A walk through nucleon sigma term

–Dispersive and BχPT approaches–

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Workshop on Near-threshold J/ψ photoproduction

SCNT-IMP-CAS, Huizhou, Feb. 19-23, 2024

Outline

Introduction

Why nucleon sigma term?

- Anatomy of the nucleon mass
  - Trace of the energy-momentum tensor

$$T^{\mu}_{\mu} = \left[\frac{\beta_{QCD}}{2g} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \sum_q \gamma_m m_q \bar{q} q \right] + \sum_q m_q \bar{q} q$$

    - Explicit chiral symmetry breaking term (classical)
    - γ_m anomalous dimension of the mass operator
    - Trace anomaly owing to quantum effects
  - Nucleon mass budget

$$m_N = \langle N | \underbrace{\frac{\beta_{QCD}}{2g} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \gamma_m \sum_q m_q \bar{q} q}_{Trace\ \ anomaly} + \underbrace{(m_u \bar{u} u + m_d \bar{d} d + m_s \bar{s} s)}_{sigma\ \ term} | N \rangle$$

Various applications of nucleon sigma term

- Dark matter detection [Hill & Solon 2012]
  - scalar coupling of the nucleon

$$\langle N|m_q\bar{q}q|N\rangle = m_N f_q$$

  - Dark matter of scalar nature

$$\mathcal{L}_{\chi q} = C\frac{m_q}{\Lambda^4}\bar{\chi}\chi\bar{q}q \quad \Longrightarrow \quad \sigma_{\chi N} = \frac{m_{\chi N}^2}{\pi\Lambda^6}\left|\sum_q C_q f_q\right|^2$$

- Condensate in nuclear matter [Kaiser 2008]

$$\frac{\langle\Psi|\bar{q}q|\Psi\rangle}{\langle 0|\bar{q}q|0\rangle} = 1 - \frac{\rho}{F_\pi^2}\left\{\frac{\sigma_{\pi N}}{M_\pi^2}\left[1 - \frac{3k_f^2}{10m_N^2} + \frac{9k_f^4}{56m_N^4}\right] + \cdots\right\}$$

Basics of nucleon sigma term

Origin of nucleon sigma term

Originate in the current-algebra study of meson-nucleon scattering

- Meson-nucleon scattering

$$M_a(q) + N(p) \to M_b(q') + N(p')$$

- Off-mass-shell amplitude Reya 1974

$$T_{ba}(\nu, t, q^2, q'^2) = i\frac{(q^2 - m_a^2)}{m_a^2 f_a}\frac{(q^2 - m_b^2)}{m_b^2 f_b} \times \int \mathrm{d}^4x\mathrm{d}^4y\, e^{iq'\cdot x}e^{-iq\cdot y}\langle p'|\, T\left\{\partial_\mu A_b^\mu(x)\partial_\nu A_a^\nu(y)\right\}|p\rangle$$

  - LSZ reduction formular
  - PCAC hypothesis : $\partial_\mu A_a^\mu(x) = m_a^2 f_a \phi_a(x)$
  - $\phi_a(x)$: interpolating fields of Nambu-Goldstone bosons

Low-energy theorem

- (Generalized) Ward-Takahashi identity

$$\int \mathrm{d}^4x\mathrm{d}^4y\, e^{iq'\cdot x}e^{-iq\cdot y}\langle p'|\, T\left\{\partial_\mu A_b^\mu(x)\partial_\nu A_a^\nu(y)\right\}|p\rangle$$
$$= \int \mathrm{d}^4x\mathrm{d}^4y\, e^{iq'\cdot x}e^{-iq\cdot y}\left\{q'_\mu q_\nu\langle p'|\, T\left\{A_b^\mu(x)A_a^\nu(y)\right\}|p\rangle\right.$$
$$- iq'_\mu\langle p'|[A_b^\mu(x), A_a^0(y)]|p\rangle\delta(x_0 - y_0)$$
$$\left. - \langle p'|[A_b^0(x), \partial_\nu A_a^\nu(y)]|p\rangle\delta(x_0 - y_0)\right\}$$

$\Longrightarrow$ meson-nucleon sigma commutator

- Low-energy Theorem

$$T_{ba}(0,0,0,0) = \frac{-1}{f_b f_a}\, i\int d^4x\langle p'|[A_b^0(x), \partial_\nu A_a^\nu(y)]|p\rangle\delta(x_0) \equiv \frac{-1}{f_b f_a}\,\sigma_{NN}^{ba}$$

