A coupled channel analysis of the $X(3872)$ lineshape

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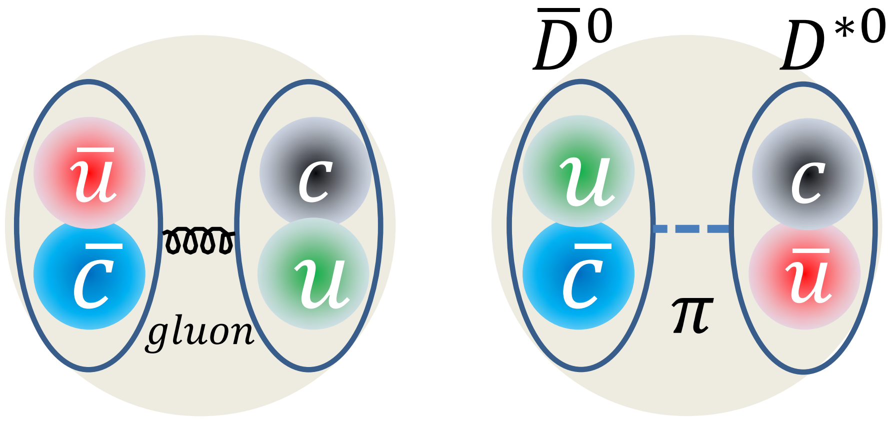
Two decades have past, the nature of the $X(3872)$ remains the subject of intense debate.



Compact tetraquark   Hadronic molecule

Important properties

- $M(X) \approx M(\bar{D}^0) + M(D^{*0})$
- $\Gamma < 1$ MeV

} Target of this work

- $J^{pc} = 1^{++}$, Isospin singlet (no charged partner has been found)
- $B(J/\psi\rho) \approx B(J/\psi\omega)$, isospin break
- In pp collisions, it is produced more copiously through "prompt" processes, rather that through the decay of B mesons

Various theories about its nature

- Conventional charmonium $\chi_{c1}(2P)$
- Hadronic molecule (deuteron)
- Compact tetraquark (diquark - anti diquark)
- Hybrid charmonium (ccbar and gluons)
- Hadro-charmonium

Global fit [Chunhua Li, Chang-Zheng Yuan, PRD 100(2019) 094003]

Parameter index	Decay mode	Branching fraction
1	$X(3872) \to \pi^+\pi^- J/\psi$	$(4.1^{+1.9}_{-1.1})\%$
2	$X(3872) \to D^{*0}\bar{D}^0 + c.c.$	$(52.4^{+25.3}_{-14.3})\%$
3	$X(3872) \to \gamma J/\psi$	$(1.1^{+0.6}_{-0.3})\%$
4	$X(3872) \to \gamma\psi(3686)$	$(2.4^{+1.3}_{-0.8})\%$
5	$X(3872) \to \pi^0\chi_{c1}$	$(3.6^{+2.2}_{-1.6})\%$
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8	$B^0 \to X(3872)K^0$	$(1.1^{+0.5}_{-0.4}) \times 10^{-4}$
	$X(3872) \to$ unknown	$(31.9^{+18.1}_{-31.5})\%$

$X(3872)$ decays

[Chunhua Li, Chang-Zheng Yuan, PRD 100(2019) 094003]

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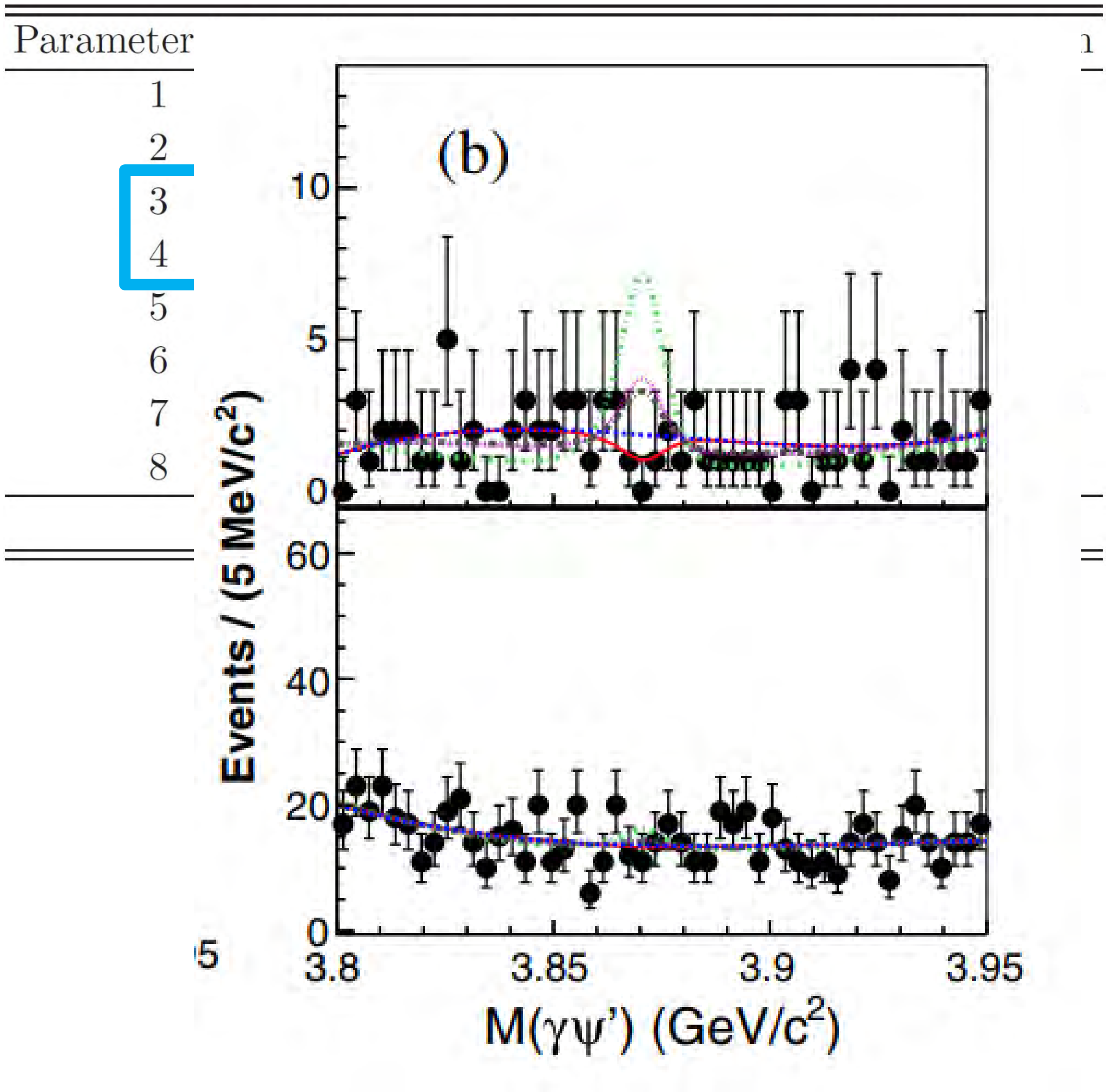
- $R = \frac{\Gamma(\gamma\psi')}{\Gamma(\gamma J/\psi)} \simeq 2.2$
- For a pure $D^0\bar{D}^{*0}$, $\gamma\psi'$ should be suppressed, $R \sim 10^{-3}$
  [E.S. Swanson, Phys. Lett. B 598 (2004) 197]
- For a pure $\chi_{c1}(2P)$, the predictions are in a wide range
- A small $\chi_{c1}(2P)$ component (5% - 12%) may explain
  [Feng-Kun Guo et al., PLB 742 (2015) 394-398]

[Chunhua Li, Chang-Zheng Yuan, PRD 100(2019) 094003]



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- $R < 0.59,\ 90\%\ \mathrm{C.L.}$
  [BESIII, PRL 124, 242001 (2020)]

- The pionic transitions $\chi_{c1}(2P) \to \pi^0\chi_{c1}$ proceed through the isospin breaking by the light quark masses. [S. Dubynskiy, PRD 77, 014013 (2008)]
- Suppressed relative to $\pi\pi\chi_{c1}$
- Disfavor the $\chi_{c1}(2P)$ interpretation

[Chunhua Li, Chang-Zheng Yuan, PRD 100(2019) 094003]

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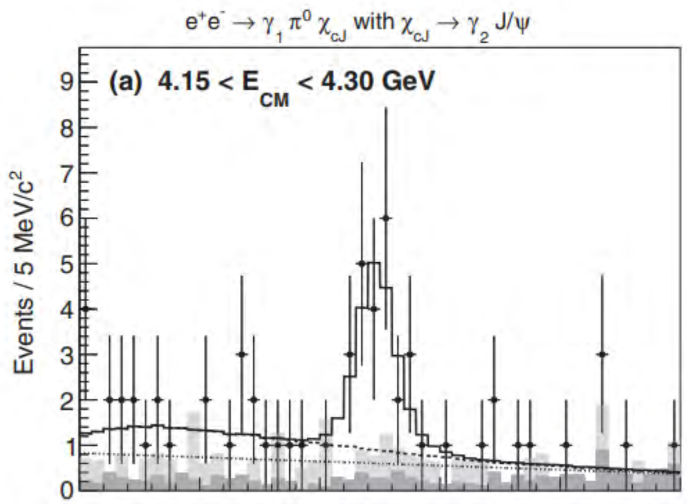

[BESIII, PRL 122, 202001 (2019)]

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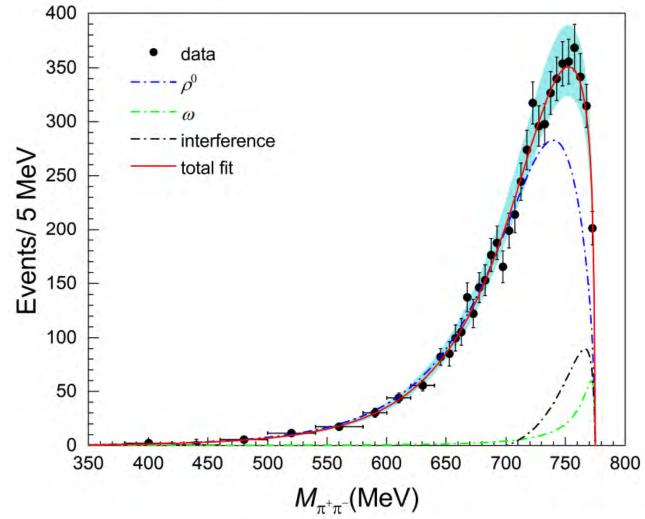
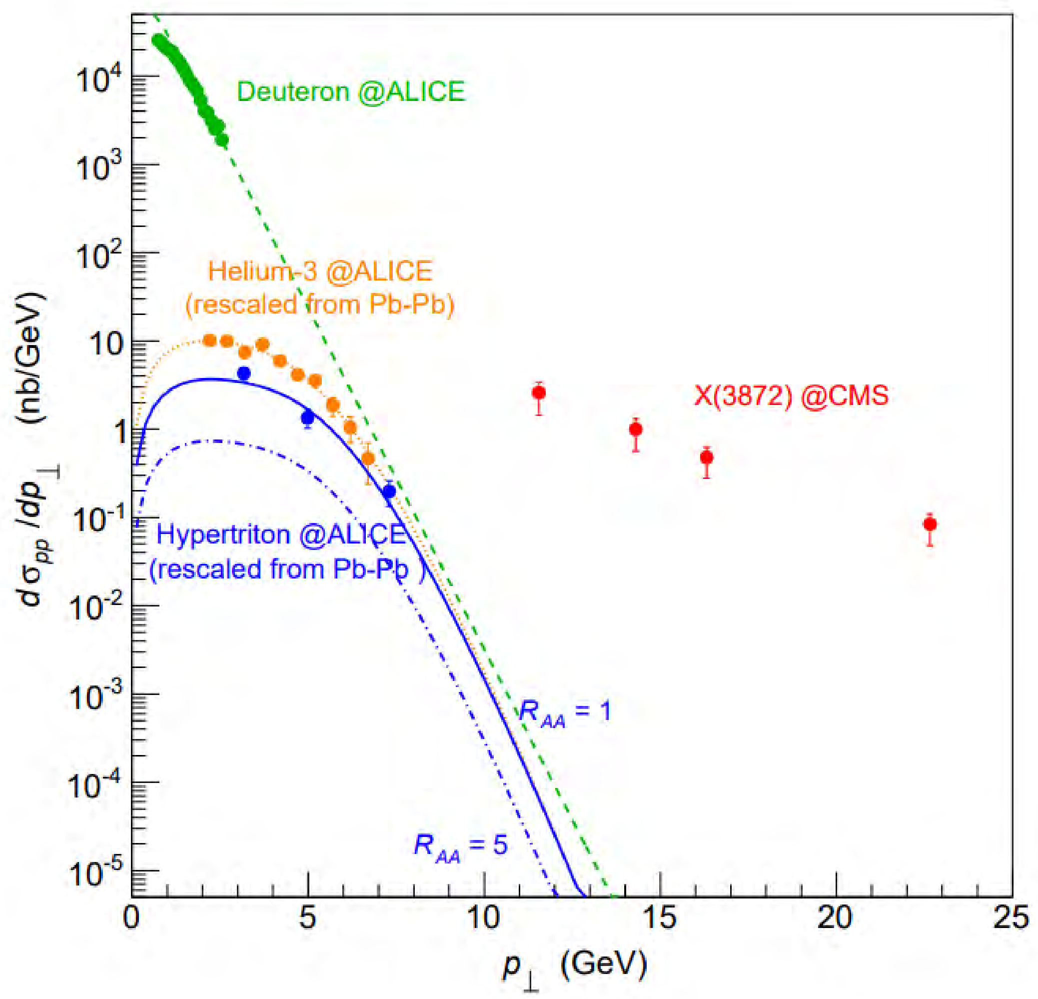
- $\frac{\Gamma(\rho J/\psi)}{\Gamma(\omega J/\psi)} \simeq 1$
- For a pure $\chi_{c1}(2P)$, $\rho J/\psi$ should be suppressed (isospin breaking)
- In molecular picture, an enhancement of isospin violation can be naturally produced [F. Guo et al., Rev.Mod.Phys. 90 (2018) 1, 015004]
- A large ω interference is observed in $\pi^+\pi^-$ mode, the ratio need to be reconsidered

[LHCb, arxiv:2204.12597 [hep-ex]]



[A. Esposito, Phys. Rev. D 92, 034028, 2015]

