



FDC partial wave analysis software and excited-baryon study at BESIII

Ronggang Ping

IHEP pingrg@ihep.ac.cn

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FDC, FDC-PWA, FDC-Tensorflow

- FDC: Feynman Diagram Calculation
- FDC-PWA: FDC partial wave analysis
- FDC-Tensorflow tutorial

FDC Homepage

FDC is a package to do Feynman Diagram Calculation

The Project started in 1994 and aimed at developing a package to calculate Feynman Diagram automatically. The following parts have been finished already:

Construct the Lagrangian and deduce Feynman rules automatically

Generation of all Feynman diagrams and amplitudes for a given process.

Manipulate the amplitudes of these diagrams and generation of the expression of the total squared amplitude Deal phase space integration automatically.

This page shows part of the results generated by FDC system.

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FDC-PWA homepage: FDC-PWA for Partial Wave Analysis method

FDCHQHP(FDC Heavy Quarkonium HadroProduction)

<u>Manual</u>

되

Progress in FDC project

Jian-Xiong Wang (Beijing, Inst. High Energy Phys.) (Jul, 2004)

Published in: Nucl.Instrum.Meth.A 534 (2004) 241-245 • Contribution to: ACAT 03 • e-Print: hep-ph/0407058 [hep-ph]

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#2

Baryonic Spectroscopy Program at BES/BEPC



Excited baryon in J/ψ and ψ' decays

Data sets: 10 billion J/ψ , 3 billion $\psi(2S)$

X	$Br(J/\psi \to X)$	$Br(\psi' \to X)$
$N\overline{N}\pi$	$(9.7 \pm 0.6) \times 10^{-3}$	$(7.6 \pm 0.6) \times 10^{-4}$
$p \bar{p} \pi^+ \pi^-$	$(6.0 \pm 0.5) \times 10^{-3}$	$(6.0 \pm 0.4) \times 10^{-4}$
$N\overline{N}\eta$	$(4.18 \pm 0.36) \times 10^{-3}$	$(0.58 \pm 0.13) \times 10^{-4}$
$\Lambda\overline{\Lambda}\eta$	$(0.16 \pm 0.02) \times 10^{-3}$	$(0.25 \pm 0.04) \times 10^{-4}$
$pK^{-}\overline{\Lambda}$ + c. c.	$(0.86 \pm 0.11) \times 10^{-3}$	$(1.00 \pm 0.04) \times 10^{-4}$
$pK^-\overline{\Sigma}{}^0$	$(0.29 \pm 0.08) \times 10^{-3}$	$(0.17 \pm 0.02) \times 10^{-4}$
$\Sigma\overline{\Lambda}\pi$	$(0.83 \pm 0.07) \times 10^{-3}$	$(1.54 \pm 0.04) \times 10^{-4}$

Big challenge for PWA Software : FDC-tf package

Advancements in FDC-PWA Development

Theoretical formalism and Monte Carlo study of partial wave analysis for J / $^{\#1}$ psi --> p anti-p omega

W.H. Liang (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys. and Guangxi Normal U. and Natl. Lab. Heavy Ion Accel., Lanzhou), P.N. Shen (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys. and Guangxi Normal U. and Natl. Lab. Heavy Ion Accel., Lanzhou), J.X. Wang (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys. and Guangxi Normal U. and Natl. Lab. Heavy Ion Accel., Lanzhou), B.S. Zou (CCAST World Lab, Beijing and Beijing, Inst. High Energy Phys. and Guangxi Normal U. and Natl. Lab. Heavy Ion Accel., Lanzhou) (2002)

Published in: J.Phys.G 28 (2002) 333-343 19-23 February 2024

 $\begin{array}{cccc} \end{tabular} \end{t$

$\mathcal{L} = \bar{\psi}_1 \Gamma \psi_2 A$

 Table 2. The transformation properties of some operators.

