

Probing quark OAM in exclusive π^0 production in ep collisions

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Based on the papers: [2312.01309](#), [2304.05784](#) ; collaborators: Shohini Bhattacharya, Duxin Zheng

[Workshop on Near-Threshold Production of Heavy Quarkonium](#), huizhou, 19-23 Feb., 2024

Outline:

- Background
- Probing Quark OAM via exclusive π^0 production
- Constraining the gluon GTMD via exclusive π^0 production
- Summary

Spin decompositions of proton

◆ “Spin crisis”

$$\Delta\Sigma(Q^2 = 10.7 \text{ GeV}^2) = 0.060 \pm 0.047 \pm 0.069 \quad 1988, \text{ EMC}$$

➤ Jaffe-Manohar decomposition

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + l_q + \Delta g + l_g$$

 $\Delta\Sigma \approx 0.3$  $\Delta g \approx 0.2$

Jaffe, Manohar, 1990
Canonical OAM

➤ Ji decomposition

$$\frac{1}{2} = J_q + J_g = \frac{1}{2} \Delta\Sigma + L_q + J_g$$

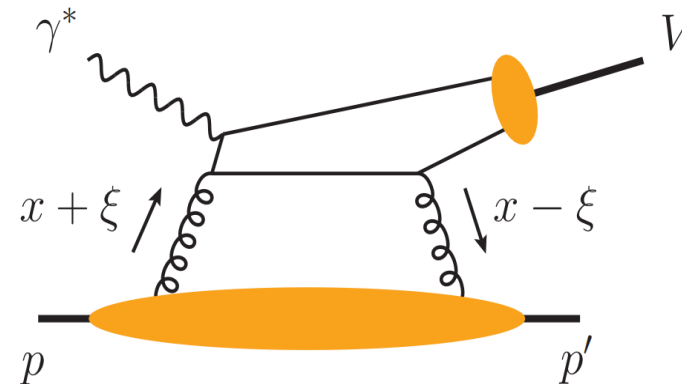
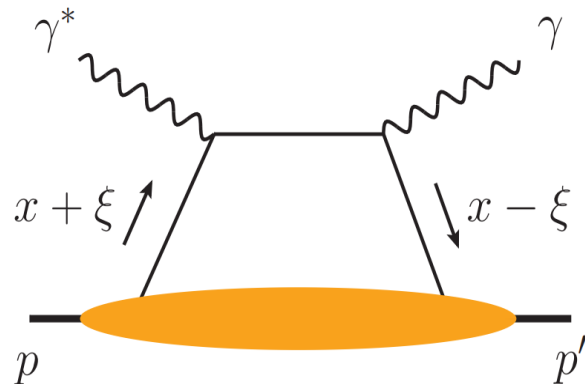
X.D. Ji, 1996
Kinematic OAM

Probing kinematic parton OAM

➤ Probing kinematic OAM: $J_{q,g} = \frac{1}{2} \int_{-1}^1 dx x [H_{q,g}(x, \xi, 0) + E_{q,g}(x, \xi, 0)]$ X.D. Ji, 1996

◆ Subtracting the spin contribution(helicity distribution)

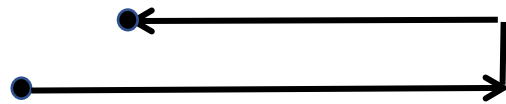
➤ GPDs, accessible in exclusive processes



Canonical parton OAM and the GTMD

➤ Canonical OAM and GTMD:
$$L^q(x, \xi) = - \int d^2 k_{\perp} \frac{k_{\perp}^2}{M^2} F_{1,4}^q(x, k_{\perp}, \xi, \Delta_{\perp} = 0)$$

