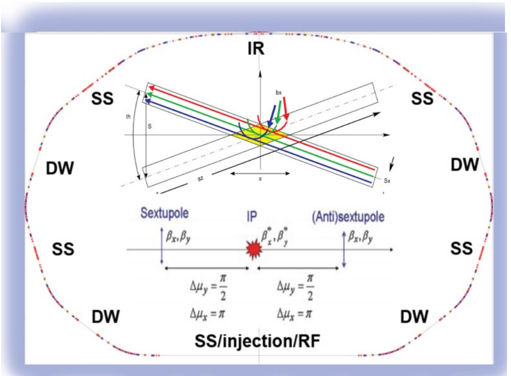
τ Physics
Opportunities at the STCF

Antonio Pich

IFIC, Univ. Valencia - CSIC



ITP CAS, Beijing, China
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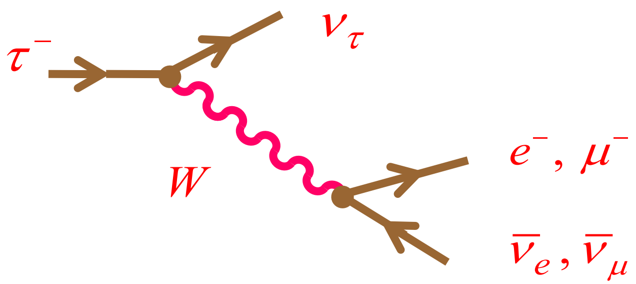
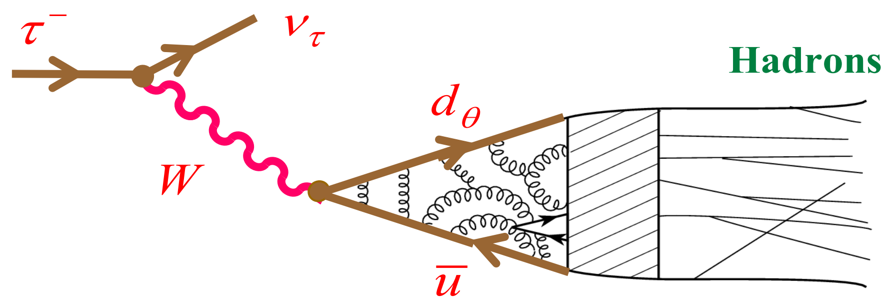
A.P., Prog. Part. Nucl. Phys. 75 (2014) 41

τ Physics

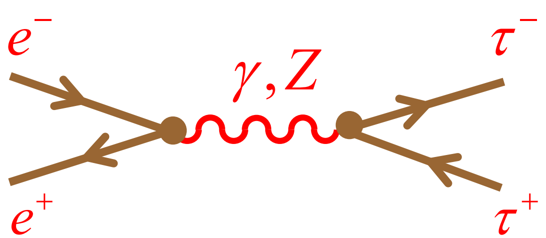
Decay



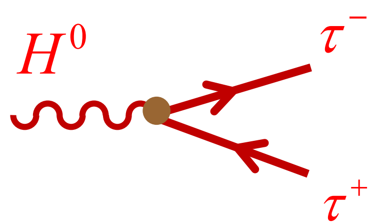
QCD

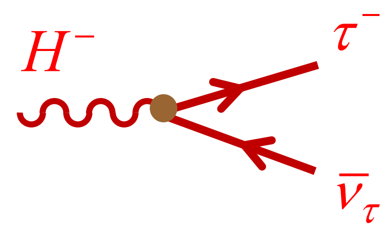


Production



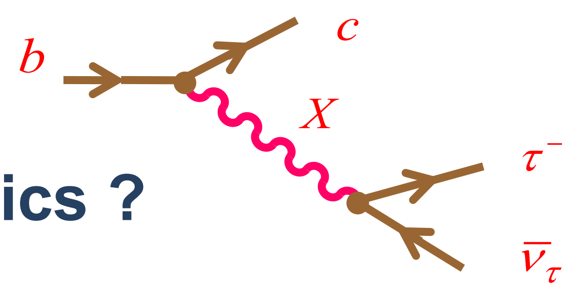
Higgs Interactions

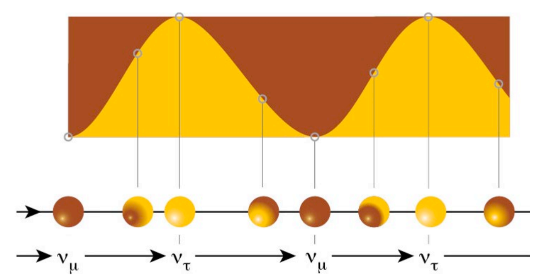




?

New Physics ?





Neutrinos

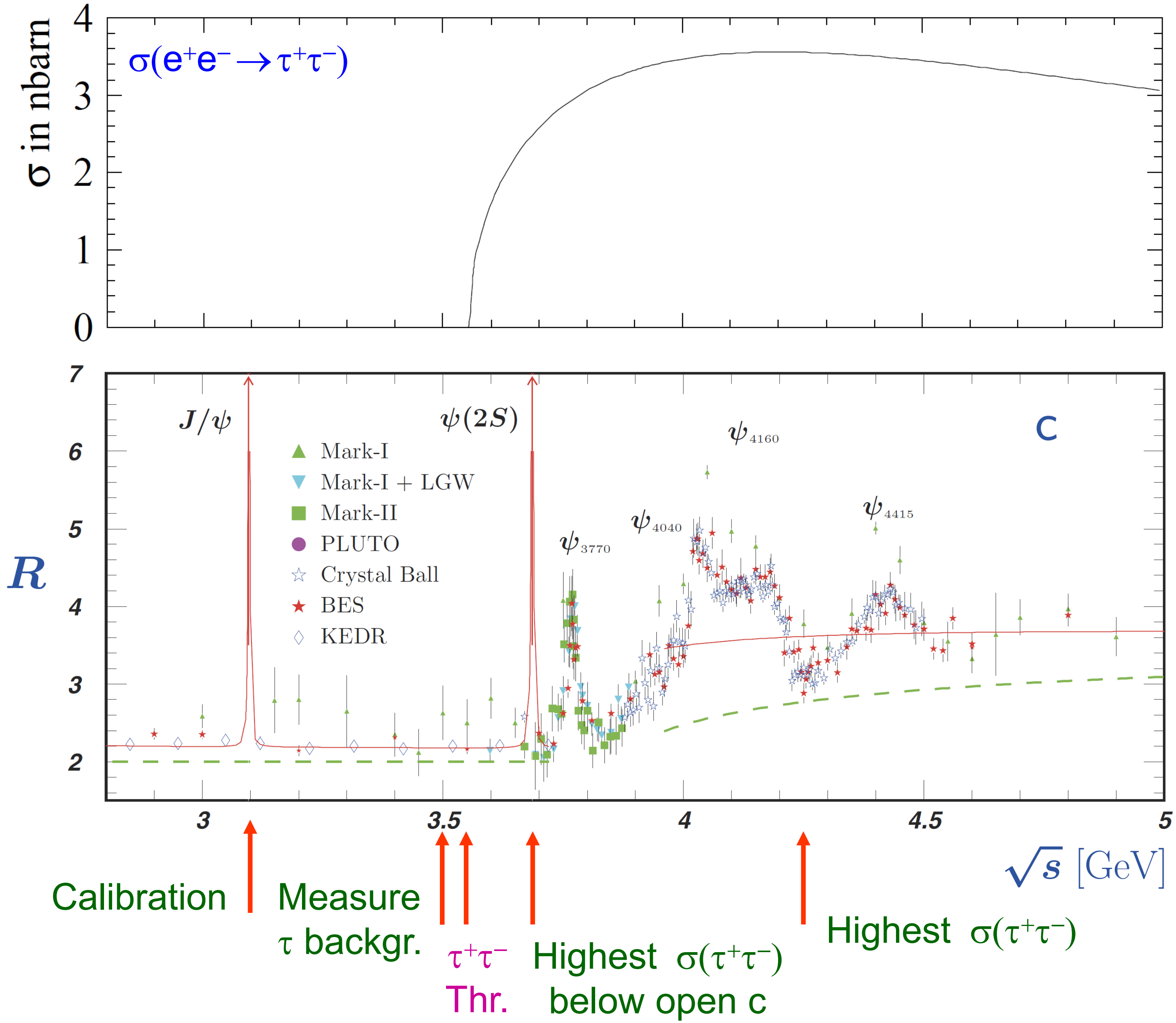
τ Data Samples

ALEPH:	$3.3 \cdot 10^5$	reconstructed τ decays
BaBar / Belle:	$1.4 \cdot 10^9$	$\tau^+\tau^-$ pairs
Belle-II:	$4.6 \cdot 10^{10}$	$\tau^+\tau^-$ pairs
sτcF:	$2.1 \cdot 10^{10}$	$\tau^+\tau^-$ pairs (10^8 near threshold)

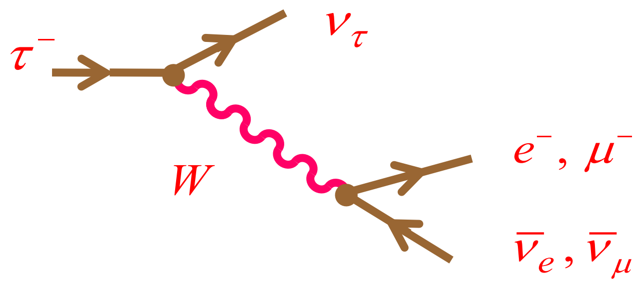
Luminosity (10^{35} cm^{-2} s^{-1}) is important. Systematics also!

Advantages of the threshold región:

- Ability to measure backgrounds (running below threshold)
- Free of heavy quark backgrounds
- Single-Tagging ➡ Precise measurement of absolute branching fractions
- Monochromatic spectra for two-body decays (π, K)


σ in nbarn
σ(e⁺e⁻ → τ⁺τ⁻)
0
1
2
3
4
R
2
3
4
5
6
7
J/ψ
ψ(2S)
ψ3770
ψ4040
ψ4160
ψ4415
c
Mark-I
Mark-I + LGW
Mark-II
PLUTO
Crystal Ball
BES
KEDR
3
3.5
4
4.5
5
√s [GeV]
Calibration
Measure τ backgr.
τ⁺τ⁻ Thr.
Highest σ(τ⁺τ⁻) below open c
Highest σ(τ⁺τ⁻)

LEPTONIC DECAYS

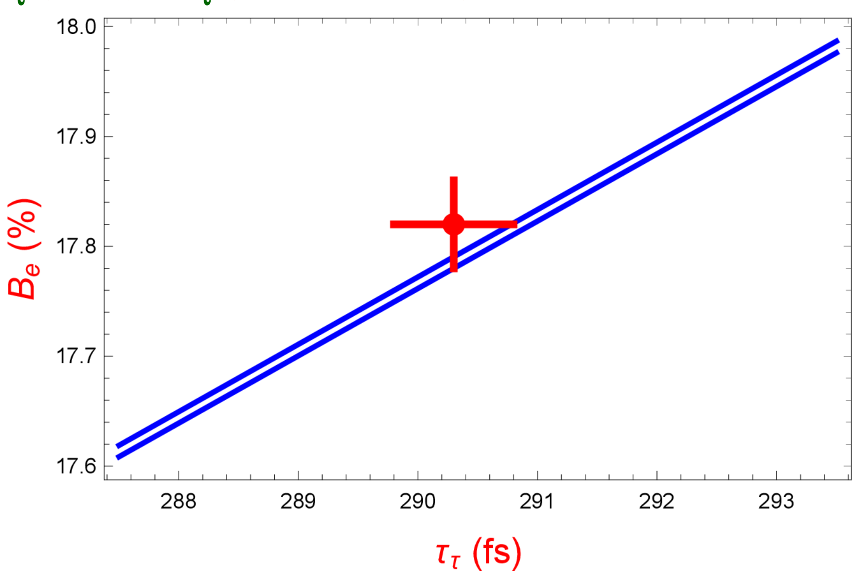


$$\Gamma(\tau \to \nu_\tau \, l \, \bar{\nu}_l) = \frac{G_F^2 m_\tau^5}{192\pi^3} f(m_l^2/m_\tau^2) \left(1+\delta_{RC}\right)$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

$$B_e = \frac{B_\mu}{0.972564 \pm 0.000003} = \frac{\tau_\tau}{(1632.3 \pm 0.5)\times 10^{-15}\,\text{s}}$$

τ_τ (Belle), m_τ (BesIII, BelleII)



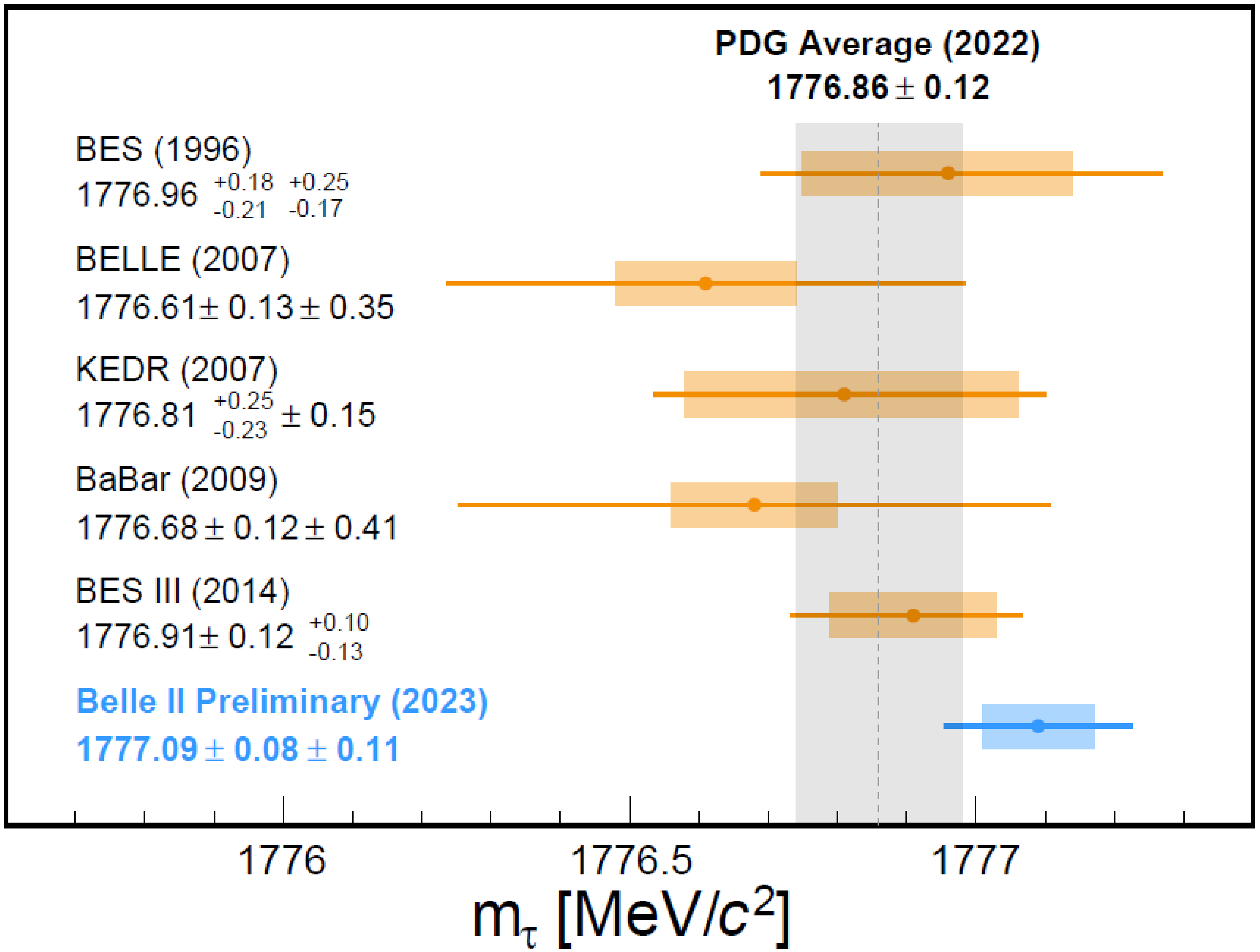
$$\left(B_\mu/B_e\right)_{\text{exp}} = 0.9762 \pm 0.0028$$

