Quark Confinement for Multi-Quark Systems: An Application to Fully-Charmed Tetraquark States

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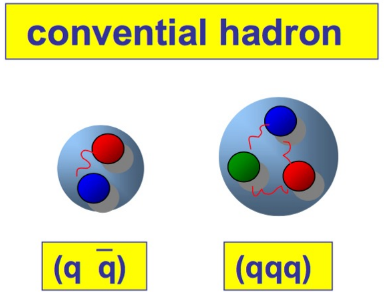
Based on Phys. Rev. D. 100, 096013, Phys. Rev. D 104, 036016, Phys. Rev. D 106, 096005, arXiv: 2307.04310 (to appear in PRD Letter)

Outline

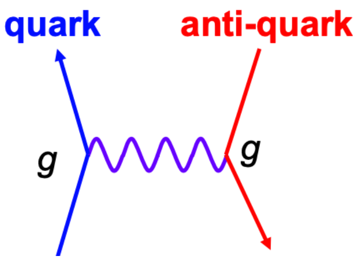
- Background
- Conventional quark model: the S-wave and P-wave tetraquark states.
- Identifying the resonance: Complex scaling method.
- Investigation of the confinement mechanisms: Novel string-like confinement model
- Summary

Classical Quark model

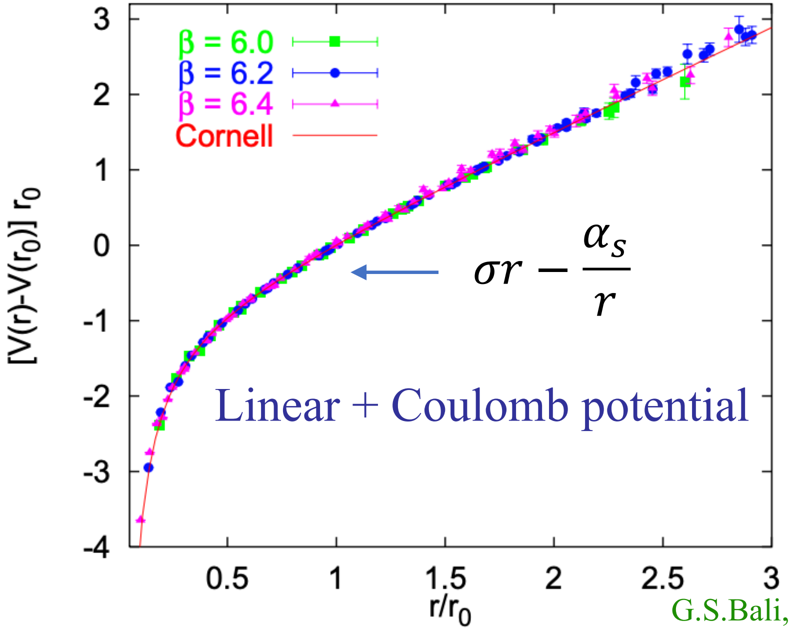
- Classical Quark model (QM):



- Quark-antiquark: one gluon exchange + string



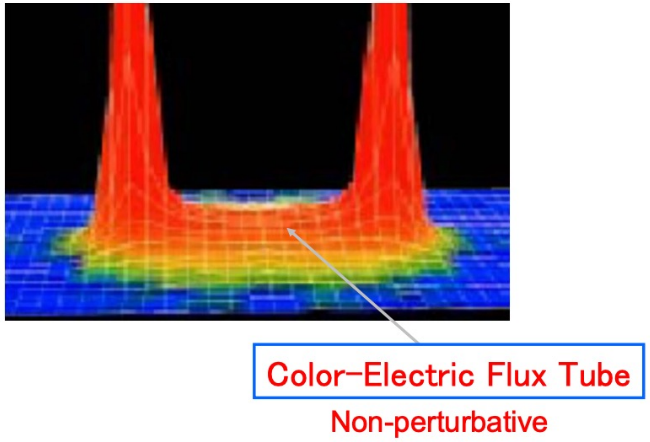
- Quark-antiquark static potential from Lattice QCD

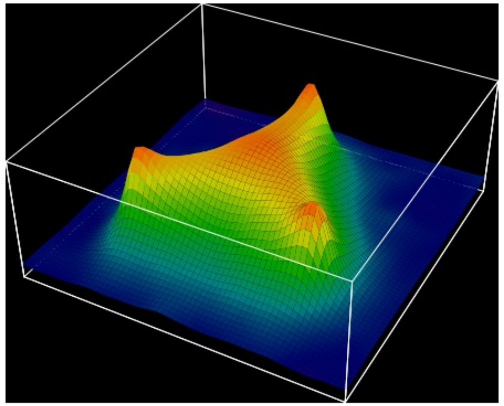


G.S.Bali, Phys. Rep. 343(2001) 1

- Lattice QCD shows string configurations

G.S. Bali



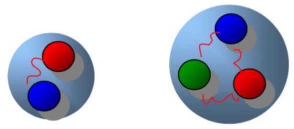


H. Ichie et al., Nucl. Phys. A 721,899

Exotic Multiquark Hadrons

- Classical Quark model (QM):

convential hadron

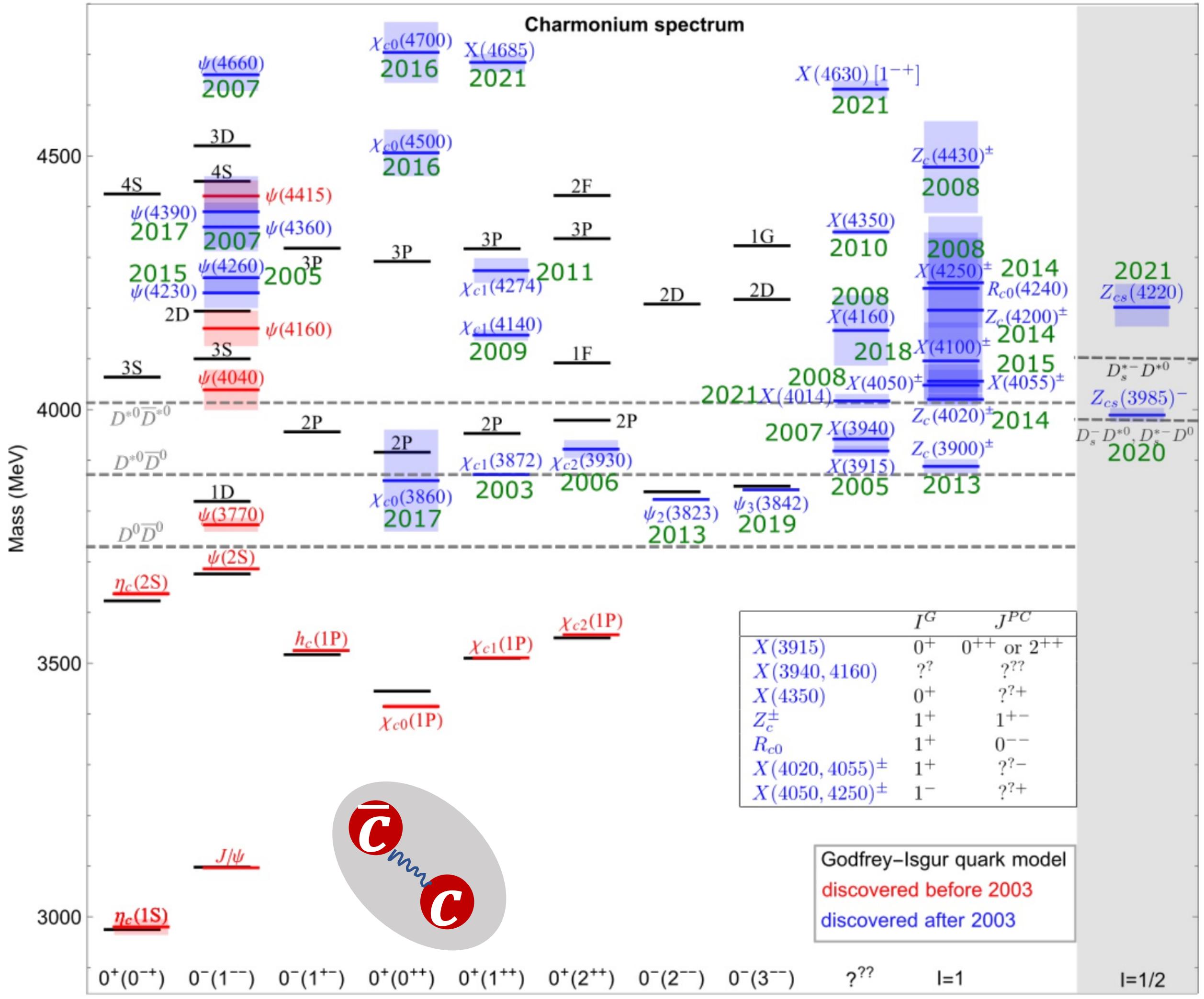


$(q\ \bar{q})$ (qqq)

QCD-Allowed Exotic Hadrons

Quarkonium | Hadronic molecule | Tetraquark | Hadro-Quarkonium | Pentaquark | …

Color configurations in multi-quark and associated color confinement dynamics are not determined uniquely

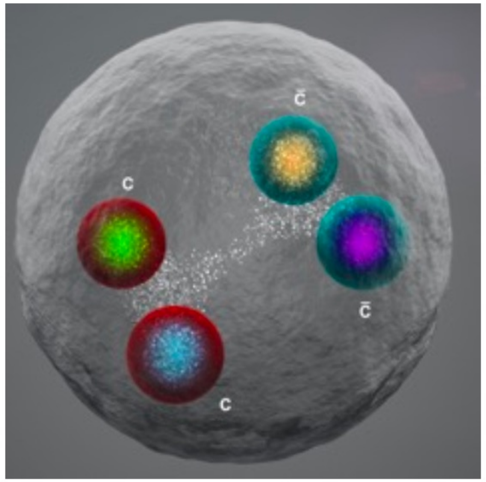


$D^{*}_{s0}(2317)$ &X(3872)@2003,…, P_c @2019, X(6900)@2020, T^{+}_{cc}@2021

Fully-heavy tetraquark

- The fully heavy tetraquark state $T_{Q_1Q_2\bar{Q}_3\bar{Q}_4}(Q=c,b)$ is a good candidate for a **compact** tetraquark state.

- Theoretical works started in1970s. (*More details are referred to Prog.Part.Nucl.Phys. 107 (2019) 237-320.*)
  PRL 36 (1976) 1266, Z.Phys.C7 (1981) 317, PRD 25 (1982) 2370



✓ ***The tension in the existence*** of the stable (bound) fully heavy tetraquark state:

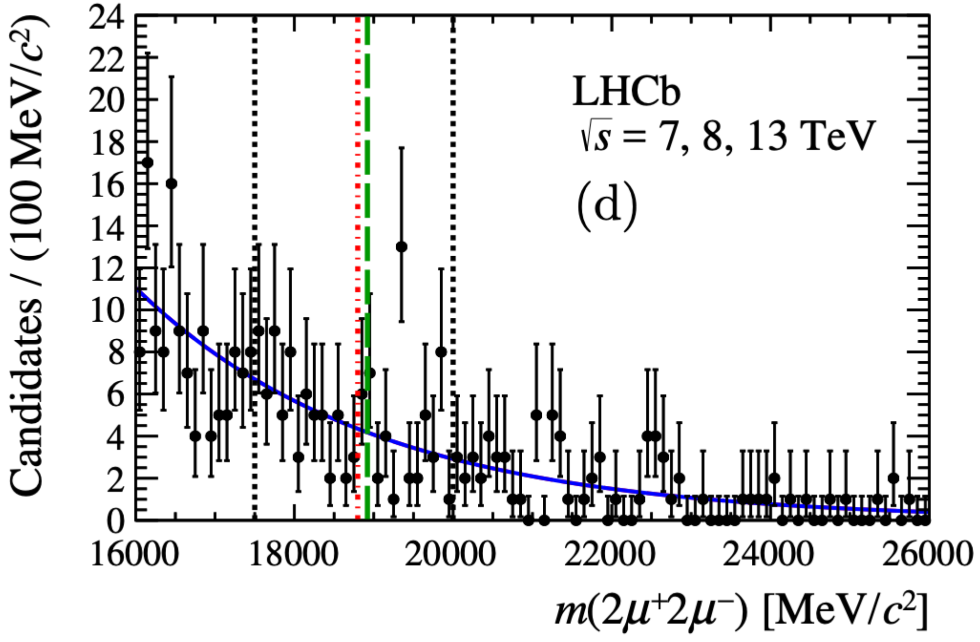
✦ Stable $QQ\bar{Q}\bar{Q}$ states exist: $bb\bar{b}\bar{b}\sim 18-20$ GeV, $cc\bar{c}\bar{c}\sim 5-7$ GeV:
arXiv:1612.00012, Eur. Phys. J. C 78, 647, EPJ Web Conf. 182, 02028,
Phys. Lett. B 718, 545, Phys. Rev. D 70, 014009 ...

✦ Negative: no bound $QQ\bar{Q}\bar{Q}$ states exist.
Phys. Rev. D 97, 094015, Phys. Rev. D.97.054505, Phys. Rev. D. 100, 096013, ...

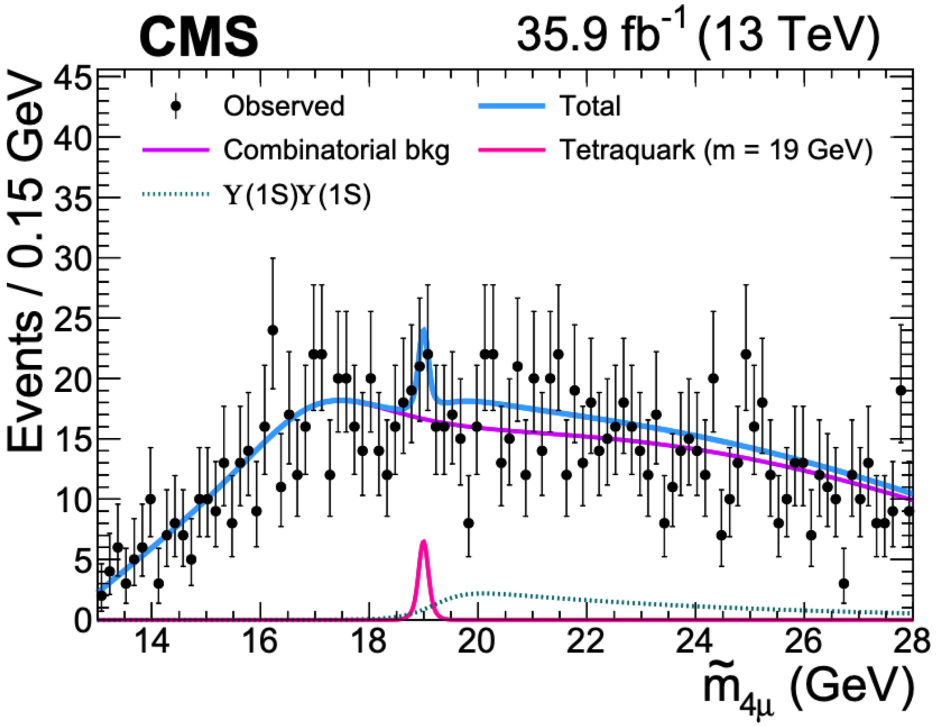
✓ ***Existence*** of the ***resonant*** $T_{Q_1Q_2\bar{Q}_3\bar{Q}_4}$ and ***the mass spectrum.***

Experimental search for $T_{QQ\bar{Q}\bar{Q}}$

- ***No significant excess observed for $T_{bb\bar{b}\bar{b}}$.***



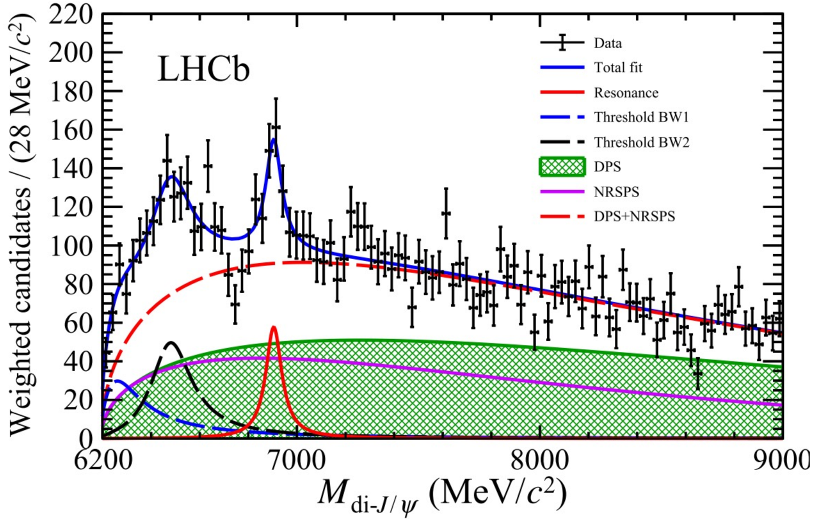
LHCb, JHEP 1810, 086 (2018).

