Interaction of fully heavy hadrons

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Long-term fruitful collaboration with my colleagues and co-authors is gratefully acknowledged!

- X. K. Dong, V. Baru, F. K. Guo, C. Hanhart, A. Nefediev, "Coupled-Channel Interpretation of the LHCb Double-J/ψ Spectrum and Hints of a New State Near the $J/\psi J/\psi$ Threshold," Phys. Rev. Lett. **126**, 132001 (2021)
- X. K. Dong, V. Baru, F. K. Guo, C. Hanhart, A. Nefediev, B. S. Zou, "Is the existence of a $J/\psi J/\psi$ bound state plausible?," Sci. Bull. **66**, 2462 (2021)
- X. K. Dong, F. K. Guo, A. Nefediev, J. T. Castellà, "Chromopolarizabilities of fully-heavy baryons," Phys. Rev. D **107**, 034020 (2023)

Motivation: Near-threshold fully heavy exotic states

Why to study?

- Encodes information on internal structure of heavy hadrons
- Relevant for molecular model
- Allows making predictions for exotic states

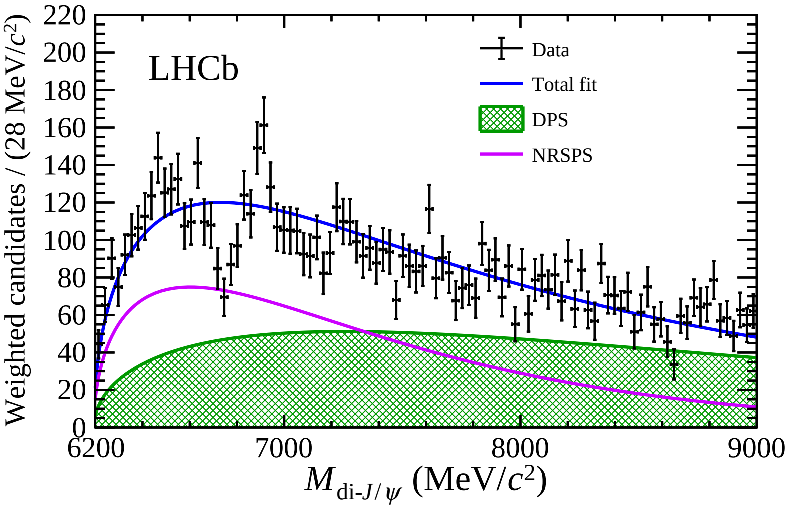
Do we have adequate theoretical approaches?

- Effective field theory
- Coupled channels
- Multipole expansion in QCD

Do we have promising candidates?

- Di-charmonium production at LHC
- Lattice simulations of heavy di-baryons

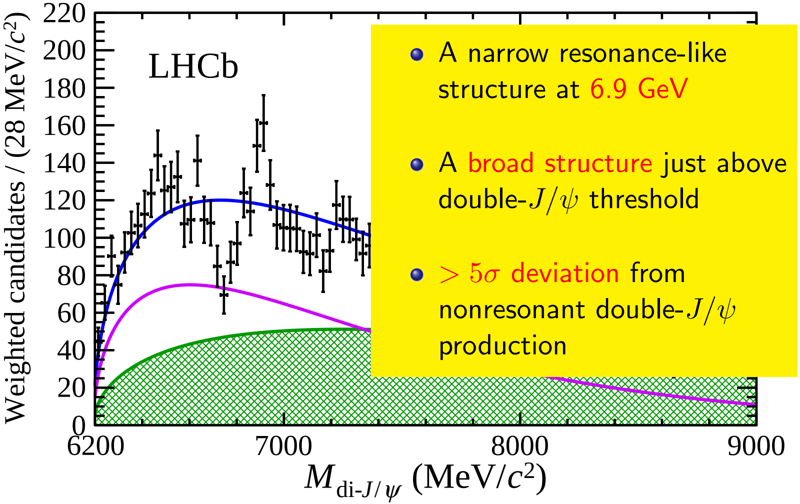
Double-J/ψ production @ LHCb (Sc.Bull. 65 (2020) 1983)



