Lattice Calculation of g_V, g_A in Chroma

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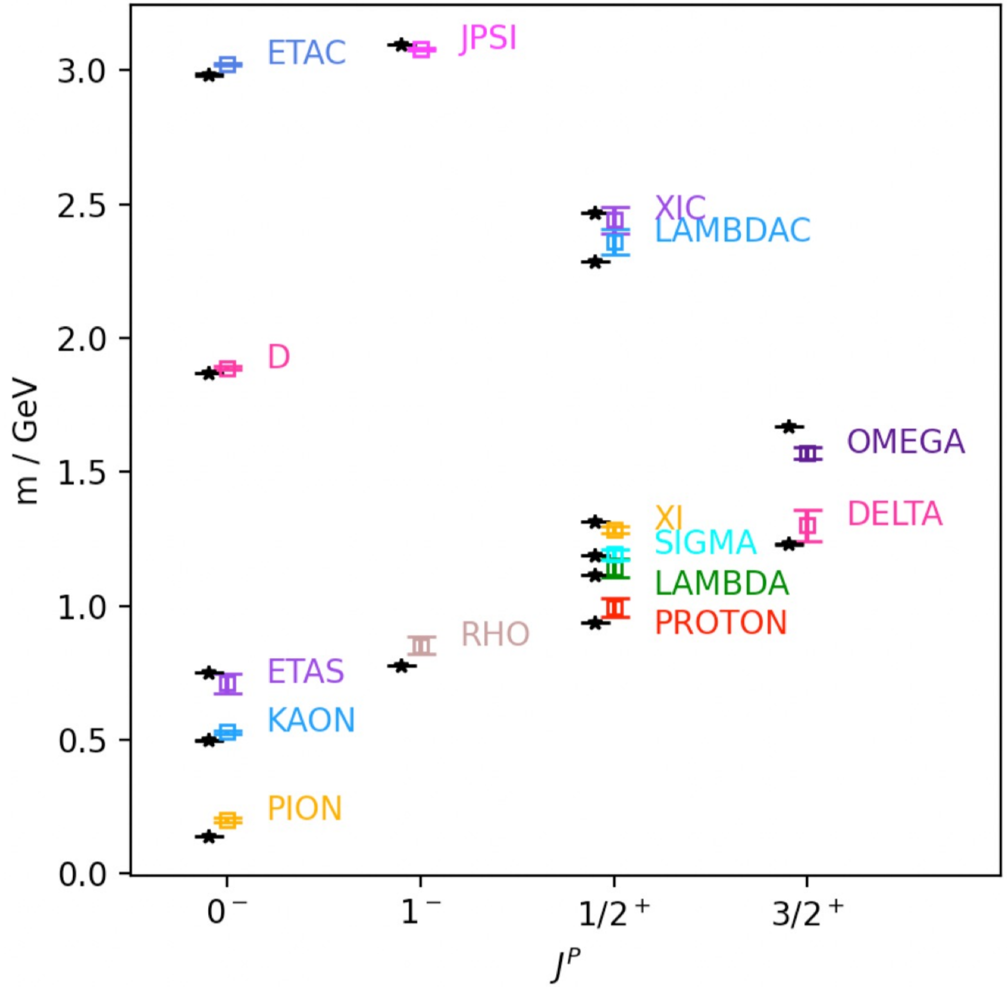
• Outline

- Introduction
- Meson and Parton 3-point correlators
- Program of 3pt and sequential source/propagator
- 3pt Numerical analysis
- Practice

- **Introduction**

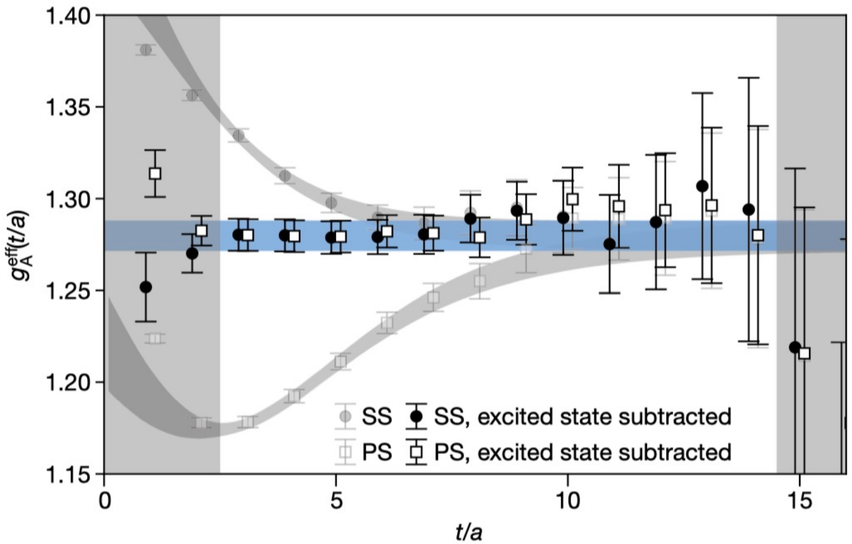


Local 2-point correlators

Hadron Spectrum

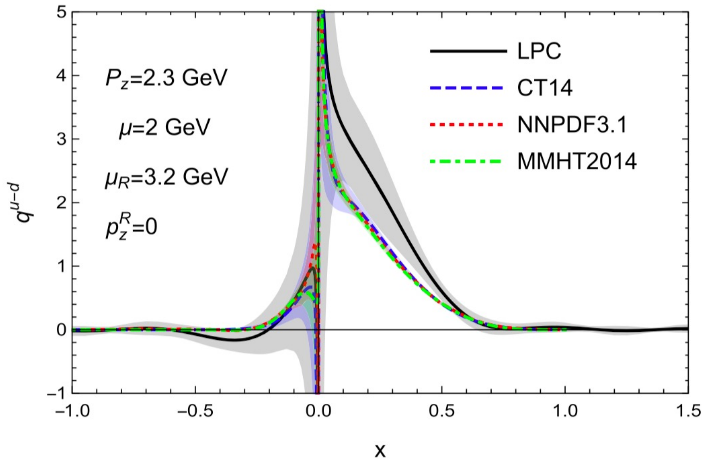
• Introduction

- **Local 3pt → Form factors**

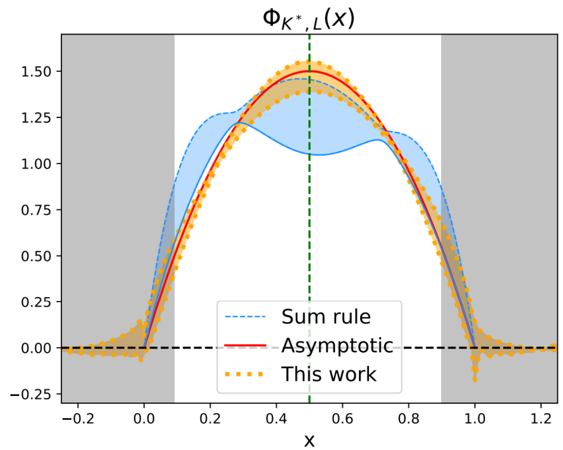


Nature 558, 91–94 (2018)

- **Non-local 2/3pt → Parton distribution functions**

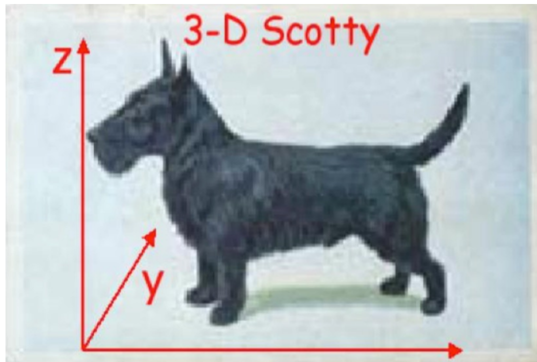


LPC, PRD 101 034020(2020)



LPC, PRL127 (2021), 062002

- **GPDs and TMDs**
- **…**



LATTICE

More and More Powerful !

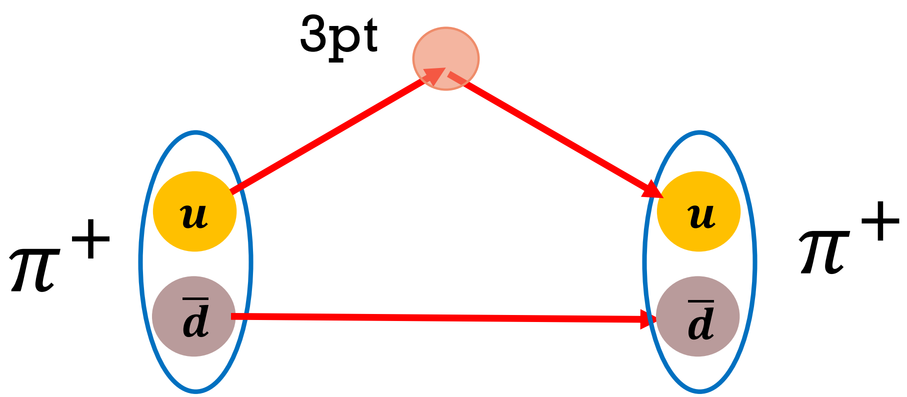
- **Meson and 3-point correlators**

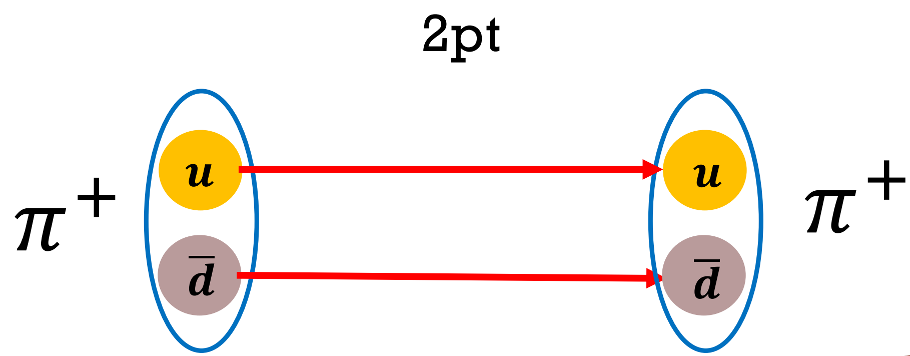
- **The interpolator of Pion:**

$$O_{\pi^+}(x) \;=\; \bar{d}(x)\gamma_5 u(x) \;=\; \bar{\psi}^{d,a}_{\alpha}(x)(\gamma_5)_{\alpha\beta}\psi^{u,a}_{\beta}(x),$$

