Hadron Physics Online Forum
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Method for measuring the proton charge radius from the time-like region

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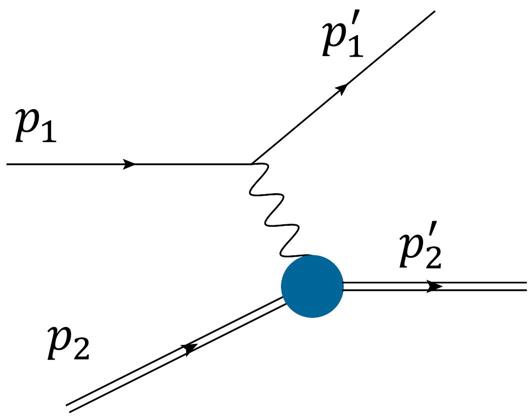
ITP, CAS

Based on Yong-Hui Lin, FKG, U.-G. Meißner, arXiv:2309.07850 [hep-ph]

HAPOF-79, 2023.10.10

Electric form factor

- Electron scattering off a charge distribution



charge density

$$\propto \int d^3r \left[\underbrace{\int d^3r' \frac{e^2\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}}_{} \right] e^{-i\vec{q}\cdot\vec{r}}$$

$$\propto \frac{e^2}{\vec{q}^{\,2}} \int d^3r'\, \rho(\vec{r}\,')\, e^{-i\vec{q}\cdot\vec{r}\,'}$$

$$= \frac{e^2}{-q^2} \underbrace{\int d^3r'\, \rho(\vec{r}\,')\, e^{-i\vec{q}\cdot\vec{r}\,'}}$$

in the Breit frame, $q^2 = -\vec{q}^{\,2}$

Form factor $F(q^2) = F(-\vec{q}^{\,2})$ is the Fourier transform of the charge density in the Breit frame

$$\rho(\vec{r}) = \int \frac{d^3q}{(2\pi)^3} F(-\vec{q}^{\,2})\, e^{-i\vec{q}\cdot\vec{r}}$$

- Charge radius

$$\langle r^2 \rangle = \frac{\int d^3\vec{r}\, r^2 \rho(\vec{r})}{\int d^3\vec{r}\, \rho(\vec{r})} = -6\frac{F'(0)}{F(0)} \Rightarrow F(-\vec{q}^{\,2}) = F(0)\left(1 - \frac{\langle r^2\rangle}{6}\vec{q}^{\,2} + \cdots\right)$$

we have used $\int d^3\vec{r}\,\rho(\vec{r}) = F(0)$ and $\int d^3\vec{r}\, r^2\rho(\vec{r}) = -6F'(0) \equiv -6\frac{dF(-\vec{q}^{\,2})}{d\vec{q}^{\,2}}\Big|_{\vec{q}^{\,2}=0}$

Proton EM form factor

- Nucleon electromagnetic form factor

$$\langle N(p')|J^{\nu}_{\text{em}}|N(p)\rangle = \bar{u}(p')\left[\gamma^{\nu}F_1(q^2) - \frac{iF_2(q^2)}{2m_N}\sigma^{\mu\nu}q_{\mu}\right.$$
$$\left. + i(\gamma^{\nu}q^2\gamma_5 - 2m_N q^{\nu}\gamma_5)F_A(q^2) - \frac{F_3(q^2)}{2m_N}\sigma^{\mu\nu}q_{\mu}\gamma_5\right]u(p)$$

Lorentz invariant form factors (FFs)

F_1: Dirac FF; F_2: Pauli FF; F_A: P-violating anapole FF; F_3: P, CP-violating electric dipole FF

Sachs FFs $(t = q^2)$ Ernst, Sachs, Wali, PR 119, 1105 (1960); Sachs, PR 126, 2256 (1962)

$$G_E(t) = F_1(t) + \frac{t}{4m^2}F_2(t), \qquad G_M(t) = F_1(t) + F_2(t)$$

Fourier transforms of the charge and magnetization distributions in the Breit frame

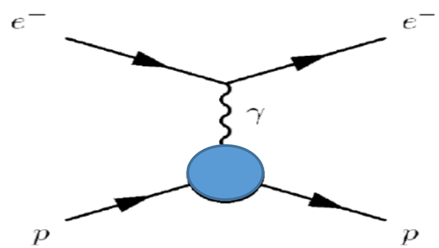
$$G_E(t) = G_E(0)\left(1 + \frac{\langle r_E^2\rangle}{6}t + \cdots\right)$$

$$G_E(0) = e_N \quad (\text{charge}), \qquad G_M(0) = \mu_N \quad (\text{magnetic moment})$$

Therefore, $\langle r_E^2\rangle$ needs to be measured at as small t as possible

Proton EM form factor

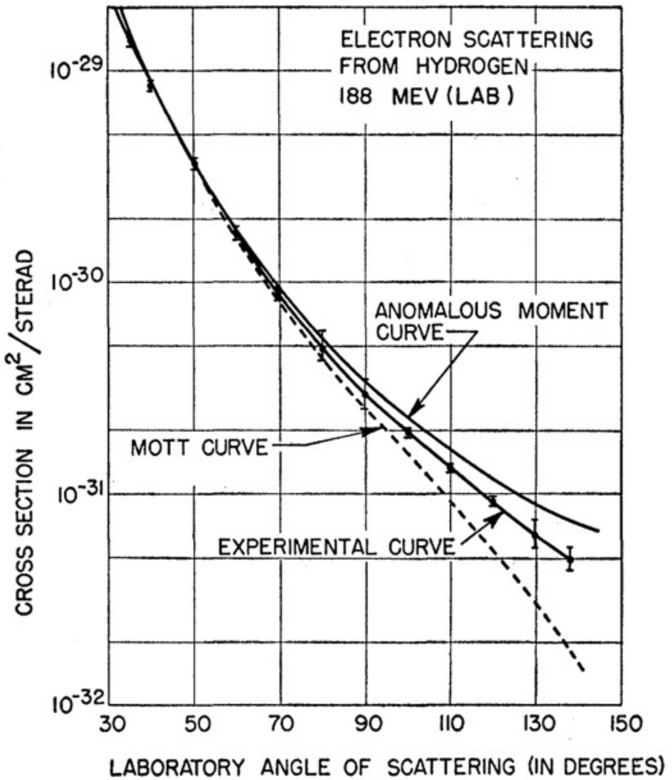
- Electron scattering



Electron Scattering from the Proton

Robert Hofstadter and Robert W. McAllister
Phys. Rev. **98**, 217 – Published 1 April 1955

well by the following choices of size. At 188 Mev, the data are fitted accurately by an rms radius of $(7.0\pm2.4)\times10^{-14}$ cm. At 236 Mev, the data are well fitted by an rms radius of $(7.8\pm2.4)\times10^{-14}$ cm. At 100 Mev the data are relatively insensitive to the radius but the experimental results are fitted by both choices given above. The 100-Mev data serve therefore as a valuable check of the apparatus. A compromise value fitting all the experimental results is $(7.4\pm2.4)\times10^{-14}$ cm. If the proton were a spherical ball of charge, this rms radius would indicate a true radius of 9.5×10^{-14} cm, or in round numbers 1.0×10^{-13} cm. It is to be noted that if our interpretation is correct the Coulomb law of force has not been violated at distances as small as 7×10^{-14} cm.



