF

Twist-2 matching

$$u_1(x,\xi_h,P_{h\perp}) = g_s^2\frac{1}{\vec{P}_{h\perp}^2}\int\frac{dz}{z^2}\left[2C_F\underline{d_g(z)q(y)}\frac{z^2}{y^2}(x^2+y^2) + \underline{d_{\bar{q}}(z)g(y)}\frac{\xi_h}{zy^3}(z^2x^2+\xi_h^2)\right]$$

$$l_{1L}(x,\xi_h,P_{h\perp}) = g_s^2\frac{1}{\vec{P}_{h\perp}^2}\int\frac{dz}{z^2}\left[2C_F\underline{d_g(z)\Delta q(y)}\frac{z^2}{y^2}(x^2+y^2) + \underline{d_{\bar{q}}(z)\Delta g(y)}\frac{\xi_h}{y^2}(xz-\xi_h)\right]$$

$$y = x + \xi_h/z$$

Scales as $1/P_{h\perp}^2$

Accessing the twist-2 unpolarized/longitudinal polarized PDF in TFR

Collinear matching of fracture functions

■ Matching of transverse polarized fracture function

$$\mathcal{M}_{ij} = \frac{(\gamma^-)_{ij}}{2N_c}\left[u_1(x,\xi_h,P_{h\perp}) - \frac{P_{h\perp}\cdot \tilde{S}_\perp}{M}\boxed{u_{1T}^h(x,\xi_h,P_{h\perp})}\right] + \frac{(\gamma_5\gamma^-)_{ij}}{2N_c}\left[S_L l_{1L}(x,\xi_h,P_{h\perp}) - \frac{P_{h\perp}\cdot S_\perp}{M}\boxed{l_{1T}^h(x,\xi_h,P_{h\perp})}\right] + \cdots$$

T-odd

Absorptive parts required! (single-spin asymmetry)

T-even

No absorptive part required

Collinear matching of fracture functions

■ Matching of transverse polarized fracture function

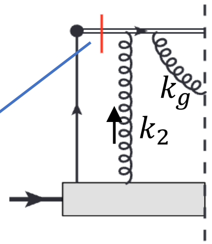
$$\mathcal{M}_{ij} = \frac{(\gamma^-)_{ij}}{2N_c}\left[u_1(x,\xi_h,P_{h\perp}) - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}\boxed{u_{1T}^h(x,\xi_h,P_{h\perp})}\right] + \frac{(\gamma_5\gamma^-)_{ij}}{2N_c}\left[S_L l_{1L}(x,\xi_h,P_{h\perp}) - \frac{P_{h\perp}\cdot S_\perp}{M} l_{1T}^h(x,\xi_h,P_{h\perp})\right] + \cdots$$

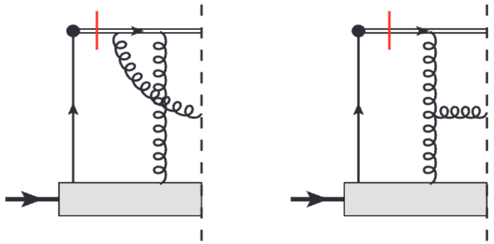
T-odd

Absorptive parts required! (single-spin asymmetry)

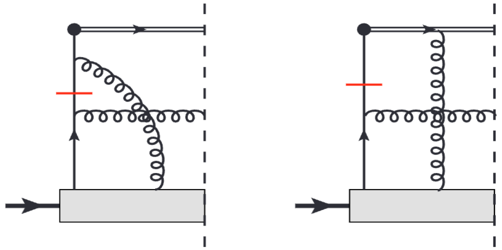
(Prof. Yoshida's talk)

$$\boxed{\frac{1}{k_g^+ - k_2^+ + i\epsilon} \to -i\pi\delta\left(k_g^+ - k_2^+\right)}$$

