Hadron Spectroscopy and Interactions from Lattice QCD

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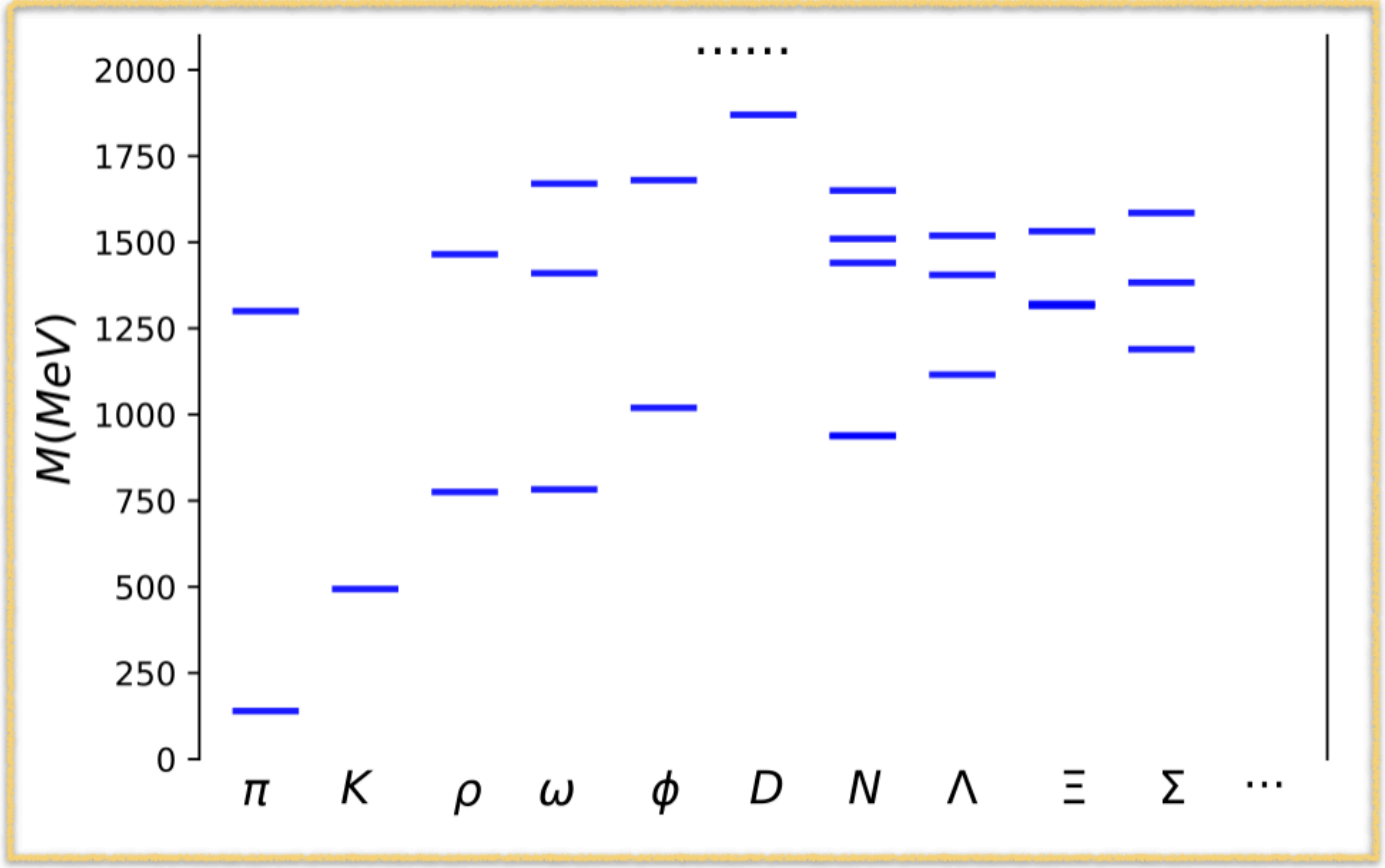
Spectroscopy: What and Why



- ✦ Obtain hadron masses
- ✦ Understand how hadrons are built from quarks and gluons.