$\Gamma =$	i	γ5	γ_{μ}	$\gamma_{\mu}\gamma_{5}$	$\sigma_{\mu u}$	$\sigma_{\mu u}\gamma_5$	$g_{\mu u}$
$\gamma_0 \Gamma^+ \gamma_0 =$	—i	$-\gamma_5$	γ_{μ}	$\gamma_{\mu}\gamma_{5}$	$\sigma_{\mu u}$	$-\sigma_{\mu u}\gamma_5$	$g_{\mu u}$
$C(\gamma_0 \Gamma^+ \gamma_0)^T C^{-1} =$	—i	$-\gamma_5$	$-\gamma_{\mu}$	$\gamma_{\mu}\gamma_{5}$	$-\sigma_{\mu u}$	$\sigma_{\mu u}\gamma_5$	$g_{\mu u}$
$\gamma_0 \Gamma^P \gamma_0 =$	i	$-\gamma_5$	γ_{μ}	$-\gamma_{\mu}\gamma_{5}$	$\sigma_{\mu u}$	$-\sigma_{\mu u}\gamma_5$	$g_{\mu u}$

Covariant tensor amplitude in FDC-PWA

- Tensor form of vertex generation by phenomenological Lagrangian (strong intera.)
 - conserve P, C parity, isospin, strangeness, charm, baryon and lepton numbers

- an example of
$$J/\psi \to \Lambda\left(\frac{1}{2}^+\right)\overline{\Lambda}\left(\frac{1}{2}^-\right)$$



P, *C*, *CPT* symmetry transformation: $\mathcal{L}^P = \mathcal{L}$, $\mathcal{L}^C = \mathcal{L}$, $\mathcal{L} = \mathcal{L}^{\dagger}$

Progress in the Development of FDC-PWA

"To do a good job, one must first sharpen one's tools"

- ✓ GPU Implementation of Feynman Diagram Computation (FDC) System : FDC + Tensorflow
- \checkmark FDC-TF physics analyses
 - 1. PWA $J/\psi \to \pi^+\pi^-\Sigma^+\overline{\Sigma}^-$
 - 2. $PWAJ/\psi \rightarrow p\bar{p}K^+K^-$
 - 3. PWA $J/\psi \rightarrow \phi \eta \eta$
 - 4. PWA $J/\psi \rightarrow \phi \eta \eta'$
 - 5. PWA $\psi' \to \pi^0 \Sigma^+ \overline{\Sigma}^-$

Upgrade of FDC and Expansion of Two Major Functions

- 1. Hyperon weak decays
- 2. Radiative decays

Partial Wave Analysis Theory-Experiment Joint Workshop, 3/9/2022, China Advanced Science and Technology Center





Tensor Algorithm for FDC Amplitudes

- $|\mathcal{M}(\text{event}_{v})|^{2} = \overline{\Sigma}_{s_{1},...,s_{j}} |\Sigma_{k}c_{k} a_{v,k}|^{2}$ $= C_{k,l} A_{v,k,l} \text{ (dumb index rule)}$ with $C_{k,l} = c_{k} c_{l}^{*}$, $A_{v,k,l} = \overline{\Sigma}_{s_{1},...,s_{j}} (a_{v,k}a_{v,l}^{*})$ c_{k} : k-th parameter, $a_{v,k}$: k-th term of amplitude for event v
- $A_{v,k,l}$ calculated by FDC, and stored in memory (limitation form GPU memory)
- Amplitude reduction in Tensorflow

Structure of the FDC-TF software package



• Signal yields and statistical errors

For mode *i*:
$$N_i \pm \delta N_i$$
,
where $N_i = r_i \left(N_{obs} - N_{bkg} \right)$ with $r_i = \frac{\sigma_i}{\sigma_{tot}}$ $-V^{-1} = \begin{pmatrix} \frac{\partial^2 lnL}{\partial x_1^2} & \cdots & \frac{\partial^2 lnL}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 lnL}{\partial x_n \partial x_1} & \cdots & \frac{\partial^2 lnL}{\partial x_n^2} \end{pmatrix}$

 V_{mn} : Covariant matrix calculated by MINUIT. If failed, then calculated by Hessian matrix determined by tf.GradientTape or numerical calculations.

Mass resolution for narrow resonance

 $|BW(x)|^2 = |BW(x')|^2 \otimes R(x', x)$

R(x'x): parametrized with 3 Breit-Wigner function, determined with zero-width resonance. Multi-Gaussian function parametrization is under developed.