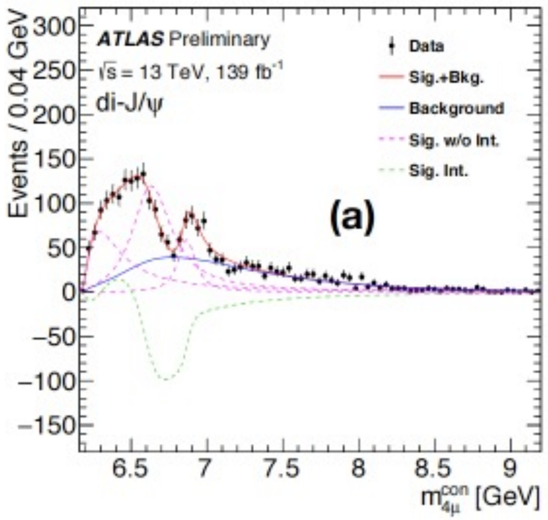


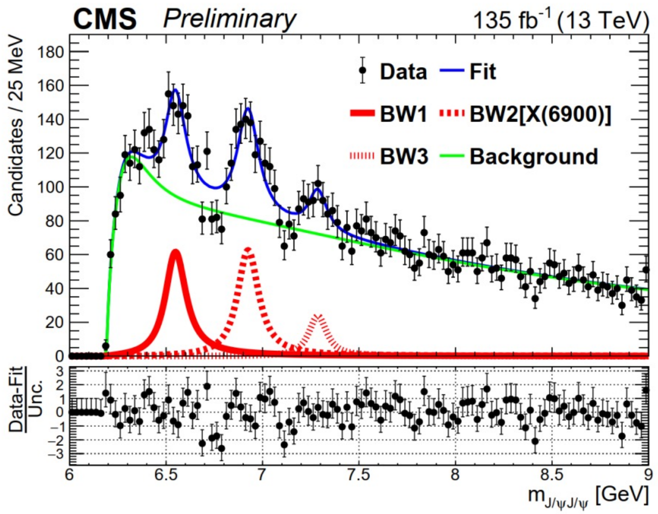
CMS, PLB 808 (2020) 135578

Experimental search for $T_{cc\bar{c}\bar{c}}$

- Observation of structure $T_{cc\bar{c}\bar{c}}$ in di-J/ψ channel





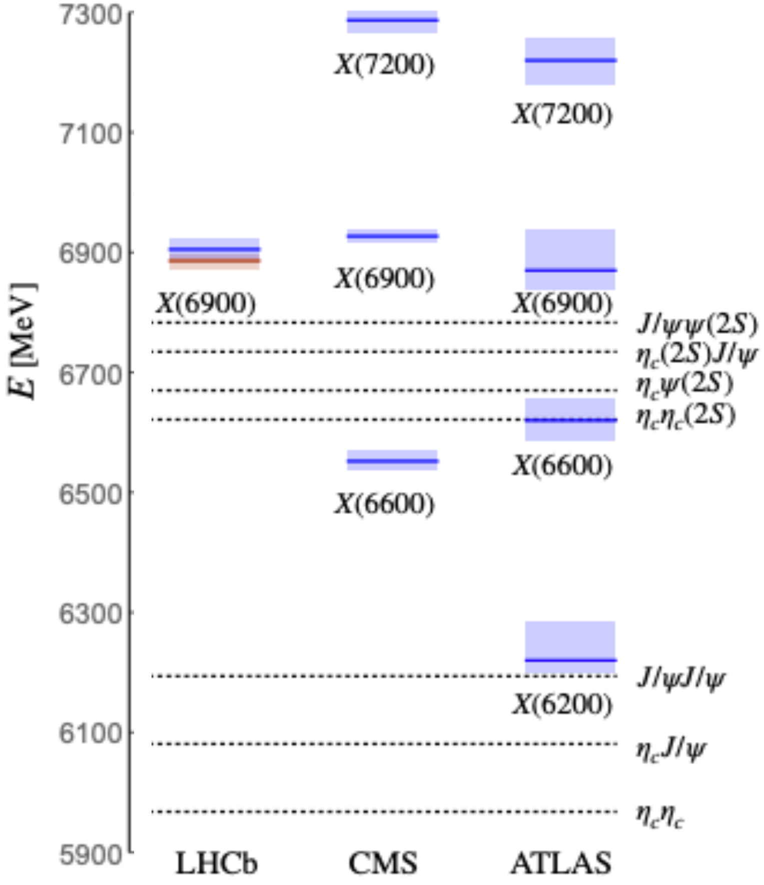


LHCb, Science Bulletin 65 (2020) 198 3.

K. Y. on behalf of the CMS Collaboration, https: //agenda.infn.it/event/28874/contributions/ 170300/.

E. B.-T. on behalf of the ATLAS Collaboration, https://agenda.infn.it/event/28874/ contributions/170298/.

J/ψ-J/ψ resonances observed in experiments



Experimental search for $T_{cc\bar{c}\bar{c}}$

- Observation of structure $T_{cc\bar{c}\bar{c}}$ in di-J/ψ and $J/\psi\psi(2S)$ channel

Update of results in ATLAS

	M	Γ	Observable channels
$X(6200)$	$6.22 \pm 0.05^{+0.04}_{-0.05}$	$0.31 \pm 0.12^{+0.07}_{-0.08}$	di-J/ψ
$X(6600)$	$6.62 \pm 0.03^{+0.02}_{-0.01}$	$0.31 \pm 0.09^{+0.06}_{-0.11}$	
$X(6900)$	$6.87 \pm 0.03^{+0.06}_{-0.01}$	$0.12 \pm 0.04^{+0.03}_{-0.01}$	
	$6.78 \pm 0.36^{+0.35}_{-0.54}$	$0.39 \pm 0.11^{+0.11}_{-0.07}$	$J/\psi\psi(2S)$
$X(7200)$	$7.22 \pm 0.03^{+0.02}_{-0.03}$	$0.10^{+0.13+0.06}_{-0.07-0.05}$	

E. B.-T. on behalf of the ATLAS Collaboration, https://agenda.infn.it/event/28874/ contributions/170298/.

di-J/ψ	model A	model B
m_0	$6.41 \pm 0.08^{+0.08}_{-0.03}$	$6.65 \pm 0.02^{+0.03}_{-0.02}$
Γ_0	$0.59 \pm 0.35^{+0.12}_{-0.20}$	$0.44 \pm 0.05^{+0.06}_{-0.05}$
m_1	$6.63 \pm 0.05^{+0.08}_{-0.01}$	—
Γ_1	$0.35 \pm 0.11^{+0.11}_{-0.04}$	
m_2	$6.86 \pm 0.03^{+0.01}_{-0.02}$	$6.91 \pm 0.01 \pm 0.01$
Γ_2	$0.11 \pm 0.05^{+0.02}_{-0.01}$	$0.15 \pm 0.03 \pm 0.01$
$\Delta s/s$	$\pm 5.1\%^{+8.1\%}_{-8.9\%}$	—
J/ψ+ψ(2S)	model α	model β
m_3 or m	$7.22 \pm 0.03^{+0.01}_{-0.03}$	$6.96 \pm 0.05 \pm 0.03$
Γ_3 or Γ	$0.09 \pm 0.06^{+0.06}_{-0.03}$	$0.51 \pm 0.17^{+0.11}_{-0.10}$
$\Delta s/s$	$\pm 21\% \pm 14\%$	$\pm 20\% \pm 12\%$

Model A: Three resonances

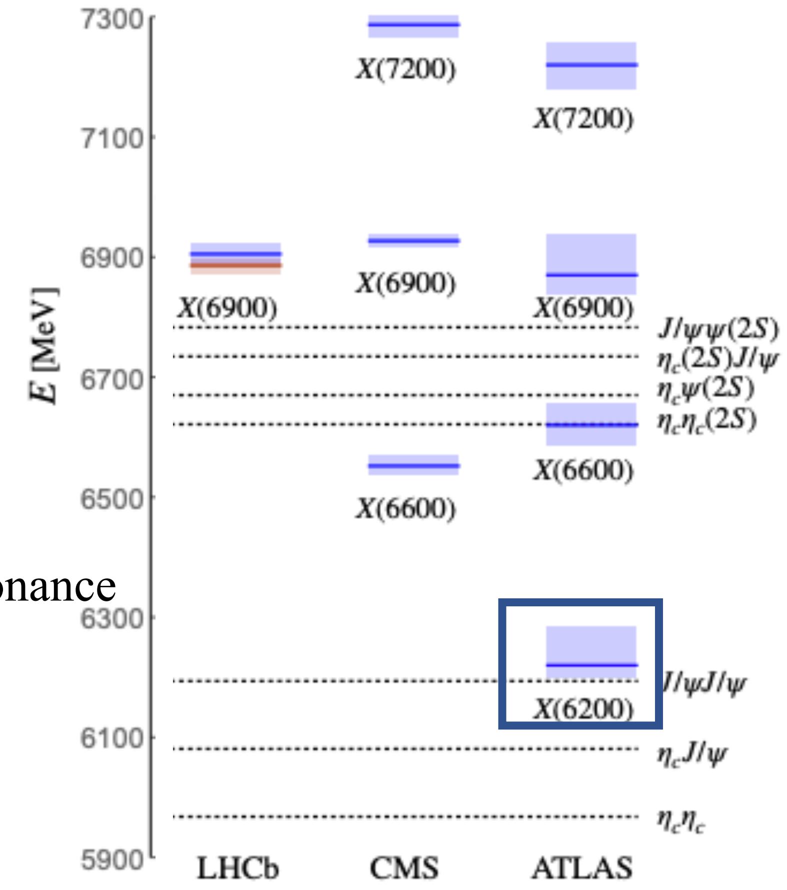
Model B: Two resonances

Model α: A model + a standalone fourth resonance

Model β: A single resonance in $J/\psi\psi(2S)$

arXiv:2304.08962v1

J/ψ-J/ψ resonances observed in experiments



Theoretical interpretations

- The predicted ground S-wave $T_{cc\bar{c}\bar{c}}$: $(6.3, 6.5)$ GeV.

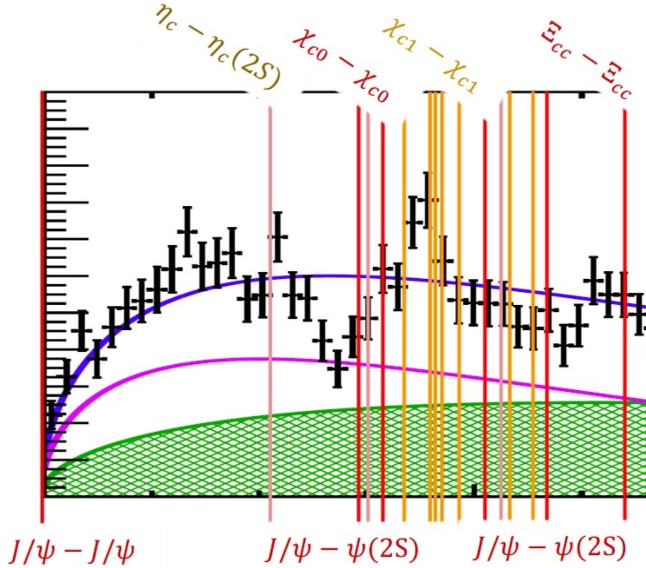
- X(6900): Radial & P-wave excitation?

Phys. Rev. D 104, 116029 (2021).
arXiv:2207.07537 [hep-ph].
Phys. Rev. D 105, 014006 (2022).
Phys. Rev. D 104, 036016 (2021).
Phys. Rev. D 104, 014020 (2021).
arXiv:2104.08814 [hep-ph].
….

	J^{PC}	$M^1_{\rm th}$	$M^2_{\rm th}$	[43]	[44]	[47]	[34]	[33]	[41]	[49]	[37,57]
$cc\bar{c}\bar{c}$	0^{++}	6.377	6.371	5.966	6.192 ± 0.025	6.001	⋯	⋯	6.038	6.470	6.44 ± 0.15
		6.425	6.483							6.558	
	1^{+-}	6.425	6.450	6.051	⋯	6.109	⋯	⋯	6.101	6.512	6.37 ± 0.18
	2^{++}	6.432	6.479	6.223	⋯	6.166	⋯	⋯	6.172	6.534	6.37 ± 0.19
$bb\bar{b}\bar{b}$	0^{++}	19.215	19.243	18.754	18.826 ± 0.025	18.815	18.72 ± 0.02	18.69 ± 0.03	⋯	19.268	18.45 ± 0.15
		19.247	19.305							19.305	
	1^{+-}	19.247	19.311	18.808	⋯	18.874	⋯	⋯	⋯	19.285	18.32 ± 0.17
	2^{++}	19.249	19.325	18.916	⋯	18.905	⋯	⋯	⋯	19.295	18.32 ± 0.17
$bb\bar{c}\bar{c}(cc\bar{b}\bar{b})$	0^{++}	12.847	12.886	⋯	⋯	12.571	⋯	⋯	⋯	12.935	⋯
		12.866	12.946							13.023	
	1^{+-}	12.864	12.924	⋯	⋯	12.638	⋯	⋯	⋯	12.945	⋯
	2^{++}	12.868	12.940	⋯	⋯	12.673	⋯	⋯	⋯	12.956	⋯

- The dynamical rescattering mechanism of double-charmonium.

Phys. Rev. D 103, 034024 (2021).
Phys. Rev. Lett. 126, 132001 (2021) .
arXiv:2011.11374 [hep-ph].
Phys. Rev. D 103, 071503 (2021) .
Sci. Bull. 66, 2462 (2021).
arXiv:2206.13867 [hep-ph].
…



From Yanxi Zhang's talk

Conventional quark model

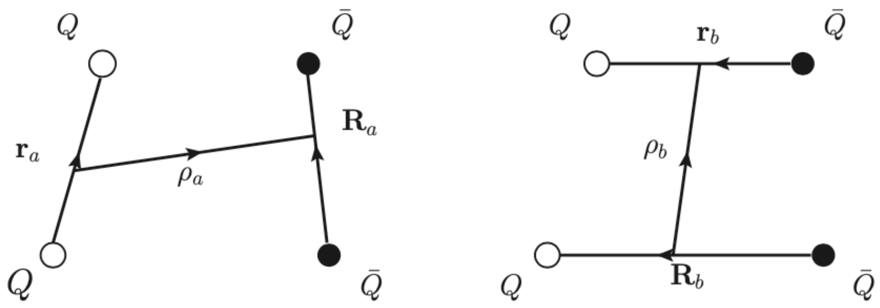
- Four body system: ***two independent color singlet states are allowed***

$$\mathbf{3}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\bar{\mathbf{3}} = 2\times\mathbf{1}\oplus 4\times\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

✓ Diquark-antidiquark: (QQ)-$(\bar{Q}\bar{Q})$:

$\bar{3}_c\otimes 3_c = 1_c$ and $6_c\otimes\bar{6}_c = 1_c$.

✓ Meson-Meson: $(Q\bar{Q})$-$(Q\bar{Q})$



$$|\mathbf{1}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{8}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{8}}(Q_2\bar{Q}_4)_{\mathbf{8}}\rangle,$$

Or

$$|\mathbf{1}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{1}'\rangle \equiv |(Q_1\bar{Q}_4)_{\mathbf{1}}(Q_2\bar{Q}_3)_{\mathbf{1}}\rangle,$$

Conventional quark model

- Four body system: ***two independent color singlet states are allowed***

$$\mathbf{3}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\bar{\mathbf{3}} = 2\times\mathbf{1}\oplus 4\times\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

✓ Diquark-antidiquark: (QQ)-$(\bar{Q}\bar{Q})$:

$\bar{3}_c \otimes 3_c = 1_c$ and $6_c \otimes \bar{6}_c = 1_c$.