  - soft pion limit: $q'_\mu \to 0,\ q_\nu \to 0$

Chiral symmetry breaking and sigma term

- Sigma term $=$ Signal of χSB

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\chi SB}$$

  - $\mathcal{H}_0$ – chiral symmetry conserving (χSC) part: $[Q_a^5, \mathcal{H}_0] = 0$
  - $\mathcal{H}_{\chi SB}$ – chiral symmetry breaking (χSB) part: $\partial_\nu A_a^\nu(x) = i[\mathcal{H}_{\chi SB}, Q_a^5]$
  - General form of sigma term

$$\sigma_{NN}^{ba} = \langle p'|[Q_b^5(0), [Q_a^5(0), \mathcal{H}_{\chi SB}(0)]]|p\rangle$$

- Sigma term in QCD

$$\mathcal{H}_{\chi SB}^{QCD} = \bar{q}\mathcal{M}q\ , \quad \mathcal{M} = \mathrm{diag}\{m_u, m_d, m_s\}\ , \quad q = (u, d, s)^T\ .$$

$$\boxed{\sigma_{NN}^{ba} = \langle p'|\bar{q}\{\frac{\lambda_b}{2}, \{\frac{\lambda_a}{2}, \mathcal{M}\}\}q|p\rangle}$$

Pion (Kaon) -nucleon sigma term

- Pion-nucleon sigma term

$$\sigma_{\pi N} = \frac{1}{3}\sum_{a=1}^{3}\sigma_{NN}^{aa} = \hat{m}\langle N|\bar{u}u + \bar{d}d|N\rangle$$

- Kaon-nucleon sigma term

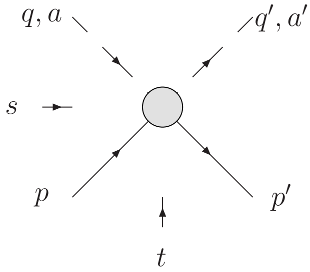
$$\sigma_{KN} = \frac{1}{4}\sum_{a=4}^{7}\sigma_{NN}^{aa} = \frac{\hat{m}+m_s}{4}\langle N|\bar{u}u + \bar{d}d + 2\bar{s}s|N\rangle$$

  - strangeness sigma term: $\sigma_s = m_s\langle N|\bar{s}s|N\rangle$
  - relationship: $\sigma_{KN} = \frac{\hat{m}+m_s}{4}(\frac{\sigma_{\pi N}}{\hat{m}} + 2\frac{\sigma_s}{m_s})$

Pion-nucleon sigma term

Pion-nucleon scattering

- Kinematics



$$\nu = \frac{s-u}{2m_N} = \frac{q\cdot(p'+p)}{2m_N}$$

$$\nu_B = \frac{t-2M_\pi^2}{4m_N} = \frac{-q'\cdot q}{2m_N}$$

- *Isospin struture*

$$T^{ba}_{\pi N}(\nu,t,q^2,q'^2) = \delta^{ba}T^+ + \frac{1}{2}[\tau^b,\tau^a]\,T^- \;,$$

- Lorentz decomposition $(D^\pm = A^\pm + \nu B^\pm)$

$$T^\pm = \bar{u}'\{A^\pm + \frac{1}{2}(\not{q}' + \not{q})B^\pm\}u = \bar{u}'\{D^\pm - \frac{1}{4m_N}[\not{q}',\not{q}]B^\pm\}u\;.$$

Pion-nucleon scattering

- Off-mass shell amplitude (WT identity)

$$T^{ba}_{\pi N}(\nu, t, q^2, q'^2) = T^{ba}_{pv} + \frac{q^2 + q'^2 - M_\pi^2}{F_\pi^2 M_\pi^2}\sigma(t)\delta^{ba} + \frac{i\epsilon^{bac}}{F_\pi^2}\frac{q'^\mu + q^\mu}{2}V^c_\mu(t) + q'^\mu q^\mu r^{ba}_{\mu\nu}$$