- **Take the electromagnetic form factor as an example:**

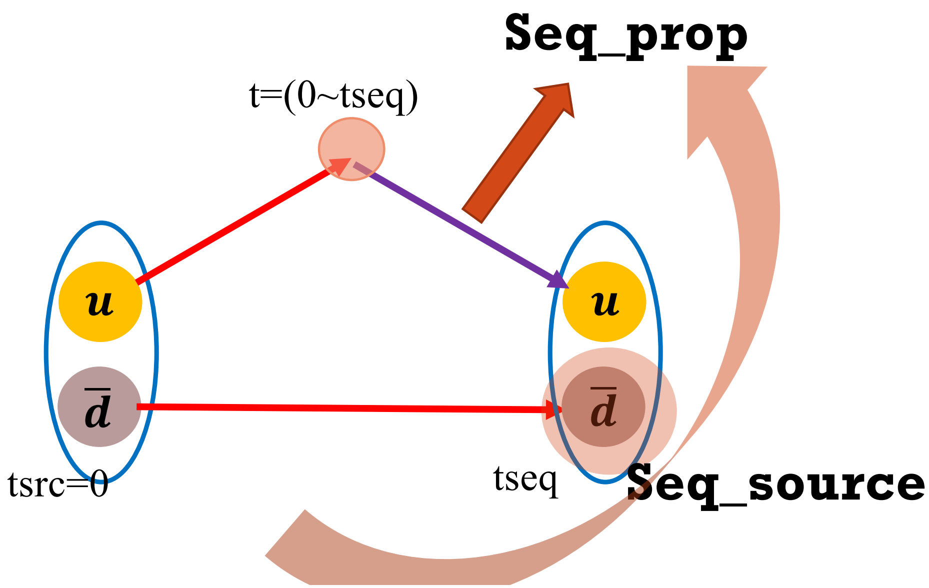
$$F_{\pi\pi} \;=\; Tr[\langle\pi|\bar{q}\gamma_t q|\pi\rangle]$$





- # Meson and 3-point correlators

$$
\begin{aligned}
\langle O_M(x_2)\mathcal{O}_\Gamma^{uu}(x)\bar{O}_M(0)\rangle &= \Big\langle \bar\psi_\alpha^{d,a}(x_2)\Gamma_{\alpha\beta}\psi_\beta^{u,a}(x_2)\left[\bar\psi_\mu^{u,c}(x)(\Gamma)_{\mu\nu}\psi_\nu^{u,c}(x)\right]\bar\psi_{\alpha'}^{u,b}(0)\Gamma_{\alpha'\beta'}\psi_{\beta'}^{d,b}(0)\Big\rangle \\
&= \Big\langle \bar\psi_\alpha^{d,a}(x_2)\Gamma_{\alpha\beta}\psi_\beta^{u,a}(x_2))\Big[\bar\psi_\mu^{u,c}(x)\Gamma_{\mu\nu}\psi_\nu^{u,c}(x)\Big]\bar\psi_{\alpha'}^{u,b}(0)\Gamma_{\alpha'\beta'}\psi_{\beta'}^{d,b}(0)\Big\rangle \\
&= Tr\Big[G^{d,ba}_{\beta'\alpha}(0,x_2)\Gamma_{\alpha\beta}G^{u,ac}_{\beta\mu}(x_2,x)\Gamma_{\mu\nu}G^{u,cb}_{\nu\alpha'}(x,0)\Gamma_{\alpha'\beta'}\Big] \\
&= Tr\Big[\left[G^d(0,x_2)\Gamma\right]_{\beta'\beta}G^u_{\beta\mu}(x_2,x)\Gamma_{\mu\nu}G^u_{\nu\alpha'}(x,0)\Gamma_{\alpha'\beta'}\Big]
\end{aligned}
$$



- **Sequential source：**

$$X_{\beta'\beta} = \left[G^d(0,x_2)\Gamma\right]_{\beta'\beta},$$

- **Sequential propagator：**

$$G^{seq}_{\beta'\mu}(0,x) = \left[\left[G^d(0,x_2)\Gamma\right]_{\beta'\beta}G^u_{\beta\mu}(x_2,x),\right.$$

- **3pt trace：**

$$3pt = Tr\left[adj(\gamma^5 G^{seq}(0,x)\gamma^5)_{\beta'\mu}\Gamma_{\mu\nu}G^u_{\nu\alpha'}(x,0)\Gamma_{\alpha'\beta'}\right].$$

- **Parton/Neutron 3-point correlators**

$$\langle p_2 | A^\mu(0) | p_1 \rangle = \bar{U}(p_2)[\gamma^\mu \gamma_5 G_A(\Delta^2) + \frac{\gamma_5 \Delta^\mu}{2M_N} G_P(\Delta^2)] U(p_1)$$

$$A^\mu(0) = \bar{\psi}_f(0)\gamma^\mu \gamma_5 \psi_f(0)$$

- **The interpolator of Parton/Neutron:**

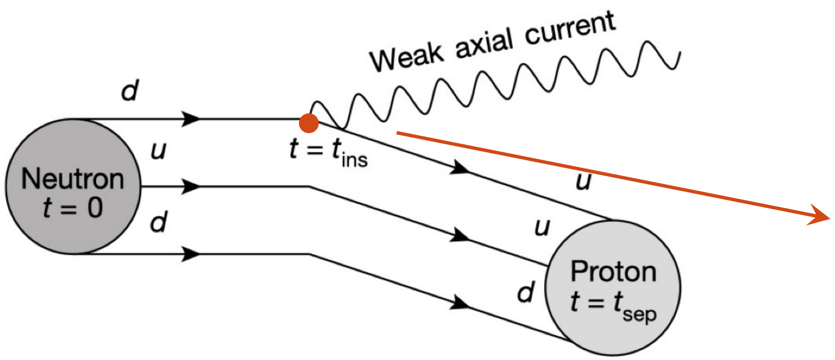
$$\overline{\mathcal{N}}(x) = \epsilon_{a',b',c'} \left(\bar{u}^{a'}_{\alpha'}(x)(C\gamma^5)_{\alpha'\beta'} \bar{d}^{b'}_{\beta'}(x) \right) \bar{u}^{c'}_{\gamma'}(x),$$

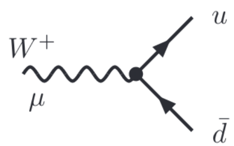
$$\mathcal{N}(y) = -\epsilon_{abc} \left(u^a_\alpha(y)(C\gamma^5)_{\alpha\beta} d^b_\beta(y) \right) u^c_\gamma(y)$$

- **The definition of g_V and g_A**

$$g_V = \mathrm{Tr}[\mathcal{P}^n \langle N|\bar{q}\gamma_4 q|N\rangle], \; g_A = \mathrm{Tr}[\mathcal{P}^p_3 \langle N|\bar{q}\mathrm{i}\gamma_3\gamma_5 q|N\rangle],$$

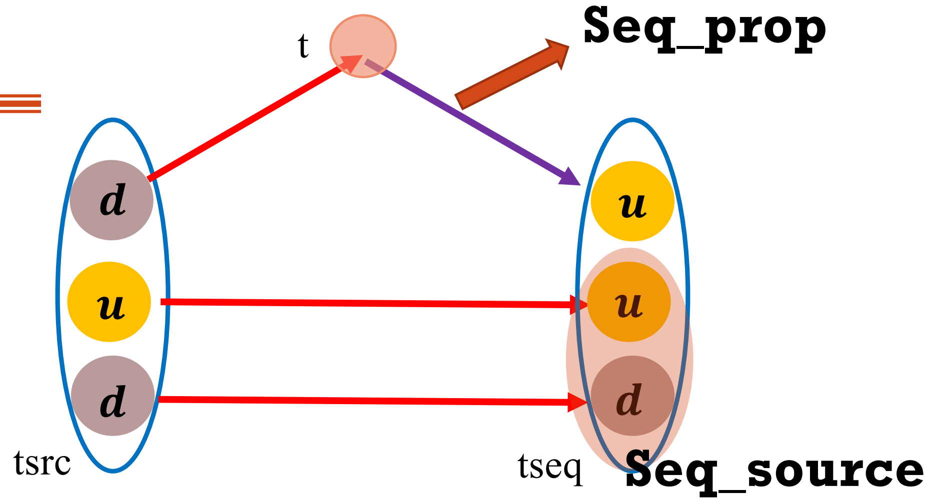
P^n / P^p : projection operators





$$\frac{ig}{2\sqrt{2}}\gamma_\mu(1-\gamma_5)V_{ud}$$

- **Parton/Neutron 3-point correlators**



Similar but much more complex

1: upper quark
2: down quark
abcd: color

$$\mathcal{O}^{12}_{\Gamma}(x) = \bar{\psi}^{1,d}_{\mu}(x)(\Gamma)_{\mu\nu}\psi^{2,d}_{\nu}(x)$$

$$T_{\gamma'\gamma}\left\langle \chi^{121}_{\gamma}\ (x_2)\ \mathcal{O}^{12}_{\Gamma}(x)\bar{\chi}^{122}_{\gamma'}(0)\right\rangle = -\epsilon^{abc}\epsilon^{a'b'c'}T_{\gamma'\gamma}$$

