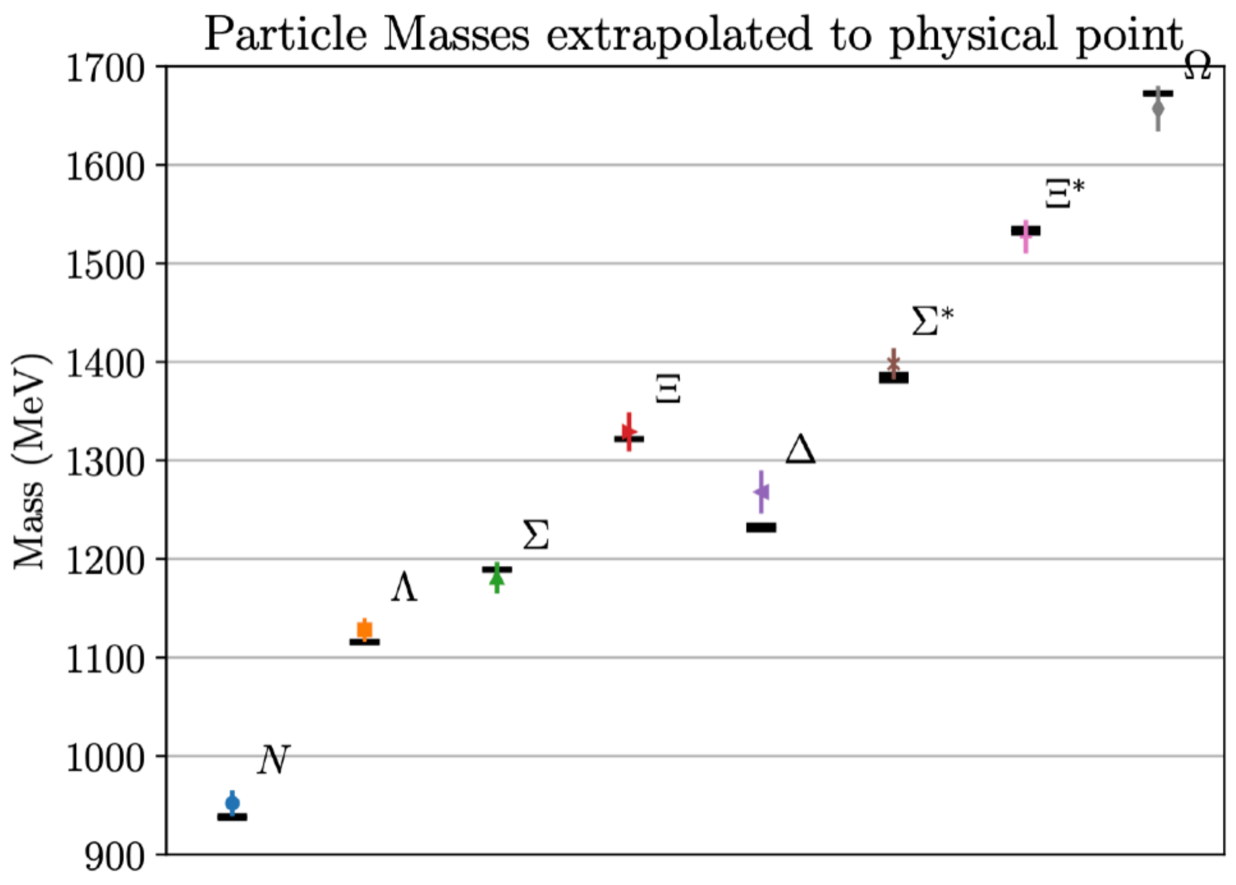
Trace anomaly contribution to the hadron mass

Yi-Bo Yang

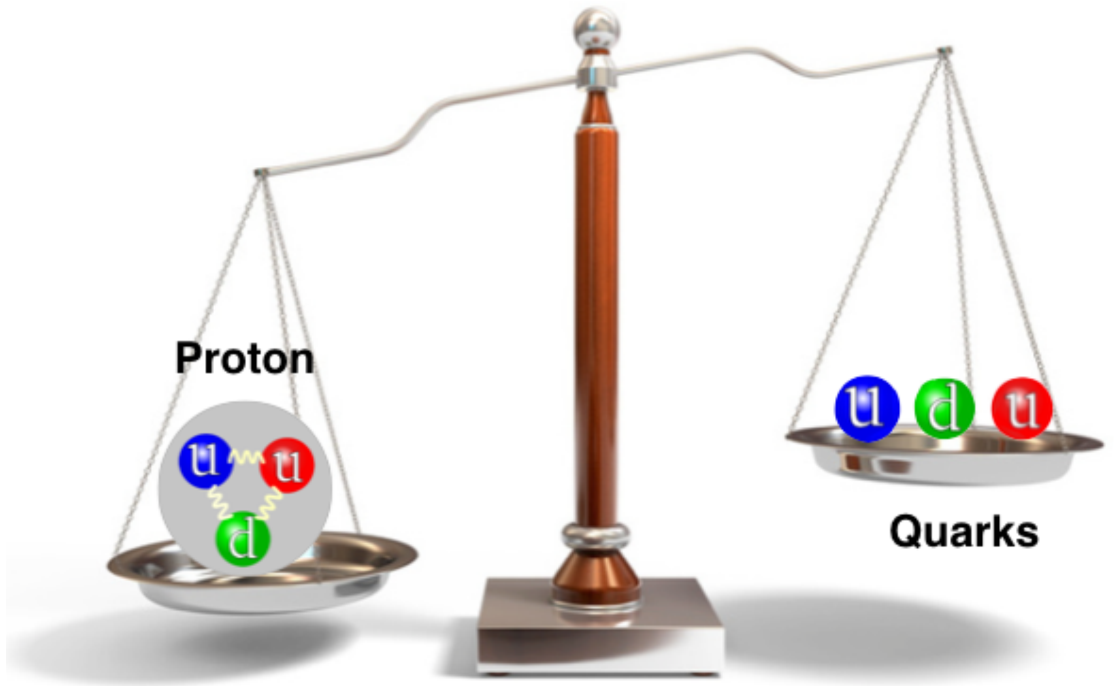
2024/02/20

Outline

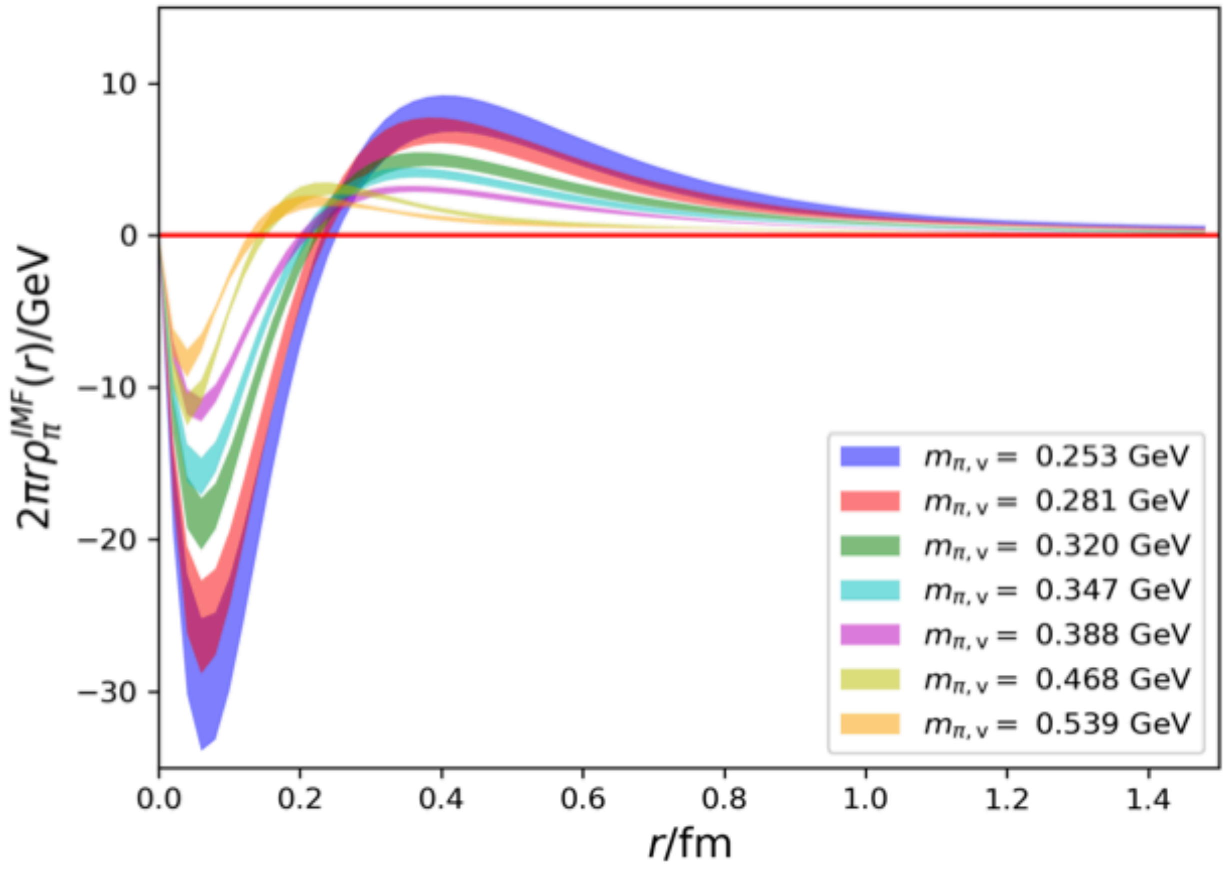
Hadron mass and trace anomaly



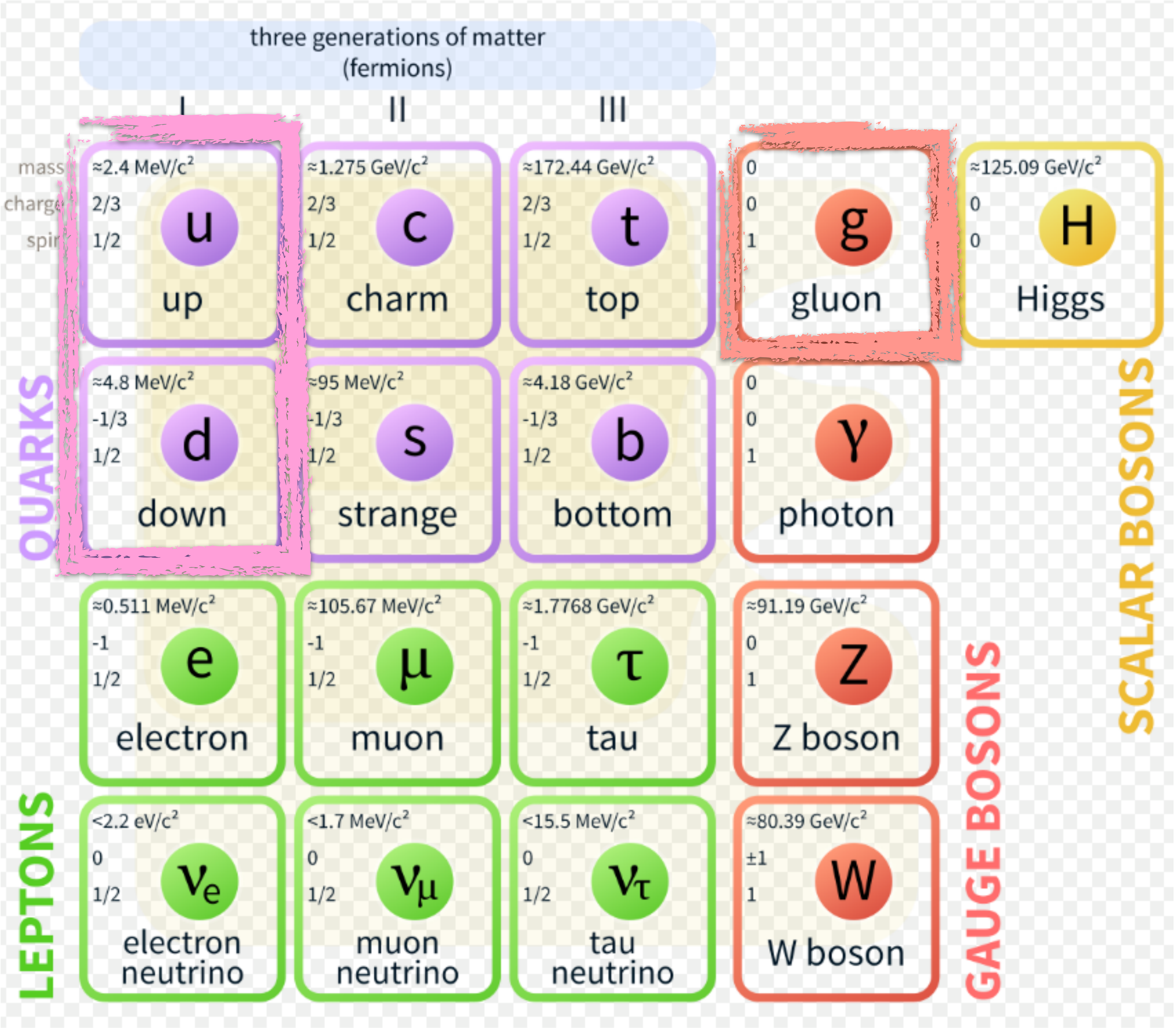
More information from direct calculations



Indirect calculations



Standard *model* of the elementary particles



- Most of the mass of the earth comes from the nucleon (proton and neutron) which are formed by **the quarks** and **gluon;**
- **Higgs** provided the quark masses;
- *Gluon doesn't couple with Higgs, and then is massless.*

The light quark masses

u-QUARK MASS

INSPIRE search

The u-, d-, and s-quark masses are estimates of so-called ``current-quark masses,'' in a mass- independent subtraction scheme such as $\overline{\text{MS}}$. The ratios m_u/m_d and m_s/m_d are extracted from pion and kaon masses using chiral symmetry. The estimates of d and u masses are not without controversy and remain under active investigation. Within the literature there are even suggestions that the u quark could be essentially massless. The s-quark mass is estimated from SU(3) splittings in hadron masses.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of μ = 2 GeV. Results quoted in the literature at μ = 1 GeV have been rescaled by dividing by 1.35. The values of ``Our Evaluation'' were determined in part via Figures 1 and 2.