✓ Meson-Meson: $(Q\bar{Q})$-$(Q\bar{Q})$

$$|\mathbf{1}\rangle = \sqrt{\frac{1}{3}}|\bar{\mathbf{3}}\rangle + \sqrt{\frac{2}{3}}|\mathbf{6}\rangle,$$
$$|\mathbf{8}\rangle = -\sqrt{\frac{2}{3}}|\bar{\mathbf{3}}\rangle + \sqrt{\frac{1}{3}}|\mathbf{6}\rangle,$$
$$|\mathbf{1}'\rangle = -\sqrt{\frac{1}{3}}|\bar{\mathbf{3}}\rangle + \sqrt{\frac{2}{3}}|\mathbf{6}\rangle.$$

$$|\mathbf{1}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{8}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{8}}(Q_2\bar{Q}_4)_{\mathbf{8}}\rangle,$$

Or

$$|\mathbf{1}\rangle \equiv |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{1}'\rangle \equiv |(Q_1\bar{Q}_4)_{\mathbf{1}}(Q_2\bar{Q}_3)_{\mathbf{1}}\rangle,$$

Either two of them

- ***Not orthogonal***

$$\langle\mathbf{1}'|\mathbf{1}\rangle = \frac{1}{3}$$

Formalism

- Hamiltonian:

$$H = H_0 + \frac{\lambda_i}{2} \cdot \frac{\lambda_j}{2} [V_{\rm cen}^{(0)}(r_{ij}) + V_{\rm so}^{(1)}(r_{ij}) + V_{\rm tens}^{(1)}(r_{ij})]$$

$$H_0 = \sum_{i=1}^{4} \frac{\mathbf{p}_i^2}{2m_i} + \sum_i m_i - T_G.$$

- $V_{cen}^{(0)}$: Color Coulomb+linear confinement + hyperfine

$$V_{\rm cen}^{(0)}(r_{ij}) = \frac{\alpha_s}{r_{ij}} - \frac{3}{4} b r_{ij} - \frac{8\pi\alpha_s}{3m_i m_j}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r_{ij}^2} \mathbf{s}_i \cdot \mathbf{s}_j.$$

Phys. Rev. D 72 (2005) 054026

- $V_{so}^{(1)} + V_{tens}^{(1)}$: spin-orbital and tensor interactions.

$$V_{\rm so}^{(1)}(r_{ij}) = V_{\rm so}^{v}(r_{ij}) + V_{\rm so}^{s}(r_{ij}).$$

$$V_{\rm so}^{v}(r_{ij}) = \frac{1}{r_{ij}} \frac{dV_{\rm Coul}}{dr_{ij}} \frac{1}{4}\left[\left(\frac{1}{m_i^2} + \frac{1}{m_j^2} + \frac{4}{m_i m_j}\right)\mathbf{L}_{ij} \cdot \mathbf{S}_{ij} + \left(\frac{1}{m_i^2} - \frac{1}{m_j^2}\right)\mathbf{L}_{ij} \cdot (\mathbf{s}_i - \mathbf{s}_j)\right]$$

$$V_{\rm so}^{s}(r_{ij}) = -\frac{1}{r_{ij}} \frac{dV_{\rm lin}}{dr_{ij}} \left(\frac{\mathbf{L}_{ij} \cdot \mathbf{s}_i}{2m_i^2} + \frac{\mathbf{L}_{ij} \cdot \mathbf{s}_j}{2m_j^2}\right)$$

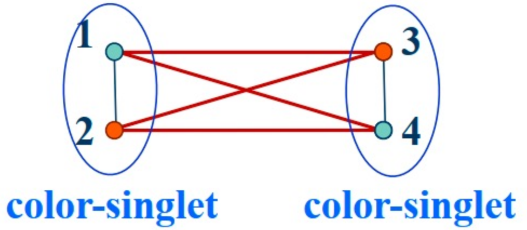
Phys. Rev. D 32, 189

$$V_{\rm tens}^{(1)}(r_{ij}) = -\left(\frac{\partial^2}{\partial r_{ij}^2} - \frac{1}{r_{ij}} \frac{\partial}{\partial r_{ij}}\right) \frac{V_{\rm Coul}}{3m_i m_j} \mathcal{S}_{ij}$$

Charmonium state $\bar{c}c$

parameter		Mass spectrum (MeV)			
		$^{2S+1}L_J$	Meson	EXP	THE
α_s	0.5461	$^1S_0$	η_c	2983.9	2984
b [GeV^2]	0.1452	$^3S_1$	J/ψ	3096.9	3092
m_c [GeV]	1.4794	$^3P_0$	χ_{c0}	3414.7	3426
σ [GeV]	1.0946	$^3P_1$	χ_{c1}	3510.7	3506
		$^1P_1$	$h_c(1P)$	3525.4	3516
		$^3P_2$	χ_{c2}	3556.2	3556
		$^1S_0$	$\eta_c(2S)$	3637.5	3634
		$^3S_1$	$\psi(2S)$	3686.1	3675
		$^3S_1$	$\psi(3S)$	4039.0	4076
		$^3S_1$	$\psi(4S)$	4421.0	4412

PDG



Gaussian Expansion Method

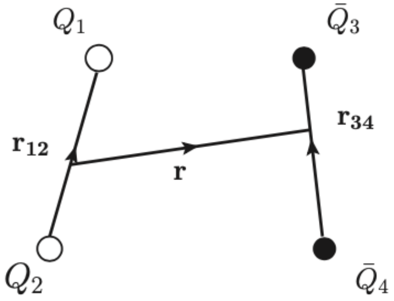
- Few-body problem: Gaussian expansion method

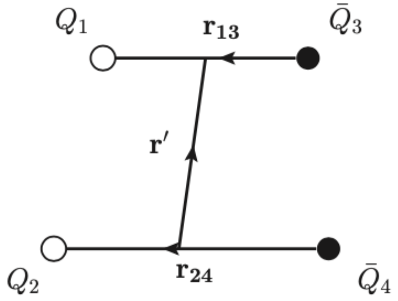
Prog. Part. Nucl. Phys. 51 223-307

$$\psi_{JJ_z} = \sum \left[\varphi_{n_a J_a}(\mathbf{r}_{12}, \beta_a) \otimes \varphi_{n_b J_b}(\mathbf{r}_{34}, \beta_b) \otimes \phi_{N L_{ab}}(\mathbf{r}, \beta)\right]_{JJ_z} ,$$

- Basic wave function of each Jacobi coordinate

$$\varphi_{n_a J_a M_a} = [\phi_{n_a l_a}(\mathbf{r}_{12}, \beta_a)\chi_{s_a}]^{J_a}_{M_a} \chi_f \chi_{c_a}$$





- $\chi_{s,f,c}$: the wave function in the spin, flavor, and color space.
- Gaussian function:

$$\phi_{n_a l_a}(r_{12}, \beta_a) = \left\{\frac{2^{l_a+2}(2\nu_{n_a})^{l+3/2}}{\sqrt{\pi}(2l_a+1)!!}\right\}^{1/2} r_{12}^{l_a} e^{-\nu_{n_a} r_{12}^2}$$

- Calculate the mass spectrum in two stages.

✓ $H = H_0 + V^{(0)}_{cen}$ (OGE Coulomb + confinement+ hyperfine) & Schrödinger equation to obtain $\psi^0_{JJ_z}$.

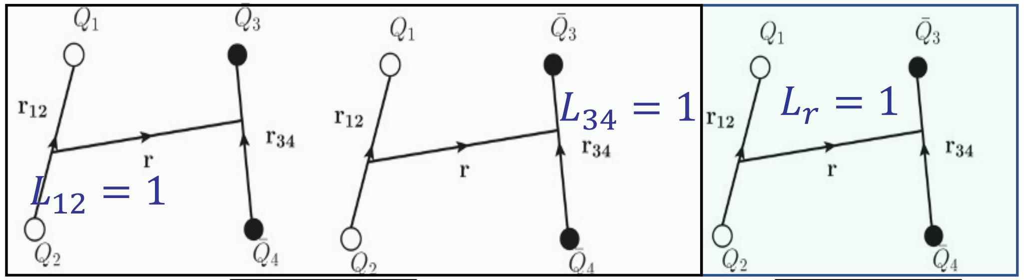
✓ $H = H_0 + V^{(0)}_{cen} + V^{(1)}_{so} + V^{(1)}_{tens}$ & diagonalizing the Hamiltonian matrix in the basis of $\psi^0_{JJ_z}$.

Diquark-antidiquark configuration

- The S-wave $T_{cc\bar{c}\bar{c}}$ state: $L_{12} = L_{34} = L_r = 0$.

0^{++}	$[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}]^0_{1_c}$	1^{+-}	$[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}]^1_{1_c}$
	$[[QQ]^0_{6_c}[\bar{Q}\bar{Q}]^0_{\bar{6}_c}]^0_{1_c}$	2^{++}	$[[QQ]^1_{\bar{3}_c}[\bar{Q}\bar{Q}]^1_{3_c}]^2_{1_c}$

- P-wave state: λ-and ρ- mode excitations.



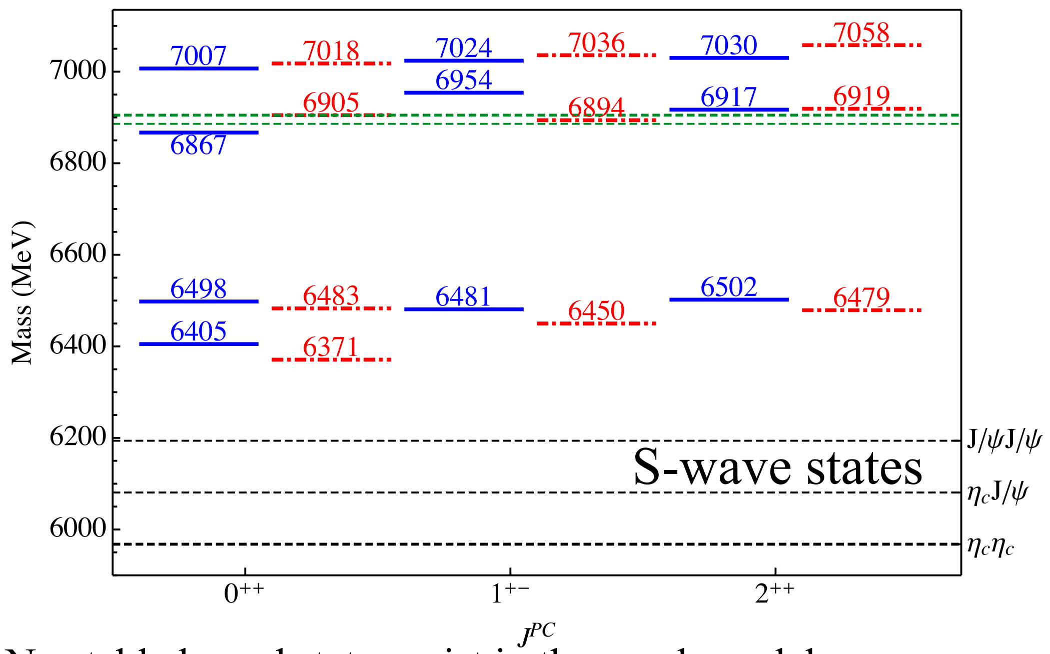
ρ-mode **λ-mode**

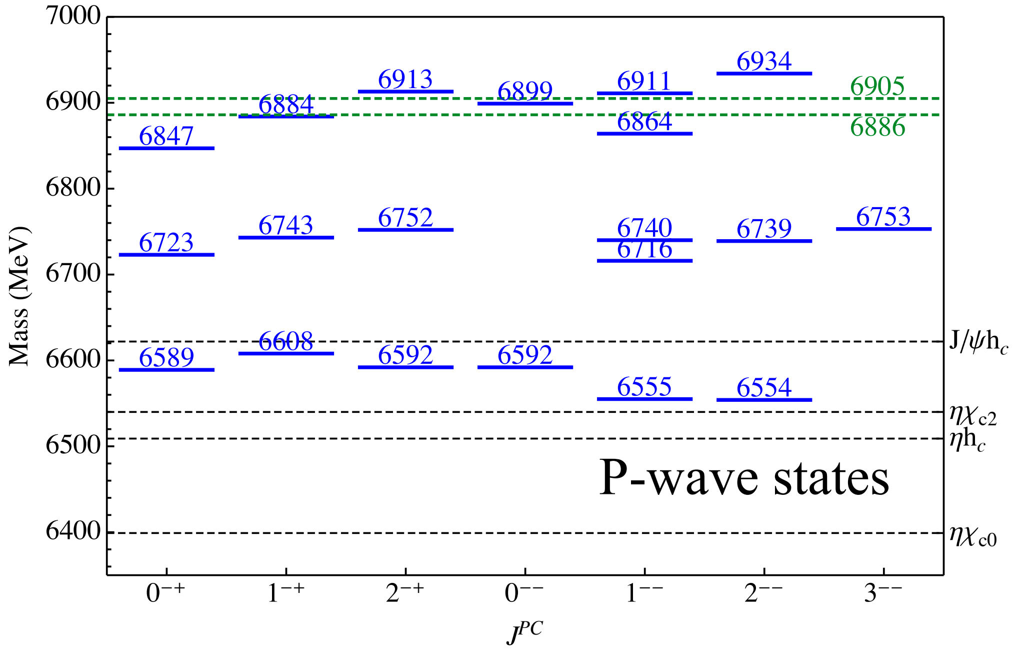
TABLE II. The color-flavor-spin configurations of the QQ ($\bar{Q}\bar{Q}$) diquark (antidiquark). The scripts "S" and "A" represent the exchange symmetry and antisymmetry for the identical particles, respectively.

Flavor	S-wave ($L = 0$)	Spin	Color		J^P
S	S	S($S_{QQ} = 1$)	$\bar{3}_c$(A)	$[QQ]^1_{\bar{3}_c}$	1^+
S	S	A($S_{QQ} = 0$)	6_c(S)	$[QQ]^0_{6_c}$	0^+
Flavor	P-wave ($L = 1$)	Spin	Color		
S	A	S($S_{QQ} = 1$)	6_c(S)	$[[QQ]^1_{6_c}, \rho]^0_{6_c}$	0^-
				$[[QQ]^1_{6_c}, \rho]^1_{6_c}$	1^-
				$[[QQ]^1_{6_c}, \rho]^2_{6_c}$	2^-
S	A	S($S_{QQ} = 0$)	$\bar{3}_c$(A)	$[[QQ]^0_{\bar{3}_c}, \rho]^1_{\bar{3}_c}$	1^-

Phys. Rev. D. 100, 096013

Results





- No stable bound states exist in the quark models.
- The lowest fully charmed tetraquark state : **in mass region (6.5, 6.7, 6.9) GeV**
- $X(6900)$: wide S-wave states $J^{PC} = 0^{++}$ or 2^{++} or narrow P-wave states with $J^{PC} = 1^{-+}$ or 2^{-+}.
- **Redundancy states :**

✓ The finite number of the bases → discrete eigenvalues of the scattering states

✓ Multiquark states with large decay widths → hard to observe

Phys. Rev. D. 100, 096013
Phys. Rev. D. 104, 036016

Complex scaling method (CSM): $T_{cc\bar{c}\bar{c}}$

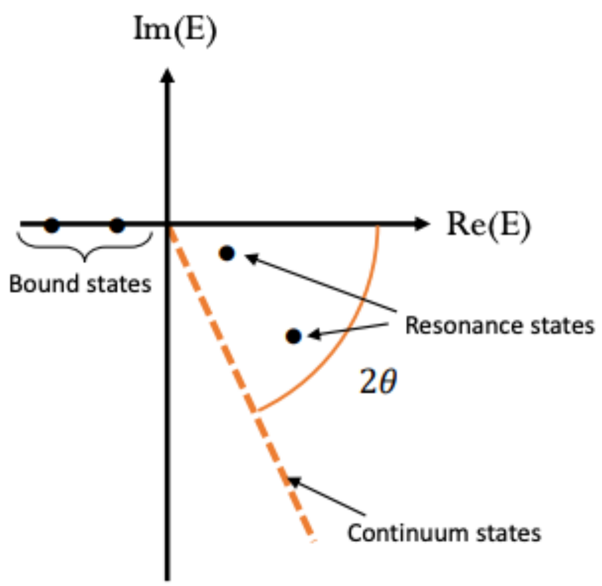
- Complex scaling method

$$\boldsymbol{r} \to \boldsymbol{r} e^{i\theta}, \quad \boldsymbol{q} \to \boldsymbol{q} e^{-i\theta}$$

$$H_\theta \Phi_\theta = E_\theta \Phi_\theta,$$

with

$$H_\theta = H\left(\boldsymbol{r} e^{i\theta}, \boldsymbol{q} e^{-i\theta}\right)$$
$$= \sum_{i=1}^{4} \frac{\mathbf{p}_i^2}{2m_i} e^{-i2\theta} + \sum_i m_i + \sum_{i<j=1}^{4} \frac{\lambda_i}{2}\frac{\lambda_j}{2} V_{ij}(\boldsymbol{r}_{ij} e^{i\theta})$$



S.Aoyama et al. PTP. 116, 1 (2006).
T. Myo et al. PPNP. 79, 1 (2014)
N. Moiseyev, Physics reports 302, 212 (1998)

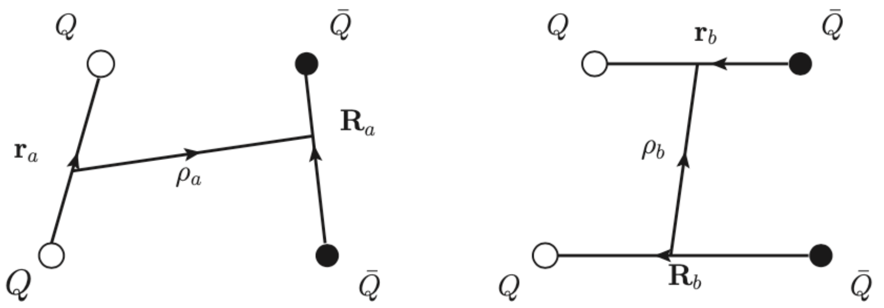
- Complex scaling method for tetraquark state:

Diquark-antidiquark

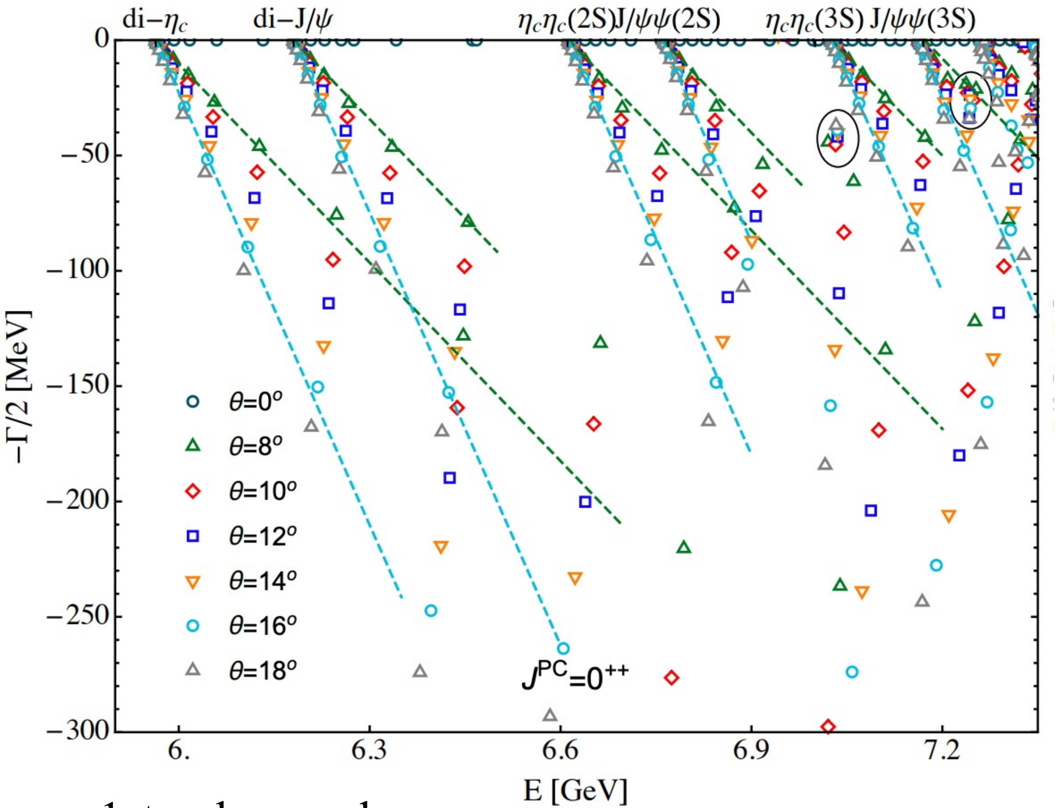
Coupled-channel effect

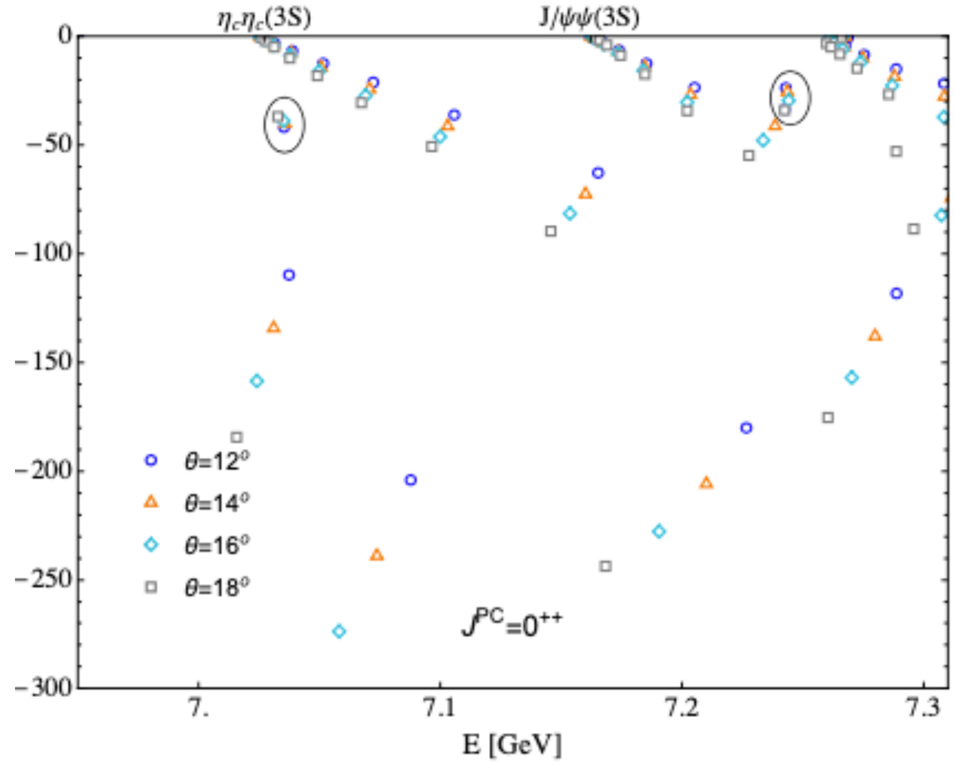
…

Meson-meson scattering channels (di-J/ψ, $J/\psi\ \psi$…)



$T_{cc\bar{c}\bar{c}}$





- 1st pole: good convergency

Phys. Rev. D 106, 096005

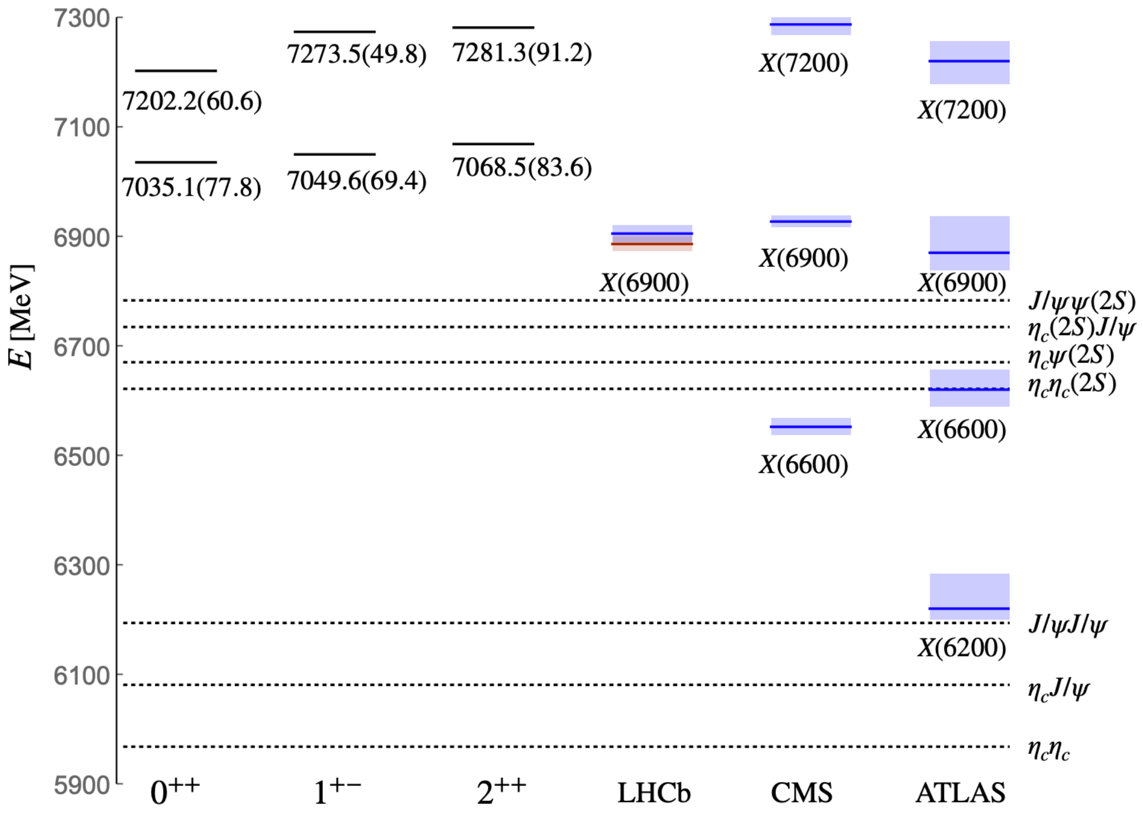
- 2nd pole: quite close to the threshold lines with a scaling angle in the (8, 10) degree

✓ Higher states are more difficult to describe.

$$\Phi^{\theta}_{\text{Res}} \sim \exp(iK_R e^{-i\theta_R} \cdot re^{i\theta}) = \exp(iK_R re^{i(\theta-\theta_R)})$$
$$= \exp\left[iK_R r \cdot \cos(\theta-\theta_R)\right] \cdot \exp\left[-K_R r \cdot \sin(\theta-\theta_R)\right] \to \text{damping with } \theta > \theta_R$$

T. Myo et al. PPNP. 79, 1 (2014)

$T_{cc\bar{c}\bar{c}}$



		M	Γ	Observable channels
LHCb model I [12]	$X(6900)$	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	di-J/ψ
LHCb model II [12]		$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	
CMS [14]	$X(6600)$	$6552 \pm 10 \pm 12$	$124 \pm 29 \pm 34$	di-J/ψ
	$X(6900)$	$6927 \pm 9 \pm 5$	$122 \pm 22 \pm 19$	
	$X(7200)$	$7287 \pm 19 \pm 5$	$95 \pm 46 \pm 20$	
ATLAS [15]	$X(6200)$	$6.22 \pm 0.05^{+0.04}_{-0.05}$	$0.31 \pm 0.12^{+0.07}_{-0.08}$	di-J/ψ
	$X(6600)$	$6.62 \pm 0.03^{+0.02}_{-0.01}$	$0.31 \pm 0.09^{+0.06}_{-0.11}$	
	$X(6900)$	$6.87 \pm 0.03^{+0.06}_{-0.01}$	$0.12 \pm 0.04^{+0.03}_{-0.01}$	
		$6.78 \pm 0.36^{+0.35}_{-0.54}$	$0.39 \pm 0.11^{+0.11}_{-0.07}$	$J/\psi\psi(2S)$
	$X(7200)$	$7.22 \pm 0.03^{+0.02}_{-0.03}$	$0.10^{+0.13+0.06}_{-0.07-0.05}$	

- ***1st pole VS X(6900):***

✓ 100 MeV higher mass & consistent decay width

- ***2nd pole: a candidate for X(7200).***

- ***Absence of the lower $X(6600)$ state.***

✓ a wide resonance asymptote will oscillate very strongly in the complex plane.

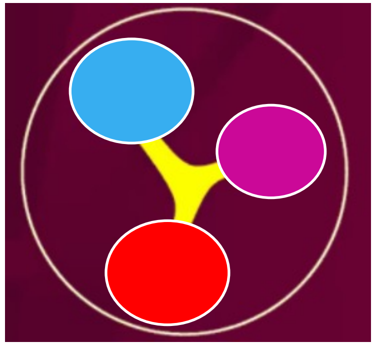
- The confinement mechanism~ br.

Phys. Rev. D 106, 096005

Investigation of the confinement mechanisms

- Conventional quark-quark confinement potential form $\bar{Q}Q$ meson: $V(r) \sim br$
- Application to baryons (qqq): $V = -\frac{3}{4}\sigma\sum_{i<j}(T_i \cdot T_j)r_{ij}$ (Δ-shape) + Y-shape?
- Direct application to $T_{Q_1Q_2\bar{Q}_3\bar{Q}_4}$: $V = -\frac{3}{4}\sigma\sum_{i<j}(T_i \cdot T_j)r_{ij}$

V. Dmitrasinovic et al., Eur. Phys. J. C 62, 383-397 (2009)



Two bases: $|1\rangle = |(1\bar{3})(2\bar{4})\rangle\phi((r_{13}, r_{24}, R)$

$|1'\rangle = |(1\bar{4})(2\bar{3})\rangle\phi(r_{14}, r_{23}, R')$

$$\text{NM}=\begin{pmatrix}\langle 1|1\rangle & \langle 1|1'\rangle \\ \langle 1'|1\rangle & \langle 1'|1'\rangle\end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{pmatrix}$$

$$\langle V\rangle = \begin{pmatrix} 2a & \frac{1}{3}\left(2a+2b-2\sqrt{a^2+b^2}\right) \\ \frac{1}{3}\left(2a+2b-2\sqrt{a^2+b^2}\right) & 2b \end{pmatrix}$$

✓Problem: ***long-range color van der Waals*** between color singlet mesons,

$$V_{\text{cvdW}} = \frac{|\langle \mathbf{8}|V_{\text{QM}}|\mathbf{1}\rangle|^2}{\Delta E} \propto -\frac{1}{R^3}$$

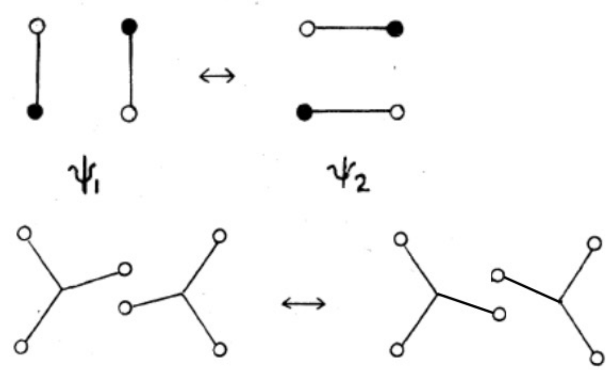
T. Appelquist et al Phys. Lett. B77, 405 (1978)

String Flip-Flop model

- "Reconnection of strings and quark matter"

$$V_{\text{string}} = \sigma \times \min_{\text{links}} \sum r_{\text{link}}$$

H. Miyazawa, PRD20, 2953 (1979).



- "String Flip-Flop" --Strings can make a transition to another configuration when they touch each other.

- ***long-range color van der Waals*** between color singlet mesons disappear.

H. Miyazawa, PR D20, 2953 (1979)
N. Isgur, J. E. Paton, Phys. Lett. B 124, 247 (1983)
M. Oka, Phys. Rev. D 31, 2274 (1985).
J. Vijande, et. Al. Phys. Rev. D 85, 014019 (2012).

- The lattice QCD may choose the adiabatic potential of the configuration with the shortest string lengths to minimize the string tension energy – Flip-Flop model

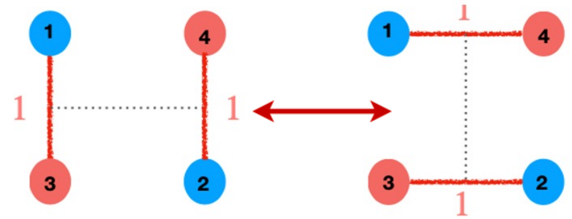
F. Okiharu, et al. PRD72 (2005) 014505
C. Alexandrou . et al. Nucl. Phys. A 518, 723-751 (1990)
F. Okiharu .et al. J. Mod. Phys. 7, 774-789 (2016)

String Flip-Flop model

$$V_{\rm FF} = \sigma\, {\rm Min}\left[r_{13}+r_{24}, r_{14}+r_{23}\right].$$

• The flip-flop potential model may not be satisfactory for color SU(3): choice of color configurations has some ambiguity

$$r_{13}+r_{24} = r_{14}+r_{23}$$



Hidden color channel automatically mixed

• $|\mathbf{1}\rangle$ and $|\mathbf{1}'\rangle$ are not smoothly connected in SU(3), because the overlap of $|\mathbf{1}\rangle$ and $|\mathbf{1}'\rangle$ is not complete.

only the $1/N_c$ part of $|\mathbf{1}\rangle$ can go directly to $|\mathbf{1}'\rangle$.

• The transition between two color configurations is dynamically generated, and the HC channel can be treated as an independent configuration.