NRSPS=NonResonant Single Parton Scattering

DPS=Double Parton Scattering

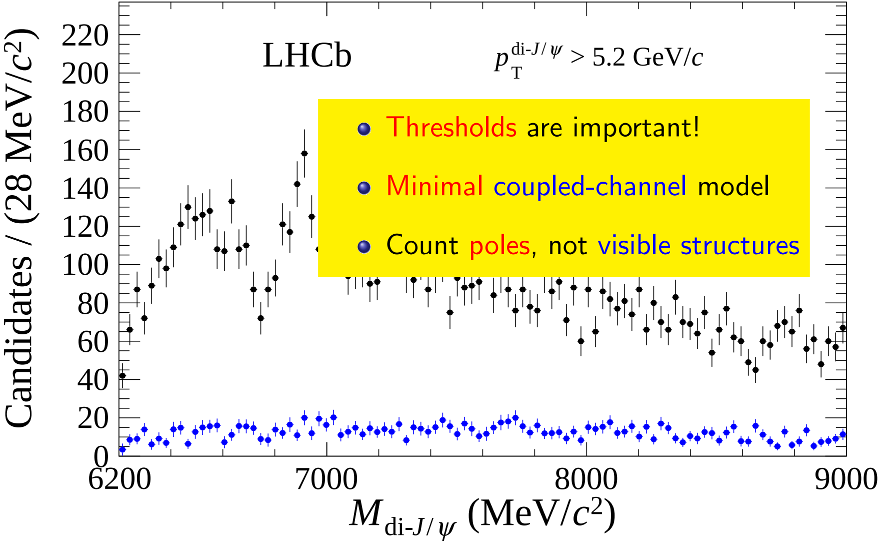
Double-J/ψ production @ LHCb (Sc.Bull. 65 (2020) 1983)



- A narrow resonance-like structure at 6.9 GeV
- A broad structure just above double-J/ψ threshold
- $> 5\sigma$ deviation from nonresonant double-J/ψ production

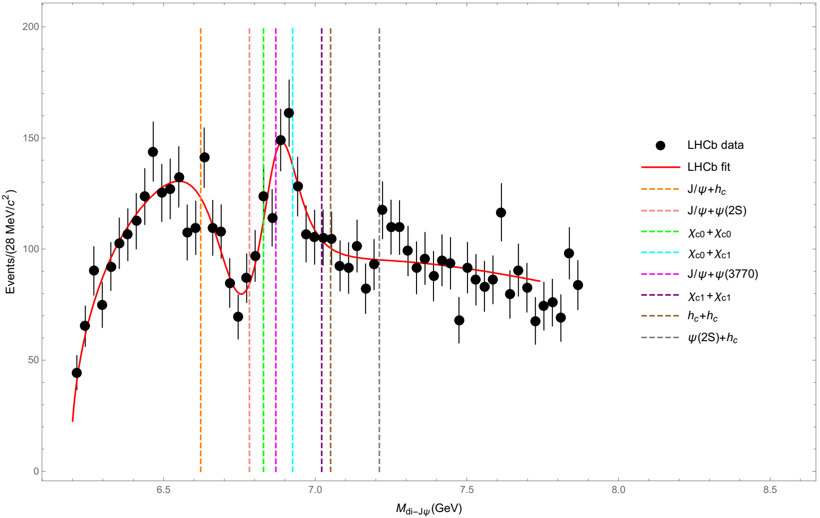
NRSPS=NonResonant Single Parton Scattering
DPS=Double Parton Scattering