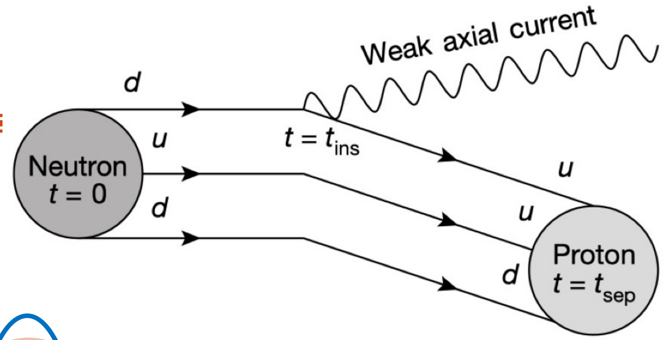
$$+\left\langle \psi^{1,a}_{\alpha}(x_2)(C\gamma^5)_{\alpha\beta}\psi^{2,b}_{\beta}(x_2)\psi^{1,c}_{\gamma}(x_2)\left[\bar{\psi}^{1,d}_{\mu}(x)\Gamma_{\mu\nu}\psi^{2,d}_{\nu}(x)\right]\bar{\psi}^{1,a'}_{\alpha'}(0)(C\gamma^5)_{\alpha'\beta'}\bar{\psi}^{2,b'}_{\beta'}(0)\bar{\psi}^{2,c'}_{\gamma'}(0)\right\rangle$$

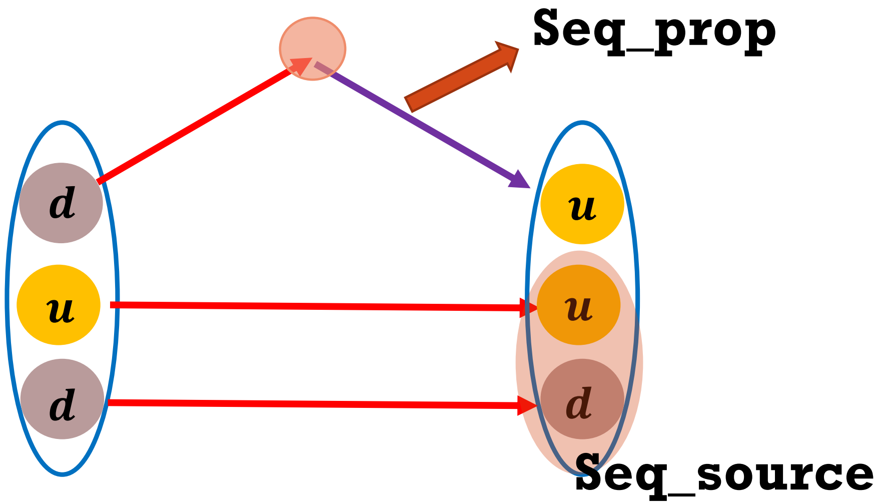
$$+\left\langle \psi^{1,a}_{\alpha}(x_2)(C\gamma^5)_{\alpha\beta}\psi^{2,b}_{\beta}(x_2)\psi^{1,c}_{\gamma}(x_2)\left[\bar{\psi}^{1,d}_{\mu}(x)\Gamma_{\mu\nu}\psi^{2,d}_{\nu}(x)\right]\bar{\psi}^{1,a'}_{\alpha'}(0)(C\gamma^5)_{\alpha'\beta'}\bar{\psi}^{2,b'}_{\beta'}(0)\bar{\psi}^{2,c'}_{\gamma'}(0)\right\rangle$$

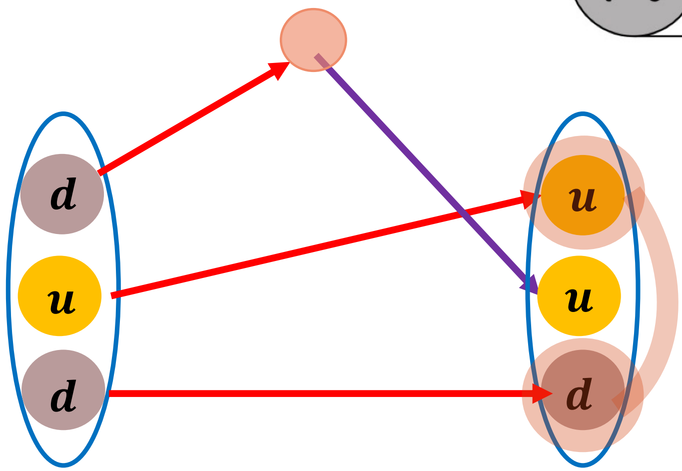
$$+\left\langle \psi^{1,a}_{\alpha}(x_2)(C\gamma^5)_{\alpha\beta}\psi^{2,b}_{\beta}(x_2)\psi^{1,c}_{\gamma}(x_2)\left[\bar{\psi}^{1,d}_{\mu}(x)\Gamma_{\mu\nu}\psi^{2,d}_{\nu}(x)\right]\bar{\psi}^{1,a'}_{\alpha'}(0)(C\gamma^5)_{\alpha'\beta'}\bar{\psi}^{2,b'}_{\beta'}(0)\bar{\psi}^{2,c'}_{\gamma'}(0)\right\rangle$$

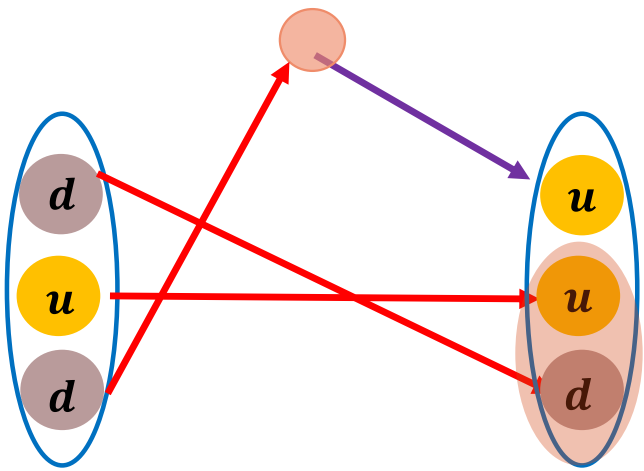
$$+\left\langle \psi^{1,a}_{\alpha}(x_2)(C\gamma^5)_{\alpha\beta}\psi^{2,b}_{\beta}(x_2)\psi^{1,c}_{\gamma}(x_2)\left[\bar{\psi}^{1,d}_{\mu}(x)\Gamma_{\mu\nu}\psi^{2,d}_{\nu}(x)\right]\bar{\psi}^{1,a'}_{\alpha'}(0)(C\gamma^5)_{\alpha'\beta'}\bar{\psi}^{2,b'}_{\beta'}(0)\bar{\psi}^{2,c'}_{\gamma'}(0)\right\rangle$$

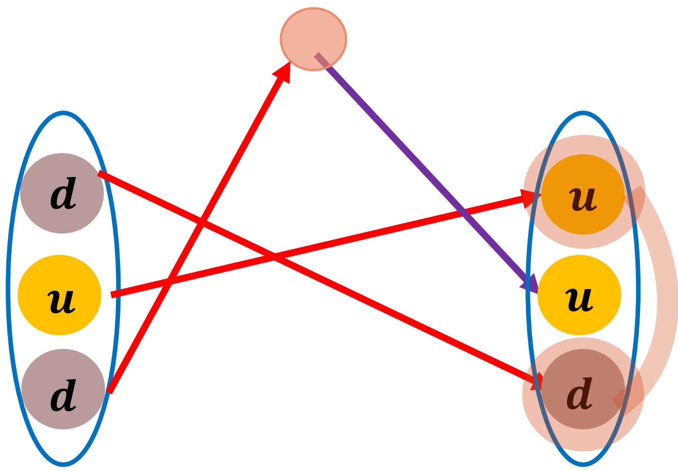
- **Parton/Neutron 3-point correlators**



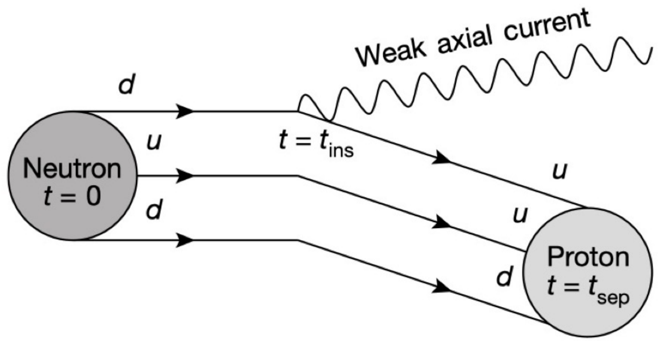








- **Parton/Neutron 3-point correlators**



- **Sequential source：**

$$X(x_2,0)_{all} = \Big[T * Tr_s\left[qC_{13}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right]\right] + T * qC_{24}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right]$$
$$+ qC_{13}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right] * T + qC_{14}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\,T\right]\Big]$$

- **Sequential propagator：**

$$G^{seq}(x,0) = X(x_2,0)_{all} G^1(x_2,x)$$

- **3pt trace：**

$$g_V: \Gamma = \gamma_4$$
$$g_A: \Gamma = \gamma_3\gamma_5$$

$$3pt = Tr\left[adj(\gamma^5 G^{seq}(x,0)\gamma^5)\Gamma G^2(x,0)\right].$$

More details can be checked in note.