VALUE (MeV)	*DOCUMENT ID*		*TECN*
$2.16^{+0.49}_{-0.26}$	**OUR EVALUATION**		
2.6 ±0.4	1 DOMINGUEZ	2019	THEO
2.130 ±0.041	2 BAZAVOV	2018	LATT
2.27 ±0.06 ±0.06	3 FODOR	2016	LATT
2.36 ±0.24	4 CARRASCO	2014	LATT

d-QUARK MASS

INSPIRE search

See the comment for the u quark above.

We have normalized the $\overline{\text{MS}}$ masses at a renormalization scale of μ = 2 GeV. Results quoted in the literature at μ = 1 GeV have been rescaled by dividing by 1.35. The values of ``Our Evaluation'' were determined in part via Figures 1 and 2.

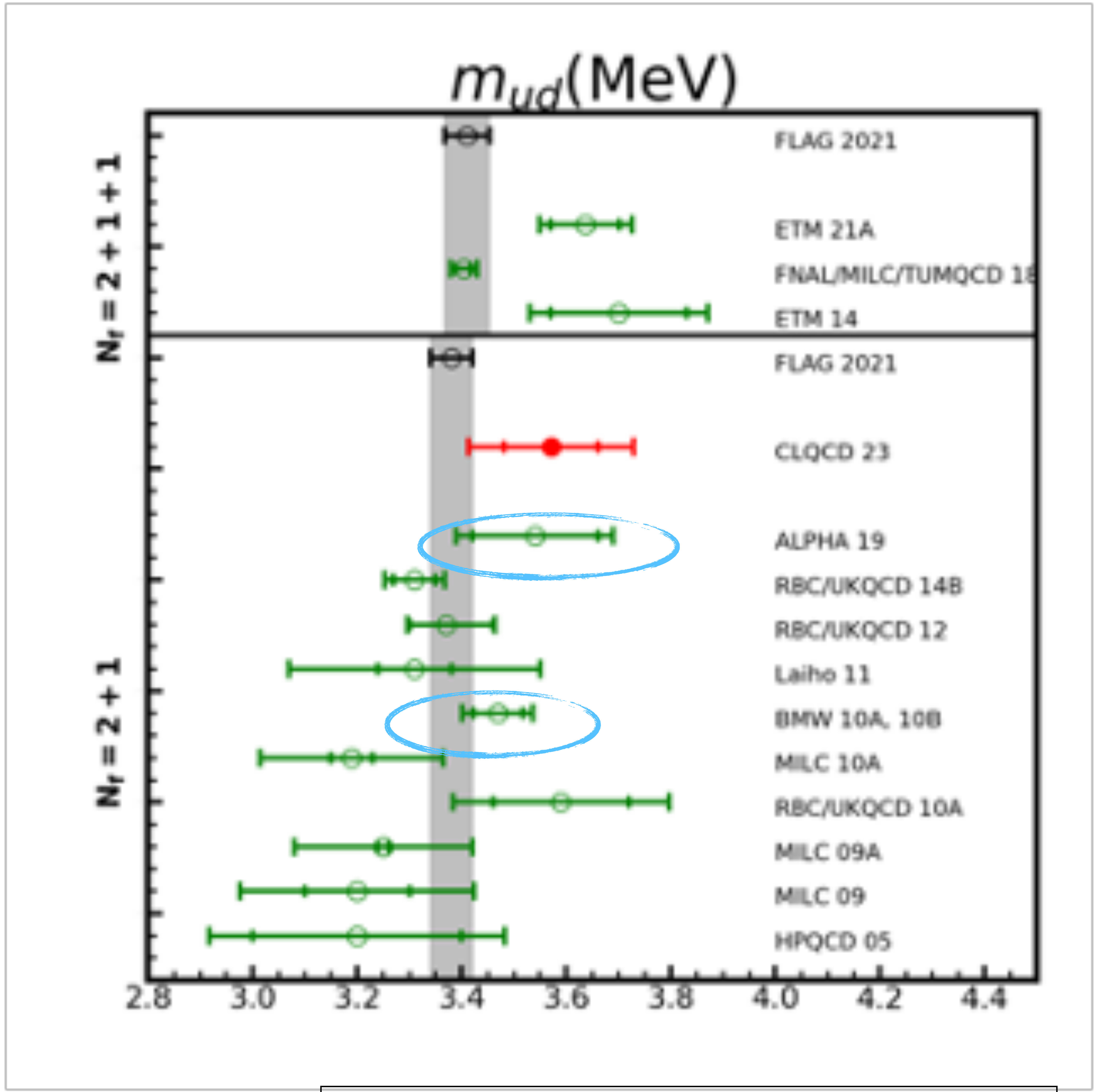
VALUE (MeV)	*DOCUMENT ID*		*TECN*
$4.67^{+0.48}_{-0.17}$	**OUR EVALUATION**		
5.3 ±0.4	1 DOMINGUEZ	2019	THEO
4.675 ±0.056	2 BAZAVOV	2018	LATT
4.67 ±0.06 ±0.06	3 FODOR	2016	LATT
5.03 ±0.26	4 CARRASCO	2014	LATT

P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

on PDG Live

- The u and d quark masses on PDG Live are ∼2.2 and 4.7 MeV respectively, at MS-bar 2 GeV.
- The values will be 135% at 1 GeV, and 84.6% at 4 GeV.
- They are majorly constrained by the Lattice QCD calculation.
- If we consider the RI/MOM scheme under the Landau gauge at $\mu = 0$ GeV, the light quark can have a mass ~300 MeV. Its meaning is still "**not without controversy and remain under active investigation**".

The light quark masses

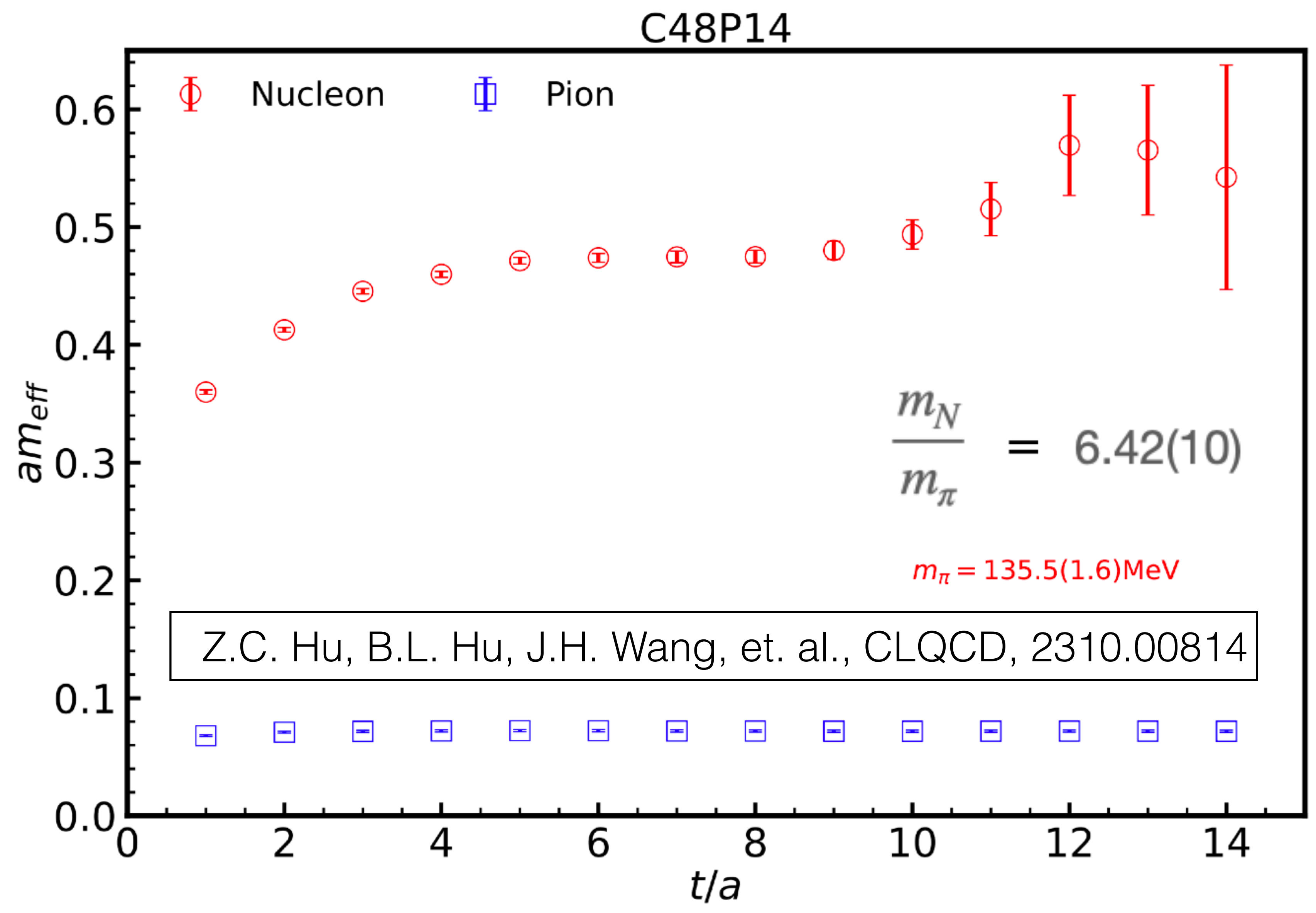


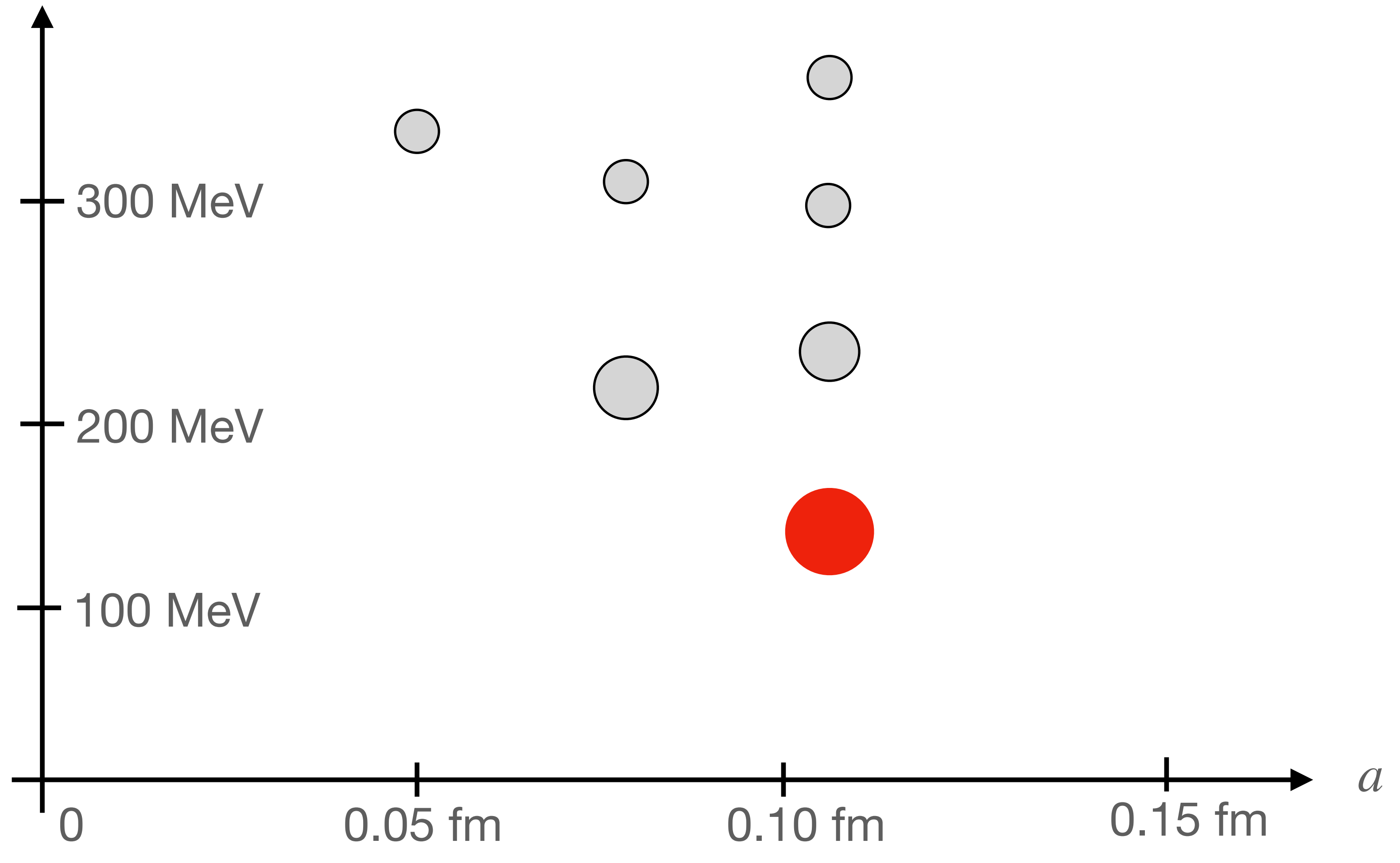
Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

From lattice QCD

- Lattice QCD treats the quark masses and strong coupling constant as free parameters.
- One can calculate the pion mass with different quark mass, and then requires the charged pion mass to be 134.98 MeV to determine **the light quark mass**;
- The bare quark mass is converted to MS-bar 2 GeV with non-perturbative renormalization.
- **The first CLQCD prediction of light quark mass has been carried out using the CLQCD ensembles last year, including the systematic uncertainty from the different non-perturbative renormalization schemes.**