Non-BF: 0.9725 ± 0.0039

BaBar '10: 0.9796 ± 0.0039

$$B_e^{\text{univ}} = \left(17.812 \pm 0.022\right)\%$$

Preliminary Belle-II measurement of m_τ

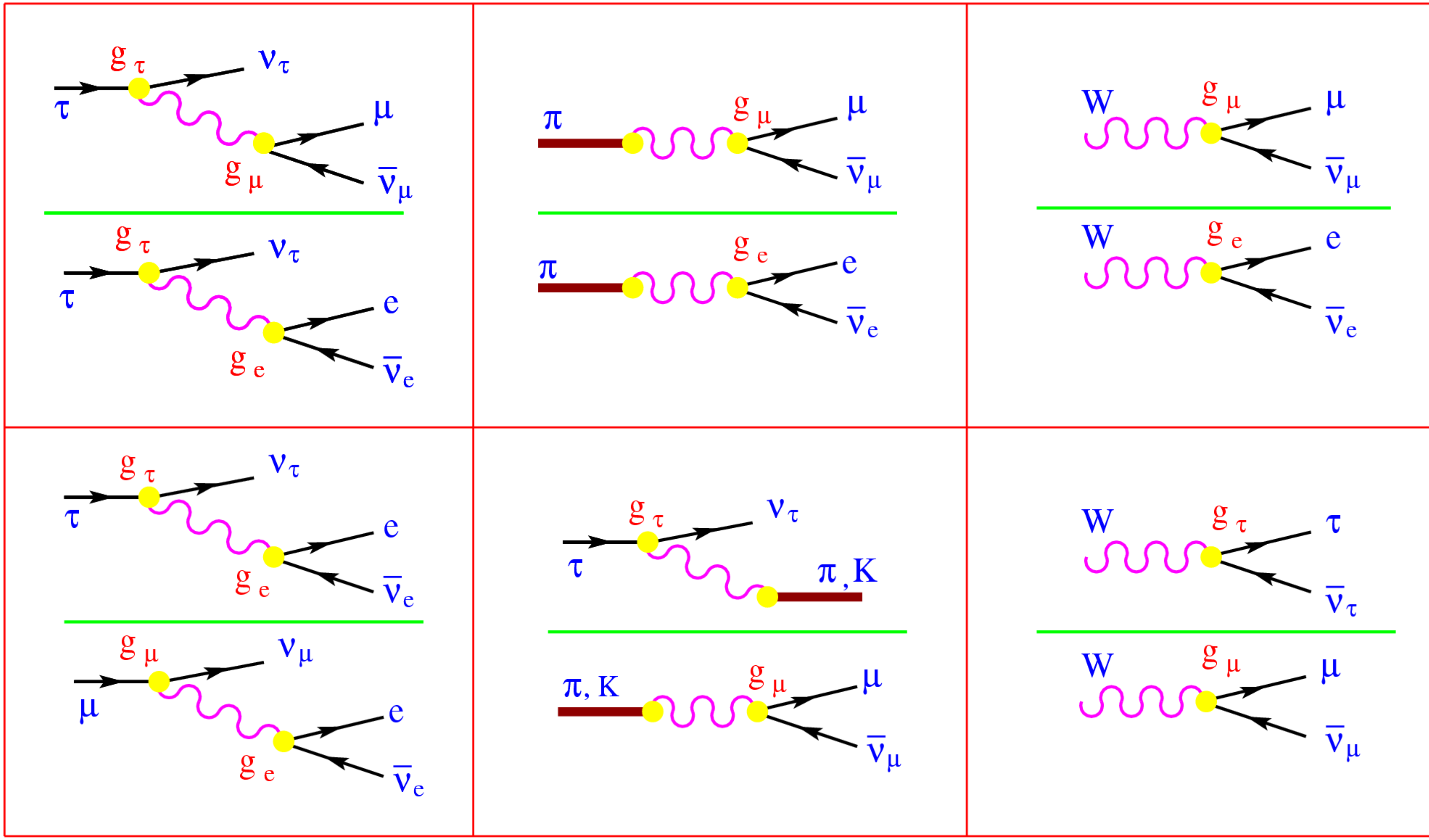


$m_\tau = (1776.96 \pm 0.09)$ MeV

Lepton Universality

$$\frac{g_\mu}{g_e}$$

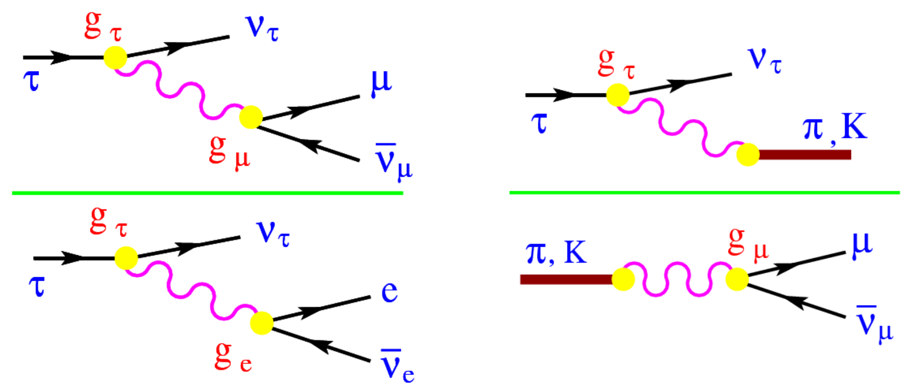
$$\frac{g_\tau}{g_\mu}$$



Lepton Universality

$$|g_\mu / g_e|$$

$B_{\tau\to\mu}/B_{\tau\to e}$	1.0019 ± 0.0014
$B_{\pi\to\mu}/B_{\pi\to e}$	1.0010 ± 0.0009
$B_{K\to\mu}/B_{K\to e}$	0.9978 ± 0.0018
$B_{K\to\pi\mu}/B_{K\to\pi e}$	1.0010 ± 0.0025
$B_{W\to\mu}/B_{W\to e}$	1.001 ± 0.003



$$|g_\tau / g_e|$$

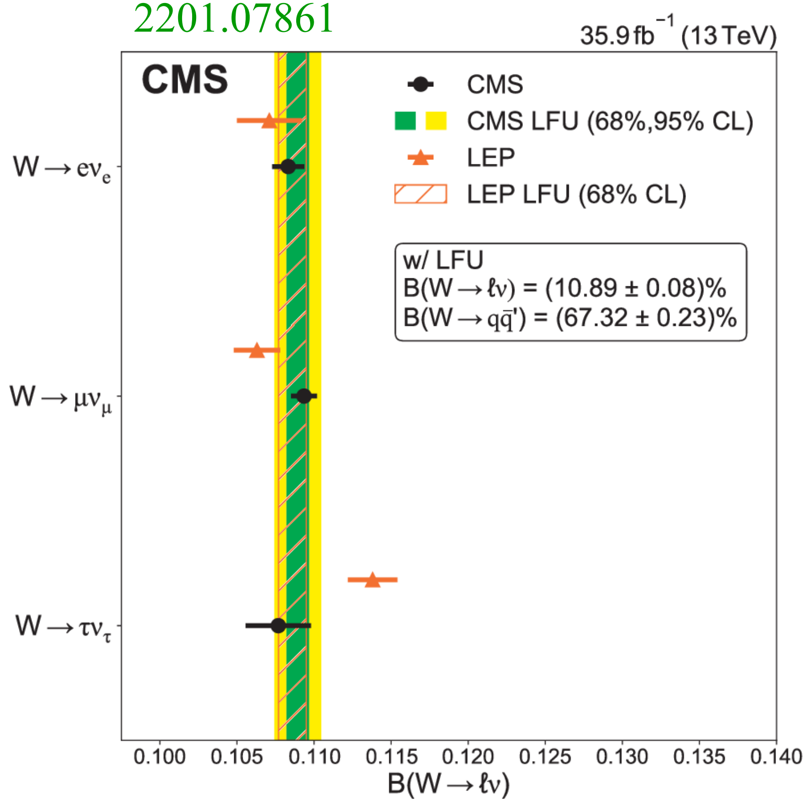
$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0027 ± 0.0014
$B_{W\to\tau}/B_{W\to e}$	1.007 ± 0.010

$$|g_\tau / g_\mu|$$

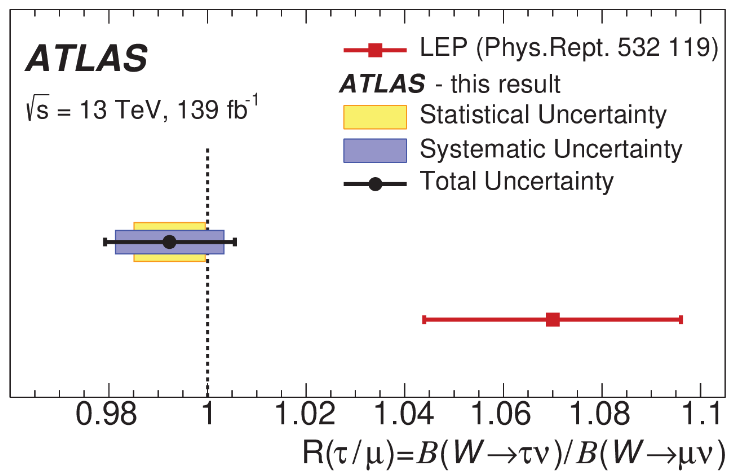
$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	1.0009 ± 0.0014
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	0.9959 ± 0.0038
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.986 ± 0.008
$B_{W\to\tau}/B_{W\to\mu}$	1.001 ± 0.010

Lepton Universality in W decays

2201.07861



2007.14040



	CMS	LEP	ATLAS	LHCb	CDF	D0
$R_{\mu/e}$	1.009 ± 0.009	0.993 ± 0.019	1.003 ± 0.010	0.980 ± 0.012	0.991 ± 0.012	0.886 ± 0.121
$R_{\tau/e}$	0.994 ± 0.021	1.063 ± 0.027	—	—	—	—
$R_{\tau/\mu}$	0.985 ± 0.020	1.070 ± 0.026	0.992 ± 0.013	—	—	—
$R_{\tau/\ell}$	1.002 ± 0.019	1.066 ± 0.025	—	—	—	—

Lorentz Structure: $\ell^- \to \ell'^- \bar{\nu}_{\ell'} \nu_\ell$

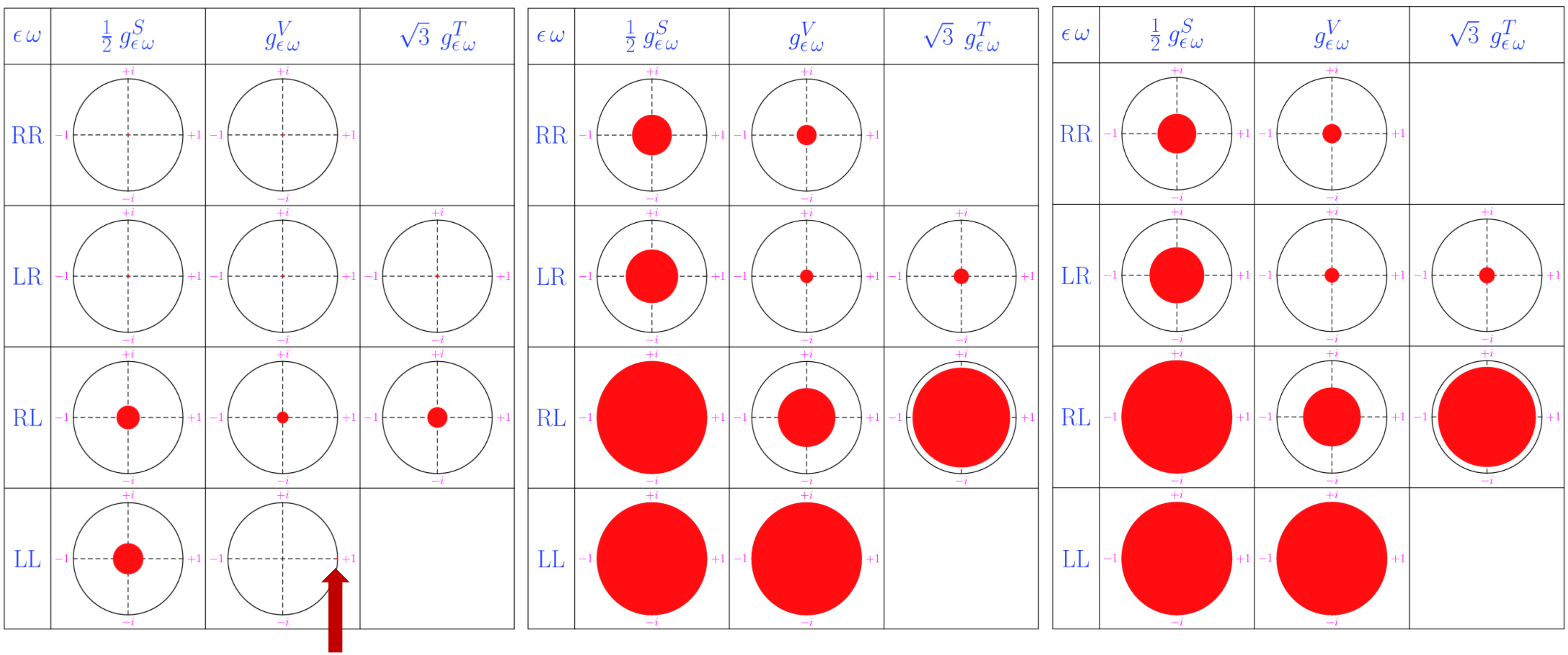
Effective Hamiltonian:

$$\mathcal{H} = 4\,\frac{G_{\ell'\ell}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\overline{\ell'_\epsilon}\Gamma^n (\nu_{\ell'})_\sigma\right] \left[\overline{(\nu_\ell)_\lambda}\Gamma_n \ell_\omega\right]$$

Normalization:

$$\Gamma \propto \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2 + |g^S_{LR}|^2 + |g^S_{LL}|^2\right) + 3\left(|g^T_{RL}|^2 + |g^T_{LR}|^2\right) + \left(|g^V_{RR}|^2 + |g^V_{RL}|^2 + |g^V_{LR}|^2 + |g^V_{LL}|^2\right) \equiv 1$$

$\mu \to e\,\bar{\nu}_e \nu_\mu$ $\qquad$ $\tau \to \mu\,\bar{\nu}_\mu \nu_\tau$ $\qquad$ $\tau \to e\,\bar{\nu}_e \nu_\tau$



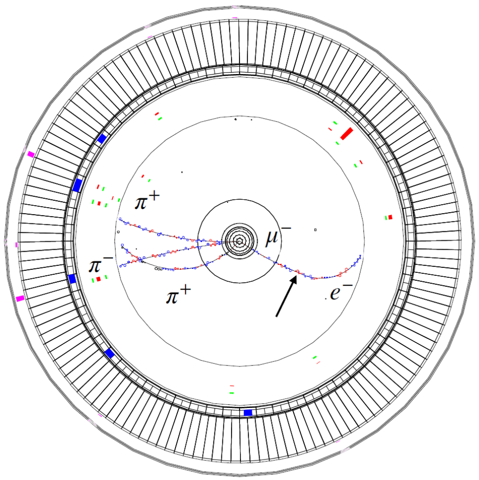
$|g^V_{LL}| > 0.960$ $\quad$ (90% CL)

High-precision τ data needed!

μ^- Longitudinal Polarization in $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$

Probability to decay into a right-handed muon:

$$Q_{\mu_R} = Q_{RR} + Q_{RL} = \frac{1}{4}\left(|g^S_{RR}|^2 + |g^S_{RL}|^2\right) + 3|g^T_{RL}|^2 + |g^V_{RR}|^2 + |g^V_{RL}|^2 = \frac{1}{2}(1-\xi')$$



MC $\tau^+\tau^-$ event

Belle, 2303.10570

Tiny probability of muon decaying inside the detector compensated by huge statistics

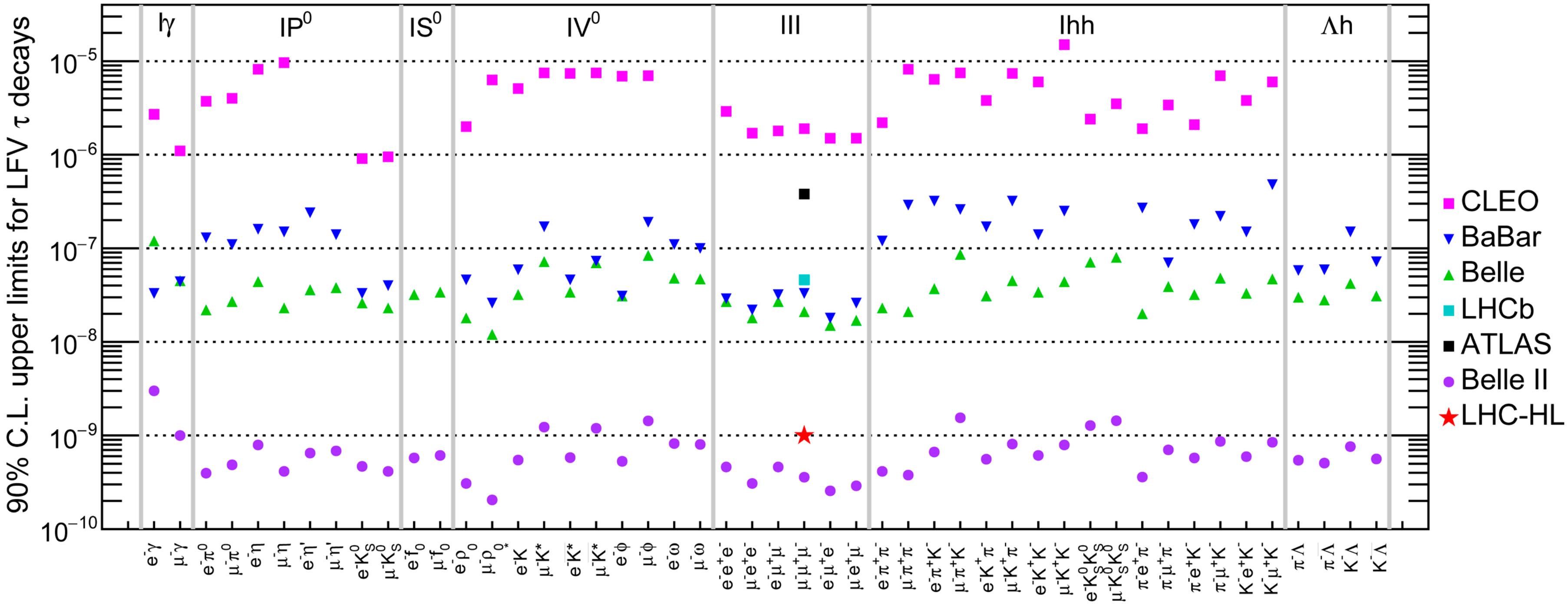
$$\xi' = 0.22 \pm 0.94 \pm 0.42$$

$$\Rightarrow \quad Q_{\mu_R} \leq 1.23 \quad (90\%\ \mathrm{CL})$$

Not yet constraining. Error dominated by statistics…

Bounds on Lepton Flavour Violation

τ Decays (90% CL)