Proton charge radius

- Spectroscopy method:

  measuring the charge radius from Lamb shift of (muonic) hydrogen atom

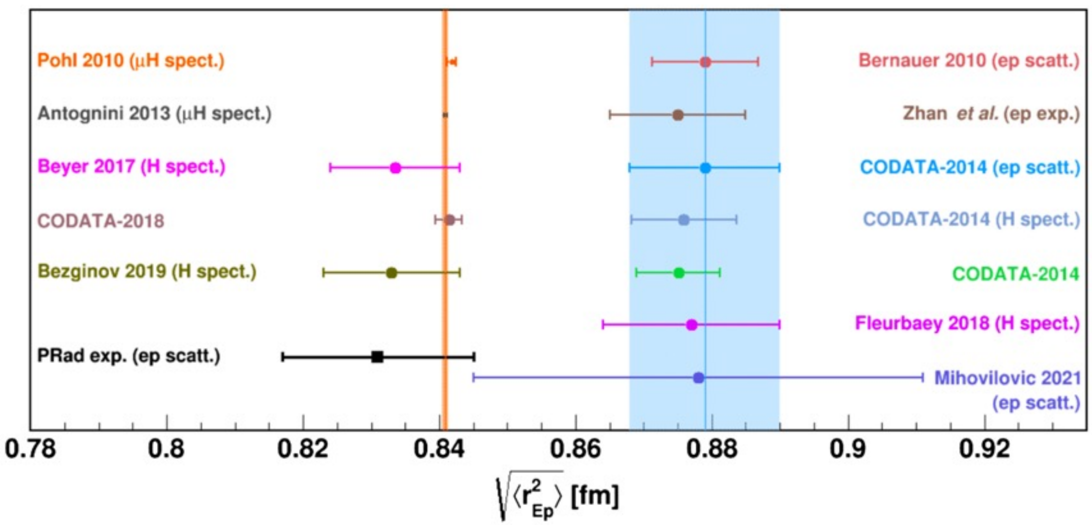


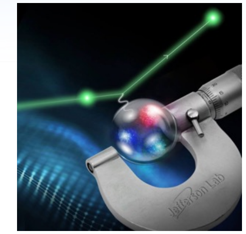
FIG. 18. The proton charge radius $\langle r^2_{Ep} \rangle^{1/2}$ as extracted from electron-scattering and spectroscopic experiments since 2010 and before 2020 together with CODATA-2014 and CODATA-2018 recommended values. Note the reinterpreted result from the Mainz ISR experiment was scheduled for publication in 2021. From Jingyi Zhou.

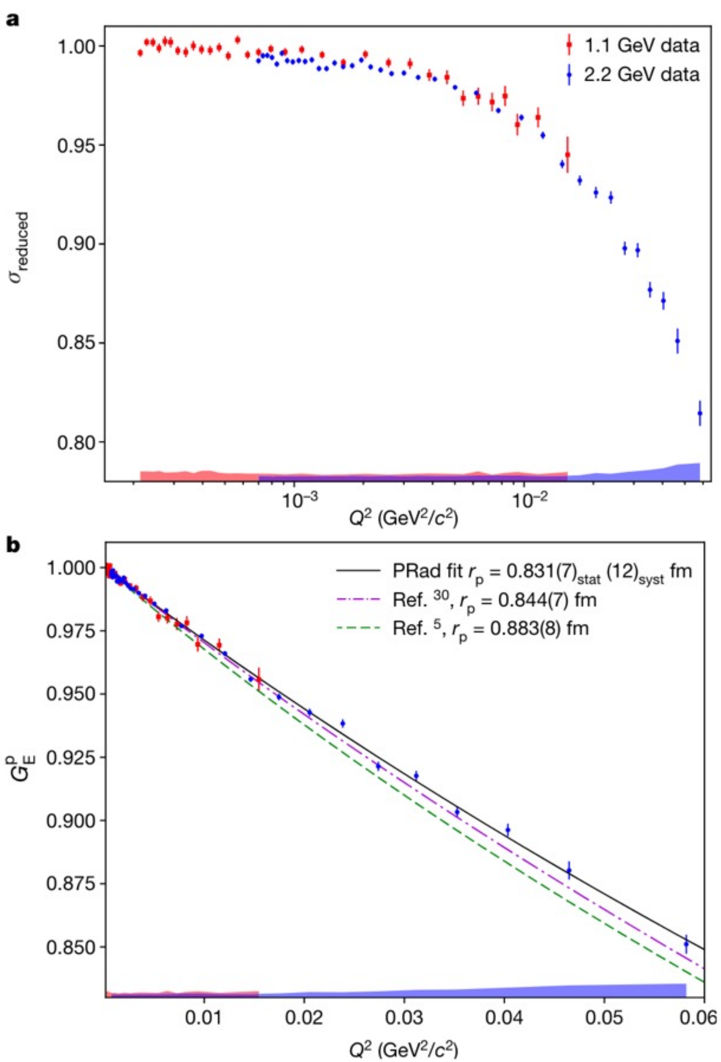
H. Gao, M. Vanderhaeghen, RMP 94, 015002 (2022)

PRad measurement

- ep scattering with $Q^2 \in [2.1\times10^{-4}, 0.06]$ GeV^2 in the spacelike region

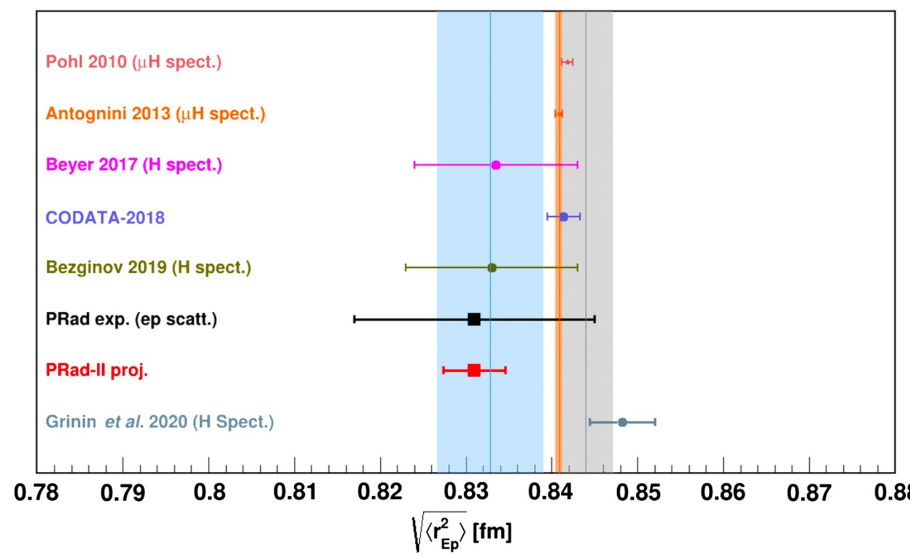
W. Xiong et al. [PRad], Nature 575, 147 (2019)