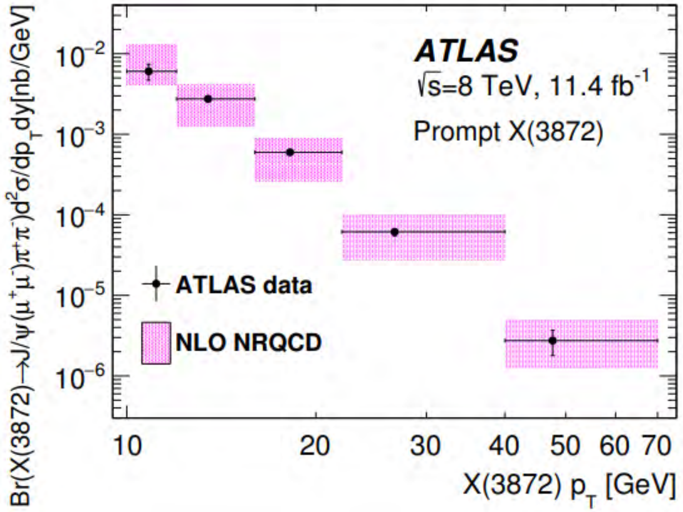


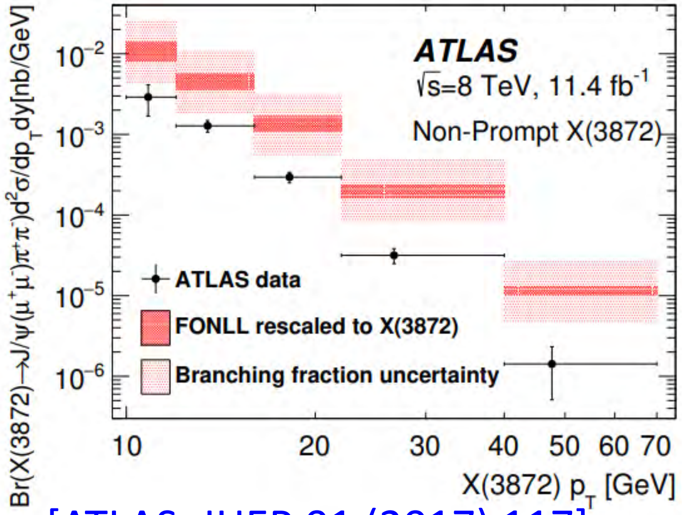
- Prompt and non-prompt productions
- There is a difference in the probability of producing X(3872) and that of producing light nuclei, in high energy collisions at large p_T

- Disfavor pure molecular interpretation

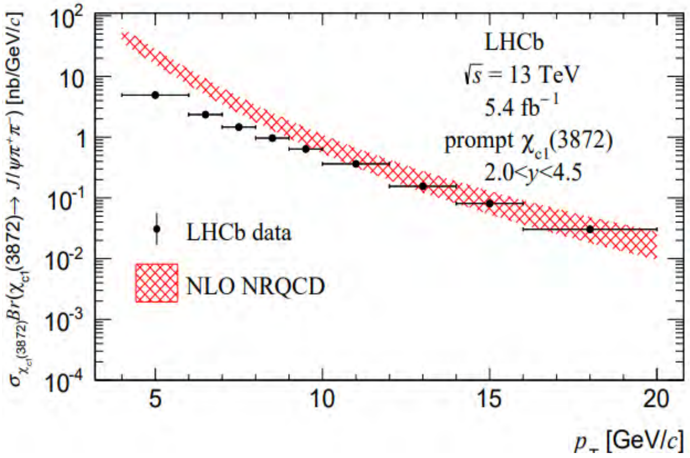
- Prompt production rate is consistent with NLO NRQCD predictions with the $X(3872)$ modelled as a mixture of $\chi_{c1}(2P)$ and a $D^0\overline{D}^{*0}$ molecular, assuming production proceeds dominantly through its χ'_{c1} component. [C. Meng, PRD 96 (2017) 7, 074014]

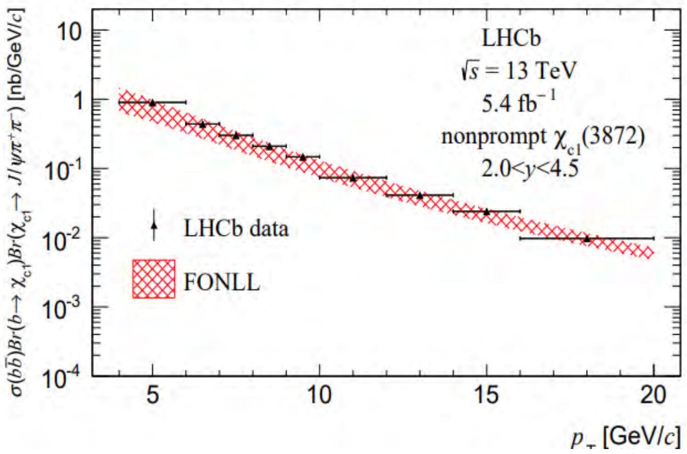
- Inconsistent in low pt region, which may be due to the problem of the fixed-order NRQCD calculation that may not be applicable for the region with small pT (pT $\sim$ 5 GeV) and large forward rapidity





[ATLAS, JHEP 01 (2017) 117]



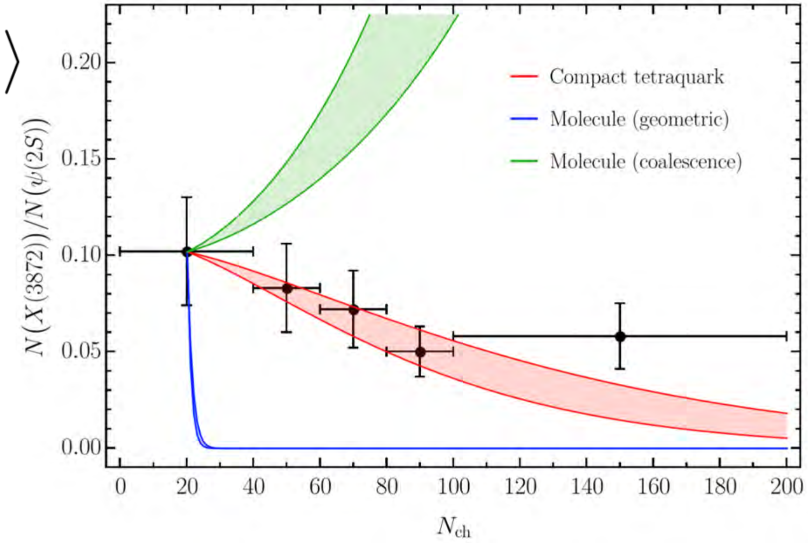


[LHCb, JHEP 01 (2022) 131]

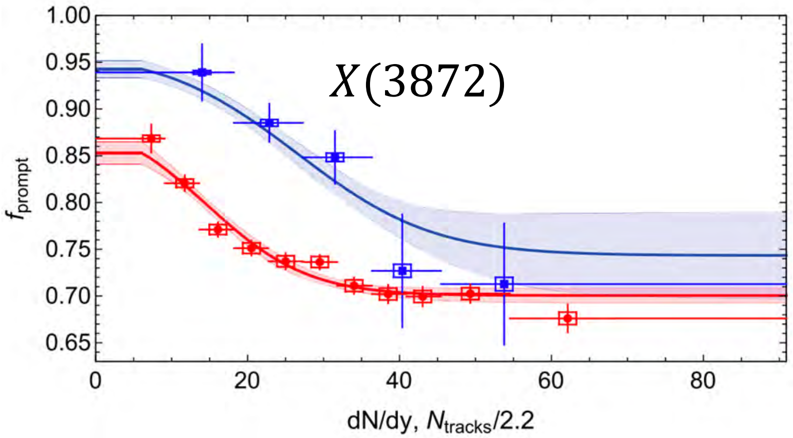
$X(3872)$ production vs multiplicity

$$\langle v\sigma \rangle \approx \pi \langle r^2 \rangle$$

- Prompt production: X(3872) could be broken by comovers
- Survival rate decreases with multiplicity
- Disfavor molecular interpretation
- Can be explained by molecular interpretation



[A. Esposito, EPJC 81 (2021) 7, 669]



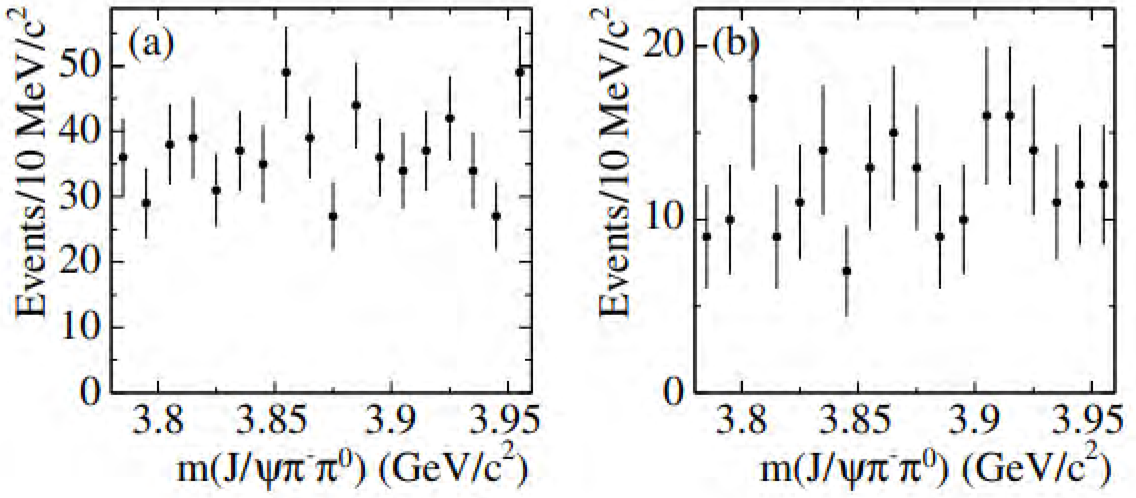
[E. Braaten, PRD 103 (2021) 7, L071901]

[Belle, PRD 84, 052004 (2011)]
[BaBar, PRD 71, 031501 (2005)]

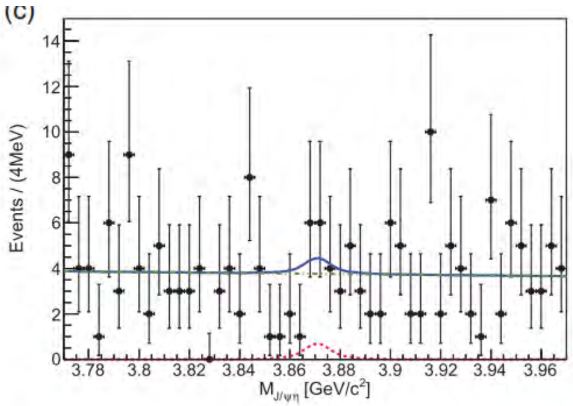
$X^{\pm} \to J/\psi\pi^{\pm}\pi^{0}$

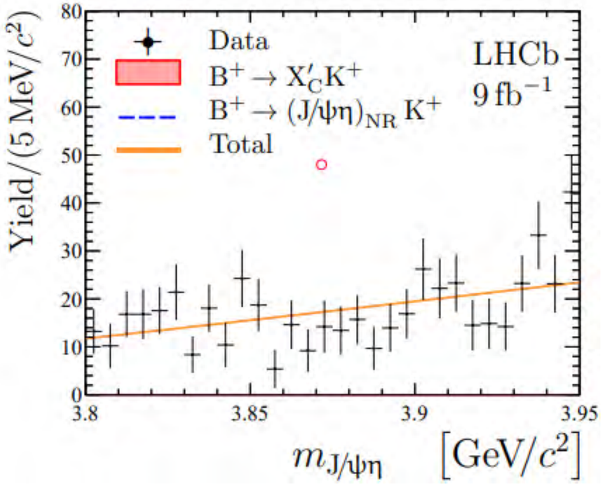
[Belle, PTEP 2014 (2014) 4, 043C01]
[LHCb, JHEP 04 (2022) 046]

$\bar{X} \to J/\psi\eta$



- Negative result for charged and C-odd partners
- Disfavor compact tetraquark interpretation





$X(3872)$: $[cu][\bar{c}\bar{u}]$
$X^{\pm}$: $[cu][\bar{c}\bar{d}]$?

[Belle, PRD 84, 052004 (2011)]
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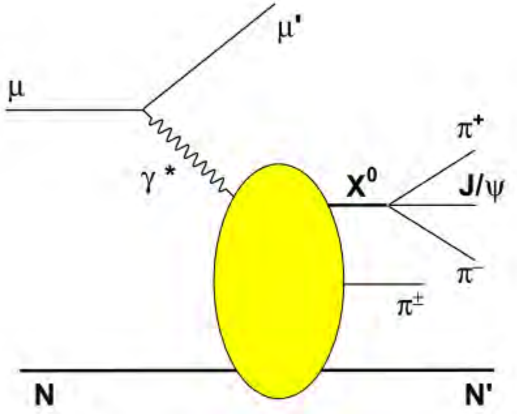
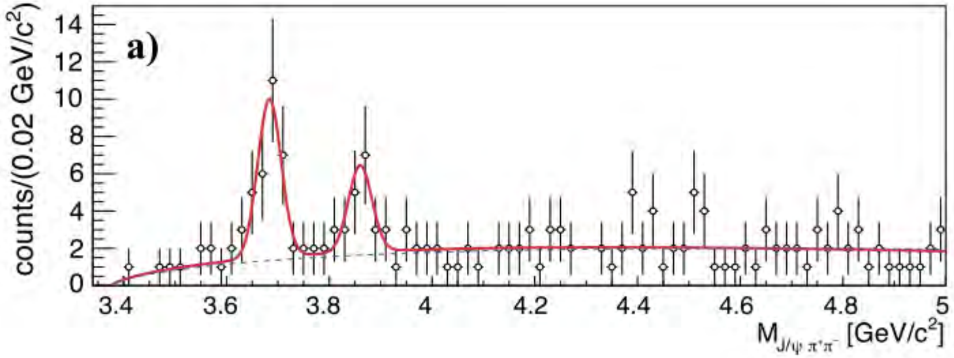
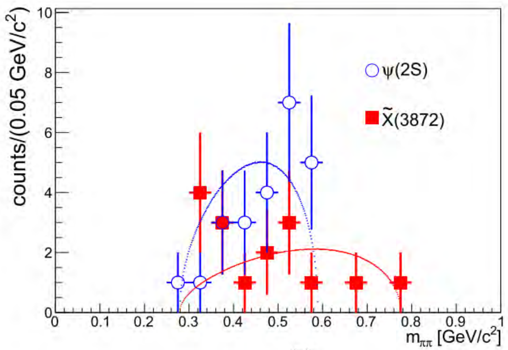
$\bar{X} \to J/\psi\eta$

[Compass, PLB 783 (2018) 334-340] $\bar{X} \to J/\psi\pi^{+}\pi^{-}$ $4.1\,\sigma$

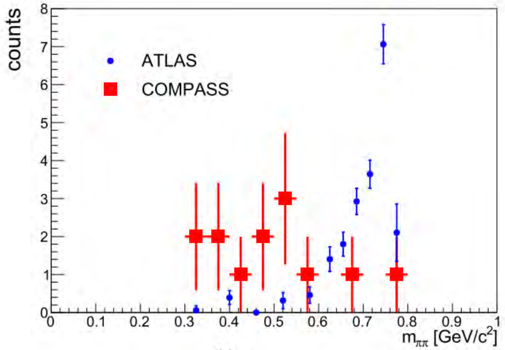
$M = 3860 \pm 10.4$ MeV
$\Gamma < 51$ MeV







(a)



(b)

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[Chunhua Li, Chang-Zheng Yuan, PRD 100(2019) 094003]

- Two important decay modes:
  - $\pi^+\pi^-J/\psi$: Pure charged daughter particles, higher selection efficiency, lower background, narrower peak.
  - $D^{*0}\bar{D}^0 + c.c.$: Major decay mode, the opening threshold will strongly distort the lineshape.

