Resolving negative cross section of quarkonium hadroproduction using soft gluon factorization

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Based on: Chen, Ma, Meng, Phys.Rev.D 108 (2023) 1, 014003

Workshop on Near-Threshold Production of Heavy Quarkonium
19-23 February 2024

Outline

I. Introduction

II. Soft gluon factorization

III. Phenomenological studies

IV. Summary

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Introduction

➢ NRQCD factorization Bodwin, Braaten, Lepage, PRD, 1995

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle\mathcal{O}_n^H\rangle$$

$d\hat{\sigma}_n$: production of a heavy quark pair in state $n(^{2S+1}L_J^{[c]})$.

$\langle\mathcal{O}_n^H\rangle$: the hadronization of $Q\overline{Q}(n)$ to H ;

can be ordered in powers of v;

universality.

➢ A glory history

- Solved IR divergences in P-wave quarkonium decay
- Explained ψ' surplus
- Explained χ_{c2}/χ_{c1} production ratio
- ....

➢ Difficulty

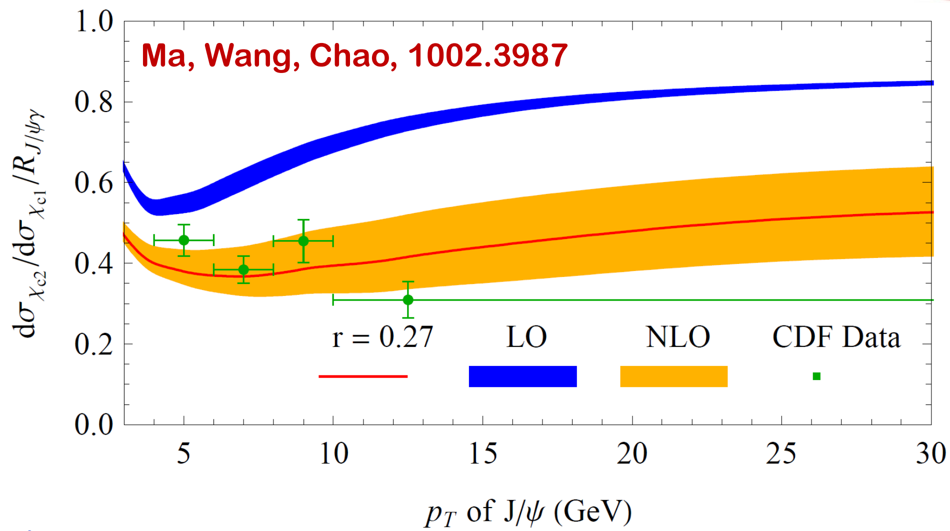
- Polarization puzzle
- Universality problem

➢ Difficulty : negative cross sections

□ Explain χ_{cJ} production

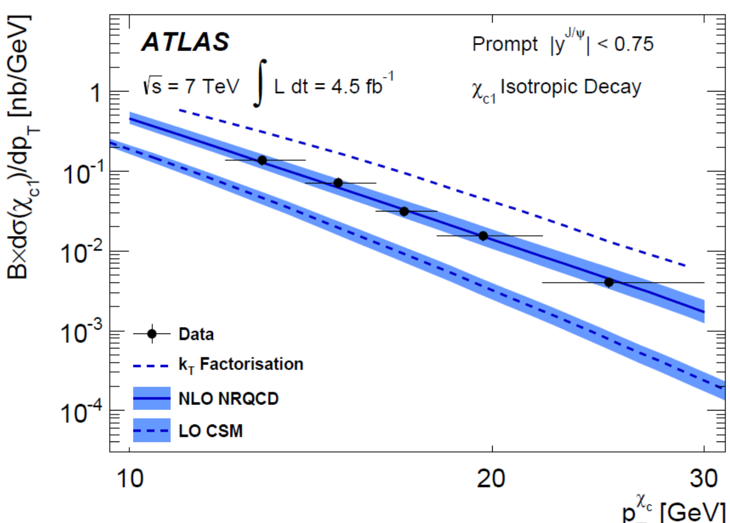
- The ratio $R_{\chi_c} = \sigma_{\chi_{c2}}/\sigma_{\chi_{c1}}$