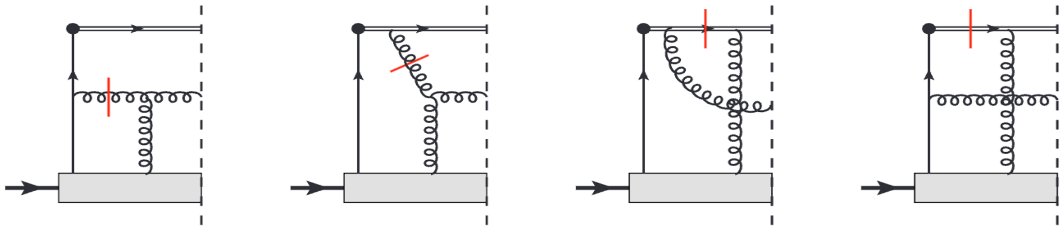




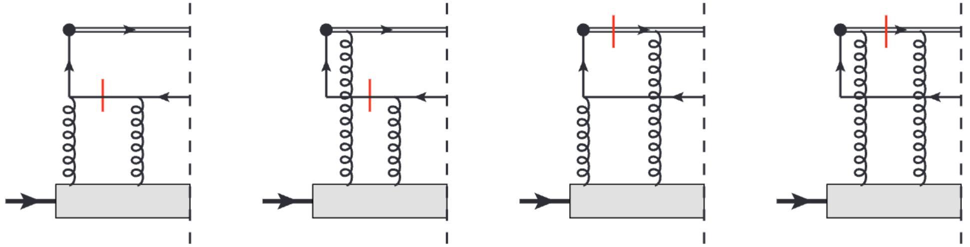
Hard-pole



Soft-fermion-pole



Soft-gluon-pole



pole from 3-gluon correlations

Collinear matching of fracture functions

Matching of transverse polarized fracture function

Matching results after collinear expansion

$$\frac{1}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\Big|_{HP} = g_s^2\frac{N_c}{P_{h\perp}^4}\int\frac{dz}{z^2}d_g(z)\frac{z^2}{y}\Big[\xi_h T_\Delta(y,x)-(\xi_h+2xz)T_F(y,x)\Big]$$

$$\frac{1}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\Big|_{SFP} = g_s^2\frac{1}{N_c}\frac{1}{P_{h\perp}^4}\int\frac{dz}{z^2}d_g(z)\frac{x\xi_h z}{y^3}\Big[(xz-\xi_h)T_F(y,0)-(\xi_h+xz)T_\Delta(y,0)\Big]$$

Gauge invariant!

$$\frac{1}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\Big|_{SGP} = \frac{g_s^2N_c}{P_{h\perp}^4}\int\frac{dz}{z^2}d_g(z)\frac{1}{y^3}\Big[z^3(y^3+3x^2y-2x^3)T_F(y,y)-y\xi_h z^2(y^2+x^2)\frac{\partial T_F(y,y)}{\partial y}\Big]$$

$$\begin{aligned}\frac{1}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\Big|_{3G} = \frac{4\pi g_s^2\xi_h^2}{P_{h\perp}^4}\int\frac{dz}{z^2}d_{\bar q}(z)\frac{1}{zy^5}\Big\{&2(\xi_h^2+2x^2z^2-\xi_h xz)\big[N(y,y)-O(y,y)\big]\\ &-2(\xi_h^2+4x^2z^2-3\xi_h xz)\big[N(y,0)+O(y,0)\big]+y(\xi_h-xz)^2\frac{d}{dy}\big[N(y,0)+O(y,0)\big]\\ &-y(\xi_h^2+x^2z^2)\frac{d}{dy}\big[N(y,y)-O(y,y)\big]\Big\}\end{aligned}$$

Twist-3 PDFs
$\otimes$ twist-2 FFs

Scales as $1/P_{h\perp}^4$

$T_F(x_1,x_2)$, $T_\Delta(x_1,x_2)$, $O(x_1,x_2)$, $N(x_1,x_2)$: gauge invariant twist-3 collinear PDFs.

E-T, PLB 150, 383; Q-S, PRL 67, 2264.
Ji, PLB 289, 137. Beppu et al., PRD 82, 034005.

Contents

Summary & Outlook

Summary

- TFR is complementary to CFR for describing the SIDIS process and studying the nucleon structure.
- We have calculated the TFR SIDIS for polarized lepton beam and nucleon target up to twist-3 in tree level of pQCD. The results for structure functions and azimuthal asymmetries are given using the gauge-invariant fracture functions.
- The matching of fracture functions to PDFs and FFs up to twist-3 is studied in large hadron transverse momentum region.