$$r_p = 0.831 \pm 0.007_{\rm stat} \pm 0.012_{\rm syst}\ \mathrm{fm}$$

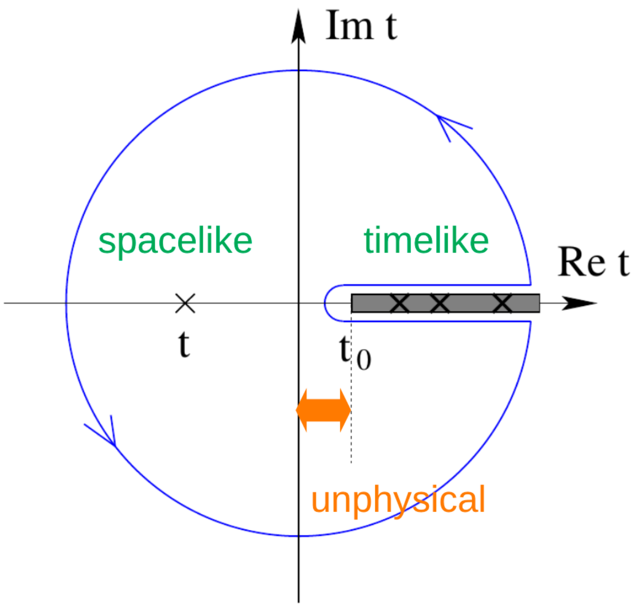
- Prad-II will cover $Q^2 \in [4\times10^{-5}, 0.06]$ GeV^2



H. Gao, M. Vanderhaeghen, RMP 94, 015002 (2022);
A. Gasparian et al. [PRad-II], arXiv:2009.10510;
private communication with W.-Z. Xiong

Dispersive approach

Y.-H. Lin, HAPOF-28



$$F(t) = \frac{1}{\pi}\int_{t_0}^{\infty} \frac{\mathrm{Im}\, F(t')}{t' - t - i\epsilon}\, dt'$$

Ingredients: multiple cuts

(start from t_0)

&& vector meson poles

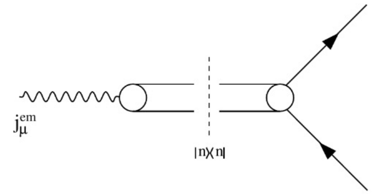
The spectral function $\mathrm{Im}F(t)$ are central quantities.

Dispersive approach

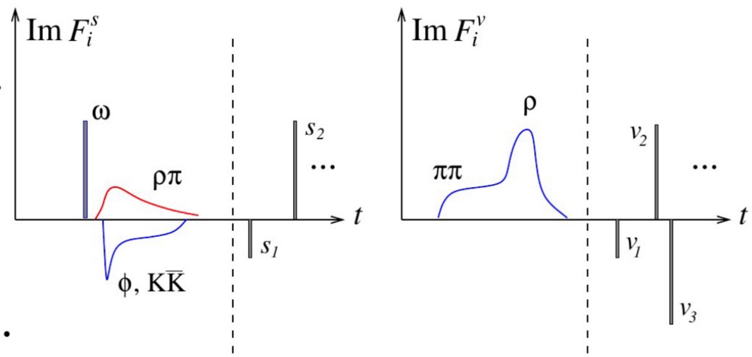
Y.-H. Lin, HAPOF-28

- Spectral Decomposition (lower energy part)
  - Crossing symmetry $\langle N(p')|j_\mu^{\rm em}|N(p)\rangle \longleftrightarrow \langle N(p)\bar N(\bar p)|j_\mu^{\rm em}|0\rangle$
  - Spectral decomposition G. F. Chew, *et al.* PhysRev110, 265(1958)

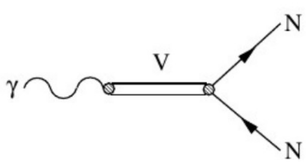
$$\mathrm{Im}\langle N(p)\bar N(\bar p)|j_\mu^{\rm em}|0\rangle \sim \sum_n \langle N(p)\bar N(\bar p)|n\rangle\langle n|j_\mu^{\rm em}|0\rangle$$



  - Intermediate mass states $|n\rangle$
    - Isoscalar: $3\pi, 5\pi, \ldots, K\bar K, \pi\rho, \ldots$
    - Isovector: $2\pi, 4\pi, \ldots$



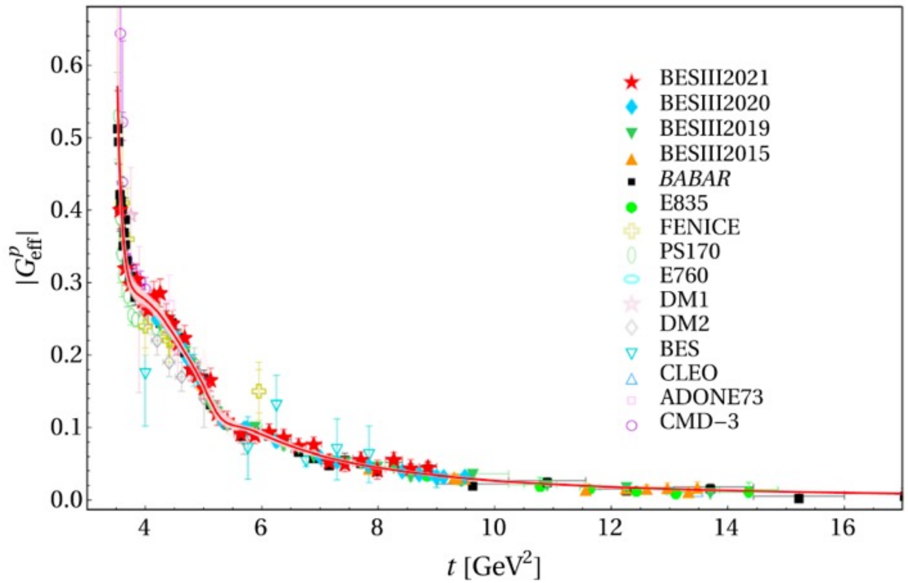
- Vector meson dominance



Higher mass state $s_1, s_2, \ldots$
$v_1, v_2, \ldots$

Dispersive approach

Y.-H. Lin, H.-W. Hammer, U.-G. Meißner, PRL 128, 052002 (2022)