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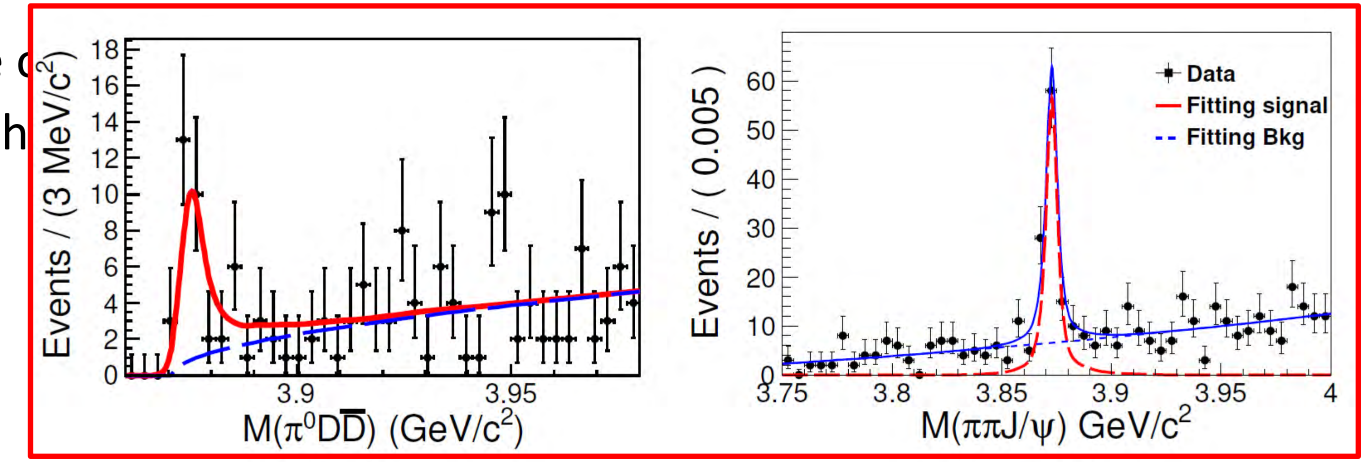


[BES III, PRL 122, 232002(2019)], [BES III, PRL 124, 242001(2020)]

We have well established samples for both modes, we can perform a simultaneous fit!

Construct the lineshape in a model independent way

Correlate the number of signal events as a function of lineshape parameters

Model the detector effect based on the MC simulation and calibrate it by control samples

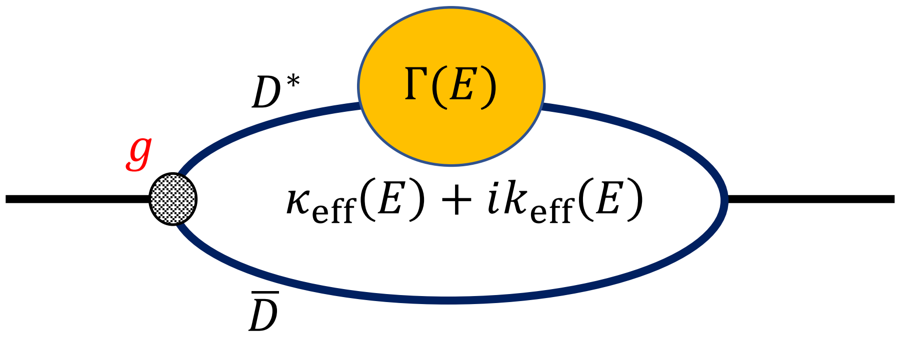
Unbinned maximum likelihood fit simultaneously to the $D^0\bar{D}^0\pi^0$ and $\pi^+\pi^-J/\psi$ samples

Investigate the result

$$\frac{d\mathrm{Br}(D^0\bar{D}^0\pi^0)}{dE} = \boldsymbol{B}\frac{1}{2\pi} \times \frac{g * k_{\mathrm{eff}}(E)}{|D(E)|^2} \times \mathrm{Br}(D^{*0} \to D^0\pi^0)$$

$$\frac{d\mathrm{Br}(\pi^+\pi^- J/\psi)}{dE} = \boldsymbol{B}\frac{1}{2\pi} \times \frac{\Gamma_{\pi^+\pi^- J/\psi}}{|D(E)|^2}$$

[C. Hanhart, PRD 81, 094028]



Composite particle with one unstable constituent

$$D(E) = E - E_X + \frac{1}{2} g * \left(\kappa_{\mathrm{eff}}(E) + ik_{\mathrm{eff}}(E) + \kappa^c_{\mathrm{eff}}(E) + ik^c_{\mathrm{eff}}(E)\right) + \frac{i}{2}\Gamma_0$$

$$k_{\mathrm{eff}}(E) = \sqrt{\mu_p}\sqrt{\sqrt{(E-E_R)^2 + \Gamma^2/4} + E - E_R}$$

$$\kappa_{\mathrm{eff}}(E) = -\sqrt{\mu_p}\sqrt{\sqrt{(E-E_R)^2 + \Gamma^2/4} - E + E_R}$$

$$+\sqrt{\mu_p}\sqrt{\sqrt{(E_X-E_R)^2 + \Gamma_X^2/4} - E_X + E_R}$$

$$\Gamma_0 = \Gamma_{\pi^+\pi^- J/\psi} + \Gamma_{known} + \Gamma_{unknown}$$

$$E_X = M_X - (m_{D^0} + m_{\bar{D}^0} + m_{\pi^0})$$

- Due to the limited statistics, $\boldsymbol{\Gamma_{unknown}/\Gamma_{\pi^+\pi^- J/\psi}}$ is fixed [Chunhua Li, Chang-Zheng Yuan, PRD 100(2019) 094003]
- It brings us the largest uncertainty

[C. Hanhart, PRD 81, 094028]

$$\frac{d\mathrm{Br}(D^0\bar{D}^0\pi^0)}{dE} = \boldsymbol{B}\frac{1}{2\pi}\times\frac{g * k_{\mathrm{eff}}(E)}{|D(E)|^2}\times \mathrm{Br}(D^{*0}\to D^0\pi^0)$$

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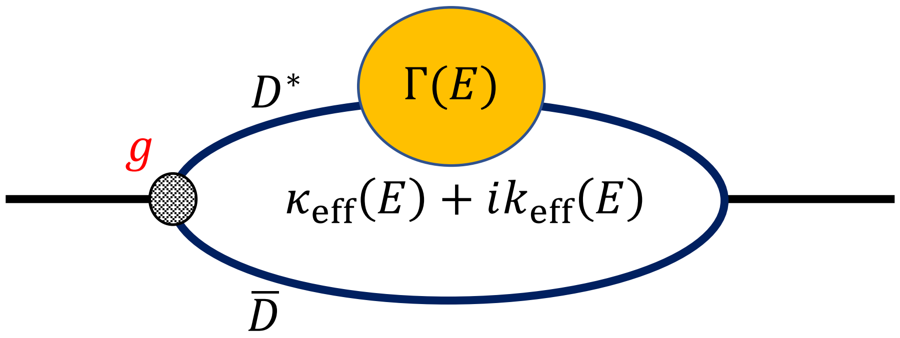

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$$\sqrt{\sqrt{(E-E_R)^2+\Gamma^2/4} - E + E_R} - i\sqrt{\sqrt{(E-E_R)^2+\Gamma^2/4} + E - E_R} = \sqrt{-2(E-E_R+i\Gamma/2)}$$

$$\Gamma_0 = \Gamma_{\pi^+\pi^- J/\psi} + \Gamma_{known} + \Gamma_{unknown}$$

$$E_X = M_X - (m_{D^0} + m_{\bar{D}^0} + m_{\pi^0})$$

$$\boldsymbol{\Gamma(E) = \Gamma_R\left(\mathrm{Br}(D^{*0}\to D^0\pi^0)\left(\frac{E}{E_R}\right)^{\frac{3}{2}} + \mathrm{Br}(D^{*0}\to D^0\gamma)\right)}$$

$$\boldsymbol{\Gamma_R = 55.9\ \mathrm{keV}}$$

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$$\frac{d\mathrm{Br}(D^0\bar{D}^0\pi^0)}{dE} = \boldsymbol{B}\frac{1}{2\pi}\times\frac{g * k_{\mathrm{eff}}(E)}{|D(E)|^2}\times \mathrm{Br}(D^{*0}\to D^0\pi^0)$$

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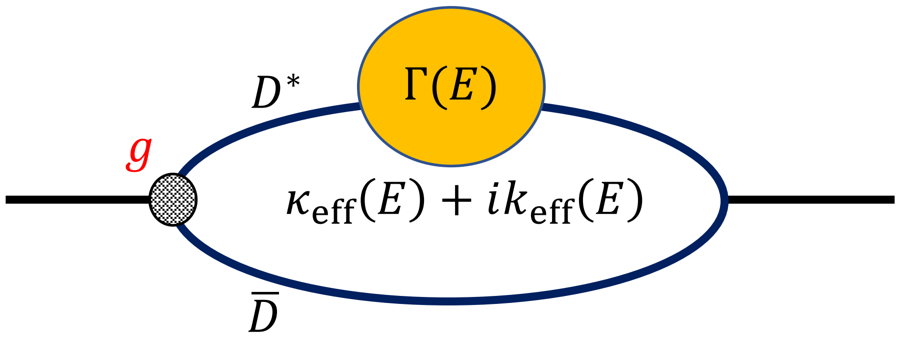

Composite particle with one unstable constituent

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⋆ It brings us the largest uncertainty

Key features:

- Model independent
- Including the $D^*\bar{D}$ self energy terms
- Including the width of D^*
- Including the coupled channel effect
- Fit parameters: g, $\Gamma_{\pi^+\pi^- J/\psi}$, M_X

BESIII

$$2\,Im[D(E)] = g * (k_{\text{eff}} + k^{c}_{\text{eff}}) + \Gamma_{\pi^+\pi^- J/\psi} + \Gamma_{known} + \Gamma_{unknown}$$

The produced numbers of events in a fitting range (E_{min}, E_{max}) are:

$$\mu^{prod}_{X(3872)} = \int_{E_{min}}^{E_{max}} dE\, \frac{\boldsymbol{B}}{2\pi} * \frac{2\,Im[D(E)]}{|D(E)|^2}$$

$$\mu^{prod}_{D^0\bar{D}^0\pi^0} = \mathrm{Br}(D^{*0} \to D^0\pi^0) \times \int_{E_{min}}^{E_{max}} dE\, \frac{\boldsymbol{B}}{2\pi} * \frac{g * k_{\text{eff}}}{|D(E)|^2}$$

$$\mu^{prod}_{\pi^+\pi^- J/\psi} = \int_{E_{min}}^{E_{max}} dE\, \frac{\boldsymbol{B}}{2\pi} * \frac{\Gamma_{\pi^+\pi^- J/\psi}}{|D(E)|^2}$$

$$\mu_{D^0\bar{D}^0\pi^0} = \epsilon_{D^0\bar{D}^0\pi^0} \times R_{D^0\bar{D}^0\pi^0} \times \mu^{prod}_{X(3872)}$$

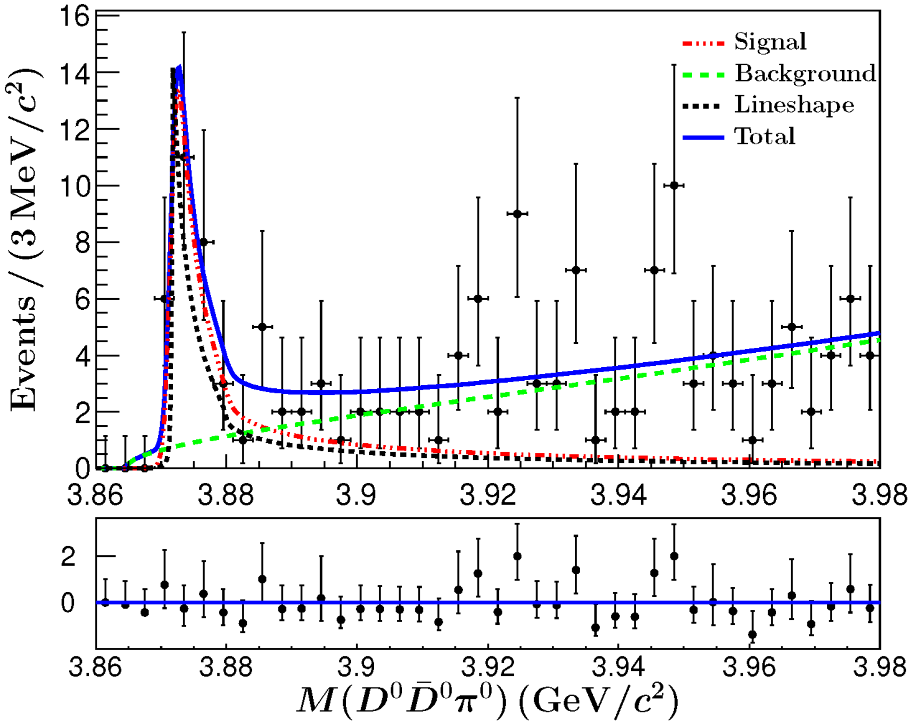
$$\mu_{\pi^+\pi^- J/\psi} = \epsilon_{\pi^+\pi^- J/\psi} \times R_{\pi^+\pi^- J/\psi} \times \mu^{prod}_{X(3872)}$$

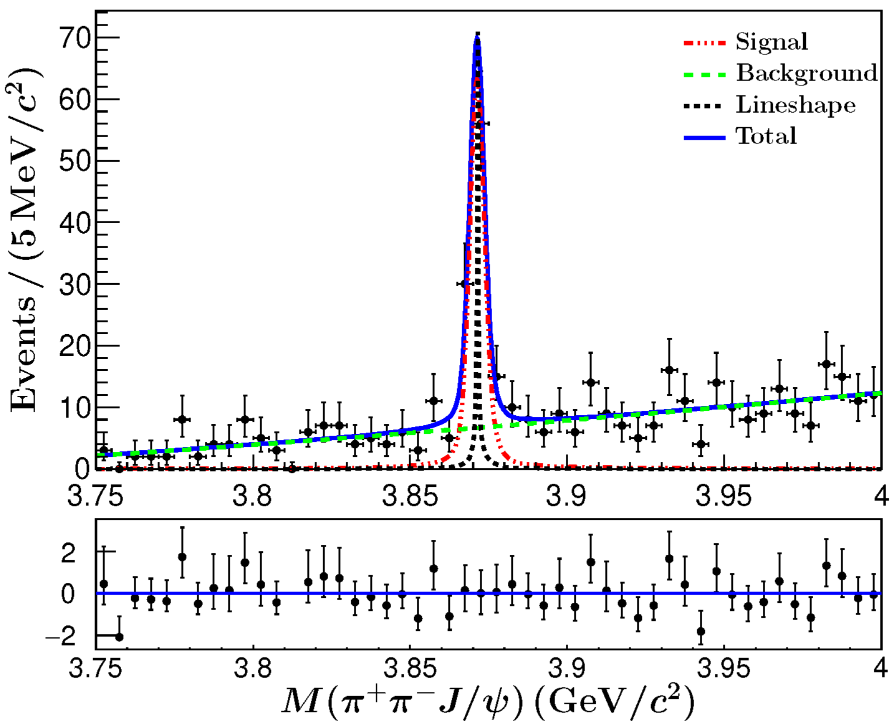
ϵ : efficiency and branching fractions correction

$$R_{D^0\bar{D}^0\pi^0} = \mathrm{Br}\left(D^{*0} \to D^0\pi^0\right) \times \frac{\int_{E_{min}}^{E_{max}} dE\, \frac{g * k_{\text{eff}}}{|D(E)|^2}}{\int_{E_{min}}^{E_{max}} dE\, \frac{2\,Im[D(E)]}{|D(E)|^2}}$$

$$R_{\pi^+\pi^- J/\psi} = \frac{\int_{E_{min}}^{E_{max}} dE\, \frac{\Gamma_{\pi^+\pi^- J/\psi}}{|D(E)|^2}}{\int_{E_{min}}^{E_{max}} dE\, \frac{2\,Im[D(E)]}{|D(E)|^2}}$$