$\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ (MEG, 90% CL) , $\mathrm{Br}(\mu \to 3e) < 1.0 \times 10^{-12}$ (SINDRUM, 90% CL)

$\mathrm{Br}(K_L \to \mu e) < 4.7 \times 10^{-12}$ (BNL-E871, 90% CL) , $\mathrm{Br}(K^+ \to \pi^+\mu^+e^-) < 1.3 \times 10^{-11}$ (BNL-E865, 90% CL)

$\mathrm{Br}(B^0 \to e\mu) < 1.0 \times 10^{-9}$ (LHCb, 90% CL) , $\mathrm{Br}(D^0 \to e\mu) < 1.3 \times 10^{-8}$ (LHCb, 90% CL)

$\mathrm{Br}(Z^0 \to e\mu) < 7.5 \times 10^{-7}$ (ATLAS, 95% CL) , $\mathrm{Br}(H \to e\mu) < 6.1 \times 10^{-5}$ (ATLAS, 95% CL)

$\mathrm{Br}(Z^0 \to e\tau) < 5.0 \times 10^{-6}$ (ATLAS, 95% CL) , $\mathrm{Br}(H \to e\tau) < 2.2 \times 10^{-3}$ (CMS, 95% CL)

$\mathrm{Br}(Z^0 \to \mu\tau) < 6.5 \times 10^{-6}$ (ATLAS, 95% CL) , $\mathrm{Br}(H \to \mu\tau) < 1.5 \times 10^{-3}$ (CMS, 95% CL)

CP Asymmetry

$$A_\tau \equiv \frac{\Gamma\left(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S \nu_\tau\right)} = \left(-3.6 \pm 2.3 \pm 1.1\right) \cdot 10^{-3}$$

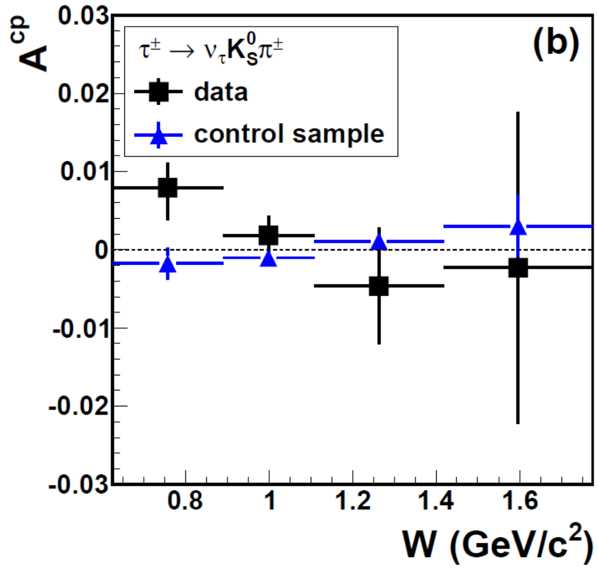
BaBar'11 $\left(\geq 0\,\pi^0\right)$

$$A_\tau^{\rm SM}\left(\tau^+ \to \pi^+ K_S \bar{\nu}_\tau\right) = \left(3.6 \pm 0.1\right) \cdot 10^{-3}$$

Bigi-Sanda, Grossman-Nir

2.8 σ discrepancy

Belle does not see any asymmetry at the 10^{-2} level



$$A_i^{\rm CP} \simeq \left\langle \cos\beta \cos\psi \right\rangle_i^{\tau^-} - \left\langle \cos\beta \cos\psi \right\rangle_i^{\tau^+}$$

bins (i) of $W = \sqrt{Q^2}$

$\beta = K_S$ direction in hadronic rest frame

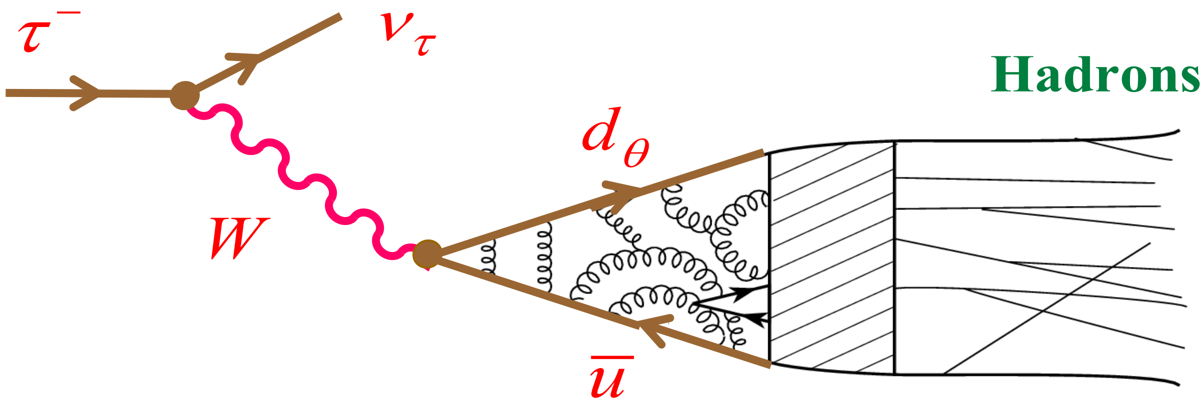
$\psi = \tau$ direction

BaBar signal incompatible (with EFT)
with other sets of flavour data

Cirigliano-Crivellin-Hoferichter, 1712.06595

Rendón-Roig-Toledo, 1902.08143

HADRONIC TAU DECAY



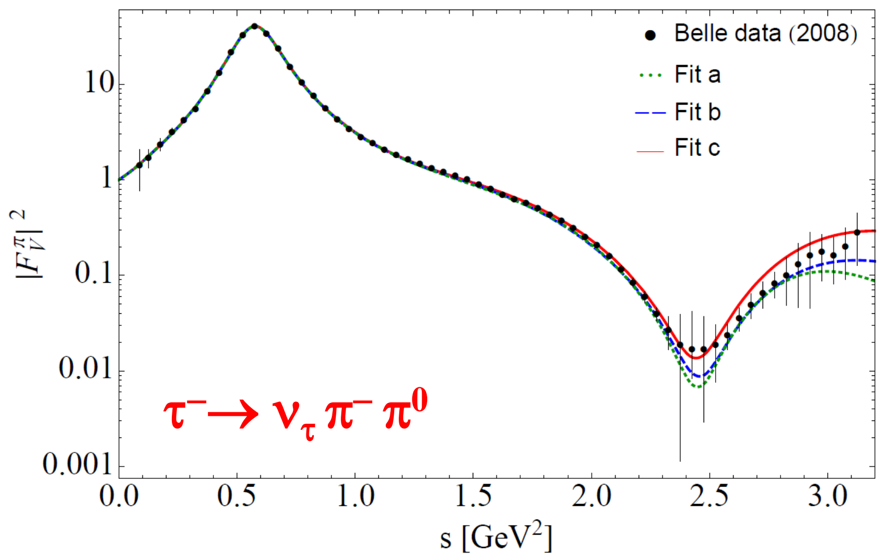
$$d_\theta = V_{ud}\, d + V_{us}\, s$$

Only lepton massive enough to decay into hadrons

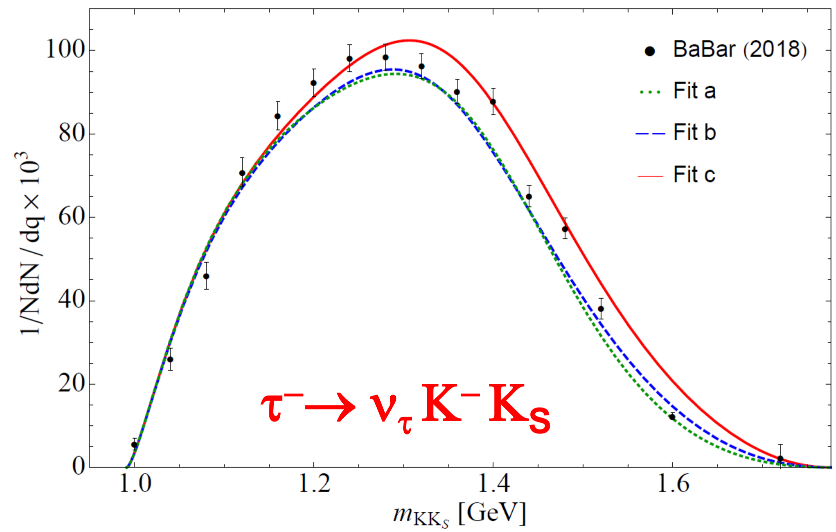
$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau\, e^-\, \overline{\nu}_e\right)} \approx N_C \quad ; \quad R_\tau = \frac{1 - B_e - B_\mu}{B_e^{\text{univ}}} = 3.6381 \pm 0.0075$$

$$R_\tau = \frac{1}{B_e^{\text{univ}}} - 1.972564 = 3.6417 \pm 0.0070 \quad ; \quad R_\tau = \frac{\text{Br}\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{B_e^{\text{univ}}} = 3.6343 \pm 0.0082$$

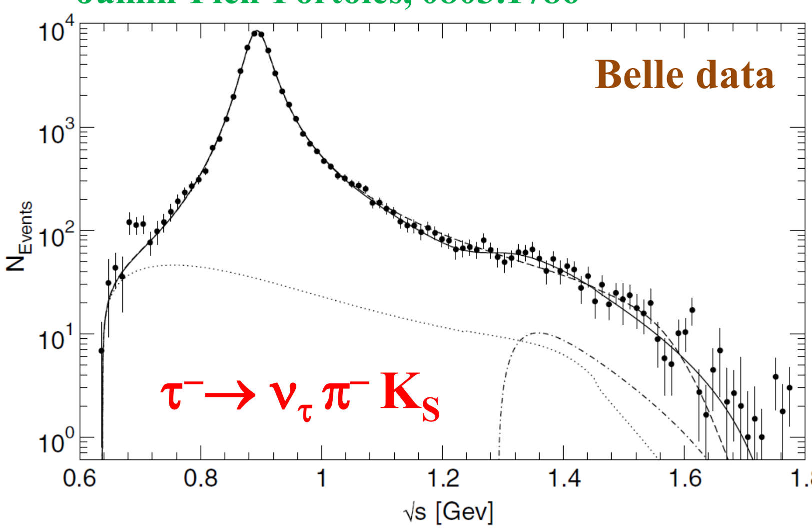
Invariant Mass Spectra



Gónzàlez-Solís – Roig, 1902.02273



Jamin-Pich-Portolés, 0803.1786

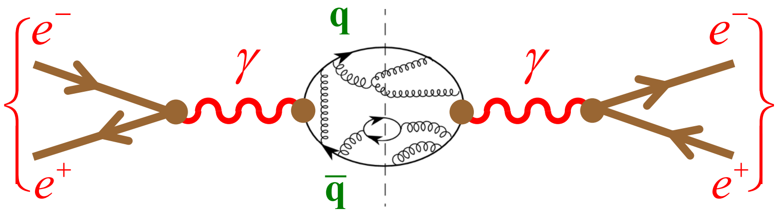


Useful tests of QCD Dynamics

Form Factors

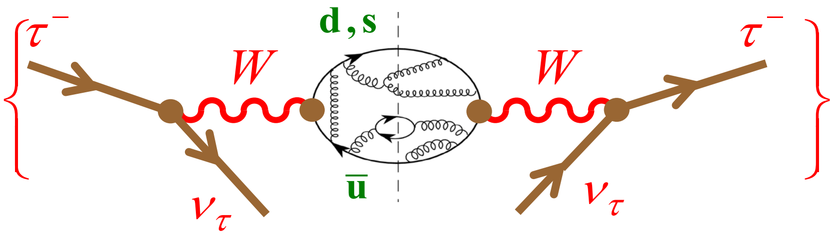
Non-perturbative parameters

Resonance Chiral Theory (RχT)

$\sigma \sim \mathrm{Im}$ 

$$\frac{\sigma(e^+e^- \to \mathrm{had})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi\, \mathrm{Im}\,\Pi_{\mathrm{em}}(s)$$

$$\Pi^{\mu\nu}_{\mathrm{em}}(q) \equiv i \int d^4x\; e^{iqx}\, \langle 0 | T[J^\mu_{\mathrm{em}}(x) J^\nu_{\mathrm{em}}(0)] | 0 \rangle \;=\; \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi_{\mathrm{em}}(q^2)$$

$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}$ 

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \mathrm{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right] + |V_{us}|^2 \left[\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right]$$

$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i \int d^4x\; e^{iqx}\, \langle 0 | T[J^\mu_{ij}(x) J^\nu_{ij}(0)^\dagger] | 0 \rangle = \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi^{(1)}_{ij,J}(q^2) + q^\mu q^\nu\, \Pi^{(0)}_{ij,J}(q^2)$$

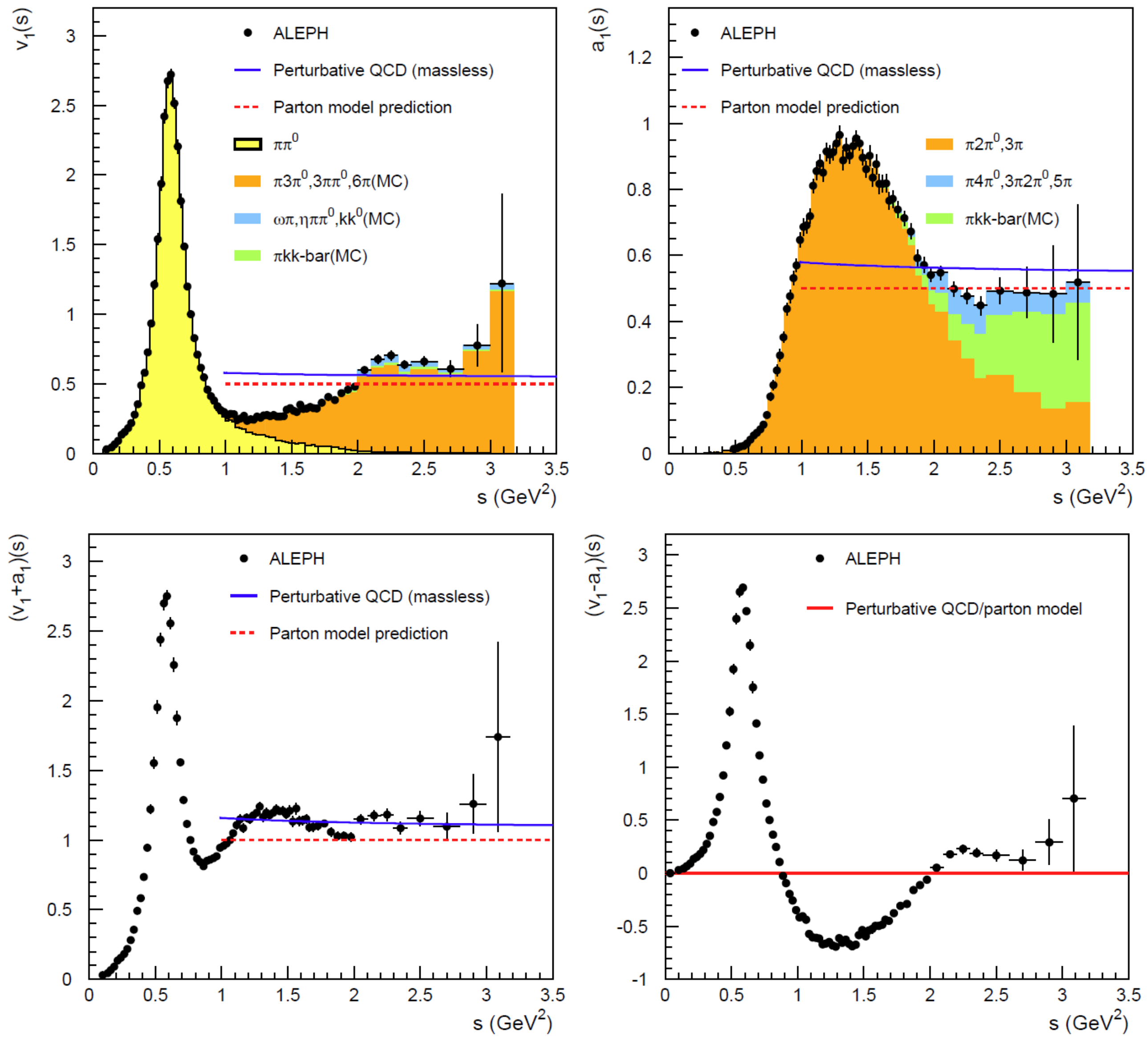
SPECTRAL FUNCTIONS

Davier et al, 1312.1501

$$v_1(s) = 2\pi\, \mathrm{Im}\, \Pi^{(0+1)}_{ud,V}(s)$$

$$a_1(s) = 2\pi\, \mathrm{Im}\, \Pi^{(0+1)}_{ud,A}(s)$$

Better data needed

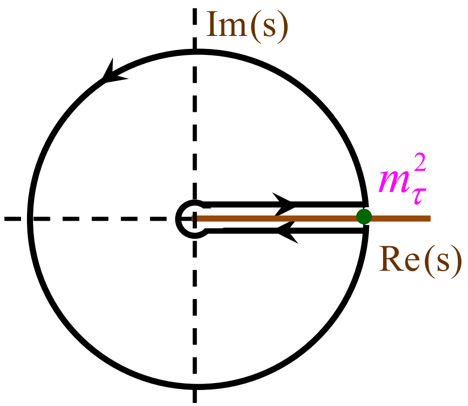


QCD Prediction of R_τ

Braaten-Narison-Pich'92

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\text{Im}\,\Pi^{(1)}(x m_\tau^2) + \text{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$

