# Angular distributions of heavy quark in DIS

Guang-Peng Zhang(张广鹏)

云南大学(Yunnan University)

近代物理研究所惠州研究部,18/11/2023

## Outline

- Kinematics for heavy quark production in DIS;
- Possible angular distributions;
- One-loop corrections: real and virtual;
- Numerical results for EIcC energy;
- Summary and Future work.

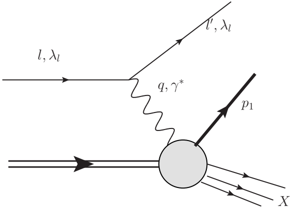
# Kinematics

$$e(l, \lambda_l) + h_A(P_A, \lambda_h) \to e(l') + Q(p_1) + X(\text{undetected hadrons})$$

DIS variables:

$$S_{pl} = (p_A + l)^2, x_B = \frac{Q^2}{2p_A \cdot q}, y_l = \frac{p_A \cdot q}{p_A \cdot l} = \frac{Q^2}{x_B S_{pl}}$$

$$Q^2 = -q^2 = -(l - l')^2, \text{Heavy quark Mass} : m$$




Photon is virtual:

$Q^2 > 0$,

$Q \sim m \gg \Lambda_{QCD}(\sim 300\text{MeV})$

Ranges of DIS variables: Diehl&Sapeta, EPJC41(2005)515

$$0 \leq y_l \leq 2\frac{\sqrt{1+\gamma^2}-1}{\gamma^2}, \gamma = \frac{2x_B M_p}{Q}.$$

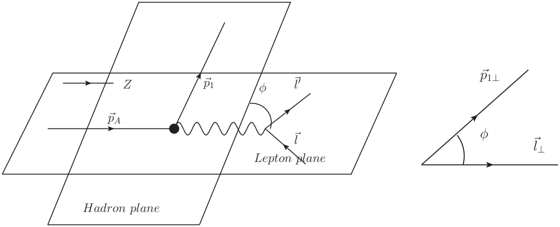
- High energy limit: $Q \gg M_p \Rightarrow 0 \leq y_l \leq 1$.
- Threshold condition: $(p_A + q)^2 \geq 4m^2 \Rightarrow x_B \leq \frac{Q^2}{Q^2+4m^2}$.
- DIS range:

$$S_{pl} \geq Q^2 + 4m^2, \; \frac{Q^2}{S_{pl}} \leq x_B \leq \frac{Q^2}{Q^2+4m^2}, \; y_l = \frac{Q^2}{x_B S_{pl}}.$$

Hadron variables : $z = \frac{p_A \cdot p_1}{p_A \cdot q} > 0$, $y = \frac{q \cdot p_1}{p_A \cdot q}$, $x = x_B$

**Azimuthal angle :** $\phi$

$\gamma^* N$ frame




Light-cone coordinates: $a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$, $a^\mu = (a^+, a^-, a^\mu_\perp)$,

$a^2 = 2a^+a^- + a^2_\perp$, $a^2_\perp = -\vec{a}^2_\perp < 0$. $q^\mu, p^\mu_A$ are longitudinal.

$(y, z)$ is equivalent to $(p^2_{1\perp}, Y)$:

$$z = e^{-Y} \frac{E_{1\perp}}{Q} \sqrt{\frac{x}{1-x}}, \; y = -xz + \frac{x}{z} \frac{E^2_{1\perp}}{Q^2},$$

$$Y = \frac{1}{2} \ln \frac{p^0_1 + p^z_1}{p^0_1 - p^z_1}, \; E_{1\perp} = \sqrt{m^2 + \vec{p}^2_{1\perp}}$$

Threshold: $p_X^2 = (p_A + q - p_1)^2 \geq m^2 \Rightarrow 0 < x + y + z \leq 1 \Rightarrow$

$$E_{1\perp} \leq \frac{Q}{2}\sqrt{\frac{1-x}{x}},\ \rho_\perp \equiv \sqrt{1 - \frac{4x}{1-x}\frac{E_{1\perp}^2}{Q^2}}$$

$$\frac{1-\rho_\perp}{2} \leq z \leq \frac{1-\rho_\perp}{2}, \text{or } \frac{1}{2}\ln\frac{1-\rho_\perp}{1+\rho_\perp} \leq Y \leq \frac{1}{2}\ln\frac{1+\rho_\perp}{1-\rho_\perp}$$

Range of $z$ is symmetric about $z = 1/2$.

DIS range:

$$S_{pl} \geq Q^2 + 4m^2,\ \frac{Q^2}{S_{pl}} \leq x_B (= x) \leq \frac{Q^2}{Q^2 + 4m^2},\ y_l = \frac{Q^2}{x_B S_{pl}}$$

Differential cross section(5 dimension):

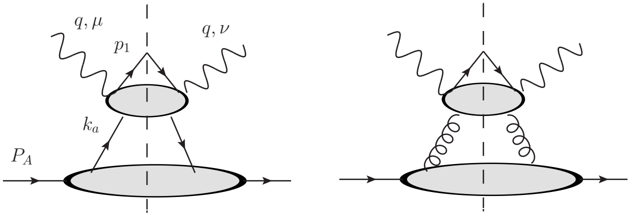
$$\frac{d\sigma}{dx_B dQ^2 dY d^2 p_{1\perp}} = \frac{\alpha_{em}^2}{32\pi^3 x_B^2 S_{pl}^2 Q^2} L_{\mu\nu} W^{\mu\nu}.$$

$$L^{\mu\nu} = \sum_{\lambda'} \bar{u}(l', \lambda')\gamma^\mu u(l, \lambda_l)\bar{u}(l, \lambda_l)\gamma^\nu u(l', \lambda')$$
$$= 2[l'^\mu l^\nu + l'^\nu l^\mu - l' \cdot l g^{\mu\nu} + i\lambda_l \epsilon^{l'\mu l\nu}],$$
$$W^{\mu\nu} = \int d^4\xi e^{iq\cdot\xi} \sum_X \langle P_A \lambda_h | j^\nu(\xi) | p_1 X\rangle\langle X p_1 | j^\mu(0) | P_A \lambda_h\rangle$$