C. Lorce, B. Pasquini, 2011; Y. Hatta, 2012



Canonical
 $M\mathbf{v} + e\mathbf{A}$

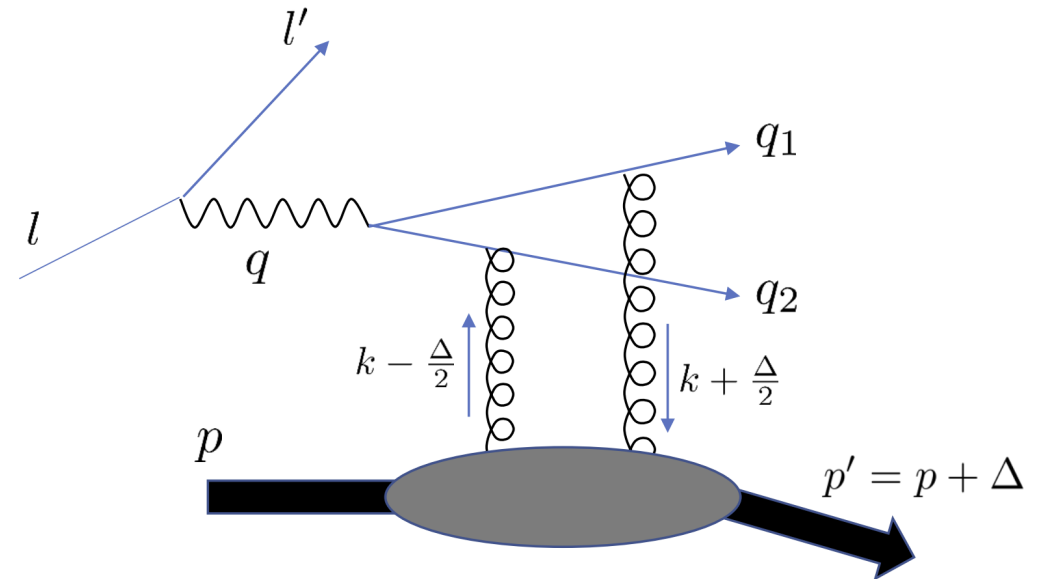
V.S.



Kinematical
 $M\mathbf{v}$

◆ Probe F 1,4 in diffractive di-jet production

X. Ji, F. Yuan, and Y. Zhao, 2017; Y. Hatta, Y. Nakagawa, F. Yuan, Y. Zhao, and B. Xiao, 2017; S. Bhattacharya, R. Boussarie, and Y. Hatta 2022



Generalized TMDs

$$W_{\lambda, \lambda'}^q [\Gamma](P, \Delta, x, \vec{k}_\perp) = \int \frac{dz^- d^2 \vec{z}_\perp}{2(2\pi)^3} e^{ik \cdot z} \langle p', \lambda' | \bar{q}(-\frac{z}{2}) \Gamma \mathcal{W}(-\frac{z}{2}, \frac{z}{2}) q(\frac{z}{2}) | p, \lambda \rangle \Big|_{z^+=0}$$

S. Meissner, A. Metz, M. Schlegel and K. Goeke, 2008

◆ which is the Fourier transform of Wigner distribution:

$$\rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} W^{[\Gamma]}(\vec{\Delta}_\perp, \vec{k}_\perp, x, \vec{S})$$

Parametrization:

$$W_{\lambda, \lambda'}^q [\gamma^+] = \frac{1}{2M} \bar{u}(p', \lambda') \left[F_{1,1}^q + \frac{i\sigma^{i+} k_\perp^i}{P_+} F_{1,2}^q + \frac{i\sigma^{i+} \Delta_\perp^i}{P_+} F_{1,3}^q + \frac{i\sigma^{ij} k_\perp^i \Delta_\perp^j}{M^2} F_{1,4}^q \right] u(p, \lambda)$$

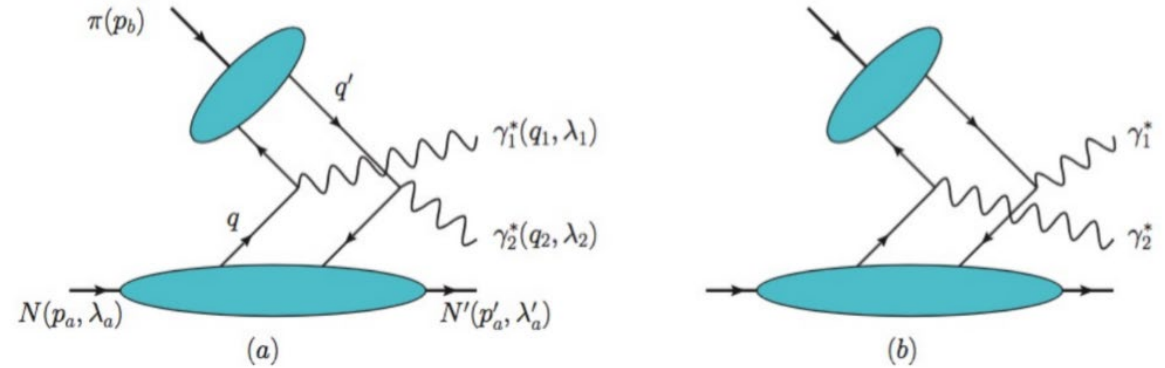
$\vec{b}_\perp \times \vec{k}_\perp$ Orbital angular momentum

$$W_{\lambda, \lambda'}^q [\gamma^+ \gamma_5] = \frac{1}{2M} \bar{u}(p', \lambda') \left[-\frac{i\epsilon^{ij} k_\perp^i \Delta_\perp^j}{M^2} G_{1,1}^q + \frac{i\sigma^{i+} \gamma_5 k_\perp^i}{P_+} G_{1,2}^q + \frac{i\sigma^{i+} \gamma_5 \Delta_\perp^i}{P_+} G_{1,3}^q + i\sigma^{+-} \gamma_5 G_{1,4}^q \right] u(p, \lambda)$$

How to measure parton Wigner distribution

- Quark case: exclusive double DY process

Only ERBL region!