Development of FDC-TF Applications

• Simultaneous fit to multiple data sets

Object function for data set *i*: $S_i = -(ln\mathcal{L}_{dt}^i - ln\mathcal{L}_{bkg}^i)$ Minimized object function: $S = \sum_i S_i$ where S_i calculated by one GPU card, dispatched by CPU muti-threads

• One channel decay with running width

$$\begin{split} BW(s,M_0,\Gamma_0) &= \frac{1}{s - M_0^2 - iM_0\Gamma(s)} \quad \text{with} \quad \Gamma(m) = \Gamma_0 \left(\frac{q}{q_0}\right)^{2l+1} \frac{m_0}{m} B_l'^2(q,q_0,d). \\ B_0'(q,q_0,d) &= 1, \\ B_1'(q,q_0,d) &= \sqrt{\frac{1 + (q_0d)^2}{1 + (qd)^2}}, \\ B_2'(q,q_0,d) &= \sqrt{\frac{9 + 3(q_0d)^2 + (q_0d)^4}{9 + 3(qd)^2 + (qd)^4}}, \\ B_3'(q,q_0,d) &= \sqrt{\frac{225 + 45(q_0d)^2 + 6(q_0d)^4 + (q_0d)^6}{225 + 45(qd)^2 + 6(qd)^4 + (qd)^6}}, \\ B_4'(q,q_0,d) &= \sqrt{\frac{11035 + 1575(q_0d)^2 + 135(q_0d)^4 + 10(q_0d)^6 + (q_0d)^8}{11035 + 1575(qd)^2 + 135(qd)^4 + 10(qd)^6 + (qd)^8}}, \end{split}$$

• Baryon resonance : couple channel running width

For example: N(1535):

$$BW(s, M_0, \Gamma_0) = \frac{1}{s - M_0^2 - iM_0 \Gamma(s)}, \text{ with } \Gamma(s) = \Gamma_0 \left(0.5 \frac{\rho_{\pi N}(s)}{\rho_{\pi N}(M_0^2)} + 0.5 \frac{\rho_{\eta N}(s)}{\rho_{\eta N}(M_0^2)} \right)$$

关键词。	N1440width	N1520width	N1535width	N1650width	N1700width	÷
共振态。	N(1440) $\frac{1}{2}^{+} e^{2}$	N(1520) $\frac{3}{2} +$	N(1535) $\frac{1}{2} +$	N(1650) $\frac{1}{2}$	N(1700) $\frac{3}{2}$	÷
关键词↩	N1710width	N1720width₽	L1380width	L1405width	L1520width₽	÷
共振态↔	$N(1710) \frac{1}{2}^{+} e^{-1}$	$N(1720) \frac{3}{2}^{+}$	$\Lambda(1380) \frac{1}{2} \bullet$	$\Lambda(1405) \frac{1}{2} *$	$\Lambda(1520) \frac{3}{2} +$	÷
关键词₽	L1600width₽	L1670width₽	D1232width	D1600width₽	D1620width	÷
共振态↔	$\Lambda(1600) \frac{1}{2}^{+} +$	$\Lambda(1670) \frac{1}{2} *$	$\Delta(1232) \frac{3}{2}^{+} e^{3}$	$\Delta(1600) \frac{3}{2}^{+}$	$\Delta(1620) \frac{1}{2} \bullet$	÷
关键词↩	D1700width+	S1385width₽	S1660width₽	S1670width	S1750width₽	÷
共振态↔	$\Delta(1700) \frac{3}{2} \leftrightarrow$	$\Sigma(1385) \frac{3}{2}^{+} e^{-2}$	$\Sigma(1660) \frac{1}{2}^{+} e^{-2}$	$\Sigma(1670) \frac{3}{2} \phi$	$\Sigma(1750) \frac{1}{2}^{-1}$	¢
关键词₽	S1910width	X1530width	c,	C.	сь С	÷
共振态₽	$\Sigma(1910) \frac{3}{2} e^{-2}$	$\Xi(1530) \frac{3}{2}^{+} e^{-3}$	¢	C4	τ,	¢