CLQCD ensembles





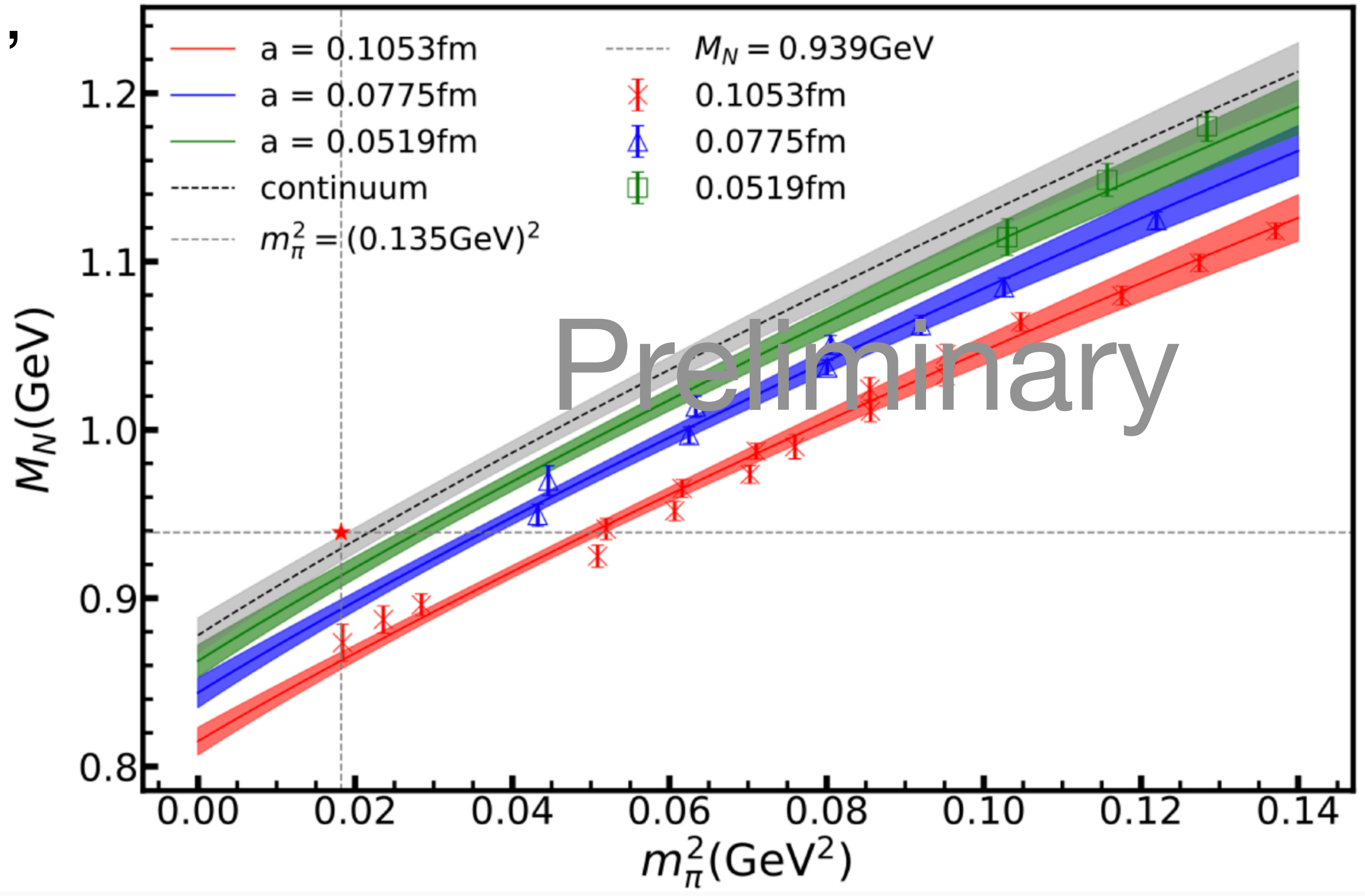
Nucleon mass v.s. pion mass

- At $a = 0.105(3)$ fm, we have

$$m_\pi = 135.5(1.6)\ \text{MeV},$$

$$m_N = 870(12)\ \text{MeV}.$$

- m_N are ~7% smaller than the physical value, and can reach the physical value after the continuum extrapolation.



Quark mass dependence

of different hadron masses

- The $\alpha_s(m_Z)$ dependence of most hadrons around the chiral limit is somehow similar to the nucleon case: $m_H = c_0 + c_1 m_q^{\overline{\rm MS}(2{\rm GeV})} + \mathcal{O}(m_q^2)$ (based on the chiral perturbative theory).

- But the pseudo-scalar meson around the chiral limit is different:

$$m_{PS} \propto \sqrt{m_q^{\overline{\rm MS}(2{\rm GeV})}}$$

  and then approach zero when $m_q \to 0$.

- Usually we consider pseudo-scalar meson around the chiral limit, pion, as a goldstone boson of the spontaneously chiral symmetry breaking and then almost massless.

Trace anomaly

Defintion

- The trace of QCD energy momentum tensor (EMT),

$$T_{\mu\nu} = \frac{i}{2}\bar{\psi}\overleftrightarrow{D}_{(\mu}\gamma_{\nu)}\psi + \frac{1}{4}g_{\mu\nu}F^2 - F_{\mu\rho}F_{\rho\nu}$$

is just the quark mass term $m\bar{\psi}\psi$ at the classical level.

- But with the quantum corrections, the quantum correction changes the trace term into:

J.Collins et,al. PRD16(1977) 438

$$T^{\mu}_{\mu} = \left[1 + \frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right]m\bar{\psi}\psi + \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right]F^2,$$

where the terms proportional to α_s are the QCD quantum corrections.

- Then the hadron mass can be decomposed into three pieces:

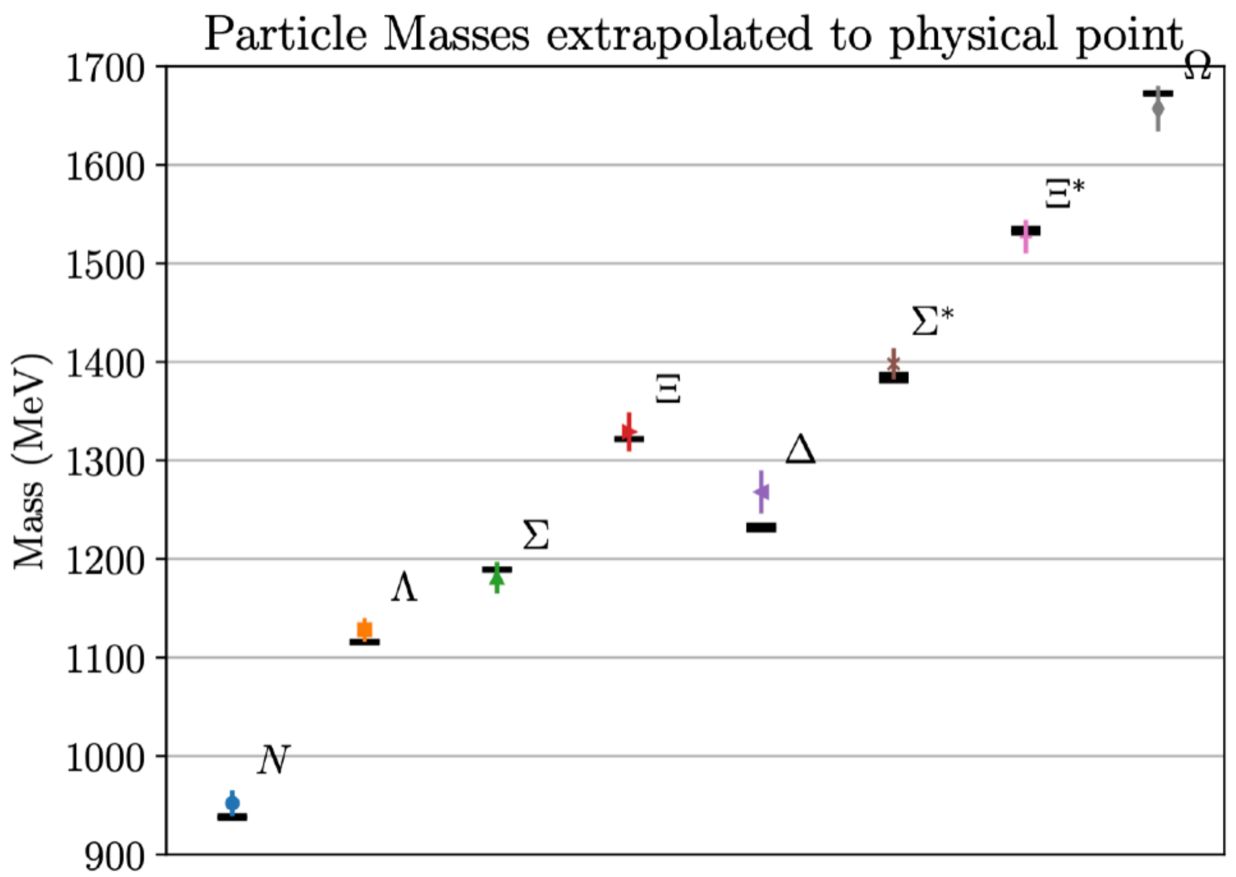
M.A. SHIFMAN et,al. PLB78(1978)

$$m_N = m\langle\bar{\psi}\psi\rangle_N + \left[\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right]m\langle\bar{\psi}\psi\rangle_N + \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right]\langle F^2\rangle_N.$$

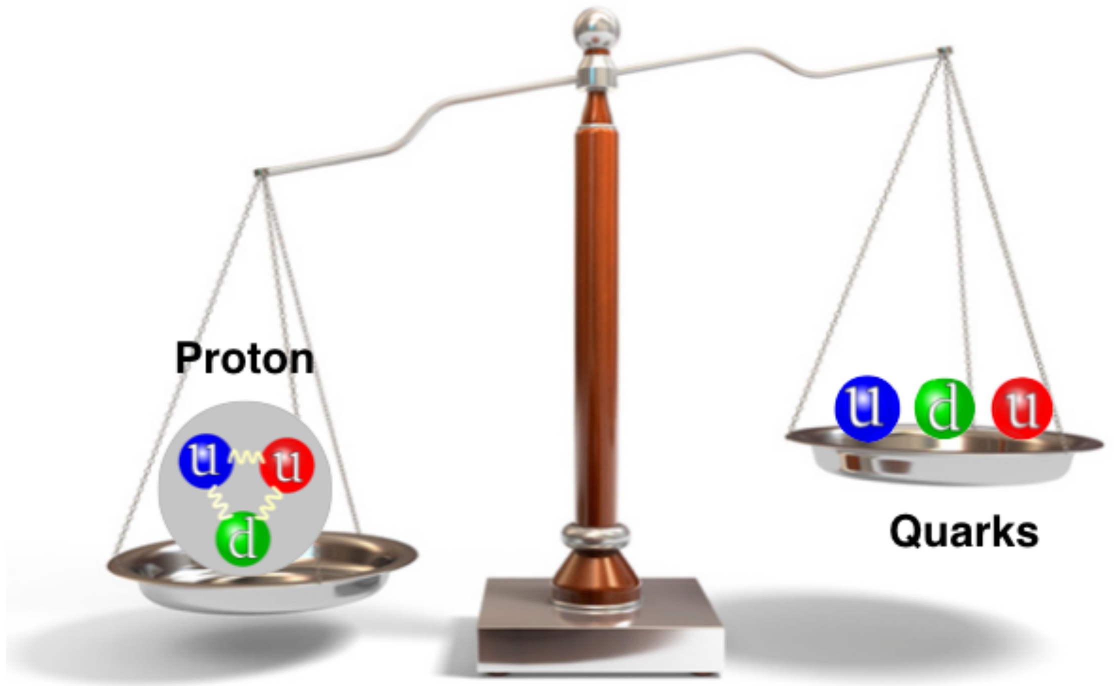
YBY, et. al., χQCD Collaboration, PRD94(2016)054503

Outline

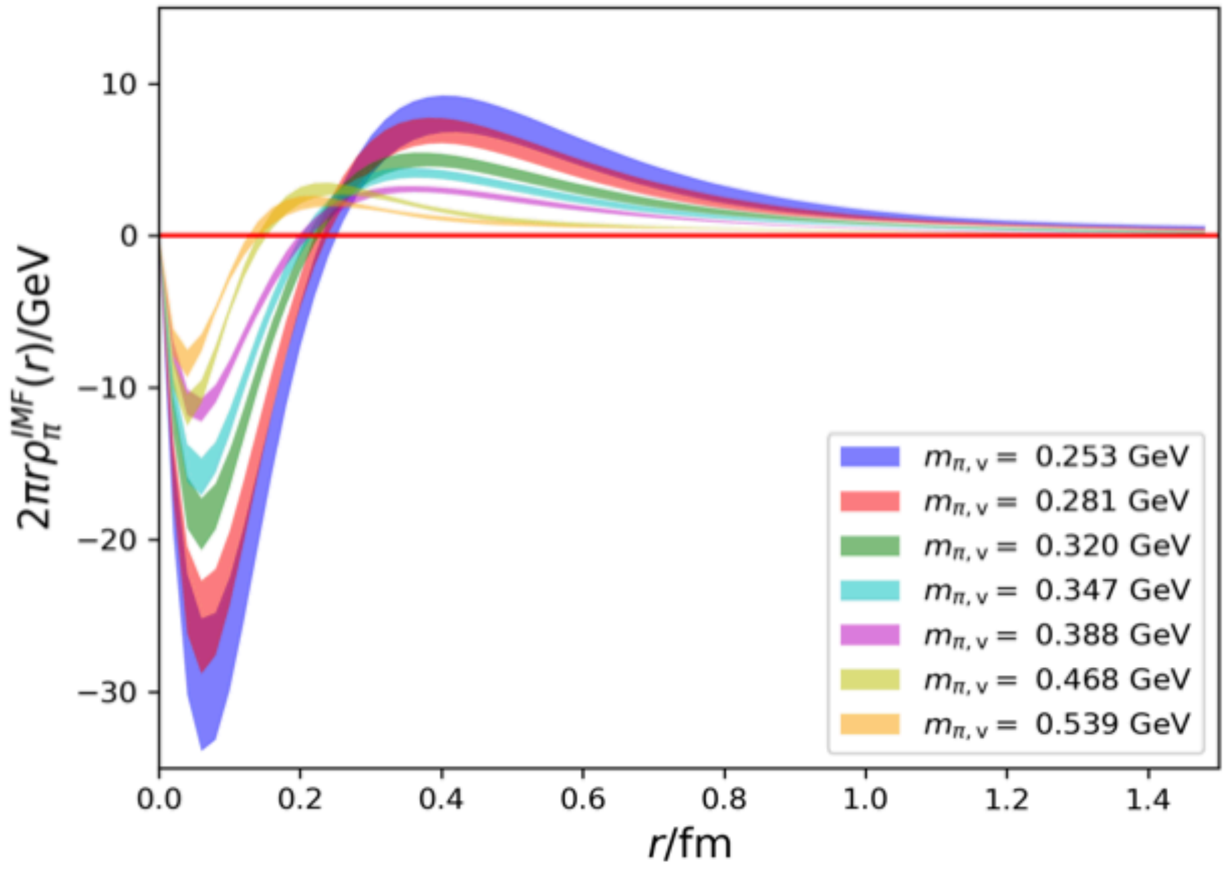
Hadron mass and trace anomaly



More information from direct calculations



Indirect calculations



Indirect calculation

Sum rule and Feynman-Hellman theorem

- The quark condensate in the hadron can be extracted from the Feynman-Hellman theorem:

$$\langle \bar{\psi}\psi \rangle_H = \frac{\partial m_H(m_q)}{\partial m_q};$$

- Thus the trace anomaly contribution to the hadron mass can be extracted as

$$m_H - m_q \frac{\partial m_H(m_q)}{\partial m_q} = \langle H_a \rangle_H \equiv \Big[\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\Big] m \langle \bar{\psi}\psi \rangle_H + \Big[\Big(-\frac{11}{8\pi} + \frac{N_f}{12\pi}\Big)\alpha_s + \mathcal{O}(\alpha_s^2)\Big] \langle F^2 \rangle_H.$$