Double-J/ψ production: Theoretical data analysis

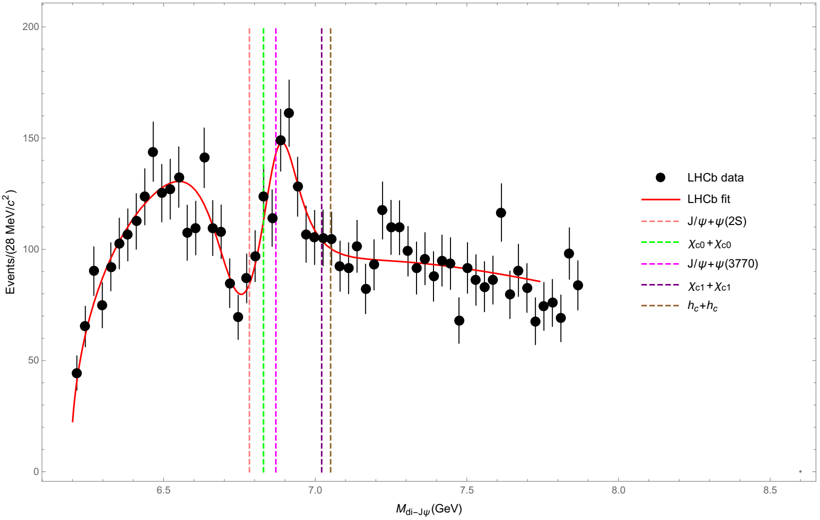


- Thresholds are important!
- Minimal coupled-channel model
- Count poles, not visible structures

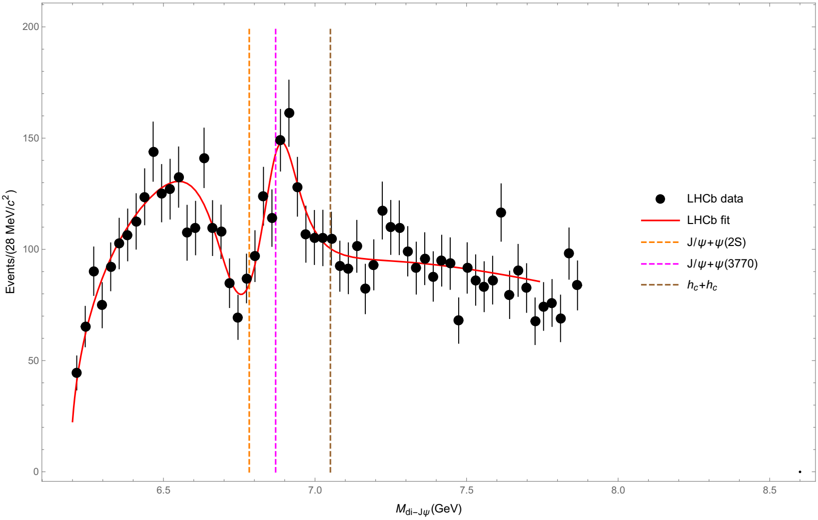
All channels



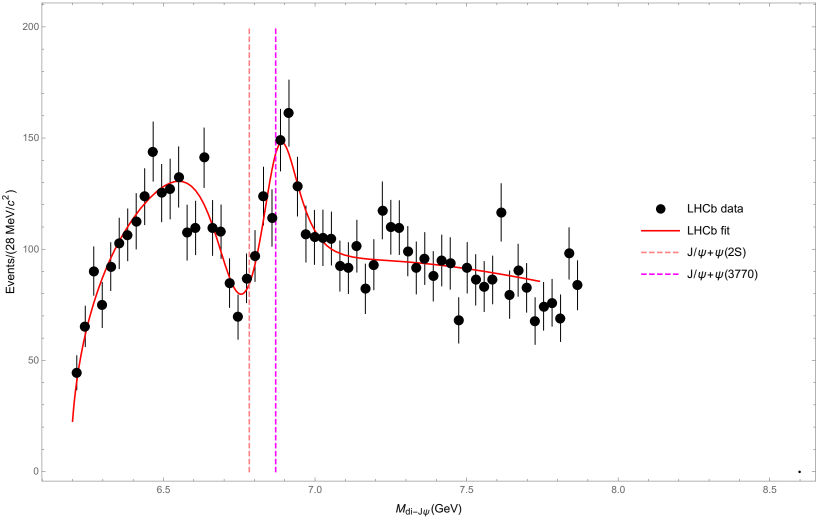
Only S-wave channels (no $J/\psi h_c$, $\psi(2S)h_c$, $\chi_{c0}\chi_{c1}$)