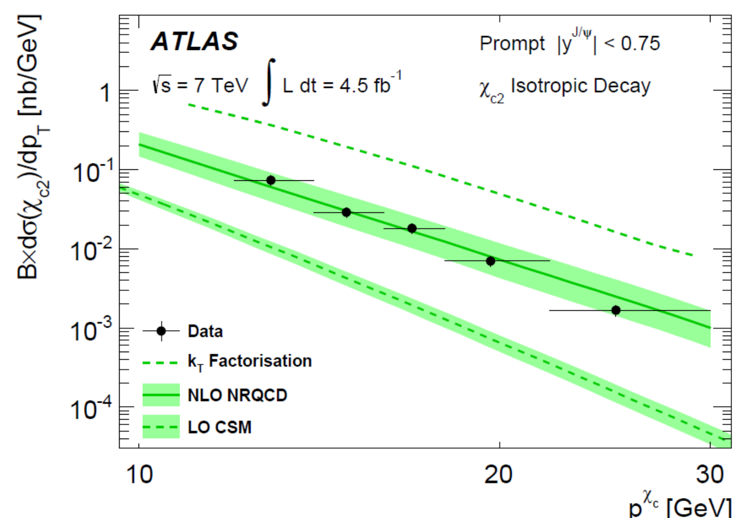
  LO NRQCD: $R_{\chi_c} = 5/3$



- The differential cross sections

ATLAS, 1404.7035

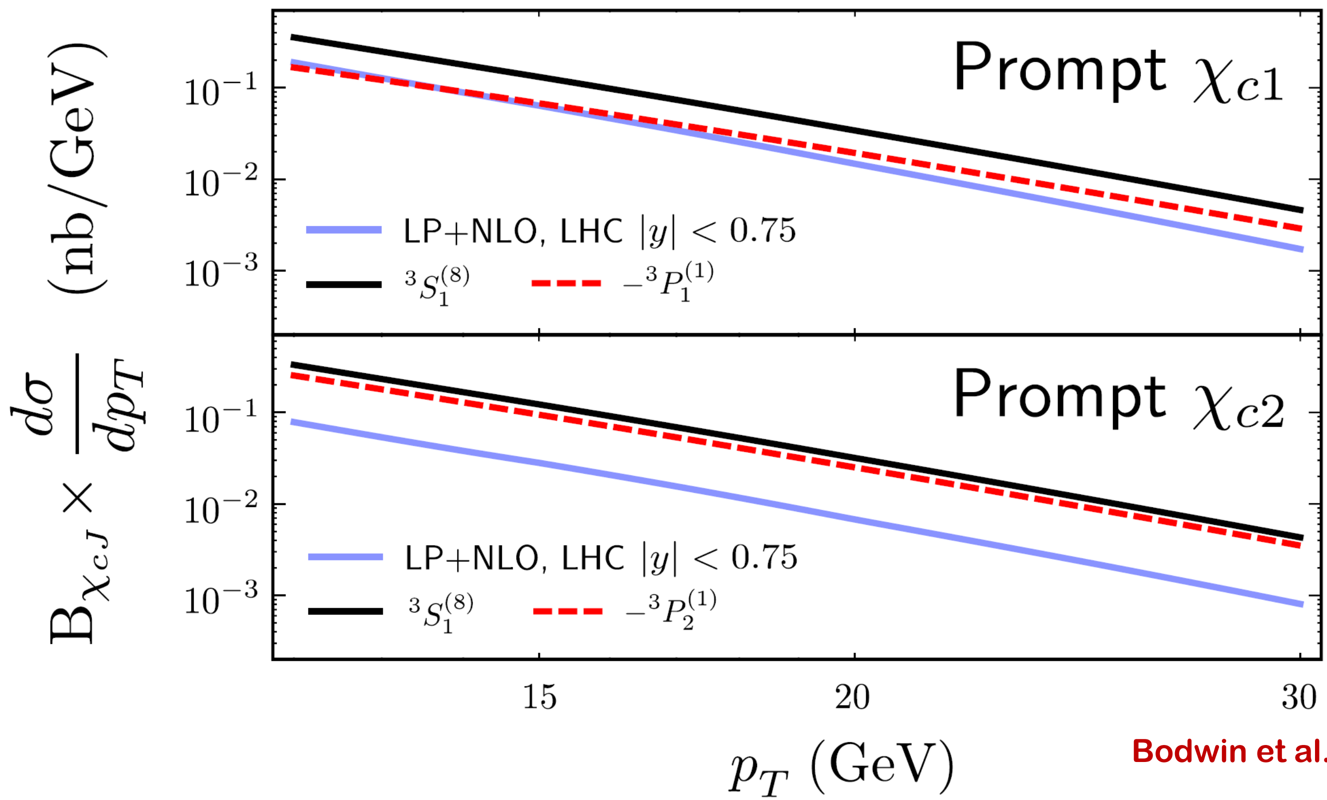




□ There are substantial cancellations between $^3S_1^{[8]}$ and $^3P_J^{[1]}$

$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}[^3S_1^{[8]}]\langle\mathcal{O}^{\chi_{c0}}(^3S_1^{[8]})\rangle + (2J+1)d\hat{\sigma}[^3P_J^{[1]}]\frac{\langle\mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle}{m_c^2}$$