$j^\mu = \sum_q e_q \bar{\psi}\gamma^\mu\psi, \; q = u, d, s, c, b$

Similar process: Laenen,Riemersma&Smith, NPB392(1993)162, 4-dim, $\phi$ integrated, unpolarized; Hekhorn&Stratmann,2105.13944, Anderle *et al*,2110.04489, $\phi$ integrated, double polarized;

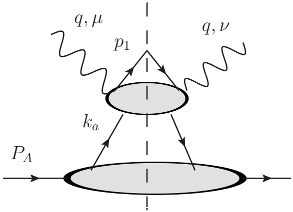
## Twist-2 factorization of hadronic tensor $W^{\mu\nu}$.

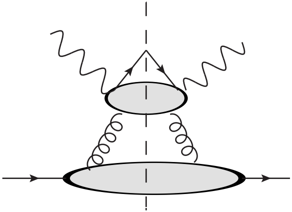



Collinear region: $k_a^\mu = (k_a^+, k_a^-, k_{a\perp}) \sim Q(1, \lambda^2, \lambda)$, $\lambda \ll 1$.

Twist-2: $k_a^\mu \to \hat{k}_a^\mu = (k_a^+, 0, 0) = (x_a p_A^+, 0, 0)$ in hard part.

$$W_q^{\mu\nu} = \int d^4 k_a H_{q,ji}^{\mu\nu}(k_a, q) \int \frac{d^4\xi}{(2\pi)^4} e^{ik_a\cdot\xi} \langle P_A \lambda_h | \bar{\psi}_j(0) \psi_i(\xi) | P_A \lambda_h \rangle$$



$$W_q^{\mu\nu} = \int d^4 k_a H_{q,ji}^{\mu\nu}(k_a, q) \int \frac{d^4\xi}{(2\pi)^4} e^{ik_a\cdot\xi} \langle P_A\lambda_h|\bar{\psi}_j(0)\psi_i(\xi)|P_A\lambda_h\rangle$$

$$\simeq \int dk_a^+ H_{q,ji}^{\mu\nu}(\hat{k}_a, q) \int dk_a^- d^2 k_{a\perp} \int \frac{d^4\xi}{(2\pi)^4} e^{ik_a\cdot\xi} \langle P_A\lambda_h|\bar{\psi}_j(0)\psi_i(\xi)|P_A\lambda_h\rangle$$

$$+ \text{(higher twist)}$$

$$= \int dk_a^+ H_{q,ji}^{\mu\nu}(\hat{k}_a, q) \int \frac{d\xi^-}{2\pi} e^{ik_a^+\xi^-} \langle P_A\lambda_h|\bar{\psi}_j(0)\psi_i(\xi)|P_A\lambda_h\rangle$$

$$+ \text{(higher twist)}$$

Only $k_a^+$ is preserved in twist-2(Leading twist).

Gluon contribution is similar:

$$W_g^{\mu\nu} = \int dk_a^+ H_{g,\beta\alpha}^{\mu\nu}(\hat{k}_a, q) \int \frac{d\xi^-}{2\pi} e^{ik_a^+\xi^-} \langle P_A \lambda_h | G_\perp^\beta(0) G_\perp^\alpha(\xi) | P_A \lambda_h \rangle$$

$$+ \text{(higher twist)}$$

At leading twist, $\alpha, \beta$ are transverse.

PDFs:

$$\int \frac{d\xi^-}{2\pi} e^{ik_a^+\xi^-} \langle P_A \lambda_h | \bar{\psi}_j(0) \psi_i(\xi) | P_A \lambda_h \rangle$$

$$= \frac{(1_c)_{ij}}{2N_c} \left[ \gamma^- f_1^q(x_a) + \gamma_5 \gamma^- \lambda_h g_1^q(x_a) \right]_{ij};$$

$$\int \frac{d\xi^-}{2\pi} e^{ik_a^+\xi^-} \langle P_A \lambda_h | G_{b\perp}^\beta(0) G_{a\perp}^\alpha(\xi^-) | P_A \lambda_h \rangle$$

$$= \frac{\delta_{ab}}{2(N_c^2 - 1) x_a P_A^+} \left[ \frac{2}{2-d} g_\perp^{\alpha\beta} f_1^g(x_a) + \lambda_h (-i) \epsilon_\perp^{\beta\alpha} g_1^g(x_a) \right].$$

$k_a^+ = x_a P_A^+, d = 4 - \epsilon.$

Factorized form of hadronic tensor:

$$W_g^{\mu\nu} = \int \frac{dx_a}{x_a} \bar{H}^{\mu\nu}_{g,\alpha\beta} \Big[ g_\perp^{\beta\alpha} \frac{2}{2-d} f_1^g(x_a) - i\lambda_h \epsilon_\perp^{\beta\alpha} g_1^g(x_a) \Big]$$

$$W_q^{\mu\nu} = \int \frac{dx_a}{x_a} \Big[ \bar{H}_q^{\mu\nu,\alpha\beta} \frac{1}{d-2} g_\perp^{\alpha\beta} f_1^q(x_a) + \Delta \bar{H}_q^{\mu\nu,\alpha\beta} \frac{-i}{4} \epsilon_{\perp,\alpha\beta} \lambda_h g_1^q(x_a) \Big]$$

# Angular distributions: $\phi$ dependence

$\bar{H}^{\mu\nu,\alpha\beta}$ satisfies:

- $\alpha, \beta$ are transverse;
- No $\gamma_5$ or $\epsilon-$tensor;
- QED gauge invariance: $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$ or $q_\mu \bar{H}^{\mu\nu,\alpha\beta} = q_\nu \bar{H}^{\mu\nu,\alpha\beta} = 0$;
- **Depends on only one transverse vector $p_{1\perp}^{\mu}$!**

# Angular distributions: $\phi$ dependence

Tensor structure:

$$\bar{H}^{\mu\nu}_{\alpha\beta} = H_{X_{ij}} X_{ij}^{\mu\nu\alpha\beta} + H_{Y_i} Y_i^{\mu\nu\alpha\beta} + H_{Z_i} Z_i^{\mu\nu\alpha\beta} + H_{V_i} V_i^{\mu\nu\alpha\beta}$$

$H_{X_{ij}}$ etc depend on $p_{1\perp}^2$ only, in addition to $k_a \cdot q$, $k_a \cdot p_1$, $q \cdot p_1$.
Thus, they are $\phi$ independent!