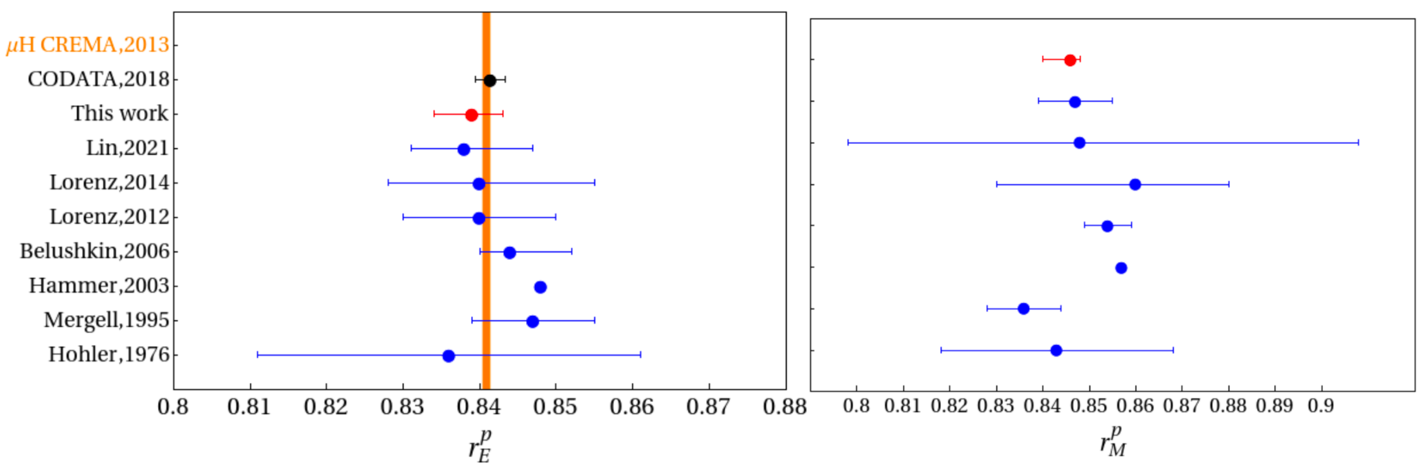
$$|G_{\rm eff}| \equiv \sqrt{\frac{|G_E|^2 + \xi |G_M|^2}{1+\xi}}$$

$$r_E^p = 0.839 \pm 0.002^{+0.002}_{-0.003}\,\text{fm},$$

$$r_M^p = 0.846 \pm 0.001^{+0.001}_{-0.005}\ \text{fm}$$

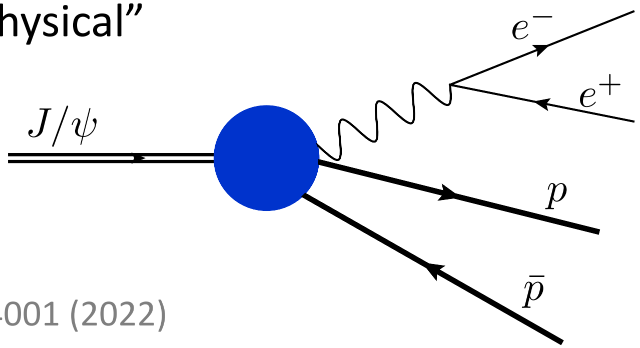
- Comparing to existing DR determination

Y.-H. Lin, HAPOF-28



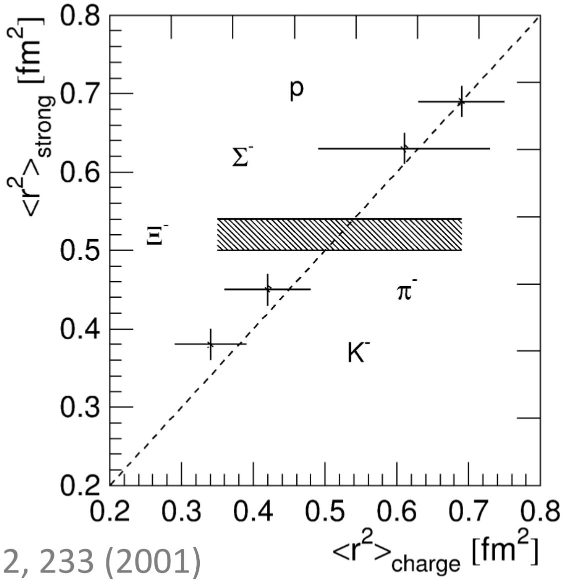
Dalitz decay

- Possibility to measure the proton FFs in the time-like "unphysical" region?
  - Dalitz decay $J/\psi \to p\bar{p}e^+e^-$ by measuring the e^+e^- distribution



  - BESIII has 10^{10} J/ψ BESIII, CPC 46, 074001 (2022)
  - STCF can collect 3.4×10^{12} J/ψ per year STCF, Front. Phys. 19, 14701 (2024) [arXiv:2303.15790]
  - Can reach very small $q^2 \sim 4\,m_e^2 = 1.05\times10^{-6}\ \mathrm{GeV}^2$
    - e^+ and e^- can be efficiently detected as long as they have transverse momenta larger than a few tens of MeV (~50 MeV at BESIII from H.-B. Li)
    - Collinear e^+e^- ⇒ threshold kinematics
  - More similar Dalitz decays:
    - $J/\psi \to \pi^+\pi^-e^+e^-, K^+K^-e^+e^-$
    - $J/\psi \to \Xi^-\bar{\Xi}^+e^+e^-,\ J/\psi \to \Sigma^\pm\bar{\Sigma}^\mp e^+e^-$
    - …

Among all hyperons, only the Σ^- charge radius was measured: 0.78 ± 0.10 fm, with Σ^- beam scattering off atomic electrons



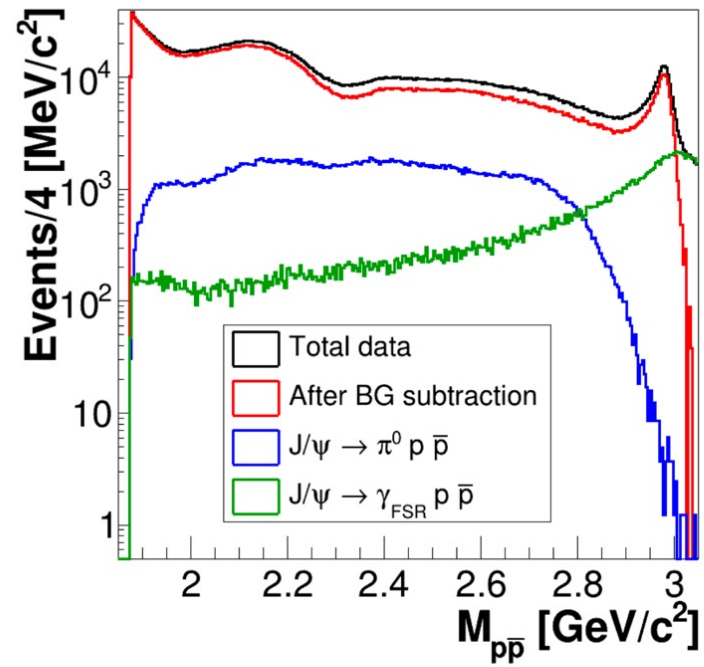
SELEX (E781), PLB 522, 233 (2001)