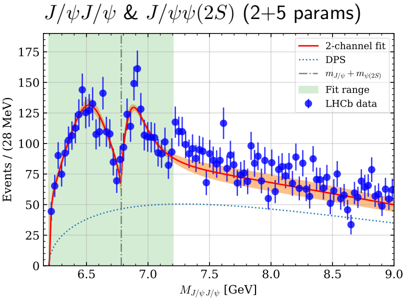
No heavy exchanges (no $\chi_{c0}\chi_{c0}$, $\chi_{c1}\chi_{c1}$)

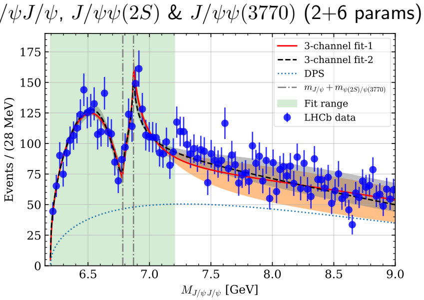


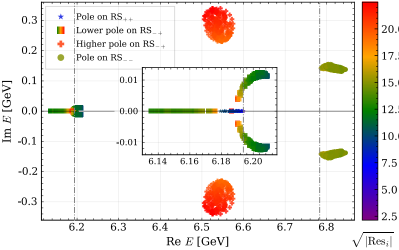
Only HQSS-allowed channels (no $h_c h_c$)

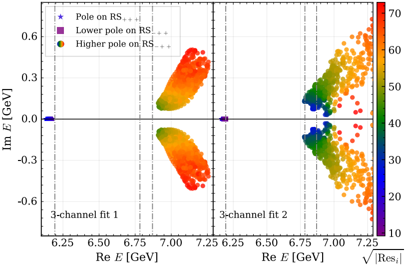


Fits & poles


$J/\psi J/\psi$ & $J/\psi\psi(2S)$ (2+5 params)
2-channel fit
DPS
$m_{J/\psi}+m_{\psi(2S)}$
Fit range
LHCb data
Events / (28 MeV)
$M_{J/\psi J/\psi}$ [GeV]


$J/\psi J/\psi$, $J/\psi\psi(2S)$ & $J/\psi\psi(3770)$ (2+6 params)
3-channel fit-1
3-channel fit-2
DPS
$m_{J/\psi}+m_{\psi(2S)/\psi(3770)}$
Fit range
LHCb data
Events / (28 MeV)
$M_{J/\psi J/\psi}$ [GeV]