# Angular distributions: $\phi$ dependence

10 $X_{ij}$:

$$X_{ij}^{\mu\nu,\alpha\beta} = \tilde{a}_i^{\mu\nu} \tilde{b}_j^{\alpha\beta},$$

where

$$\tilde{a}_1 = g_\perp^{\mu\nu}, \; \tilde{a}_2 = \frac{1}{p_{1\perp}^2}\left[p_{1\perp}^\mu p_{1\perp}^\nu - \frac{1}{d-2} g_\perp^{\mu\nu} p_{1\perp}^2\right],$$

$$\tilde{a}_3 = p_{1\perp}^\mu \tilde{p}^\nu + p_{1\perp}^\nu \tilde{p}^\mu, \; \tilde{a}_4 = \tilde{p}^\mu \tilde{p}^\nu,$$

$$\tilde{a}_5 = p_{1\perp}^\mu \tilde{p}^\nu - p_{1\perp}^\nu \tilde{p}^\mu,$$

$$\tilde{b}_1 = g_\perp^{\alpha\beta}, \; \tilde{b}_2 = \frac{1}{p_{1\perp}^2}\left[p_{1\perp}^\alpha p_{1\perp}^\beta - \frac{1}{d-2} g_\perp^{\alpha\beta} p_{1\perp}^2\right],$$

where $\tilde{p}^\mu = p_A^\mu - \frac{p_A \cdot q}{q^2} q^\mu$, satisfying $q \cdot \tilde{p} = 0$.

# Angular distributions: $\phi$ dependence

3 $Y_i$:

$$
\begin{aligned}
Y_1^{\mu\nu,\alpha\beta} =& g_\perp^{\mu\alpha} g_\perp^{\nu\beta} - g_\perp^{\mu\beta} g_\perp^{\nu\alpha}, \\
Y_2^{\mu\nu,\alpha\beta} =& \frac{1}{p_{1\perp}^2}\left[\left(g_\perp^{\mu\alpha} p_{1\perp}^{\nu} p_{1\perp}^{\beta} - g_\perp^{\nu\alpha} p_{1\perp}^{\mu} p_{1\perp}^{\beta}\right) - (\alpha \leftrightarrow \beta)\right], \\
Y_3^{\mu\nu,\alpha\beta} =& \frac{1}{p_{1\perp}^2}\left[\left(g_\perp^{\mu\alpha} p_{1\perp}^{\nu} p_{1\perp}^{\beta} - g_\perp^{\nu\alpha} p_{1\perp}^{\mu} p_{1\perp}^{\beta}\right) + (\alpha \leftrightarrow \beta)\right].
\end{aligned}
$$

# Angular distributions: $\phi$ dependence

3 $Z_i$:

$$
\begin{aligned}
Z_1^{\mu\nu,\alpha\beta} =& g_\perp^{\mu\alpha} g_\perp^{\nu\beta} + g_\perp^{\mu\beta} g_\perp^{\nu\alpha} - g_\perp^{\mu\nu} g_\perp^{\alpha\beta} \frac{2}{d-2}, \\
Z_2^{\mu\nu,\alpha\beta} =& \frac{1}{p_{1\perp}^2} \Big[ \Big( g_\perp^{\mu\alpha} p_{1\perp}^\nu p_{1\perp}^\beta + g_\perp^{\nu\alpha} p_{1\perp}^\mu p_{1\perp}^\beta + (\alpha \leftrightarrow \beta) \Big) \\
& - p_{1\perp}^\mu p_{1\perp}^\nu g_\perp^{\alpha\beta} \frac{4}{d-2} \Big], \\
Z_3^{\mu\nu,\alpha\beta} =& \frac{1}{p_{1\perp}^2} \Big[ \Big( g_\perp^{\mu\alpha} p_{1\perp}^\nu p_{1\perp}^\beta + g_\perp^{\nu\alpha} p_{1\perp}^\mu p_{1\perp}^\beta - (\alpha \leftrightarrow \beta) \Big) \Big].
\end{aligned}
$$

# Angular distributions: $\phi$ dependence

4 $V_i$:

$$
\begin{aligned}
V_1^{\mu\nu,\alpha\beta} =& g_\perp^{\mu\alpha}\tilde{p}^\nu p_{1\perp}^\beta + g_\perp^{\nu\alpha}\tilde{p}^\mu p_{1\perp}^\beta + (\alpha \leftrightarrow \beta),\\
V_2^{\mu\nu,\alpha\beta} =& g_\perp^{\mu\alpha}\tilde{p}^\nu p_{1\perp}^\beta - g_\perp^{\nu\alpha}\tilde{p}^\mu p_{1\perp}^\beta - (\alpha \leftrightarrow \beta),\\
V_3^{\mu\nu,\alpha\beta} =& g_\perp^{\mu\alpha}\tilde{p}^\nu p_{1\perp}^\beta - g_\perp^{\nu\alpha}\tilde{p}^\mu p_{1\perp}^\beta + (\alpha \leftrightarrow \beta),\\
V_4^{\mu\nu,\alpha\beta} =& g_\perp^{\mu\alpha}\tilde{p}^\nu p_{1\perp}^\beta + g_\perp^{\nu\alpha}\tilde{p}^\mu p_{1\perp}^\beta - (\alpha \leftrightarrow \beta).
\end{aligned}
$$