Dalitz decay

Problems:

- Requires final-state radiation (FSR) virtual photon, only a small portion from the whole decay events
  - method subtracting the major background and/or partial-wave analysis
- For FSR photon, measures transition FFs from some intermediate state A to $p\gamma^*$, proton is only part of A
  - to identify a region dominated by the proton pole
  - For large $\mathrm{m}_{p\bar{p}}$, both $m_{p\gamma^*}$ and $m_{\bar{p}\gamma^*}$ are small, proton and antiproton pole dominance may work

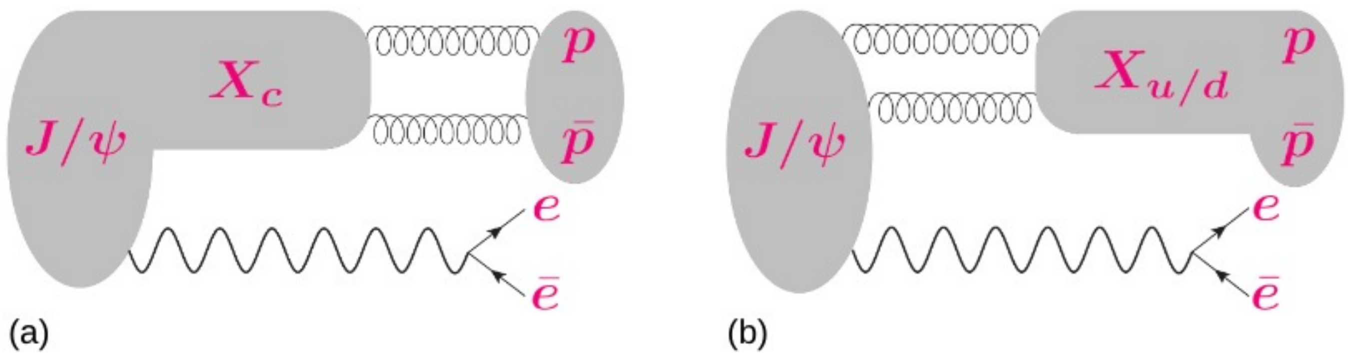
$J/\psi \to p\bar{p}\gamma$



R. Kappert, PhD thesis, Groningen U. (2022)

Decay mechanisms

- Virtual photon emitted
  - from (anti-)charm quark, type X: diagrams (a) and (b)
    - $c\bar{c} \rightarrow$ two gluons
      - type-X_c: η_c
      - type-$X_{u/d}$: light meson resonances such as $X(1835)$, …
    - isospin symmetric: $\mathcal{A}_X(p\bar{p}e^+e^-) = \mathcal{A}_X(n\bar{n}e^+e^-)$ up to $\mathcal{O}(1\%)$
      - Isospin breaking effects: from quark mass difference $\mathcal{O}\left(\frac{m_d - m_u}{\Lambda_{\text{QCD}}}\right)$ or from virtual photons $\mathcal{O}(\alpha)$
      - Similarly, $\mathcal{A}_X(\Xi^-\bar{\Xi}^+e^+e^-) = \mathcal{A}_X(\Xi^0\bar{\Xi}^0e^+e^-)$, …

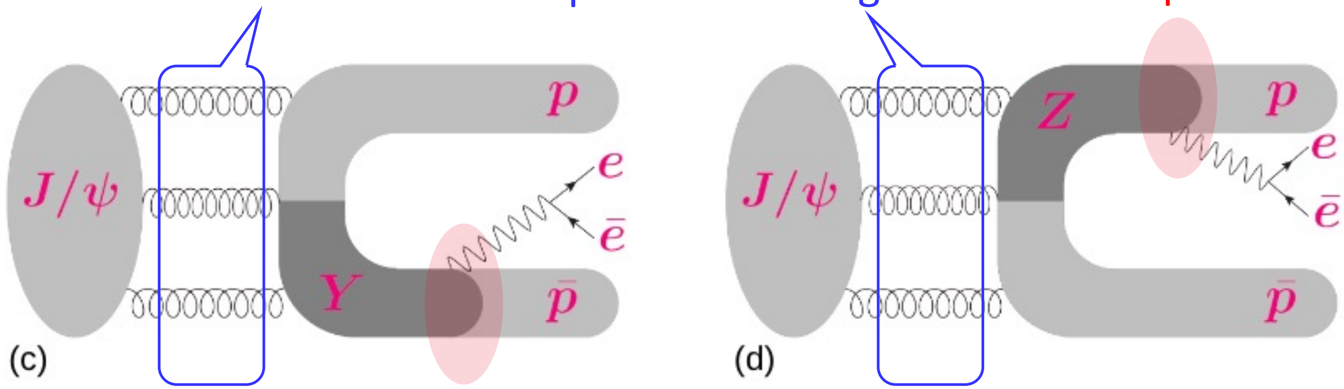


Decay mechanisms

- Virtual photon emitted
  - from anti-light and light quarks, types Y and Z: diagrams (c) and (d)
    - three gluons or a virtual photon
    - FSR $\gamma^* \to e^+e^-$
    - if proton is replaced by neutron, the FSR contribution is negligible at small q^2: zero charge, $\langle (r_E^n)^2 \rangle = -0.1155(17)\ \mathrm{fm}^2$ PDG2023
    - Neglecting CP violation: proton FF = antiproton FF

can also be a virtual photon exchange

contains proton FF



contains antiproton FF

Subtraction of background

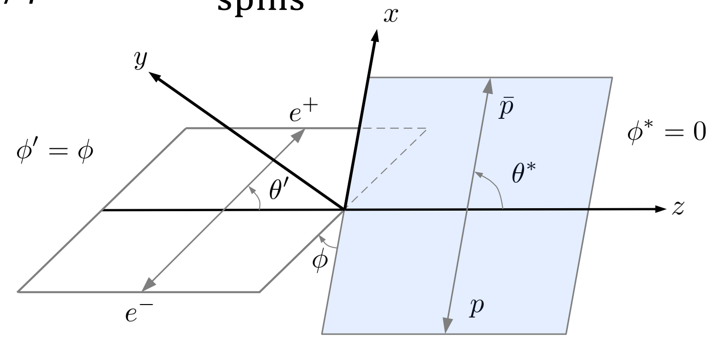
- Differential decay widths

$$\frac{d\Gamma(J/\psi \to p\bar{p}e^+e^-)}{dm_{e^+e^-}dm_{p\bar{p}}d\cos\theta_p^* d\cos\theta_e' d\phi} = \frac{|\vec{k}_{e^+e^-}||\vec{k}_p^*||\vec{k}_{e^-}'|}{(2\pi)^6 16 M_{J/\psi}^2} \frac{C(q^2)}{3} \sum_{\text{spins}} |\mathcal{M}|^2$$