$x \equiv s/m_\tau^2$



$$R_\tau = 6\pi i \oint_{|x|=1} dx\,(1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2)\right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

OPE

$$R_\tau = N_C\, S_{\text{EW}} \left(1 + \delta_{\text{P}} + \delta_{\text{NP}}\right) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$S_{\text{EW}} = 1.0201\ (3)$; $\delta_{\text{NP}} = -0.0064 \pm 0.0013$

Marciano-Sirlin, Braaten-Li, Erler — Fitted from data (Davier et al)

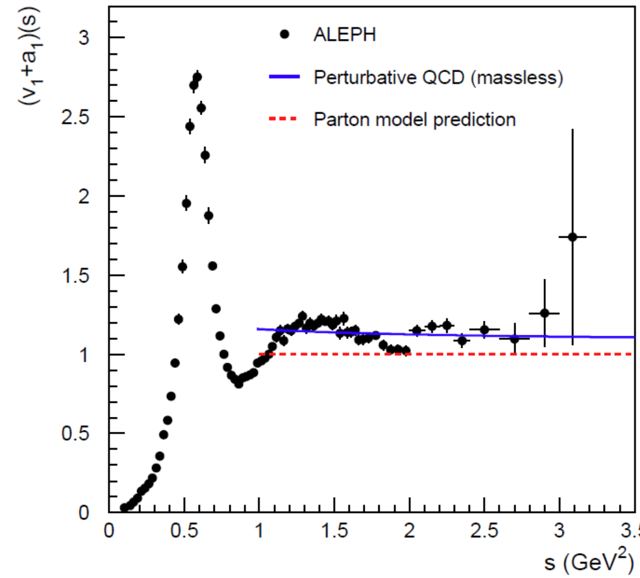
$$\delta_{\text{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\% \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Baikov-Chetyrkin-Kühn

Spectral Function Distribution

Moments:

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$



Sensitivity to power corrections (k,l)

The non-perturbative contribution to R_τ can be obtained from the invariant-mass distribution of the final hadrons

Detailed analyses by ALEPH, CLEO and OPAL

$$\delta_{\rm NP} = -0.0064 \pm 0.0013$$

$$\alpha_s(m_\tau^2) = 0.332 \pm 0.005_{\rm exp} \pm 0.011_{\rm th}$$

Davier et al., 1312.1501
(ALEPH data)

Exhaustive Analysis of ALEPH Data

Rodríguez-Sánchez, Pich, 1605.06830

Method (V + A)	$\alpha_s(m_\tau^2)$		
	CIPT	FOPT	Average
ALEPH moments[1]	$0.339\,^{+0.019}_{-0.017}$	$0.319\,^{+0.017}_{-0.015}$	$0.329\,^{+0.020}_{-0.018}$
Mod. ALEPH moments[2]	$0.338\,^{+0.014}_{-0.012}$	$0.319\,^{+0.013}_{-0.010}$	$0.329\,^{+0.016}_{-0.014}$
$A^{(2,m)}$ moments[3]	$0.336\,^{+0.018}_{-0.016}$	$0.317\,^{+0.015}_{-0.013}$	$0.326\,^{+0.018}_{-0.016}$
s_0 dependence[4]	0.335 ± 0.014	0.323 ± 0.012	0.329 ± 0.013
Borel transform[5]	$0.328\,^{+0.014}_{-0.013}$	$0.318\,^{+0.015}_{-0.012}$	$0.323\,^{+0.015}_{-0.013}$
Combined value	$\mathbf{0.335 \pm 0.013}$	$\mathbf{0.320 \pm 0.012}$	$\mathbf{0.328 \pm 0.013}$

$$\longrightarrow \quad \alpha_s(M_Z^2) = 0.1197 \pm 0.0015$$

1) $\omega_{kl}(x) = (1+2x)\,(1-x)^{2+k}\,x^l \qquad (k,l) = (0,0),\ (1,0),\ (1,1),\ (1,2),\ (1,3)$

2) $\tilde{\omega}_{kl}(x) = (1-x)^{2+k}\,x^l \qquad (k,l) = (0,0),\ (1,0),\ (1,1),\ (1,2),\ (1,3)$

3) $\omega^{(2,m)}(x) = (1-x)^2 \sum_{k=0}^{m} (k+1)\,x^k = 1 - (m+2)\,x^{m+1} + (m+1)x^{m+2} \quad , \quad 1 \leq m \leq 5$

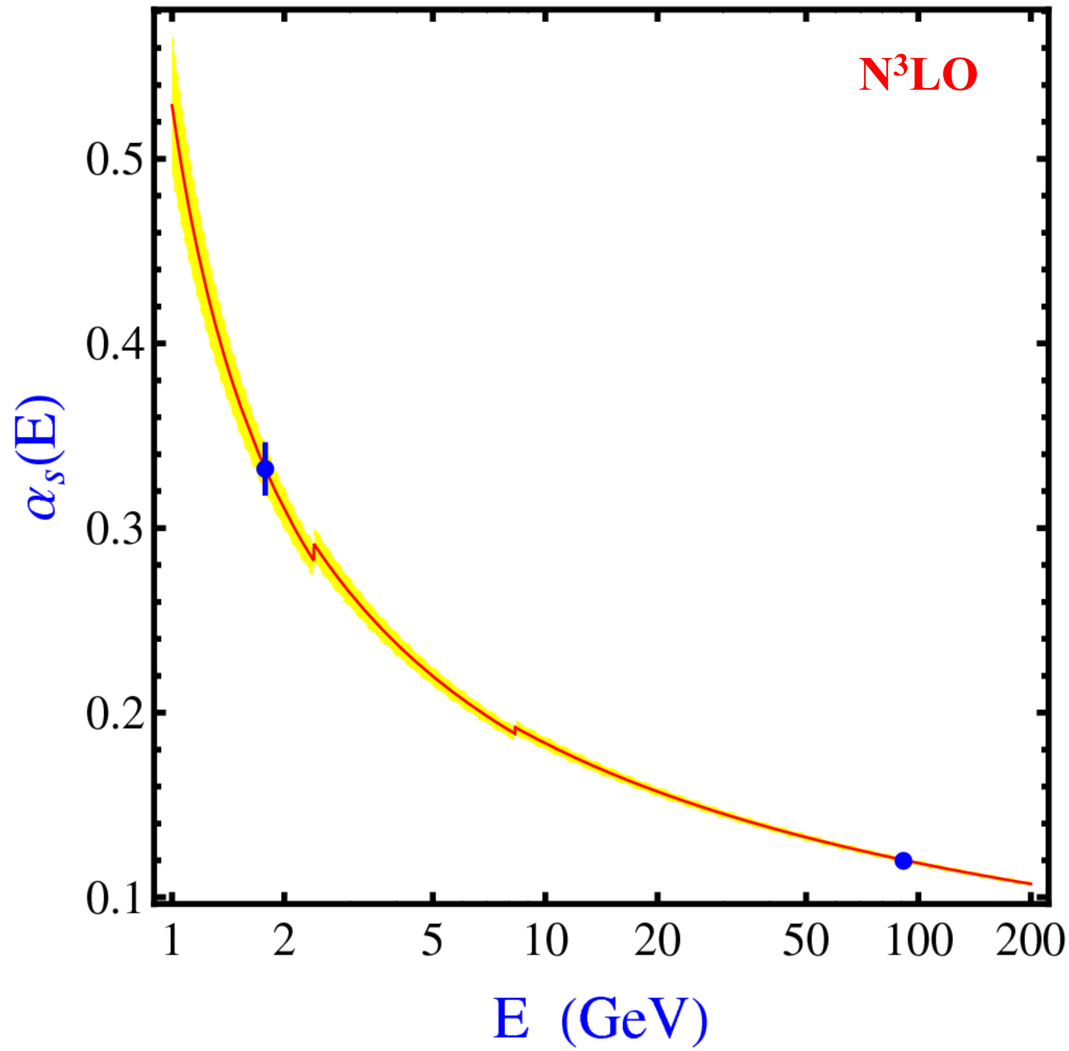
4) $\omega^{(2,m)}(x) \qquad 0 \leq m \leq 2 \quad ,$ 1 single moment in each fit

5) $\omega_a^{(1,m)}(x) = (1 - x^{m+1})\,\mathrm{e}^{-ax} \qquad 0 \leq m \leq 6$

α_s at N^{3}LO from τ and Z

Rodríguez-Sánchez, Pich, 1605.06830

$\alpha_s(m_\tau^2) = 0.328 \pm 0.013$

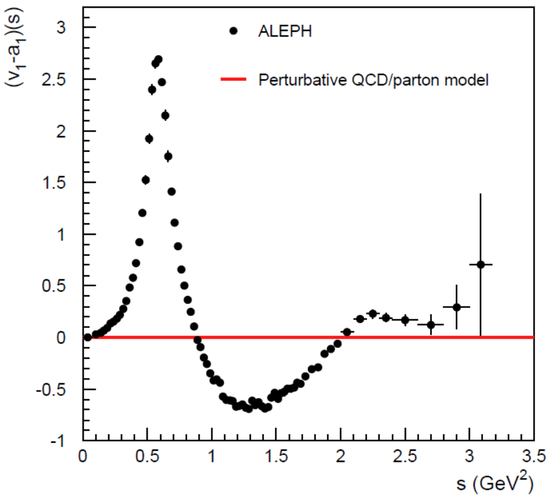


$\alpha_s(M_Z^2) = 0.1197 \pm 0.0015$

$\alpha_s(M_Z^2)_{Z\,\mathrm{width}} = 0.1199 \pm 0.0029$

Very precise test of Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0002 \pm 0.0015_\tau \pm 0.0029_Z$$



Chiral Sum Rules

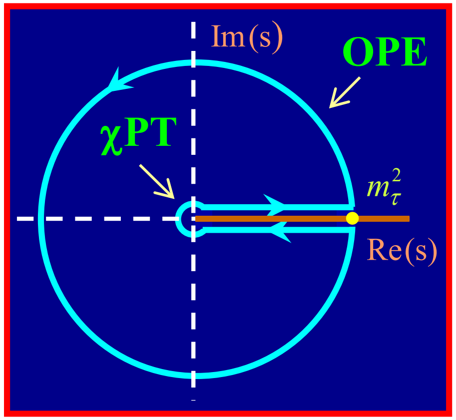
$$\Pi(s) \equiv \Pi_{VV}(s) - \Pi_{AA}(s)$$

Pure non-perturbative quantity

$$\lim_{s\to\infty} s^2\,\Pi(s) = 0 \quad\rightarrow\quad \Pi^{\rm OPE}(s) = -\frac{O_6}{s^3} + \frac{O_8}{s^4} - \cdots$$

χPT (s→0): $$\Pi(s) = \frac{2F^2}{s} - 8\,L_{10}^r(\mu^2) + \frac{1}{16\pi^2}\left(\frac{5}{3} - \ln\frac{-s}{\mu^2}\right) + 16\,C_{87}^r(\mu^2)\,\frac{s}{F^2} + \cdots$$

$$\int_{s_{\rm th}}^{s_0} ds\;\omega(s)\,\frac{1}{\pi}\,{\rm Im}\,\Pi(s) + \frac{1}{2\pi i}\oint_{|s|=s_0} ds\;\omega(s)\,\Pi(s) \;=\; 2\,f_\pi^2\,\omega(m_\pi^2) + {\rm Res}[\omega(s)\Pi(s), s=0]$$



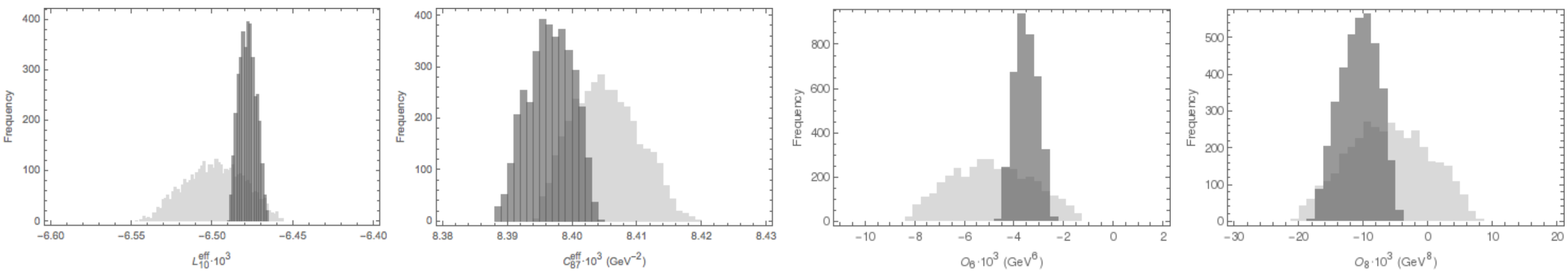
Statistical analysis:

González-Pich-Rodríguez, 1602.06112

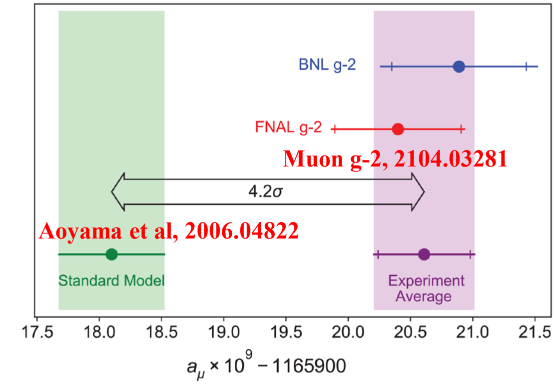
$$C_{87}^{\rm eff} = (8.40 \pm 0.18)\cdot 10^{-3}\ {\rm GeV}^{-2} \qquad O_6 = (-3.6 \pm 0.7)\cdot 10^{-3}\ {\rm GeV}^6$$

$$L_{10}^{\rm eff} = (-6.48 \pm 0.05)\cdot 10^{-3}\ . \qquad O_8 = (-1.0 \pm 0.4)\cdot 10^{-2}\ {\rm GeV}^8$$