Novel string-like confinement potential

- Three bases: States with different string configurations are orthogonal

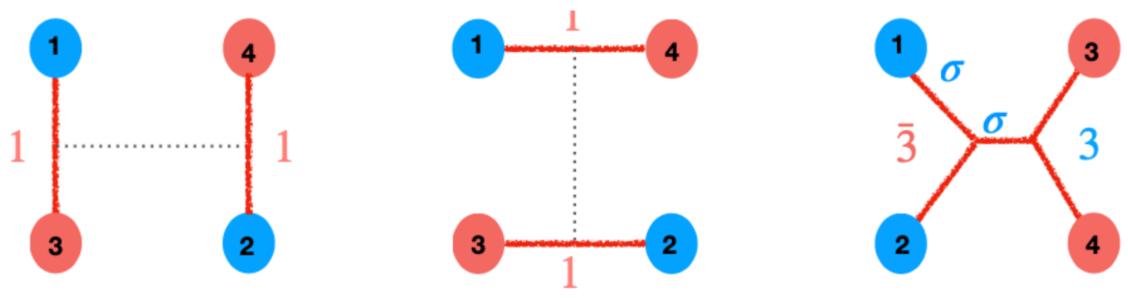
$$|\mathbf{1}\rangle\rangle \equiv |(Q_1 \to \bar{Q}_3)_{\mathbf{1}}(Q_2 \to \bar{Q}_4)_{\mathbf{1}}\rangle$$

$$|\mathbf{1}'\rangle\rangle \equiv |(Q_1 \to \bar{Q}_4)_{\mathbf{1}}(Q_2 \to \bar{Q}_3)_{\mathbf{1}}\rangle.$$

$$|\mathbf{hc}\rangle\rangle \equiv |(Q_1 \leftrightarrow Q_2)_{\bar{\mathbf{3}}} \leftarrow (\bar{Q}_3 \leftrightarrow \bar{Q}_4)_{\mathbf{3}}\rangle,$$

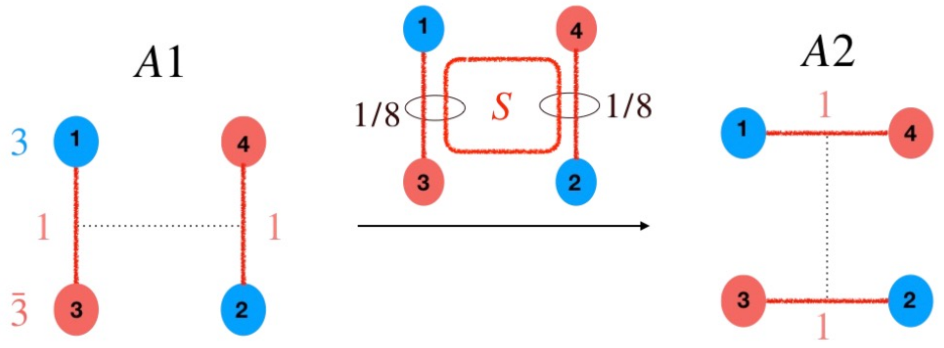
$$\langle\langle \mathbf{1}'|\mathbf{1}\rangle\rangle = 0.$$

$$\langle\langle \mathbf{1}|\mathbf{hc}\rangle\rangle = \langle\langle \mathbf{1}'|\mathbf{hc}\rangle\rangle = 0.$$



Phys. Rev. D.37.2431
Nucl. Phys. A 505, 655-669.
Prog. Theor. Phys. Suppl. 137, 21-42.

- Minimal surface area S: N-body force



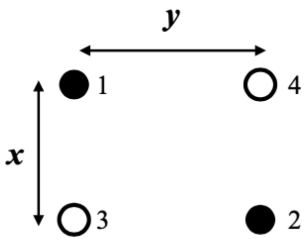
$$V_{\mathrm{ST}} = \begin{pmatrix} \sigma(r_{13}+r_{24}) & \kappa e^{-\sigma S} & \kappa' e^{-\sigma S} \\ \kappa e^{-\sigma S} & \sigma(r_{14}+r_{23}) & -\kappa' e^{-\sigma S} \\ \kappa' e^{-\sigma S} & -\kappa' e^{-\sigma S} & \frac{\sigma}{4}[r_{13}+r_{24}+r_{14}+r_{23}+2(r_{12}+r_{34})] \end{pmatrix}$$

$\kappa' = \sqrt{8}\kappa$ confinement range: $a \sim 1/\sqrt{\sigma} \sim 0.45$ fm

$0 \le \kappa' = \sqrt{8}\kappa \le 2\sigma a \qquad \kappa \le 0.3\mathrm{GeV}$

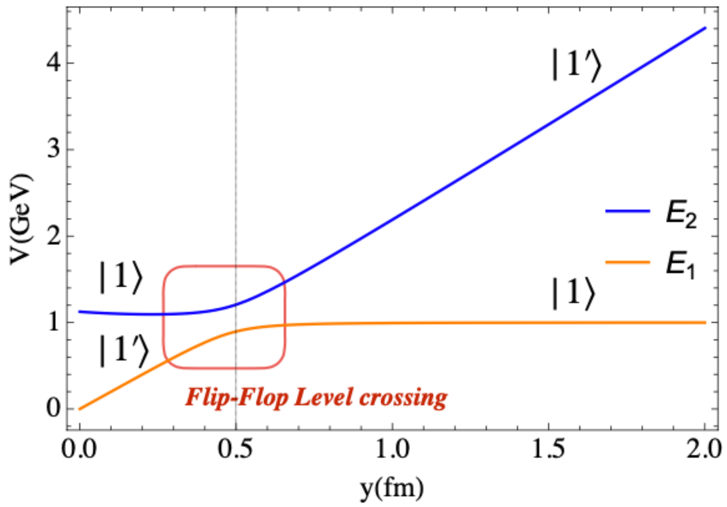
A toy model: Conventional QM VS String-like potential

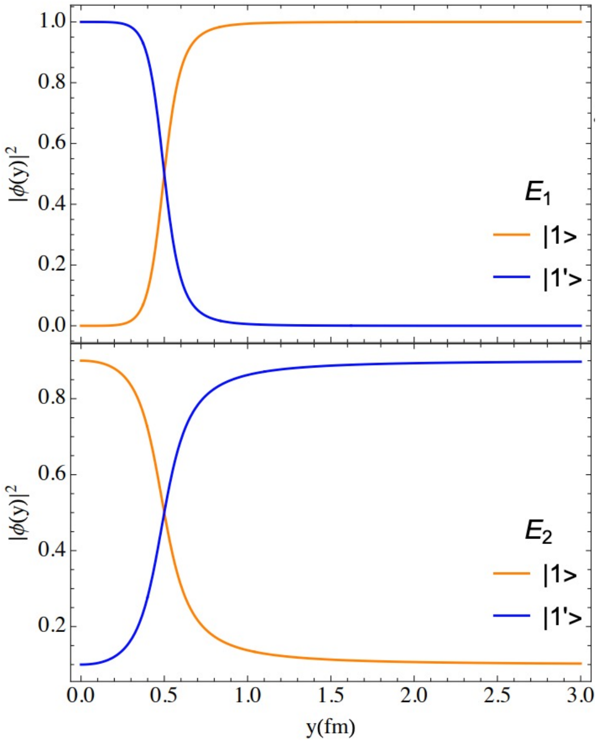
- Born-Oppenheimer (BO) potential: The quark positions are fixed.

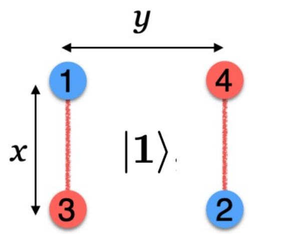


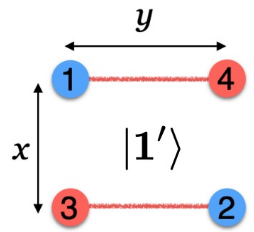
Fixed $x = 0.5$ fm

$$\tilde{V}_{\mathrm{QM}}(x,y) = \begin{pmatrix} 2\sigma x & \frac{2}{3}\sigma(x+y-\sqrt{x^2+y^2}) \\ \frac{2}{3}\sigma(x+y-\sqrt{x^2+y^2}) & 2\sigma y \end{pmatrix}$$



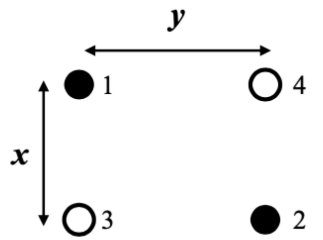




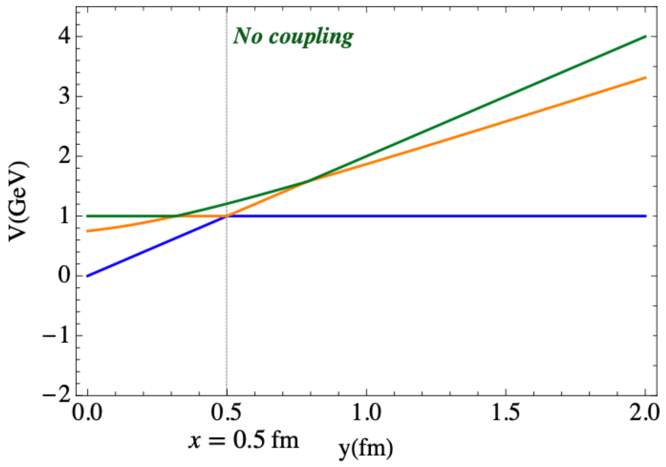


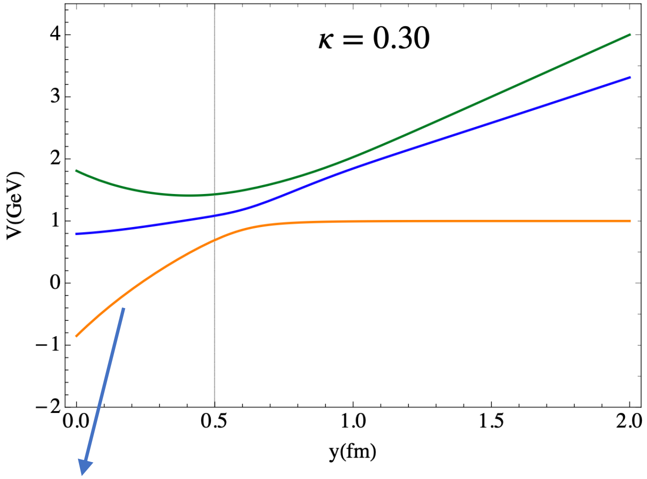
A toy model: Conventional QM VS String-like potential

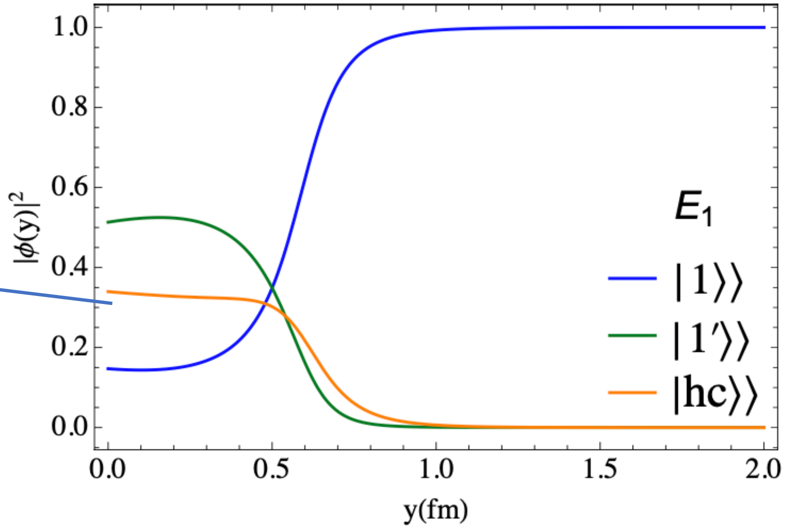
- Born-Oppenheimer (BO) potential: The quark positions are fixed.



Fixed $x = 0.5$ fm







Mixing induced a strong attraction at short distances with important mixing of the hidden color (hc) state.

$$V_{\rm ST}(x,y) = \begin{pmatrix} 2\sigma x & \kappa e^{-\sigma xy} & \kappa' e^{-\sigma xy} \\ \kappa e^{-\sigma xy} & 2\sigma y & -\kappa' e^{-\sigma xy} \\ \kappa' e^{-\sigma xy} & -\kappa' e^{-\sigma xy} & \sigma(\frac{x+y}{2} + \sqrt{x^2+y^2}) \end{pmatrix}$$

Novel string-like potential: $T_{cc\bar{c}\bar{c}}$

- Application to the $T_{cc\bar{c}\bar{c}}$ states:
- Parameters are same as the conventional QM → reproduce the two meson thresholds
- ***Replace the linear confinement by the string-like confinement***

$$H = H_0 + \sum_{i,j} \frac{\lambda_i}{2} \cdot \frac{\lambda_j}{2} V_{\mathrm{SR}}\left(r_{ij}\right) + V_{\mathrm{ST}}$$

$$H_0 = \sum_{i=1}^{4} \frac{\mathbf{p}_i^2}{2m_i} + \sum_i m_i - T_G$$

$$V_{\mathrm{SR}}\left(r_{ij}\right) = \frac{\alpha_s}{r_{ij}} - \frac{8\pi\alpha_s}{3m_i m_j}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r_{ij}^2} \mathbf{s}_i \cdot \mathbf{s}_j$$

$$V_{\mathrm{ST}} = \begin{pmatrix} \sigma\left(r_{13}+r_{24}\right) & \kappa e^{-\sigma S} & \kappa' e^{-\sigma S} \\ \kappa e^{-\sigma S} & \sigma\left(r_{14}+r_{23}\right) & -\kappa' e^{-\sigma S} \\ \kappa' e^{-\sigma S} & -\kappa' e^{-\sigma S} & \frac{\sigma}{4}\left[r_{13}+r_{24}+r_{14}+r_{23}+2\left(r_{12}+r_{34}\right)\right] \end{pmatrix}$$

with

$$S = \frac{1}{4}\left(r_{13}^2 + r_{24}^2 + r_{14}^2 + r_{23}^2\right)$$

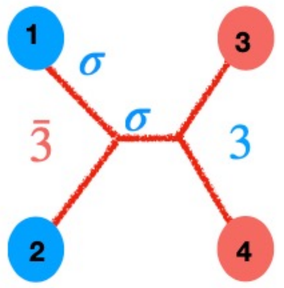
→ ***N-body force***

Novel string-like potential: $T_{cc\bar{c}\bar{c}}$

- Wave function expansion:

$$\Psi(1,2,3,4) = \psi_{\mathbf{1}}|\mathbf{1}\rangle\rangle + \psi_{\mathbf{1}'}|\mathbf{1}'\rangle\rangle + \psi_{\mathbf{hc}}|\mathbf{hc}\rangle\rangle$$

$$\Psi(1,2,3,4) = \psi_{\mathrm{S}}\frac{1}{\sqrt{2}}(|\mathbf{1}\rangle\rangle - |\mathbf{1}'\rangle\rangle) + \psi_{A}\frac{1}{\sqrt{2}}(|\mathbf{1}\rangle\rangle + |\mathbf{1}'\rangle\rangle)$$
$$+\psi_{S,\mathbf{hc}}|\mathbf{hc}\rangle\rangle,$$

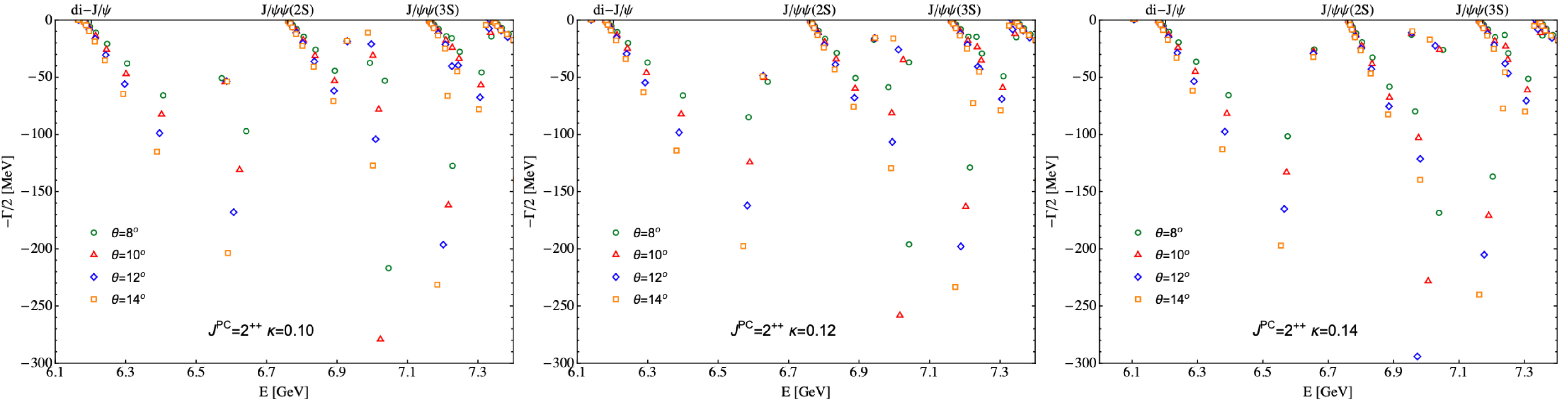


- The confinement potentials are independent of the color representation.