• Projection

The definition of g_V and g_A with vector and axial vector current

$$g_V = \mathrm{Tr}[\mathcal{P}^n \langle N|\bar{q}\gamma_4 q|N\rangle],\ g_A = \mathrm{Tr}[\mathcal{P}_3^p \langle N|\bar{q}\mathrm{i}\gamma_3\gamma_5 q|N\rangle],$$

$$\frac{1+\gamma_4}{2} = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix},\ \mathrm{i}\gamma_3\gamma_5 = \begin{bmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{bmatrix},$$

$$\mathcal{P}^n = \frac{1+\gamma_4}{2}$$

$$\mathcal{P}_i^p = \frac{1+\gamma_4}{2}\mathrm{i}\gamma_i\gamma_5$$

How to choose a projection operator to project both gV and gA ?

- **Projection**

$$g_V = \mathrm{Tr}[\mathcal{P}^n \langle N|\bar{q}\gamma_4 q|N\rangle], \; g_A = \mathrm{Tr}[\mathcal{P}_3^p \langle N|\bar{q}\mathrm{i}\gamma_3\gamma_5 q|N\rangle],$$

$$\mathcal{P}^n = \frac{1+\gamma_4}{2}$$

$$\mathcal{P}_i^p = \frac{1+\gamma_4}{2}\mathrm{i}\gamma_i\gamma_5$$

- Decompose the 4-components Dirac spinor into four pieces $q^+_{\frac{1}{2}}, q^+_{-\frac{1}{2}}, q^-_{\frac{1}{2}}, q^-_{-\frac{1}{2}}$

$$g_V = \langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{\frac{1}{2}} q^\pm_{\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle + \langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{-\frac{1}{2}} q^\pm_{-\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle + \langle N^+_{-\frac{1}{2}}|\bar{q}^\pm_{\frac{1}{2}} q^\pm_{\frac{1}{2}}|N^+_{-\frac{1}{2}}\rangle + \langle N^+_{-\frac{1}{2}}|\bar{q}^\pm_{-\frac{1}{2}} q^\pm_{-\frac{1}{2}}|N^+_{-\frac{1}{2}}\rangle,$$
$$g_A = \langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{\frac{1}{2}} q^\pm_{\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle - \langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{-\frac{1}{2}} q^\pm_{-\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle - \langle N^+_{-\frac{1}{2}}|\bar{q}^\pm_{\frac{1}{2}} q^\pm_{\frac{1}{2}}|N^+_{-\frac{1}{2}}\rangle + \langle N^+_{-\frac{1}{2}}|\bar{q}^\pm_{-\frac{1}{2}} q^\pm_{-\frac{1}{2}}|N^+_{-\frac{1}{2}}\rangle.$$

Assume

$$\langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{\frac{1}{2}} q^\pm_{\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle = \langle N^+_{-\frac{1}{2}}|\bar{q}^\pm_{-\frac{1}{2}} q^\pm_{-\frac{1}{2}}|N^+_{-\frac{1}{2}}\rangle \quad \langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{-\frac{1}{2}} q^\pm_{-\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle = \langle N^+_{-\frac{1}{2}}|\bar{q}^\pm_{\frac{1}{2}} q^\pm_{\frac{1}{2}}|N^+_{-\frac{1}{2}}\rangle$$

Considering

$$\mathrm{Tr}[\mathcal{P}^n \langle N|\bar{q}\gamma_5\gamma_z|N\rangle] = 0, \; \mathrm{Tr}[\mathcal{P}_3^p \langle N|\bar{q}\gamma_4|N\rangle] = 0,$$

- Finally, the projection can be chosen as:

More details can be checked in note.

$$g_V = 2(\langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{\frac{1}{2}} q^\pm_{\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle + \langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{-\frac{1}{2}} q^\pm_{-\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle) = \mathrm{Tr}[\mathcal{P}^n(1+\mathrm{i}\gamma_3\gamma_5)\langle N|\bar{q}\gamma_4 q|N\rangle],$$
$$g_A = 2(\langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{\frac{1}{2}} q^\pm_{\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle - \langle N^+_{\frac{1}{2}}|\bar{q}^\pm_{-\frac{1}{2}} q^\pm_{-\frac{1}{2}}|N^+_{\frac{1}{2}}\rangle) = \mathrm{Tr}[\mathcal{P}^n(1+\mathrm{i}\gamma_3\gamma_5)\langle N|\bar{q}\mathrm{i}\gamma_3\gamma_5 q|N\rangle].$$

• Projection

Form factor (3pt)

$$3pt = Tr\left[adj(\gamma^5 G^{seq}(x,0)\gamma^5)\Gamma G^2(x,0)\right].$$

$g_V: \Gamma = \gamma_4$
$g_A: \Gamma = \gamma_3\gamma_5$

Sequential Prop

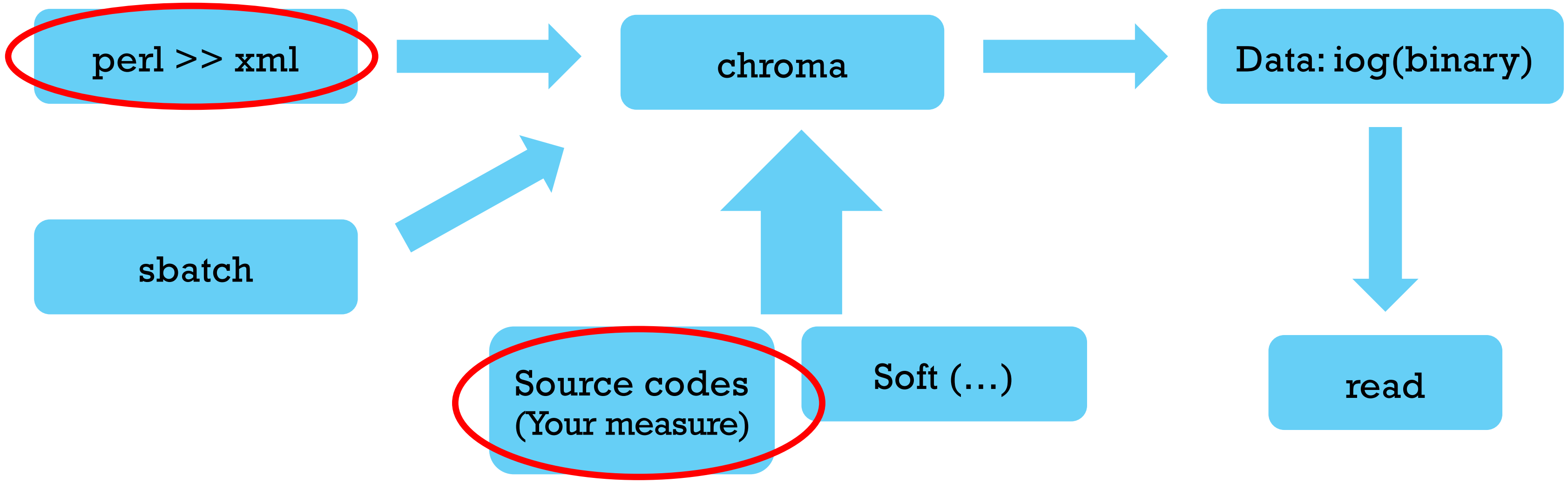
$$G^{seq}(x,0) = X(x_2,0)_{all} G^1(x_2,x)$$

Sequential Source

$$T = (\frac{1+\gamma_4}{2})(1+\gamma_3\gamma_5)$$

$$X(x_2,0)_{all} = \Big[T * Tr_s\left[qC_{13}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right]\right] + T * qC_{24}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right]$$
$$+ qC_{13}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right] * T + qC_{14}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\,T\right]\Big]$$

- **Code for sequential source and propagator**



- # Code for sequential source and propagator

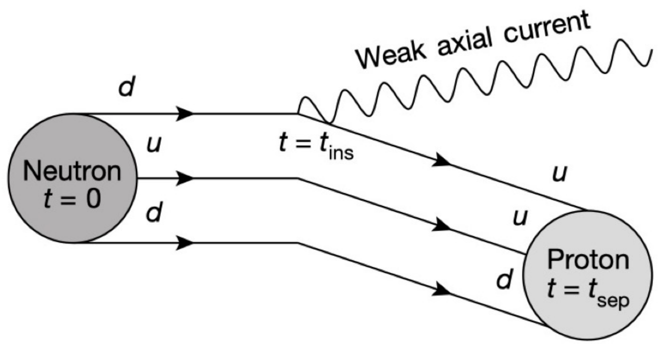
```
<elem>
  <Name>SEQSOURCE_FAST</Name>
  <SmearedProps>true</SmearedProps>
  <multi_tSinks>${tseq}</multi_tSinks>
  <Frequency>1</Frequency>
  <Param>
    <version>2</version>
    <SeqSource>
      <version>1</version>
      <SeqSourceType>WEAK_CURRENT_gAgV_CUR</SeqSourceType>
      <j_decay>3</j_decay>
      <t_sink>0</t_sink>
      <sink_mom>0 0 0</sink_mom>
    </SeqSource>

    <NamedObject>
      <prop_ids>
        <elem>smeared_L_quark_propagator</elem>
        <elem>smeared_L_quark_propagator</elem>
      </prop_ids>
      <seqsource_id>
        <elem>seqsrc_tseq_${tseq}</elem>
      </seqsource_id>
      <gauge_id>default_gauge_field</gauge_id>
    </NamedObject>
 </elem>
```