# Angular distributions: $\phi$ dependence

$$W_g^{\mu\nu} = \int \frac{dx_a}{x_a} \bar{H}_{g,\alpha\beta}^{\mu\nu} \Big[ g_\perp^{\beta\alpha} \frac{2}{2-d} f_1^g(x_a) - i\lambda_h \epsilon_\perp^{\beta\alpha} g_1^g(x_a) \Big]$$

$$W_q^{\mu\nu} = \int \frac{dx_a}{x_a} \Big[ \bar{H}_q^{\mu\nu,\alpha\beta} \frac{1}{d-2} g_\perp^{\alpha\beta} f_1^q(x_a) + \Delta \bar{H}_q^{\mu\nu,\alpha\beta} \frac{-i}{4} \epsilon_{\perp,\alpha\beta} \lambda_h g_1^q \Big]$$

$$L^{\mu\nu} W_{\mu\nu} \sim H_{X_{ij}} X_{ij}^{\mu\nu,\alpha\beta} L_{\mu\nu} \{ g_{\perp\alpha\beta}, \epsilon_{\perp\alpha\beta} \} + \cdots$$

After contracted with leptonic tensor $L^{\mu\nu}$, which contains $l_\perp^\mu$, $X_{ij}$ etc give all possible $\phi$ distributions.

9 of these tensors give nonzero distributions, for **unpolarized** or **longitudinally polarized** proton. They are:

$$X_{11}, X_{21}, X_{31}, X_{41}, X_{51}, Y_2, Z_3, V_2, V_4.$$

# Angular distributions: $\phi$ dependence

$$L_{\mu\nu}W^{\mu\nu}\Big|_{UU}$$
$$= Q^2 \int \frac{dx_a}{x_a}\Big\{\vec{a}_U \cdot \vec{b}_{X_{11}} \frac{-4(y_l^2 - 2y_l + 2)}{y_l^2} + \vec{a}_U \cdot \vec{b}_{X_{41}} \frac{2(1-y_l)Q^2}{(\tilde{p}^2)^2 x_B^2 y_l}$$
$$+ \vec{a}_U \cdot \vec{b}_{X_{21}} \frac{4(y_l - 1)}{y_l} \cos 2\phi$$
$$+ \vec{a}_U \cdot \vec{b}_{X_{31}} \frac{4(y_l-2)Q}{|\vec{p}_{1\perp}|\tilde{p}^2} \frac{\sqrt{1-y_l}}{x_B y_l^2} \cos\phi\Big\},$$

where $\vec{a} \cdot \vec{b}_{X_{11}}$ is the combination of gluon PDF and quark PDF.

For example,

$$\vec{a} \cdot \vec{b}_{X_{11}} = \frac{1}{(2-\epsilon)^2}\Big[\frac{2X_{11}^{\mu\nu\alpha\beta}}{\epsilon - 2} H^g_{\mu\nu\alpha\beta} f_1^g(x_a) + \frac{X_{11}^{\mu\nu\alpha\beta}}{2-\epsilon} H^q_{\mu\nu\alpha\beta} f_1^q(x_a)\Big]$$

# Angular distributions: $\phi$ dependence

$$
\begin{aligned}
L_{\mu\nu}W^{\mu\nu}\Big|_{LL} =& -4Q^2\lambda_l\lambda_h\int\frac{dx_a}{x_a}\Big\{\vec{a}_L\cdot\vec{b}_{Y_2}\frac{y_l-2}{y_l} \\
&+\vec{a}_L\cdot\vec{b}_{V_2}\frac{Q}{|\vec{p}_{1\perp}|\tilde{p}^2}\frac{\sqrt{1-y_l}}{x_B y_l}\cos\phi\Big\}, \\
L_{\mu\nu}W^{\mu\nu}\Big|_{UL} =& i\lambda_h Q^2\int\frac{dx_a}{x_a}\Big\{\vec{a}_L\cdot\vec{b}_{Z_3}\frac{8(1-y_l)}{y_l}\sin 2\phi \\
&+\vec{a}_L\cdot\vec{b}_{V_4}\frac{Q|\vec{p}_{1\perp}|}{p_{1\perp}^2\tilde{p}^2}\frac{4(y_l-2)\sqrt{1-y_l}}{x_B y_l^2}\sin\phi\Big\}, \\
L_{\mu\nu}W^{\mu\nu}\Big|_{LU} =& 4i\lambda_l\frac{Q^3}{|\vec{p}_{1\perp}|\tilde{p}^2}\frac{\sqrt{1-y_l}}{x_B y_l}\int\frac{dx_a}{x_a}\Big\{\vec{a}_U\cdot\vec{b}_{X_{51}}\sin\phi\Big\},
\end{aligned}
$$

# Angular distributions: $\phi$ dependence

Known semi-inclusive DIS results: Diehl&Sapeta, EPJC41(2005)515

$$\frac{d\sigma}{dx_B dQ^2 d\phi d\psi} \sim \frac{\sigma_{++}^{++} + \sigma_{++}^{--}}{2} + \varepsilon \sigma_{00}^{++} - \varepsilon \cos(2\phi) \mathrm{Re} \sigma_{+-}^{++}$$
$$- \sqrt{\varepsilon(1+\varepsilon)} \cos\phi \mathrm{Re}(\sigma_{+0}^{++} + \sigma_{+0}^{--})$$
$$- P_l \sqrt{\varepsilon(1-\varepsilon)} \sin\phi \mathrm{Im}(\sigma_{+0}^{++} + \sigma_{+0}^{--})$$
$$- S_L \left[ \varepsilon \sin(2\phi) \mathrm{Im} \sigma_{+-}^{++} + \sqrt{\varepsilon(1+\varepsilon)} \sin\phi \mathrm{Im}(\sigma_{+0}^{++} - \sigma_{+0}^{--}) \right]$$
$$+ S_L P_l \left[ \sqrt{1-\varepsilon^2} \frac{\sigma_{++}^{++} - \sigma_{++}^{--}}{2} - \sqrt{\varepsilon(1-\varepsilon)} \cos\phi \mathrm{Re}(\sigma_{+0}^{++} - \sigma_{+0}^{--}) \right].$$