$$|\mathcal{M}|^2 = |\mathcal{M}_{Y+Z}|^2 + 2\,\mathrm{Re}(\mathcal{M}_{Y+Z}\mathcal{M}_X^*) + |\mathcal{M}_X|^2$$

$$i\mathcal{M}_{(i)} = H_{(i)}^{\mu} \frac{-ig_{\mu\nu}}{q^2} \underbrace{\left[-ie\bar{u}_{s_{e^-}}(p_1)\gamma^\nu v_{s_{e^+}}(p_2)\right]}_{\text{leptonic part}}$$

hadronic part



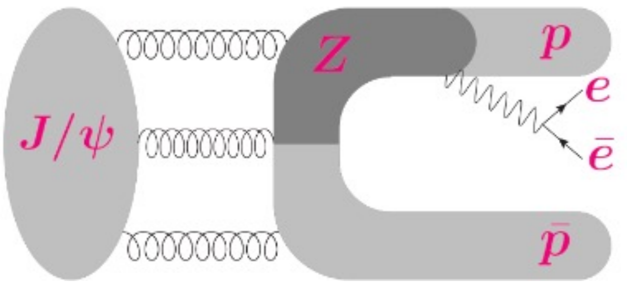
- Sommerfeld factor resums poles of e^+e^- Coulomb bound states:

$$C(q^2) = \frac{y}{1 - e^{-y}}, \qquad y = \frac{\pi\alpha m_e}{k_e'}$$

- Background subtraction
  - For $J/\psi \to n\bar{n}e^+e^-$: $\mathcal{M} \approx \mathcal{M}_X$
    - Background subtraction can in principle be achieved by subtracting out the $J/\psi \to n\bar{n}e^+e^-$ (properly normalized) event distribution
  - Signal part: $|\mathcal{M}_{\text{signal}}|^2 \equiv |\mathcal{M}_{Y+Z}|^2 + 2\,\mathrm{Re}(\mathcal{M}_{Y+Z}\mathcal{M}_X^*)$ all contains proton FF in the specific kinematic region

Selection of kinematic region

$$|p\rangle\langle p| + |N\pi\rangle\langle N\pi| + \cdots$$



- Identify a region dominated by the proton and antiproton poles
  - Large $m_{p\bar{p}}$ $\Rightarrow$ small $m_{p\gamma^*}$ and $m_{\bar{p}\gamma^*}$ $\Rightarrow$ (anti-)proton dominance
  - Approximate $|N\pi\rangle\langle N\pi| + \cdots$ by the lowest $N\pi$ resonance Δ^+: $J/\psi \to \Delta^+ p + \mathrm{c.c.}$, $\Delta^+ \to p\gamma^*$, check the region where the Δ contribution can be neglected

$$\frac{dR_{N/(N+\Delta)}}{dm_{e^+e^-}dm_{p\bar{p}}} = \int d\cos\theta_p^*\, d\cos\theta_e'\, d\phi\, \frac{d\Gamma_{Y+Z}^{N}}{d\Gamma_{Y+Z}^{N+\Delta}}$$

Selection of kinematic region

- Hadronic part contains
  - $J/\psi \to N\bar{N}, \Delta\bar{N}$ with covariant obtital-spin scheme

B.-S. Zou, D. Bugg, EPJA 16, 537 (2003);
H.-J. Jing et al., JHEP 06, 039 (2023)

$$\Gamma^{\mu}_{J/\psi N\bar{N}}(r,p_0) = g_S\left(\gamma^\mu - \frac{r^\mu}{M_{J/\psi}+2m_N}\right) + g_D e^{i\delta_1}\left(\gamma_\nu - \frac{r_\nu}{M_{J/\psi}+2m_N}\right)t^{\mu\nu}$$

$$\Gamma^{\mu\alpha}_{J/\psi\Delta\bar{N}}(r,p_0) = f_S\gamma_5 g^{\mu\alpha} + f_D e^{i\delta_2}\gamma_5 t^{\mu\alpha}$$

  - ✓ g_D/g_S: fixed to the $\Gamma_S/\Gamma_{S+D} \in [0.851, 0.915]$ ratios from $J/\psi \to \Lambda\bar{\Lambda}, \Sigma^+\bar{\Sigma}^-, \psi(2S) \to \Sigma^+\bar{\Sigma}^-$

S.-M. Wu, J.-J. Wu, B.-S. Zou, PRD 104, 054018 (2021)

  - ✓ assuming $f_D/f_S = g_D/g_S$
  - ✓ $\mathcal{B}(J/\psi \to p\bar{p}) = 2.12\times10^{-3}$,
  - ✓ $\mathcal{B}(J/\psi \to \Delta\bar{p}) < 10^{-4}$

  - Proton FFs and $\Delta \to N\gamma^*$ transition FFs

$$\Gamma^{\mu}_{\gamma NN}(q) = ie\left(\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m_N}q_\nu F_2(q^2)\right)$$