tseq=3,4,5,6…
Extract ground state
Statistics

u,d quark propagator

Sequential source id



- # Code for sequential source and propagator

source_code/
simple_baryon_seqsrc_w.cc

```
    LatticePropagator
    WeakCurrentgAgV::operator()(const multi1d<LatticeColorMatrix>& u,
                                const multi1d<ForwardProp_t>& forward_headers,
                                const multi1d<LatticePropagator>& quark_propagators)
    {
      START_CODE();
      if ( Nc != 3 ){    /* Code is specific to Ns=4 and Nc=3. */
        QDPIO::cerr<<" code only works for Nc=3 and Ns=4\n";
        QDP_abort(111) ;
      }
#if QDP_NC == 3

      checkArgs2("WeakCurrentgAgV", quark_propagators);

      LatticePropagator src_prop_tmp;

      src_prop_tmp += T * transposeSpin( quarkContract13(quark_propagators[0] * Cg5, Cg5 * quark_propagators[1]) );
      src_prop_tmp += T * quarkContract24(quark_propagators[0] * Cg5, Cg5 * quark_propagators[1]);
      src_prop_tmp += quarkContract13(quark_propagators[0] * Cg5, Cg5 * quark_propagators[1]) * T;
      src_prop_tmp += quarkContract14(quark_propagators[0] * Cg5, Cg5 * quark_propagators[1] * T);


      END_CODE();

      return projectBaryon(src_prop_tmp,
                           forward_headers);
#else
      LatticePropagator q1_tmp;

      q1_tmp = zero ;
      return q1_tmp ;
#endif
    }
```

$$X(x_2,0)_{all} = \Big[T * Tr_s\left[qC_{13}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right]\right] + T * qC_{24}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right]$$
$$+ qC_{13}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\right] * T + qC_{14}\left[G^1(x_2,0)\,C\gamma^5, C\gamma^5 G^2(x_2,0)\,T\right]\Big]$$

- # Code for sequential source and propagator

```
<elem>
  <Name>PROPAGATOR</Name>
  <Frequency>1</Frequency>
  <Param>
    <version>10</version>
    <quarkSpinType>FULL</quarkSpinType>
    <obsvP>true</obsvP>
    <numRetries>1</numRetries>
      <FermionAction>
        <FermAct>UNPRECONDITIONED_CLOVER</FermAct>
        <Mass>${l_mass}</Mass>
        <clovCoeff>1.160920226</clovCoeff>
        <FermState>
            <Name>STOUT_FERM_STATE</Name>
            <rho>0.125</rho>
            <n_smear>1</n_smear>
            <orthog_dir>-1</orthog_dir>
            <FermionBC>
              <FermBC>SIMPLE_FERMBC</FermBC>
              <boundary>1 1 1 -1</boundary>
            </FermionBC>
          </FermState>
      </FermionAction>
      <InvertParam>
        <invType>CG_INVERTER</invType>
        <RsdCG>1.0e-5</RsdCG>
        <MaxCG>1000</MaxCG>
      </InvertParam>
  </Param>
```

sequential source

```
  <NamedObject>
    <gauge_id>default_gauge_field</gauge_id>
    <source_id>seqsrc_tseq_${tseq}</source_id>
    <prop_id>prop_P0_tseq_${tseq}</prop_id>
  </NamedObject>
</elem>
```

sequential propagator

- **Code for 3pt contraction**

```
<elem>
  <annotation>
     Compute the measurements you build
  </annotation>
  <Name>Measure_gAgV</Name>
  <Param>
    <cfg_serial>${conf}</cfg_serial>
    <gAgV_curr>
      <elem>gV</elem>
      <elem>gA</elem>
    </gAgV_curr>
    <l_prop>L_quark_propagator</l_prop>
    <seq_prop>Seq_propagator</seq_prop>
    <file_name>${prefix}/Data/3pt_${conf}_tseq${tseq}.dat.iog</file_name>
   </Param>
</elem>
```

source_code/
inline_gAgV.cc

Notice, we need Seq_propagator with tseq=X to calculate 3pt with tseq=X.

- **Code for 3pt contraction**

```
int offset = 0;
for ( int i = 0; i < operator_no; i++ )
{
  LatticeComplex corr = zero;

  if (current_list[i] == "gV")
  {
    corr = trace(adj(Gamma(15) * Seq_prop * Gamma(15)) * Gamma(8) * L_prop);
  }
  else if (current_list[i] == "gA")
  {
    corr = trace(adj(Gamma(15) * Seq_prop * Gamma(15)) * Gamma(11) * L_prop);
  }
  else
  {
    QDPIO::cerr << "Unknown current name: " << current_list[i] << std::endl;
    QDP_abort(1);
  }

  multi1d<DComplex> hsum = sumMulti( Phases * corr,
  if(Layout::primaryNode())
  for (int t=0; t < tlen; ++t)
  {
    res.data[offset*tlen*2 + 2*t] = hsum[t].elem().elem().elem().real();
    res.data[offset*tlen*2 + 2*t + 1] = hsum[t].elem().elem().elem().imag();
  }
  offset++;
}
```

$$3pt = Tr\left[adj(\gamma^5 G^{seq}(x,0)\gamma^5)\Gamma G^2(x,0)\right].$$

$g_V: \Gamma = \gamma_4$

$g_A: \Gamma = \gamma_3\gamma_5$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
I	gx	gy	sigma_xy	gz	sigma_xz	sigma_yz	g5gt	gt	gxgt	gygt	gzg5	gzgt	-gyg5	gxg5	g5

- **Numerical results**

2pt

10000	0	0	5.31117e+05	-8765.75771
10000	0	1	2.21282e+05	-8227.89489
10000	0	2	1.06889e+05	-2610.62397
10000	0	3	5.05592e+04	588.67583
10000	0	4	2.40119e+04	692.12786
10000	0	5	1.20391e+04	509.16560
10000	0	6	5980.93762	349.13651
10000	0	7	2662.30027	270.86883
10000	0	8	1150.40579	181.80283
10000	0	9	483.86196	87.01916
10000	0	10	215.93367	37.30380
10000	0	11	114.10741	9.33224
10000	0	12	59.12059	0.74162
10000	0	13	29.84378	-2.70125
10000	0	14	15.30184	-2.63824
10000	0	15	8.49545	-1.66347
10000	0	16	4.81531	-0.96898
10000	0	17	2.63622	-0.75496
10000	0	18	1.43256	-0.52317
10000	0	19	0.82791	-0.30946
10000	0	20	0.47876	-0.20447
10000	0	21	0.24117	-0.14941
10000	0	22	0.09989	-0.08732
10000	0	23	0.03981	-0.04756

3pt

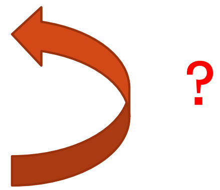
tseq

10000	0	0	8471.84128	480.33251
10000	0	1	8581.06612	500.72560
10000	0	2	8548.30248	541.39885
10000	0	3	8513.15431	498.13305
10000	0	4	8356.67814	568.18339
10000	0	5	8090.22817	525.22246
10000	0	6	31.43189	5.81184
10000	0	7	-0.86867	-0.32147
10000	0	8	-0.29755	2.99280
10000	0	9	1.46579	1.20732
10000	0	10	2.19900	-3.59126
10000	0	11	-0.47264	-0.24265
10000	0	12	-0.23717	0.77605
10000	0	13	-0.58696	-0.52249
10000	0	14	-0.55057	-0.33018
10000	0	15	0.50290	0.23213
10000	0	16	-0.34352	-0.29705
10000	0	17	-0.29338	0.11848
10000	0	18	0.08039	0.08380
10000	0	19	0.02947	0.09560
10000	0	20	0.05329	-0.03709
10000	0	21	0.00317	0.03806
10000	0	22	0.03399	0.01742
10000	0	23	2.50527e-05	0.00276

• Reduction and 3pt/2pt

What we want:

$$g_V = \mathrm{Tr}\left[T^n \left\langle P \left| \bar{q}\gamma_4 q \right| N \right\rangle\right], g_A = \mathrm{Tr}\left[T_3^p \left\langle P \left| \bar{q} i\gamma_3\gamma_5 q \right| N \right\rangle\right]$$



What we calculated (correlation function):

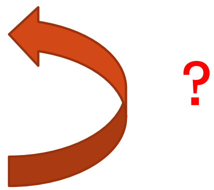
$$C_{3,\mathcal{O}}^{N\to P}\left(q^2, t, t_{\text{seq}}\right) = \int d^3\vec{x_2} d^3\vec{x}\, T^{\gamma'\gamma} \left\langle \psi_\gamma^{(P)}\left(\vec{x_2}, t_{\text{seq}}\right) \mathcal{O}^\mu(\vec{x}, t) \bar{\psi}_{\gamma'}^{(N)}(\vec{0}, 0)\right\rangle$$

$$\mathcal{O}_\Gamma^{12}(x) = \bar{\psi}_\mu^{1,d}(x)(\Gamma)_{\mu\nu}\psi_\nu^{2,d}(x), \quad T^{\gamma'\gamma} = \frac{1+\gamma_4}{2}(1+i\gamma_3\gamma_5).$$

- **Reduction and 3pt/2pt**

What we want:

$$g_V = \mathrm{Tr}\left[T^n \left\langle P \left| \bar{q}\gamma_4 q \right| N \right\rangle\right], g_A = \mathrm{Tr}\left[T_3^p \left\langle P \left| \bar{q} i \gamma_3 \gamma_5 q \right| N \right\rangle\right]$$



What we calculated (correlation function):

$$C_{3,\mathcal{O}}^{N \to P}\left(q^2, t, t_{\text{seq}}\right) = \int d^3\vec{x_2} d^3\vec{x}\, T^{\gamma'\gamma} \left\langle \psi_\gamma^{(P)}\left(\vec{x_2}, t_{\text{seq}}\right) \mathcal{O}^\mu(\vec{x}, t) \bar{\psi}_{\gamma'}^{(N)}(\vec{0}, 0) \right\rangle$$

$$\mathcal{O}_\Gamma^{12}(x) = \bar{\psi}_\mu^{1,d}(x)(\Gamma)_{\mu\nu}\psi_\nu^{2,d}(x), \qquad T^{\gamma'\gamma} = \frac{1+\gamma_4}{2}(1 + i\gamma_3\gamma_5).$$