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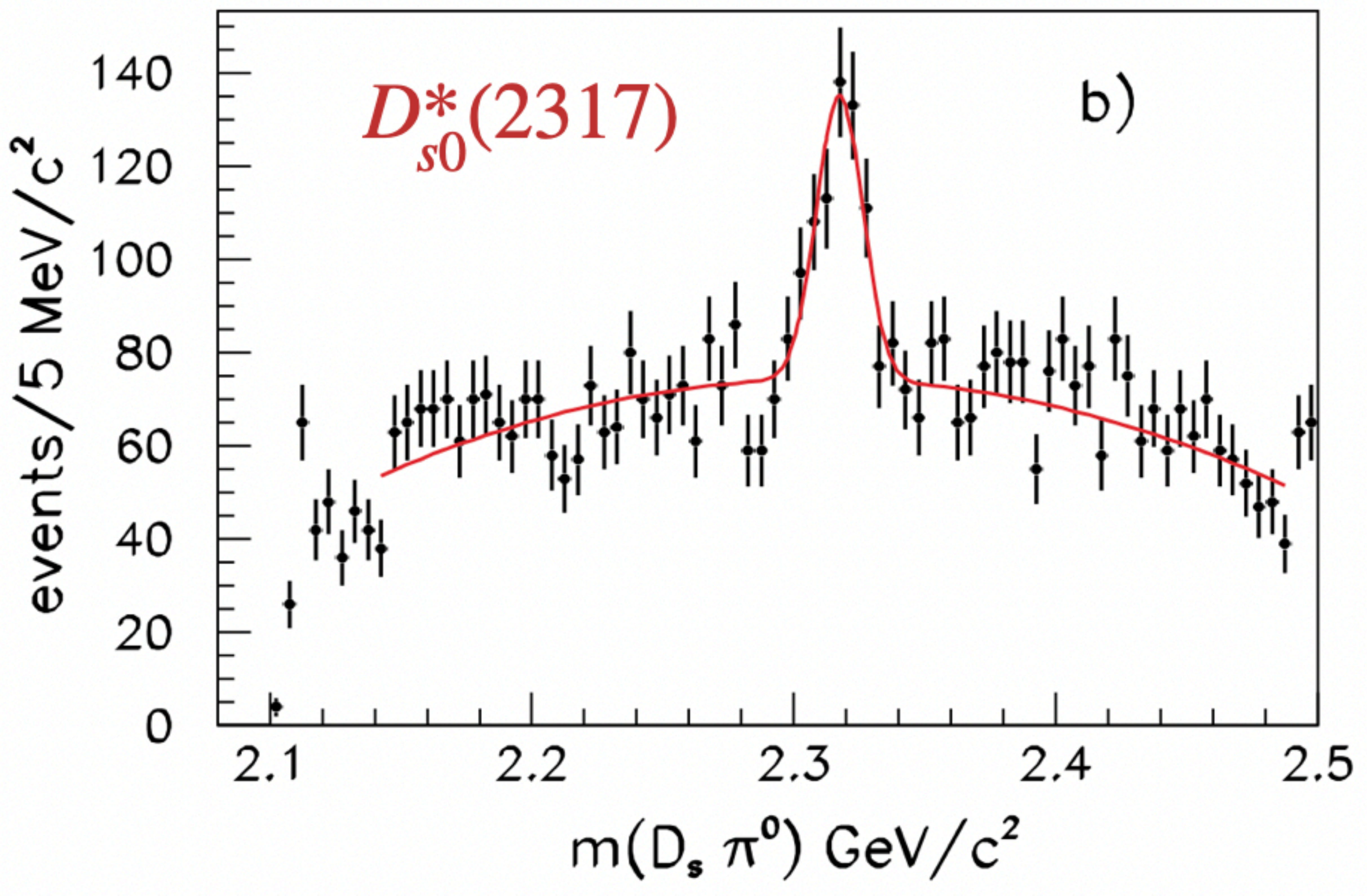

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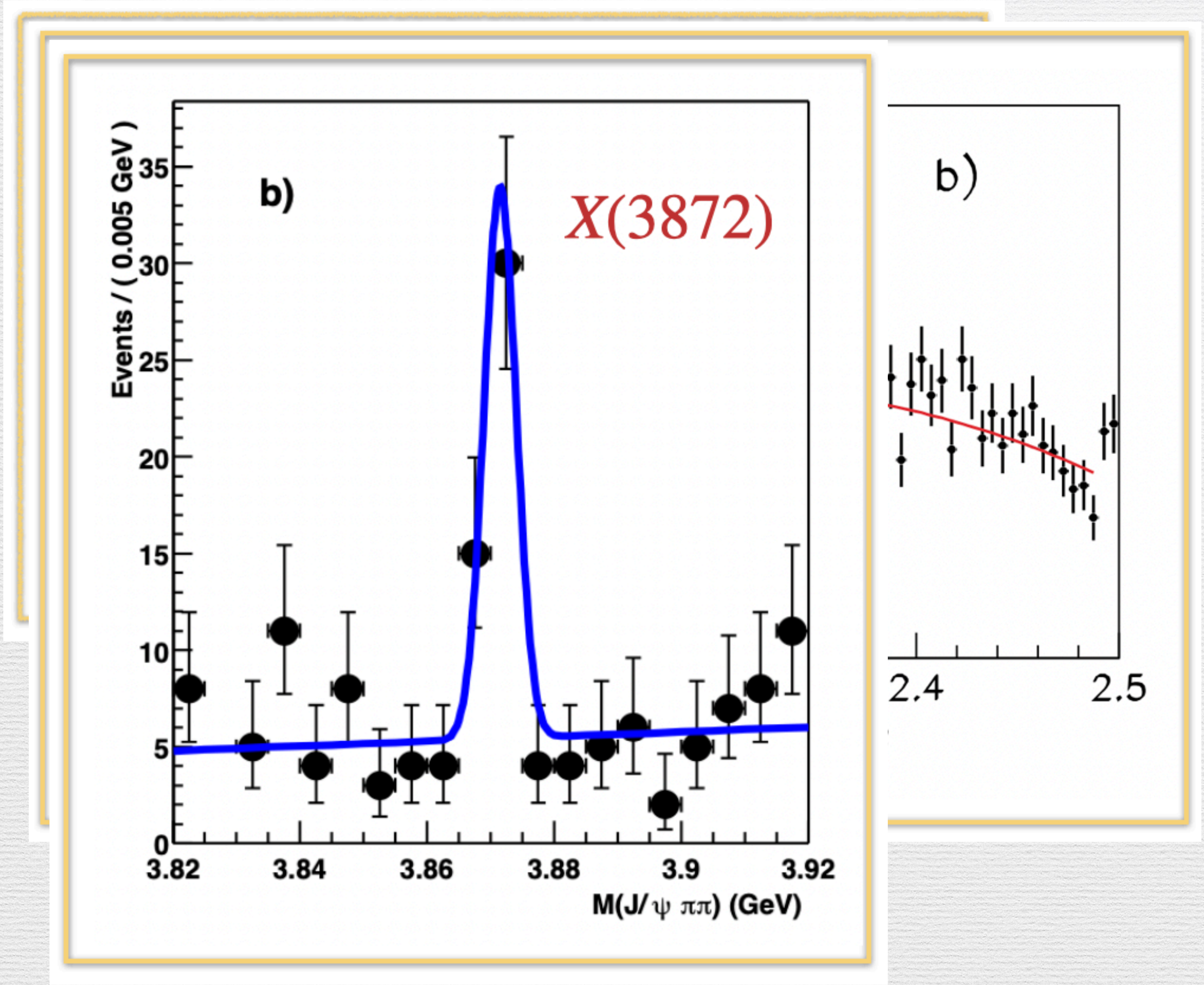