- If $m_H = c_0 + c_1 m_q + \mathcal{O}(m_q^2)$, then $\langle H_a \rangle_H = c_0 + \mathcal{O}(m_q^2)$; If $m_H = \sqrt{c_1 m_q + \mathcal{O}(m_q^2)}$, then $\langle H_a \rangle_H = \frac{1}{2} m_H (1 + \mathcal{O}(m_q))$.

- But the clover fermion used by the CLQCD ensemble introduces additive chiral symmetry breaking and then one need to verify the Feynman-Hellman theorem actually holds.

Trace anomaly

F^2 contribution

$$m_H = m\langle\bar{\psi}\psi\rangle_H + [\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)]m\langle\bar{\psi}\psi\rangle_H + [(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)]\langle F^2\rangle_H\,.$$

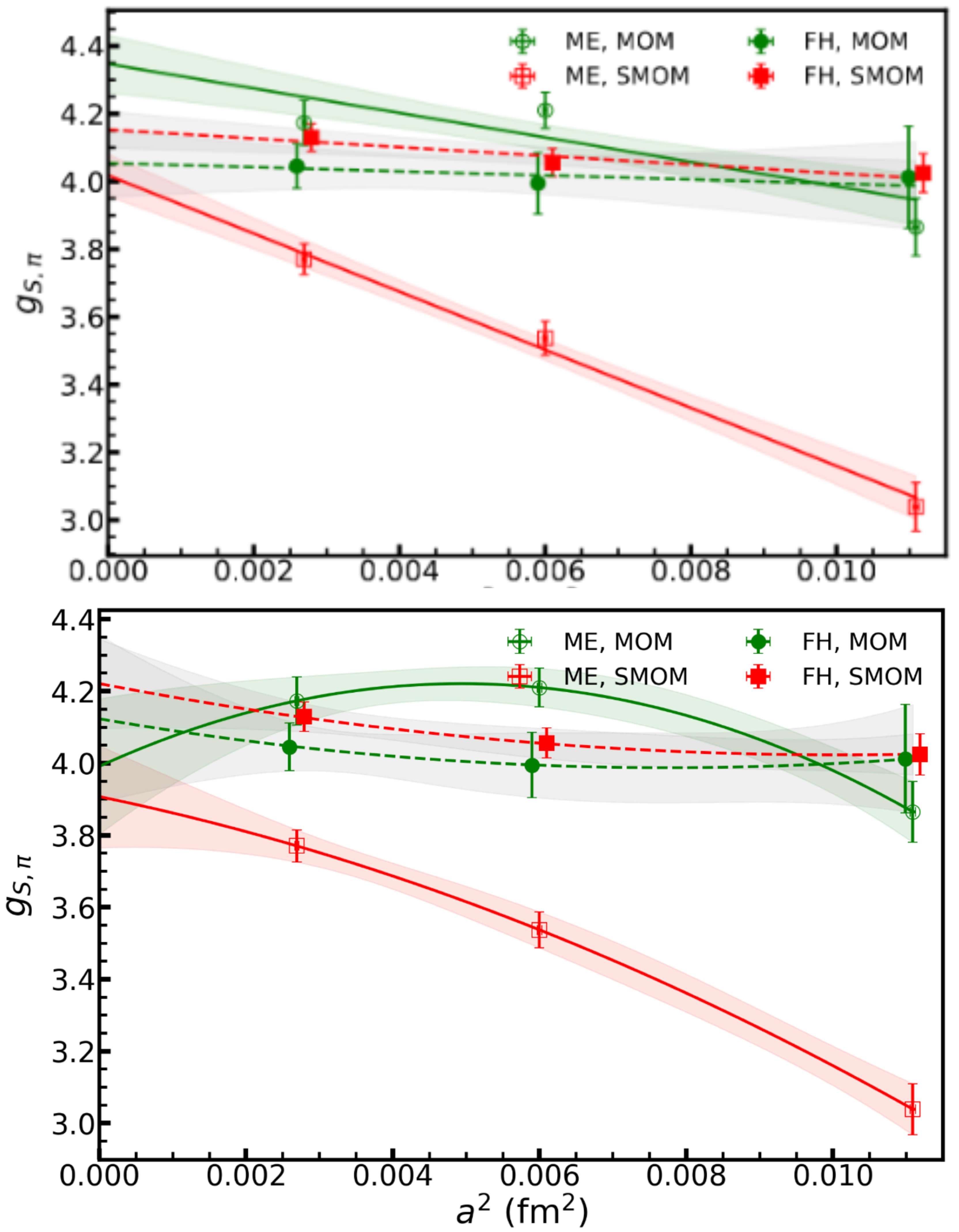
- $\langle F^2\rangle_H$ can related to the heavy quark matrix element in the heavy quark limit:

$$m_Q\bar{\psi}_Q\psi_Q \xrightarrow[m_Q\to\infty]{} -\frac{\alpha_s}{12\pi}F^2 + \mathcal{O}(\alpha_s^2)\,.$$

- Thus the heavy quark correlation $m_Q\langle\bar{\psi}_Q\psi_Q\rangle_H$ to the hadron *H* is proportional to $\langle F^2\rangle_H$.
- In the chiral limit, $\langle F^2\rangle_N$ should contributes all the nucleon mass, ~800 MeV.
- On the other hand, $\langle F^2\rangle_\pi$ should be zero in the chiral limit. With $\gamma_m = 1 + \frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2) \sim 0.3 + \mathcal{O}(\alpha_s^4)$ at MS-bar 2 GeV, $\langle F^2\rangle_\pi$~50 MeV.

Chiral symmetry breaking and renormalization

Restore of chiral symmetry in the continuum



- Renormalized quark mass $m_q^R = Z_A/Z_P m_q^{\rm PC}$ with 317 MeV pion mass at three lattice spacings:
- Feynman-Hellman theorem can extract $g_{S,\pi} = \langle \bar{u}u \rangle_\pi$ as

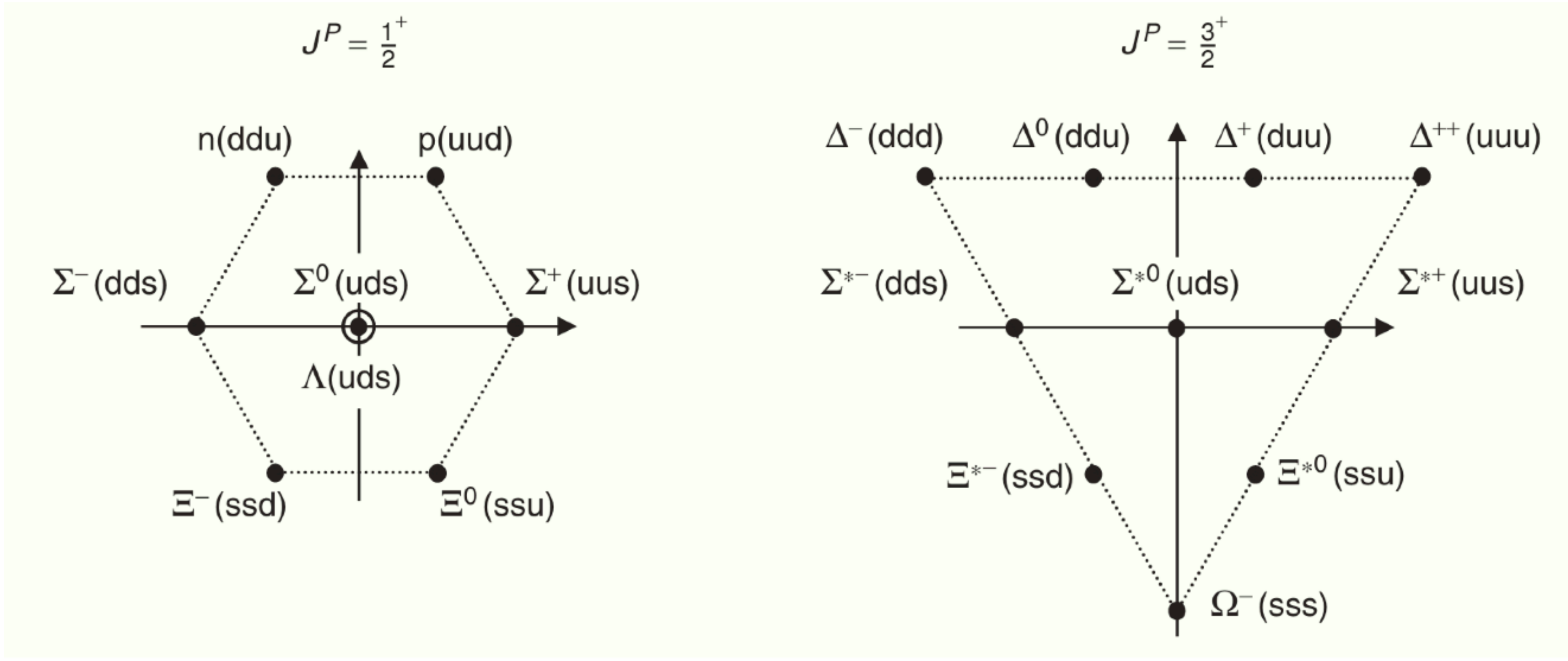
$$g_{S,\pi}^{\rm FH} = \frac{1}{2}\frac{\partial m_\pi(m_q)}{\partial m_q} \simeq \frac{Z_P}{Z_A}\frac{m_\pi}{4m_q^{\rm PC}} + \mathcal{O}(m_q, a^2)$$

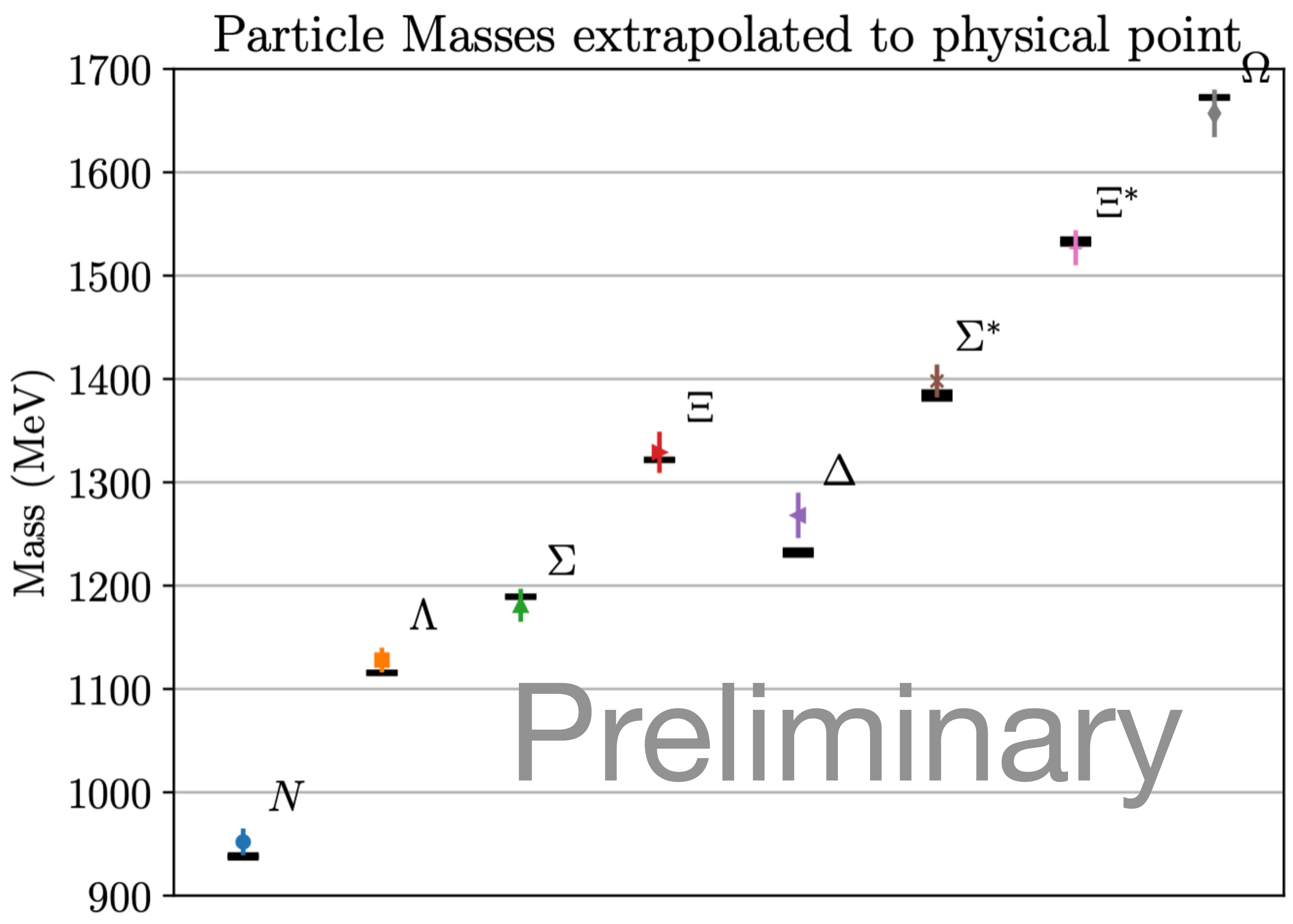
which is 4.04(6)(12) for $m_\pi = 317\ \mathrm{MeV}$ in the continuum.