# Angular distributions: $\phi$ dependence

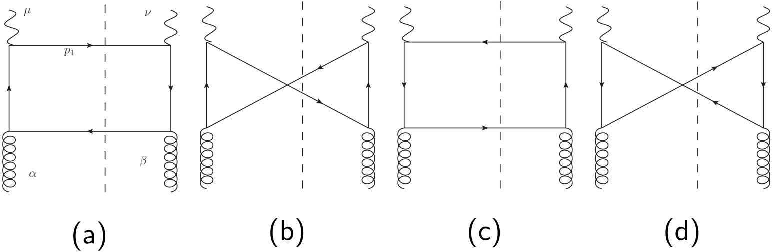
Comparison with Diehl&Sapeta, EPJC41(2005)515

1. UU case:

$$\vec{a}_U \cdot \vec{b}_{X_{11}} \sim \frac{1}{2}(\sigma^{++}_{++} + \sigma^{--}_{++}),\ \vec{a}_U \cdot \vec{b}_{X_{41}} \sim \sigma^{++}_{00},$$
$$\vec{a}_U \cdot \vec{b}_{X_{21}} \sim \mathrm{Re}\sigma^{++}_{+-},\ \vec{a}_U \cdot \vec{b}_{X_{31}} \sim \mathrm{Re}(\sigma^{++}_{+0} + \sigma^{--}_{+0}).$$

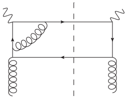
2. UL case: $\vec{a}_L \cdot \vec{b}_{Z_3} \sim \mathrm{Im}\sigma^{++}_{+-},\ \vec{a}_L \cdot \vec{b}_{V_4} \sim \mathrm{Im}(\sigma^{++}_{+0} - \sigma^{--}_{+0})$
3. LU case: $\vec{a}_U \cdot \vec{b}_{X_{51}} \sim \mathrm{Im}(\sigma^{++}_{+0} + \sigma^{--}_{+0})$
4. LL case:
$\vec{a}_L \cdot \vec{b}_{Y_{12}} \sim \frac{1}{2}(\sigma^{++}_{++} - \sigma^{--}_{++}),\ \vec{a}_L \cdot \vec{b}_{V_2} \sim \mathrm{Re}(\sigma^{++}_{+0} - \sigma^{--}_{+0})$

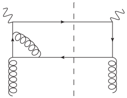
# Tree Diagrams

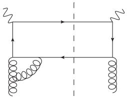


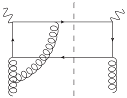

Tree level diagrams in DIS, contributing to heavy quark production.

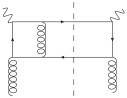
# Virtual corrections

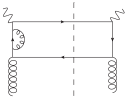

(a)

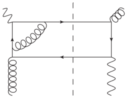

(b)

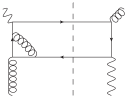

(c)

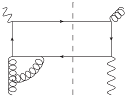

(d)(No reverse)

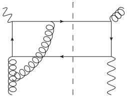

(e)

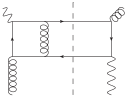

(f)

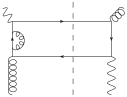

(g)


(h)

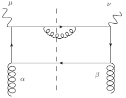

(i)


(j)(No reverse)

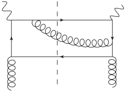

(k)


(l)

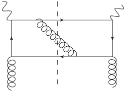
# Real corrections



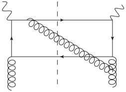

(a)(No cc)



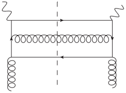
(b)



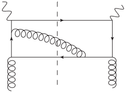
(c)



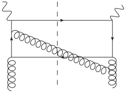
(d)



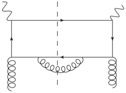
(e)(No cc)



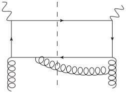
(f)



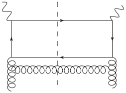
(g)



(h)(No cc)

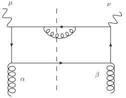


(i)

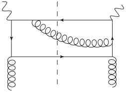


(j)(No cc)

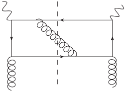
# Real corrections



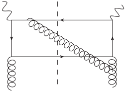

(a)(No cc)



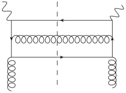
(b)



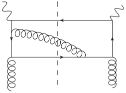
(c)



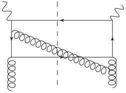
(d)



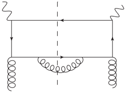
(e)(No cc)



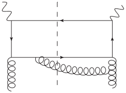
(f)



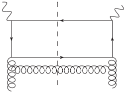
(g)



(h)(No cc)

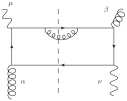


(i)

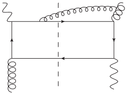


(j)(No cc)

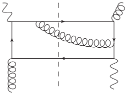
# Real corrections



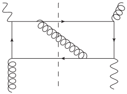

(a)



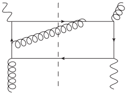
(b)



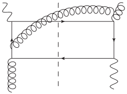
(c)



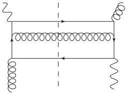
(d)



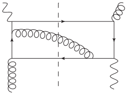
(e)



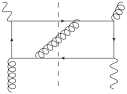
(f)



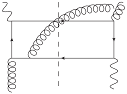
(g)



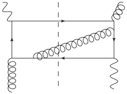
(h)



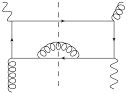
(i)



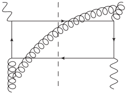
(j)