Only one new parameter $\mu^{prod}_{X(3872)}$





$$g = 0.16 \pm 0.10^{+1.12}_{-0.11}$$
$$\Gamma_0 = \left(2.67 \pm 1.77^{+8.01}_{-0.82}\right)\ \mathrm{MeV}$$
$$M_X = \left(3871.63 \pm 0.13^{+0.06}_{-0.05}\right)\ \mathrm{MeV}$$

Large systematic uncertainty from $\Gamma_{unknow}/\Gamma_{\pi^+\pi^-J/\psi}$

BESIII

Sources:

- The width of D^{*0}
- The uncertainties of the efficiency corrections
- The mass resolution model
- The background shape
- The mass of D^0
- The uncertainties of the center-of-mass energies
- The input e^+e^- cross sections and decay models in MC simulation

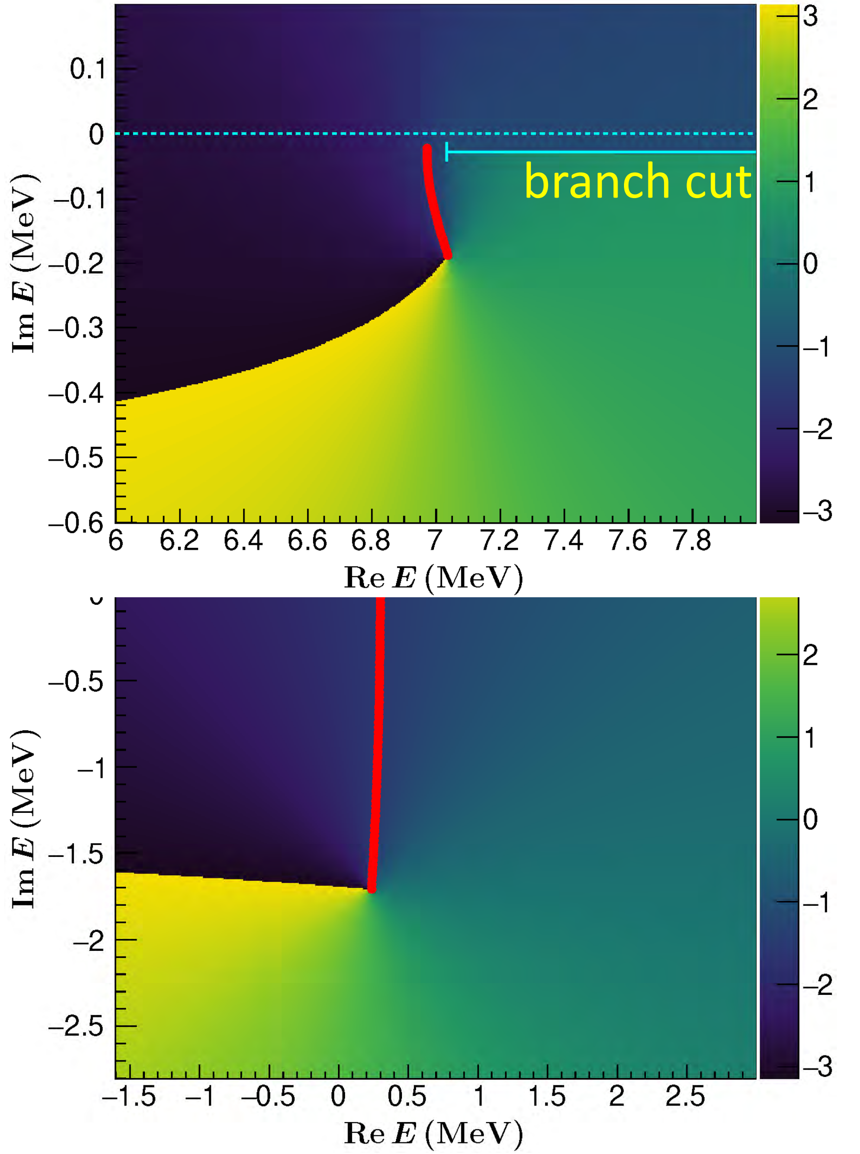
- The largest uncertainty α: the value $(\Gamma_{known} + \Gamma_{unknown})/\Gamma_{\pi^+\pi^- J/\psi}$ is changed in range (4.2, 21.8), mostly due to the lack of knowledge about the $X(3872)$ absolute branching fractions

Source	g	Γ_0 (MeV)	M_X (MeV)
α	$+1.08\ -0.10$	$+6.54\ -0.65$	$+0.05\ -0.04$
$\Gamma_{D^{*0}}$	$-$	$+0.05\ -0.07$	$-$
Efficiency	$+0.05\ -0.03$	$+0.35\ -0.24$	$-$
Resolution	$-$	±0.02	$-$
Background	$+0.05$	$+0.51\ -0.24$	±0.01
$M(D^0)$	$-$	$+0.11\ -0.09$	±0.03
$E_{\rm cms}$	$+0.29$	$+4.57$	-0.01
Simulation	±0.02	±0.26	±0.01
Sum	$+1.12\ -0.11$	$+8.01\ -0.82$	$+0.06\ -0.05$

- Due to causality, the scattering amplitude should be analytic over the complex energy plane, up to poles and branch cuts
- The pole locations can reveal the intrinsic properties of the particle
- Two sheets with respect to $D^{*0}\overline{D}^0$ branch cut
  - Sheet I: $E - E_X - g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$
  - Sheet II: $E - E_X + g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$

- $E_{\mathrm{I}} = (7.04 \pm 0.15^{+0.07}_{-0.08}) + (-0.19 \pm 0.08^{+0.14}_{-0.19})i$ MeV
- $E_{\mathrm{II}} = \left(0.26 \pm 5.74^{+5.14}_{-38.32}\right) + (-1.71 \pm 0.90^{+0.60}_{-1.96})i$ MeV

$M(D^0\overline{D}^{*0}) - M(D^0\overline{D}^0\pi^0) = 7.0332$ MeV



Pole search

- Due to causality, the scattering amplitude should be analytic over the complex energy plane, up to poles and branch cuts
- The pole locations can reveal the intrinsic properties of the particle
- Two sheets with respect to $D^{*0}\bar{D}^0$ branch cut
  - Sheet I: $E - E_X - g\sqrt{-2\mu(E - E_R + i\Gamma/2)}$
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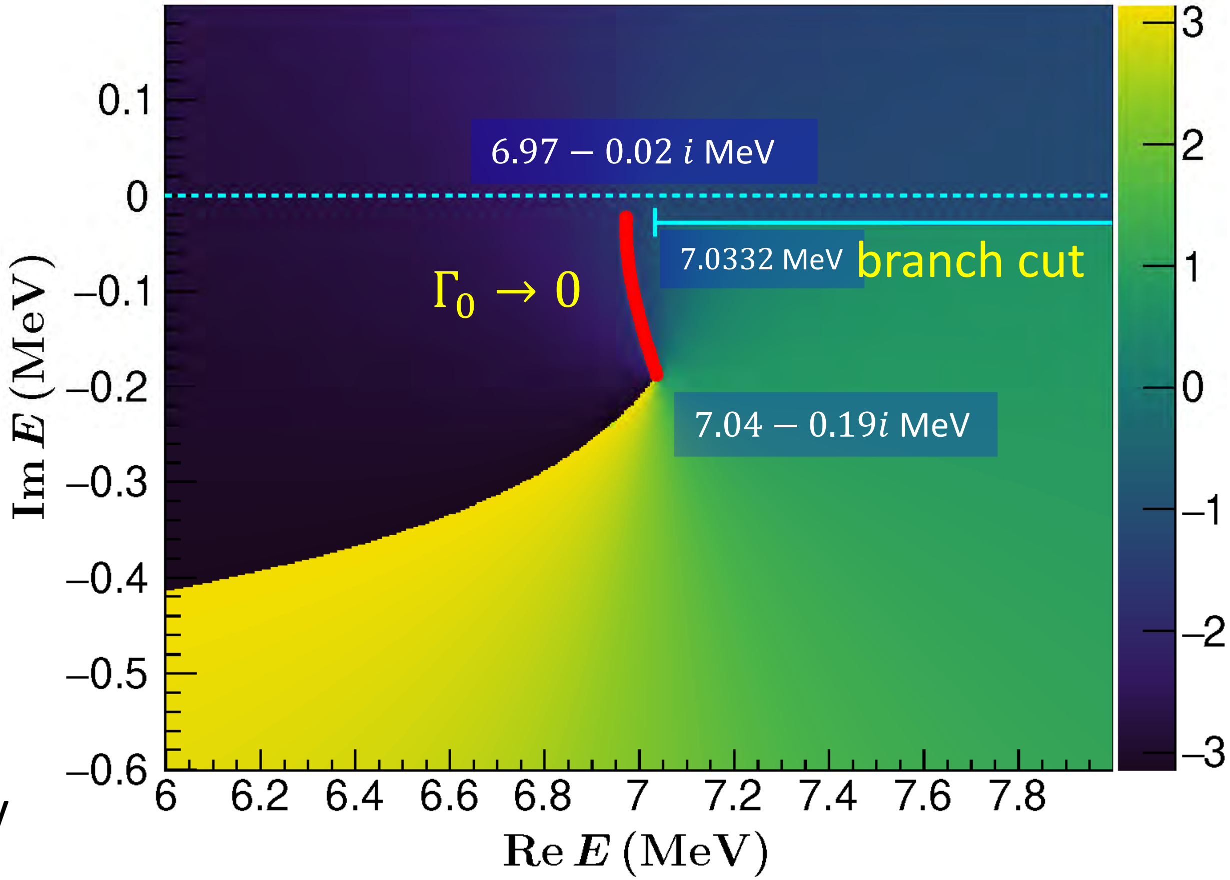
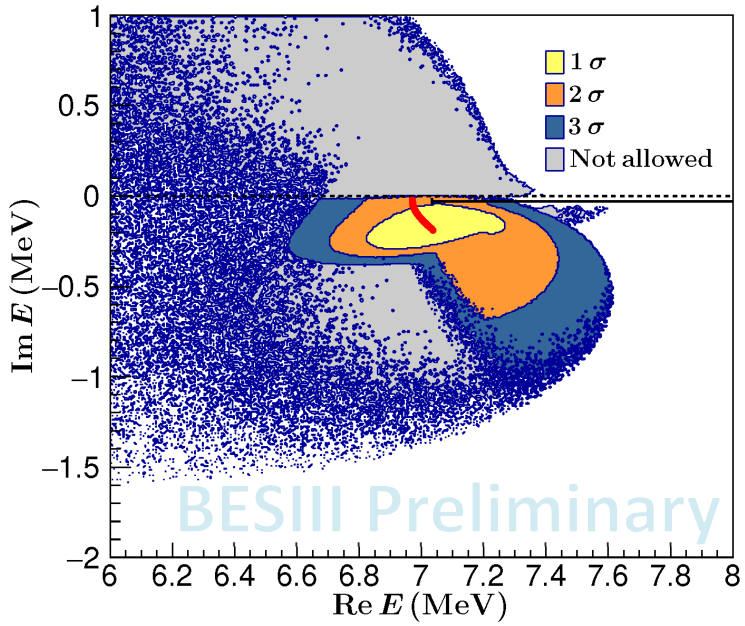
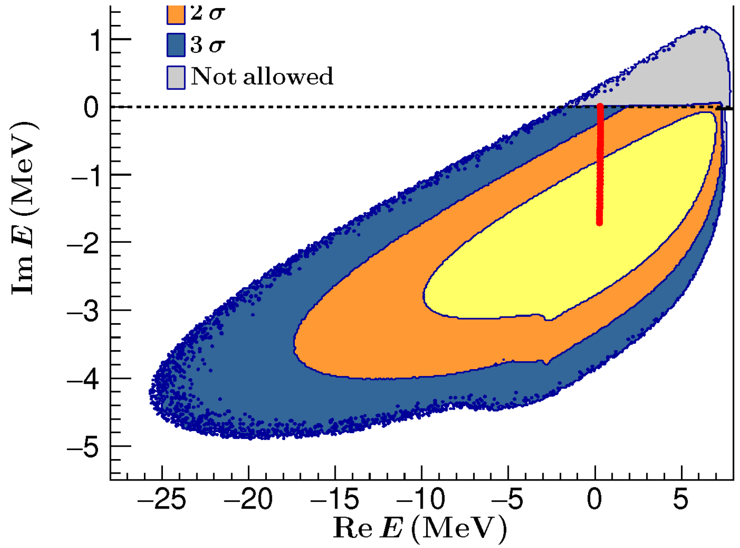
- $E_{\mathrm{I}} = (7.04 \pm 0.15^{+0.07}_{-0.08}) + (-0.19 \pm 0.08^{+0.14}_{-0.19})i$ MeV
- $E_{\mathrm{II}} = (0.26 \pm 5.74^{+5.14}_{-38.32}) + (-1.71 \pm 0.90^{+0.60}_{-1.96})i$ MeV
- E_{I} is much closer to the threshold, should play a dominant role in the X(3872) confinement mechanism