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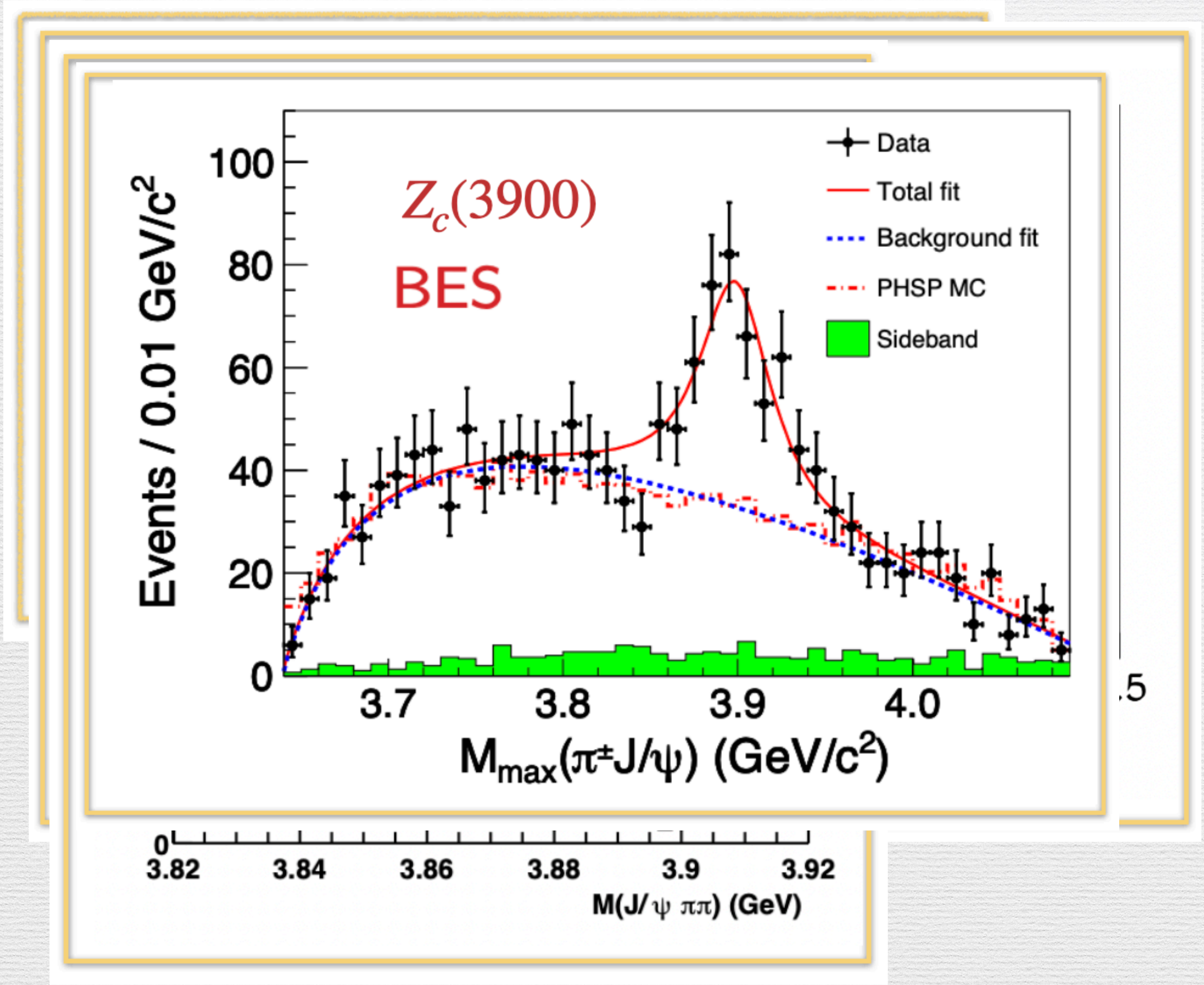

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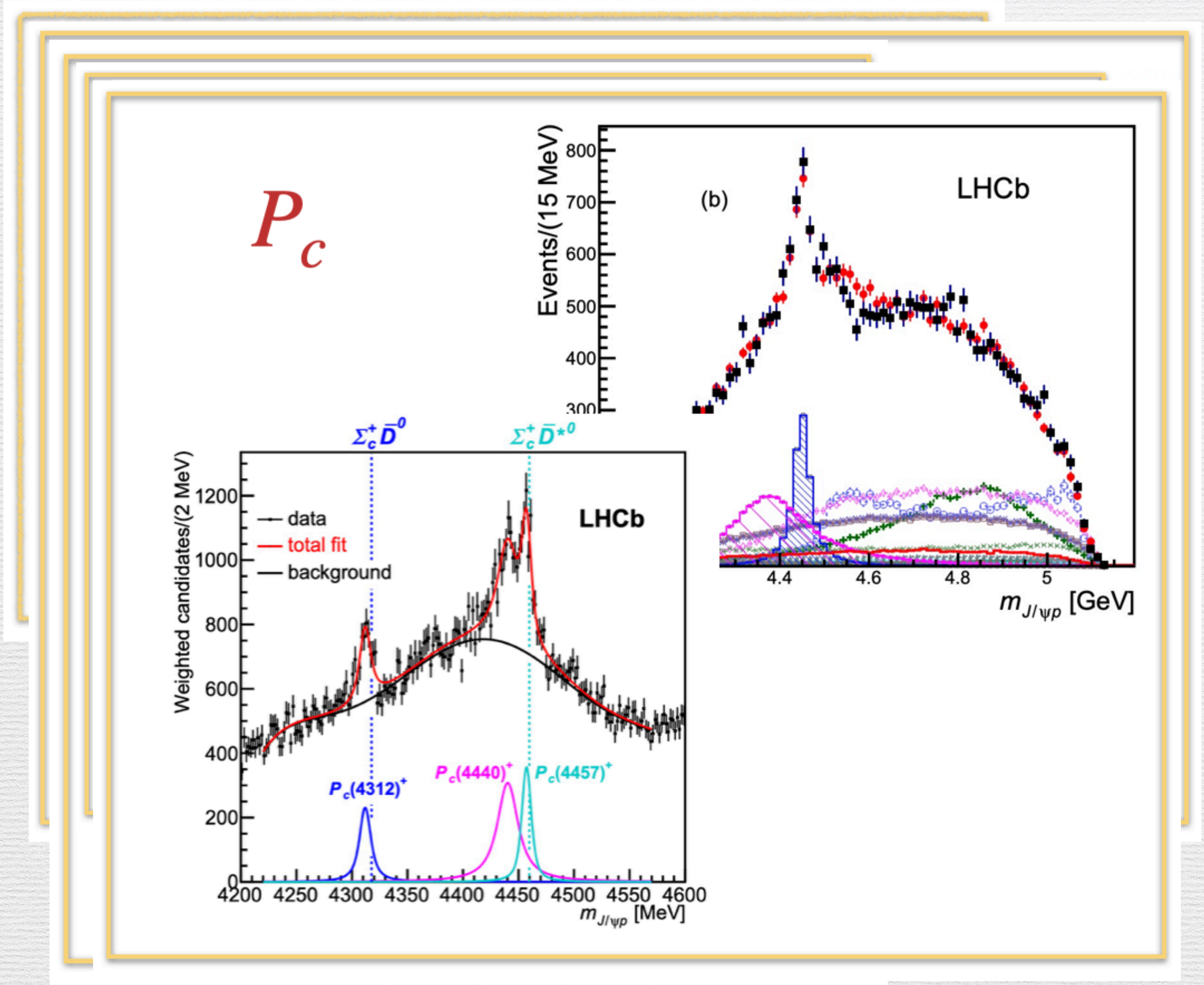