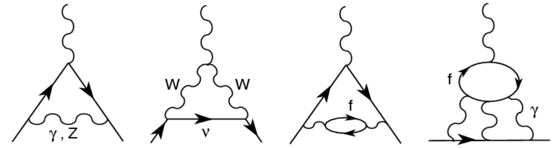
Non-pinched & pinched weights



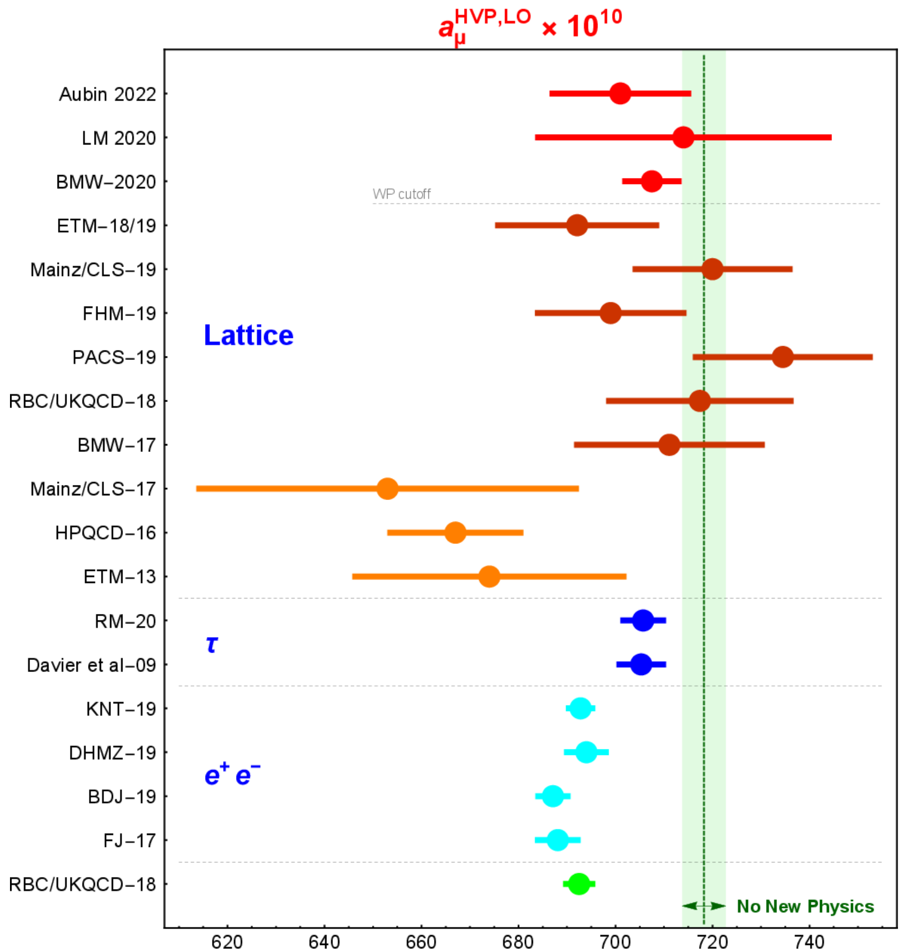
μ Anomalous Magnetic Moment

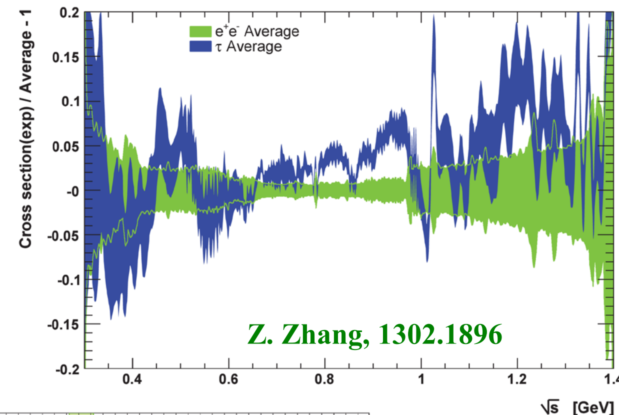


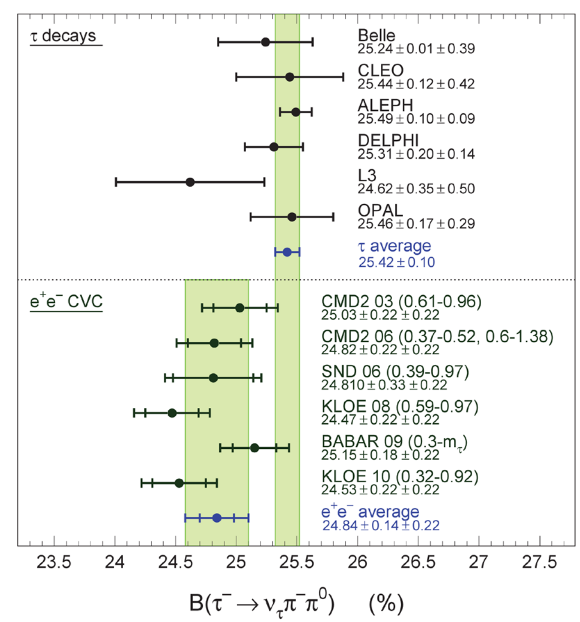
$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2 m_\mu^2}{9\pi^2} \int_{s_{\mathrm{th}}}^{\infty} \frac{ds}{s^2}\, \hat{K}(s)\, R(s)$$



Dominated (75%) by 2π

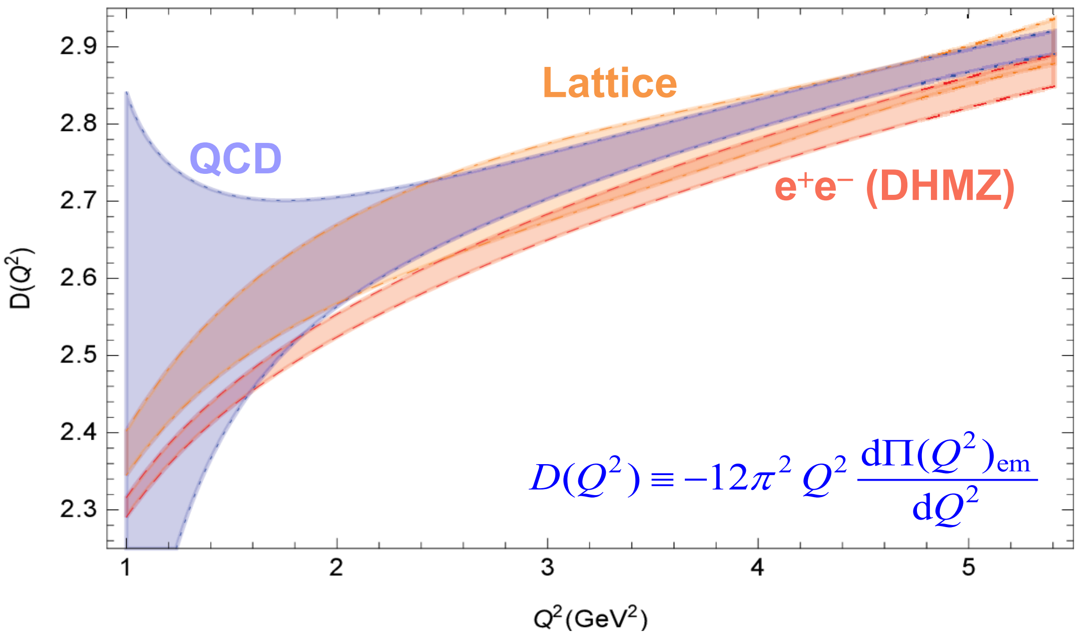






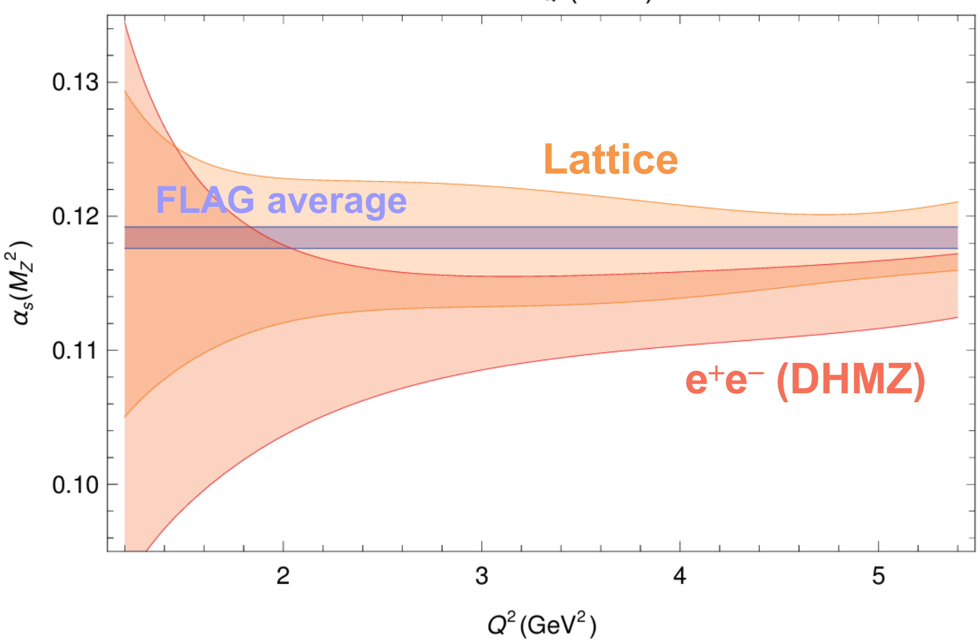
M. Davier
Tau 10

Euclidean Adler Function $(Q^2=-q^2)$



M. Davier, D. Díaz-Calderón, B. Malaescu, A. Pich, A. Rodríguez-Sánchez, Z. Zhang, 2302.01359

2σ discrepancy between e⁺e⁻ data & QCD

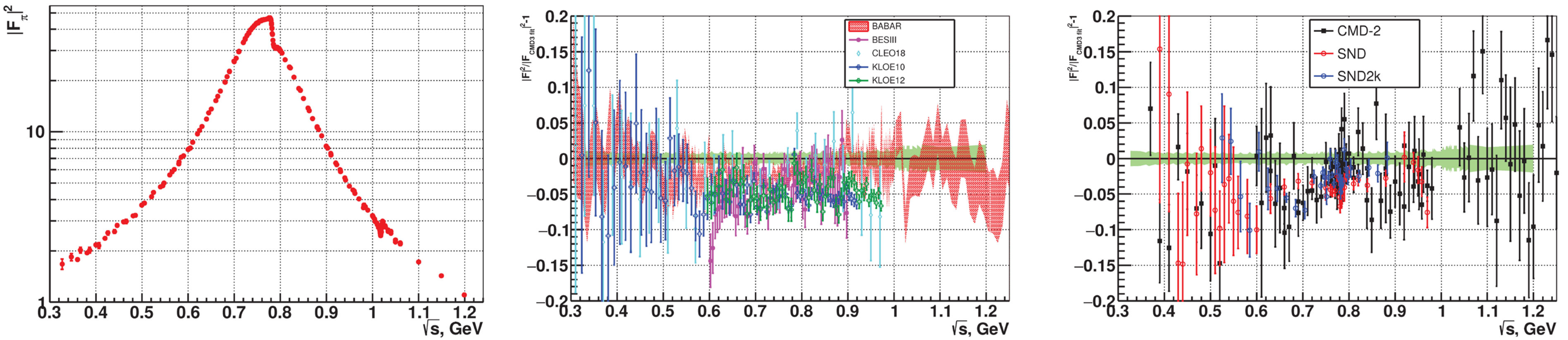


FLAG average: $\alpha_s(M_Z^2) = 0.1184 \pm 0.0008$

$$\alpha_s(M_Z^2) = \begin{cases} 0.1136 \pm 0.0025 & (e^+e^- \text{ data}) \\ 0.1179 \pm 0.0025 & (\text{Lattice}) \end{cases}$$

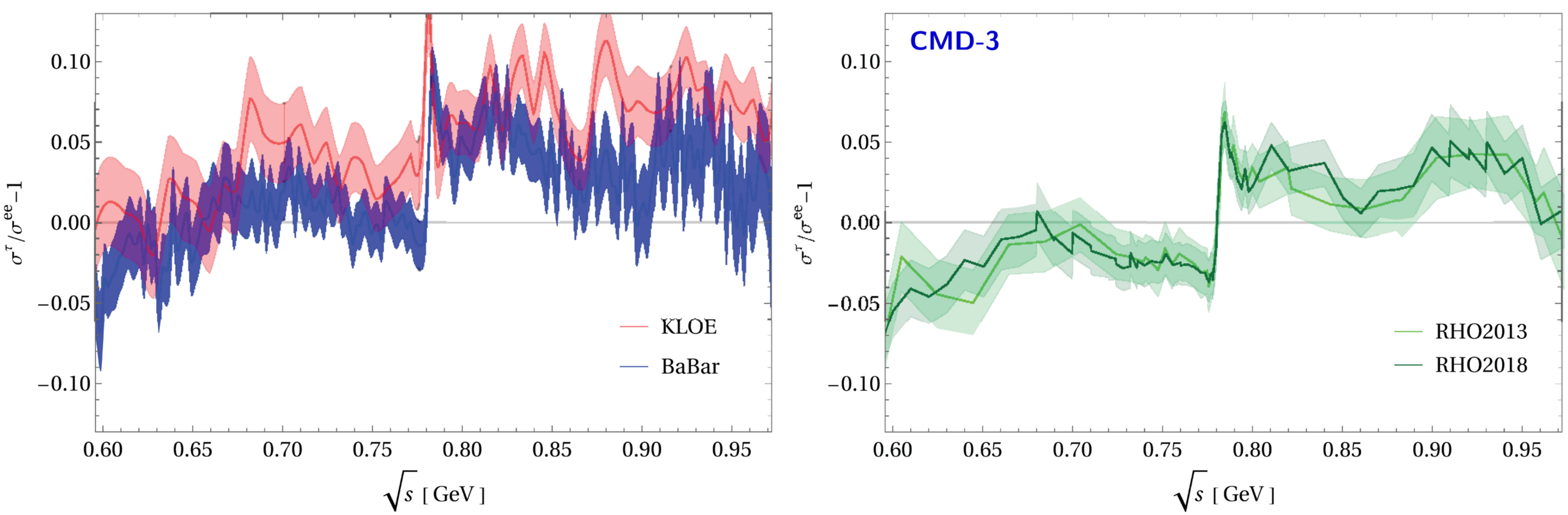
2023 CMD-3 $e^+e^- \to \pi^+\pi^-$ Data

2302.08834

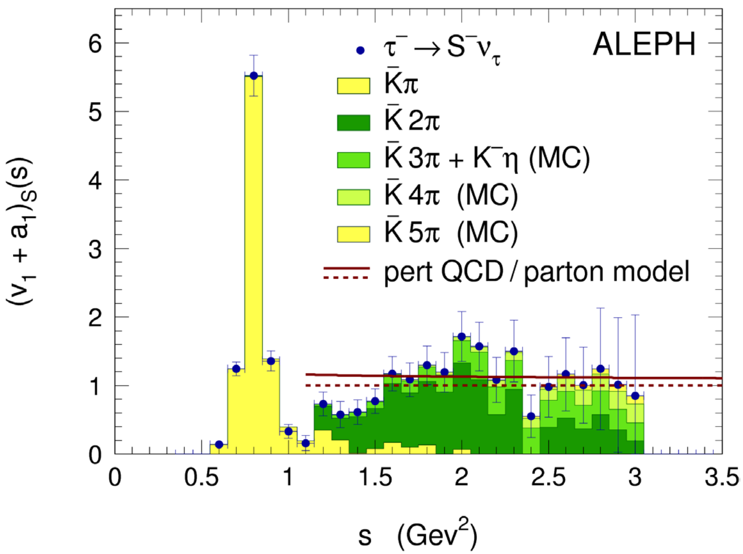


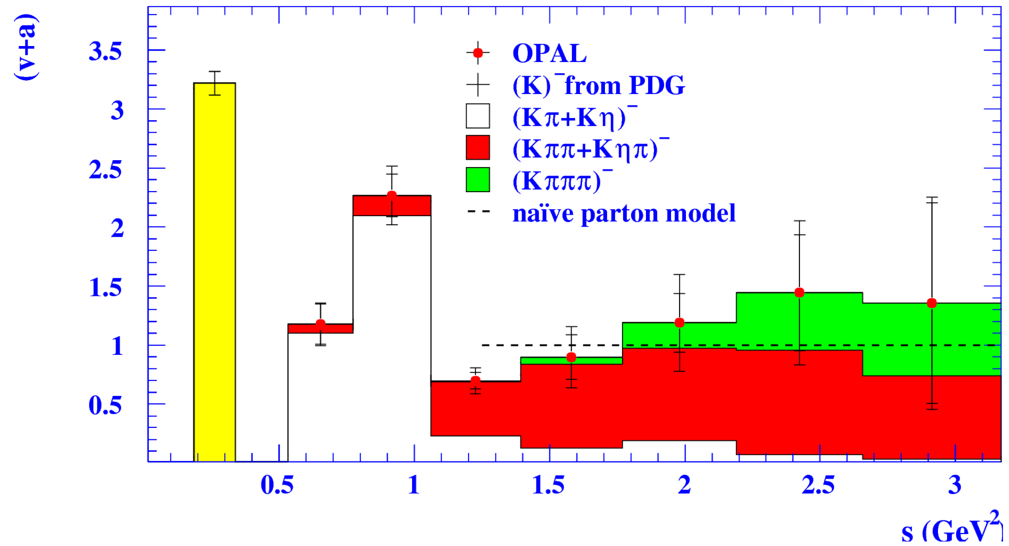
$\tau \to 2\pi\nu_\tau$ & $e^+e^- \to 2\pi$ Spectral Functions

Masjuan-Miranda-Roig 2305.20005



Strange Spectral Function





Very low statistics. Large experimental uncertainties

Sensitive to SU(3) breaking: m_s , V_{us}

V_{us} Determination

Gámiz-Jamin-Pich-Prades-Schwab

$$|V_{us}|^2 = \frac{R_{\tau,us}}{\frac{R_{\tau,ud}}{|V_{ud}|^2} - \delta R_{\tau,\mathrm{th}}}$$

$$\delta R_{\tau,\mathrm{th}} \approx 24\, \frac{m_s^2(m_\tau^2)}{m_\tau^2}\, \Delta(\alpha_s)$$

$$\delta R_{\tau,\mathrm{th}} \equiv \underbrace{0.1544\,(37)}_{\mathrm{J=0}} + \underbrace{0.084\,(33)}_{m_s(2\ \mathrm{GeV}) = 93.0\,(8.5)\ \mathrm{MeV}} = 0.238\,(33)$$

$$R_{\tau,uq} = \Gamma\left(\tau^- \to \nu_\tau \bar{u} q\right) / \Gamma\left(\tau^- \to \nu_\tau \bar{\nu}_e e^-\right)$$