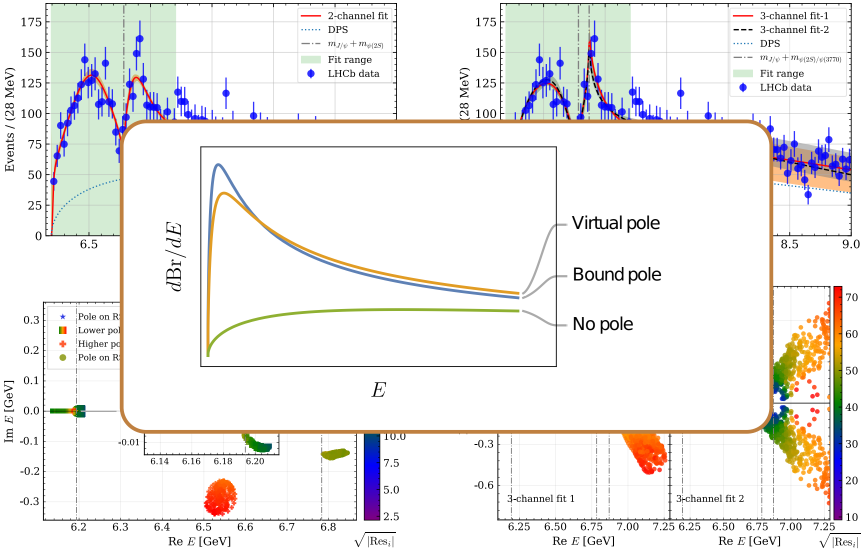

Pole on RS$_{++}$
Lower pole on RS$_{-+}$
Higher pole on RS$_{-+}$
Pole on RS$_{--}$
Im E [GeV]
Re E [GeV]
$\sqrt{|\mathrm{Res}_i|}$


Pole on RS$_{+++}$
Lower pole on RS$_{-++}$
Higher pole on RS$_{-++}$
3-channel fit 1
3-channel fit 2
Im E [GeV]
Re E [GeV]
$\sqrt{|\mathrm{Res}_i|}$

Fits & poles

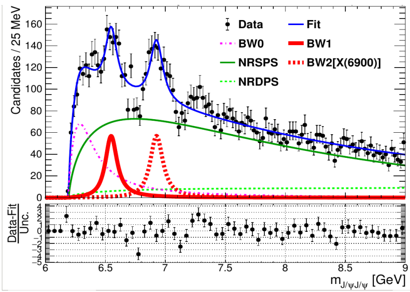
$J/\psi J/\psi$ & $J/\psi\psi(2S)$ (2+5 params)

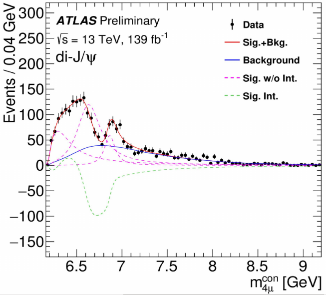
$J/\psi J/\psi$, $J/\psi\psi(2S)$ & $J/\psi\psi(3770)$ (2+6 params)



Data from CMS and ATLAS

- Analyse CMS and ATLAS data in the double-J/ψ channel





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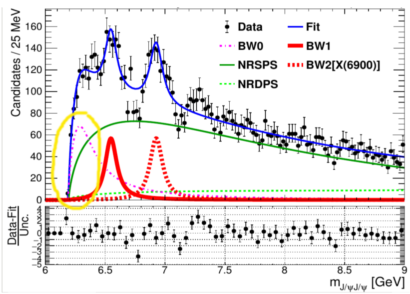

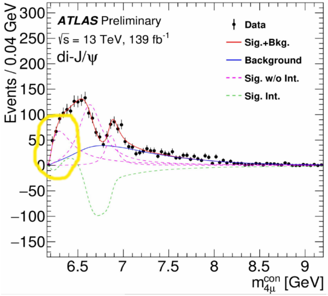

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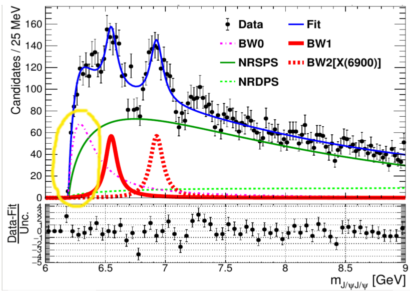
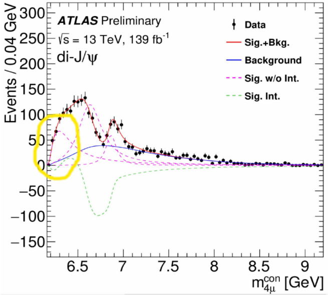




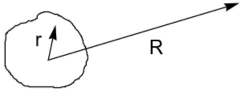
- Analyse data in the complimentary $\psi(2S)J/\psi$ channel

- Combined analysis of all data sets (?)