Bhattacharya, Metz, ZJ 2017

- Gluon case: exclusive double quarkonium production.

$$\begin{aligned}
 & \frac{1}{2}(\tau_{UL} + \tau_{LU}) \\
 & \approx 2 \operatorname{Im} \left\{ -\frac{1}{M^2} (\varepsilon_{\perp}^{ij} \Delta q_{\perp}^i \Delta a_{\perp}^j) C[\vec{\beta}_{\perp} \cdot \vec{k}_{a\perp} F_{1,4}(x_a, \vec{k}_{a\perp}) F_{1,1}(x_b, \vec{k}_{b\perp})] C[F_{1,1}^*(x_a, \vec{p}_{a\perp}) F_{1,1}^*(x_b, \vec{p}_{b\perp})] \right. \\
 & \quad \left. + \frac{1}{M^2} (\varepsilon_{\perp}^{ij} \Delta q_{\perp}^i \Delta a_{\perp}^j) C[\vec{\beta}_{\perp} \cdot \vec{k}_{a\perp} G_{1,1}(x_a, \vec{k}_{a\perp}) G_{1,4}(x_b, \vec{k}_{b\perp})] C[G_{1,4}^*(x_a, \vec{p}_{a\perp}) G_{1,4}^*(x_b, \vec{p}_{b\perp})] \right\} \\
 & \approx 2 \operatorname{Im} \left\{ -\frac{1}{M^2} (\varepsilon_{\perp}^{ij} \Delta q_{\perp}^i \Delta a_{\perp}^j) C[\vec{\beta}_{\perp} \cdot \vec{k}_{a\perp} F_{1,4}(x_a, \vec{k}_{a\perp}) F_{1,1}(x_b, \vec{k}_{b\perp})] C[F_{1,1}^*(x_a, \vec{p}_{a\perp}) F_{1,1}^*(x_b, \vec{p}_{b\perp})] \right\}
 \end{aligned}$$

Bhattacharya, Vikash, Metz, Jeng, ZJ 2018

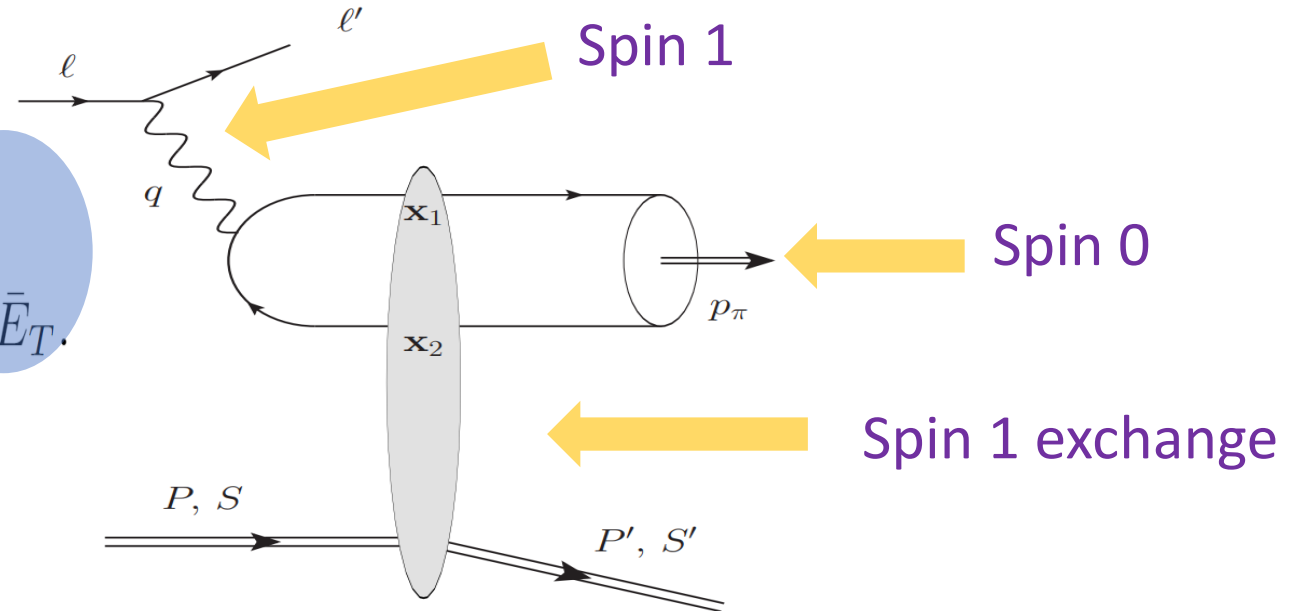
Exclusive π^0 production in ep collisions