Summary & Outlook

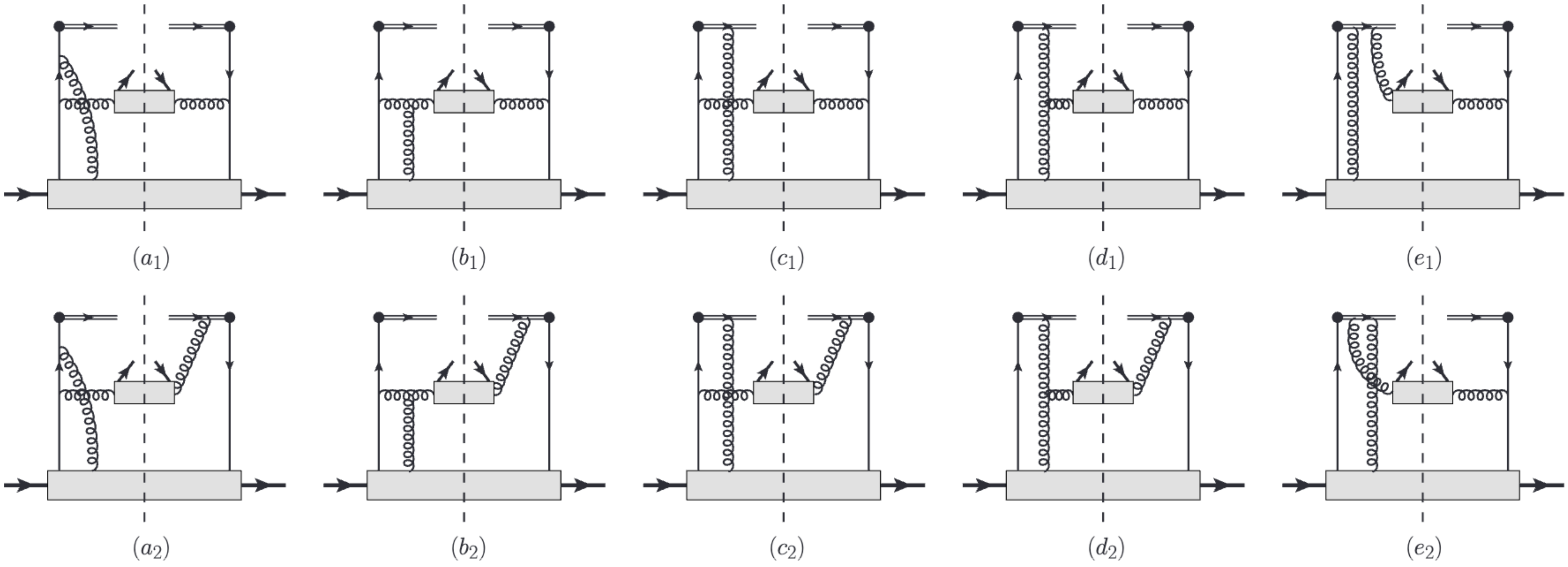
■ Outlook

- Twist-4 contributions to TFR SIDIS? NLO corrections, especially gluon contributions to structure functions? … …
- Model calculations, phenomenological studies … …
- Looking forward to experimental progress. Physics on SIDIS @ EicC … …

谢谢！

Matching of transverse polarized fracture function

$$\mathcal{M}_{ij} = \frac{(\gamma^-)_{ij}}{2N_c}\left[u_1(x,\xi_h,P_{h\perp}) - \frac{P_{h\perp}\cdot\tilde{S}_\perp}{M}u_{1T}^h(x,\xi_h,P_{h\perp})\right] + \frac{(\gamma_5\gamma^-)_{ij}}{2N_c}\left[S_L l_{1L}(x,\xi_h,P_{h\perp}) - \frac{P_{h\perp}\cdot S_\perp}{M}\boxed{l_{1T}^h(x,\xi_h,P_{h\perp})}\right] + \cdots$$



Three parton correlation contributions to the matching of l_{1T}^h

Collinear matching of fracture functions

■ Matching of transverse polarized fracture function

$$\frac{1}{M}l_{1T}^{h}\Big|_{2G+3G} = \frac{-16\pi\alpha_s\xi_h^2}{(k_\perp^2)^2}\int\frac{dzdx_2}{z^2y^3}d_{\bar{q}}(z)\Bigg\{\frac{xz}{\pi y}T_F(x_2,x_2+y) - \frac{2xz}{y(y-x_2)}\Big[N(y,x_2)-N(y-x_2,y)+2N(y-x_2,-x_2)\Big]$$
$$+\frac{1}{x_2^2(y-x_2)}\bigg(x_2\xi_h\Big[O(y-x_2,y)-N(y-x_2,y)\Big]+(2\xi_h y+yx_2z-y^2z-\xi_h x_2)\Big[N(y,x_2)+O(y,x_2)\Big]$$
$$+y(z(y+x_2)-2\xi_h)\Big[N(y-x_2,-x_2)+O(y-x_2,-x_2)\Big]\bigg)\Bigg\}$$

$$\frac{1}{M}l_{1T}^{h}\Big|_{q\bar{q}+qG\bar{q}} = \frac{1}{2P_{h\perp}^4}\int\frac{dz}{z^2}d_g(z)\bigg[\Big(H_{2p,T}(x,\xi_h)q_T(y)+H_{2p,\partial}(x,\xi_h)q_\partial(y)\Big)$$
$$+\frac{2}{\pi}\int dx_2\Big(T_F(y,x_2)H(x,\xi_h,x_2)+T_\Delta(y,x_2)H_A(x,\xi_h,x_2)\Big)\bigg]$$

$$H(x,\xi,x_2) = \frac{2g_s^2\xi_h z^2}{yx_2(y-x_2)}\left[C_A\frac{x^2-x_2y}{x_2-x}+2C_F\frac{1}{y}(xy-x_2(x+y))\right],$$
$$H_A(x,\xi,x_2) = \frac{2g_s^2\xi_h z^2}{yx_2(x_2-y)}\left[C_A\frac{(x^2+x_2y)(x_2+y-2x)}{(x_2-x)(x_2-y)}+2C_F\frac{1}{y}(x(2x-y)+x_2(x+y))\right].$$