BESII: $J/\psi \rightarrow pK^-\overline{\Lambda} + c.c.$ PRL 93 (2004) 112002



Motivation

- Observed at BESII in 2004
- Similar structure was seen in several B meson and charmonium decays
- Investigated theoretically under scenario of <u>quark</u> <u>model</u>, <u>FSI</u> and <u>chiral</u> <u>effective theory</u>



X(2075) in $e^+e^- \rightarrow \gamma^* \rightarrow pK^-\overline{\Lambda}+c.c.$

- X(2075)[K(2075)] first observed in $J/\psi \rightarrow pK^-\Lambda + c.c.$, but spinparity not measured.
- Similar evidence found in $B \to p\Lambda \pi$, and $\psi', \chi_{cJ} \to pK^-\Lambda$.
- Near $p\Lambda_c$ threshold, an enhancement also observed in $B^- \rightarrow \Lambda_c^+ \bar{p}\pi^-$



X(2075) in $e^+e^- \rightarrow \gamma^* \rightarrow pK^-\overline{\Lambda} + c.c.$



PRL131, 151901 (2023)

- $X(2075), J^P = 1^+$
- $M_{pole} = (2084^{+4}_{-2} \pm 9) \text{ MeV}$

•
$$\Gamma_{pole} = (58^{+4}_{-3} \pm 25) \text{ MeV}$$

	$\Delta \ln \mathcal{L}$	Δndf	Significance
1 ⁺ over 0 ⁻	40.6	4	8.3
1 ⁺ over 1 ⁻	30.2	2	7.5
1 ⁺ over 2 ⁺	44.8	2	9.2
1 ⁺ over 2 ⁻	13.8	0	5.3
=	1 <i>A C</i>	N. T. 7 / 2\	
$\sqrt{s} (\text{GeV})$	$M_{\rm pole}$ (MeV/c^2)	$\Gamma_{\rm pole}$ (MeV)
4.008	208	5 ± 14	$50{\pm}16$
4.178	208	5 ± 6	$62{\pm}10$
4.226	2088	$8{\pm}10$	68 ± 12
4.258	208	$3{\pm}11$	48 ± 10
4.416	2088	$8{\pm}13$	$56{\pm}12$
4.682	2095	$2{\pm}10$	$54{\pm}10$
Average	208	86 ± 4	56 ± 5

Check significance with toy MC method



	$\Delta \ln \mathcal{L}$	$\Delta n df$	Significance
1^+ over 0^-	40.6	4	8.3
1^+ over 1^-	30.2	2	7.5
1^+ over 2^+	44.8	2	9.2
1^+ over 2^-	13.8	0	5.3

- ✓ The statistical significances of 1⁺ over 0⁻, 1⁻, and 2⁺ are obtained with $\Delta \ln L = \ln L^{1^+} - \ln L^{J^P}$ and Δndf
- ✓ For 1⁺ over 2⁻, the ∆ndf is assumed to be 1, following PRL 115 (2015), 072001.
- ✓ The approach based on MC simulation is checked. The $t = -2 \ln(L^{2^-}/L^{1^+})$ distribution suggests a statistical significance 5.6 σ

X(2075) Argand plot



- From 2.05 2.13 GeV with step size 20 MeV
- BW function replaced with a complex number A_i in point i
- Cubic interpolation between two points
- No conclusion whether *X*(2075) exhibits the characteristics of a resonance or not.

check alternative J^P



check alternative J^P



Data favor 1⁺ hypothesis





Data at other five points



Σ^* and Λ^* in $J/\psi \to \overline{\Lambda}\pi^{\pm}\Sigma^{\mp} + c.c.$

BESIII, PRD108, 112012 (2023)