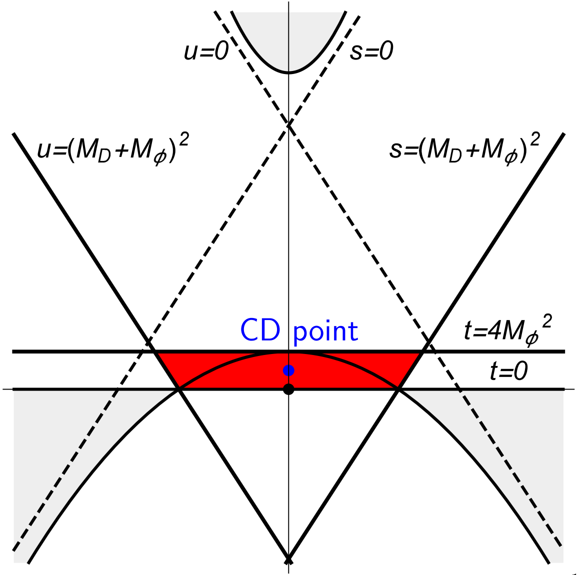
  - Pseudovector Born term: T^{ba}_{pv}
  - scalar form factor: $\sigma(t) = \langle N|\hat{m}(\bar{u}u + \bar{d}d)|N\rangle$
    - contribute to D^+ only
    - $\sigma_{\pi N} = \sigma(0)$
  - Rewind of the low energy theorem:

$$D^+(0,0,0,0) = D^+_{pv}(0,0,0,0) - \frac{1}{F_\pi^2}\sigma_{\pi N}$$

$\Longrightarrow$ Unphysical pion masses and unphysical scattering region!

Cheng-Dashen point

- CD point: $(\nu = 0, t = M_\pi^2)$
- Adler consistency condition



Cheng-Dashen Theorem

- *Cheng-Dashen theorem* [Cheng, Dashen, 1971], [Brown, Pardee, Peccei, 1971]

$$F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2, q^2 = M_\pi^2, q'^2 = M_\pi^2) = \sigma(t = 2M_\pi^2) + \Delta_R$$

  - The remainder Δ_R is of order M_π^4
  - Sub-threshold expansion:

$$\bar{D}^+(\nu, t) = \bar{d}_{00}^+ + \bar{d}_{10}^+ \nu^2 + \bar{d}_{01}^+ t + \cdots$$

  Therefore (Δ_D curvature term)

$$F_\pi^2 \bar{D}^+(\nu, 2M_\pi^2) = \bar{d}_{00}^+ + 2\bar{d}_{01}^+ M_\pi^2 + \Delta_D$$

  - Scalar form factor: $\sigma(2M_\pi^2) = \sigma(0) + \Delta_\sigma$

- Sigma term

$$\sigma_{\pi N} = F_\pi^2 \underbrace{(d_{00}^+ + 2M_\pi^2 d_{01}^+)}_{RS\ equation} + \underbrace{(\Delta_D - \Delta_\sigma)}_{Dis.\ Relation} - \underbrace{\Delta_R}_{B\chi PT}$$

Roy-Steiner equation determination

- Basic formula: $$\sigma_{\pi N} = F_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+) + \Delta_D - \Delta_\sigma - \Delta_R$$
- Subthreshold parameters output of the RS equations:

$$d_{00}^+ = -1.36(3) M_\pi^{-1} \qquad \text{[KH: } -1.46(10) M_\pi^{-1}\text{]}$$

$$d_{01}^+ = \ 1.16(3) M_\pi^{-3} \qquad \text{[KH: } \ 1.14(2)\ M_\pi^{-3}\text{]}$$

- $\Delta_D - \Delta_\sigma = (1.8 \pm 0.2)$ MeV — Hoferichter, Ditsche, Kubis, UGM (2012)
- $\Delta_R \lesssim 2$ MeV — Bernard, Kaiser, UGM (1996)
- Isospin breaking in the CD theorem shifts $\sigma_{\pi N}$ by $+3.0$ MeV

$$\Rightarrow \quad \boxed{\sigma_{\pi N} = (59.1 \pm 1.9_{\rm RS} \pm 3.0_{\rm LET})\ \text{MeV} = (59.1 \pm 3.5)\ \text{MeV}}$$

[NB: recover $\sigma_{\pi N} = 45$ MeV if KH80 scattering lengths are used]

[Slide by Ulf-G. Meissner]

χPT — low-energy EFT of QCD

- Low-energy strong interaction
  - Tools: current algebra $\Longrightarrow$ QCD $\Longrightarrow$ χPT
  - Guiding principles: analyticity, unitarity, crossing symmetry, ...

There exists a unique amplitude, no matter which method is employed!