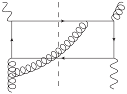
(k)



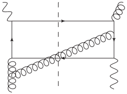
(l)



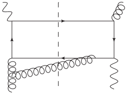
(m)



(n)

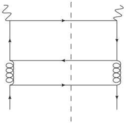


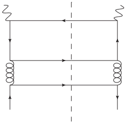
(o)

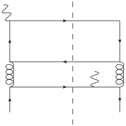


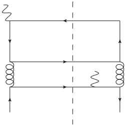
(p)

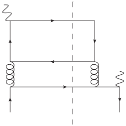
# Real corrections

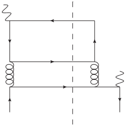

(a)(No cc)

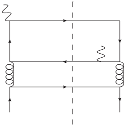

(b)(No cc)

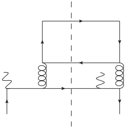

(c)

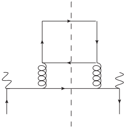

(d)

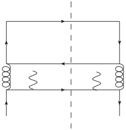

(e)


(f)


(g)


(h)


(i)(No cc)


(j)(No cc)

# Loop integral reduction

FIRE: A.Smirnov&V.Smirnov,1302.5885, by IBP(Integration by part)

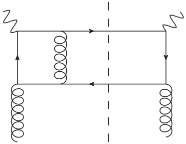
$$0 = \int d^n l \frac{\partial}{\partial l^\mu} f(l, p_i).$$

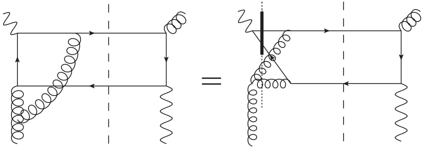
Virtual loops: Tensor integrals are reduced to linear combination of standard A,B,C,D functions;

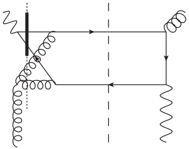
$$\int \frac{d^n k_g}{N_1^{i_1} N_2^{i_2} N_3^{i_3} N_4^{i_4}} \to \int \frac{d^n k_g}{N_1 N_2 N_3 N_4}, \int \frac{d^n k_g}{N_1 N_2 N_3}, \int \frac{d^n k_g}{N_1 N_2}, \int \frac{d^n k_g}{N_1}.$$

All A,B,C,D functions are recalculated and then continued to DIS region, in order to extract the imaginary parts of them. Results are consistent with Eills& Zanderighi, JHEP02(2008)002

Only Abelian diagrams have imaginary parts:

Im  $\neq 0$.



Im  $= 0$ : the cut is unphysical

# Loop integral reduction

Real loops:

$$\int d^n k_g \delta(N_3)\delta(N_4)\frac{1}{N_1^{i_1} N_2^{i_2}},\ \ i_1, i_2 \neq 0,\ \ N_3 = k_g^2, N_4 = (W - k_g)^2 - m^2.$$

Formal Replacement

$$\delta(N_3) \to \frac{i}{2\pi}\Big[\frac{1}{k_g^2 + i\epsilon} - \frac{1}{k_g^2 - i\epsilon}\Big],$$
$$\delta(N_4) \to \frac{i}{2\pi}\Big[\frac{1}{(W - k_g)^2 - m^2 + i\epsilon} - \frac{1}{(W - k_g)^2 - m^2 - i\epsilon}\Big]$$

Then, the integral becomes

$$\int d^n k_g \frac{1}{N_1^{i_1} N_2^{i_2} N_3 N_4}$$ : reduction like virtual integrals.

# Loop integral reduction

Real loops:

$$\int d^n k_g \delta(N_3)\delta(N_4)\frac{1}{N_1^{i_1} N_2^{i_2}}$$
$$\to \int \frac{d^n k_g}{N_1^{i_1} N_2^{i_2} N_3 N_4}$$
$$\to \int \frac{d^n k_g}{N_1 N_2 N_3 N_4}, \int \frac{d^n k_g}{N_1 N_3 N_4}, \int \frac{d^n k_g}{N_2 N_3 N_4}, \int \frac{d^n k_g}{N_3 N_4}$$
$$\to \int d^n k_g \frac{\delta(N_3)\delta(N_4)}{N_1 N_2}, \int d^n k_g \frac{\delta(N_3)\delta(N_4)}{N_1}, \int d^n k_g \frac{\delta(N_3)\delta(N_4)}{N_2},$$
$$\int d^n k_g \delta(N_3)\delta(N_4).$$

If $1/N_3$ or $1/N_4$ is absent, the reduced integral is identified as zero.

Calculation of these master integrals are done in the frame $\vec{W}=0$, with $W^\mu = k_a + q - p_1$. Beenakker *et al*, PRD40(1989)1;Zhang, EPJC80(2020)345;

$$\int \frac{d^n k_g}{(2\pi)^{n-2}} \frac{\delta(k_g^2)\delta((W-k_g)^2-m^2)}{[(k_g+p_1)^2-m^2](k_g-k_a)^2} \sim \int \frac{(k_g^0)^{-1-\epsilon} d\Omega_{n-1}}{[\Delta_3 - \hat{p}_1\cdot\hat{k}_g][1-\hat{k}_a\cdot\hat{k}_g]},$$

$|\hat{p}_1| = |\hat{k}_a| = |\hat{k}_g| = 1$, $\Delta_3 = p_1^0/|\vec{p}_1| > 1$. $k_g^0 = (W^2-m^2)/(2\sqrt{W^2})$.

Soft divergence: $k_g^0 \to 0$;

$$(k_g^0)^{-1-\epsilon} = \left(\frac{k_a\cdot q}{\sqrt{W^2}}\right)^{-1-\epsilon}(\tau_x)^{-1-\epsilon},\ 0\le\tau_x<1,$$

$$(\tau_x)^{-1-\epsilon} = \frac{-1}{\epsilon}\delta(\tau_x) + \left(\frac{1}{\tau_x}\right)_+ - \epsilon\left(\frac{\ln\tau_x}{\tau_x}\right)_+.$$

Collinear divergence: $1-\hat{k}_a\cdot\hat{k}_g = 0$, or $\hat{k}_a||\hat{k}_g \to \frac{1}{\epsilon}$.