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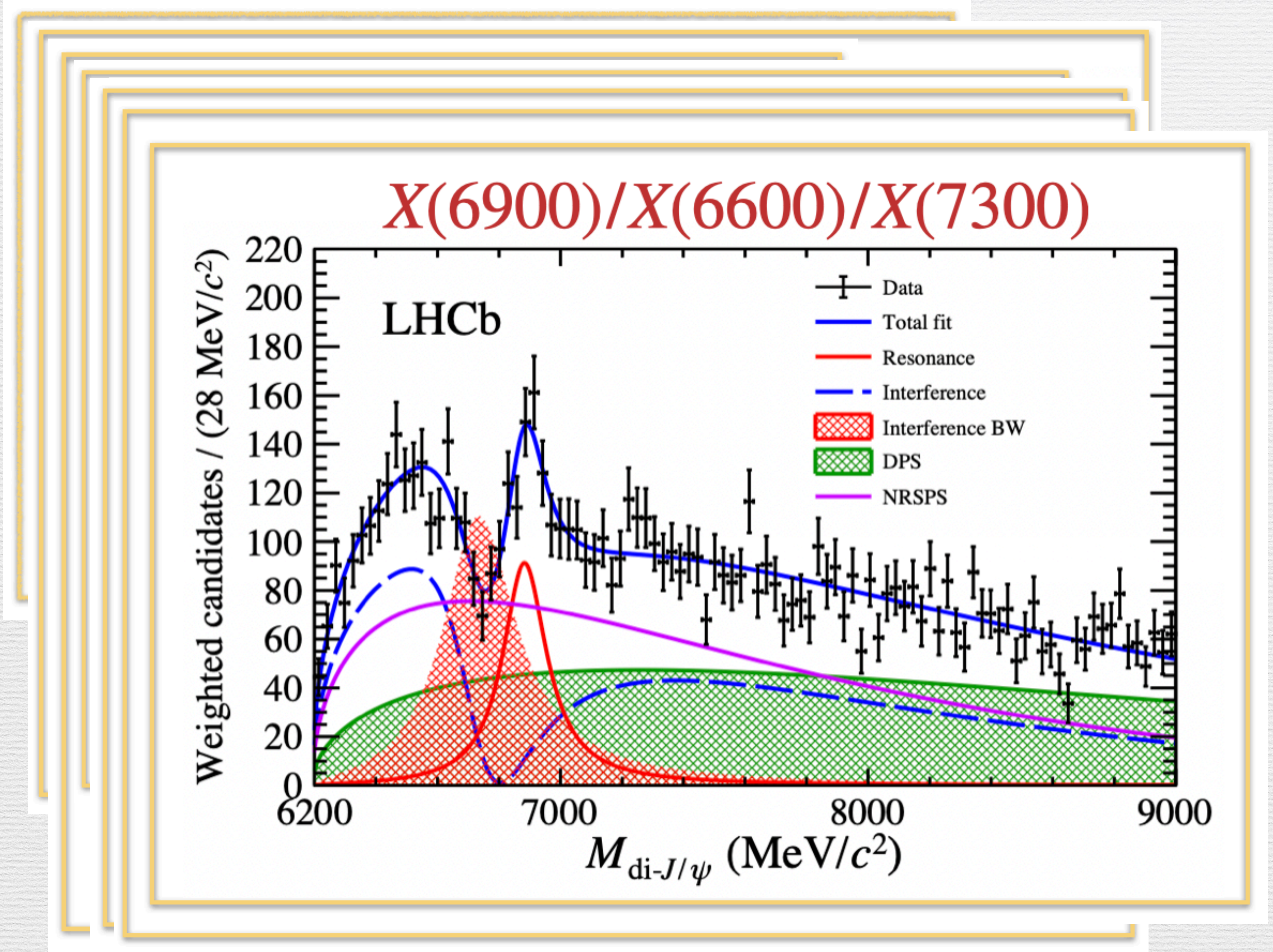