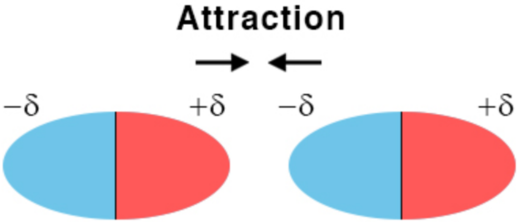
Multipole expansion in QED

Field produced by charge distribution at $R \gg r$

$$\Phi(R) = \frac{Q}{R} + \text{Dipole} + \text{Quadrupole} + \ldots$$



If $Q = 0$, dipole term provides leading contribution



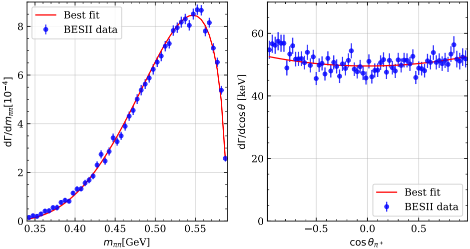
Polarisability measures how easily neutral object is distorted by electric field

Multipole expansion for charmonia

Multipole expansion for soft-gluon exchanges ($r_{\bar{Q}Q} \ll \Lambda_{\rm QCD}^{-1}$): $H_{\rm int} \approx -\frac{1}{2}\zeta_a \boldsymbol{r}\cdot\boldsymbol{E}^a$

(Gottfried'1977,Voloshin'1978,Peskin'1979,...Voloshin&Sibirtsev'2005)

$$\beta_{\psi\psi'} = \frac{1}{9}\langle\psi'|\boldsymbol{r}\frac{1}{\hat{H}_O - M}\boldsymbol{r}|\psi\rangle \Longrightarrow \begin{cases} \mathcal{M}(\psi(2S)\to J/\psi\pi\pi) = \beta_{\psi(2S)J/\psi}\langle\pi\pi|\boldsymbol{E}^a\cdot\boldsymbol{E}^a|0\rangle \\ \boxed{V_{J/\psi J/\psi}(r) \propto \beta^2_{J/\psi J/\psi} \propto \xi^2} \quad \left(\xi = \frac{\beta_{J/\psi J/\psi}}{\beta_{\psi(2S)J/\psi}}\right) \end{cases}$$



$$\boxed{|\beta_{\psi(2S)J/\psi}| \approx 1.81\ \text{GeV}^{-3}}$$

(Dong et al.'2021)

Chromopolarisability of J/ψ

- Simple estimate

$$\xi \sim I_{J/\psi J/\psi}/I_{\psi(2S)J/\psi} \sim 10 \qquad I_{\psi'\psi} = \langle\psi'|\, e^{-i\Delta \boldsymbol{q}_c\cdot\boldsymbol{r}/2}\,|\psi\rangle$$

(Dong et al.'2021)

- More advanced estimate

$$a_0^{\rm th}(J/\psi\pi \to J/\psi\pi) \approx 0.0036\xi \text{ fm}$$

(Dong et al.'2021)

$$|a_0^{\rm lat}(J/\psi\pi \to J/\psi\pi)| \sim 0.01 \text{ fm} \implies \xi \simeq 3$$

(Yokokawa et al'2006,Liu et al.'2008)

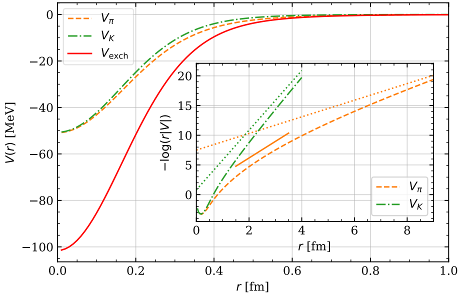
- Quark model calculation (approximation: J/ψ = Coulombic system)

$$\beta_{J/\psi J/\psi} = \frac{1}{9}\langle J/\psi|\boldsymbol{r}\frac{1}{\hat{H}_O - M_{J/\psi}}\boldsymbol{r}|J/\psi\rangle = \frac{1}{9}\int\frac{d^3p}{(2\pi)^3}\int_0^\infty r dr \frac{u_{J/\psi}(r)v_{p,l=1}(r)}{2m_c + \frac{p^2}{m_c} - M_{J/\psi}}$$

$$= \frac{0.93}{\alpha_s^4 m_c^3} = 19^{+15}_{-14}\ \text{GeV}^{-3} \qquad \implies \qquad 3 \lesssim \xi \lesssim 19$$