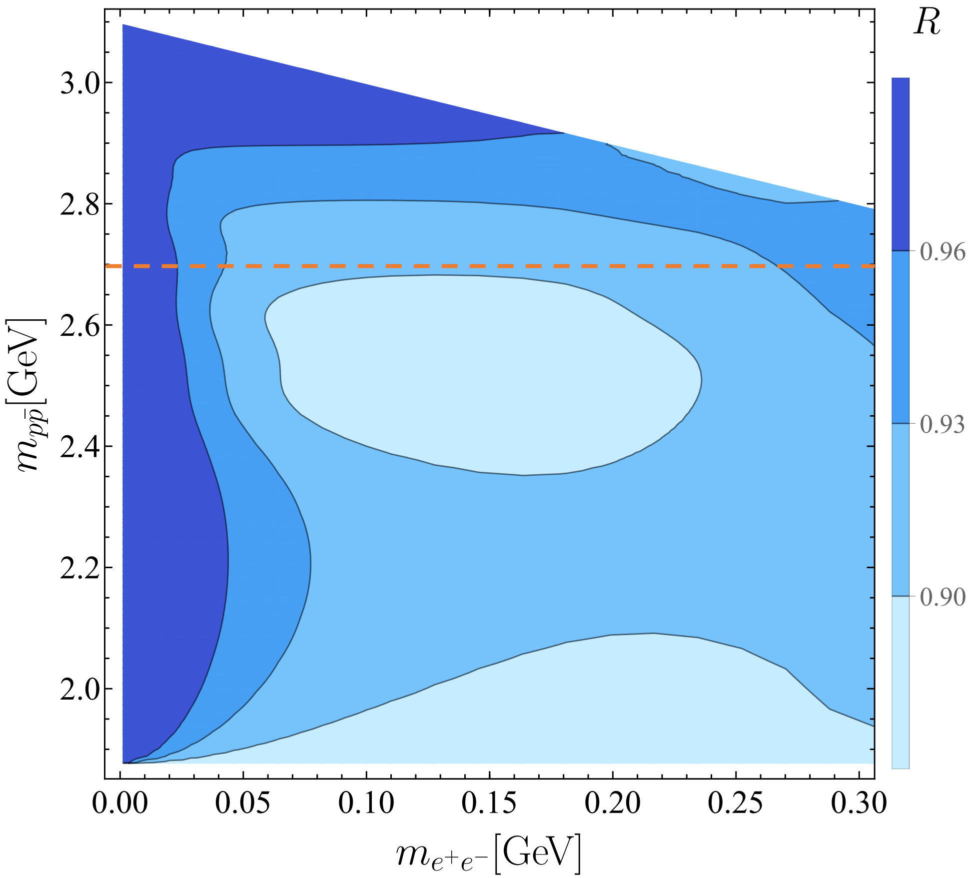
$$\Gamma^{\alpha\mu}_{\gamma\Delta N}(q,p_\Delta) = ie\sqrt{\frac{2}{3}}\frac{3(m_N+m_\Delta)}{2m_N\left((m_\Delta+m_N)^2-q^2\right)}g_M^\Delta(q^2)\epsilon^{\alpha\mu\rho\sigma}p_{\Delta,\rho}q_\sigma$$

dominated by magnetic-dipole term

J. Guttmann, M. Vanderhaeghen, PLB 719, 136 (2013); L. Tiator et al., EPJ ST 198, 141 (2011)

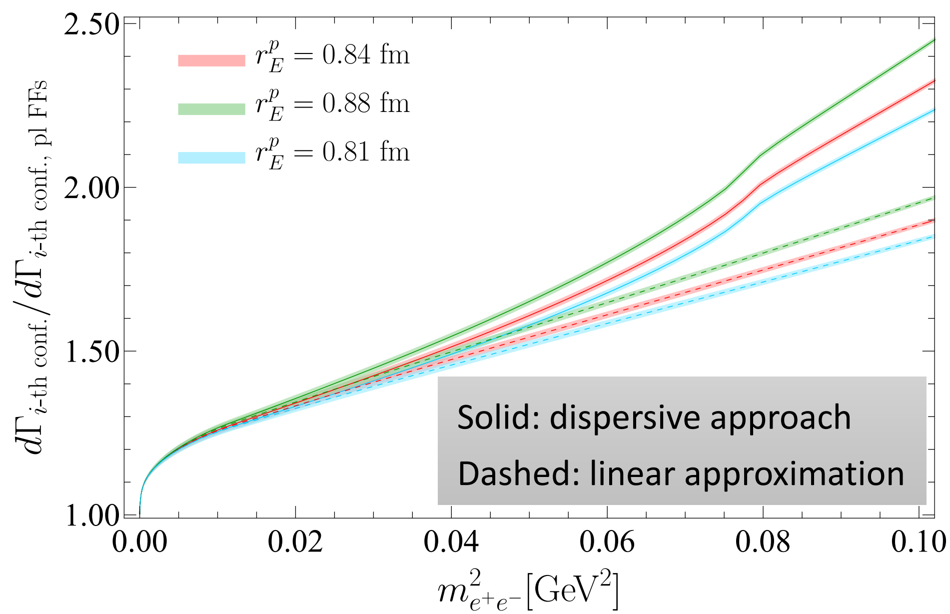
Selection of kinematic region

- Lower bound of the ratio $\frac{dR_{N/(N+\Delta)}}{dm_{e\bar{e}}dm_{p\bar{p}}}$ from types Y+Z
  - always larger than 90% for $m_{p\bar{p}} > 2.7\ \mathrm{GeV}$



Sensitivity study

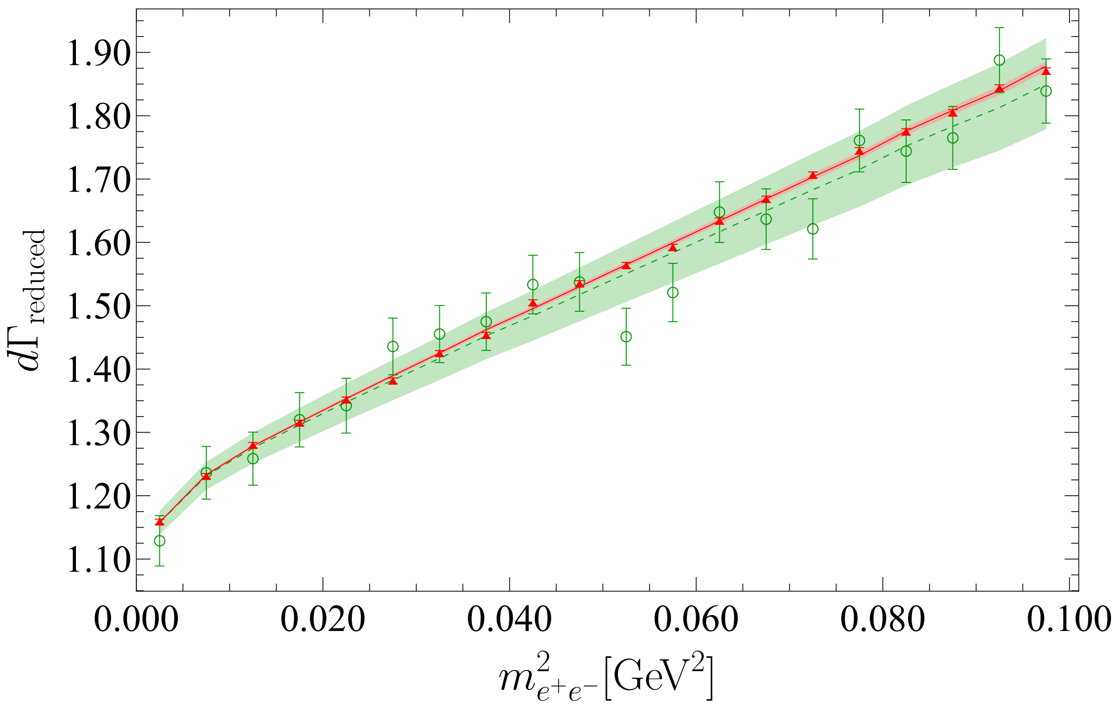
- Estimate of the number of events
  - Consider only the signal part $\left|\mathcal{M}_{\text{signal}}\right|^2 \equiv |\mathcal{M}_{Y+Z}|^2 + 2\,\mathrm{Re}(\mathcal{M}_{Y+Z}\mathcal{M}_X^*)$
  - For type-X, consider only the η_c contribution
  - 10^4 events for $m_{e^+e^-} < 0.3$ GeV, $m_{p\bar{p}} > 2.7$ GeV
- Sensitivity to the proton charge radius r_E^p of the $m_{e^+e^-}$ distribution normalized to a pointlike-proton assumption