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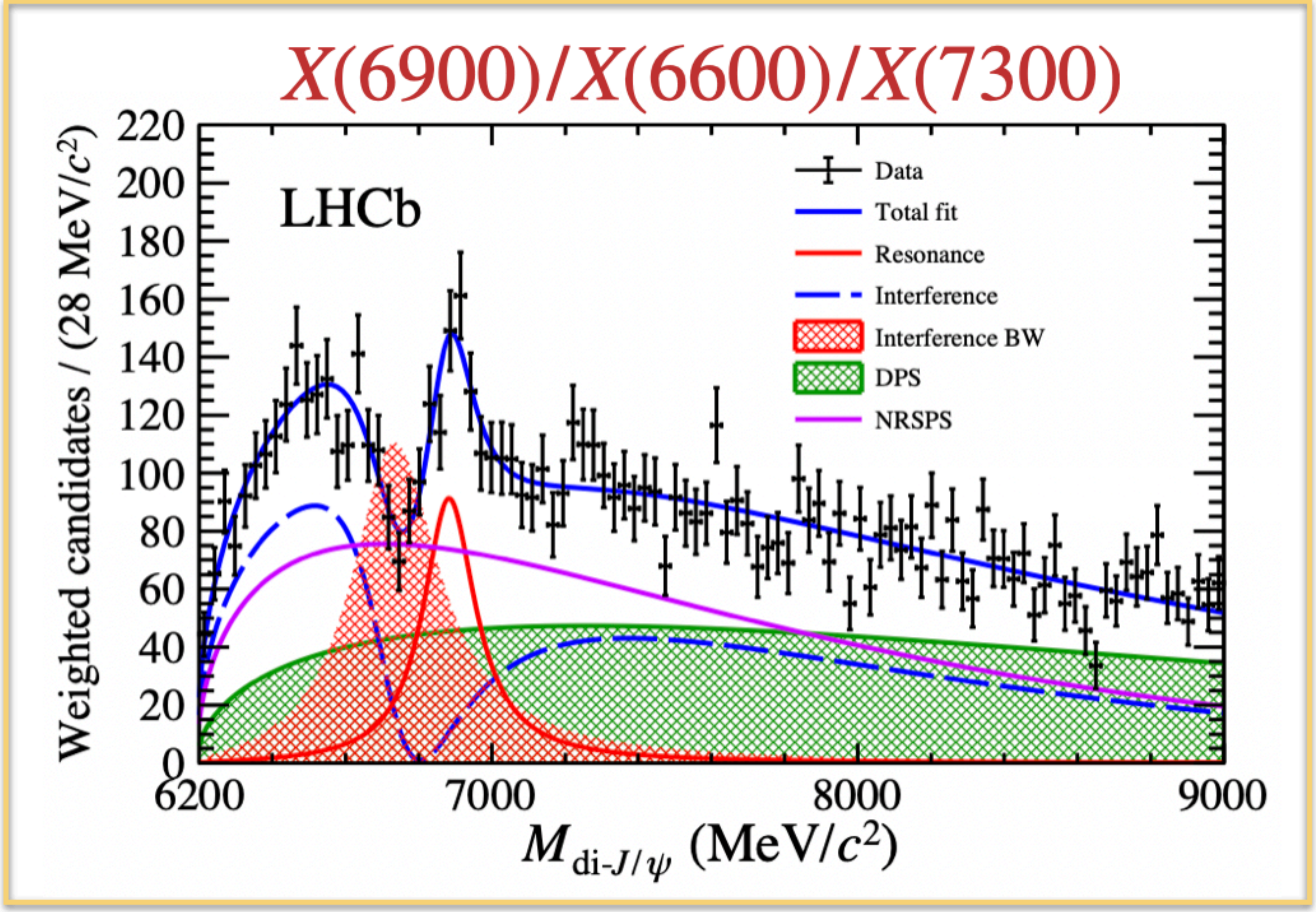
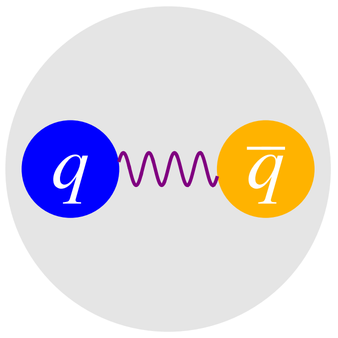


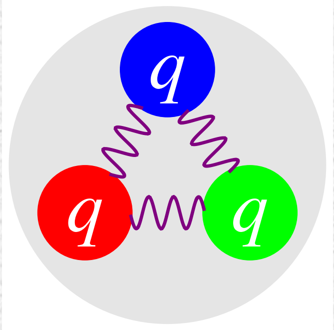
- ✦ Obtain hadron masses
- ✦ Understand how hadrons are built from quarks and gluons.