➤ In the forward limit, helicity flip process!

$$M_{0-,++} = \langle H_T \rangle \propto \int_{-1}^1 d\bar{x} \mathcal{H}_{0-,++}(\bar{x}, \dots) H_T;$$

$$M_{0+,++} = \langle \bar{E}_T \rangle \propto \frac{\sqrt{-t'}}{4m} \int_{-1}^1 d\bar{x} \mathcal{H}_{0-,++}(\bar{x}, \dots) \bar{E}_T.$$

Give access to the chiral odd GPDs



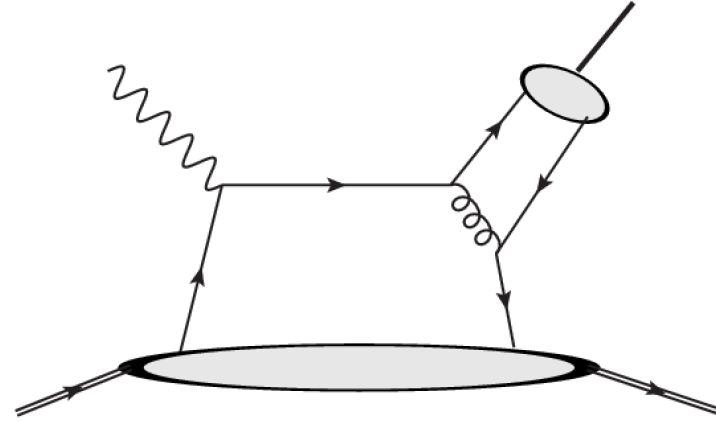
L. Frankfurt, P. Pobylitsa, M. Polyakov, and M. Strikman, 1999

S. Goloskokov, Y. p. Xie, X. r. Chen, 2022

Twist-3 correction to DA, G. Duplancić, P. Kroll, K. Passek-K., and L. Szymanowski, 2024

Helicity non-flip production: quark channel

➤ Vanishes at leading twist!



➤ Non-forward region, allowing pion to carry 1 unit OAM, twist-3 effect.

◆ Collinear expansion to isolate twist-3 contribution:

$$H(k_{\perp}, \Delta_{\perp}) = H(k_{\perp} = 0, \Delta_{\perp} = 0) + \left. \frac{\partial H(k_{\perp}, \Delta_{\perp} = 0)}{\partial k_{\perp}^{\mu}} \right|_{k_{\perp}=0} k_{\perp}^{\mu} + \left. \frac{\partial H(k_{\perp} = 0, \Delta_{\perp})}{\partial \Delta_{\perp}^{\mu}} \right|_{\Delta_{\perp}=0} \Delta_{\perp}^{\mu} + \dots$$

Angular correlations

➤ Scattering amplitudes
depending on different correlations

$$\mathcal{M}_1 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2 - 1)2\xi}{N_c^2 \sqrt{1 - \xi^2}} \delta_{\lambda\lambda'} \frac{\epsilon_\perp \times \Delta_\perp}{Q^2} \{\mathcal{F}_{1,1} + \mathcal{G}_{1,1}\},$$

$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2 - 1)2\xi}{N_c^2 \sqrt{1 - \xi^2}} \delta_{\lambda, -\lambda'} \frac{M \epsilon_\perp \cdot S_\perp}{Q^2} \{\mathcal{F}_{1,2} + \mathcal{G}_{1,2}\}$$

$$\mathcal{M}_4 = \frac{ig_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2 - 1)2\xi}{N_c^2 \sqrt{1 - \xi^2}} \lambda \delta_{\lambda\lambda'} \frac{\epsilon_\perp \cdot \Delta_\perp}{Q^2} \{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\} \left(\begin{array}{l} S_\perp^\mu = (0^+, 0^-, -i, \lambda) \end{array} \right)$$



$$\mathcal{F}_{1,1} = \int_{-1}^1 dx \frac{x^2 \int d^2 k_\perp F_{1,1}^{u+d}(x, \xi, \Delta_\perp, k_\perp)}{(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1 + z^2 - z)}{z^2(1 - z)^2}, \quad (8)$$

$$\mathcal{G}_{1,1} = \int_{-1}^1 dx \int_0^1 dz \frac{\phi_\pi(z)(x^2 + 2x^2 z + \xi^2)}{z^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int d^2 k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^{u+d}(x, \xi, \Delta_\perp, k_\perp), \quad (9)$$

$$\mathcal{F}_{1,2} = \int_{-1}^1 dx x \frac{\xi(1 - \xi^2) \int d^2 k_\perp k_\perp^2 F_{1,2}^{u+d}(x, \xi, \Delta_\perp, k_\perp)}{M^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1 + z^2 - z)}{z^2(1 - z)^2}, \quad (10)$$