- Bands almost invisible: insensitive to model details
- Linear approximation to the FF $\left(1 + \frac{\langle r_E^2 \rangle}{6} t\right)$ tends to overshoot the result, only reliable for very low q^2 region

Sensitivity study

- Monte Carlo simulation assuming a $1+\frac{\langle r_E^2\rangle}{6}t$ FF
  - Synthetic data using the von Neumann rejection method with $r_E^p = 0.84$ fm as input
  - 10^4 events ⇒ 0.828 ± 0.040 fm (feasible at BESIII?)
  - 10^6 events ⇒ 0.846 ± 0.004 fm (events at STCF per year is a factor of ~ 340 more than all BESIII events)



Summary and outlook

- Propose to measure the proton charge radius from the time-like region using the Dalitz decay $J/\psi \to p\bar{p}e^+e^-$
  - Can reach $|q^2| \sim 1.05\times10^{-6}\ \mathrm{GeV}^2$, smaller than all ep scattering experiments
- Simple MC simulation
  - statistical uncertainty at BESIII
  - statistical uncertainty at STCF on par with that of PRad-II
  - maybe too optimistic, full-fledged simulation?
- Applicable to the charge radii of other stable hadrons, e.g., $\Xi^-, \Sigma^\pm, K^\pm, \pi^\pm$, ... Results on the hyperons are rare so far
- Analysis of real data?

Thank you for your attention!

Proton charge radius

- Spectroscopy method

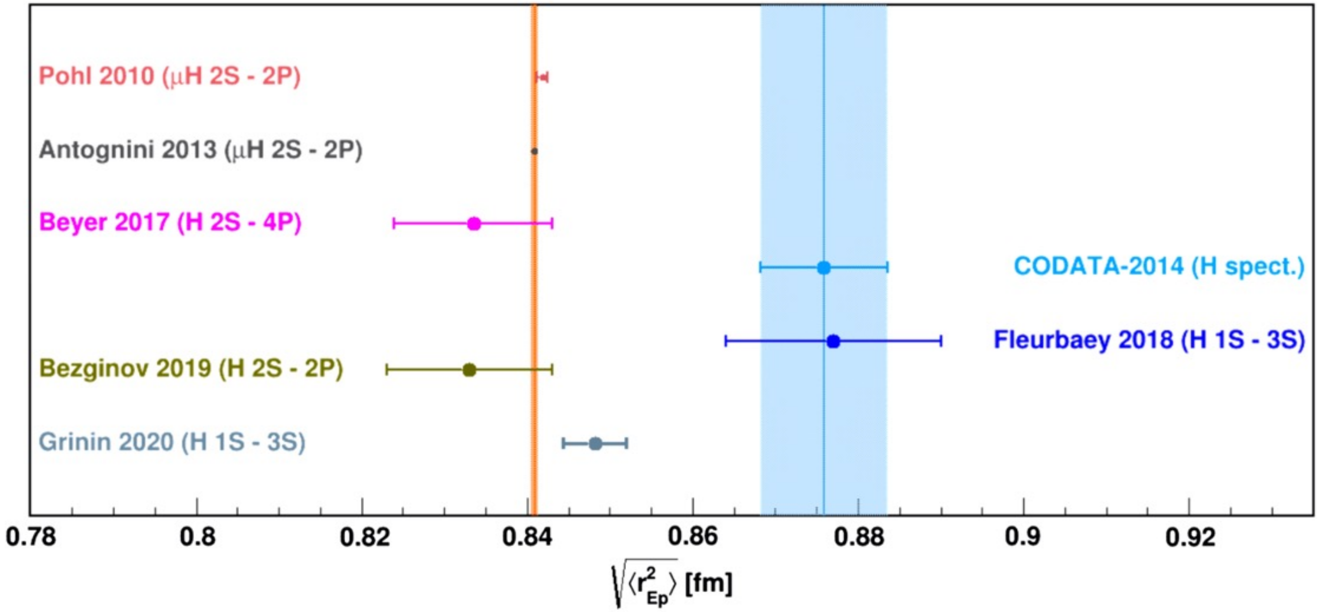


FIG. 21. The latest proton charge radius results from ordinary hydrogen spectroscopic measurements together with muonic hydrogen results and the CODATA-2014 recommended value based on ordinary hydrogen spectroscopy. From Jingyi Zhou.

TABLE IV. Summary of proton charge radius results from muonic and ordinary hydrogen spectroscopic measurements published since 2010.

Experiment	Type	Transition(s)	$\sqrt{\langle r_{Ep}^2 \rangle}$ (fm)	r_∞ (m^{-1})
Pohl 2010	μH	$2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$	0.841 84(67)	
Antognini 2013	μH	$2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ $2S_{1/2}^{F=0} - 2P_{3/2}^{F=1}$	0.840 87(39)	
Beyer 2017	H	$2S - 4P$ with $(1S - 2S)$	0.8335(95)	10 973 731.568 076 (96)
Fleurbaey 2018	H	$1S - 3S$ with $(1S - 2S)$	0.877(13)	10 973 731.568 53(14)
Bezginov 2019	H	$2S_{1/2} - 2P_{1/2}$	0.833(10)	
Grinin 2020	H	$1S - 3S$ with $(1S - 2S)$	0.8482(38)	10 973 731.568 226(38)

H. Gao, M. Vanderhaeghen, RMP 94, 015002 (2022)

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- 多样化地添加文献方式
- 非常多的插件
  - Zotfile: 可以利用第三方云端存储，定义复杂的文件自动重命名规则
  - Better BibTeX (BBT): 提供 citekey 功能
  - Zotero-INSPIRE: 任给 arXiv 号或 doi 或 INSPIRE 标准 citekey 等信息，可从 INSPIRE 数据库更新文献的元数据，包括发表信息、引用次数（及他引次数）等，插件页面有更多使用说明
  - Zotero-Reference
  - ...
- 可以将某个 collection 的文献信息输出 .bib 文件，并自动更新
- 结合 vs code (+ vs code 的 LaTeX workshop 插件)
  - Zotero LaTeX 插件：实现快捷键插入 Zotero 库中的任意文献

彩蛋：文献管理软件推荐 Zotero