- Renormalized $g_{S,\pi}^{\rm R,ME} = Z_S \frac{\langle \pi | S | \pi \rangle_{\rm conn}}{\langle \pi | \pi \rangle}$ based on the direct calculation:
- $g_{S,\pi}^{\rm ME}$ using RI/MOM scheme has smaller discretization error, and agree with $g_{S,\pi}^{\rm R,FH}$ within 2σ at all the lattice spacings.

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Trace anomaly





Octet and decuplet baryons

- **Octet:**
  - $m_0 = 0.90(6)$ GeV;
  - $a_l = 3.0(9), b_l = 1.6(9)$;
  - $a_s = 1.9(2), b_s = 1.4(2)$;
  - $c_l = 1.4(2), c_s = 0.1(4)$;

$$m_N = m_0 + (a_l + b_l + 2c_l)m_l + c_s m_s$$

$$m_\Lambda = m_0 + (\frac{a_l + 4b_l}{3} + 2c_l)m_l + (\frac{2a_l - b_l}{3} + c_s)m_s$$

$$m_\Sigma = m_0 + (a_l + 2c_l)m_l + (b_l + c_s)m_s$$

$$m_\Xi = m_0 + (b_l + 2c_l)m_l + (a_l + c_s)m_s$$

- **Decuplet:**
  - $\bar{m}_0 = 1.19(3)$ GeV;
  - $\bar{a} = 1.4(1)$;
  - $\bar{c}_l = 2.2(2), \bar{c}_s = 0.5(4)$.

$$m_\Delta = \bar{m}_0 + (3\bar{a} + 2\bar{c}_l)m_l + \bar{c}_s m_s$$

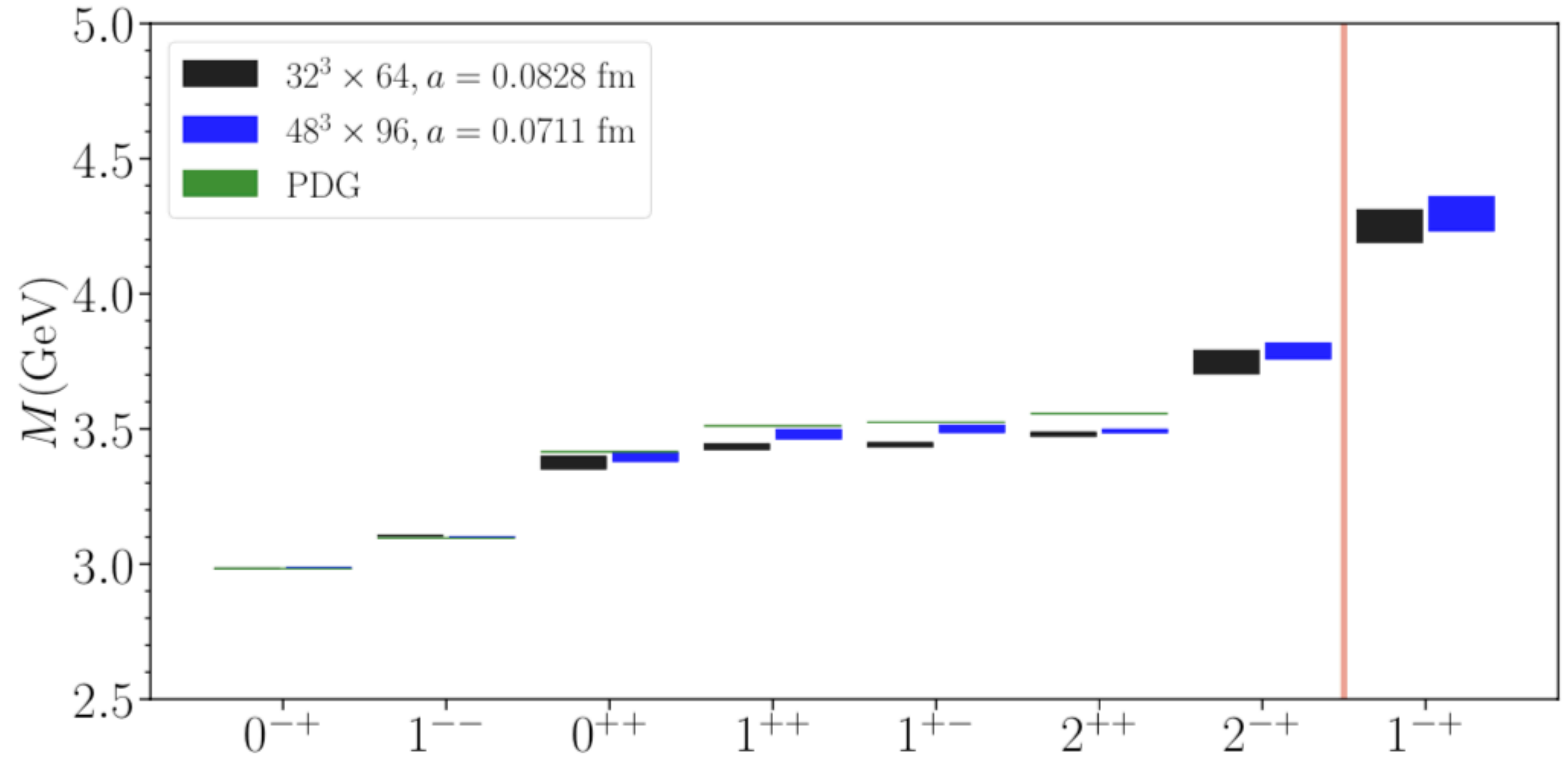
$$m_{\Sigma*} = \bar{m}_0 + (2\bar{a} + 2\bar{c}_l)m_l + (\bar{a} + \bar{c}_s)m_s$$

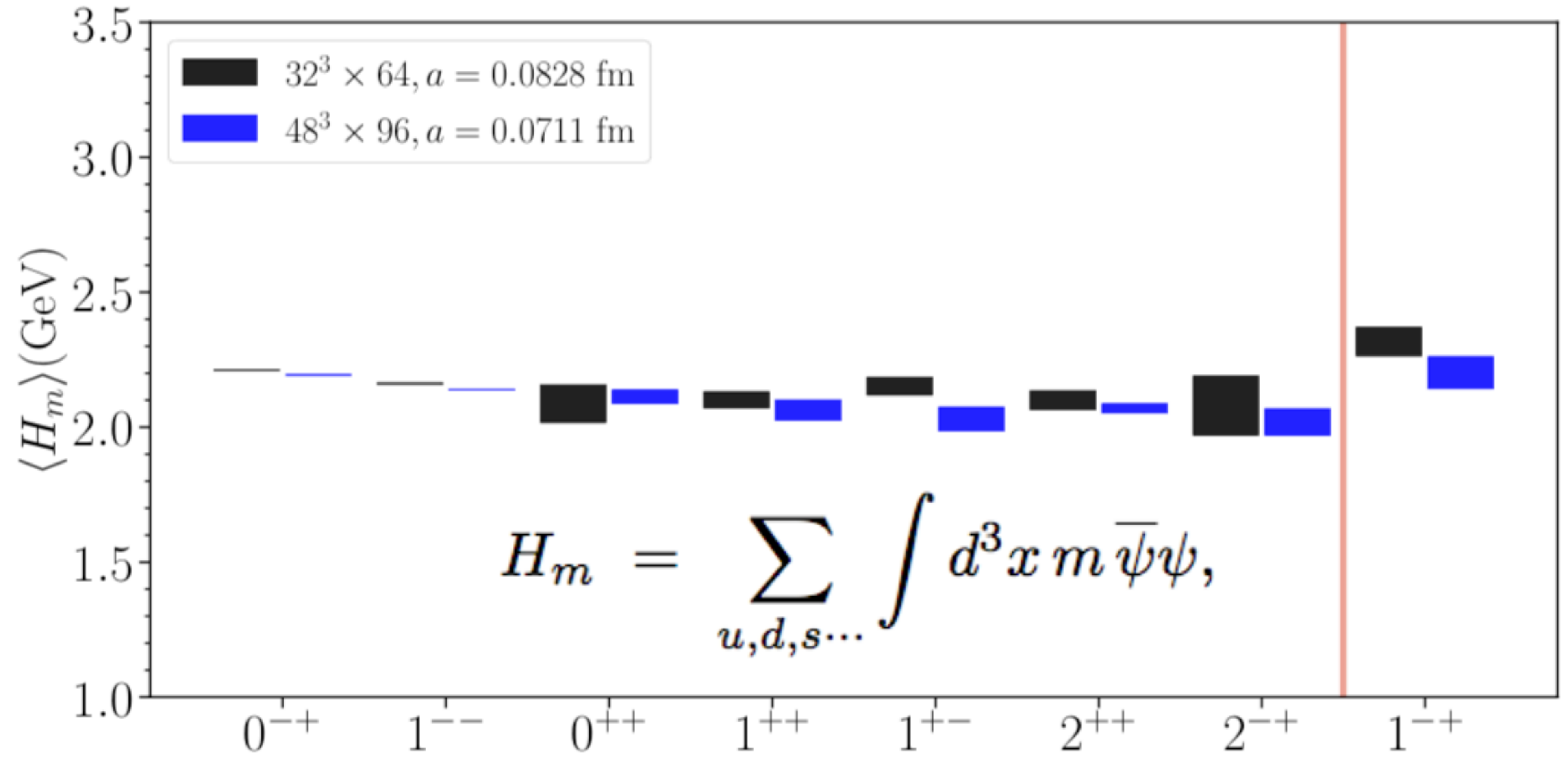
$$m_{\Xi*} = \bar{m}_0 + (\bar{a} + 2\bar{c}_l)m_l + (2\bar{a} + \bar{c}_s)m_s$$

$$m_{\Omega*} = \bar{m}_0 + 2\bar{c}_l m_l + (3\bar{a} + \bar{c}_s)m_s$$

Trace anomaly

Charmonium



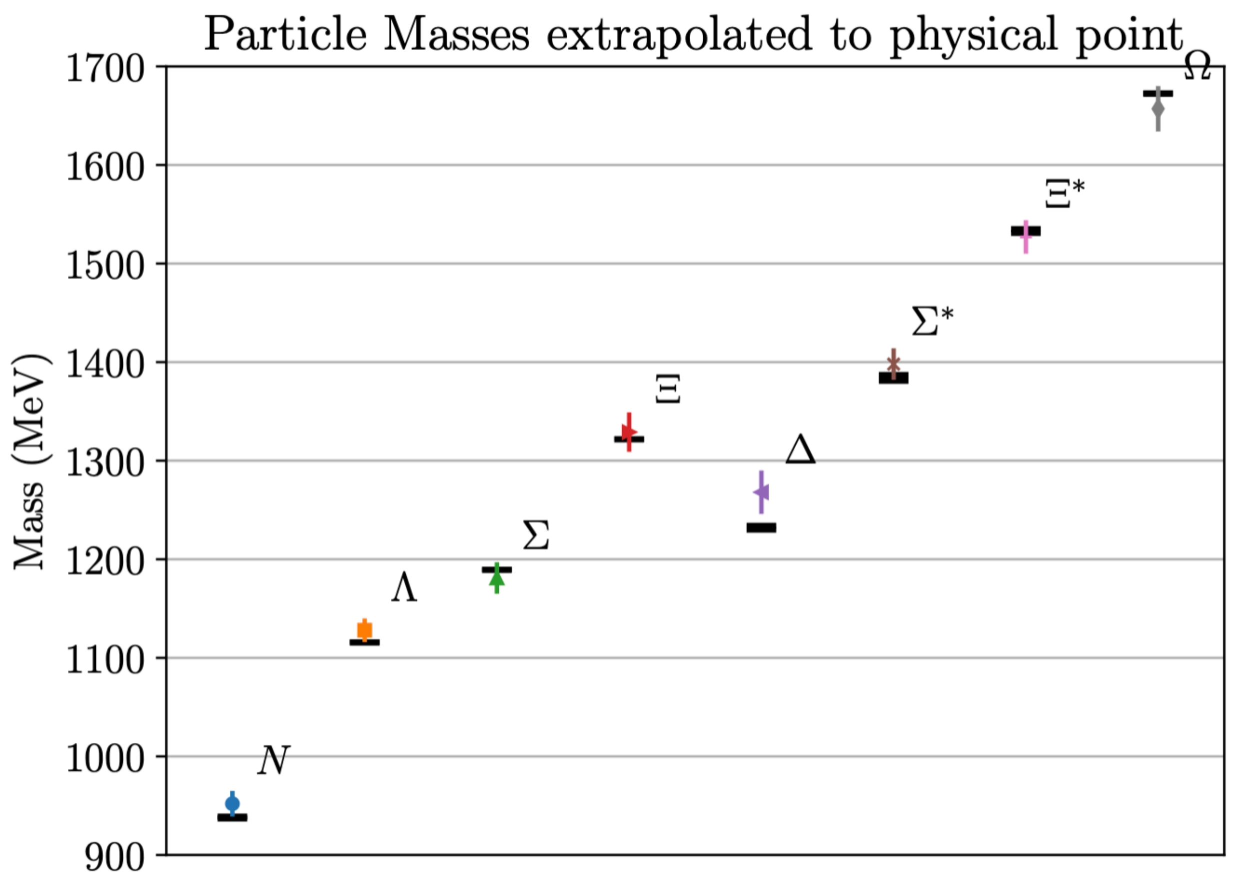


- The quark mass contribution to all the charmonium states are all around 2.3 GeV;
- Even though the hadron masses can be quite different.
- The rest contribution should comes from the trace anomaly;
- The gluon trace anomaly will be small than 100 meV in η_c, but more than 500 MeV in the P-wave charmonium.