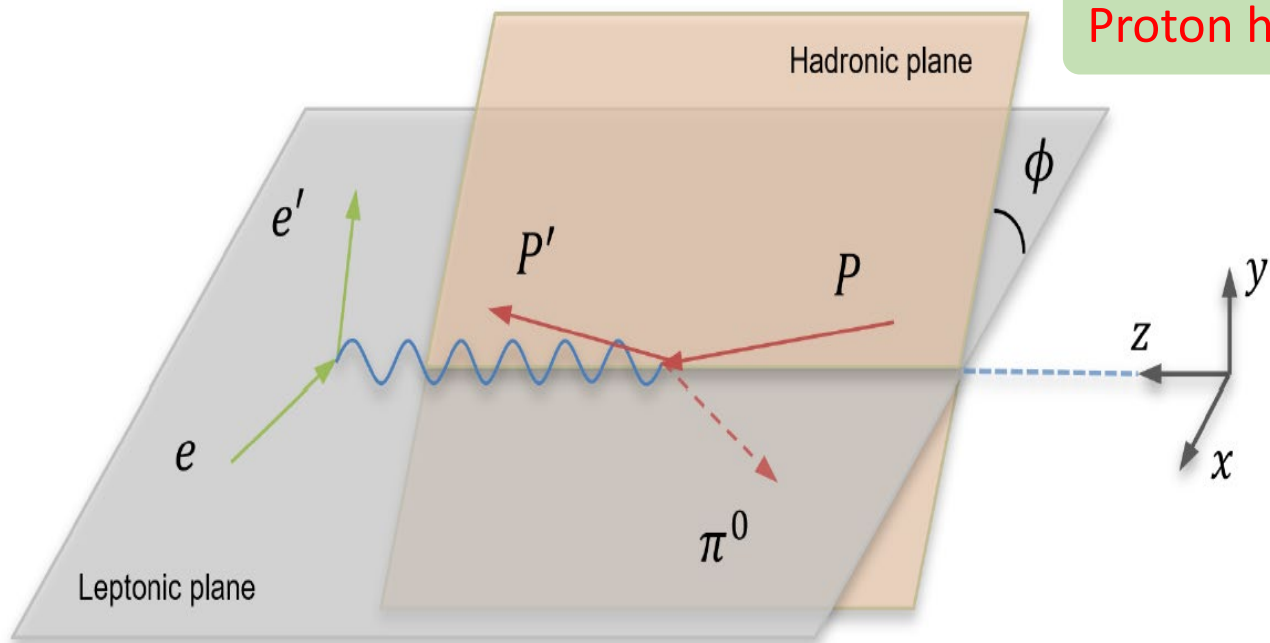
$$\mathcal{G}_{1,2} = \int_{-1}^1 dx \int_0^1 dz \frac{\phi_\pi(z)(x^2 + 2x^2 z + \xi^2)(1 - \xi^2)}{z^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int d^2 k_\perp \frac{k_\perp^2}{M^2} G_{1,2}^{u+d}(x, \xi, \Delta_\perp, k_\perp), \quad (11)$$

$$\mathcal{F}_{1,4} = \int_{-1}^1 dx \frac{x\xi \int d^2 k_\perp k_\perp^2 F_{1,4}^{u+d}(x, \xi, \Delta_\perp, k_\perp)}{M^2(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1 + z^2 - z)}{z^2(1 - z)^2}, \quad (12)$$

$$\mathcal{G}_{1,4} = \int_{-1}^1 dx \int_0^1 dz \frac{x(4\xi^2 z + \xi^2 - 2x^2 z + x^2)}{z^2 \xi (x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \phi_\pi(z) \times \int d^2 k_\perp G_{1,4}^{u+d}(x, \xi, \Delta_\perp, k_\perp). \quad (13)$$

Azimuthal dependent cross section

$$\frac{d\sigma}{dt dQ^2 dx_B d\phi} = \frac{(N_c^2 - 1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1 - \xi^2) Q^{10} (1 + \xi)} [1 + (1 - y)^2] \times \left\{ \left[|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 + 2 \frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2} + \mathcal{G}_{1,2}|^2 \right] \right. \\ \left. + \cos(2\phi) a \left[-|\mathcal{F}_{1,1} + \mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4} + \mathcal{G}_{1,4}|^2 \right] + \lambda \sin(2\phi) 2a \operatorname{Re} \left[(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) (\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^*) \right] \right\}$$