Reduction by inserting: $\quad 1 = \int \frac{d^3\vec{P}_q}{(2\pi)^3 2E_q} |q\rangle\langle q|$

$$C_{3,\mathcal{O}}^{N \to P}\left(q^2, t, t_{\text{seq}}\right) = \int d^3\vec{x} d^3\vec{y} \int \frac{d^3\vec{P}_N}{(2\pi)^3 2E_N} \frac{d^3\vec{P}_P}{(2\pi)^3 2E_P}$$
$$T^{\gamma'\gamma} \left\langle 0 \left| \psi_\gamma^{(P)}\left(\vec{x}, t_{\text{seq}}\right) \right| P \right\rangle \left\langle P \left| \mathcal{O}^\mu(\vec{y}, t) \right| N \right\rangle \left\langle N \left| \bar{\psi}_{\gamma'}^{(N)}(\vec{0}, 0) \right| 0 \right\rangle.$$

$$\begin{aligned} C_2^P &= \int d^3\vec{x} T^{\prime\gamma'\gamma} \left\langle \psi_\gamma^{(P}(\vec{x}, t) \bar{\psi}_{\gamma'}^{(P)}(\vec{0}, 0) \right\rangle \\ &= \int d^3\vec{x} T^{\prime\gamma'\gamma} \int \frac{d^3\vec{P}_q}{(2\pi)^3 2E_q} \left\langle \psi_\gamma^{(P}(\vec{x}, t) |q\rangle\langle q| \bar{\psi}_{\gamma'}^{(P)}(\vec{0}, 0) \right\rangle. \end{aligned}$$

Take the ratio:

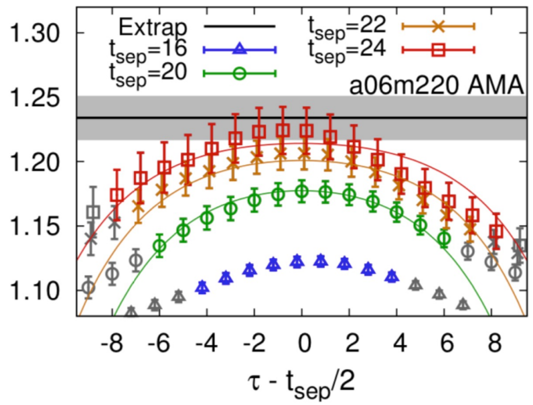
$$R_{V/A}(T, \mu) = \frac{C_3^{V/A}\left(q^2, t, t_{\text{seq}}\right)}{C_2^P(t_{seq})}.$$

• Fit to extract ground state

- **Consider lowest two energy states in the reduction process:**

$$3\text{pt} \approx z_0^2 O_{00} e^{-E_0 t_{\text{sep}}} + z_0^\dagger z_1 O_{01} e^{-E_0 t_{\text{sep}}} e^{-\Delta E t} + z_1^\dagger z_0 O_{10} e^{-E_1 t_{\text{sep}}} e^{\Delta E t} + z_1^2 O_{11} e^{-E_1 t_{\text{sep}}}$$

$$2\text{pt} \approx z_0^2 e^{-E_0 t_{\text{sep}}} + z_1^2 e^{-E_1 t_{\text{sep}}} = z_0^2 e^{-E_0 t_{\text{sep}}} \left(1 + c_1 e^{-\Delta E t_{\text{sep}}}\right)$$



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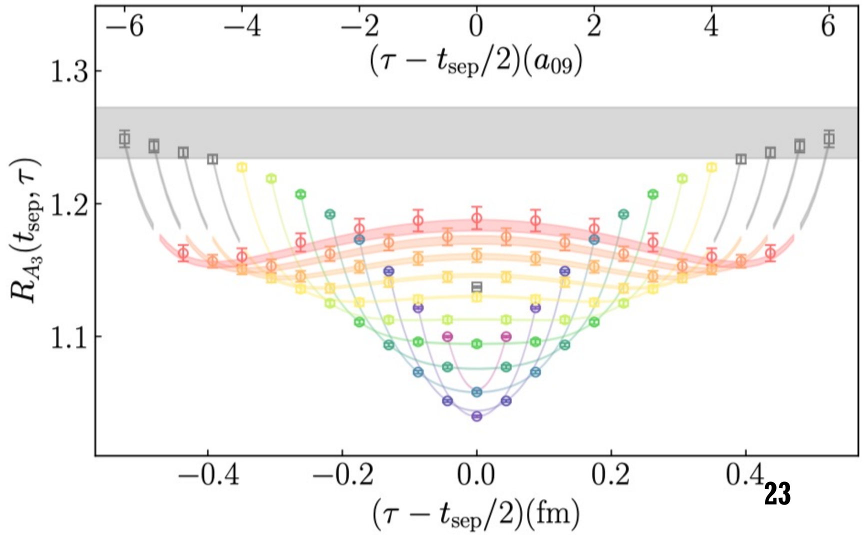
Assume $O_{01} = O_{10}$

$$\frac{3\text{pt}}{2\text{pt}} = \frac{1}{1 + c_1 e^{-\Delta E t_{\text{sep}}}} \cdot \left[O_{00} + \frac{z_0^\dagger z_1}{z_0^2} O_{01} e^{-\Delta E t} + \frac{z_1^\dagger z_0}{z_0^2} O_{10} e^{-\Delta E t_{\text{sep}}} e^{\Delta E t} + \frac{z_1^2}{z_0^2} O_{11} e^{-\Delta E t_{\text{sep}}} \right]$$

$$\approx O_{00} + \frac{z_0^\dagger z_1}{z_0^2} O_{01} e^{-\Delta E t} + \frac{z_1^\dagger z_0}{z_0^2} O_{10} e^{-\Delta E t_{\text{sep}}} e^{\Delta E t} + \frac{z_1^2}{z_0^2} O_{11} e^{-\Delta E t_{\text{sep}}}$$

$$= O_{00} \cdot \left[1 + a_1 \left(e^{-\Delta E (t_{\text{sep}} - t)} + e^{-\Delta E t}\right) + a_2 e^{-\Delta E t_{\text{sep}}}\right]$$

Ground State



Finally: $R_{3pt/2pt} \simeq O_{00}\left[1 + a_1\left(e^{-\Delta E_1 t} + e^{-\Delta E_1 (t_{\text{seq}} - t)}\right)\right]$

• Feynman-Hellmann Method

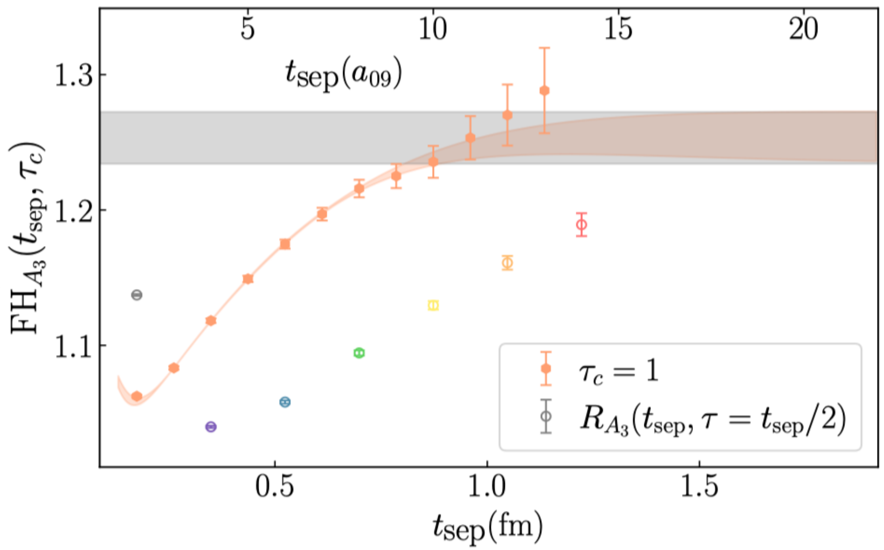
- **Summing over t at each tsep ratio:**

$$\Sigma\left(t_{\text{sep}}\right)=\sum_{t=n}^{t_{\text{sep}}-n}\frac{3\text{pt}}{2\text{pt}}=\left(O_{00}+O_{00}a_2e^{-\Delta E t_{\text{sep}}}\right)\cdot\left(t_{\text{sep}}+1-2n\right)+O_{00}a_1e^{-\Delta E t_{\text{sep}}}\sum_t e^{\Delta E t}+O_{00}a_1\sum_t e^{-\Delta E t}$$

$$\sum_{t=n}^{t_{\text{sep}}-n}e^{\Delta E t}=e^{\Delta E n}\frac{1-e^{\Delta E(t_{\text{sep}}+1-2n)}}{1-e^{\Delta E}}\approx\frac{e^{\Delta E n}}{1-e^{\Delta E}}$$

$$\Sigma\left(t_{\text{sep}}\right)=\left(O_{00}+O_{00}a_2e^{-\Delta E t_{\text{sep}}}\right)\cdot\left(t_{\text{sep}}+1-2n\right)+\frac{O_{00}a_1e^{-\Delta E(t_{\text{sep}}-n)}}{1-e^{\Delta E}}+\frac{O_{00}a_1e^{-\Delta E n}}{1-e^{-\Delta E}}$$

$$=\left(O_{00}+O_{00}a_2e^{-\Delta E t_{\text{sep}}}\right)\cdot\left(t_{\text{sep}}+1-2n\right)+\frac{O_{00}a_1e^{-\Delta E(t_{\text{sep}}-n+1)}}{e^{-\Delta E}-1}+\frac{O_{00}a_1e^{-\Delta E n}}{1-e^{-\Delta E}}$$