W. Sun, et.al, χQCD, PRD103(2021)094503

Outline

Hadron mass and trace anomaly



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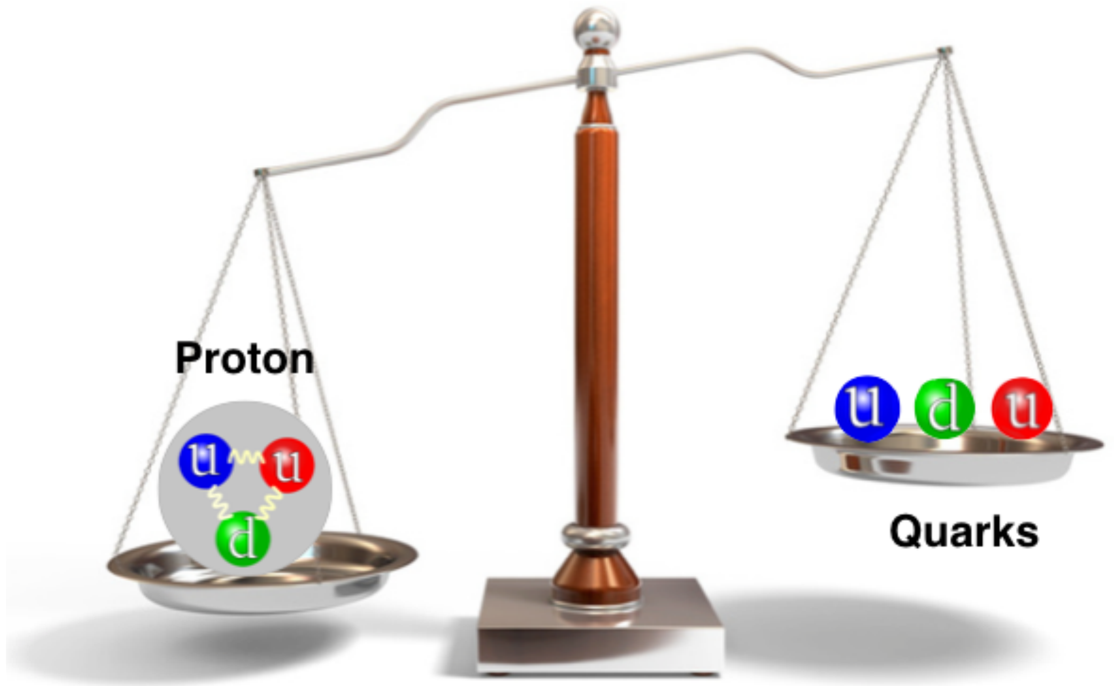

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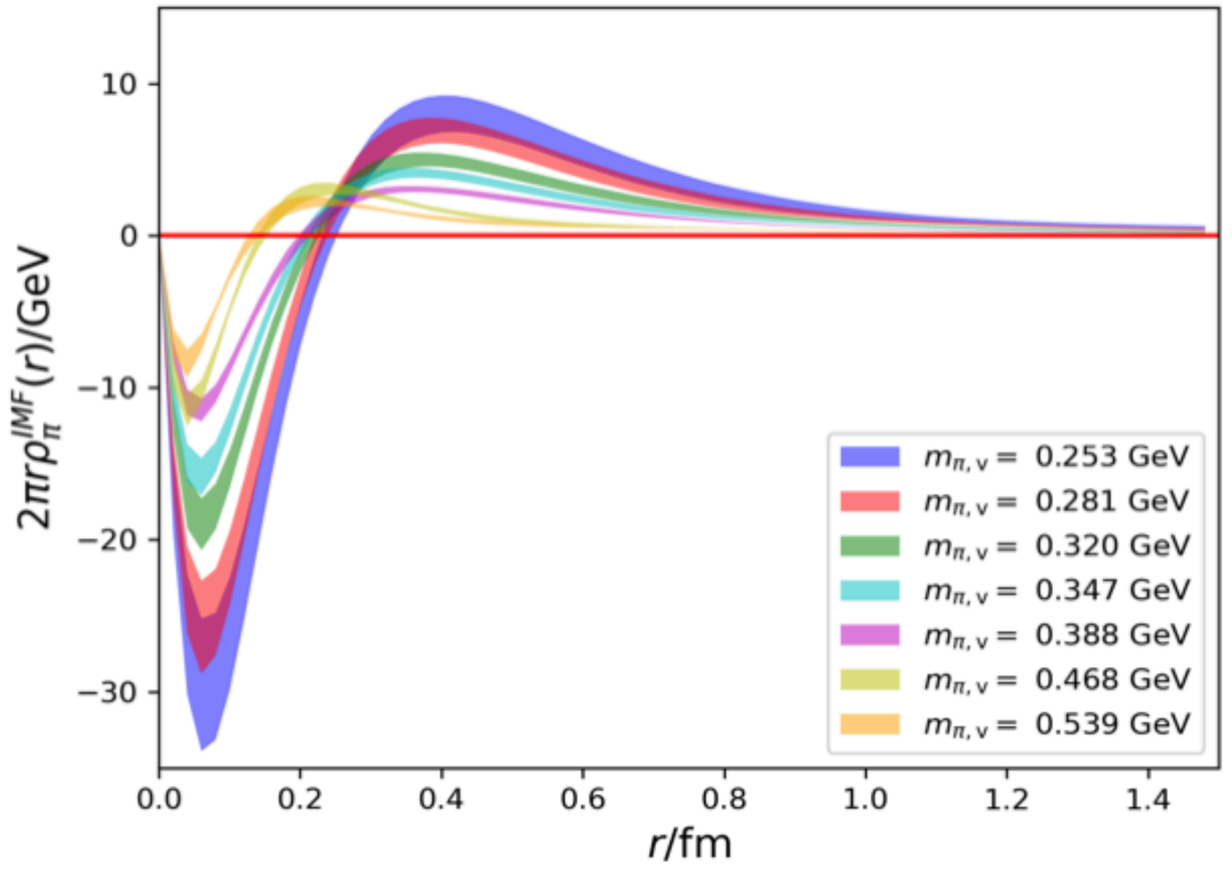


Trace anomaly

Ideal simulation strategy

- The hadron mass can be decomposed into two pieces:

$$m_N = \left[1 + \frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right] m\langle\bar{\psi}\psi\rangle_N + \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right]\langle F^2\rangle_N .$$

- The coefficients $\gamma_m = \frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)$ and $\frac{\beta}{2g} = (-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)$ should be independent to the quark mass and hadron state, while we don't know how large the higher order corrections are.

- We can calculate the $\langle\bar{\psi}\psi\rangle_H$ and $\langle F^2\rangle_H$ in two arbitrary hadrons, and use the above equation to determine γ_m and $\frac{\beta}{2g}$ non-perturbatively.

- Then we can check whether γ_m and $\frac{\beta}{2g}$ are actually quark mass and hadron independent.

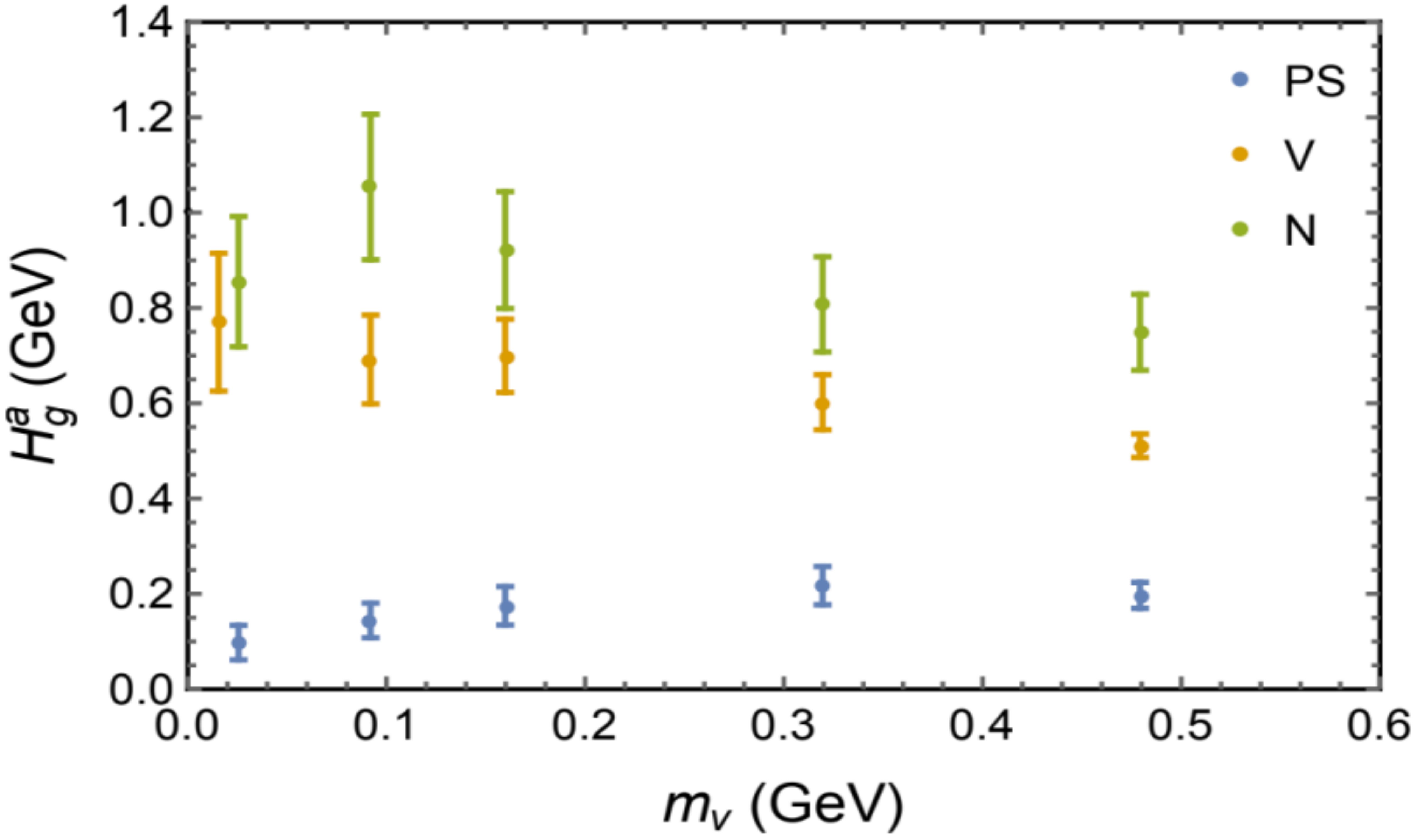
Trace anomaly

Gluon contribution

- The gluon contribution $\frac{\beta}{2g}\langle F^2\rangle_H$ to the hadron mass m_H.

- **$\frac{\beta}{2g}\langle F^2\rangle_N$ is around 800 MeV in the chiral limit $m_q \to 0$.**
- **$\frac{\beta}{2g}\langle F^2\rangle_\pi$ will be less than 100 MeV in the chiral limit $m_q \to 0$.**

- They are exact what QCD predict.



F. He, P. Sun, **YBY**, χQCD, Phys.Rev.D 104(2021)074507

Trace anomaly

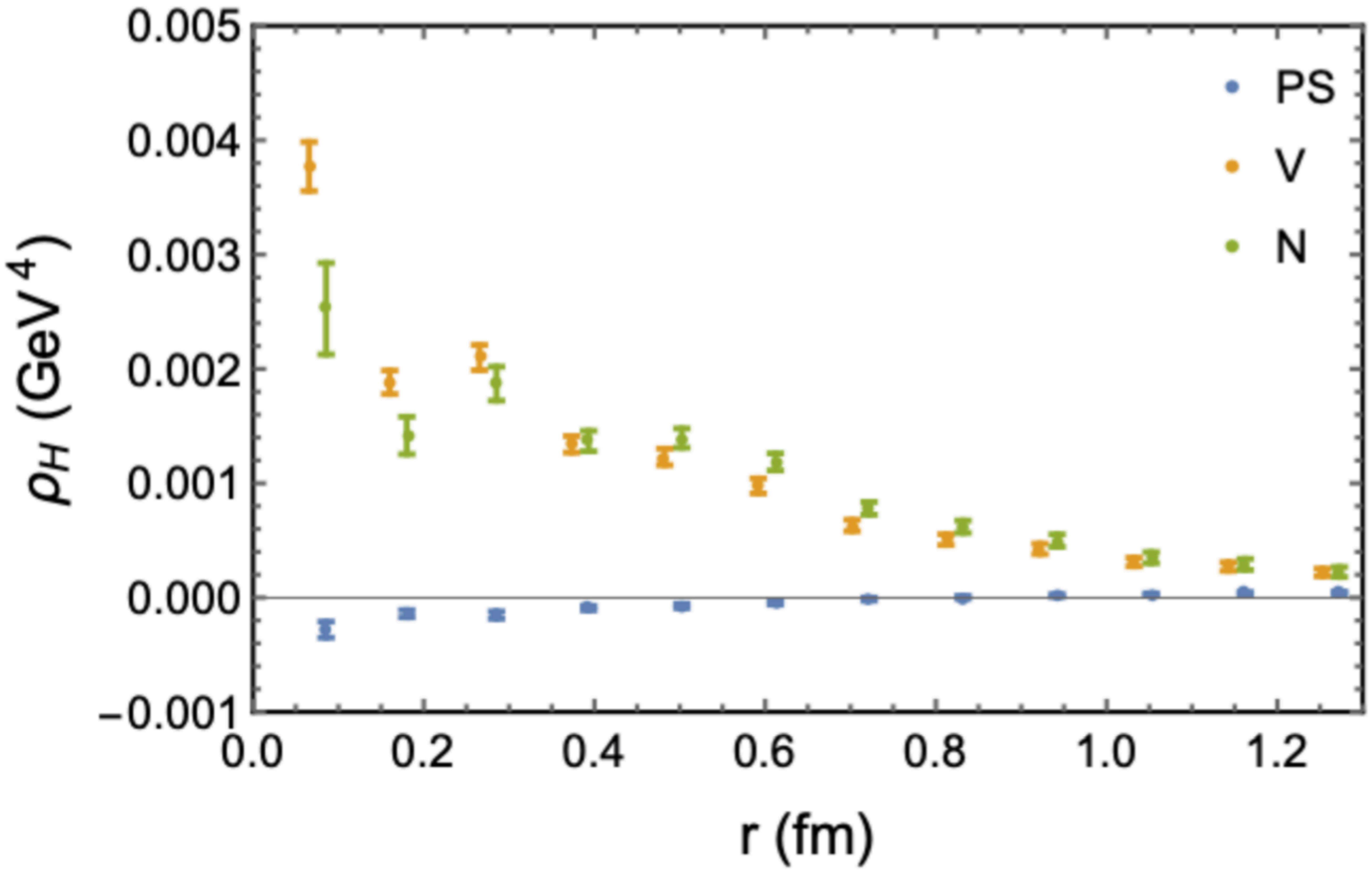
Density inside hadron

$$\rho_H(|r|) = \frac{\langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) H_a^g(t, \vec{y}+\vec{r}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \rangle}{\langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \rangle} |_{t, t_f - t \to \infty},$$

- Similar strategy as that proposed for the pion charge radius;

X. Feng. et.al, PRD101(2020)051502, arXiv:1911.04064

- In term of the normalized density, it is clear that the pion would have a negative trace anomaly hollow in its center.