Proton helicity

Distinguished experimental signature of $F_{1,4}$

$$\phi = \phi_{l_\perp} - \phi_{\Delta_\perp}$$

$$\int d^2 k_\perp \operatorname{Re}[F_{1,1}(x, \xi, \Delta_\perp, k_\perp)] \approx H(x, \xi, \Delta_\perp)$$

$$\int d^2 k_\perp \operatorname{Re}[G_{1,4}(x, \xi, \Delta_\perp, k_\perp)] \approx \tilde{H}(x, \xi, \Delta_\perp)$$

Model input for numerical estimations

➤ Endpoint singularity and discontinuity:

$$\mathcal{F}_{1,4} = \int_{-1}^1 dx \frac{x\xi \int d^2 k_{\perp} k_{\perp}^2 F_{1,4}^{u+d}(x, \xi, \Delta_{\perp}, k_{\perp})}{M^2(x + \xi - i\epsilon)^2(x - \xi + i\epsilon)^2} \int_0^1 dz \frac{\phi_{\pi}(z)(1 + z^2 - z)}{z^2(1 - z)^2}$$

◆ Model dependent method

$$\int_{\langle p_{\perp}^2 \rangle / Q^2}^{1 - \langle p_{\perp}^2 \rangle / Q^2} dz \quad \frac{1}{(x - \xi + i\epsilon)^2} \xrightarrow{\text{yellow arrow}} \frac{1}{(x - \xi - \langle p_{\perp}^2 \rangle / Q^2 + i\epsilon)^2}$$

S. V. Goloskokov and P. Kroll, 2005

I. V. Anikin, O. V. Teryaev, 2003

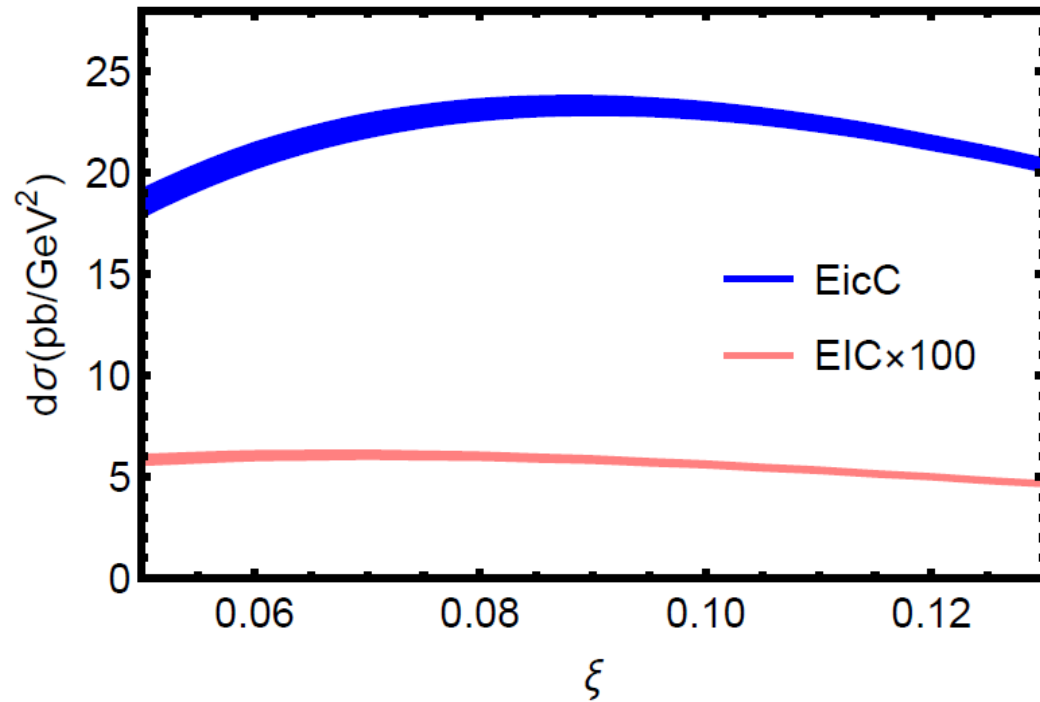
➤ Wandzura-Wilczek(WW) approximation:

$$L_q(x) \approx x \int_x^1 \frac{dx'}{x'} q(x') - x \int_x^1 \frac{dx'}{x'^2} \Delta q(x')$$