(Brambilla et al.'2016,Dong et al'2023)

Interaction potential in di-J/ψ system



$$V_{\rm tot}(r,\Lambda) = V_\pi(r,\Lambda) + V_K(r,\Lambda) = V_{\rm CT}(r,\Lambda) + V_{\rm exch}(r,\Lambda)$$

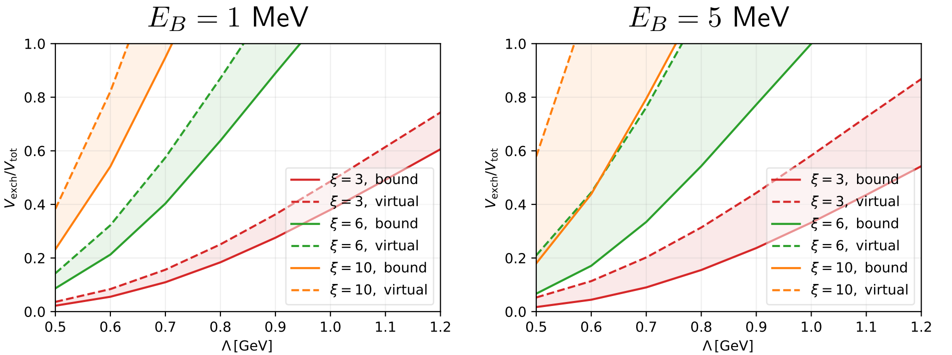
$$V_{\rm exch}(r,\Lambda) = -\frac{1}{4\pi M_{J/\psi}^2}\int \frac{d^3q}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \int_{4m_\pi^2}^{\infty} d\mu^2 \, \frac{{\rm Im}\mathcal{M}_{J/\psi J/\psi}(\mu^2)}{\mu^2+q^2} F\left(\frac{q^2+\mu^2}{\Lambda^2}\right)$$

Exchange of charmonia is suppressed as $\Lambda_{\rm QCD}^2/m_c^2$

$\Longrightarrow$ $V_{\rm CT}$ mainly comes from pion/kaon exchanges

$\Longrightarrow$ For natural $\Lambda \sim 1$ GeV $\boxed{V_{\rm CT} \sim V_{\rm exch}}$ $\Longrightarrow$ $V_{\rm exch}/V_{\rm tot} \gtrsim \frac{1}{2}$

$X(6200)$ as double-J/ψ molecule



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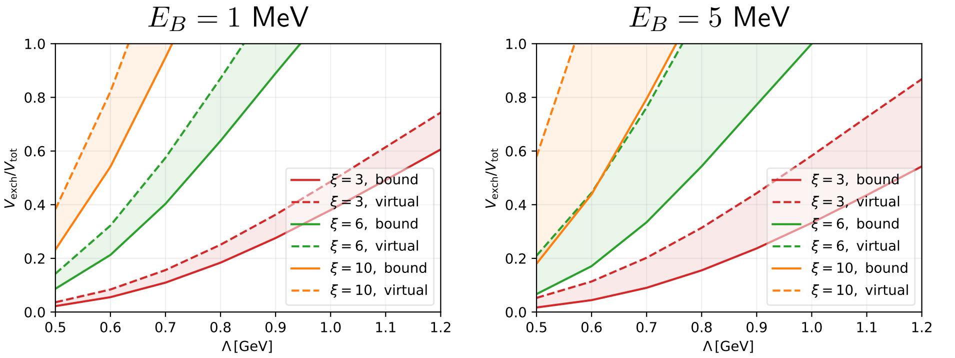
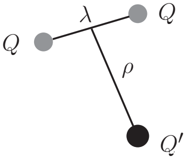
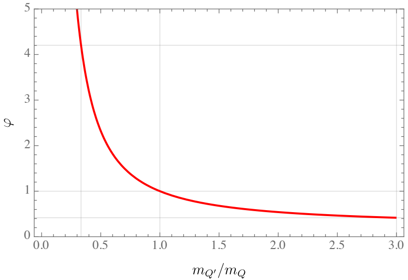