Toy MC with 9 Λ^* , 4 Σ^* and 1⁻ NR states



Program to study baryon spectroscopy at BESIII

- $\psi(3686) \rightarrow \Lambda \overline{\Lambda} \eta, \Lambda \overline{\Lambda} \pi^0$, BESIII, Phys. Rev., D106, 072006 (2022)
- $\psi(3686) \rightarrow p\bar{p}\eta, p\bar{p}\pi^0$, being reviewed in BESIII
- $\psi(3686) \rightarrow p\bar{p}K^+K^-$, being reviewed in BESIII
- $\psi(3686), J/\psi \to \overline{\Lambda}\Sigma^0\pi^0 + c.c.$, being reviewed in BESIII
- $\psi(3686) \rightarrow K^+ \overline{\Lambda} \Xi^-, K \overline{\Sigma}^0 \Xi^-, K_S^0 \overline{\Sigma}^+ \Xi^-$, being analysis
- $\psi(3686) \rightarrow \pi^0 \Sigma^+ \overline{\Sigma}^-$, being analysis
- $\psi(3686) \rightarrow p\bar{p}\phi$, being analysis
- $\psi(3686) \rightarrow pK^-\overline{\Sigma}^0 + c.c.$, being analysis
- $\chi_{c0} \rightarrow pK\overline{\Lambda} + c.c., \ p\overline{p}K^+K^-$, being analysis
- $J/\psi \to \Sigma^+ \overline{\Sigma}^- \pi^+ \pi^-$, being analysis
- $J/\psi \rightarrow pK^-\overline{\Lambda} + c.c.$, being analysis
- $J/\psi \rightarrow p\bar{p}\omega$, being analysis

.

• $J/\psi \to \eta \Lambda \overline{\Sigma}^0 + c.c.$, being analysis

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Summary and Outlook

- FDC-TF package availability
- Ongoing baryonic analyses
- Resonance lineshape: BW, running-width,K-matrix
- Continued development of multiple applications for FDC-TF
- Prospects for FDC-PWA generation code:
 - Examination of multi-step decays involving hyperons
 - Analysis of χ_{cJ} decays
 - Investigation of radiative decays

Thanks for your attention

backup

Performance test

- GPU : Tesla V100-SXM2-32GB
- decay: $\psi' \to pK^-\overline{\Lambda} + c.c.$
 - ➢ 5969 data events, and 80,000 PHSP events. 179 parameters in the fit.

24 resonances included in the fit N*(1710), N*(1870), N*(1720), Λ(1810), Λ(1800), Λ(1670), Λ(1600), Λ(1405), K₁(2075) N*(2060), Λ(2325), Λ(1890), Λ(1690), Λ(1520) K₂(2250), N*(1990), N*(2190), Λ(2110), Λ(1830) Λ(1820), N*(2250), Λ(2100), Λ(2020), Λ(2350)

Performance test (cont.)











e

 p_{1}



Fig. 11

Р

N[2190]

K _ P4

 \mathbf{p}_5







P _P₃

2250]

.p3

D4



 p_1







Fig. 19

Δ

Ρ

AT23501

 p_2 \overline{A} p_2 \overline{A} p_1 \overline{A} p_1 \overline{A} \overline{A}



Fig. 21

e

 $\mathbf{p}_{\mathbf{P}}$

p₁



N[1990]

Fig. 12

Fig. 18

Fig. 23





K _ P4





Performance test (cont.)

• $\frac{t_c}{t_g} = \frac{30}{0.07} \approx 430 : t_c(t_g)$ times cost for CPU (GPU) calculation per iteration



3.0

FDC: $-\ln L = -789.10$





Performance test (cont.)

• Check on yields ratios

Mode	FDC	FDC-tf
1	0.034	0.034
2	0.007	0.007
3	0.187	0.187
4	0.069	0.069
5	0.116	0.116
6	0.018	0.018
7	0.122	0.123
8	0.050	0.050
9	0.051	0.051
10	0.041	0.041
11	0.035	0.035
12	0.040	0.040

Mode	FDC	FDC-tf
13	0.042	0.042
14	0.021	0.021
15	0.056	0.056
16	0.523	0.523
17	0.005	0.005
18	0.614	0.611
19	0.034	0.034
20	0.023	0.023
21	0.003	0.003
22	0.137	0.138
23	0.009	0.009
24	0.011	0.006

@ /hpcfs/bes/gpupwa/pingrg/fdc/pkl2/process/fort_dbg