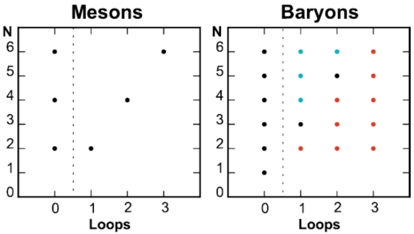
- Baryon chiral perturbation theory (BχPT)
  - Power counting breaking (PCB) problem

Feynman Diagram

⇩

$\mathcal{O}(p^N)$

How important?

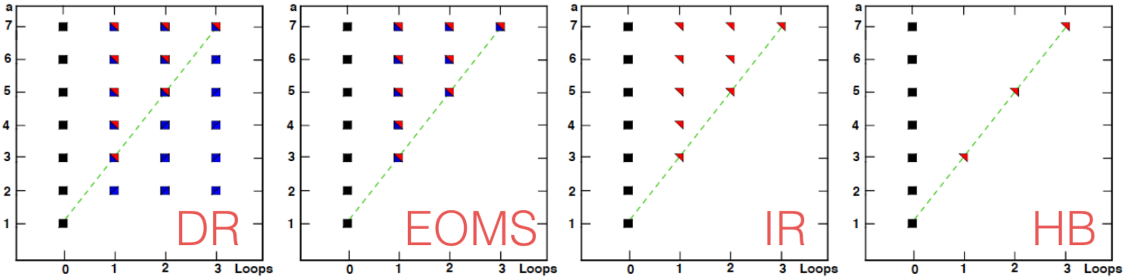


red dots denote possible PCB terms

Chiral order: $N = 4L - 2N_M - N_B + \Sigma_k k V_k$

  - Solutions: HB (Heavy baryon), IR (Infared Regularization), EOMS (Extended-on-mass-shell), ...

EOMS-BχPT



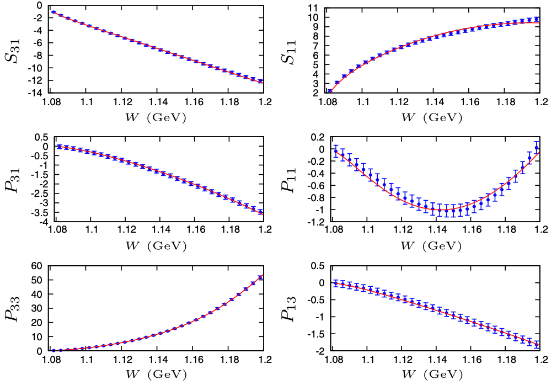
- *EOMS scheme is a two-step renormalization approach*
  - Merits:
    - $\rightarrow$ respect axiomatic S-matrix principles
    - $\rightarrow$ inherit underlying symmetries
    - $\rightarrow$ possess correct power counting
  - Experimental data $\Longrightarrow$ Low energy constants (LECs) $\Longrightarrow$ predictions

EOMS-BχPT determination: $\mathcal{O}(p^3)$

- EOMS-BχPT up to $\mathcal{O}(p^3)$ [Alarcon, et al, PRD2012]

$$\sigma_{\pi N} = -4c_1 M_\pi^2 - \frac{3g_A^2 M_\pi^3}{16\pi^3 F_\pi^2 m_N}\left[\frac{3m_N^2 - M_\pi^2}{\sqrt{4m_N^2 - M_\pi^2}} \arccos\frac{M_\pi}{2m_N} + M_\pi \log\frac{M_\pi}{m_N}\right]$$

- The only unknown LEC c_1 is common in πN scattering and $\sigma_{\pi N}$
- Experimental πN phase shifts (GW-SAID)
- $\sigma_{\pi N} = 59(7)$ MeV

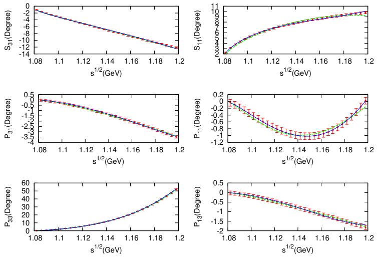


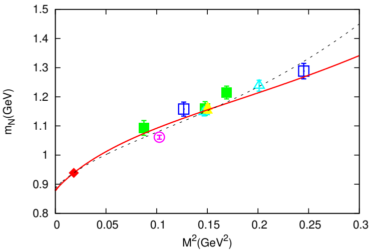
EOMS-BχPT determination: $\mathcal{O}(p^4)$