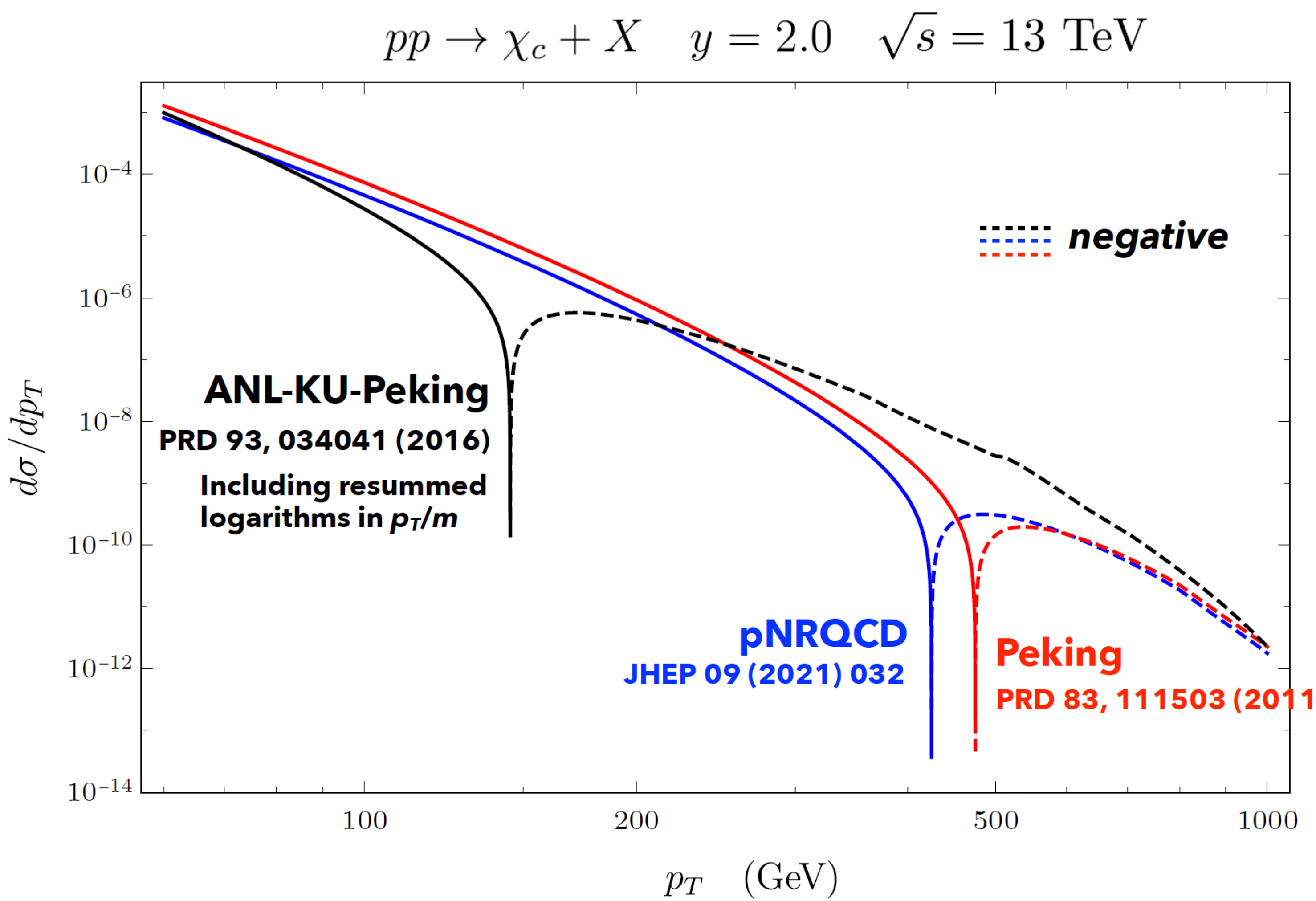
Positive **Negative**



Bodwin et al., 1509.07904

- **Perturbation unstable**

- **Cross sections turn negative at large p_T**



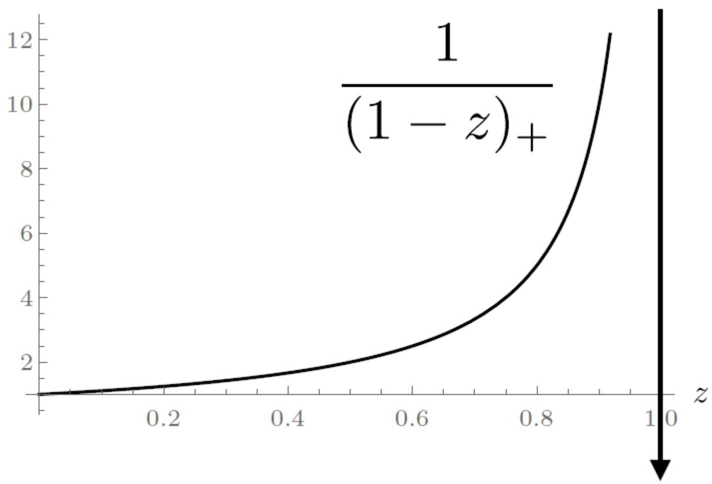
Hee Sok Chung, talk at The 15th International Workshop on Heavy Quarkonium

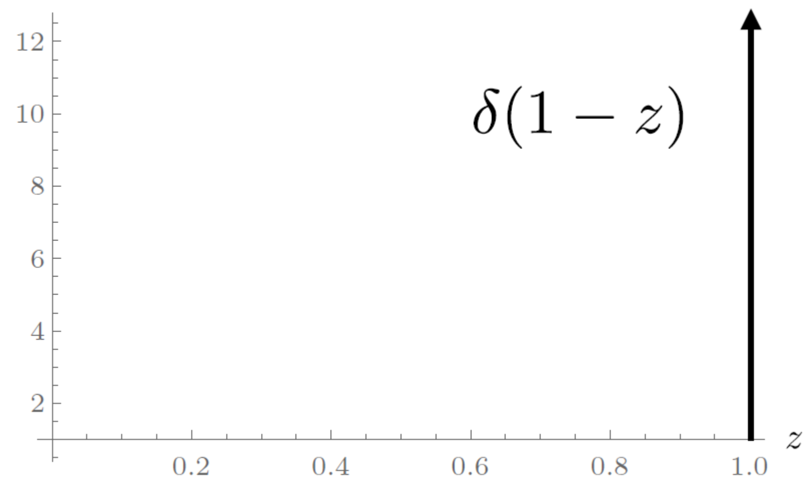
Why?