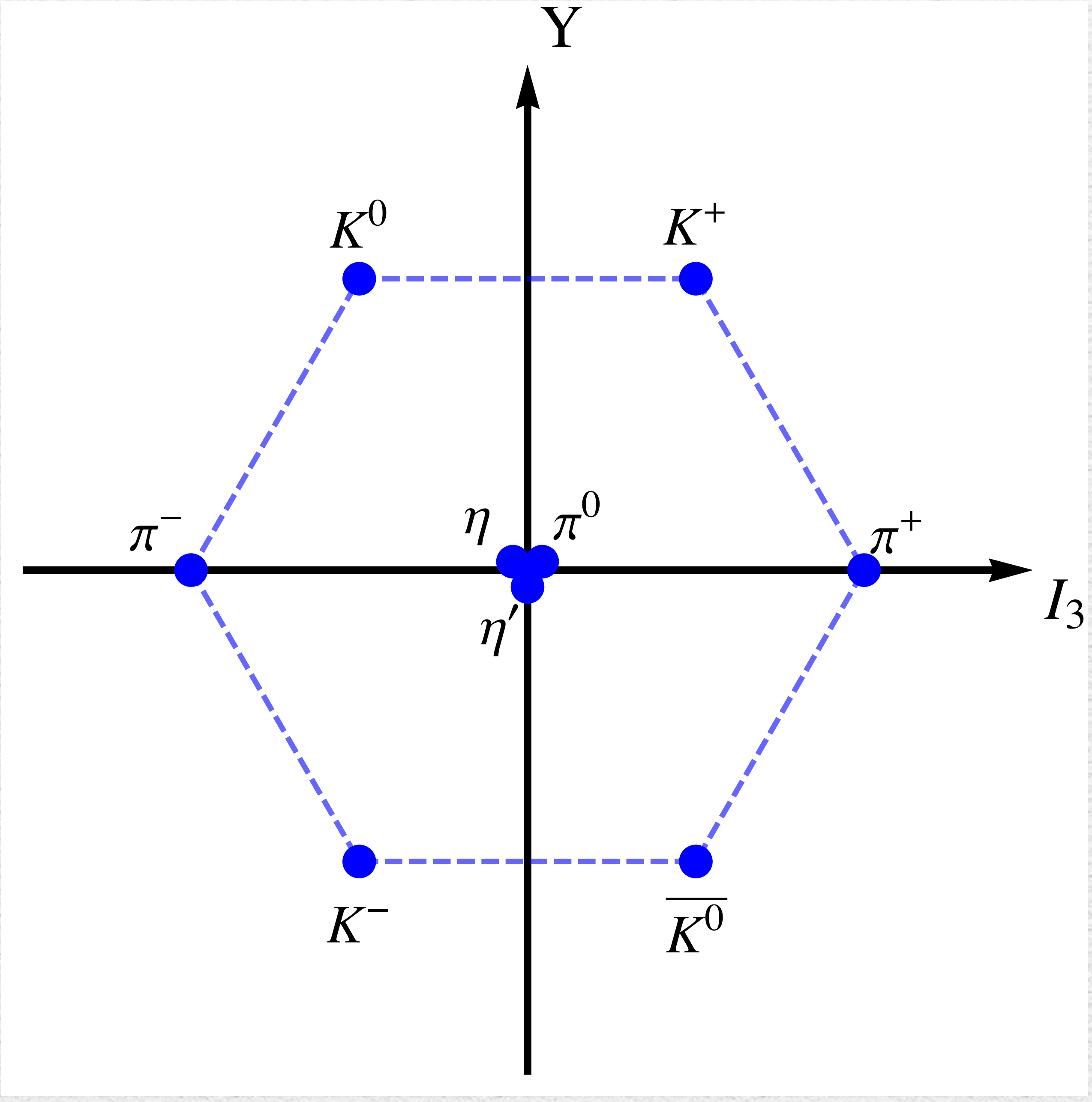
- ✦ New experimental discoveries provides opportunities and challenges for lattice QCD.

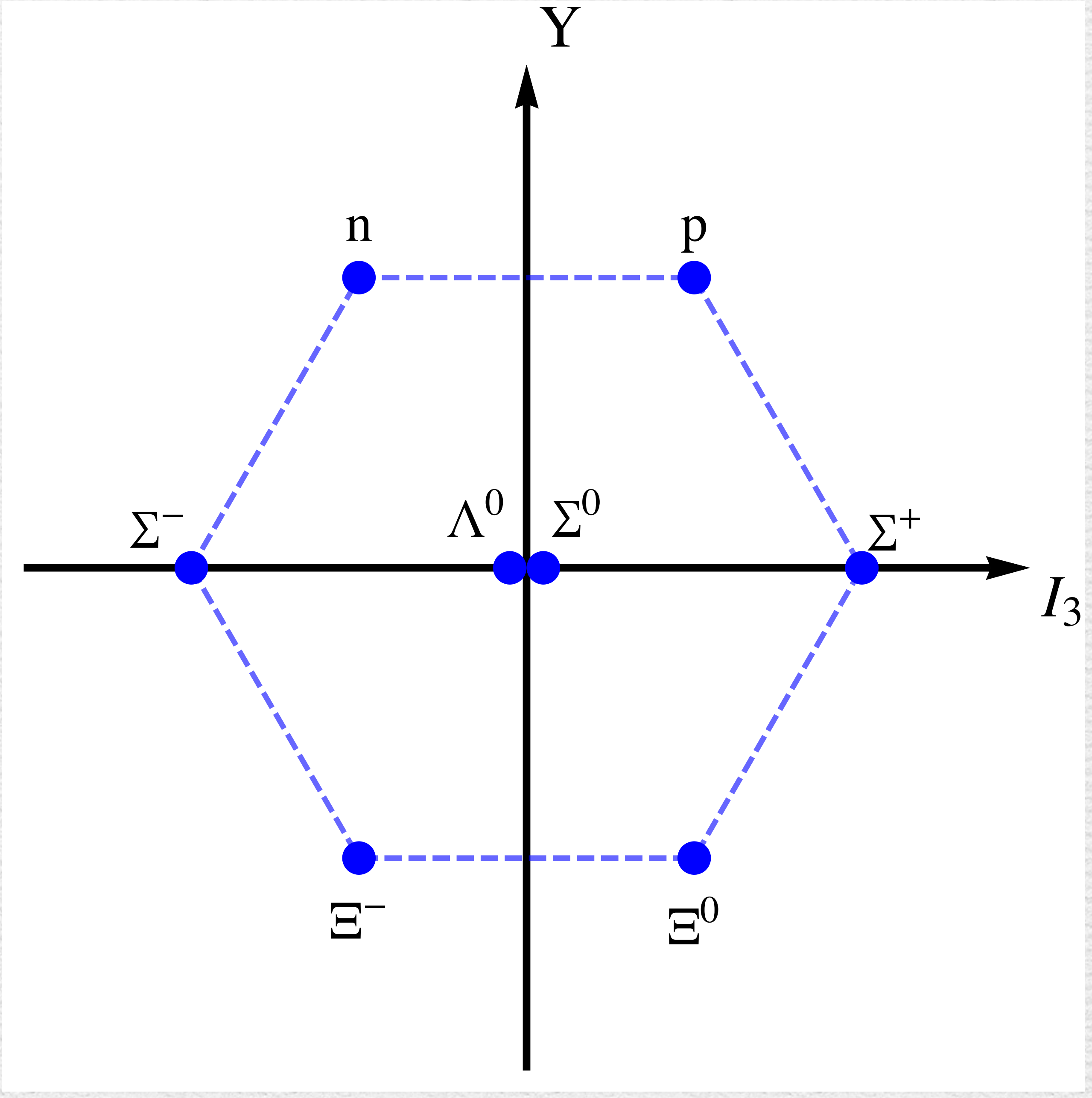
Quark Model

- "Particle zoo": through out 1950's to 1960's, large number of particles were found in scattering experiments.
- Quark model (Gell-Mann and Zweig,1964)