So, this integral contains a double pole $\frac{1}{\epsilon^2}$.

# Further ingredients:

- UV counter terms in lagrangian: $\overline{\text{MS}}+$ pole mass scheme, $\Sigma(\not{p}) + ct = 0$ if $\not{p} = m$;
- Self-energy corrections;
- Renormalization of PDF,

$$U_{i,tree}(x_a) f^g(x_a)$$
$$\to U_{i,tree}\Big[f^g(x_a) + (\frac{2}{\epsilon_{UV}} - \gamma_E + \ln(4\pi))\frac{\alpha_s}{2\pi}\int_{x_a}^{1}\frac{d\xi}{\xi}P_{gg}(\frac{x_a}{\xi})f^g(\xi)\Big].$$

DGLAP evolution:

$$\frac{\partial f_1^g(x)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}[P_{gg}\otimes f_1^g + P_{gq}\otimes f_1^q],$$
$$\frac{\partial g_1^g(x)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi}[\Delta P_{gg}\otimes g_1^g + \Delta P_{gq}\otimes g_1^q].$$

# Final hard coefficients(finite)

$$L^{\mu\nu}W_{\mu\nu} = \sum_i C_i(y_l, x, p_{1\perp}^2, \phi)\int_{x_{min}}^1 \frac{dx_a}{x_a}\Big[U_i^g f_1^g(x_a) + U_i^q f_1^q(x_a)\Big],$$

$$x_{min} = \frac{x+y}{1-z} = x\Big[1 + \frac{E_{1\perp}^2}{z(1-z)Q^2}\Big].$$

Gluon contribution:

$$U_i^g = \frac{4\pi^2 e_Q^2 \alpha_s}{2(N_c^2-1)}\Big\{\tilde{D}_{i,tree}\delta(\tau_x) + \frac{\alpha_s}{4\pi}\Big[\tilde{D}_i\delta(\tau_x) + \Big(\frac{1}{\tau_x}\Big)_+\tilde{E}_i + \Big(\frac{\ln\tau_x}{\tau_x}\Big)_+\tilde{F}_i + \frac{1}{\tau_x}\tilde{G}_i + \frac{\ln\tau_x}{\tau_x}\tilde{K}_i\Big]\Big\}.$$

$$\int_0^1 dx\Big(f(x)\Big)_+ g(x) = \int_0^1 dx f(x)(g(x) - g(0)).$$

# Final hard coefficients(finite)

Quark contribution:

$$U_i^q = \frac{\pi\alpha_s^2}{2N_c}\Big[e_Q^2\Big(\frac{\tilde{G}_i^{HH}}{\tau_x} + \frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{HH}\Big) + e_q^2\Big(\frac{\tilde{G}_i^{LL}}{\tau_x} + \frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{LL}\Big) + e_Q e_q\Big(\frac{\tilde{G}_i^{HL}}{\tau_x} + \frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{HL}\Big)\Big]$$

Anti-Quark contribution:

$$U_i^{\bar{q}} = \frac{\pi\alpha_s^2}{2N_c}\Big[e_Q^2\Big(\frac{\tilde{G}_i^{HH}}{\tau_x} + \frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{HH}\Big) + e_q^2\Big(\frac{\tilde{G}_i^{LL}}{\tau_x} + \frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{LL}\Big) - e_Q e_q\Big(\frac{\tilde{G}_i^{HL}}{\tau_x} + \frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{HL}\Big)\Big].$$

# Final hard coefficients(finite)

Charge Average: $\frac{1}{2}(d\sigma^{Q}+d\sigma^{\bar{Q}})$,

$$\sum_q f_q(x_a)U_i^q = f^g(x_a)U^g + \frac{\pi\alpha_s^2}{2N_c}\Big\{\Big(\frac{\tilde{G}_i^{HH}}{\tau_x}+\frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{HH}\Big)e_Q^2$$

$$\times\Big[(u+\bar{u})+(d+\bar{d})+(s+\bar{s})\Big]$$

$$+\Big(\frac{\tilde{G}_i^{LL}}{\tau_x}+\frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{LL}\Big)[\frac{4}{9}(u+\bar{u})+\frac{1}{9}(d+\bar{d})+\frac{1}{9}(s+\bar{s})].$$

Charge Asymmetry: $\frac{1}{2}(d\sigma^{Q}-d\sigma^{\bar{Q}})$,

$$\sum_q f_q(x_a)U_i^q = \frac{\pi\alpha_s^2}{2N_c}e_Q\Big\{\Big(\frac{\tilde{G}_i^{HL}}{\tau_x}+\frac{\ln\tau_x}{\tau_x}\tilde{K}_i^{HL}\Big)$$

$$\times\Big[\frac{2}{3}(u-\bar{u})-\frac{1}{3}(d-\bar{d})-\frac{1}{3}(s-\bar{s})\Big]\Big\}.$$

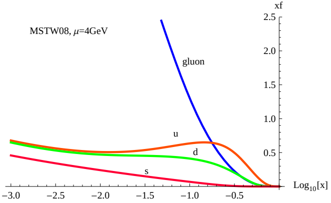
# Numerical results:

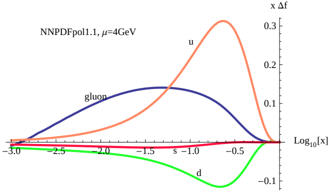
EIcC: $3.5 + 20\text{GeV}$, $\sqrt{S_{pl}} \simeq 16.7\text{GeV}$.

charm: $m = 1.4$, $\mu = 4.0$, $Q = 2.0$, $\alpha_s(\mu) = 0.236$, $\alpha_s(M_Z) = 0.120$

$x$ range: $[0.014, 0.338]$; $p_{1\perp}$ range: $[0, 6.9]$GeV.