$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}[^3S_1^{[8]}]\langle\mathcal{O}^{\chi_{c0}}(^3S_1^{[8]})\rangle + (2J+1)d\hat{\sigma}[^3P_J^{[1]}]\frac{\langle\mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle}{m_c^2}$$

$$d\hat{\sigma}[^3P_J^{[1]}] = d\hat{\sigma}_g \otimes \left\{0 \times \alpha_s + \frac{2\alpha_s^2}{27N_c m_c^5}\left[\left(\frac{Q_J}{2J+1} - \log\frac{\Lambda}{2m_c}\right)\delta(1-z) + \frac{z}{(1-z)_+} + \frac{P_J(z)}{2J+1}\right]\right\}$$

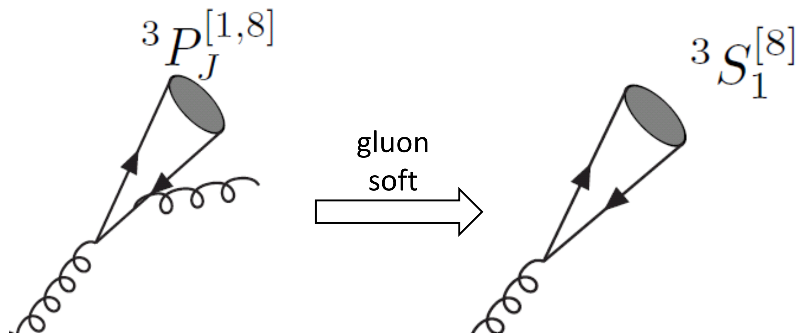
$$d\hat{\sigma}[^3S_1^{[8]}] = d\hat{\sigma}_g \otimes \frac{\pi\alpha_s}{24m_c^3}\delta(1-z)$$





Hee Sok Chung, talk at The 15th International Workshop on Heavy Quarkonium

- **Cross section at very large p_T will depend strongly on $z \to 1$ behavior of FFs**



- **Soft gluon in P-wave: factorized to S-wave matrix element**
- **Plus functions: remnants of the infrared subtraction in matching the $^3P_J^{[1]}$ SDCs**
- **Subtraction scheme: at *zero momentum*, which contributes the largest production rate. Over subtracted!**
- **Solution: soft gluon momentum should be kept during subtraction process, or resum kinematic effects to all powers in v.**

□ Soft gluon factorization: resum a dominant series of power corrections (kinematic effects) and log corrections **Ma, Chao, 1703.08402; Chen, Ma, 2005.08786.**

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Soft gluon factorization(SGF)

➢Factorization

Ma, Chao, 1703.08402;
Chen, Jin, Ma, Meng 2103.15121.

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_{n,n'} \int \frac{dz}{z^2} d\hat{\sigma}_{[nn']}(P_H/z, m_Q, \mu_f) F_{[nn']\to H}(z, M_H, m_Q, \mu_f),$$

- $d\hat{\sigma}_{[nn']}$: perturbatively calculable hard parts
- $F_{[nn']\to H}$: nonperturbative soft gluon distributions (SGDs)
- $n = {}^{2S+1}L_J^{[c]}$
- P_H : momentum of quarkonium
- M_H : mass of quarkonium
- $z = P_H^+/P^+$: the longitudinal momentum fraction with P denoting the total momentum of the intermediate $Q\bar{Q}$ pair

➢Soft gluon distributions (SGDs)

☐ Operator definition

- **Expectation values of bilocal operators in QCD vacuum**

$$F_{[nn']\to H}(z, M_H, m_Q, \mu_f) = P_H^+ \int \frac{db^-}{2\pi} e^{-iP_H^+ b^-/z} \langle 0 | [\bar{\Psi}\mathcal{K}_n \Psi]^\dagger(0) [a_H^\dagger a_H] [\bar{\Psi}\mathcal{K}_{n'}\Psi](b^-)|0\rangle_{\mathrm{S}},$$

with

$$a_H^\dagger a_H = \sum_X \sum_{J_z^H} |H+X\rangle\langle H+X|$$

$$\mathcal{K}_n(rb) = \frac{\sqrt{M_H}}{M_H + 2m} \frac{M_H + \not{P}_H}{2M_H} \Gamma_n \frac{M_H - \not{P}_H}{2M_H} \mathcal{C}^{[c]}$$

Spin project operators:

$$\Gamma_n = \sum_{L_z, S_z} \langle L, L_z; S, S_z | J, J_z \rangle \Gamma^o_{LL_z} \Gamma^s_{SS_z}$$

Color project operators:

$$\mathcal{C}^{[1]} = \frac{\mathbf{1}_c}{\sqrt{N_c}} \qquad \mathcal{C}^{[8]} = \sqrt{2} t^{\bar{a}} \, \Phi^{(A)}_{a\bar{a}}(rb)$$

Gauge link

Nayak, Qiu, Sterman, 0509021

$$\Phi_l(rb^-) = \mathcal{P}\exp\left[-ig_s\int_0^\infty \mathrm{d}\xi l\cdot A(rb^-+\xi l)\right],$$

Evaluated in ***small*** region

- **Subscript "S": evaluate the matrix element in the region where off-shellness of all particles is much smaller than heavy quark mass**

FFs in SGF

- $D_{f\to H}$: **single parton FFs**
- $\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$: **double parton FFs**
- $\hat{z} = z/x$

$$\begin{aligned} D_{f\to H}&(z,\mu_0) \\ &= \sum_{n,n'}\int\frac{\mathrm{d}x}{x}\hat{D}_{f\to Q\bar{Q}[nn']}(\hat{z};M_H/x,m_Q,\mu_0,\mu_\Lambda) \\ &\times F_{[nn']\to H}(x,M_H,m_Q,\mu_\Lambda), \end{aligned} \tag{2a}$$

$$\begin{aligned} \mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}&(z,\zeta,\zeta',\mu_0) \\ &= \sum_{n,n'}\int\frac{\mathrm{d}x}{x}\hat{\mathcal{D}}_{[Q\bar{Q}(\kappa)]\to Q\bar{Q}[nn']}(\hat{z},\zeta,\zeta';M_H/x,m_Q,\mu_0,\mu_\Lambda) \\ &\times F_{[nn']\to H}(x,M_H,m_Q,\mu_\Lambda), \end{aligned} \tag{2b}$$