Meson: 

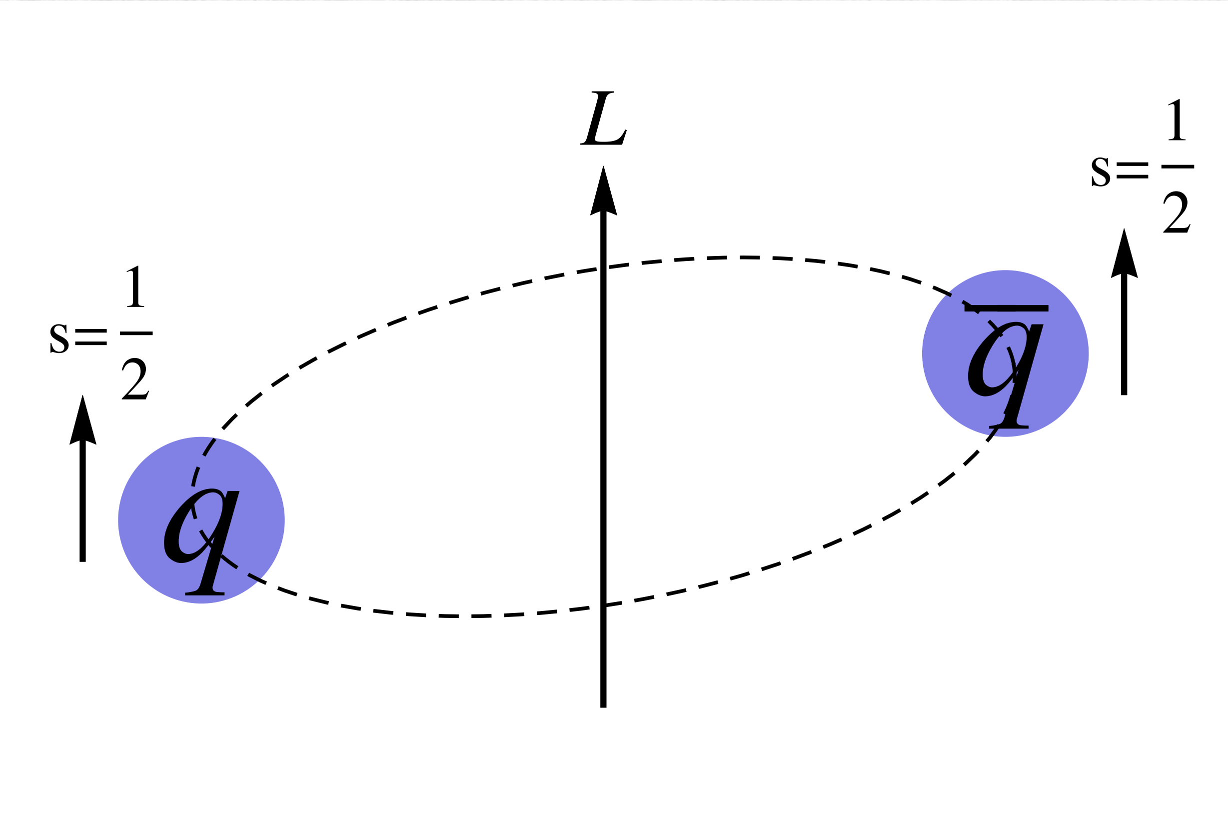
Baryon: 