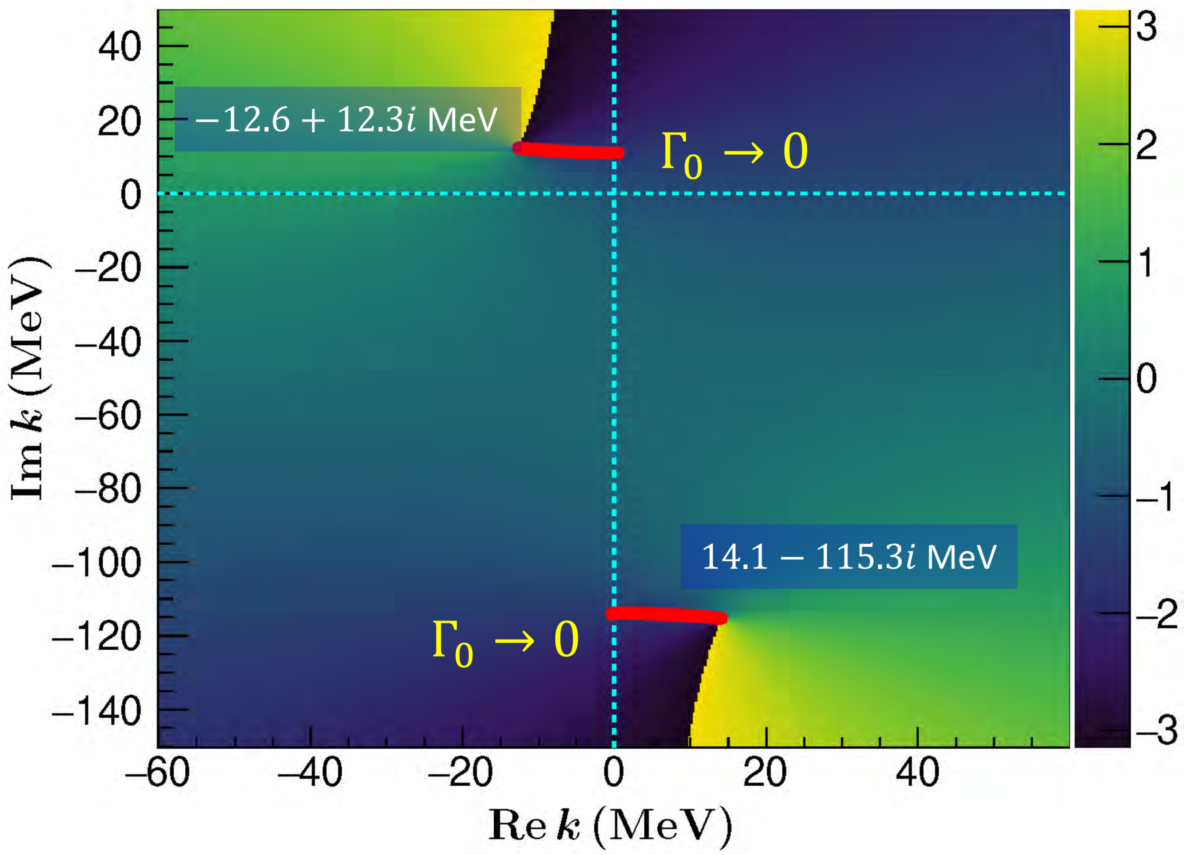
Pole search

Source	Re[$E_{\rm I}$] (MeV)	Im[$E_{\rm I}$] (MeV)	Re[$E_{\rm II}$] (MeV)	Im[$E_{\rm II}$] (MeV)
α	± 0.01	$+0.12\ -0.18$	$+4.74\ -34.20$	$+0.54\ -0.76$
$\Gamma_{D^{*0}}$	± 0.01	± 0.01	$+0.23\ -0.20$	± 0.03
Efficiency	$+0.01$	± 0.02	$+1.55\ -2.41$	$+0.13\ -0.18$
Resolution	–	–	± 0.08	± 0.01
Background	± 0.02	$+0.01$	$+0.15\ -3.02$	-0.26
$M(D^0)$	± 0.06	± 0.03	$+0.34\ -0.16$	$+0.12\ -0.11$
$E_{\rm cms}$	-0.03	-0.06	-16.79	-1.77
Simulation	± 0.01	± 0.01	± 1.15	± 0.18
Sum	$+0.07\ -0.08$	$+0.14\ -0.19$	$+5.14\ -38.32$	$+0.60\ -1.96$



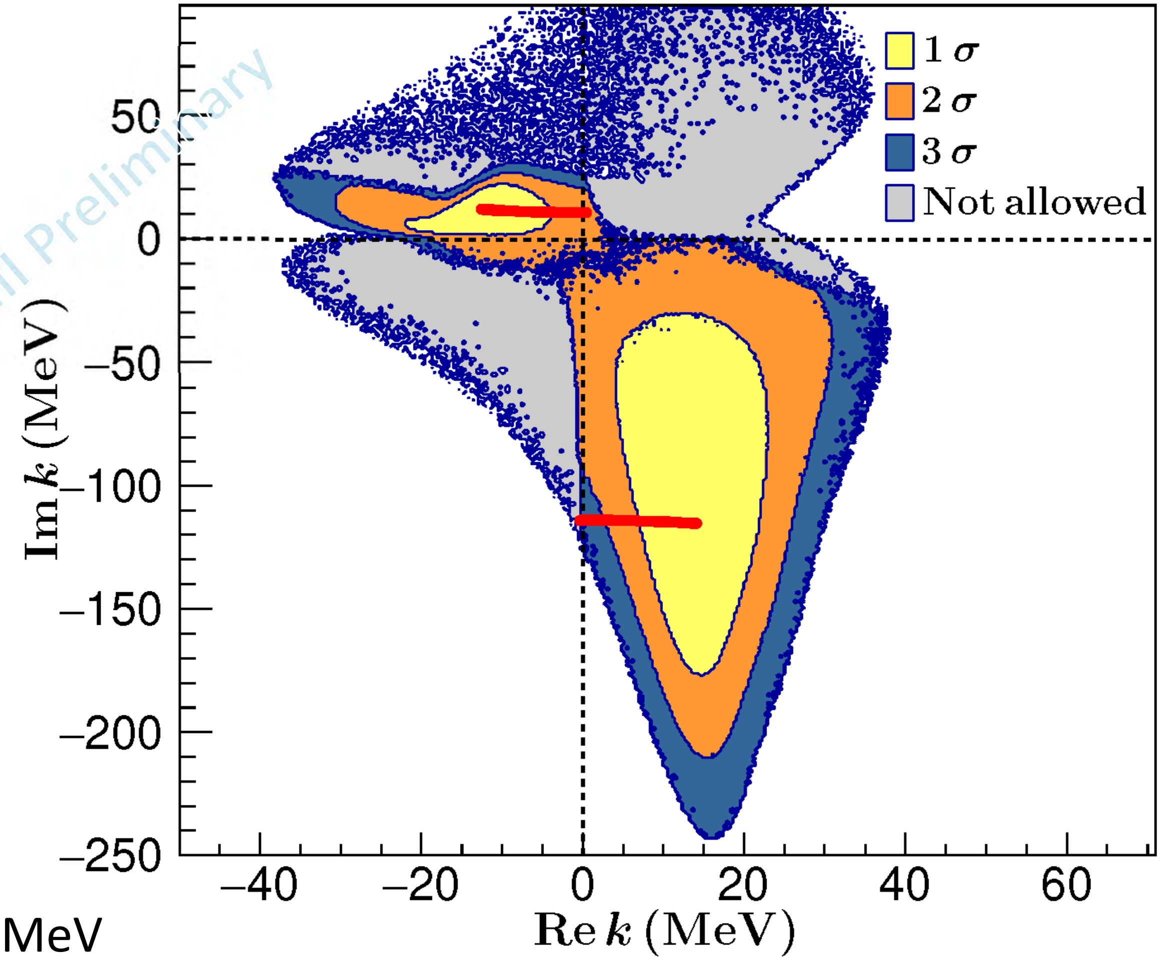


- Near the threshold, the scattering amplitude can be expanded as the power series of the momentum $k = \sqrt{2\mu(E - E_R)}$ (Effective Range Expansion, ERE)
- S-Wave $f^{-1}(E) \sim \frac{1}{a} + \frac{r_e}{2}k^2 - ik + \mathcal{O}(k^4)$
- Two poles on k-plane
  - $k^+ = -12.6 + 12.3i$ MeV
  - $k^- = 14.1 - 115.3\,i$ MeV

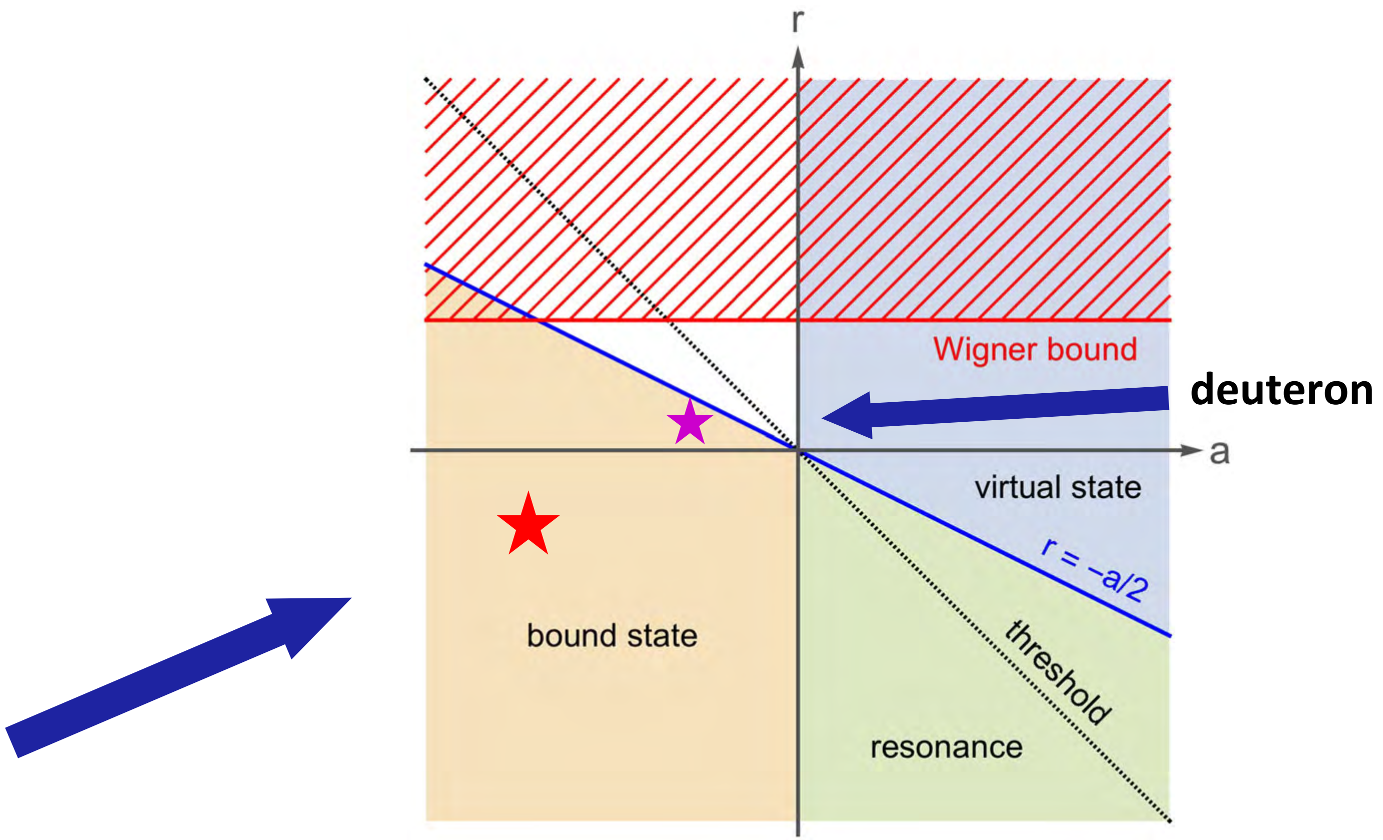


Source	Re[k^+] (MeV)	Im[k^+] (MeV)	Re[k^-] (MeV)	Im[k^-] (MeV)
α	$+5.9\ -6.0$	$+5.6\ -5.9$	$+4.9\ -1.6$	$+49.8\ -162.5$
$\Gamma_{D^{*0}}$	-0.2	$+0.7\ -0.5$	-0.1	$+2.0\ -1.7$
Efficiency	±1.0	$+0.6\ -0.7$	$+0.6\ -0.7$	$+13.7\ -18.6$
Resolution	–	–	–	±0.7
Background	±1.0	$+0.5\ -0.6$	$+0.5\ -1.0$	$+1.1\ -24.3$
$M(D^0)$	$+0.6\ -0.7$	$+1.7\ -1.9$	$+0.8\ -0.7$	$+2.6\ -1.0$
$E_{\rm cms}$	$+2.7$	$+0.2$	$+1.5$	-98.7
Simulation	±0.3	±0.8	±0.4	±9.9
Sum	$+6.6\ -6.2$	$+6.0\ -6.4$	$+5.3\ -2.1$	$+52.7\ -192.8$

- $k^+ = (-12.6 \pm 5.5^{+6.6}_{-6.2}) + (12.3 \pm 6.8^{+6.0}_{-6.0})\, i$ MeV
- $k^- = \left(14.1 \pm 5.8^{+5.3}_{-2.1}\right) + (-115.3 \pm 44.6^{+52.7}_{-192.8})\, i$ MeV



- Near the threshold, the scattering amplitude can be expanded as the power series of the momentum $k = \sqrt{2\mu(E - E_R)}$ (Effective Range Expansion, ERE)
- S-Wave $f^{-1}(E) \sim \frac{1}{a} + \frac{r_e}{2}k^2 - ik + \mathcal{O}(k^4)$
- Two poles on k-plane
    - $k^+ = -12.6 + 12.3\,i$ MeV
    - $k^- = 14.1 - 115.3\,i$ MeV
- ERE parameters
    - a: scattering length
    - r_e: effective range
- In the limit of $\Gamma_0 \to 0$ and stable D^*, the ERE parameters are determined:
    - $a = (-16.5^{+7.0}_{-27.6}{}^{+5.6}_{-27.7})$ fm
    - $r_e = (-4.1^{+0.9}_{-3.3}{}^{+2.8}_{-4.4})$ fm



[I. Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur.Phys.J.A 57 (2021) 3, 101]

The effective range expansion

[S. Weinberg, Phys. Rev. 137, B672 (1965)]

$$a = -\frac{2(1-Z)}{(2-Z)}\frac{1}{\gamma} + \mathcal{O}(\beta^{-1})$$
$$r_e = -\frac{Z}{1-Z}\frac{1}{\gamma} + \mathcal{O}(\beta^{-1})$$

Z: field renormalization constant

- $Z = 0$: pure bound (composite) state
- $Z = 1$: pure elementary state

$\beta^{-1} \approx \frac{1}{m_\pi} \approx$ 1.4 fm, for both deuteron and the *X*(3872)

$\gamma = \sqrt{2\mu E_b}$

	***X*(3872)**	**deuteron**
Nearby threshold	$D^{*0}\bar{D}^0$	$p\ n$
a	-16.5 fm	-5.41 fm
r_e	-4.1 fm	1.75 fm
Range correction	negligible	important for r_e
Z	≈ 0.18	-
$\lvert r_e/a \rvert$	0.25	0.32
$\bar{Z}_A$	0.18	0.22

$$\bar{Z}_A = 1 - \sqrt{\frac{1}{1+\lvert 2r_e/a \rvert}}$$

[Eur.Phys.J.A 57 (2021) 3, 101]

Different sign, may suggest an elementary $c\bar{c}$ core
[A. Esposito PRD 105, L031503]