Short distance hard parts at LO

$$\hat{D}^{LO,(0)}_{g\to Q\bar{Q}[^3S^{[8]}_{1,T}]}(z, M_H, \mu_0, \mu_\Lambda) = \frac{\pi\alpha_s}{(N_c^2-1)}\frac{8}{M_H^3}\delta(1-z), \quad (9a)$$

$$\hat{D}^{LO,(0)}_{g\to Q\bar{Q}[^1S^{[8]}_{0}]}(z, M_H, \mu_0, \mu_\Lambda) = \frac{8\alpha_s^2}{M_H^3}\frac{N_c^2-4}{2N_c(N_c^2-1)}\left[(1-z)\ln[1-z] - z^2 + \frac{3}{2}z\right], \quad (9b)$$

$$\hat{D}^{LO,(0)}_{g\to Q\bar{Q}[^3P^{[1]}_{0}]}(z; M_H, \mu_0, \mu_\Lambda) = \frac{32\alpha_s^2}{M_H^5 N_c}\frac{2}{9}\left[\frac{1}{36}z(837 - 162z + 72z^2 + 40z^3 + 8z^4) + \frac{9}{2}(5-3z)\ln(1-z)\right], \quad (9c)$$

- **The P-wave short distance hard parts do not include terms proportional to plus distributions**

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Phenomenological studies

➢ Collinear factorization

☐ Heavy quarkonium production at large p_T

$$
\mathrm{d}\sigma_{A+B\to H+X}(p) \approx \sum_{i,j} f_{i/A}(x_1,\mu_F) f_{j/B}(x_2,\mu_F) \Big\{ \sum_f D_{f\to H}(z,\mu_F) \otimes \mathrm{d}\hat{\sigma}_{i+j\to f+X}(\hat{P}/z,\mu_F)
$$
$$
+ \sum_\kappa \mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta,\zeta',\mu_F) \otimes \mathrm{d}\hat{\sigma}_{i+j\to[Q\bar{Q}(\kappa)]+X}(\hat{P}(1\pm\zeta)/2z, \hat{P}(1\pm\zeta')/2z,\mu_F) \Big\},
$$

Kang, Ma, Qiu, Sterman, 1401.0923

☐ Factorization of FFs

- **SGF**
- **NRQCD factorization**

☐ Nonperturbative model for SGDs

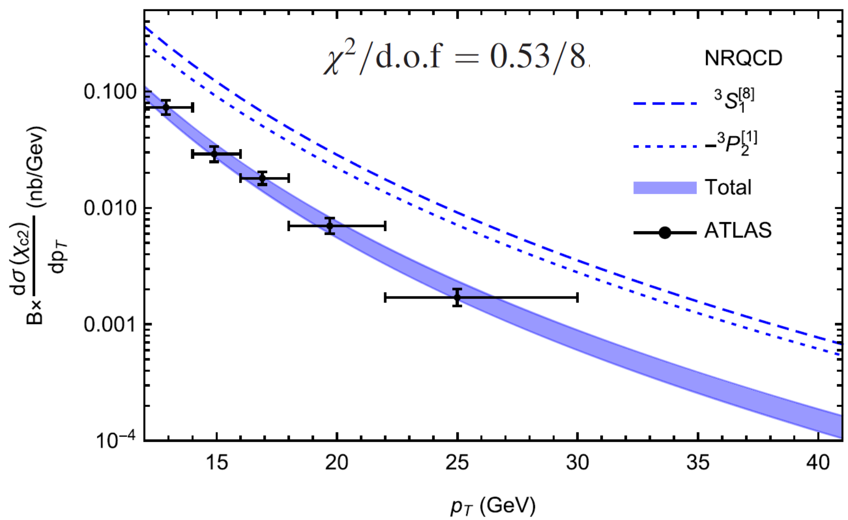
$$
F^{\mathrm{mod}}(x) = \frac{N^H \Gamma(M_H b/\bar{\Lambda})(1-x)^{b-1} x^{M_H b/\bar{\Lambda}-b-1}}{\Gamma(M_H b/\bar{\Lambda}-b)\Gamma(b)}
$$

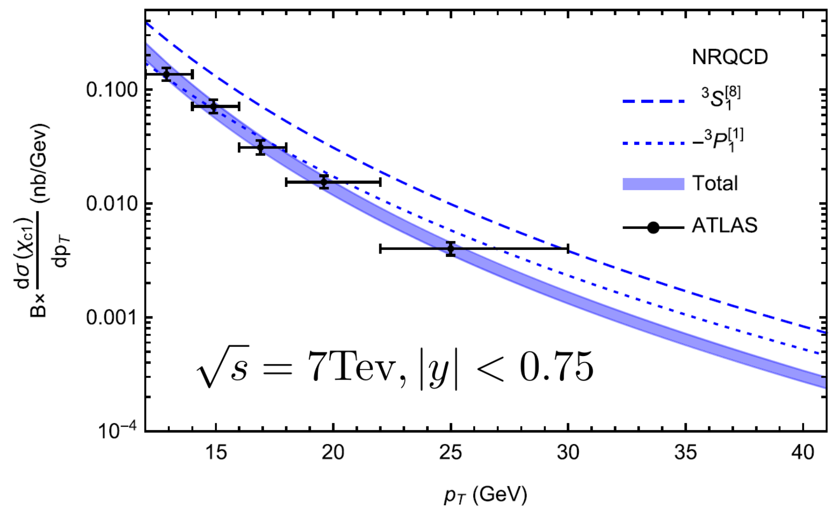
- N^H: **the normalization,** $N^H[n] \approx \langle \mathcal{O}^H(n) \rangle$
- $\bar{\Lambda}$: **the average radiated momentum in the hadronization process**
- b: **related to the second moment of model function**