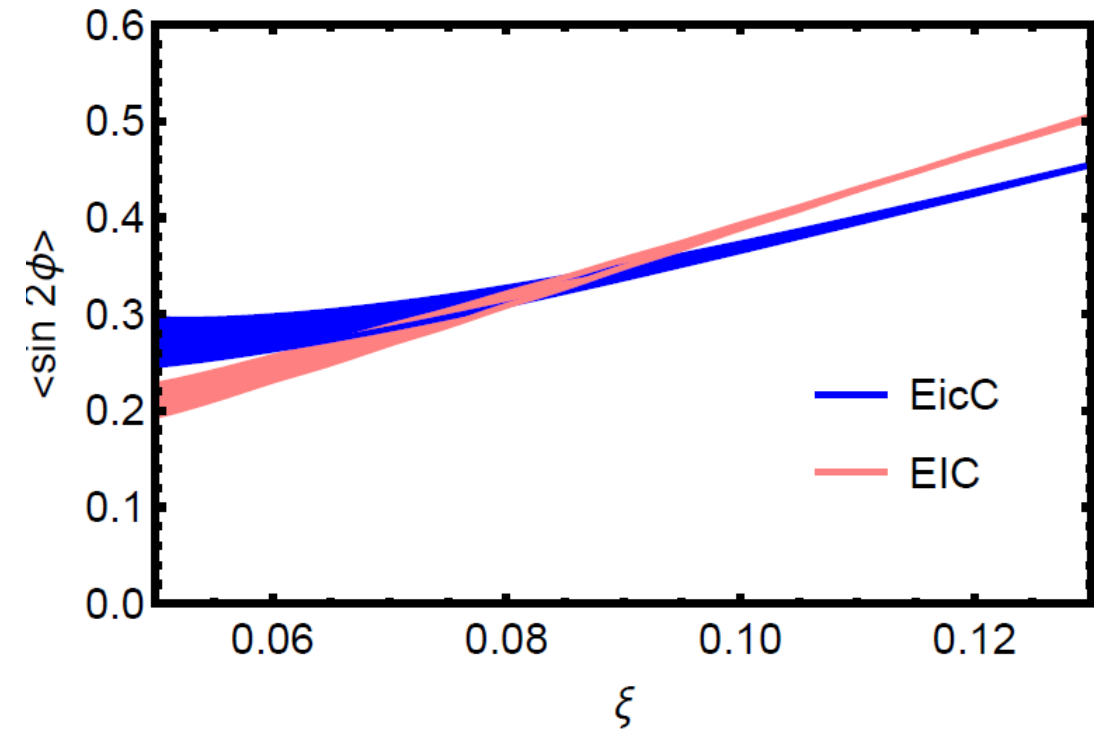
Y. Hatta and S. Yoshida, 2012

Numerical results

Unpolarized cross section:



$\sin 2\Phi$ azimuthal asymmetry



EIC: $Q^2=10$, EicC: $Q^2=3$

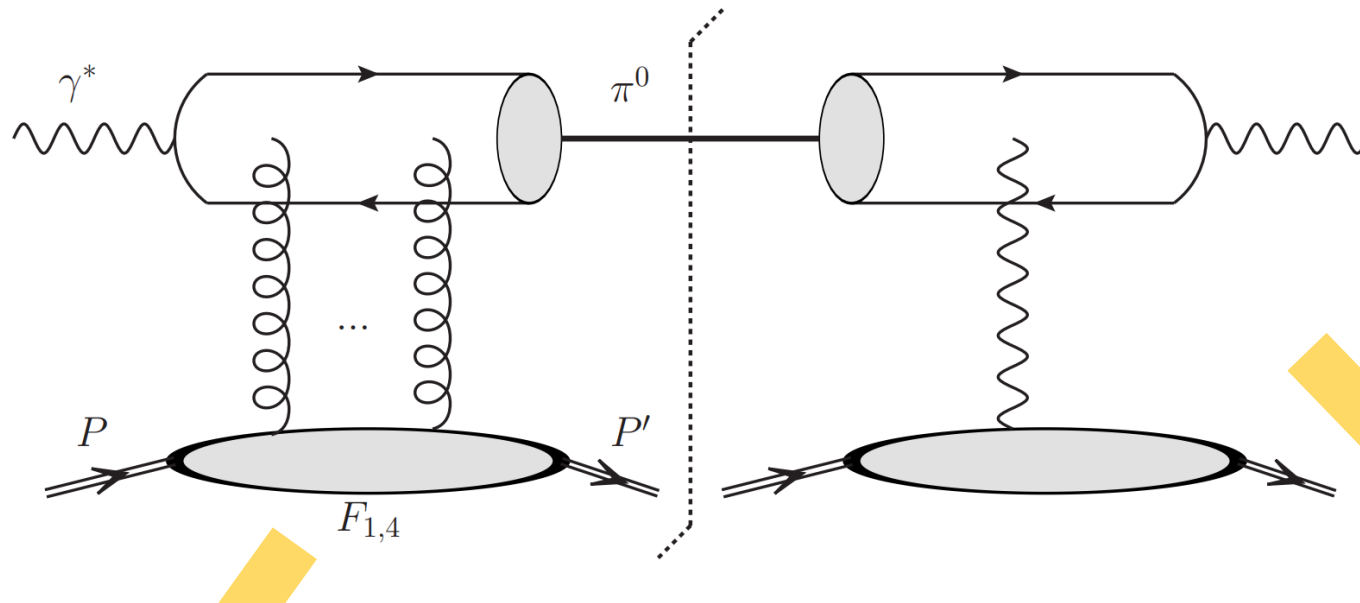
$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$$