- **Define Feynman-Hellmann factor as:**

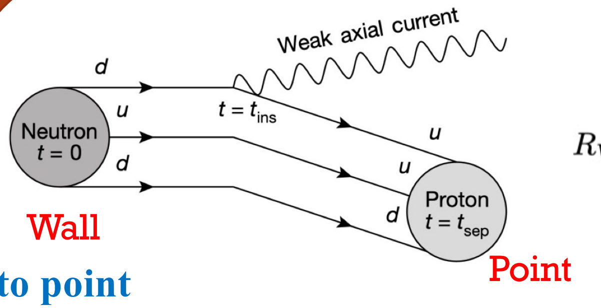
$$\text{FH}=\Sigma\left(t_{\text{sep}}+1\right)-\Sigma\left(t_{\text{sep}}\right)=O_{00}+O_{00}a_2e^{-\Delta E t_{\text{sep}}}\left[\left(t_{\text{sep}}+2-2n\right)e^{-\Delta E}-\left(t_{\text{sep}}+1-2n\right)\right]+O_{00}a_1e^{-\Delta E(t_{\text{sep}}-n+1)}$$

$$\text{FH}=O_{00}\cdot\left[1+a_1'e^{-\Delta E t_{\text{sep}}}+a_2'\cdot t_{\text{sep}}\cdot e^{-\Delta E t_{\text{sep}}}\right]$$

Without t dependence

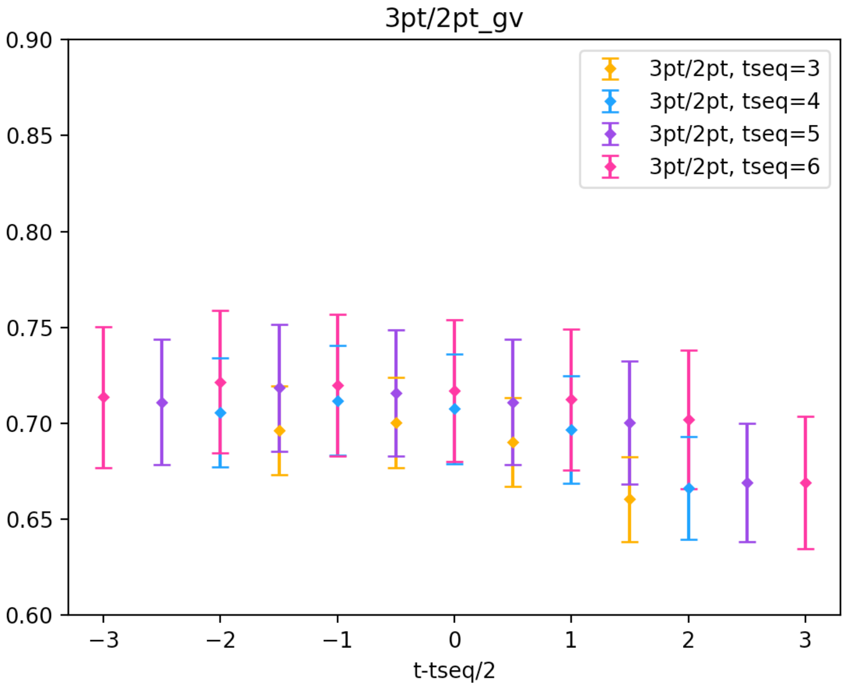
- # Date analysis

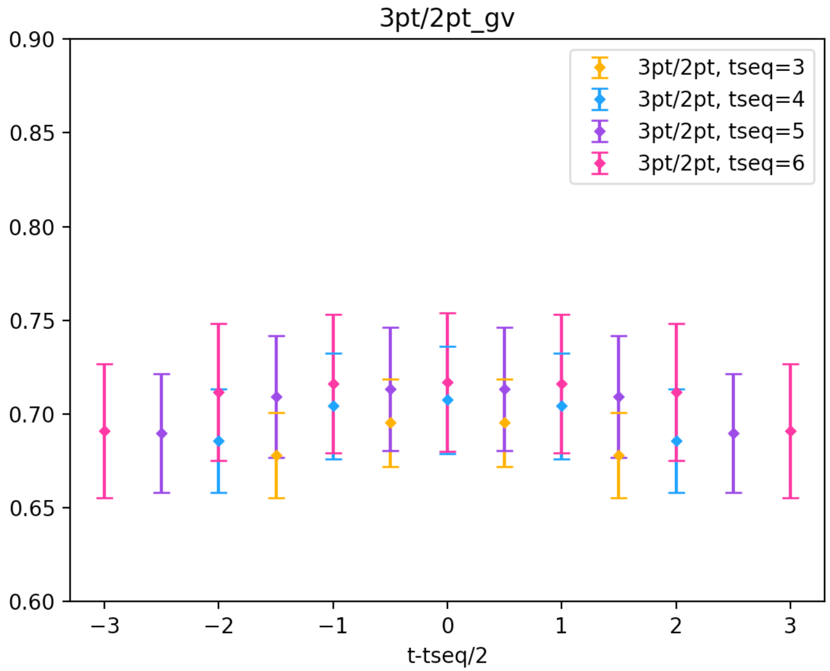
$$R_{V/A}(T,\mu) = \frac{C_3^{V/A}\left(q^2, t, t_{\text{seq}}\right)}{C_2^P\left(t_{seq}\right)}.$$



$$R_{V/A}(T,\mu) = \sqrt{\frac{C_3^{V/A}\left(q^2, t, t_{\text{seq}}\right) C_3^{V/A}\left(q^2, t_{\text{seq}} - t, t_{\text{seq}}\right)}{C_2^P\left(t_{\text{seq}}\right) C_2^P\left(t_{\text{seq}}\right)}}.$$

Asymmetries arising from wall to point





- **Date analysis**

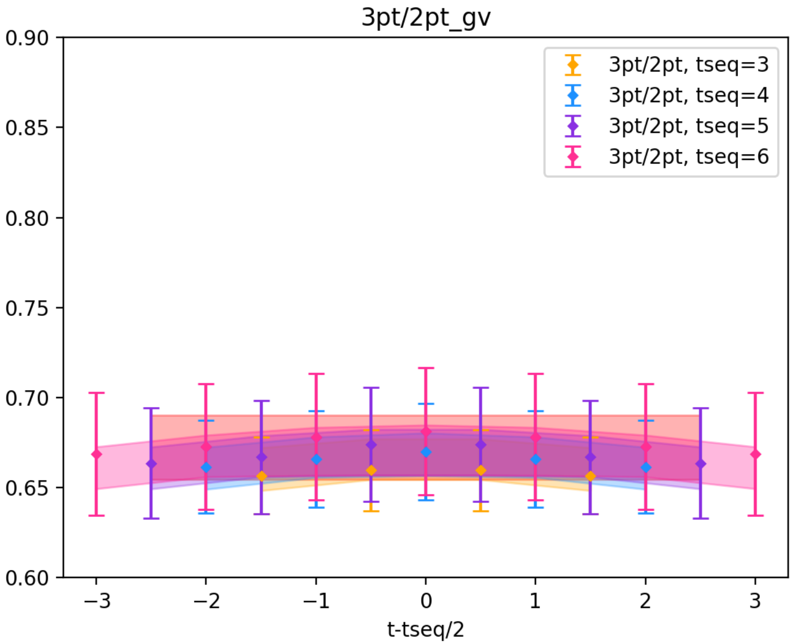
To extract the ground state

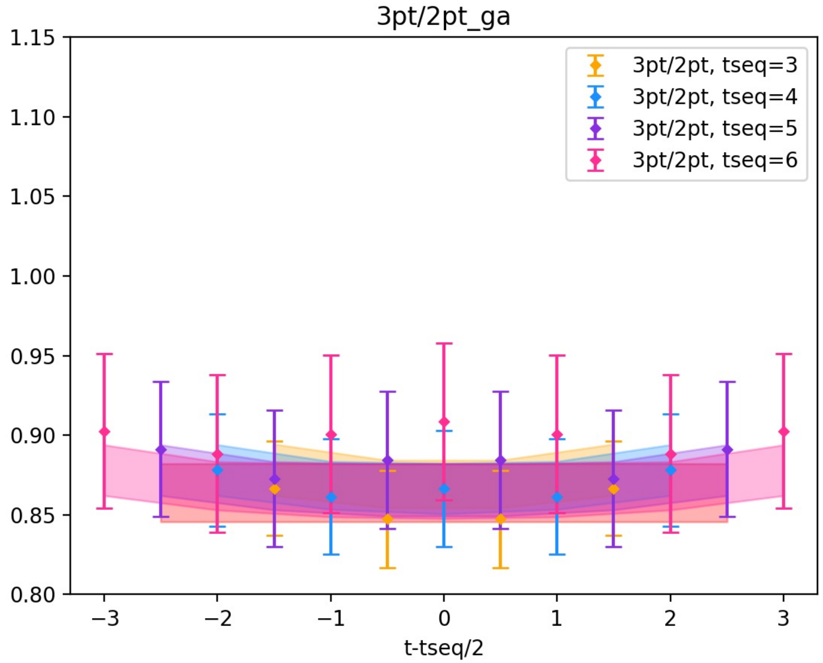
Joint fit for different t_seq:

~~$$C_{2pt} = c_0 e^{-E_0 t}\left(1 + c_1 e^{-\Delta E_1 t}\right)$$~~

$$R_{3pt/2pt} \simeq O_{00}\left[1 + a_1\left(e^{-\Delta E_1 t} + e^{-\Delta E_1 (t_{seq} - t)}\right)\right]$$