HFLAV 2022:

$$\mathrm{Br}\left(\tau^- \to \nu_\tau \bar{u} s\right) = (2.908 \pm 0.048)\%$$

$$\mathrm{Br}\left(\tau^- \to \nu_\tau \bar{u} d\right) = (61.83 \pm 0.10)\%$$

$$\mathrm{Br}\left(\tau^- \to \nu_\tau \bar{\nu}_e e^-\right)_{\mathrm{univ}} = (17.812 \pm 0.022)\%$$

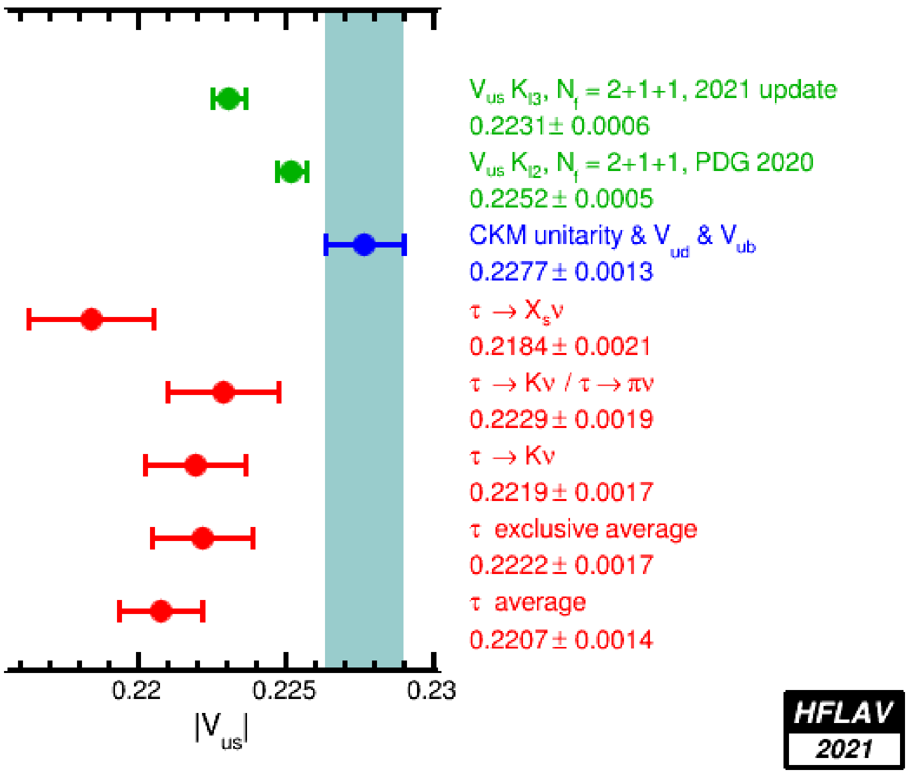
$$V_{ud} = 0.97373 \pm 0.00031$$

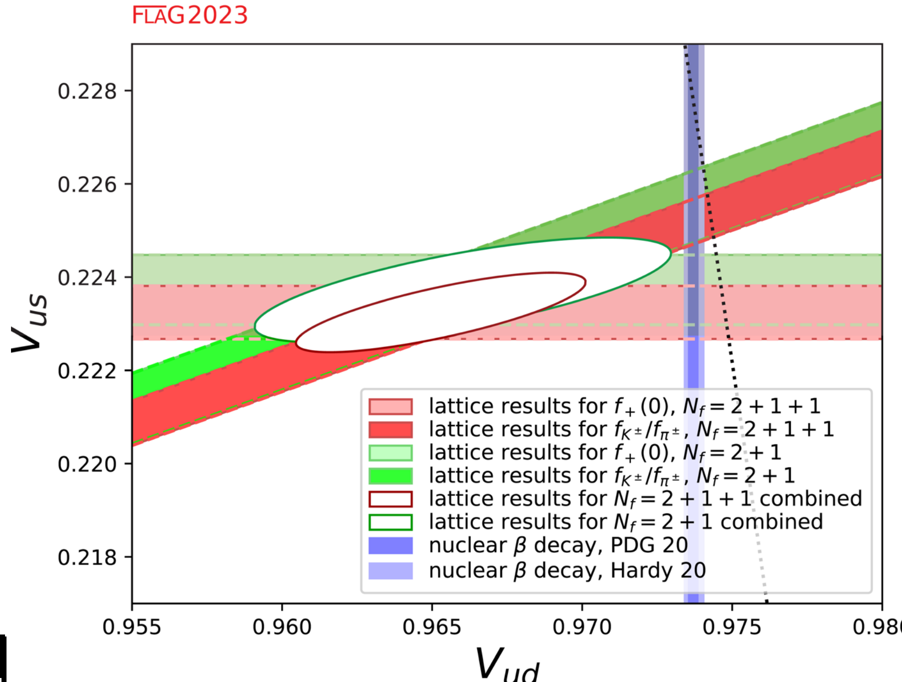
$$\Rightarrow \quad |V_{us}| = 0.2184 \pm 0.0018_{\mathrm{exp}} \pm 0.0011_{\mathrm{th}}$$

K_{l3}: $|V_{us}| = 0.2232 \pm 0.0006$ $\quad [f_+(0) = 0.9698 \pm 0.0017]$ FLAG 2021

Sizeable discrepancy. Improvements needed

V_{us} & V_{ud} Cabibbo Anomaly

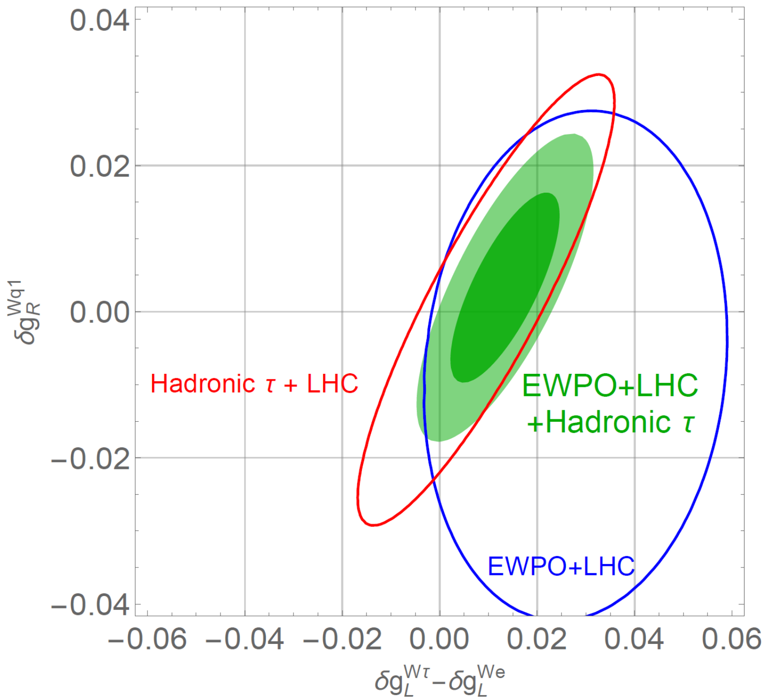




Sizeable violation of CKM unitarity

Hadronic τ Decay & New Physics

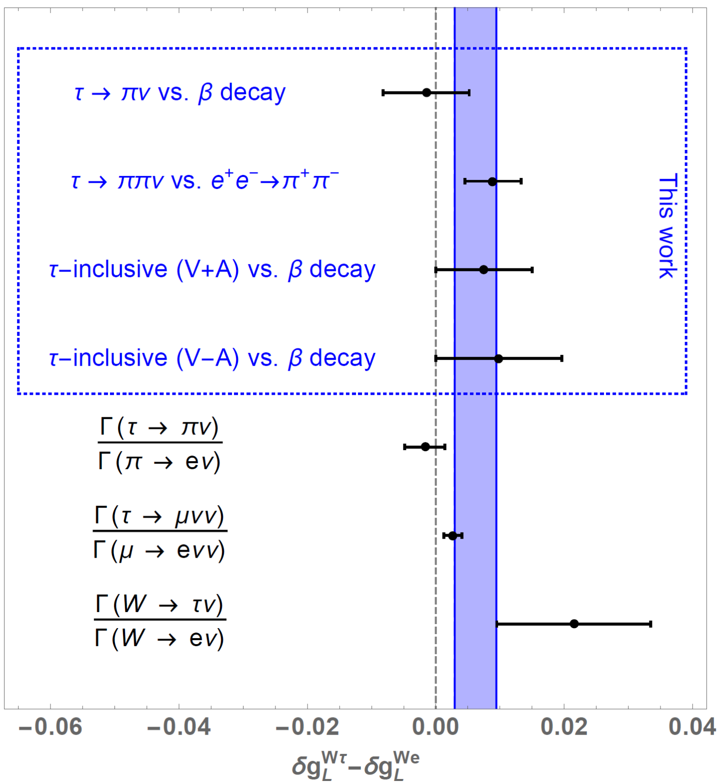
$$\mathcal{L}_{\rm eff} = -\frac{G_F V_{ud}}{\sqrt{2}} \Big[\left(1+\epsilon_L^\tau\right) \bar\tau\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar u \gamma^\mu(1-\gamma_5) d \; + \; \epsilon_R^\tau \; \bar\tau\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \bar u \gamma^\mu (1+\gamma_5) d \; + \; \bar\tau(1-\gamma_5)\nu_\tau \cdot \bar u \left[\epsilon_S^\tau - \epsilon_P^\tau\gamma_5\right] d \; + \; \epsilon_T^\tau \, \bar\tau\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \bar u \sigma^{\mu\nu}(1-\gamma_5) d \Big] + \text{h.c.}$$



Cirigliano, Falkowski, González-Alonso, Rodríguez-Sánchez, 1809.01161

$$\epsilon_L^\tau - \epsilon_L^e = \delta g_L^{W\tau} - \delta g_L^{We} - [c_{\ell q}^{(3)}]_{\tau\tau 11} + [c_{\ell q}^{(3)}]_{ee11} \qquad \epsilon_{S,P}^\tau = -\frac{1}{2}[c_{lequ} \pm c_{ledq}]^*_{\tau\tau 11}$$

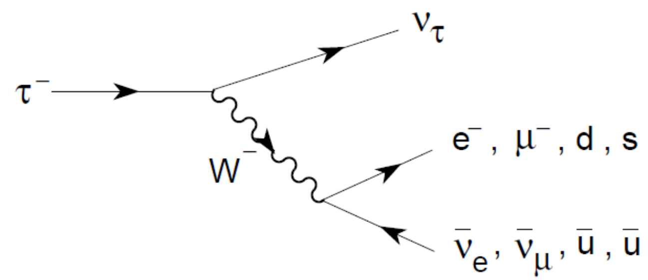
$$\epsilon_R^\tau = \delta g_R^{Wq_1}, \qquad \epsilon_T^\tau = -\frac{1}{2}[c_{lequ}^{(3)}]^*_{\tau\tau 11} \, ,$$



95% CL — 10^{-3} units

Coefficient	ATLAS $\tau\nu$	τ decays	τ and π decays
$[c_{\ell q}^{(3)}]_{\tau\tau 11}$	$[0.0, 1.6]$	$[-12.6, 0.2]$	$[-7.6, 2.1]$
$[c_{\ell equ}]_{\tau\tau 11}$	$[-5.6, 5.6]$	$[-8.4, 4.1]$	$[-5.6, 2.3]$
$[c_{\ell edq}]_{\tau\tau 11}$	$[-5.6, 5.6]$	$[-3.5, 9.0]$	$[-2.1, 5.8]$
$[c_{\ell equ}^{(3)}]_{\tau\tau 11}$	$[-3.3, 3.3]$	$[-10.4, -0.2]$	$[-8.6, 0.7]$

SUMMARY



Many interesting τ topics

- Tests of QCD and the Electroweak Theory
- Looking for Signals of New Phenomena
- Superb Tool for New Physics Searches

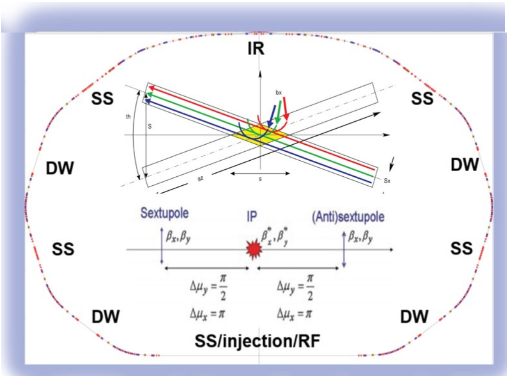
Better data samples needed

Lots of data will be produced @ Belle-II & STCF

Improving systematics brings a great reward

Clean threshold environment at the STCF

Backup



ITP CAS, Beijing, China
28 June 2023

LORENTZ STRUCTURE

$$\mathcal{H} = 4 \frac{G_{l'l}}{\sqrt{2}} \sum_{n,\epsilon,\omega} g^n_{\epsilon\omega} \left[\overline{l'_\epsilon} \Gamma^n (\nu_{l'})_\sigma\right] \left[\overline{(\nu_l)_\lambda} \Gamma_n l_\omega\right]$$

90% CL — Fetscher-Gerber, PDG2020

$\mu \to e \overline{\nu}_e \nu_\mu$

$\lvert g^S_{RR}\rvert < 0.035$	$\lvert g^V_{RR}\rvert < 0.017$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.050$	$\lvert g^V_{LR}\rvert < 0.023$	$\lvert g^T_{LR}\rvert < 0.015$
$\lvert g^S_{RL}\rvert < 0.420$	$\lvert g^V_{RL}\rvert < 0.105$	$\lvert g^T_{RL}\rvert < 0.105$
$\lvert g^S_{LL}\rvert < 0.550$	$\lvert g^V_{LL}\rvert > 0.960$	$\lvert g^T_{LL}\rvert \equiv 0$
$\lvert g^S_{LR} + 6g^T_{LR}\rvert < 0.143$	$\lvert g^S_{RL} + 6g^T_{RL}\rvert < 0.418$	
$\lvert g^S_{LR} + 2g^T_{LR}\rvert < 0.108$	$\lvert g^S_{RL} + 2g^T_{RL}\rvert < 0.417$	
$\lvert g^S_{LR} - 2g^T_{LR}\rvert < 0.070$	$\lvert g^S_{RL} - 2g^T_{RL}\rvert < 0.418$	
$Q_{RR} + Q_{LR} < 8.2 \times 10^{-4}$		

95% CL — Stahl, PDG2020

$\tau \to e \overline{\nu}_e \nu_\tau$

$\lvert g^S_{RR}\rvert < 0.70$	$\lvert g^V_{RR}\rvert < 0.17$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.99$	$\lvert g^V_{LR}\rvert < 0.13$	$\lvert g^T_{LR}\rvert < 0.082$
$\lvert g^S_{RL}\rvert < 2.01$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^T_{RL}\rvert < 0.51$
$\lvert g^S_{LL}\rvert < 2.01$	$\lvert g^V_{LL}\rvert < 1.005$	$\lvert g^T_{LL}\rvert \equiv 0$

$\tau \to \mu \overline{\nu}_\mu \nu_\tau$

$\lvert g^S_{RR}\rvert < 0.72$	$\lvert g^V_{RR}\rvert < 0.18$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.95$	$\lvert g^V_{LR}\rvert < 0.12$	$\lvert g^T_{LR}\rvert < 0.079$
$\lvert g^S_{RL}\rvert < 2.01$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^T_{RL}\rvert < 0.51$
$\lvert g^S_{LL}\rvert < 2.01$	$\lvert g^V_{LL}\rvert < 1.005$	$\lvert g^T_{LL}\rvert \equiv 0$

$\tau \to \pi \nu_\tau$

$\lvert g^V_R\rvert < 0.15$ $\quad$ $\lvert g^V_L\rvert > 0.992$

$\tau \to \rho \nu_\tau$

$\lvert g^V_R\rvert < 0.10$ $\quad$ $\lvert g^V_L\rvert > 0.995$

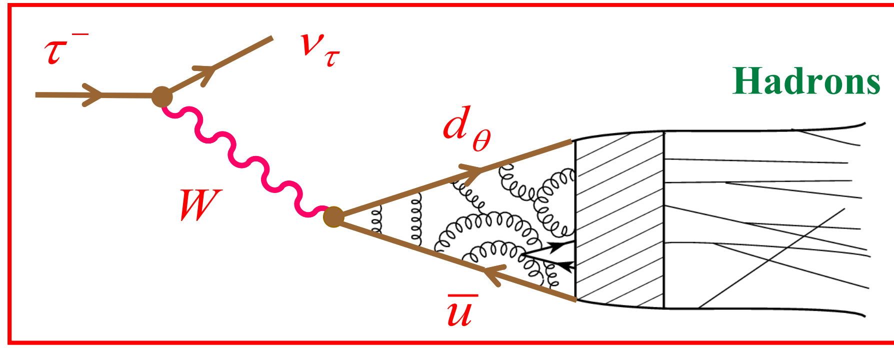
$\tau \to a_1 \nu_\tau$

$\lvert g^V_R\rvert < 0.16$ $\quad$ $\lvert g^V_L\rvert > 0.987$

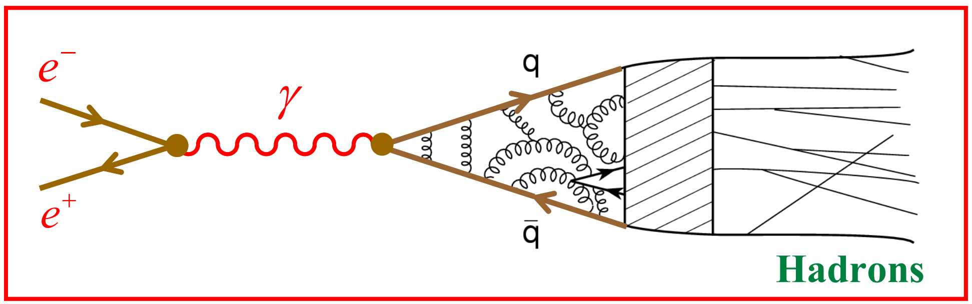
Only Lepton Massive Enough to Decay into Hadrons