- For the colored V_{SR} contributions, here we take a possible attractive configuration: $|(Q_1Q_2)_{\bar{\mathbf{3}}}(\bar{Q}_3\bar{Q}_4)_{\mathbf{3}}\rangle$

$$\langle\langle\mathbf{1}|(T_1\cdot T_3)|\mathbf{1}'\rangle\rangle = \langle\langle\mathbf{1}|(T_2\cdot T_4)|\mathbf{1}'\rangle\rangle = \langle\langle\mathbf{1}|(T_1\cdot T_4)|\mathbf{1}'\rangle\rangle$$
$$= \langle\langle\mathbf{1}|(T_2\cdot T_3)|\mathbf{1}'\rangle\rangle = -\frac{4}{3}\langle\langle\mathbf{1}|\mathbf{1}'\rangle\rangle = 0,$$
$$\langle\langle\mathbf{1}|(T_1\cdot T_2)|\mathbf{1}'\rangle\rangle$$
$$= \langle\langle\mathbf{1}|T_1\cdot(T_2+T_4) - T_1\cdot T_4|\mathbf{1}'\rangle\rangle = -\langle\langle\mathbf{1}|(T_1\cdot T_4)|\mathbf{1}'\rangle\rangle = 0,$$

Novel string-like potential: 2^{++} $T_{cc\bar{c}\bar{c}}$

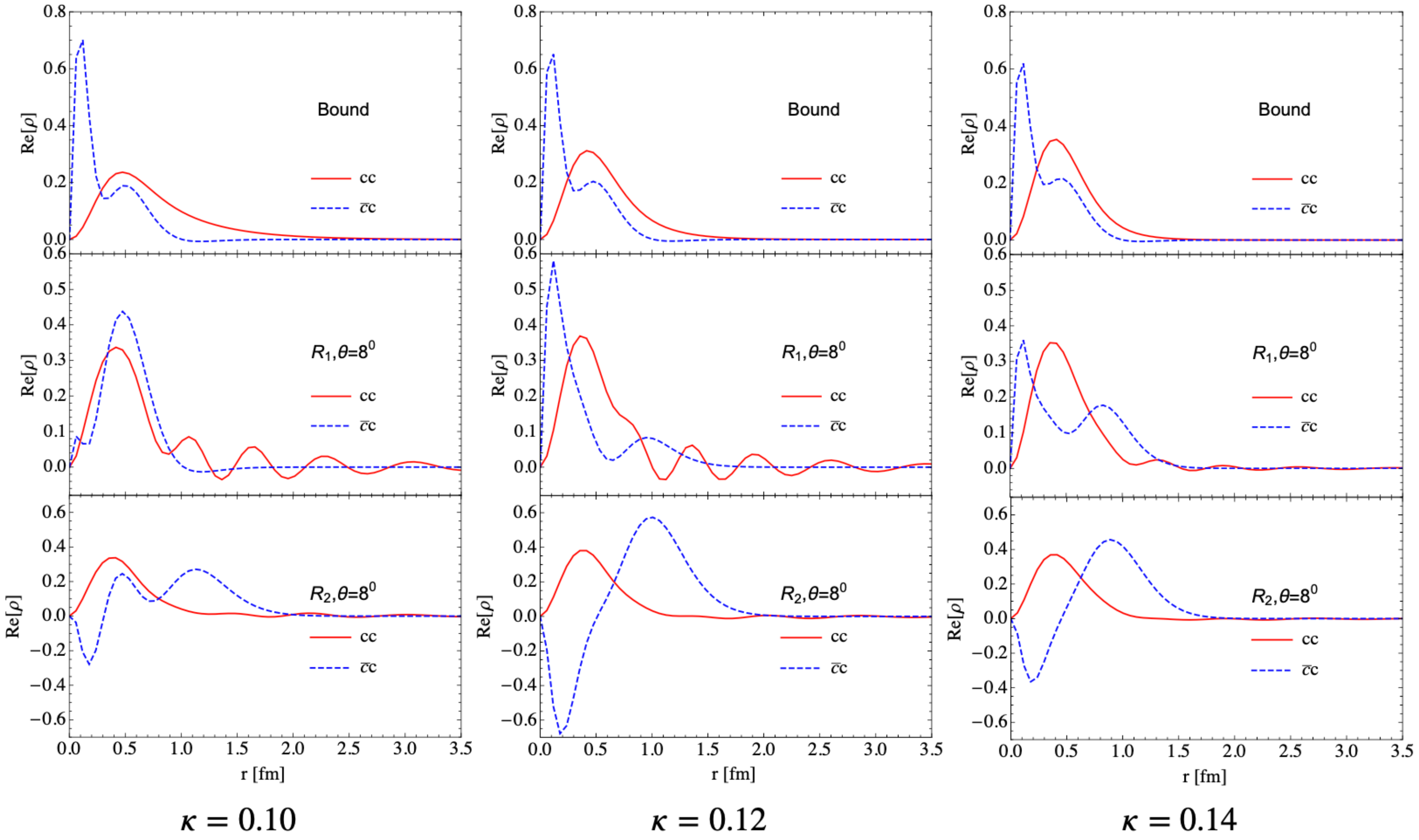


- 2^{++}: $\mathrm{E}-\Gamma/2$

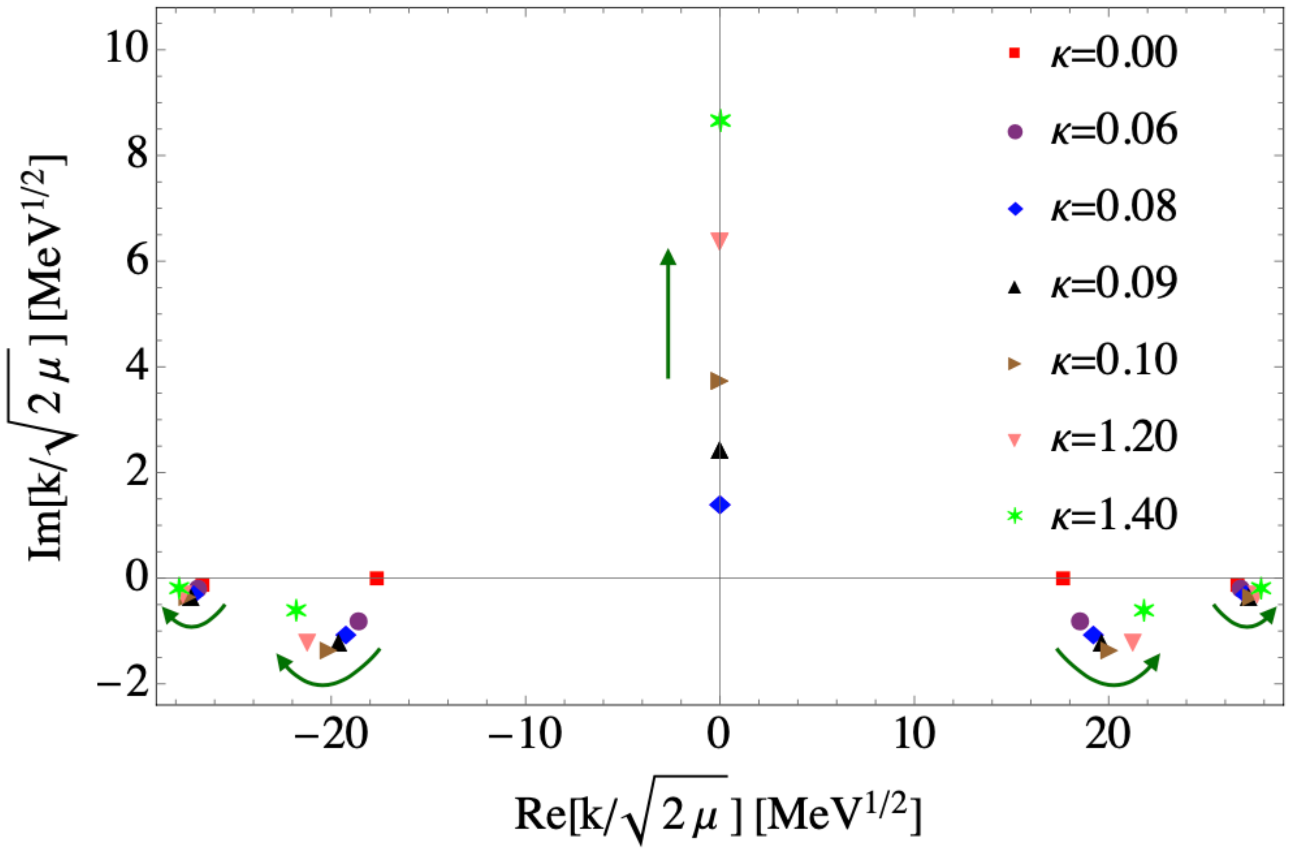
		Bound		1st Resonance	2nd Resonance
$\kappa=0.10$	$E-\frac{\Gamma}{2}I$	6166.2	$P_{MM}=88.3\%$	$6582.2-54.2I$	$6928.8-18.7I$
	ΔE	-14	$P_{cc-\bar{c}\bar{c}}=11.7\%$	402	748
$\kappa=0.12$	$E-\frac{\Gamma}{2}I$	6139.8	$P_{MM}=80.6\%$	$6630.6-50.6I$	$6943.7-16.3I$
	ΔE	-41	$P_{cc-\bar{c}\bar{c}}=19.4\%$	450	763
$\kappa=0.14$	$E-\frac{\Gamma}{2}I$	6106.1	$P_{MM}=75.2\%$	$6655.5-27.5I$	$6955.0-11.8I$
	ΔE	-74	$P_{cc-\bar{c}\bar{c}}=24.8\%$	475	774

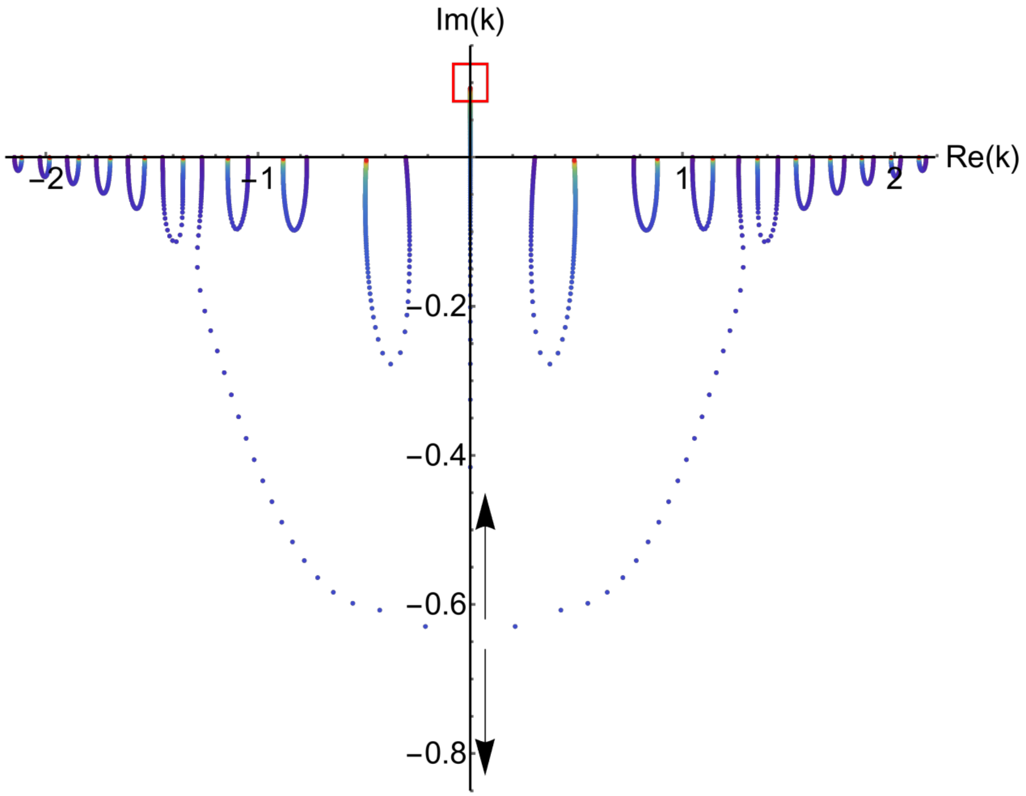
Probability Density

$$\rho(r) = \int \tilde{\psi}(r_{12}, r_{34}, r)\psi(r_{12}, r_{34}, r) d\vec{r}_{12}\vec{r}_{34}\vec{r}$$



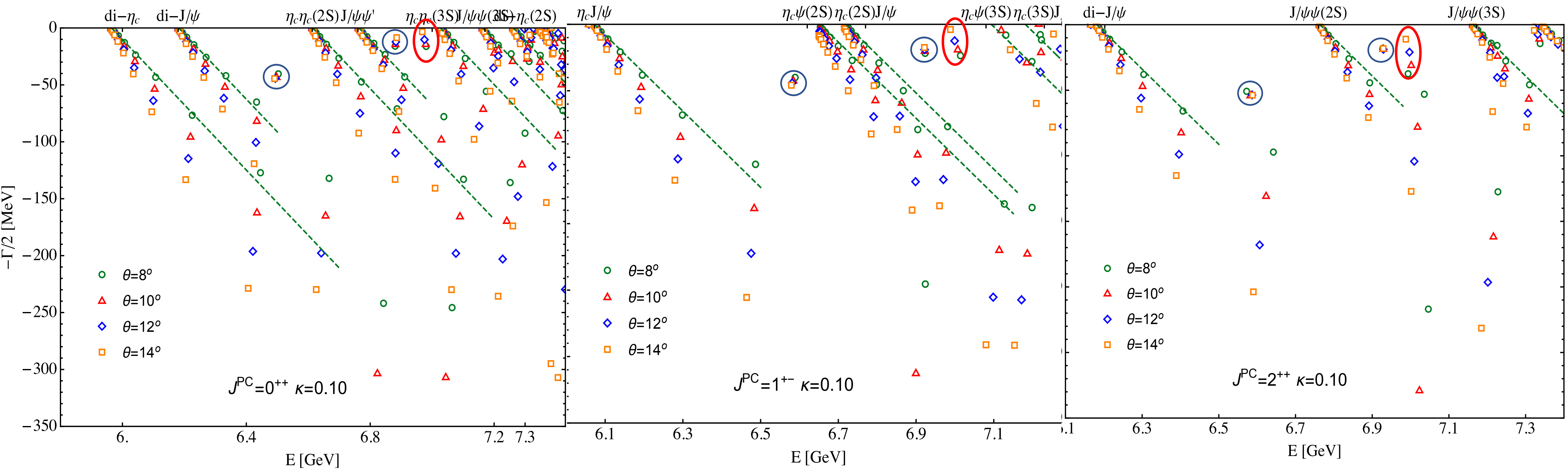
Pole trajectories





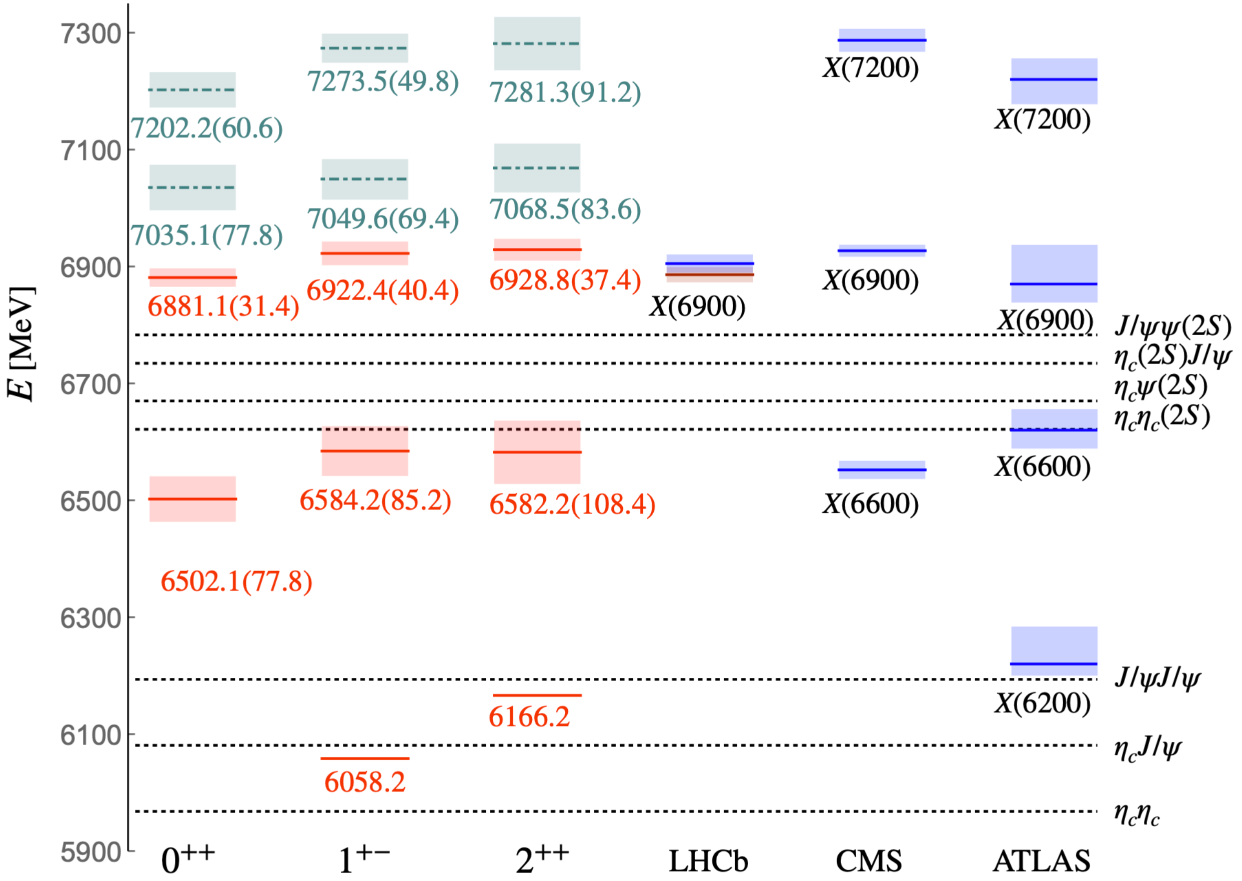
C. Hanhart et,al, Phys. Rev. D 106 (2022), 114003

Novel string-like potential: $T_{cc\bar{c}\bar{c}}$



- ***1st pole : a candidate for X(6600)***
- ***2nd pole: a candidate for X(6900).***
- ***A third pole at around 7.0 GeV?– convergency not so good.***
  For instance: 0^{++}: E=6980.4 MeV , Γ =29.0 MeV

Summary



$\kappa = 0.10$

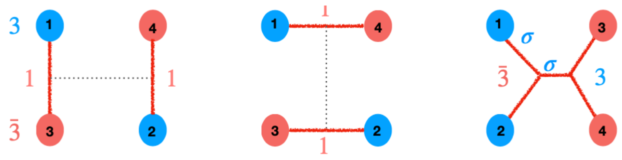
Phys. Rev. D 106, 096005 arXiv: 2307.04310

- Conventional confinement: $V = -\frac{3}{4}\sigma \sum_{i<j}(T_i \cdot T_j) r_{ij}$

✓ ***1st pole -X(6900) &2nd pole-X(7200).***

✓ ***Absence of the lower $X(6600)$ state.***

- Novel string confinement: N-body force



✓ Mixings of states induce a strong attraction.