□ We focus on the large zeta region to suppress gluon contribution.

The gluon channel

When $\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$ is small, gluon channel dominates

When $t = (p' - p)^2$ is small, the Primakoff process is important



$$\mathcal{M}_L = i \frac{g_s^2 e f_\pi}{N_c \sqrt{2}} \lambda \delta_{\lambda\lambda'} \frac{\epsilon_\perp \cdot \Delta_\perp}{M^2 Q^2} \frac{\xi^2 (1 + \xi)}{\sqrt{1 - \xi^2}} \int_{-1}^1 dx \frac{F_{1,4}^{(1)}(x, \xi, \Delta_\perp)}{(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \int_0^1 dz \frac{\phi_\pi(z)}{z(1 - z)}$$

$$\mathcal{M}_R = \delta_{\lambda\lambda'} e^3 \sqrt{2} f_\pi \frac{\sqrt{1 - \xi^2}}{1 + \xi} \frac{(\epsilon_\perp^{\gamma^*} \times \Delta_\perp)}{x_B \Delta_\perp^2} \mathcal{F}(t) \int_0^1 dz \frac{\phi_\pi(z)}{6z(1 - z)}$$

Polarization dependent cross section

$$\frac{d\Delta\sigma}{dt dQ^2 dx_B d\phi} = -\sin(2\phi) \frac{\alpha_{em}^3 \alpha_s f_\pi^2 (1-y) \xi x_B \mathcal{F}(t)}{3Q^8 N_c} \left[\int_0^1 dz \frac{\phi_\pi(z)}{z(1-z)} \right]^2 \text{Im} \left[\int_{-1}^1 dx \frac{F_{1,4}^{(1)}(x, \xi, \Delta_\perp)/M^2}{(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} \right]$$

➤ Unfortunately, due to symmetry properties:

$$F_{1,4}^{(1)}(x, \xi, \Delta_\perp) = -F_{1,4}^{(1)}(-x, \xi, \Delta_\perp)$$



$$\text{Im} \int_{-1}^1 dx \frac{\text{Re} F_{1,4}^{(1)}(x, \xi, \Delta_\perp)}{(x + \xi - i\epsilon)^2 (x - \xi + i\epsilon)^2} = 0$$

□ Only probe the imaginary part of gluon $F_{1,4}$

Unpolarized cross section in the near forward region

➤ Gluon Sivers contribution

$$\frac{d\sigma^{odderon}}{dt dQ^2 dx_B} \approx \frac{\pi^5 \alpha_{em}^2 \alpha_s^2 f_\pi^2}{8x_B N_c^2 M^2 Q^6} \left[1 - y + \frac{y^2}{2} \right] \left[\int_0^1 dz \frac{\phi_\pi(z)}{z(1-z)} \int d^2 k_\perp \frac{k_\perp^2 x f_{1T}^\perp(x, k_\perp^2)}{k_\perp^2 + z(1-z)Q^2} \right]^2$$

R. Boussarie, Y. Hatta, L. Szymanowski, and S. Wallon, 2020

➤ The Primakoff process

$$\frac{d\sigma^{Pri}}{dt dQ^2 dx_B} \approx \frac{\alpha_{em}^4 (2\pi) [1 + (1-y)^2] f_\pi^2}{x_B Q^6 \Delta_\perp^2} \frac{1-\xi}{1+\xi} \mathcal{F}^2(t) \left[\int_0^1 \frac{dz}{6z(1-z)} \phi_\pi(z) \right]^2$$

◆ The interference channel does not contribute to azimuthal averaged cross section.

Summary

- Probe quark OAM via exclusive π^0 production
- Clean background, no final state soft gluon radiations
- The asymmetry is not power suppressed
- Give the access to the DGLAP region
- Endpoint singularity and discontinuity
- Twist-3 quark-gluon correlation function is not included

Thank you for your attention!