$\tau^- \to \nu_\tau H^-$ **probes the hadronic V-A current**

$$\langle H^- | \bar{d}_\theta \gamma^\mu (1-\gamma_5) u | 0 \rangle$$



$e^+e^- \to H^0$ **probes the hadronic electromagnetic current**



$$\langle H^0 | \sum_q Q_q \, \bar{q} \gamma^\mu q | 0 \rangle$$

Isospin:

$$\frac{\Gamma(\tau^- \to \nu_\tau V^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = \frac{3\cos^2\theta_C}{2\pi\alpha^2} S_{\rm EW} \int_0^1 dx\, (1-x)^2 (1+2x)\, x \; \sigma^{I=1}_{e^+e^- \to V^0}(x m_\tau^2)$$

Perturbative $(m_q=0)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

$$K_0 = K_1 = 1 \quad , \quad K_2 = 1.63982 \quad , \quad K_3 = 6.37101 \quad , \quad K_4 = 49.07570$$

Baikov-Chetyrkin-Kühn '08

$$\Rightarrow \quad \delta_P = \sum_{n=1} K_n\; A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Power Corrections

$$\Pi^{(0+1)}_{\rm OPE}(s) \approx \frac{1}{4\pi^2}\sum_{n\ge 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

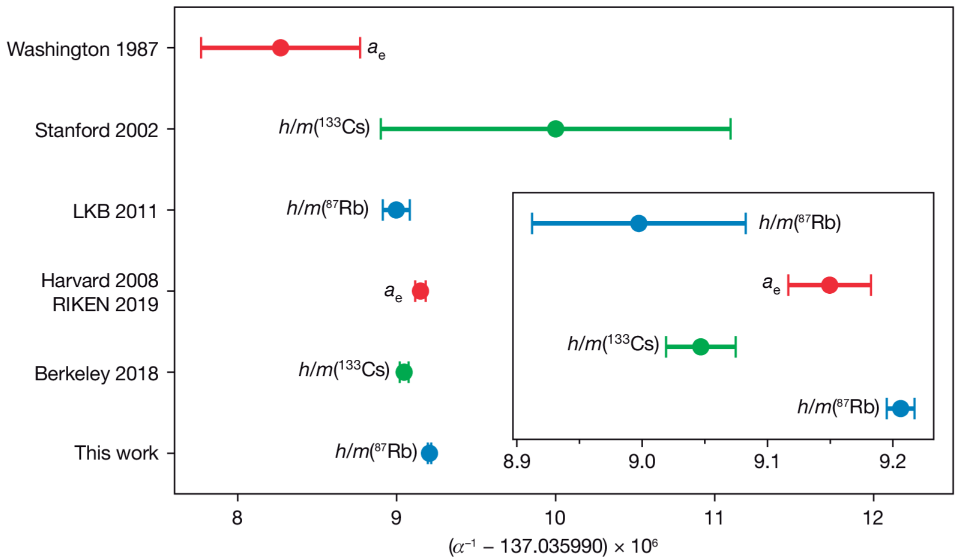
Braaten-Narison-Pich '92

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{\rm NP} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;(1-3x^2+2x^3)\sum_{n\ge 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

Suppressed by m_τ^6 **[additional chiral suppression in** $C_6\langle O_6\rangle^{V+A}$ **]**

Electron Anomalous Magnetic Moment



Morel et al, Nature 588 (2020) 61

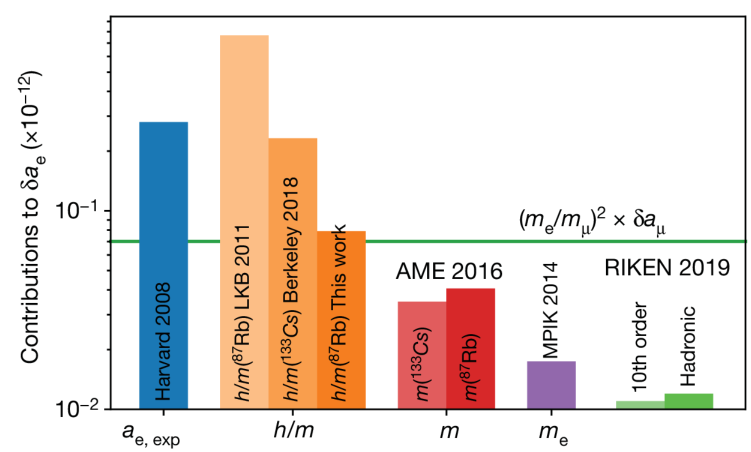
New measurement of α

$$\alpha^{-1}(\mathrm{Rb}) = 137.035\,999\,206\,(11)$$

8.1×10^{-11} accuracy

$5.8\,\sigma$ discrepancy with Cs experiment

$$\Delta a_e \equiv a_e^{\rm exp} - a_e^{\rm SM} = \begin{cases} (-8.8 \pm 3.6) \cdot 10^{-13} & (\mathrm{Cs},\ -2.4\sigma) \\ (+4.8 \pm 3.0) \cdot 10^{-13} & (\mathrm{Rb},\ +1.6\sigma) \end{cases}$$



τ Anomalous Magnetic Moment

Difficult to measure! $a_\tau^{\rm exp} = (-0.018 \pm 0.017)$ **DELPHI**

$-0.007 < a_\tau^{\rm New\ Phys} < 0.005$ **González-Springer, Santamaria, Vidal '00** **(LEP/SLD data)**

Eidelman, Passera

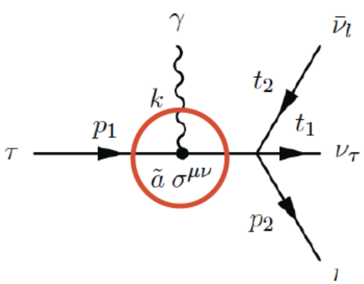
$10^8 \cdot a_\tau^{\rm th} =$	117 324	± 2	QED
	+ 47.4	± 0.5	EW
	+ 337.5	± 3.7	hvp
	+ 7.6	± 0.2	hvp NLO
	+ 5	± 3	light-by-light
	= 117 721	**± 5**	

Enhanced sensitivity to new physics: $(m_\tau/m_\mu)^2 = 283$

	Electron	Muon	Tau
a^{EW}/a^{HAD}	1/56	1/45	1/7
$a^{EW}/\delta a^{HAD}$	1.6	3	10

Essentially unknown

May be accessible at BFs through radiative leptonic decays **(Fael et al)** or with a polarized e⁻ beam **(Crivellin et al)**



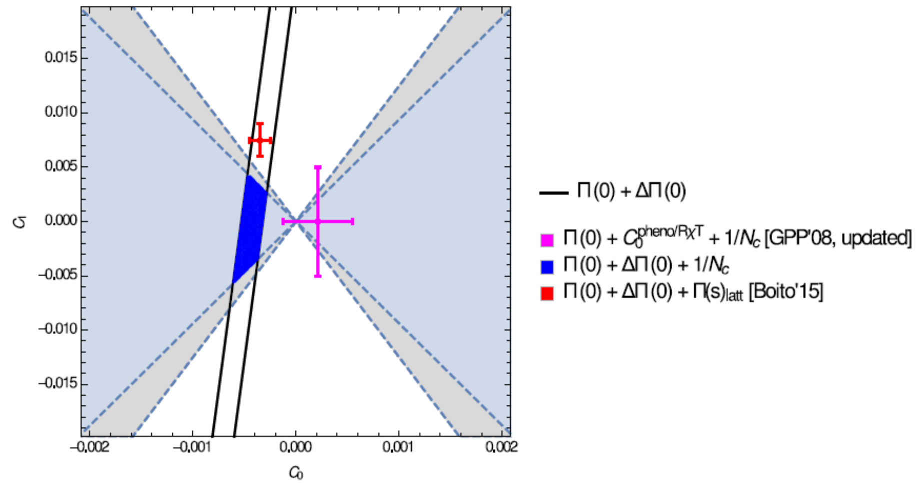
• χPT Parameters:

González-Alonso, Pich, Rodríguez-Sánchez, 1602.06112

$$L_{10}^{\rm eff} = L_{10}^r - 0.00126 + \mathcal{O}(p^6)$$

$$L_{10}^{\rm eff} = 1.53\, L_{10}^r + 0.263\, L_9^r - 0.00179 - \frac{1}{8}\left(\mathcal{C}_0^r + \mathcal{C}_1^r\right) + \mathcal{O}(p^8)$$

$$C_{87}^{\rm eff} = C_{87}^r + 0.296\, L_9^r + 0.00155 + \mathcal{O}(p^8)$$



- $\mathcal{O}(p^4)$ analysis: $L_{10}^r(M_\rho) = -(5.22 \pm 0.05) \cdot 10^{-3}$
- $\mathcal{O}(p^6)$ analysis: $L_{10}^r(M_\rho) = -(4.1 \pm 0.4) \cdot 10^{-3}$

  $C_{87}^r(M_\rho) = (5.10 \pm 0.22) \cdot 10^{-3}\ \mathrm{GeV}^{-2}$

• $\varepsilon'_K/\varepsilon_K$: $\mathcal{O}_6$ → $\langle(\pi\pi)_{I=2}|Q_8|K^0\rangle$ → e.m. penguin contribution

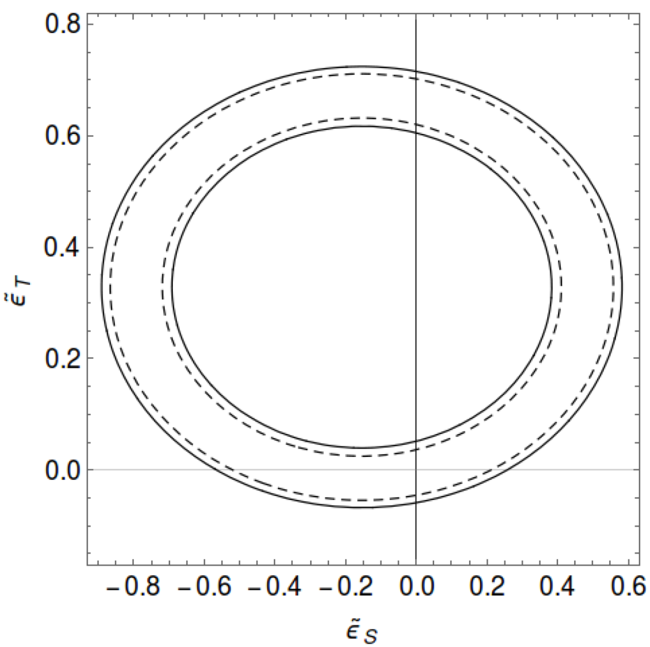
$$\left(\varepsilon'_K/\varepsilon_K\right)_{\rm EWP}^{I=2} = (-4.5 \pm 1.8)\cdot 10^{-4}$$

Pich-Rodríguez, 2102.09308

EFT analysis of $\tau \to \nu_\tau K \pi$

Rendón-Roig-Toledo, 1902.08143

$$\mathcal{L}_{cc} = -\frac{G_F V_{us}}{\sqrt{2}}(1+\epsilon_L+\epsilon_R)\Big[\bar{\tau}\gamma_\mu(1-\gamma_5)\nu_\ell \cdot \bar{u}[\gamma^\mu - (1-2\hat{\epsilon}_R)\gamma^\mu\gamma_5]s + \bar{\tau}(1-\gamma_5)\nu_\ell \cdot \bar{u}[\hat{\epsilon}_s - \hat{\epsilon}_p\gamma_5]s + 2\hat{\epsilon}_T\bar{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \cdot \bar{u}\sigma^{\mu\nu}s\Big] + \text{h.c.}$$



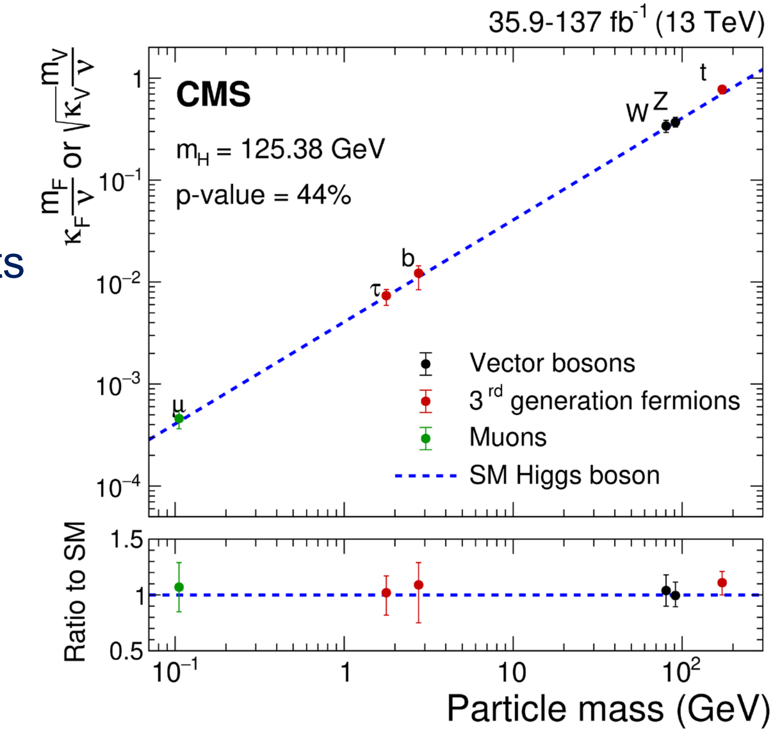
Best fit values	$\hat{\epsilon}_S$	$\hat{\epsilon}_T$	χ^2	χ^2 in the SM
Excluding $i = 5,6,7$ bins	$(1.3 \pm 0.9) \times 10^{-2}$	$(0.7 \pm 1.0) \times 10^{-2}$	$[72, 73]$	$[74, 77]$
Including $i = 5,6,7$ bins	$(0.9 \pm 1.0) \times 10^{-2}$	$(1.7 \pm 1.7) \times 10^{-2}$	$[83, 86]$	$[91, 95]$

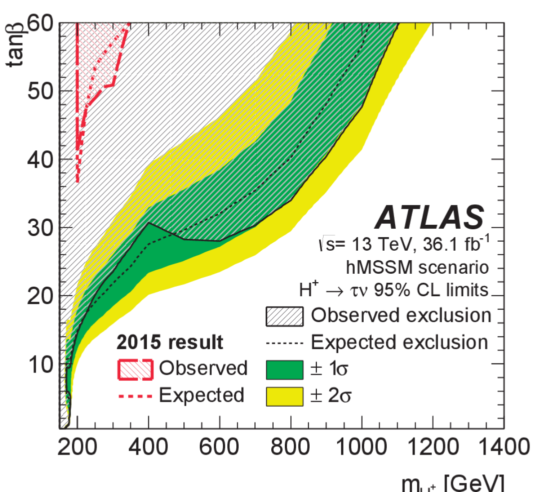
→ $\Lambda_{NP} \geq 2-5$ TeV

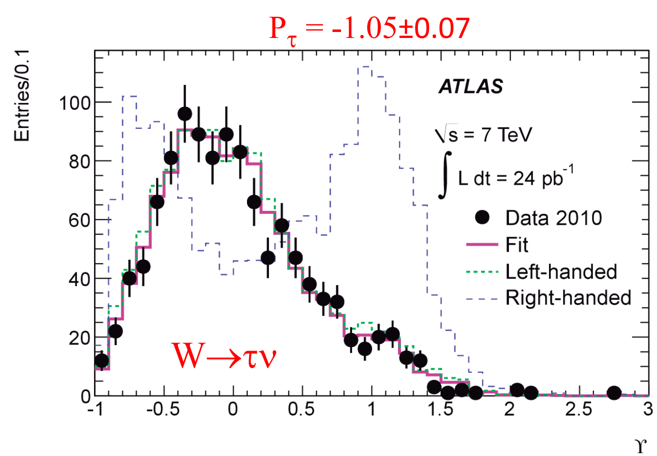
Complementary to kaon and hyperon data analyses