Conclusion: Existence of a molecular pole near the double-J/ψ threshold is consistent with our knowledge on hadron-hadron interactions

Chromopolarisability of QQQ' baryon

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$





- $Q' = Q$

$$\beta_\Omega = \beta_\Omega^{(\lambda)} + \beta_\Omega^{(\rho)} = \frac{C_\Omega}{m_Q^3 \alpha_s^4} \qquad C_\Omega = C_\Omega^{(\rho)} + C_\Omega^{(\lambda)}$$

$$C_\Omega^{(\rho)} \approx 1.1 \qquad C_\Omega^{(\lambda)} \approx 1.3$$

- $Q' \neq Q$

$$\beta_\Omega = \beta_\Omega^{(\lambda)} + \beta_\Omega^{(\rho)} = \frac{C_\Omega}{m_Q^3 \alpha_s^4} \varphi(m_{Q'}/m_Q)$$

Di-heavy-baryon molecules?

- Quarkonium $\psi = \bar{Q}Q$

$$\beta_\psi = \frac{C_\psi}{\alpha_s^4 m_Q^3} \qquad C_\psi \approx 0.93$$

(Brambilla et al.'2016,Dong et al'2023)

- Baryon $\Omega = QQQ$

$$\beta_\Omega = \frac{C_\Omega}{\alpha_s^4 m_Q^3} \qquad C_\Omega \approx 2.4 \approx 2.6 C_\psi$$

(Dong et al'2023)

- Uncertainty from hadron being not Coulombic

$$\frac{\delta\beta}{\beta} \sim \frac{\langle \sigma r \rangle}{\langle \alpha_s / r \rangle} \sim \frac{\Lambda_{\rm QCD}^2}{\alpha_s^3 m_Q^2} \sim \begin{cases} 100\% & Q = c \\ 10\% & Q = b \end{cases}$$

Di-heavy-baryon molecules?

- Quarkonium $\psi = \bar{Q}Q$

$$\beta_\psi = \frac{C_\psi}{\dots} \qquad C_\psi \approx 0.93$$

Conclusion: If $X(6200)$ is a di-J/ψ molecule, then di-Ω_{ccc} and di-Ω_{bbb} molecules are also likely to exist

Supported by recent lattice results:

- $E_B(\mathrm{di}-\Omega_{ccc}) \simeq 5..6$ MeV (Lyu et al.'2021)
- $E_B(\mathrm{di}-\Omega_{bbb}) \simeq 80..100$ MeV (Mathur et at.'2022)

- Uncertainty from hadron being not Coulombic

$$\frac{\delta\beta}{\beta} \sim \frac{\langle\sigma r\rangle}{\langle\alpha_s/r\rangle} \sim \frac{\Lambda_{\rm QCD}^2}{\alpha_s^3 m_Q^2} \sim \begin{cases} 100\% & Q=c \\ 10\% & Q=b \end{cases}$$

Conclusions

- Discovery of $X(3872)$ started new era in hadronic physics of heavy quarks
- LHC studies of double-J/ψ production opened new chapter in this book
- Data collected are analysed
  - using (minimal but realistic) coupled-channel scheme
  - preserving unitarity of multichannel scattering amplitude
  - respecting (approximate but accurate) heavy quark spin symmetry
- Existence of a pole at $J/\psi J/\psi$ threshold is predicted from data analysis
- Conjecture of molecular nature of $X(6200)$ is plausible and consistent with our knowledge of hadron-hadron interactions
- Near-threshold state in double-J/ψ channel may imply existence of double-heavy-baryon molecules