➢ Production of χ_{cJ}

☐ NRQCD factorization

- **The fitted cross sections compared with ATLAS data**





- **Define the ratio**

$$r(\chi_{c0}) \equiv \frac{\langle \mathcal{O}^{\chi_{c0}}(^3S_1^{[8]})\rangle}{\langle \mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle / m_c^2},$$

- **The cross sections**

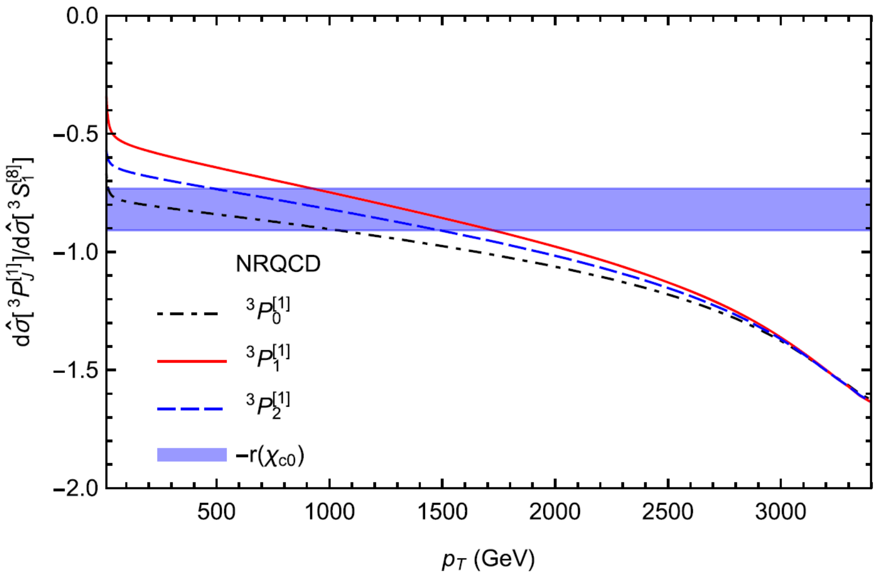
$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}[^3S_1^{[8]}]\frac{\langle \mathcal{O}^{\chi_{c0}}(^3P_0^{[1]})\rangle}{m_c^2}\left[r(\chi_{c0}) + \frac{d\hat{\sigma}[^3P_J^{[1]}]}{d\hat{\sigma}[^3S_1^{[8]}]}\right].$$

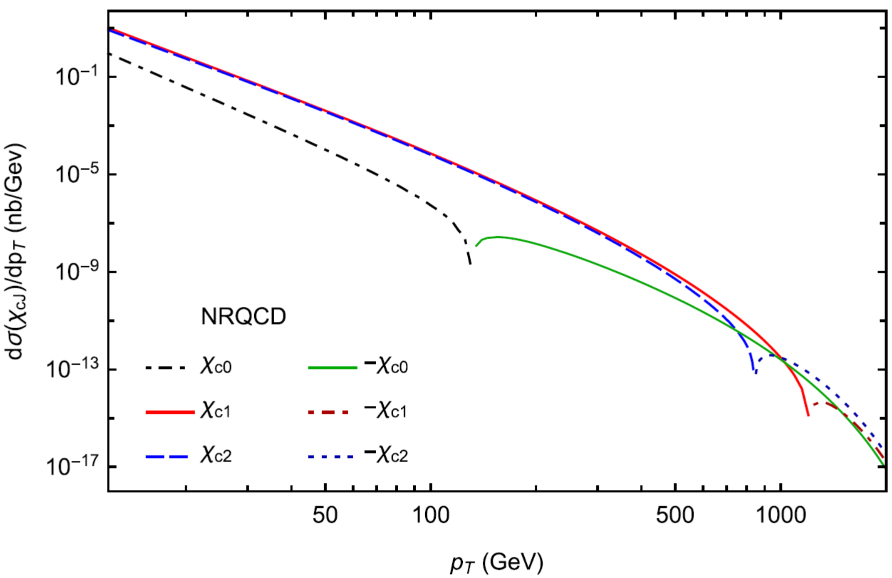
- To achieve a positive cross section, it is necessary to have

$$\frac{d\hat{\sigma}[^3P_J^{[1]}]}{d\hat{\sigma}[^3S_1^{[8]}]} > -r(\chi_{c0}).$$