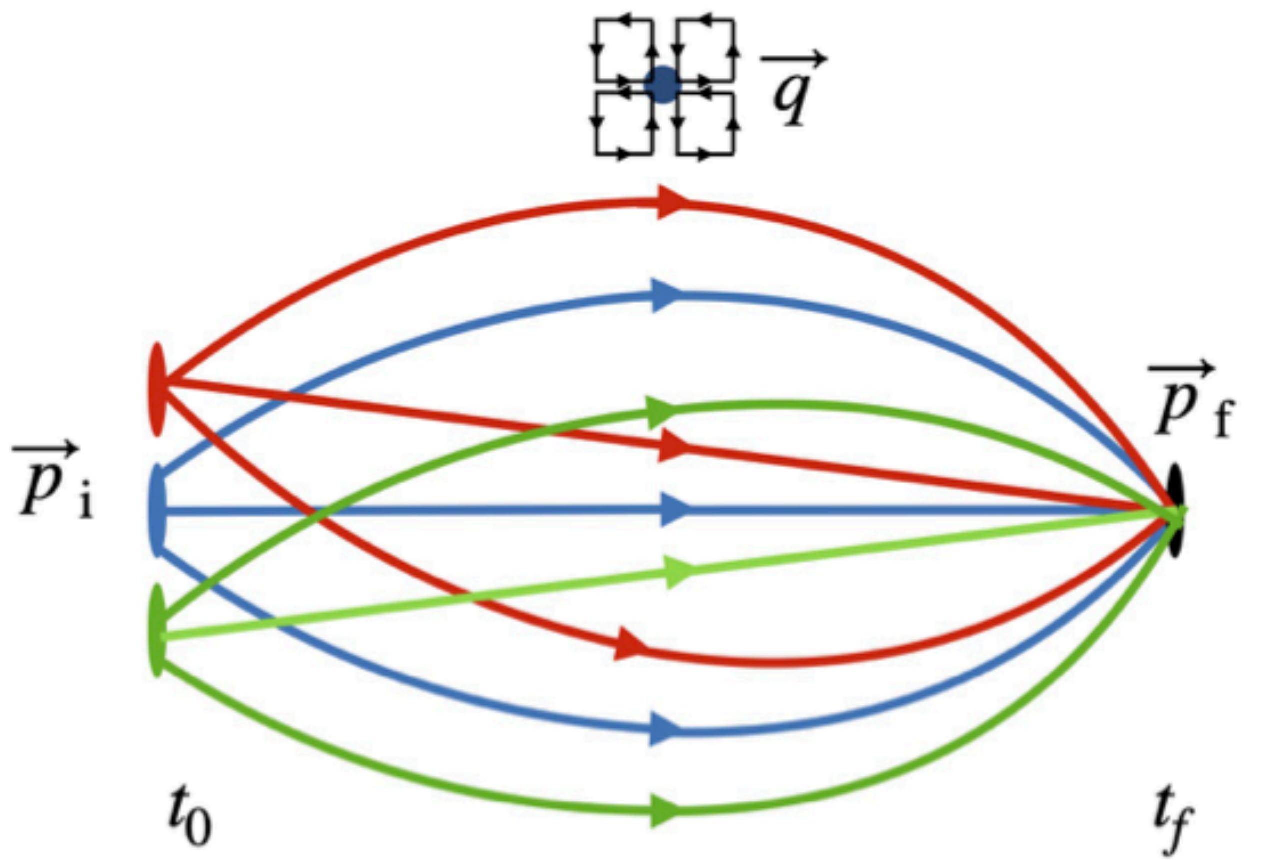
F. He, P. Sun, **YBY**, χQCD, Phys.Rev.D 104(2021)074507

Trace anomaly

form factor

$$R_{\mathrm{sqrt,H}}(t,\tau;\vec{p}_i,\vec{p}_f)=\frac{C_{\mathrm{H,3pt}}(t,\tau;\vec{p}_i,\vec{p}_f)}{C_{\mathrm{H,2pt}}(t;\vec{p}_f)}\times \sqrt{\frac{C_{\mathrm{H,2pt}}(t-\tau;\vec{p}_i)C_{\mathrm{H,2pt}}(t;\vec{p}_f)C_{\mathrm{H,2pt}}(\tau;\vec{p}_f)}{C_{\mathrm{H,2pt}}(t-\tau;\vec{p}_f)C_{\mathrm{H,2pt}}(t;\vec{p}_i)C_{\mathrm{H,2pt}}(\tau;\vec{p}_i)}}\Bigg/ \left[\frac{m_{\mathrm{H}}\mathcal{K}_{\mathrm{H,3pt}}(p_i,p_f)}{\sqrt{\mathcal{K}_{\mathrm{H,2pt}}(p_i)\mathcal{K}_{\mathrm{H,2pt}}(p_f)}}\right],$$

$$\sim G_{\mathrm{H}}(Q^2)+C_1''e^{-\Delta E_i^1\tau}+C_2''e^{-\Delta E_f^1(t-\tau)} +C_3''e^{-\Delta E_i^1\tau-\Delta E_f^1(t-\tau)} \xrightarrow{t\gg\tau\gg0} G_{\mathrm{H}}(Q^2),$$



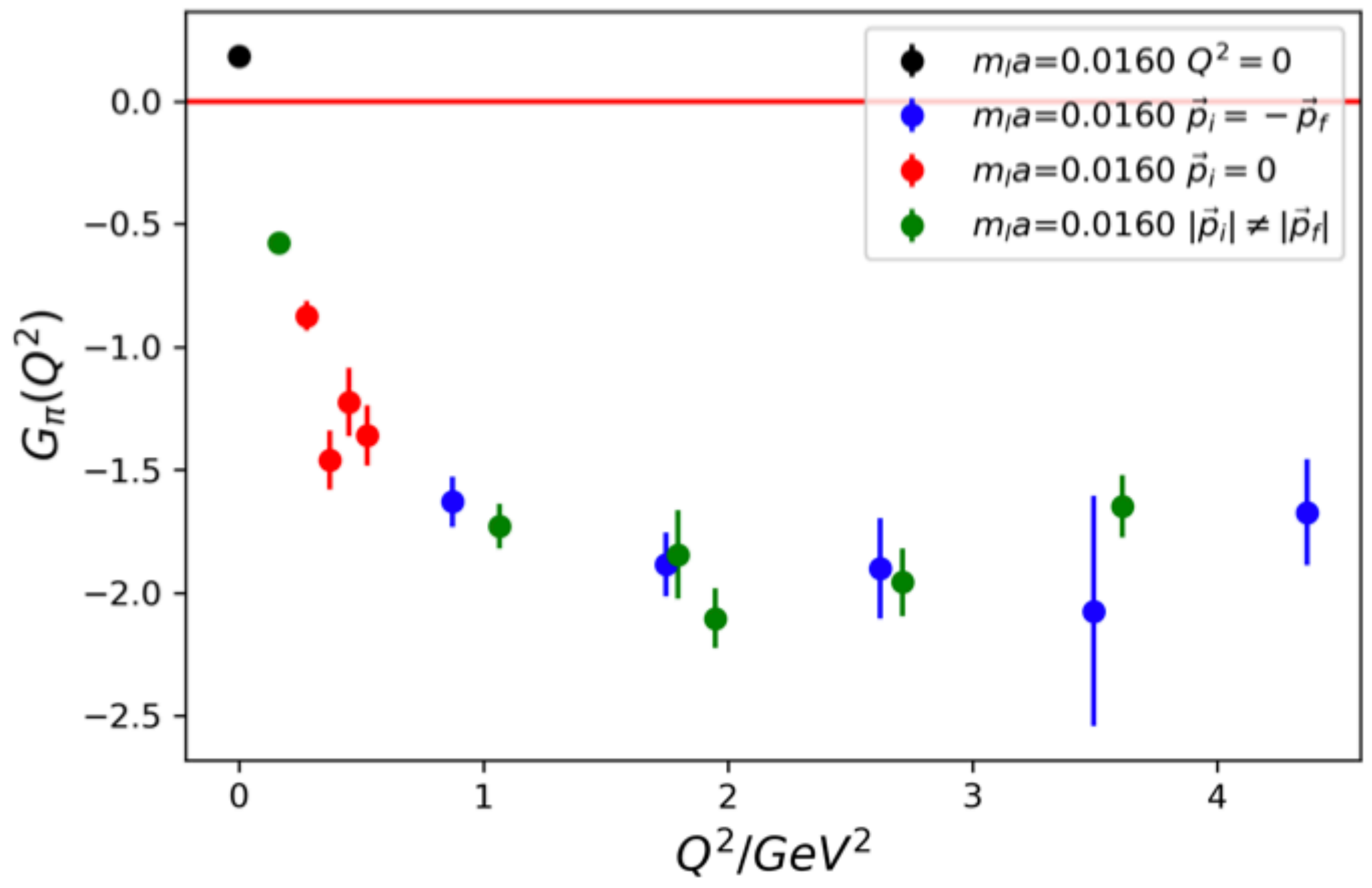
- One can also calculate the form factor $G_H(Q^2) \equiv \frac{\beta}{2g}\langle F^2(Q^2)\rangle_H/m_H$ of trace anomaly;
- And then access the 2-D spatial distribution from the Fourier transform:

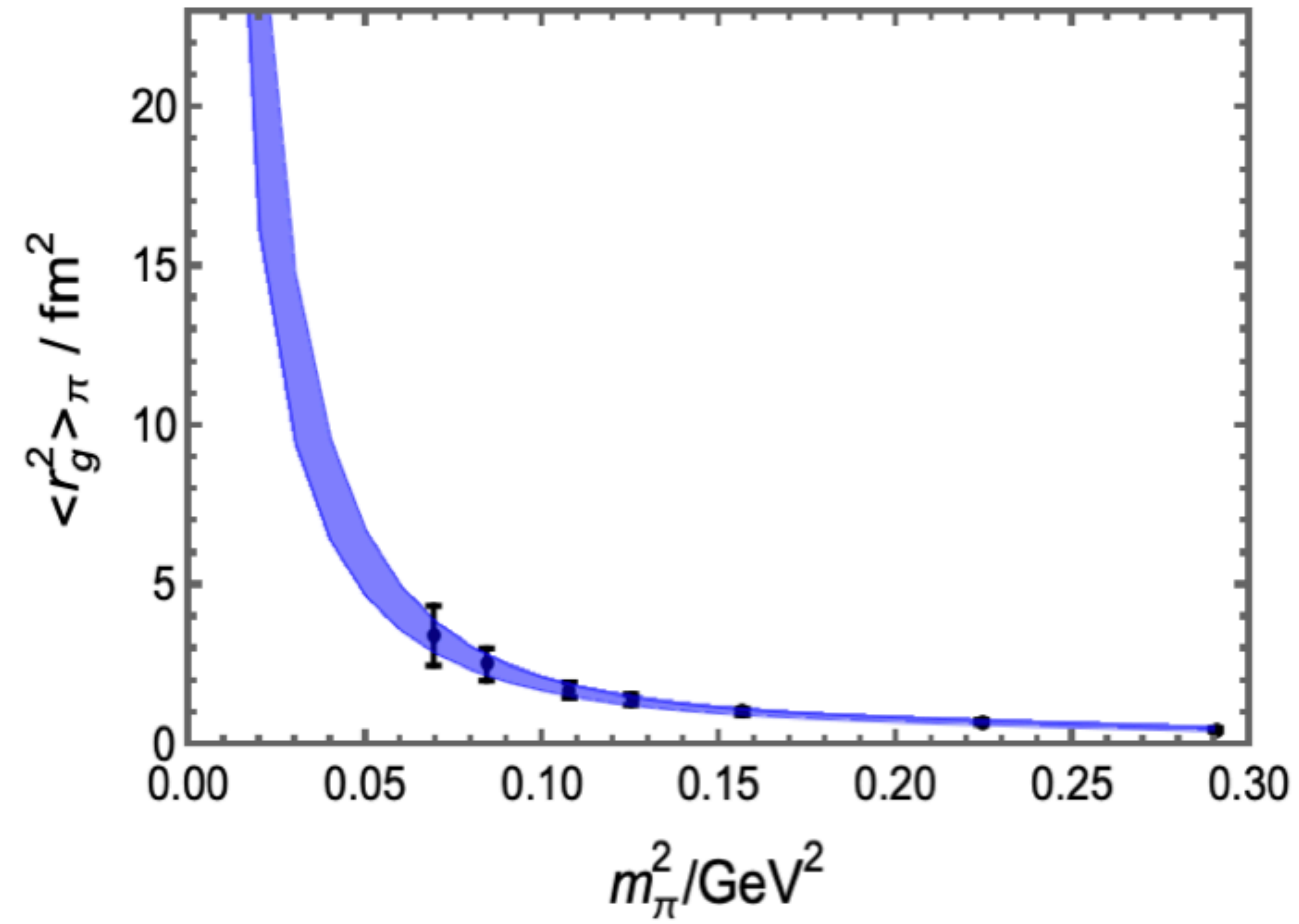
$$\rho_{\mathrm{H}}^{\mathrm{IMF}}(\mathbf{r}_\perp)=\int\frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2}e^{-i\mathbf{\Delta}_\perp\cdot\mathbf{r}_\perp}\ \tilde{G}_{\mathrm{H}}(Q^2)\Big|_{\mathbf{P}\cdot\mathbf{\Delta}=0}^{P_z\to\infty}$$