✓ ***A bound state appears.***

✓ Two candidates for ***$X(6600)$ and X(6900).***

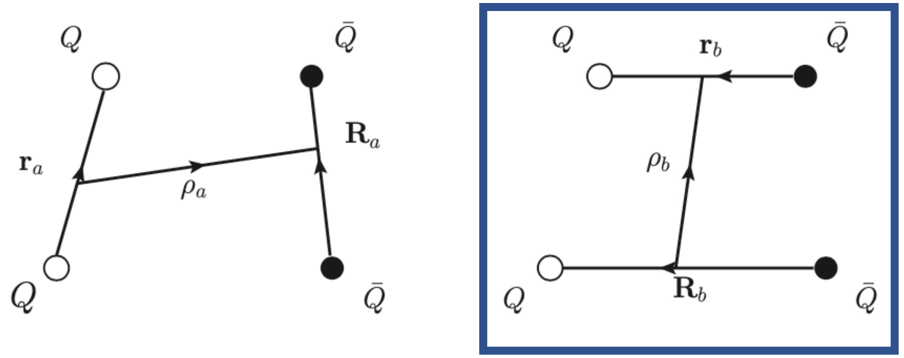
✓ ***X(7200) or X(7000)?***

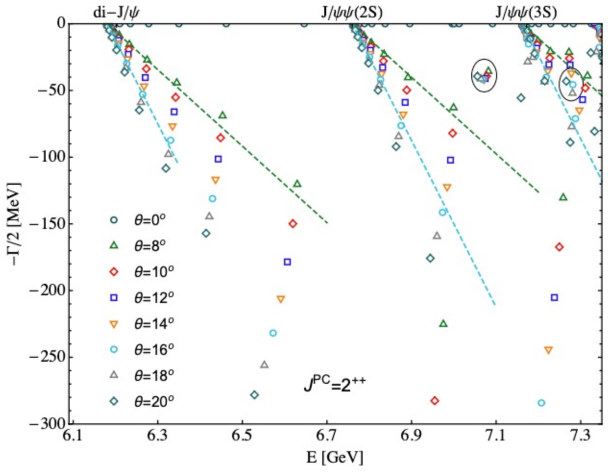
Thank you for your attention!

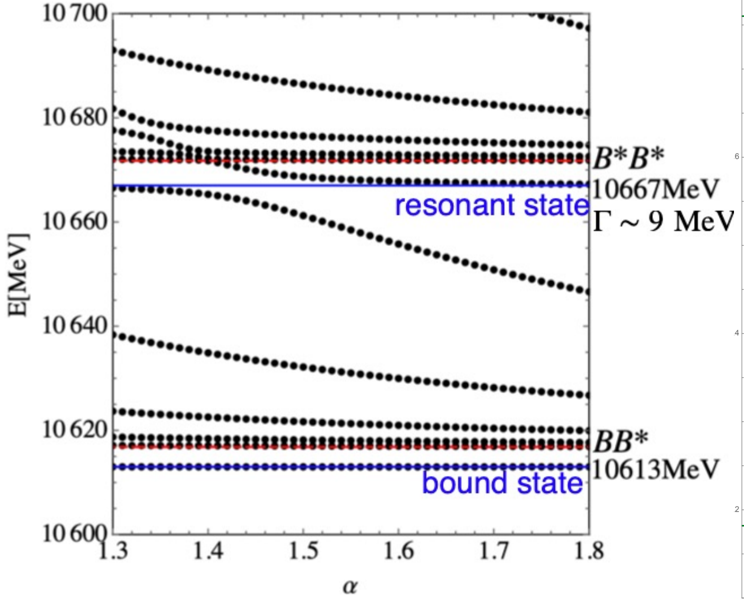
Backup slide

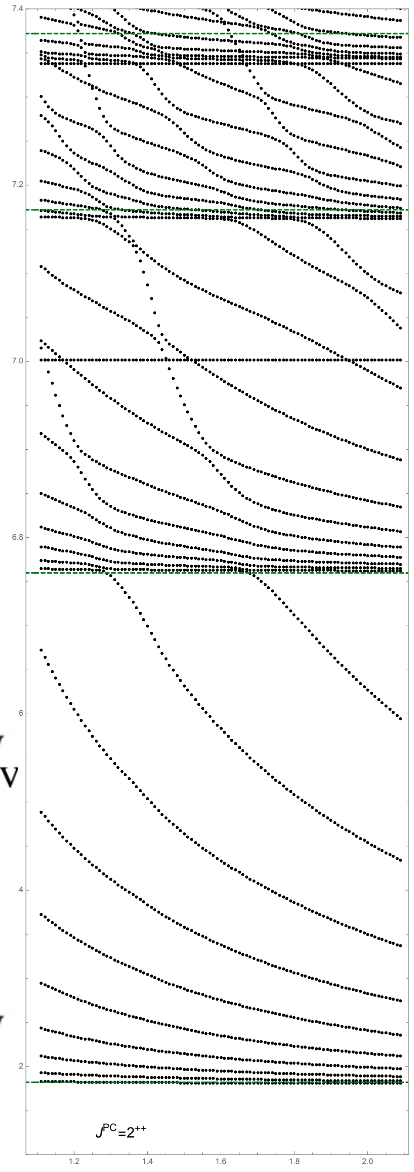
Real scaling method

• Meson-meson scattering channel









✓ scale ρ_b by multiplying α:
$\rho_b \rightarrow \alpha\rho_b$

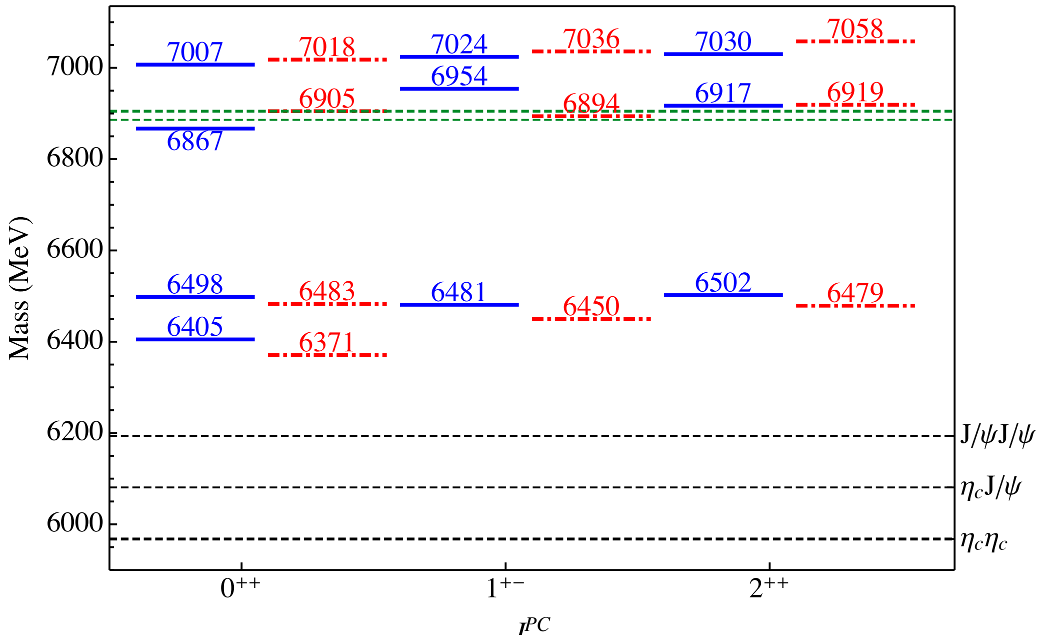
✓ stabilization plot

• Real scaling method: ***quite narrow resonances.***
✓ typical width ~ 10 MeV

Phys. Lett. B 814 (2021), 136095

S-wave $T_{cc\bar{c}\bar{c}}$

- The S-wave $T_{cc\bar{c}\bar{c}}$ state: $L_{12} = L_{34} = L_r = 0$.
- The coupling with non S-wave orbital excitations is neglected.



J^{PC}	Decay Modes
0^{++}	$\eta_c\eta_c$, $\boldsymbol{J/\psi J/\psi}$, $\chi_{c1}\eta_c$(P-wave), $J/\psi h_c(1P)$(P-wave), $J/\psi\psi(2S)$, $\chi_{c0}\chi_{c0}$
1^{+-}	$\eta_c J/\psi$, $h_c\eta_c$(P-wave), $J/\psi\chi_{c1}$(P-wave), $\eta_c\psi'$, $h_c\chi_{c0}$
2^{++}	$\boldsymbol{J/\psi J/\psi}$, $\eta_c\chi_{c1}$(P-wave), $\eta_c\chi_{c2}$(P-wave), $J/\psi h_c$(P-wave), $J/\psi\psi(2S)$, $\chi_{c0}\chi_{c2}$

TABLE IV. The mass spectrum (MeV), the percentage of different color configurations, and the root mean square radius (fm) of the *S*-wave tetraquark states.

0^{++}	Mass	$\bar{3}_c \otimes 3_c$	$6_c \otimes \bar{6}_c$	$1_c \otimes 1_c$	$8_c \otimes 8_c$	r_{12}/r_{34}	r	r_{13}/r_{24}	r'
1S	6405	31.9%	68.1%	96.9%	3.13%	0.52	0.31	0.48	0.37
	6498	67.7%	32.3%	5.7%	94.3%	0.51	0.36	0.51	0.36
2S	6867	10.6%	89.4%	80.6%	19.4%	0.65	0.35	0.58	0.46
	7007	89.7%	10.3%	26.0%	74.0%	0.49	0.47	0.59	0.35
1^{+-}	Mass	$\bar{3}_c \otimes 3_c$	$6_c \otimes \bar{6}_c$	$1_c \otimes 1_c$	$8_c \otimes 8_c$	r_{12}/r_{34}	r	r_{13}/r_{24}	r'
1S	6481	100%	0%	33.3%	66.7%	0.48	0.37	0.51	0.34
2S	6954	100%	0%	33.3%	66.7%	0.61	0.44	0.61	0.43
3S	7024	100%	0%	33.3%	66.7%	0.66	0.42	0.62	0.46
2^{++}	Mass	$\bar{3}_c \otimes 3_c$	$6_c \otimes \bar{6}_c$	$1_c \otimes 1_c$	$8_c \otimes 8_c$	r_{12}/r_{34}	r	r_{13}/r_{24}	r'
1S	6502	100%	0%	33.3%	66.7%	0.49	0.39	0.53	0.35
2S	6917	100%	0%	33.3%	66.7%	0.55	0.60	0.72	0.39
3S	7030	100%	0%	33.3%	66.7%	0.64	0.46	0.64	0.45

- 0^{++} state: an admixture of $\bar{3}_c - 3_c$ and $6_c - \bar{6}_c$ configurations.
- 0^{++} ground state: $6_c - \bar{6}_c$ component is lighter and dominates.
- No bound states exist.
- Wide S-wave $T_{cc\bar{c}\bar{c}}$: *di-* J/ψ, *di-* η_c, $\eta_c J/\psi$.
- $X(6900)$: wide S-wave states $J^{PC} = 0^{++}$ or 2^{++}.

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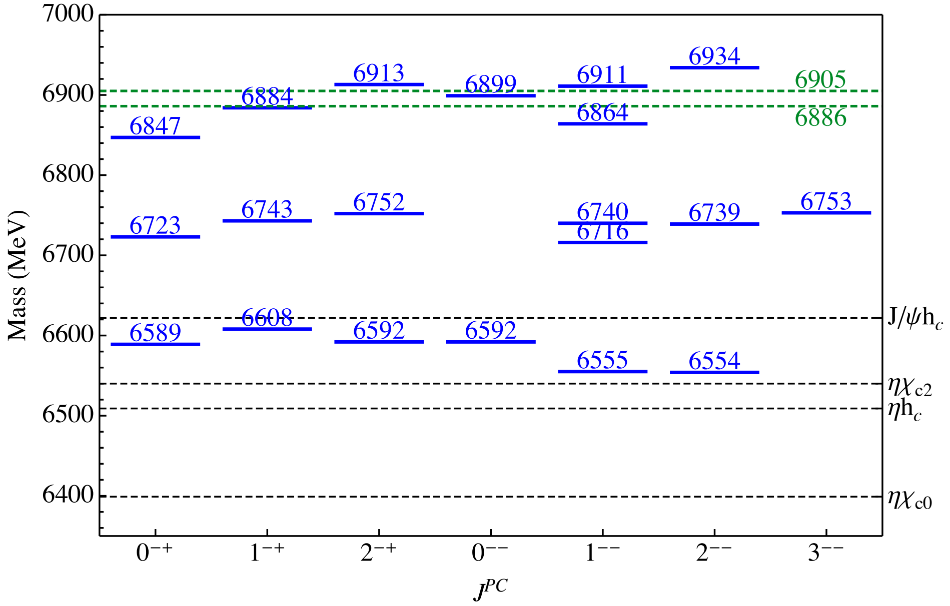
P-wave $T_{cc\bar{c}\bar{c}}$



J^{PC}	Decay modes
0^{-+}	$\boldsymbol{J/\psi J/\psi}$(P-wave), $\eta_c\chi_{c0}$, $J/\psi h_c$, $J/\psi\psi(2S)$(P-wave)
1^{-+}	$\boldsymbol{J/\psi J/\psi}$(P-wave) $J/\psi h_c$, $J/\psi\psi(2S)$(P-wave)
2^{-+}	$\boldsymbol{J/\psi J/\psi}$(P-wave), $\eta_c\chi_{c2}$, $J/\psi h_c$, $J/\psi\psi(2S)$(P-wave)
0^{--}	$\eta_c J/\psi$(P-wave), $J/\psi\chi_{c1}$, $\eta_c\psi(2S)$(P-wave)
1^{--}	$\eta_c J/\psi$(P-wave), $\eta_c h_c$, $J/\psi\chi_{c0}$, $J/\psi\chi_{c1}$, $J/\psi\chi_{c2}$, $\eta_c\psi'$(P-wave)
2^{--}	$\eta_c J/\psi$(P-wave), $J/\psi\chi_{c1}$, $J/\psi\chi_{c2}$, $\eta_c\psi'$(P-wave), $h_c\chi_{c0}$(P-wave)
3^{--}	$J/\psi\chi_{c2}$

- New narrow $T_{cc\bar{c}\bar{c}}$ tetraquark: especially $J^{PC}=0^{--}$ or 1^{-+}.
- P-wave decay channels dominated: small decay widths.
- $X(6900)$: Narrow P-wave state with $J^{PC}=1^{-+}$ or 2^{-+}.

Discussion

- For a confined charmonium $\bar{c}c$, the H in harmonic oscillator potential is

$$H = \sum_i \frac{p_i^2}{2m_i} + kr_{12}^2 = \frac{p^2}{2u_m} + \frac{u_m\omega^2}{2} r_{12}^2, \quad \text{with} \quad u_m = \frac{m_Q}{2}, \quad \omega_m = \sqrt{\frac{4k}{m_Q}},$$

- $\bar{c}c$: $m_P - m_S = \hbar\sqrt{\frac{4k}{m_Q}} \approx 400 \sim 500$ MeV.