- Left: comparison between the ratios and $-r(\chi_{c0})$

  Right: the p_T distributions when the LDMEs take the central values





- The ratios fall below the lower bound of $-r(\chi_{c0})$ at very large p_T

SGF

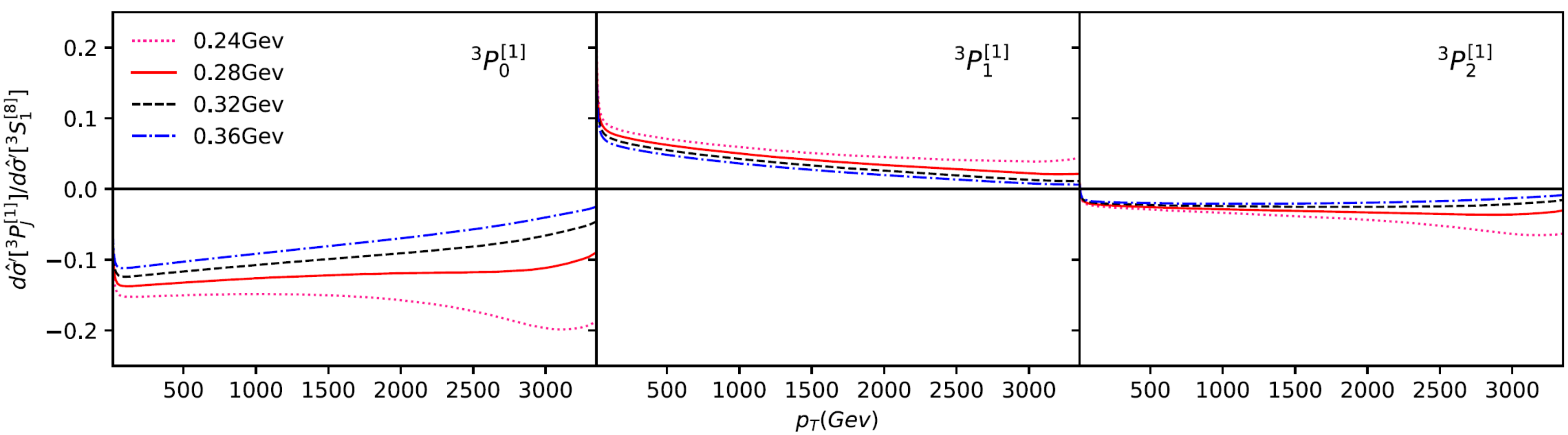
- **The cross sections**

$$d\sigma(\chi_{cJ}) = (2J+1)d\hat{\sigma}'[^3S_1^{[8]}]\frac{N^{\chi_{c0}}[^3P_0^{[1]}]}{m_c^2}\left[r'(\chi_{c0}) + \frac{d\hat{\sigma}'[^3P_J^{[1]}]}{d\hat{\sigma}'[^3S_1^{[8]}]}\right].$$

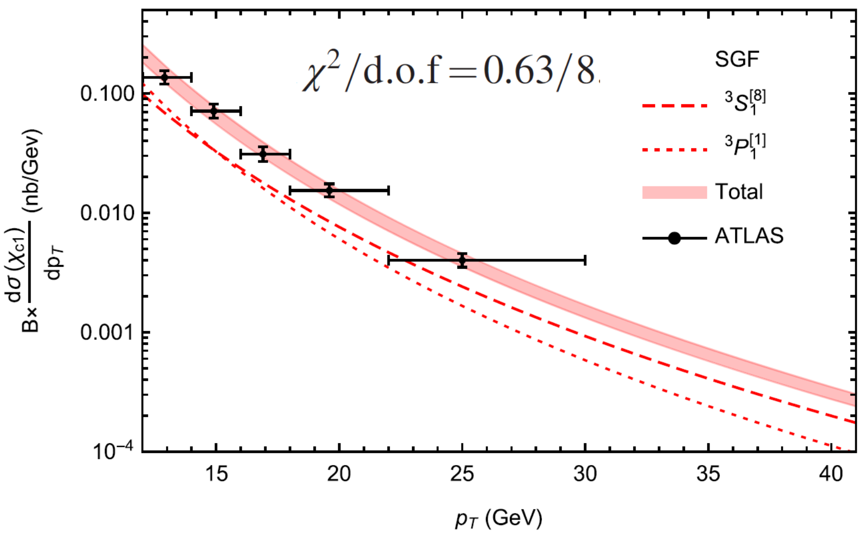
with

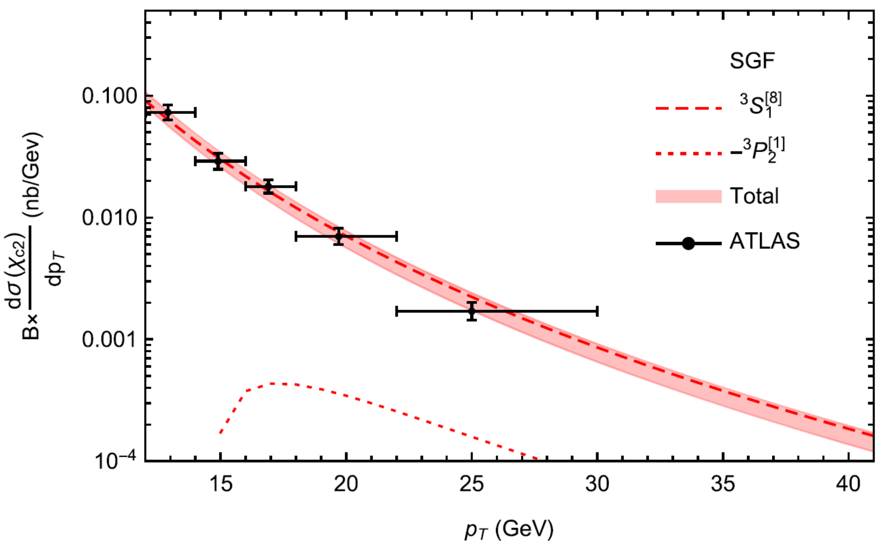
$$r'(\chi_{c0}) \equiv \frac{N^{\chi_{c0}}[^3S_1^{[8]}]}{N^{\chi_{c0}}[^3P_0^{[1]}]/m_c^2}.$$