Close to 0 but can not be solved model-independently
due to the range correction

Flatté:

$$\frac{dR(J/\psi\pi^+\pi^-)}{dE} \propto \frac{\Gamma_\rho(E)}{|D(E)|^2}$$

$$D(E) = E - E_f + \frac{i}{2}[g(k_1 + k_2) + \Gamma_\rho(E) + \Gamma_\omega(E) + \Gamma_0]$$

No substraction term like

$$\sqrt{\mu_p}\sqrt{\sqrt{(E_X - E_R)^2 + \Gamma_X^2/4} - E_X + E_R}$$

$$k_1 = \sqrt{2\mu_1 E}, \qquad k_2 = \sqrt{2\mu_2(E-\delta)} \qquad \delta = 8.2\ \text{MeV}$$

$$\Gamma_\rho(E) = f_\rho \int_{2m_\pi}^{M(E)} \frac{dm'}{2\pi} \frac{q(m',E)\Gamma_\rho}{(m'-m_\rho)^2+\Gamma_\rho^2/4}$$

$$\Gamma_\omega(E) = f_\omega \int_{3m_\pi}^{M(E)} \frac{dm'}{2\pi} \frac{q(m',E)\Gamma_\omega}{(m'-m_\omega)^2+\Gamma_\omega^2/4}$$

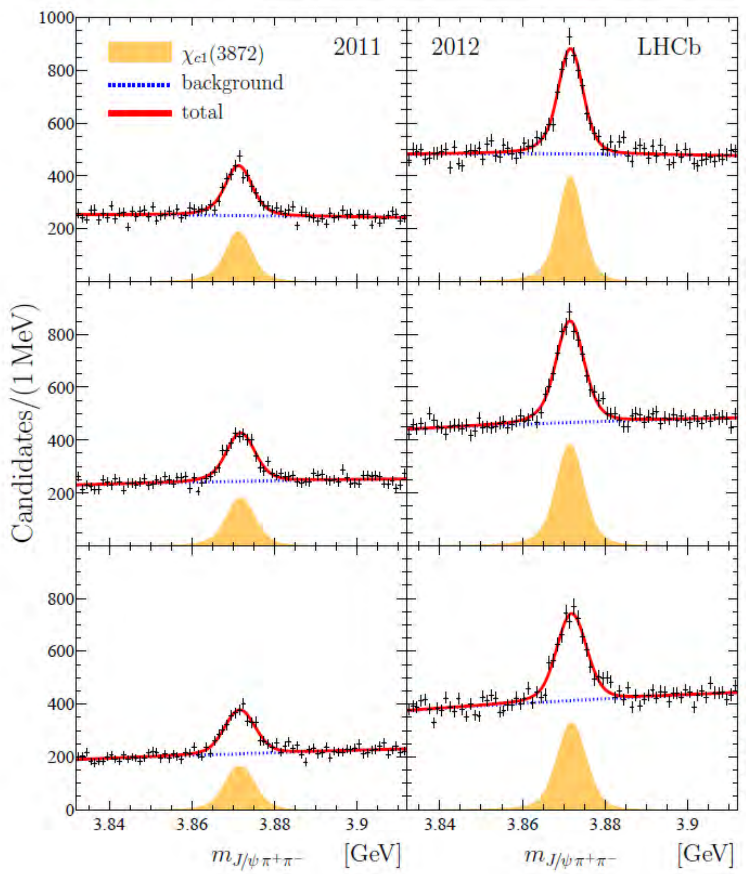
$$M(E) = E + (m_{D^0} + m_{D^{*0}}) - m_{J/\psi}$$

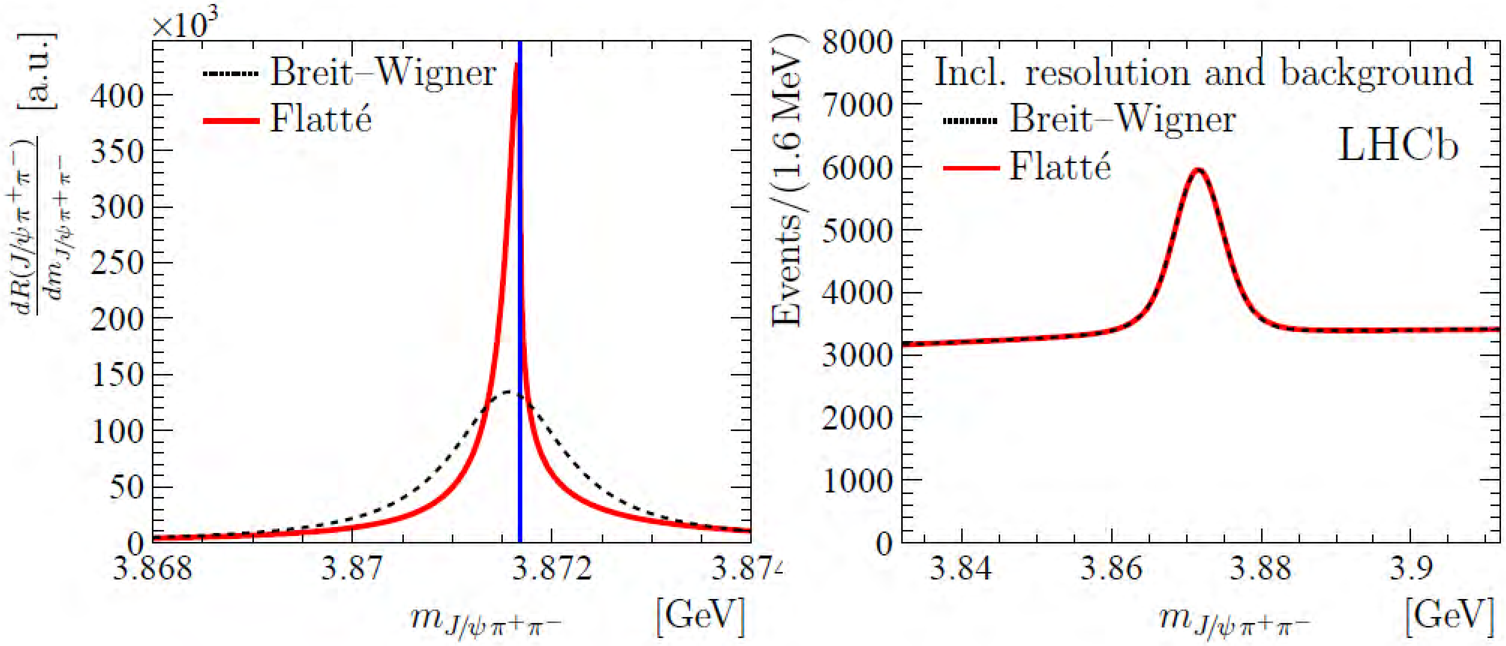
$$q(m',E) = \sqrt{\frac{\left[M^2-(m'+m_{J/\psi})^2\right]\left[M^2-(m'-m_{J/\psi})^2\right]}{4M^2}}$$

Constraints:

- $\frac{\Gamma(\pi^+\pi^- J/\psi)}{\Gamma(D^0\bar{D}^{*0})} = 0.11 \pm 0.03$
- $\Gamma(\omega J/\psi) = \Gamma(\rho J/\psi)$
- $E_f = -7.2$ MeV (3864.5)

[Belle, Babar]

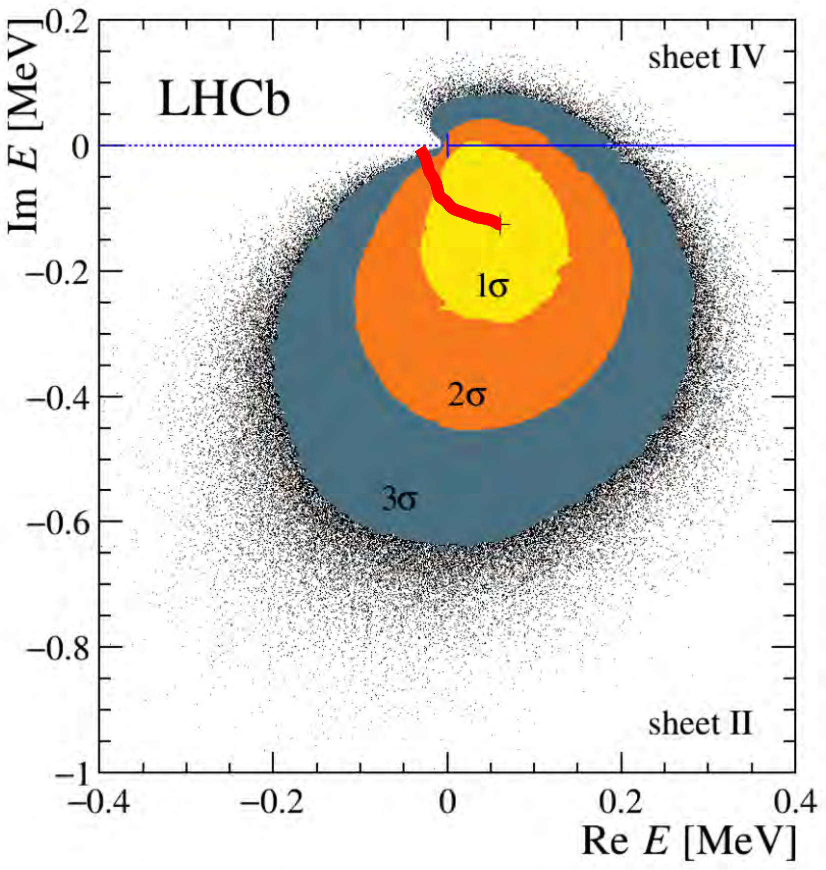




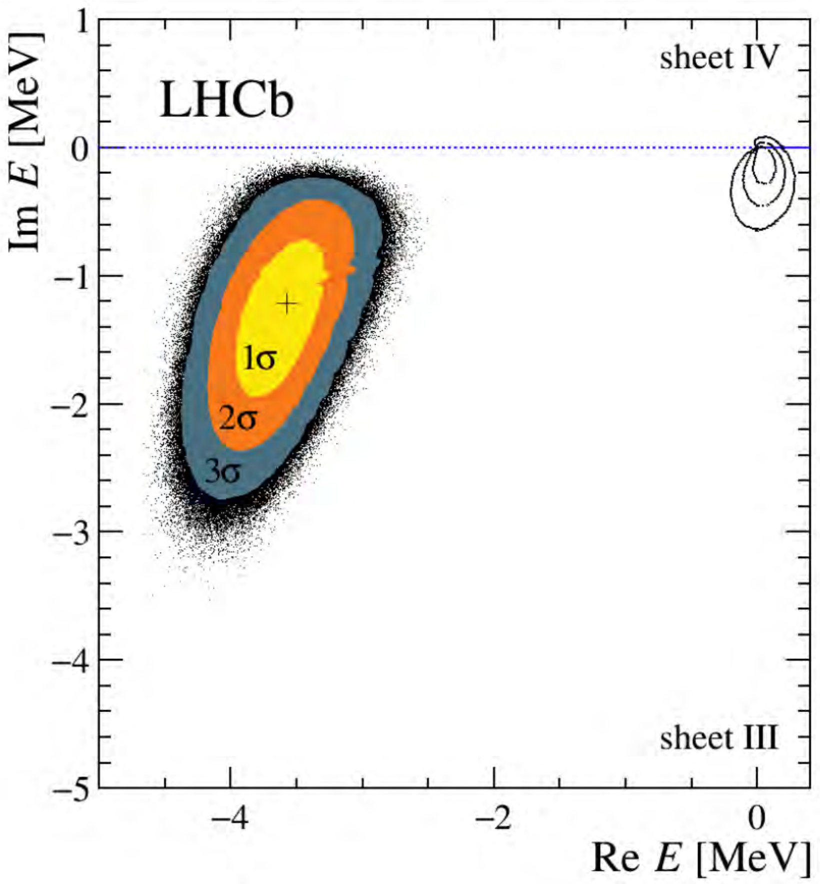
Flatté:

g	$f_\rho \times 10^3$	f_ω	Γ_0 [MeV]	m_0 [MeV]
0.108 ± 0.003	1.8 ± 0.6	0.01	1.4 ± 0.4	3864.5 (fixed)

BW:
- $m = 3871.695 \pm 0.067 \pm 0.068 \pm 0.010$ MeV
- $\Gamma_{BW} = 1.39 \pm 0.24 \pm 0.10$ MeV

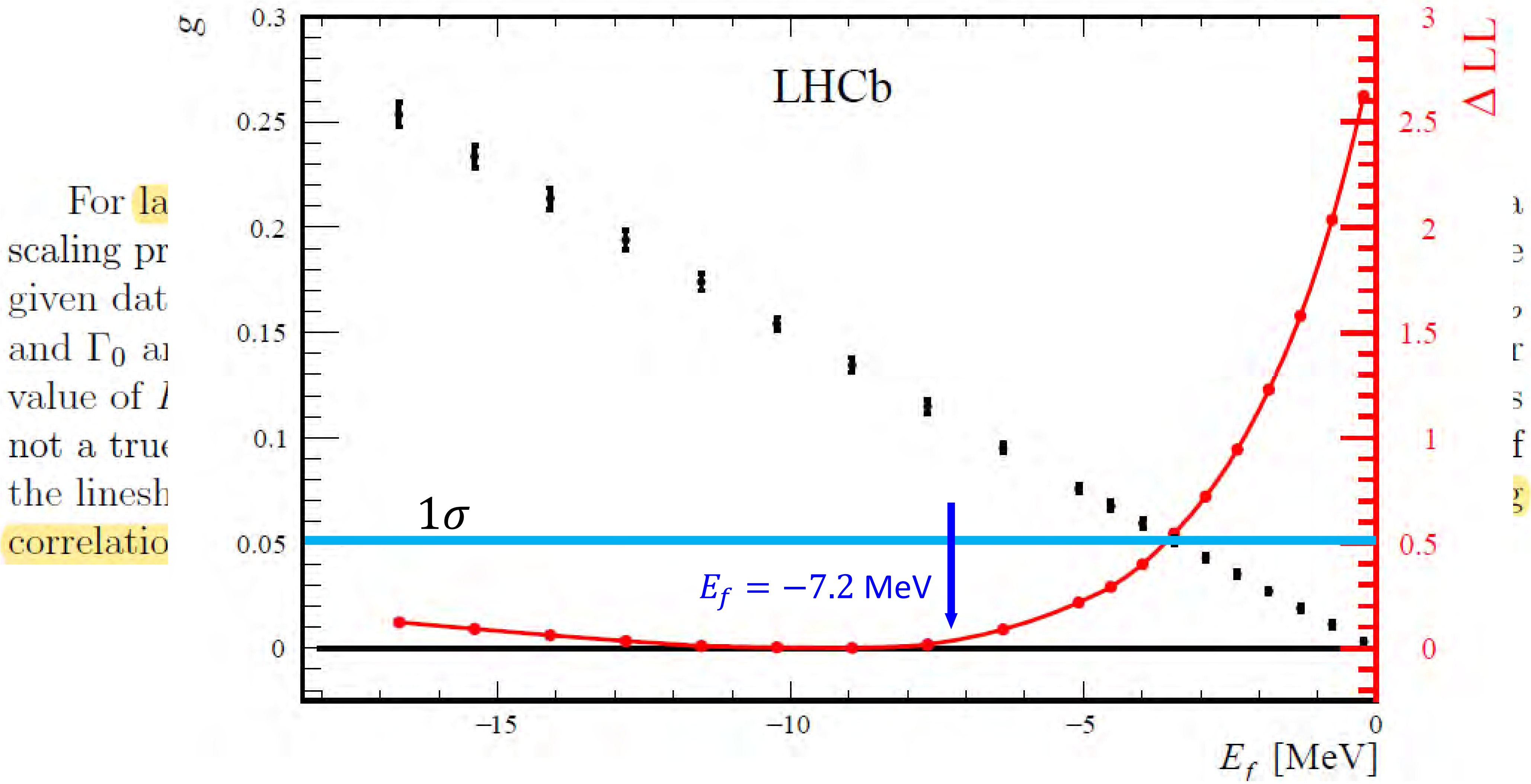


$E_{\mathrm{II}} = 7.10 - \ 0.13\ i$ MeV



$E_{\mathrm{III}} = 3.45 - \ 0.13\ i$ MeV

For large couplings to the two-body channel the Flatté parameterisation exhibits a scaling property [43] that prohibits the unique determination of all free parameters on the given data set. Almost identical lineshapes are obtained when the parameters E_f, g, f_ρ and Γ_0 are scaled appropriately. In particular, it is possible to counterbalance a lower value of E_f with a linear increase in the coupling to the $D\overline{D}^*$ channels g. While this is not a true symmetry of the parameterisation — there are subtle differences in the tails of the lineshape — in practice, within the experimental precision this effect leads to strong correlations between the parameters.