B. Wang, et.al., χQCD, arXiv:2401.05496

Trace anomaly

pion trace anomaly form factor



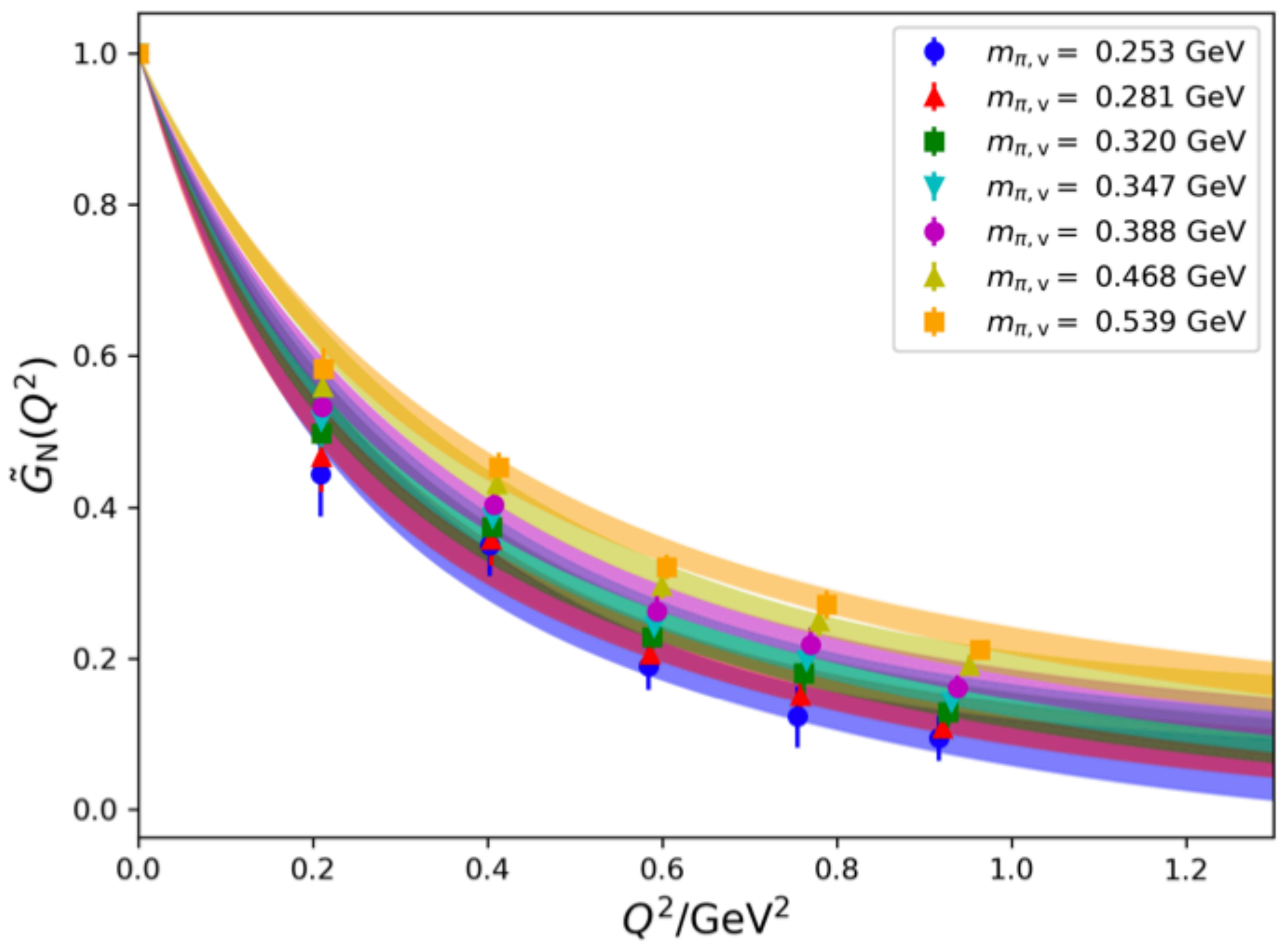


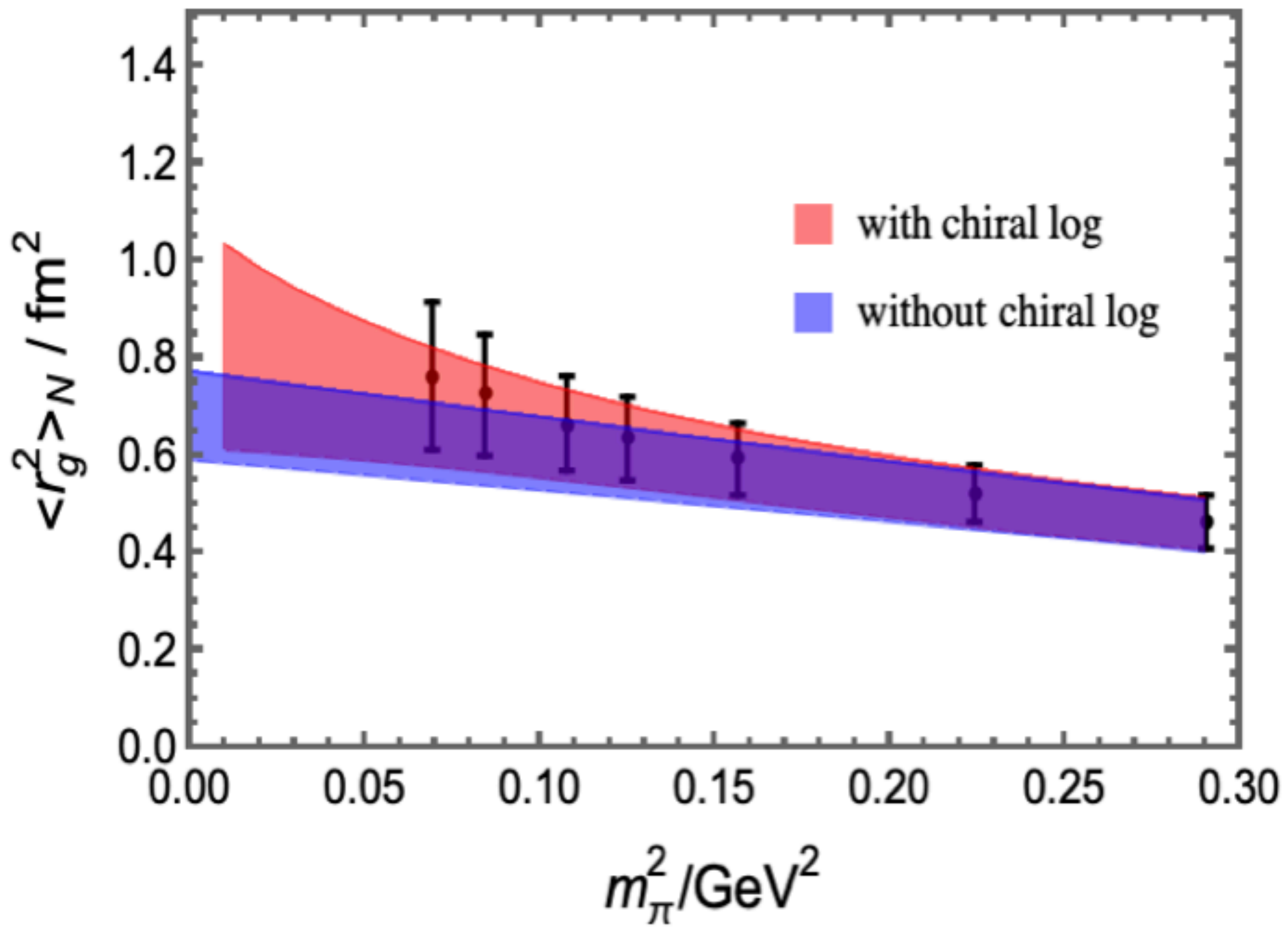
B. Wang, et.al., χQCD, arXiv:2401.05496

- χPT suggests that the form factor $G_\pi(Q^2) \sim \frac{1}{2} - \frac{1}{2m_\pi^2}Q^2 + \mathcal{O}(\alpha_s)$ and then will change the sign at $Q^2 \sim m_\pi^2$;
- The corresponding mass radius contribution $\langle r^2 \rangle_{\pi,a} = -6\frac{dG_\pi(Q^2)}{dQ^2}|_{Q^2 \to 0}$ from trace anomaly can be very large;
- Our Lattice QCD simulation reproduces this feature and also predict the form factor at much larger Q^2.

Trace anomaly

Nucleon trace anomaly form factor



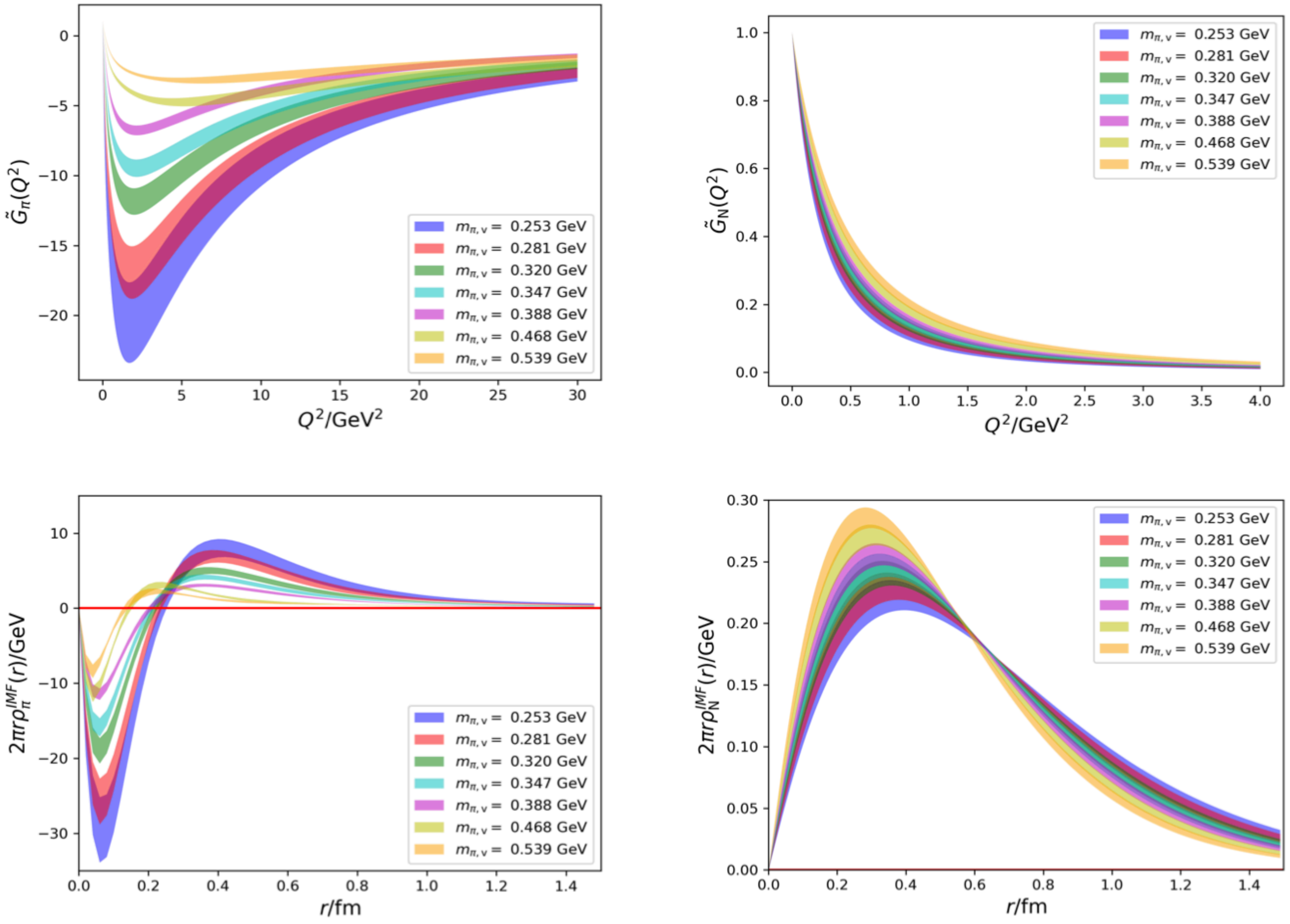


B. Wang, et.al., χQCD, arXiv:2401.05496

- Nucleon form factor $G_N(Q^2)$ is insensitive to the quark mass or pion mass, as we can expected;
- The trace anomaly contribution to the mass radius after the chiral extrapolation is 0.89(10)(07) fm.

Trace anomaly

Nucleon and pion trace anomaly distribution



- After the Fourier transform, the pion trace anomaly distribution in the infinite momentum frame also change the sign, likes the form factors;
- But the nucleon case is not.
- Further investigation would be helpful to uncover the strong interaction origin of hadron mass.

B. Wang, et.al., χQCD, arXiv:2401.05496

Summary

- The proton mass is very sensitive to the quark coupling to the massless gluon, majorly comes from the condensate of massless gluon in the nucleon;
- Lattice QCD shows that trace anomaly can have sizable contribution for all the hadrons through the quark mass dependence of hadron mass;
- Lattice QCD can also provide an accurate first principle prediction on the trace anomaly contribution for the hadron mass, and also its distribution.