- $d\hat{\sigma}'[^3P_J^{[1]}]/d\hat{\sigma}'[^3S_1^{[8]}]$ **is sensitive to the parameters** $\bar{\Lambda}$
- **Fix** $\bar{\Lambda}\left[{}^3S_1^{[8]}\right] = 0.4\text{Gev}$ **and vary** $\bar{\Lambda}\left[{}^3\mathrm{P}_J^{[1]}\right] = 0.36, 0.32, 0.28, 0.24\text{Gev}$



- A constraint relation is suggested: $\bar{\Lambda}[^3P_J^{[1]}] \geq 0.7\bar{\Lambda}[^3S_1^{[8]}]$
- We set $\bar{\Lambda}\left[\,^3S_1^{[8]}\right] = 0.4\mathrm{Gev}$ and $\bar{\Lambda}\left[\,^3\mathrm{P}_J^{[1]}\right] = 0.3\mathrm{Gev}$
- The fitted cross sections compared with ATLAS data

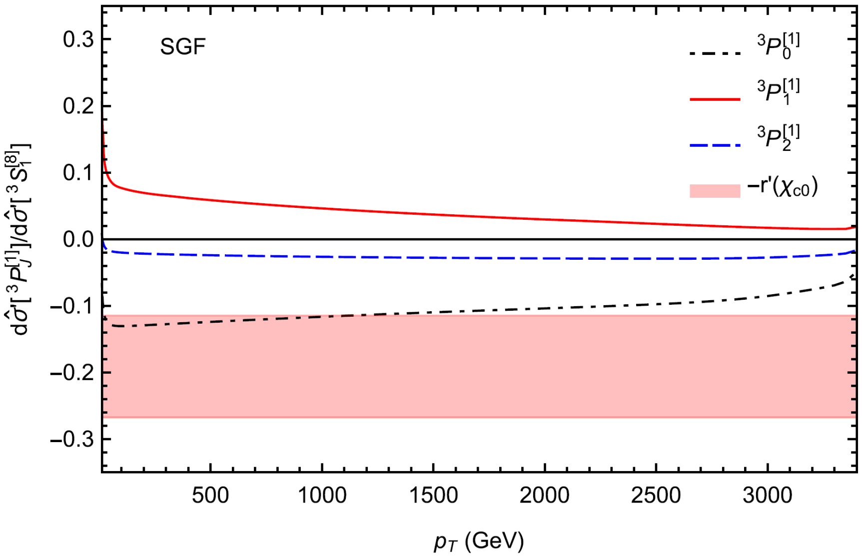


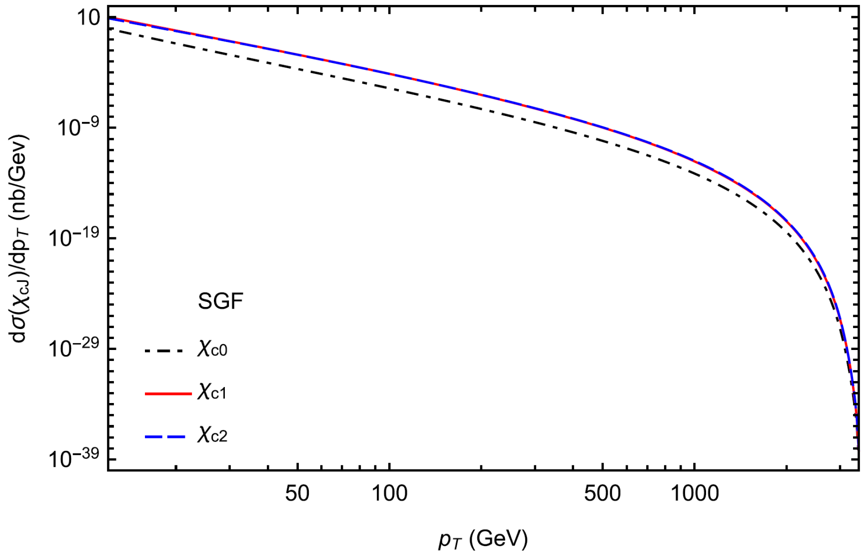


- The fit to experimental data is as good as that in NRQCD factorization

- **Left: comparison between the ratios and $-r'(\chi_{c0})$**

  Right: the p_T distributions when the parameters take the central values





- **There is a wide range of $r'(\chi_{c0})$ in which the ratios is larger than $-r'(\chi_{c0})$**
- **The negative cross section problem is resolved in SGF**

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Summary

- We studied the hadroproduction of χ_{cJ} using the SGF and NRQCD factorization;
- We confirm that the NRQCD predictions for χ_{cJ} production rates at the LHC turn negative at sufficiently large p_T ;
- Our results show that the fit to experimental data in SGF is as good as that in NRQCD factorization;
- Our results show that the negative cross section problem in NRQCD can be resolved in SGF;
- It will be very useful to apply SGF to study the polarizations of ψ(ns) production at LHC in the future.

Thank you!