LHCb
g
ΔLL
1σ
$E_f = -7.2$ MeV
E_f [MeV]
0.3
0.25
0.2
0.15
0.1
0.05
0
−15
−10
−5
0
3
2.5
2
1.5
1
0.5
For la
scaling pr
given dat
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value of
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$\eta\pi \to a_0(980) \to \eta\pi$ [V. Baru et al. EPJ A 23 (2005) 523]

$X(3872) \to \pi^+\pi^- J/\psi$

$$f_{el} = -\frac{1}{2q_\eta} \frac{\Gamma_{\eta\pi}}{E - E_f + \frac{i}{2}(\Gamma_{\eta\pi} + g_K k)}$$

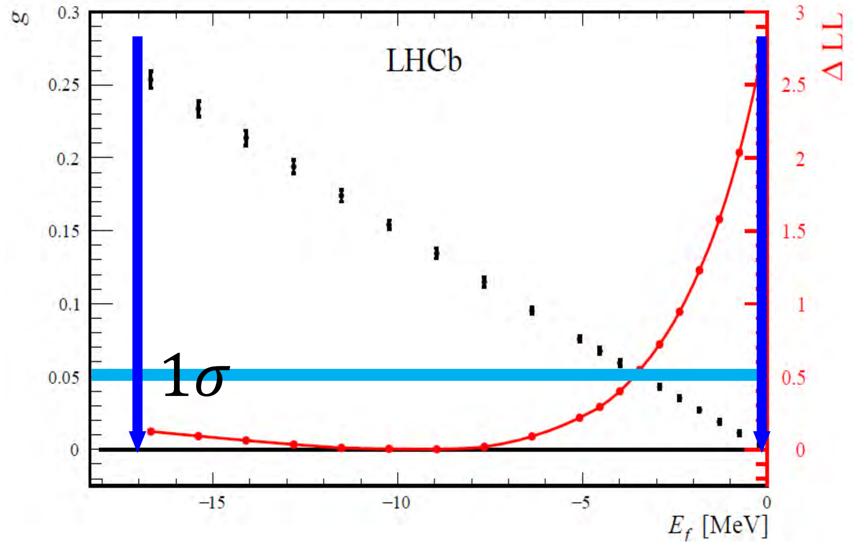
$$\sigma_{el} = 4\pi |f_{el}|^2 \sim \frac{\Gamma_{\eta\pi}^2}{\left|E - E_f\right|^2 + \frac{\left(\Gamma_{\eta\pi} + g_K k\right)^2}{4}} \quad E > 0$$

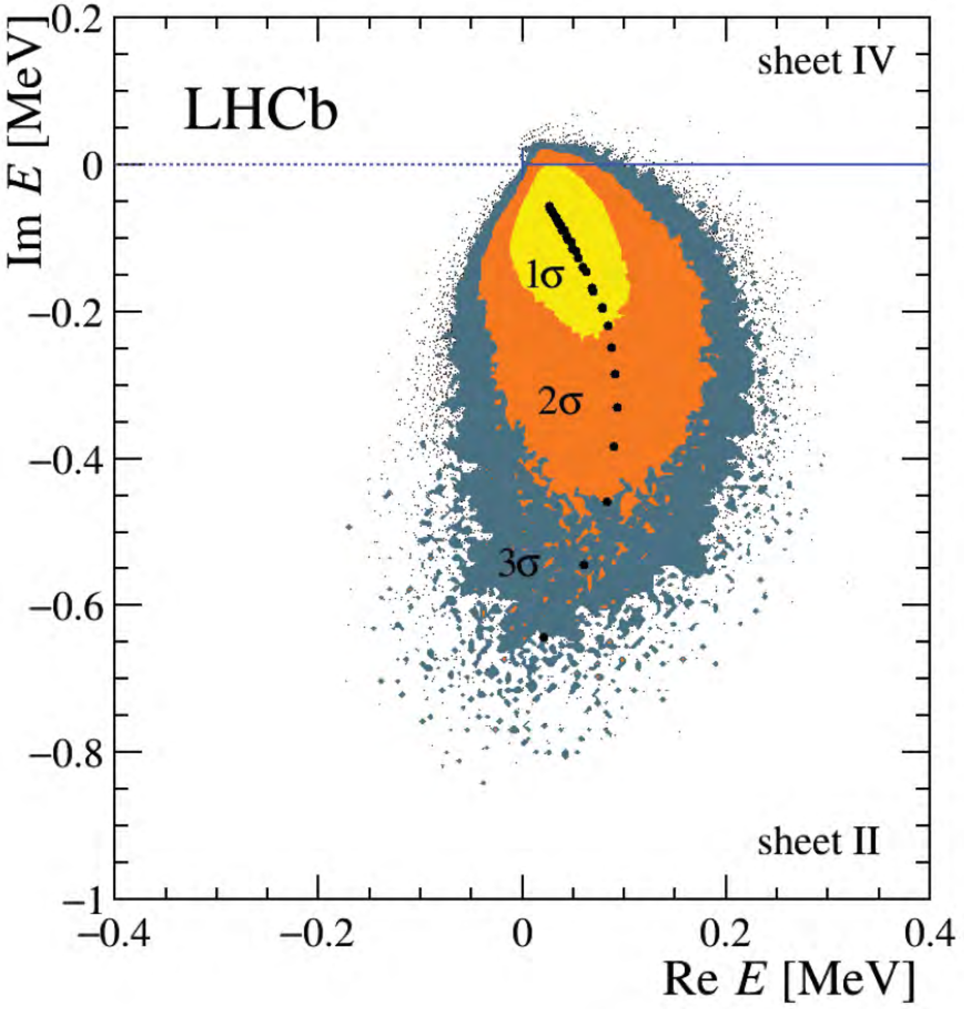
$$\frac{\Gamma_{\eta\pi}^2}{\left|E - E_f - \frac{g_K \kappa}{2}\right|^2 + \frac{\left(\Gamma_{\eta\pi}\right)^2}{4}} \quad E < 0$$

$$\frac{d\sigma}{dE} \sim \frac{gk}{\left|E - E_f - \frac{g k_c}{2}\right|^2 + \frac{(\Gamma_0 + gk)^2}{4}}$$

if the state is very narrow:

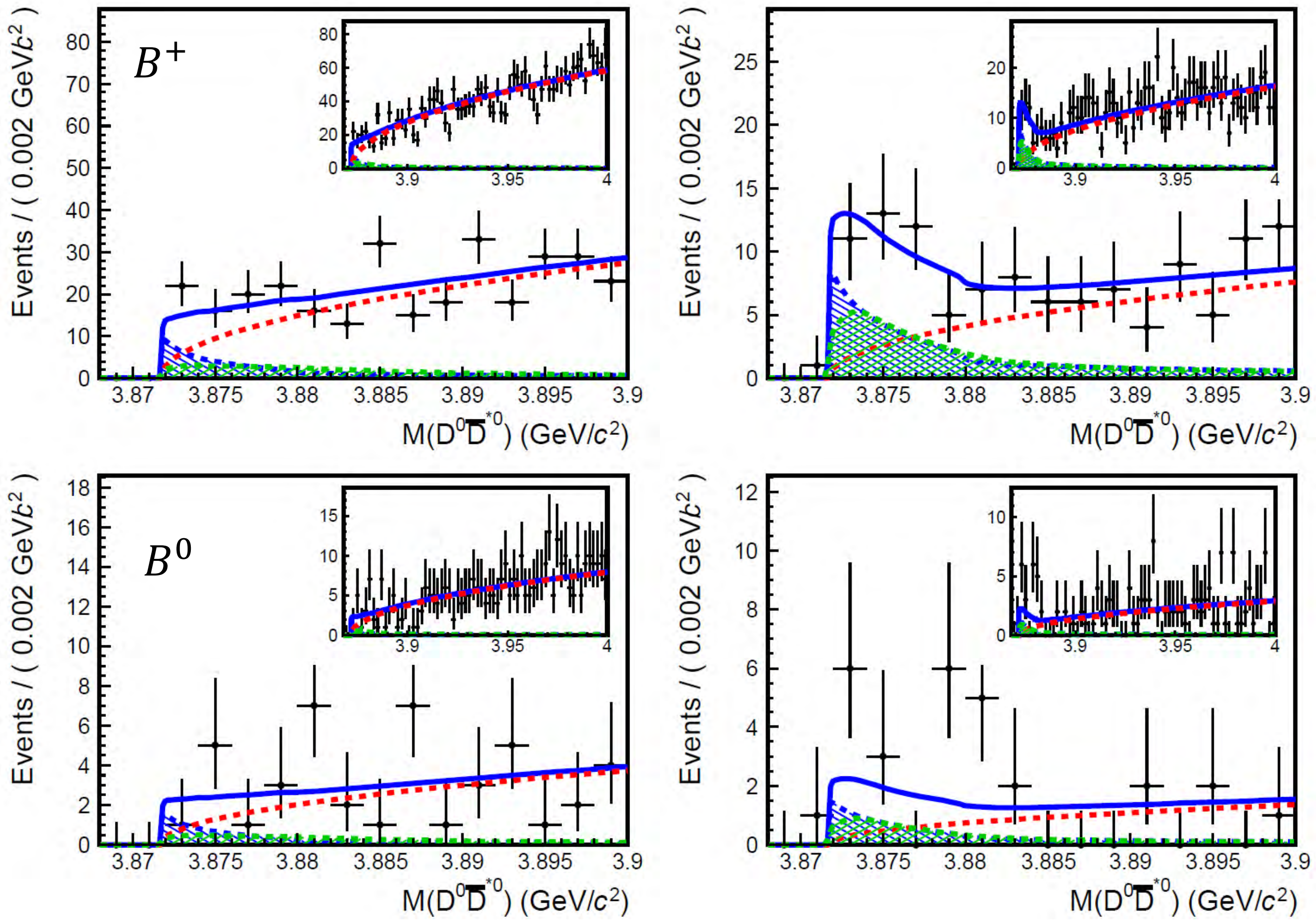
$$\frac{\Gamma_{\eta\pi}^2}{\left|E - E_f\right|^2 + \frac{\left(\Gamma_{\eta\pi} + g_K k\right)^2}{4}} \approx \frac{1}{\left|\frac{E_f}{\Gamma_{\eta\pi}}\right|^2 + \left(\frac{1}{2} + \frac{g_k}{2\Gamma_{\eta\pi}} k\right)^2}$$





- $E_f \in (-17, 0)$ MeV (black dots)
- The choices of E_f do have impact on the pole location, $\Delta(Im(E_{\mathrm{I}})) > -0.5$ MeV
- Not in LHCb's systematic uncertainties

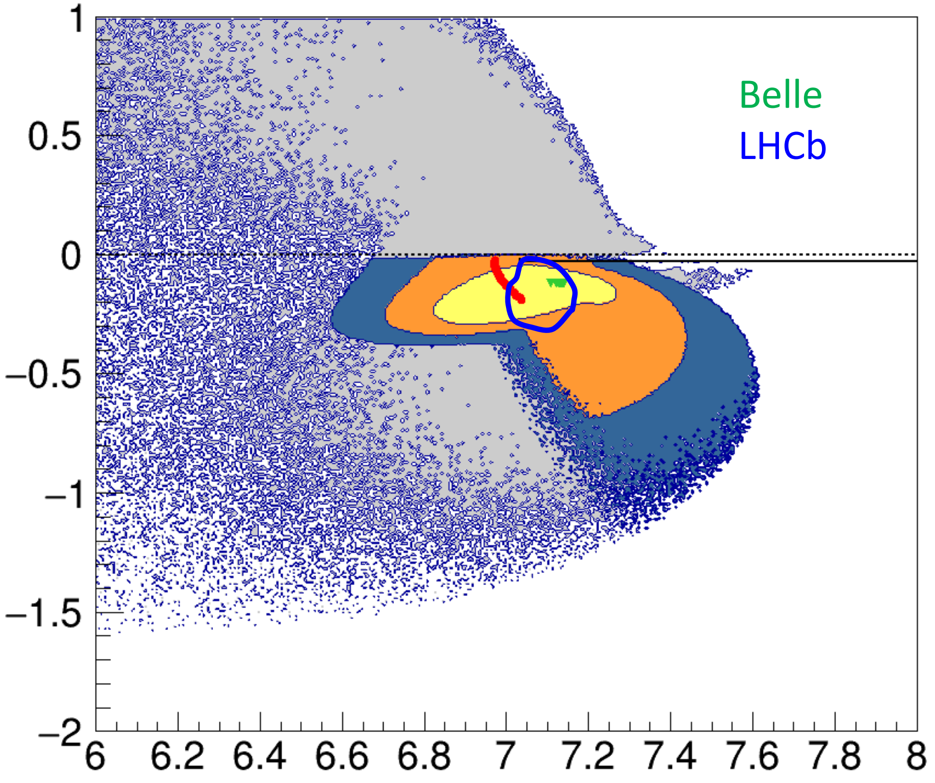
- $B^{\pm} \to K^{\pm}X, B^0 \to K^0 X$
- $X(3872) \to D^0\overline{D}^{*0}$
- Constraints:
  - $\Gamma(\omega J/\psi) = \Gamma(\rho J/\psi)$
  - $\frac{dg}{dE_f} = -15.11\ \mathrm{GeV}^{-1}$
  - $\frac{f_\rho}{E_f} = \frac{1.8\times10^{-3}}{-7.2\ \mathrm{MeV}}$
  - $\frac{\Gamma_0}{E_f} = \frac{1.4\ \mathrm{MeV}}{-7.2\ \mathrm{MeV}}$
- Only g is floated as a free parameter
- $g = 0.29^{+2.69}_{-0.15}$
- BW result:
  - $m = 3873.71^{+0.56}_{-0.50} \pm 0.13$ MeV
  - $\Gamma_0 = 5.2^{+2.2}_{-1.5} \pm 0.4$ MeV