- For a confined $T_{cc\bar{c}\bar{c}}$,

$$H = \sum_i \frac{p_i^2}{2m_i} + a_1 k(r_{12}^2 + r_{34}^2) + a_2 k'(r_{13}^2 + r_{24}^2 + r_{14}^2 + r_{23}^2)$$
$$= \frac{p_a^2}{2u_a} + \frac{p_b^2}{2u_b} + \frac{p_{ab}^2}{2u_{ab}} + \frac{u_a\omega_a^2}{2} r_{12}^2 + \frac{u_b\omega_b^2}{2} r_{34}^2 + \frac{u_{ab}\omega_{ab}^2}{2} r^2,$$

	a_1	a_2	ω_ρ	ω_λ
$\bar{3}_c - 3_c$	$\frac{1}{2}$	$\frac{1}{4}$	$\sqrt{\frac{2k+k'}{2u_a}}$	$\sqrt{\frac{-2k+5k'}{4u_a}}$
$6_c - \bar{6}_c$	$-\frac{1}{4}$	$\frac{5}{8}$	$\sqrt{\frac{2k'}{u_{ab}}}$	$\sqrt{\frac{-2k+5k'}{4u_b}}$

- If $k = k'$, $6_\rho < 3_\lambda < 3_\rho < 6_\lambda$.

- $T_{cc\bar{c}\bar{c}}$: $m_P - m_S = \hbar\omega_\lambda = \hbar\sqrt{\frac{3k}{2m_Q}} \approx 245 \sim 300$ MeV.

Dynamical calculation:

- P-wave $T_{cc\bar{c}\bar{c}}$: $6_\rho < 3_\lambda < 6_\lambda < 3_\rho$

- Small mass gap between S-wave and P-wave $T_{cc\bar{c}\bar{c}}$.

Discussion

J^{PC}		Mass	$\lvert\lambda_1^{+/-}\rangle$	$\lvert\lambda_2^{-}\rangle$	$\lvert\lambda_3^{-}\rangle$	$\lvert\rho_1^{+/-}\rangle$	$\lvert\rho_2^{+/-}\rangle$	r_{12}/r_{34}	r	r_{13}/r_{24}	r'
0^{-+}	$\begin{pmatrix} 6746-20 & -20 & -34 \\ -20 & 6599+2 & -42 \\ -34 & -42 & 6894-62 \end{pmatrix}$	6589	3.5%			92.8%	3.7%	0.62	0.33	0.60	0.50
		6723	90.4%			5.2%	4.4%	0.52	0.43	0.66	0.37
		6847	6.0%			2.1%	91.9%	0.57	0.38	0.61	0.47
0^{--}	$\begin{pmatrix} 6561+31 & -11 \\ -11 & 6913-14 \end{pmatrix}$	6592				99.9%	0.1%	0.61	0.32	0.59	0.49
		6899				0.1%	99.9%	0.58	0.38	0.61	0.48
1^{-+}	$\begin{pmatrix} 6746-3 & -4 & -6 \\ -4 & 6599+9 & 8 \\ -6 & 8 & 6894-10 \end{pmatrix}$	6608	0.1%			99.8%	0.1%	0.63	0.33	0.60	0.50
		6743	99.7%			0.1%	0.2%	0.51	0.43	0.66	0.36
		6884	0.2%			0.1%	99.7%	0.57	0.37	0.60	0.47
1^{--}	$\begin{pmatrix} 6740 & -2 & 0 & -10 & 9 \\ -2 & 6741-23 & 7 & -19 & 26 \\ 0 & 7 & 6885 & -2 & -25 \\ -10 & -19 & -2 & 6561-1 & 21 \\ 9 & 26 & -25 & 21 & 6913-28 \end{pmatrix}$	6555	0.3%	1.6%	$\sim 0\%$	97.5%	0.6%	0.61	0.32	0.59	0.49
		6716	0.6%	94.8%	0.4%	2.0%	2.1%	0.52	0.42	0.65	0.37
		6740	98.8%	0.9%	$\sim 0\%$	0.2%	0.1%	0.51	0.43	0.65	0.36
		6864	0.2%	2.2%	55.7%	0.1%	41.9%	0.62	0.35	0.64	0.47
		6911	0.1%	0.5%	43.9%	0.2%	55.3%	0.61	0.36	0.63	0.47
2^{-+}	$\begin{pmatrix} 6746+6 & 7 & 10 \\ 7 & 6599-6 & 13 \\ 10 & 13 & 6894+18 \end{pmatrix}$	6592	0.2%			99.7%	0.1%	0.63	0.33	0.60	0.50
		6752	99.4%			0.2%	0.4%	0.52	0.43	0.66	0.36
		6913	0.4%			0.2%	99.4%	0.57	0.38	0.60	0.47
2^{--}	$\begin{pmatrix} 6741-2 & 7 & -9 \\ 7 & 6561-6 & -15 \\ -9 & -15 & 6913+20 \end{pmatrix}$	6554		0.1%		99.7%	0.2%	0.61	0.32	0.59	0.49
		6739		99.6%		0.1%	0.2%	0.51	0.43	0.66	0.36
		6934		0.2%		0.2%	99.6%	0.57	0.38	0.61	0.48
3^{--}	$6741+11$	6753		100%				0.51	0.43	0.66	0.36

ATLAS

Model A: Three resonances

$$f_s(x) = \left|\sum_{i=0}^{2} \frac{z_i}{m_i^2 - x^2 - i m_i \Gamma_i(x)}\right|^2 \sqrt{1 - \frac{4m_{J/\psi}^2}{x^2}} \otimes R(\theta),$$

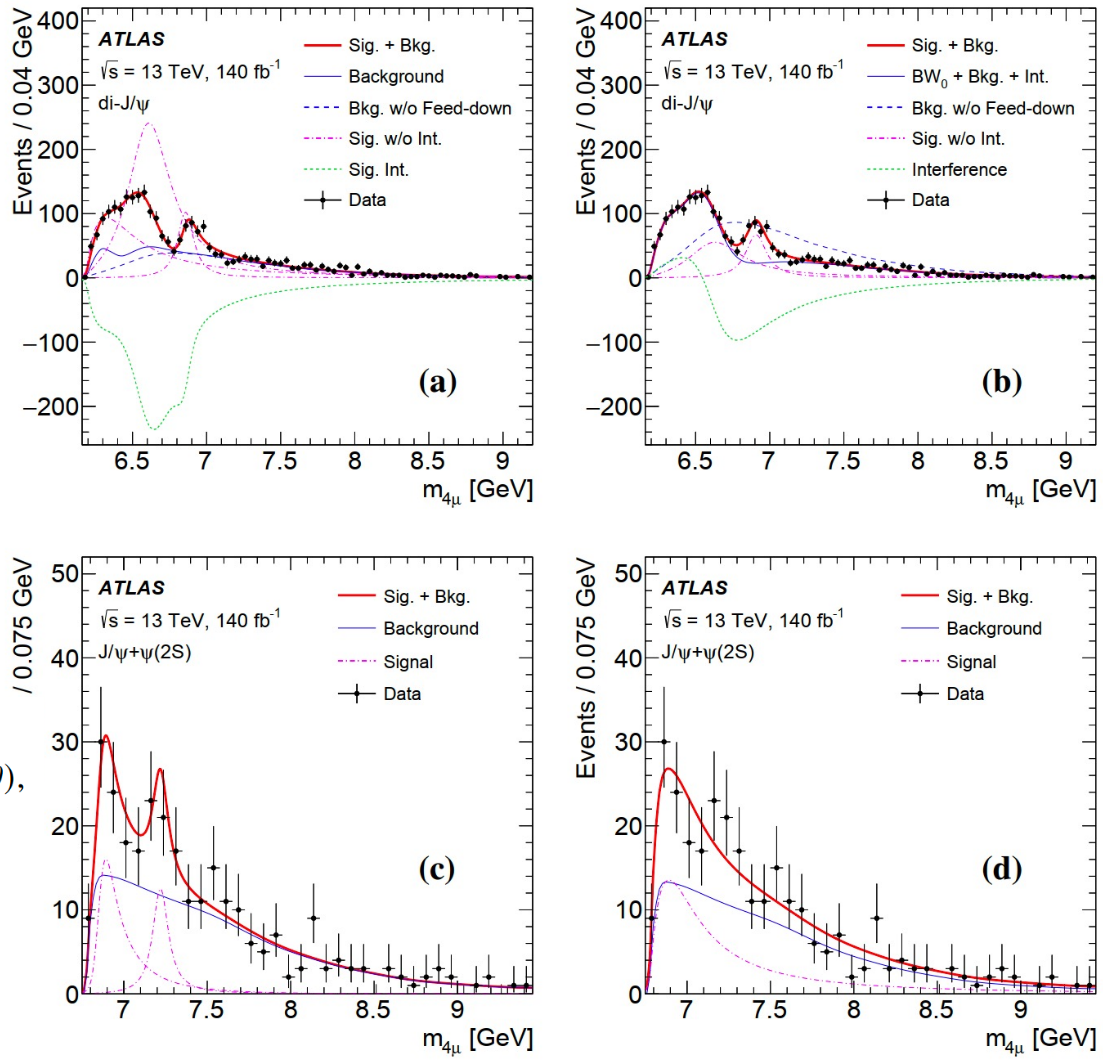
Model B: Two resonances

$$f(x) = \left(\left|\frac{z_0}{m_0^2 - x^2 - i m_0 \Gamma_0(x)} + A e^{i\phi}\right|^2 + \left|\frac{z_2}{m_2^2 - x^2 - i m_2 \Gamma_2(x)}\right|^2\right) \sqrt{1 - \frac{4m_{J/\psi}^2}{x^2}} \otimes R(\theta),$$

Model α: Two resonances: A model+standalone fourth resonance

$$f_s(x) = \left(\left|\sum_{i=0}^{2} \frac{z_i}{m_i^2 - x^2 - i m_i \Gamma_i(x)}\right|^2 + \left|\frac{z_3}{m_3^2 - x^2 - i m_3 \Gamma_3(x)}\right|^2\right) \sqrt{1 - \left(\frac{m_{J/\psi} + m_{\psi(2S)}}{x}\right)^2} \otimes R(\theta),$$

Model β: Two resonances: A single resonance in $J/\psi\psi(2S)$

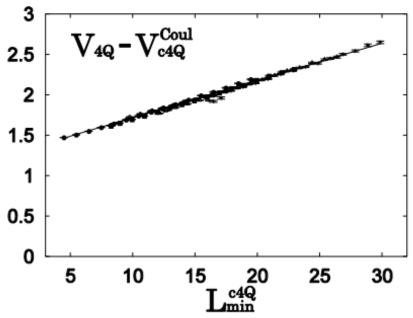


First Lattice QCD Study for Static Quark Potential in Multi-Quark System

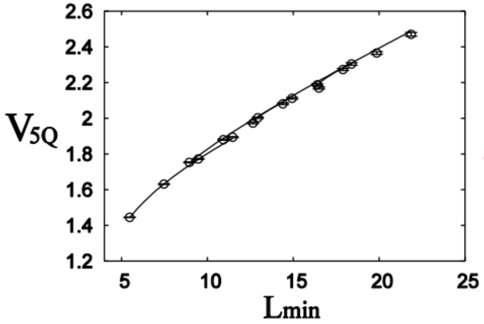
Okiharu, H.S. et al. PRL 94 (2005) 192001
Okiharu, H.S. et al. PRD72 (2005) 014505

$V_{NQ}(r)$

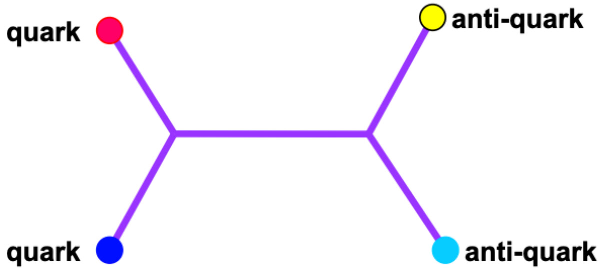
N=4,5



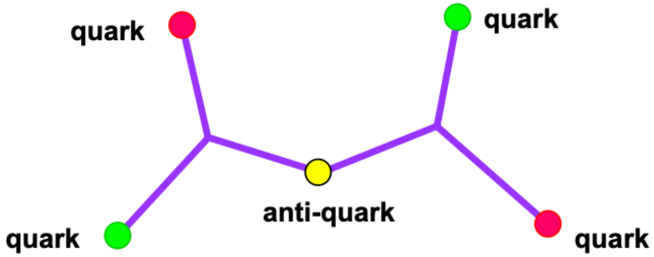
4 quark system



5 quark system



color flux tube



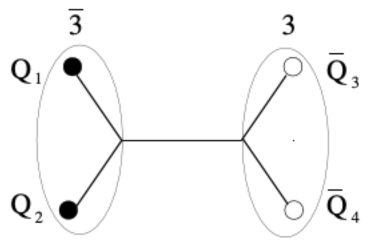
L_{min} : total length of string linking the N valence quarks

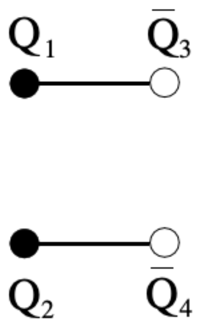
$$V_{NQ}(r) = \frac{g^2}{4\pi} \sum_{i<j}^{N} \frac{T_i^a T_j^a}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma L_{min}$$

One-Gluon-Exchange Coulomb potential **Linear potential based on string picture**

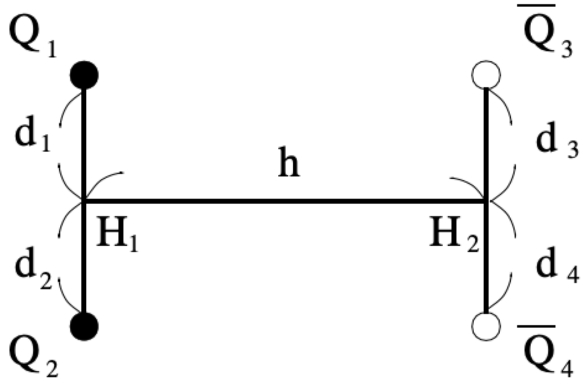
From H. Suganuma-san's talk

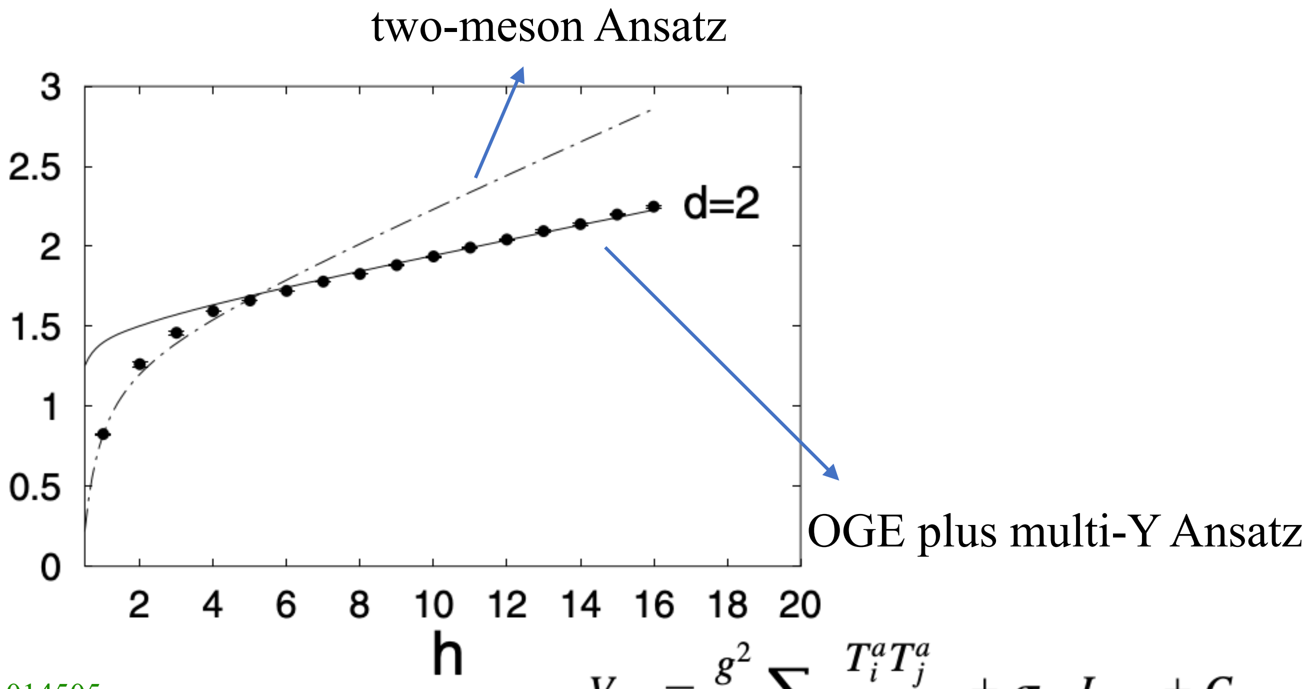
Lattice QCD





$$V_{4Q} = V_{Q\bar{Q}}(r_{13}) + V_{Q\bar{Q}}(r_{24})$$
$$= -A_{Q\bar{Q}}\left(\frac{1}{r_{13}} + \frac{1}{r_{24}}\right) + \sigma_{Q\bar{Q}}(r_{13} + r_{24}) + 2C_{Q\bar{Q}}$$





Okiharu, H.S. et al. PRD72 (2005) 014505

$$V_{4Q} = \frac{g^2}{4\pi}\sum_{i<j}\frac{T_i^a T_j^a}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{4Q}L_{\min} + C_{4Q}$$