- EOMS-BχPT up to $\mathcal{O}(p^4)$ [Chen, **DLY** and Zheng, PRD2013]

$$\sigma_{\pi N} = -4c_1 M_\pi^2 - \frac{3g_A^2 M_\pi^3}{16\pi^3 F_\pi^2 m_N}\left[\frac{3m_N^2 - M_\pi^2}{\sqrt{4m_N^2 - M_\pi^2}}\arccos\frac{M_\pi}{2m_N} + M_\pi \log\frac{M_\pi}{m_N}\right]$$
$$+ e_1 M_\pi^4 + \mathcal{C}(c_1, c_2, c_3) \times [\mathcal{O}(p^4)\ \text{loop}]$$

  - e_1 not appear in πN scattering $\Longrightarrow$ need extra lattice QCD data



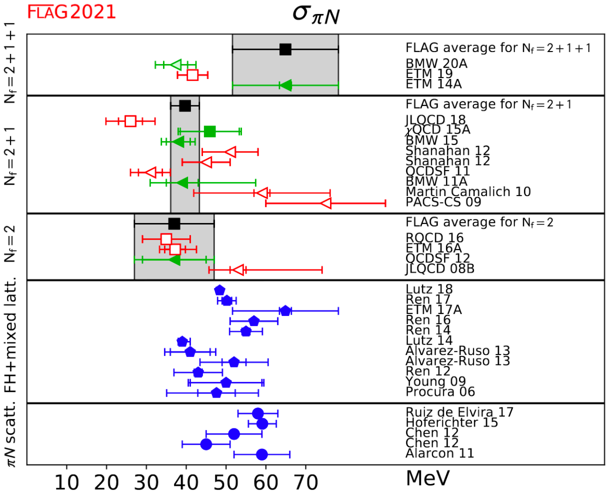


$\sigma_{\pi N} = 52(7)$ MeV (Δ-less)

$\sigma_{\pi N} = 45(6)$ MeV (Δ-full)

Comparison

- FLAG 2021: in tension with lattice QCD computation, $\sigma_{\pi N}^{lQCD} \sim 45$ MeV



Excited-state contamination?

- NME21: [Gupta, et al. PRL127(2021)]

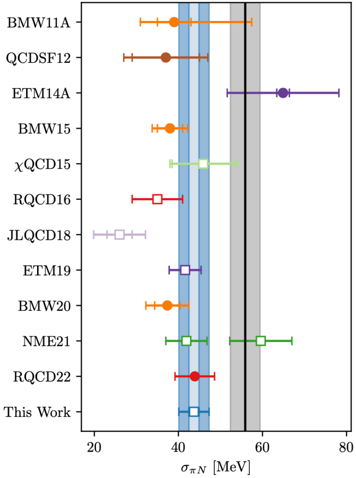
  → πN & $\pi\pi N$ excited states

$$\sigma_{\pi N} = \begin{cases} 41.9(4.9)\ \text{MeV} & \text{w/o e. s.} \\ 59.6(7.4)\ \text{MeV} & \text{w e. s.} \end{cases}$$

- Mainz23: [Agadjanov, et al. PRL131(2023)]

  → *an upward trend for $\sigma_{\pi N}$ when using priors similar to* `NME21`*, albeit not as pronounced.*

$$\sigma_{\pi N} = 43.7(3.6)\ \text{MeV}$$



Tension between lattice QCD and phenomenology still persists!

Strangeness content of the nucleon

Strangeness content of the nucleon

- Strangeness fraction

$$y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = \frac{2\hat{m}}{m_s}\frac{\sigma_s}{\sigma_{\pi N}}$$

  no strange quark contribution: $y = 0 \Leftrightarrow \sigma_s = 0$

- $\sigma_{\pi N}$ in terms of y and σ_0

$$\sigma_{\pi N} = \frac{\sigma_0}{1-y}\ , \quad \sigma_0 \equiv \hat{m}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$$

  - SU(3) operator with $(I, Y) = (0, 0)$: $\underline{3} \otimes \underline{3}^* = \underline{1} \oplus \underline{8}$

$$\mathcal{H}_m = \sum_{a=1}^{3} m_a \bar{q}_a q_a = \frac{m_s + 2\hat{m}}{3}(\bar{u}u + \bar{d}d + \bar{s}s) + \frac{\hat{m} - m_s}{3}(\bar{u}u + \bar{d}d - 2\bar{s}s)$$

    - 1st term $\rightarrow$ singlet
    - 2nd term belongs to octet IR rep. $\rightarrow$ mass splitting
    - How to obtain the chiral expression for σ_0?