$D^* \to D^0\gamma$ $D^0\pi^0$

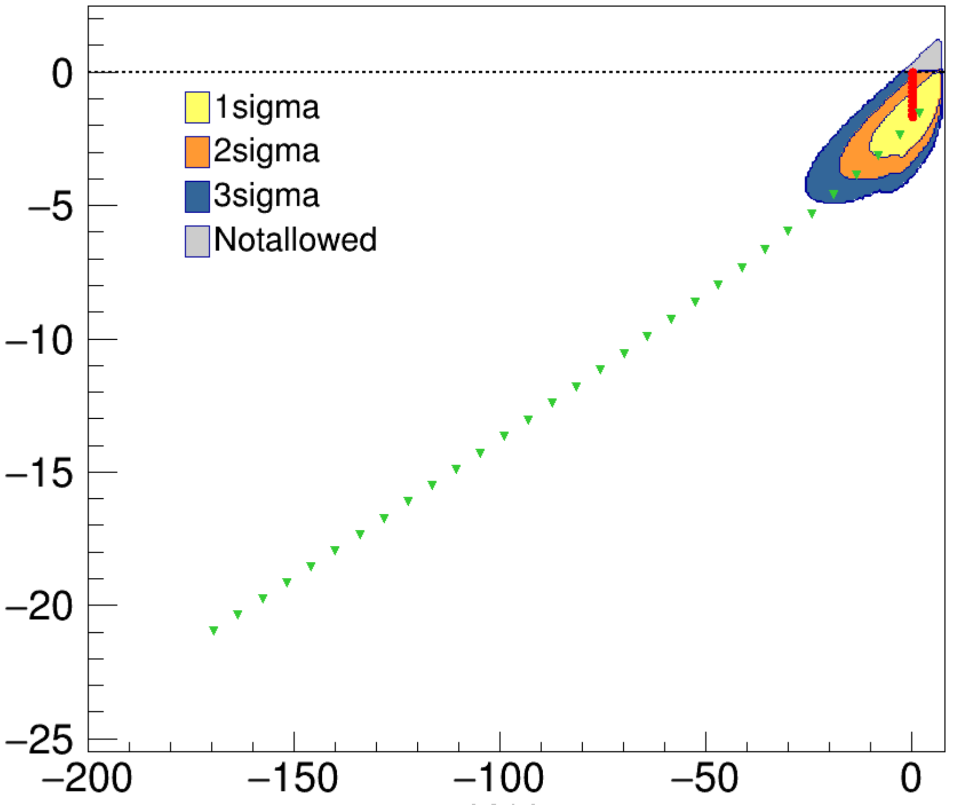
LHCb
- $m = 3871.695 \pm 0.067 \pm 0.068 \pm 0.010$ MeV
- $\Gamma_{BW} = 1.39 \pm 0.24 \pm 0.10$ MeV

sheet I



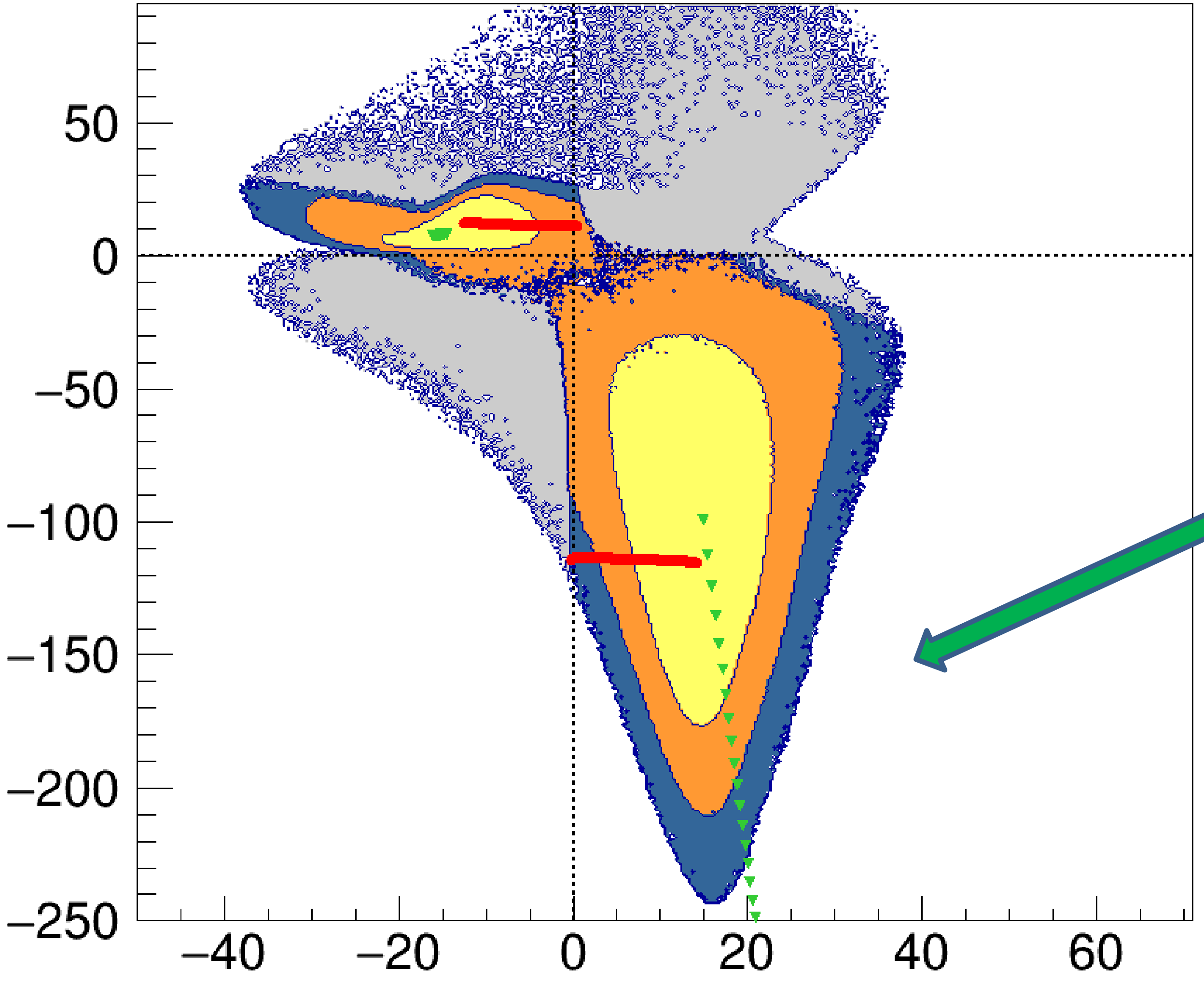
$E_{\text{I}} = 7.12 - \ 0.12\, i$ MeV

sheet II



$E_{\text{II}} = -5.82 - \ 2.84\, i$ MeV

$k-\text{plane}$



Belle

$k^{+}=-15.3+7.7\,i$ MeV
$k^{-}=17.3-158.5\,i$ MeV

$r_e=\left(-3.0^{+1.3}_{-1.5}\right)$ fm
$a=-31.2$ fm

	LHCb	Belle	This work
g	$0.108 \pm 0.003^{+0.005}_{-0.006}$	$0.29^{+2.69}_{-0.15}$	$0.16 \pm 0.10^{+1.12}_{-0.11}$
$Re[E_I]$ [MeV]	7.10	7.12	$7.04 \pm 0.15^{+0.07}_{-0.08}$
$Im[E_I]$ [MeV]	-0.13	-0.12	$-0.19 \pm 0.08^{+0.14}_{-0.19}$
$Re[k^+]$ [MeV]	-13.9	-15.3	$-12.6 \pm 5.5^{+6.6}_{-6.2}$
$Im[k^+]$ [MeV]	8.8	7.7	$12.3 \pm 6.8^{+6.0}_{-6.4}$
a (fm)	-27.1	-31.2	$-16.5^{+7.0}_{-27.6}{}^{+5.6}_{-27.7}$
r_e (fm)	-5.3	$-3.0^{+1.3}_{-1.5}$	$-4.1^{+0.9}_{-3.3}{}^{+2.8}_{-4.4}$
$\bar{Z}_A$	$0.15\ (0.33)$	$0.08^{+0.04}_{-0.03}$	$0.18^{+0.06}_{-0.17}{}^{+0.19}_{-0.16}$

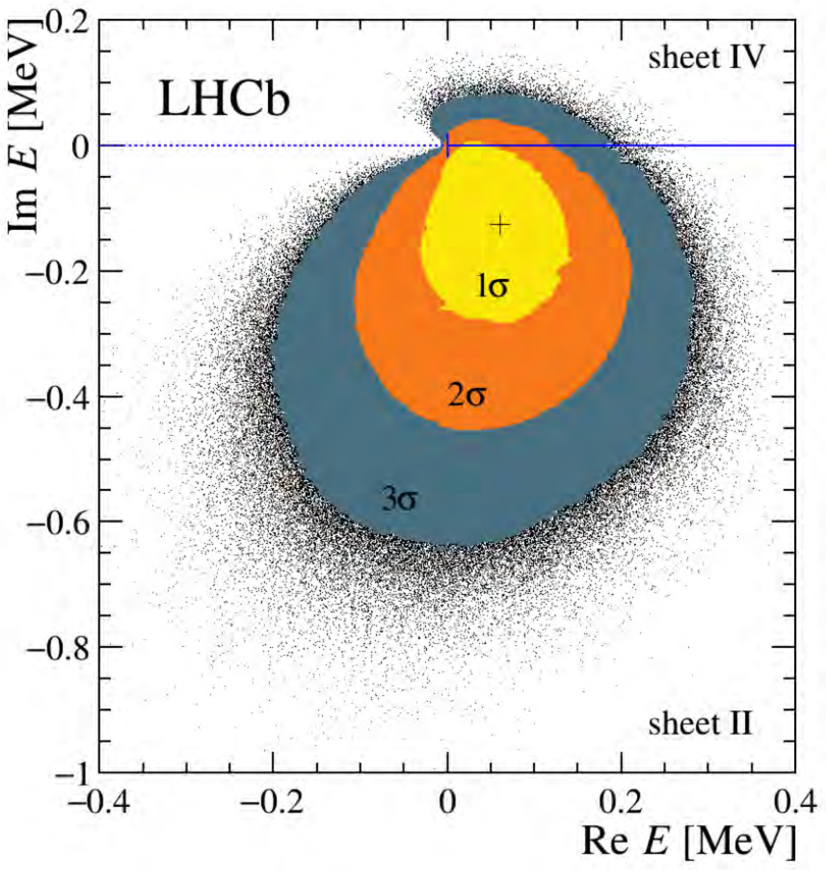
- We performed a coupled channel analysis of the $X(3872)$ Lineshape. With BESIII data, the $D^0\bar{D}^0\pi^0$ and $\pi^+\pi^-J/\psi$ samples are analyzed simultaneously for the first time.

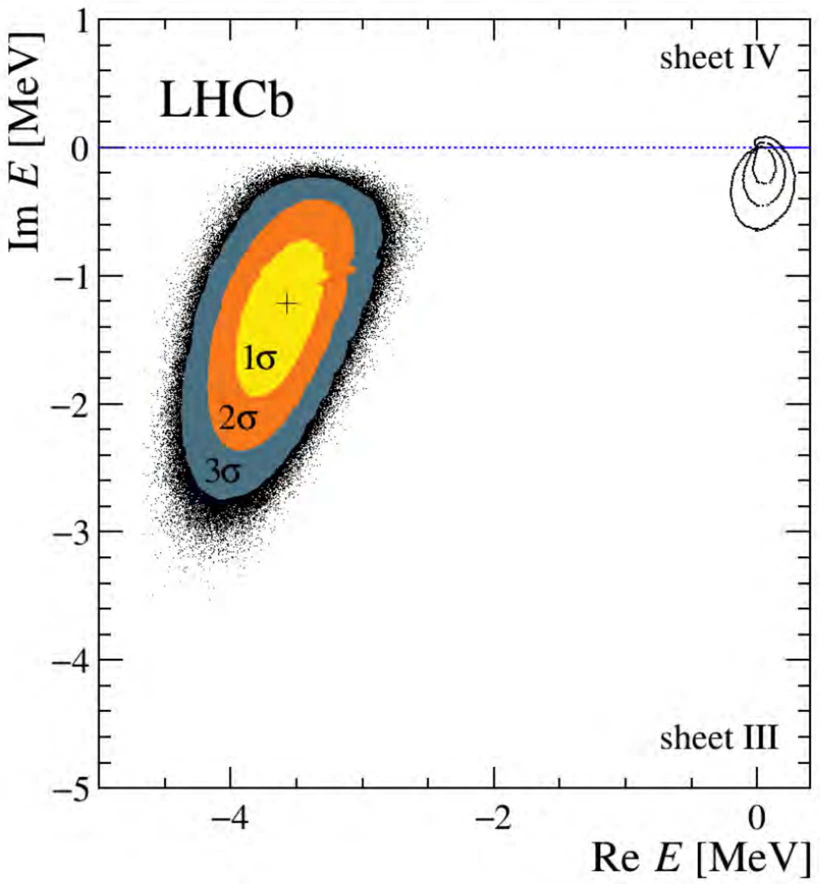
- The result is consistent with the molecular picture. Due to the uncertainty, the presence of a compact component can not be excluded.

- Future data taking and more measurements of the $X(3872)$ decay modes will help.

Thanks!

Backup slides





$$k^{+} = -13.9 + 8.8\, i \text{ MeV}$$
$$k^{-} = 14.1 - 84.5\, i \text{ MeV}$$

Estimation of the width of D^{*0}

The Lagrangian for $D^* \to D + \pi$ under HQSS is

$$\mathcal{L} = -\frac{g}{f_\pi} < \overline{H_a} H_b \gamma^\mu \gamma^5 > \partial_\mu M_{ba},$$

Where H is the heavy meson fields and the M is the matrix of pseudoscalar mesons. The partial width of $D^{*+} \to D^0 \pi^+$ reads

$$\Gamma_c = \frac{|p_\pi|}{8\pi M_{D^{*+}}^2} \times \frac{1}{3} \frac{4g^2}{f_\pi^2} |p_\pi^2| M_{D^{*+}} M_{D^0} .$$

For $D^{*0} \to D^0 \pi^0$,

$$\Gamma_0 = \frac{|p'_\pi|}{8\pi M_{D^{*0}}^2} \times \frac{1}{3} \frac{2g^2}{f_\pi^2} |p'_\pi|^2 M_{D^{*0}} M_{D^0} = \frac{|p'_\pi|^3 M_{D^{*+}}}{2|p_\pi|^3 M_{D^{*0}}} \times \Gamma_c$$

One obtains the strong decay partial width $\Gamma_0 = 35.8$ keV, the total width of D^{*0} is estimated to be

$$\Gamma_{D^{*0}} = \frac{\Gamma_0}{64.7\%} \approx 55.4 \text{ keV}$$