Mass_670 (50 fgs)

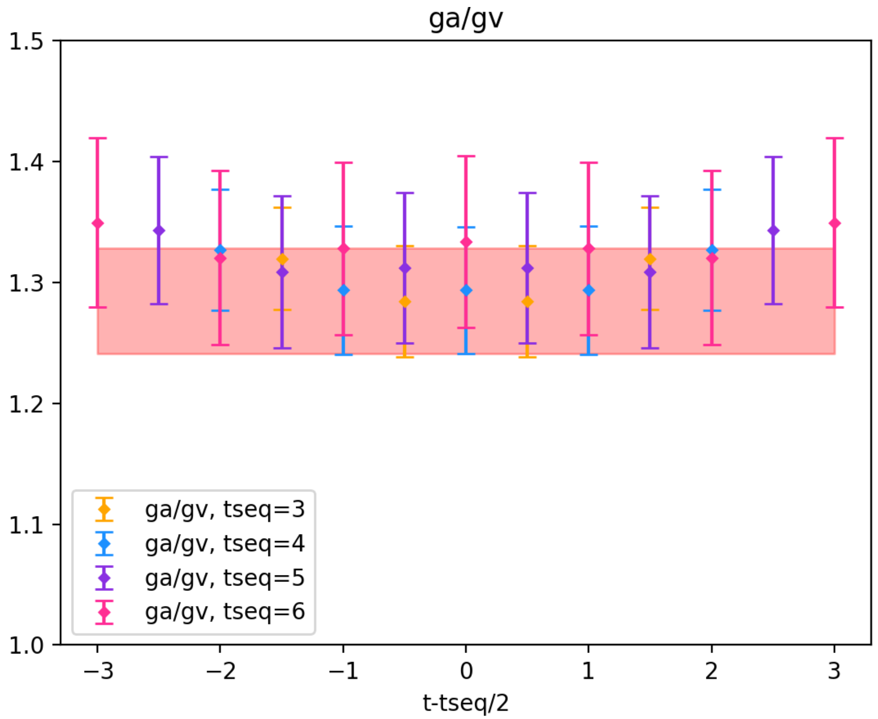




- **Date analysis**

The numerical results of g_A can be define as: $g_A = \frac{Z_A}{Z_V}\frac{\mathring{g}_A}{\mathring{g}_V}$,

- **Z_V, Z_A are renormalisation factors and $Z_V \approx Z_A$ are expected. （The chiral symmetry breaking do not conserve on clover， but is small compared to the statistical uncertainty.）**
- **$Z_V \dot{g}_V = 1$ for conservation current**



$g_A = 1.284 \pm 0.044$

Systematic uncertainties：

- Lattice spacing 0.11fm, $a \to 0$
- Pion mass 670MeV, $m_\pi \to 130MeV$
- Chiral symmetry
- …

- **Practice**

1. Use the computed quark propagator and sequential propagator to compute the 2pt and 3pt, take ratio of 3pt/2pt to obtain gA and gV

   - Generate 2pt and 3pt data

     2pt path：/dssg/home/acct-phyww/phyww/qazhang/training_camp_2023/class4_hua/ con_2pt

     3pt path：/dssg/home/acct-phyww/phyww/qazhang/training_camp_2023/class4_hua/ con_3pt

   - Data: /dssg/home/acct-phyww/phyww/qazhang/training_camp/class4_hua/Save_data

     - source_codes_Props ------ well written — Check the propagator path(Pi:670MeV)
     - source_codes_modify ------ need yourself — complete
       a) inline_gAgV.cc
       b) simple_baryon_seqsrc_w.cc

soft links in linux: ln –s ../source_codes_modify/chroma ./

- **Practice**

- Take the ratio of 3pt/2pt then do the joint fit to obtain gA and gV(data: ~/class4_hua/Save_data)

$$R_{3pt/2pt} \simeq O_{00}\left[1 + a_1\left(e^{-\Delta E_1 t} + e^{-\Delta E_1 (t_{\text{seq}} - t)}\right)\right]$$

$$R_{V/A}(T,\mu) = \sqrt{\frac{C_3^{V/A}(q^2, t, t_{\text{seq}})\, C_3^{V/A}(q^2, t_{\text{seq}} - t, t_{\text{seq}})}{C_2^P(t_{\text{seq}})\, C_2^P(t_{\text{seq}})}}.$$

3pt: tseq3(t=0~3)
tseq4(t=0~5)
tseq5(t=0~5)
tseq6(t=0~6)

- Check the number of gA /gV

plt code:

```
plt.errorbar(x, cv, err, capsize=4.0, fmt='D',  ms=3.0, color='red', label ='name')

plt.fill_between(x, cv-err, cv+err,  color='red', alpha=0.3,  label='name')

plt.savefig('./name.png')
```

- **Practice**

2pt

Cfg	Particle	t	Re	Im
10000	0	0	5.31117e+05	-8765.75771
10000	0	1	2.21282e+05	-8227.89489
10000	0	2	1.06889e+05	-2610.62397
10000	0	3	5.05592e+04	588.67583
10000	0	4	2.40119e+04	692.12786
10000	0	5	1.20391e+04	509.16560
10000	0	6	5980.93762	349.13651
10000	0	7	2662.30027	270.86883
10000	0	8	1150.40579	181.80283
10000	0	9	483.86196	87.01916
10000	0	10	215.93367	37.30380
10000	0	11	114.10741	9.33224
10000	0	12	59.12059	0.74162
10000	0	13	29.84378	-2.70125
10000	0	14	15.30184	-2.63824
10000	0	15	8.49545	-1.66347
10000	0	16	4.81531	-0.96898
10000	0	17	2.63622	-0.75496
10000	0	18	1.43256	-0.52317
10000	0	19	0.82791	-0.30946
10000	0	20	0.47876	-0.20447
10000	0	21	0.24117	-0.14941
10000	0	22	0.09989	-0.08732
10000	0	23	0.03981	-0.04756

3pt

Cfg	gv=0/ga=1	t	Re	Im
10000	0	0	8471.84128	480.33251
10000	0	1	8581.06612	500.72560
10000	0	2	8548.30248	541.39885
10000	0	3	8513.15431	498.13305
10000	0	4	8356.67814	568.18339
10000	0	5	8090.22817	525.22246
10000	0	6	31.43189	5.81184
10000	0	7	-0.86867	-0.32147
10000	0	8	-0.29755	2.99280
10000	0	9	1.46579	1.20732
10000	0	10	2.19900	-3.59126
10000	0	11	-0.47264	-0.24265
10000	0	12	-0.23717	0.77605
10000	0	13	-0.58696	-0.52249
10000	0	14	-0.55057	-0.33018
10000	0	15	0.50290	0.23213
10000	0	16	-0.34352	-0.29705
10000	0	17	-0.29338	0.11848
10000	0	18	0.08039	0.08380
10000	0	19	0.02947	0.09560
10000	0	20	0.05329	-0.03709
10000	0	21	0.00317	0.03806
10000	0	22	0.03399	0.01742
10000	0	23	2.50527e-05	0.00276

tseq

- **Practice**

$$R_{3pt/2pt} \simeq O_{00}\left[1 + a_1\left(e^{-\Delta E_1 t} + e^{-\Delta E_1 (t_{seq}-t)}\right)\right]$$

3pt: tseq3(t=0~3)
tseq4(t=0~5)
tseq5(t=0~5)
tseq6(t=0~6)

```python
def fit_function(x, paras):
    ans = {}# ratio
    for tseq in [3,4,5,6]:
        ans['tseq='+str(tseq)] =  paras['p3_C'] * (1 + paras['p3_C1'] \
                                * (np.exp(-paras['DeltaE']*x['tseq='+str(tseq)]) \
                                + np.exp(-paras['DeltaE']*(tseq - x['tseq='+str(tseq)])) ))
    return ans
```

```
Least Square Fit:
  chi2/dof [dof] = 0.019 [22]    Q = 1    logGBF = 50.996

Parameters:
           p3_C    0.672 (18)     [  1.0 (1.0) ]
          p3_C1   -0.017 (23)     [    1 (10) ]
         DeltaE     0.88 (97)     [  1.0 (1.0) ]

Fit:
         key        y[key]      f(p)[key]
----------------------------------------
    tseq=3 0     0.657 (21)     0.660 (12)
           1     0.660 (23)     0.666 (11)
           2     0.660 (23)     0.666 (11)
           3     0.657 (21)     0.660 (12)
    tseq=4 0     0.662 (26)     0.661 (12)
           1     0.666 (27)     0.667 (11)
           2     0.670 (27)     0.669 (12)
           3     0.666 (27)     0.667 (11)
           4     0.662 (26)     0.661 (12)
    tseq=5 0     0.664 (31)     0.661 (12)
           1     0.667 (32)     0.667 (11)
           2     0.674 (32)     0.670 (13)
           3     0.674 (32)     0.670 (13)
           4     0.667 (32)     0.667 (11)
           5     0.664 (31)     0.661 (12)
    tseq=6 0     0.669 (34)     0.661 (12)
           1     0.673 (35)     0.668 (12)
           2     0.678 (35)     0.670 (13)
           3     0.681 (35)     0.671 (14)
           4     0.678 (35)     0.670 (13)
           5     0.673 (35)     0.668 (12)
           6     0.669 (34)     0.661 (12)
```

- **Practice**

2. Try to compute a sequential propagator by yourself.

   - Generate a sequential propagator

     path： /dssg/home/acct-phyww/phyww/qazhang/training_camp_2023/class4_hua/ seq_prop_cg
     (source_codes_modify ------ need yourself modify: *******)

   - Calculate a 3pt data with your sequential propagator