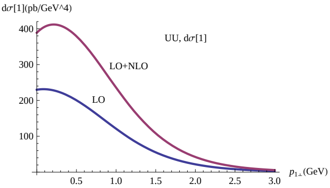
PDF sets: **MSTW08,NLO** and **NNPDFpol1.1,NLO**

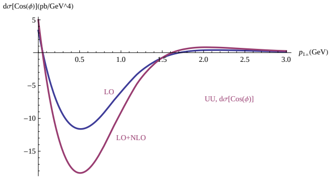


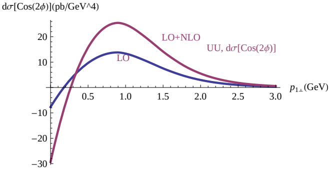



$$\sqrt{S_{pl}} = 16.7,\ Q = 2,\ \mu = 4, x = 0.02, z = 0.4\ p_{1\perp} \in [0, 3]$$

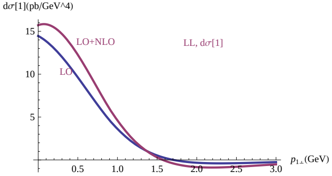
$$\frac{d\sigma}{dx_B dQ^2 dY d^2 p_{1\perp}}$$
$$= f_1^g \otimes [C_1 + C_2 \cos\phi + C_3 \cos 2\phi]$$

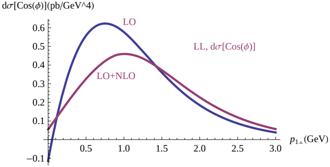


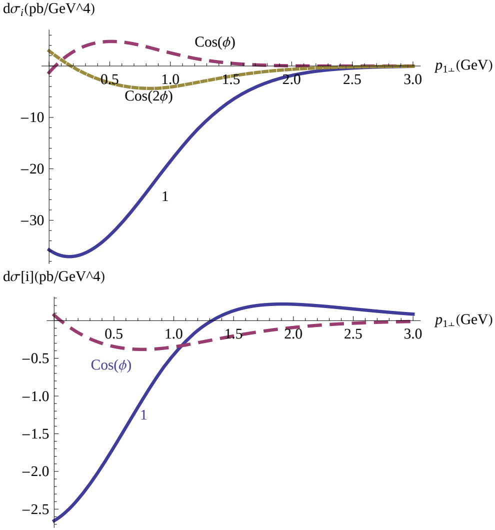
$\sqrt{S_{pl}} = 16.7,\ Q = 2,\ \mu = 4, x = 0.02, z = 0.4\ p_{1\perp} \in [0, 3]$




$$\frac{d\sigma}{dx_B dQ^2 dY d^2 p_{1\perp}} = g_1^g \otimes [C_1 + C_2 \cos\phi]$$

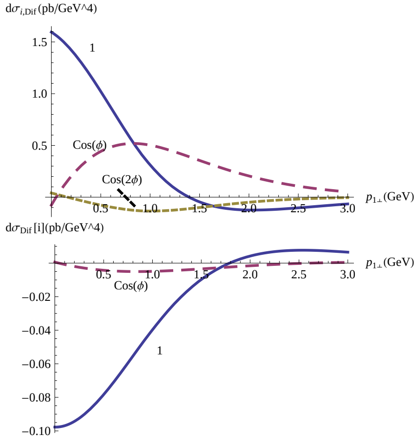
## $\sqrt{S_{pl}} = 16.7,\ Q = 2,\ \mu = 4, x = 0.02, z = 0.4\ p_{1\perp} \in [0, 3]$




Quark, UU.

Quark, LL.

## $\sqrt{S_{pl}} = 16.7,\ Q = 2,\ \mu = 4, x = 0.02, z = 0.4\ p_{1\perp} \in [0, 3]$




Charge Difference, UU.

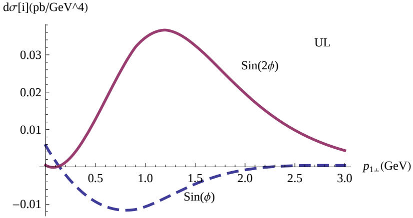
$$\frac{2}{3}(u - \bar{u}) - \frac{1}{3}(d - \bar{d}) - \frac{1}{3}(s - \bar{s})$$

Charge Difference, LL.

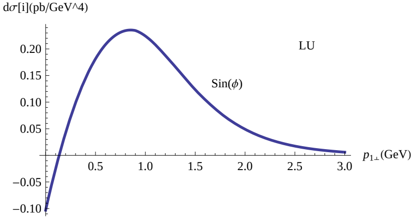
$$\frac{2}{3}(\Delta u - \Delta\bar{u}) - \frac{1}{3}(\Delta d - \Delta\bar{d}) - \frac{1}{3}(\Delta s - \Delta\bar{s})$$

$\sqrt{S_{pl}} = 16.7,\ Q = 2,\ \mu = 4, x = 0.02, z = 0.4\ p_{1\perp} \in [0, 3]$

Single Longitudinal spin asymmetry: imaginary parts of loop integrals. Higher twist correction is of twist-4, because $e(x)$ and $h_L(x)$ are chiral-odd.




$$d\sigma|_{UL} = \sin\phi C_1 \otimes g_1^g + \sin(2\phi) C_2 \otimes g_1^g$$




$$d\sigma|_{LU} = \sin\phi C_1 \otimes f_1^g$$

# Summary and outlook

- All possible Azimuthal angle distributions for longitudinally or unpolarized hadron and lepton beams;
- Give analytic calculations for one-loop QCD correction;
- Some numerical results on EIcC;
- Future work:
  1. Positivity check when $p_{1\perp}$ is large;
  2. Threshold resummation;
  3. Possible generalization to transverse spin case: twist-3.