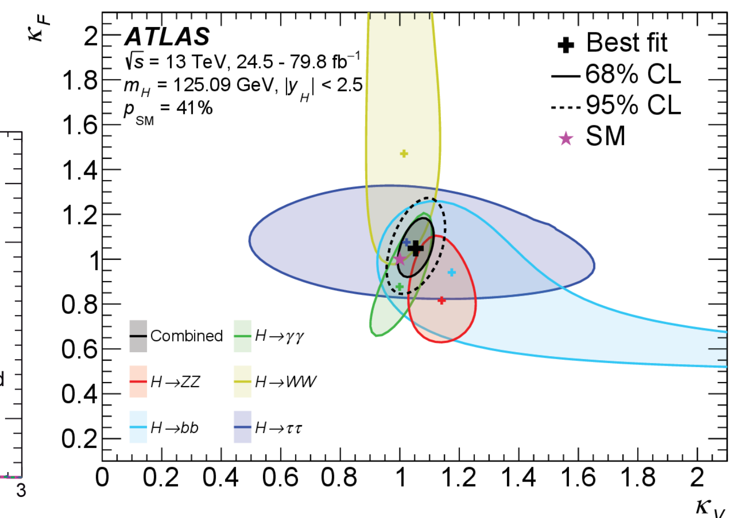
τ's @ LHC

- ❑ **Excellent signature to probe New Physics**
  - Difficult to identify light objects (Z,W$^{\pm}$) with only Jets
  - QCD Jets orders of magnitude larger
  - Must rely on leptons
- ❑ **LHC produces high-momenta τ's**
  - Tightly collimated decay products (mini-jet like)
  - Momentum reconstruction possible
- ❑ **Low multiplicity.** Good tagging efficiency
- ❑ **Heaviest lepton coupling to the Higgs** (4th H Br)
- ❑ **Polarization information**





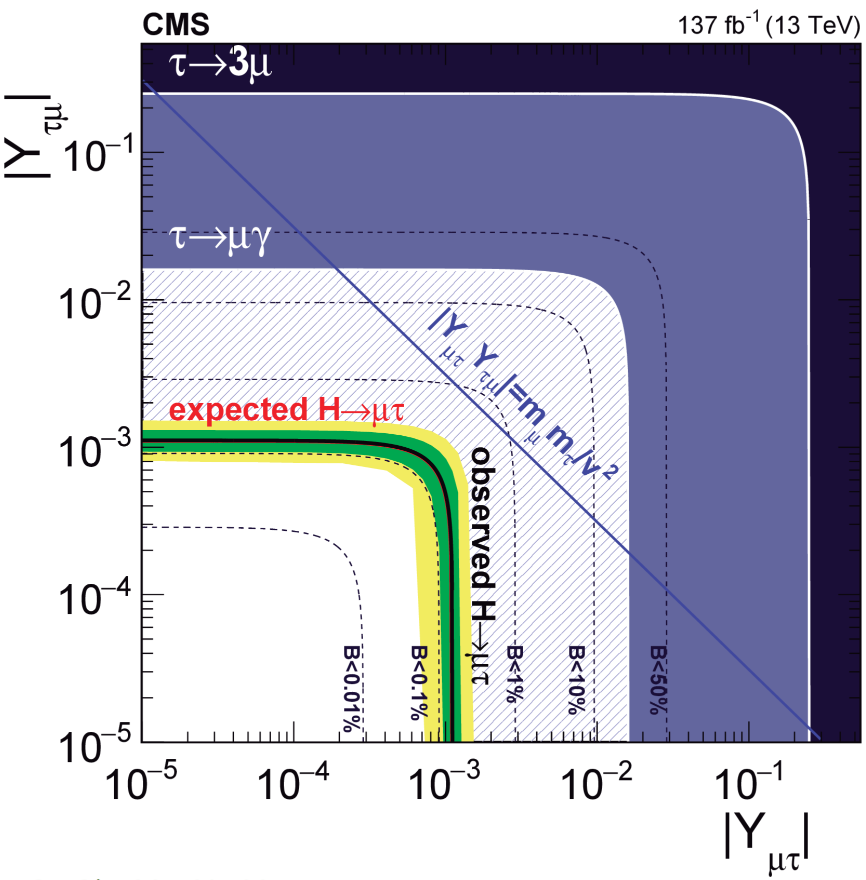




Flavour-Violating Higgs Couplings

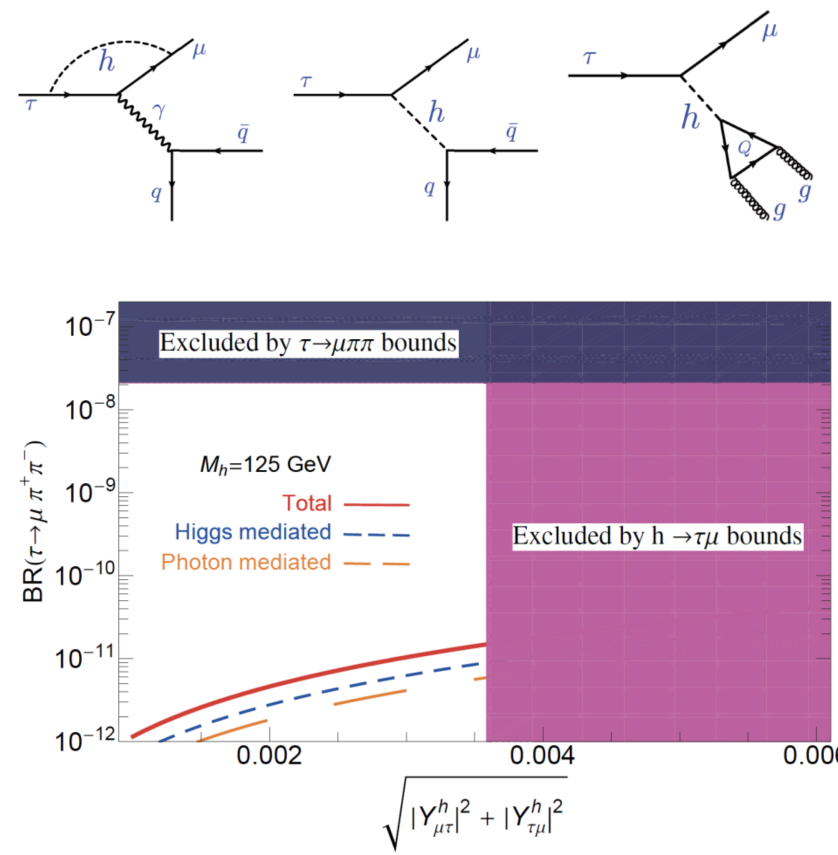
$$\mathcal{L} = -H\ \{Y_{e\mu}\,\bar{e}_L\mu_R + Y_{e\tau}\,\bar{e}_L\tau_R + Y_{\mu\tau}\,\bar{\mu}_L\tau_R + \cdots\}$$

$\mathrm{Br}\left(H \to \mu\tau\right) < 0.15\%$ (95% CL)



$\tau \to \mu\,\pi^+\pi^-$

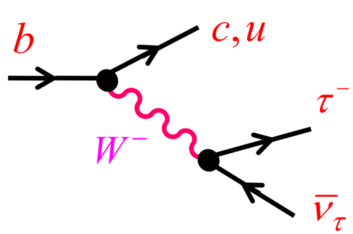
Celis et al., 1409.4439

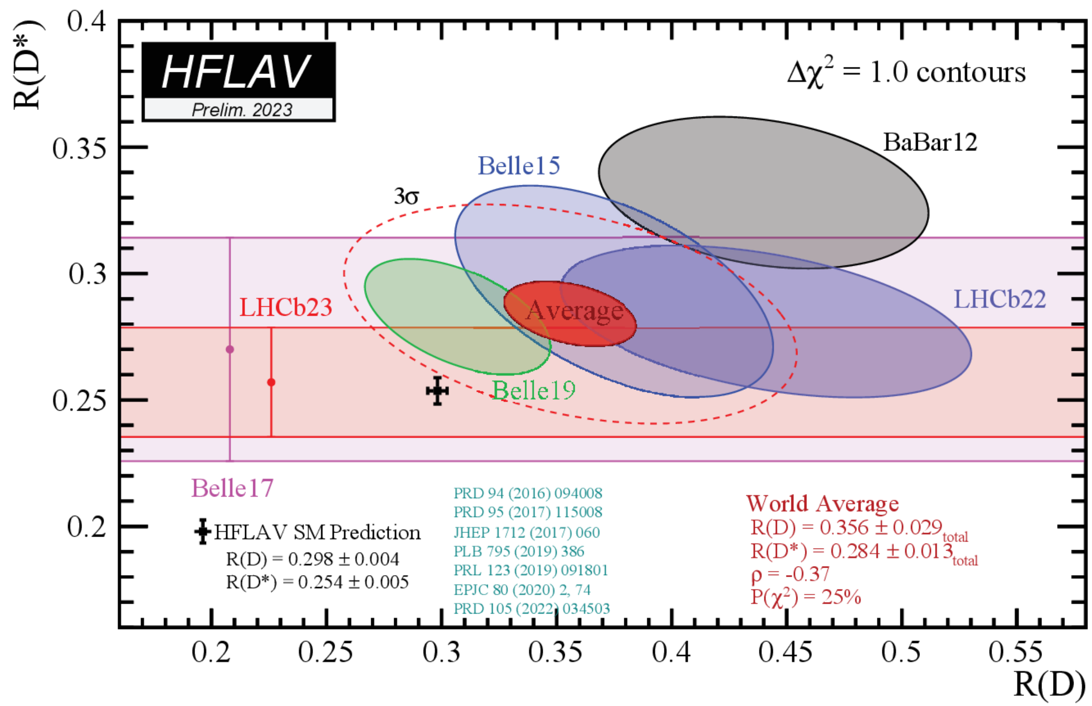


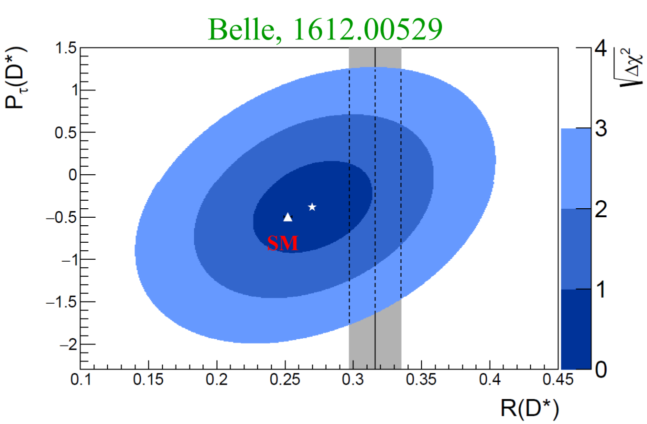
Flavour Anomaly

3.2 σ discrepancy

$$R(D^{(*)}) \equiv \frac{\mathrm{Br}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)}{\mathrm{Br}(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)}$$



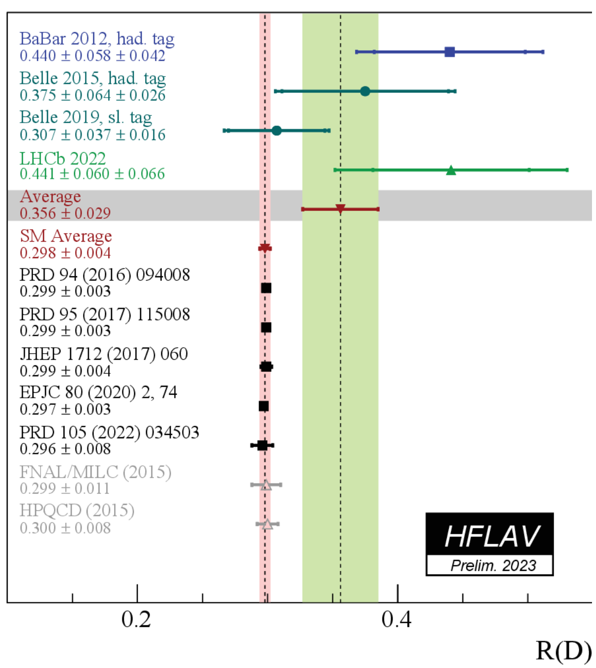


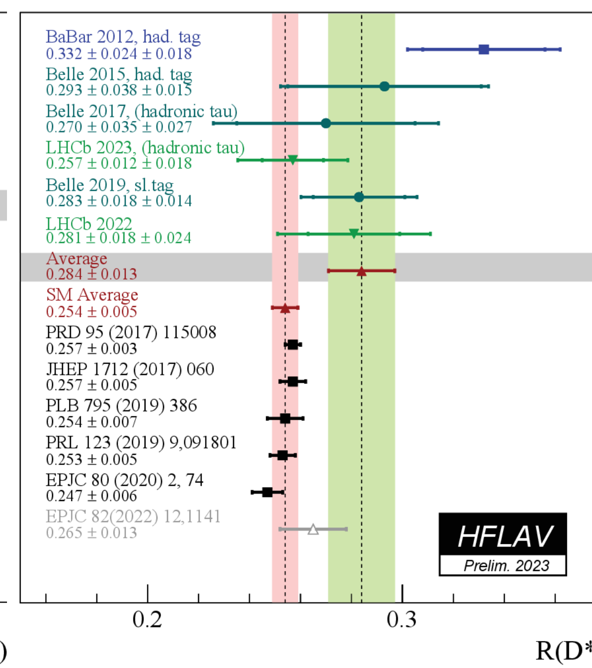


$$\mathcal{R}_{J/\psi} \equiv \frac{\mathcal{B}(B_c \to J/\psi\tau\bar{\nu}_\tau)}{\mathcal{B}(B_c \to J/\psi\mu\bar{\nu}_\mu)} = 0.71 \pm 0.17 \pm 0.18 \qquad (1.7\,\sigma)$$

$$F_L^{D^*} = 0.60 \pm 0.08 \pm 0.04 \qquad (1.6\,\sigma)$$

$$\mathcal{R}_{J/\psi}^{\mathrm{SM}} \approx 0.26 - 0.28 \qquad F_{L,\mathrm{SM}}^{D^*} = 0.455 \pm 0.003$$

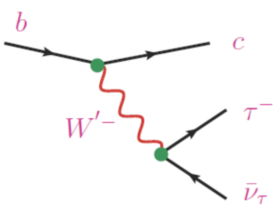




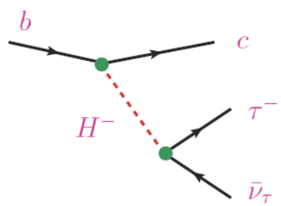
Effective Field Theory Analysis

$$\mathcal{H}_{\text{eff}}^{b\to c\ell\nu} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[(1+C_{V_L})\,\mathcal{O}_{V_L} + C_{V_R}\mathcal{O}_{V_R} + C_{S_R}\mathcal{O}_{S_R} + C_{S_L}\mathcal{O}_{S_L} + C_T\mathcal{O}_T \right] + \text{h.c.}$$

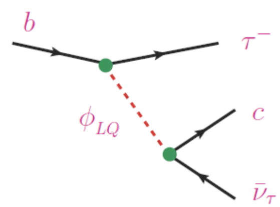
$$\mathcal{O}_{V_{L,R}} = (\bar c\,\gamma^\mu b_{L,R})\,(\bar\ell_L\gamma_\mu\nu_{\ell L})\,, \qquad \mathcal{O}_{S_{L,R}} = (\bar c\, b_{L,R})\,(\bar\ell_R\nu_{\ell L})\,, \qquad \mathcal{O}_T = (\bar c\,\sigma^{\mu\nu} b_L)\,(\bar\ell_R\sigma_{\mu\nu}\nu_{\ell L})$$



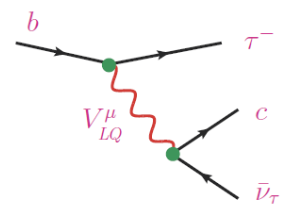
C_{V_L}



$C_{S_L}\,,\ C_{S_R} \quad (C_T)$



$C_{V_L}\,,\ C_{S_L}\,,\ C_T$



$C_{V_L}\,,\ C_{S_R}$

Many analyses (usually with single operator/mediator and partial data information)

Freytsis et al, Bardhan et al, Cai et al, Hu et al, Celis et al, Datta et al, Bhattacharya et al, Alonso et al, . . .

Global fit to all data

(q^2 distributions included)

Murgui-Peñuelas-Jung-Pich, 1904.09311

$F_L^{D^*}$, $\mathcal{B}_{10}$	Min 1	Min 2
χ^2/d.o.f.	37.4/54	40.4/54
C_{LL}^V	$0.09^{+0.13}_{-0.12}$	$0.34^{+0.05}_{-0.07}$
C_{RL}^S	$0.09^{+0.12}_{-0.61}$	$-1.10^{+0.48}_{-0.07}$
C_{LL}^S	$-0.14^{+0.52}_{-0.07}$	$-0.30^{+0.11}_{-0.50}$
C_{LL}^T	$0.008^{+0.046}_{-0.044}$	$0.093^{+0.029}_{-0.030}$

$\mathcal{B}(B_c \to \tau\bar\nu) < 10\%$

$F_L^{D^*}$ included