Hellmann-Feynman theorem

- **In QM:** Hamiltonian $H(\lambda)$ depending on external parameter λ

$$H(\lambda)|\psi(\lambda)\rangle = E(\lambda)|\psi(\lambda)\rangle\ , \quad \langle\psi(\lambda)|\psi(\lambda)\rangle = 1$$

$$\frac{\mathrm{d}E(\lambda)}{\mathrm{d}\lambda} = \frac{\mathrm{d}}{\mathrm{d}\lambda}\langle\psi(\lambda)|H(\lambda)|\psi(\lambda)\rangle = \langle\psi(\lambda)|\frac{\mathrm{d}H(\lambda)}{\mathrm{d}\lambda}|\psi(\lambda)\rangle$$

- **In QCD:** identify the quark masses as the external parameter λ

$$\frac{\mathrm{d}m_H}{\mathrm{d}m_q} = \langle H|\bar{q}q|H\rangle \qquad \text{(for hadron state } H\text{)}$$

- Elegant bridge:
  $\Longrightarrow$ relate nucleon (baryon) sigma terms to nucleon (baryon) mass

$$\sigma_{\pi N} = \hat{m}\frac{\partial m_N}{\partial \hat{m}}\ , \quad \sigma_s = m_s\frac{\partial m_N}{\partial m_s}\ .$$

Results for σ_0 and y

- SU(3) EOMS-BχPT

$$\sigma_{\pi N} = -4(2b_0 + b_D + b_F)\frac{M_\pi^2}{2} + \sigma_{\pi N}^{loops}(octet) + \sigma_{\pi N}^{loops}(decuplet)$$

$$\sigma_s = -4(2b_0 + b_D + b_F)\left[m_K^2 - \frac{M_\pi^2}{2}\right] + \sigma_s^{loops}(octet) + \sigma_s^{loops}(decuplet)$$

- Lowest order estimation: $\sigma_0^{(2)} = \frac{1}{2}\frac{M_\pi^2}{M_K^2 - M_\pi^2}(m_\Xi + m_\Sigma - 2m_N) \simeq 27$ MeV
- Necessity of decuplet: ~ 10 MeV Alarcon, et. al. PLB(2014)
- Higher orders: LECs need to be fixed by fitting, e.g.

	$\mathcal{O}(p^3)$ [Yao, et al. JHEP(2016)]	$O(p^4)$ [Severt, et al. JHEP(2019)]
σ_0 [MeV]	$58.4(0.4)(9.0)$	$61.8(31.4)(9.3)$
y [%]	$1.2(0.6)(12.1)$	~ 0

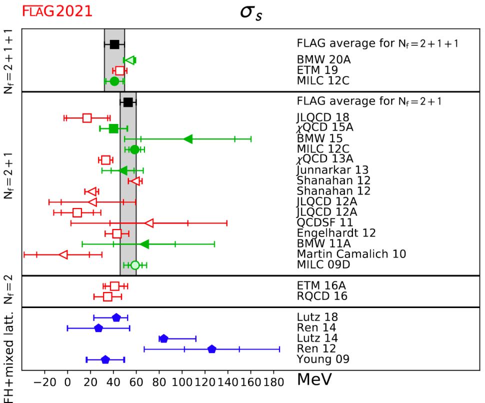
Little strangeness in the nucleon is supported.

Challenge of σ_s

- SU(3) BχPT: suffering from large theoretical uncertainty [Yao, et al. JHEP(2016)]

σ_{sN}	$\sigma_{s\Lambda}$	$\sigma_{s\Sigma}$	$\sigma_{s\Xi}$
8.5(4.4)(86.6)	166.0(3.7)(106.3)	203.6(3.9)(122.3)	342.5(3.4)(133.9)

- Lattice QCD: different collaborations vary in a large range



Summary and outlook

Summary

- Dispersive and BχPT analyses of nucleon sigma terms are briefly reviewed
- Tension exists between determinations from lattice QCD (~ 45 MeV) and phenomenology (~ 60 MeV)
- The strangeness content of the nucleon is very small but the uncertainty of σ_s is large.

Outlook

- Isospin violation, Coulomb effect, π-atom spectroscopy, $\cdots$
- A two-loop extraction of $\sigma_{\pi N}$ (in progress)
- Constraint from KN scattering for σ_s (in progress)