Quark Model

✦ Mesons($q\bar{q}$): Classified by the conserved quantum numbers J^{PC}

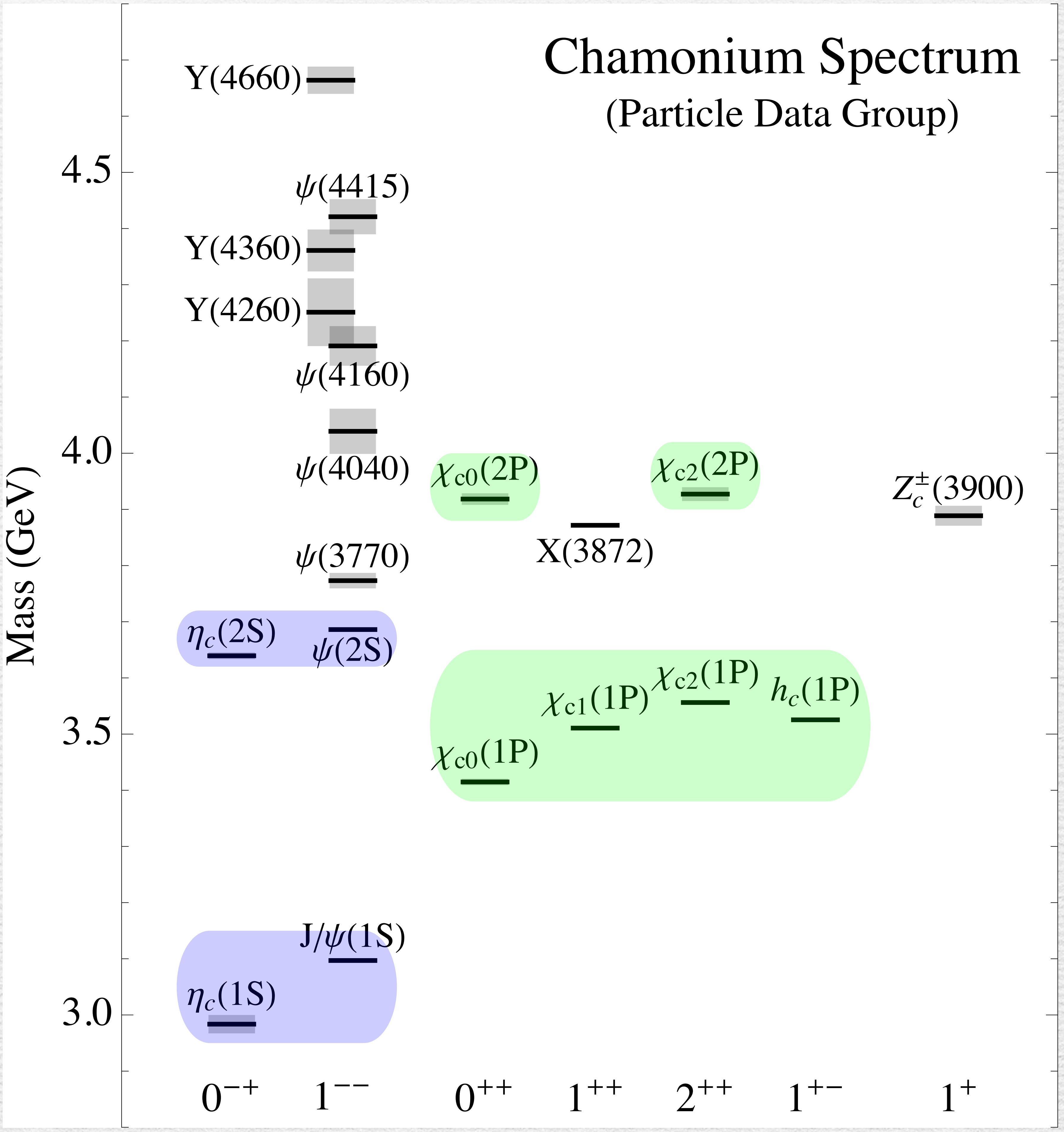


L=0 : $0^{-+}, 1^{--}$

L=1 : $(0,1,2)^{++}, 1^{+-}$

L=2 : $(1,2,3)^{--}, 2^{-+}$

$\vdots$



- Total spin:
  $|L-S| \leq J \leq |L+S|$
- Parity:
  $P = (-1)^{L+1}$
- Charge conjugation:
  $C = (-1)^{L+S}$

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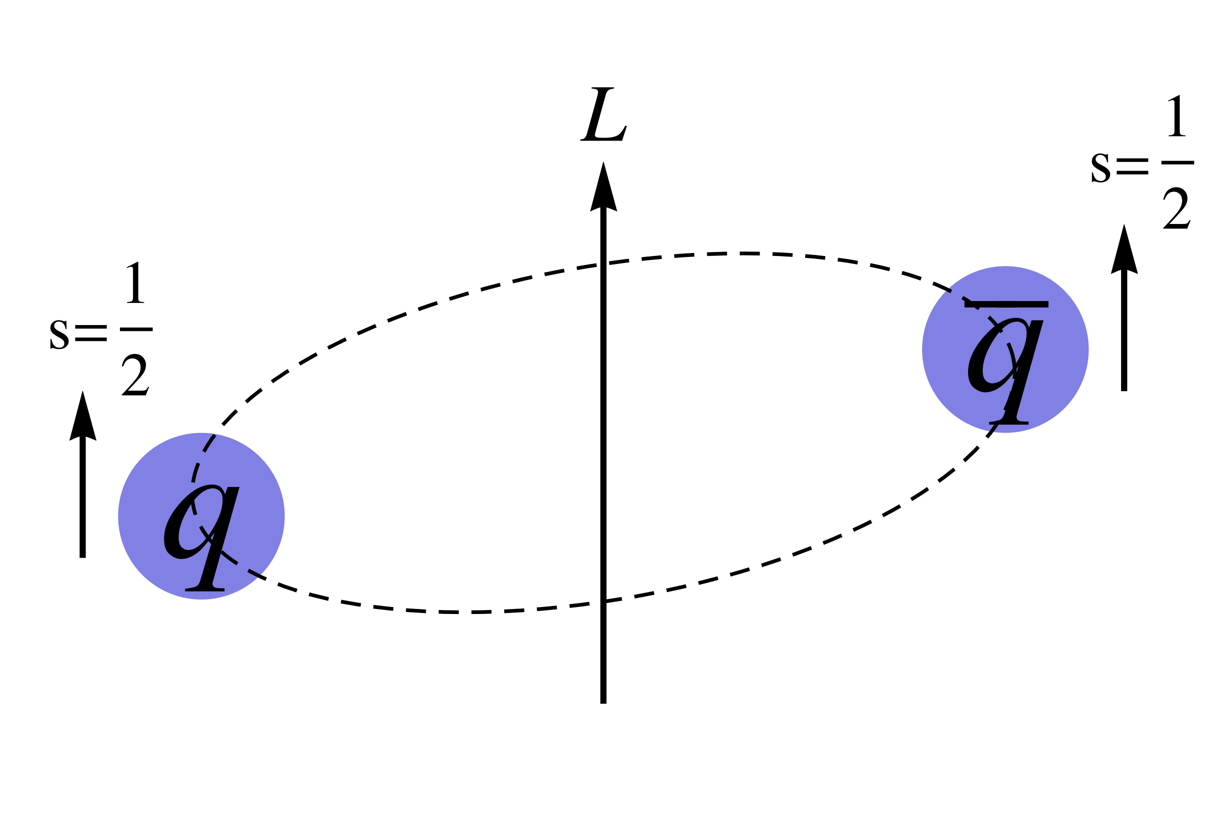

$$L=0 : 0^{-+}, 1^{--}$$
$$L=1 : (0,1,2)^{++}, 1^{+-}$$
$$L=2 : (1,2,3)^{--}, 2^{-+}$$
$$\vdots$$

Exotics : $0^{--}, 1^{-+}, 2^{+-}, \cdots$

- Total spin:
  $|L-S| \leq J \leq |L+S|$
- Parity:
  $P = (-1)^{L+1}$
- Charge conjugation:
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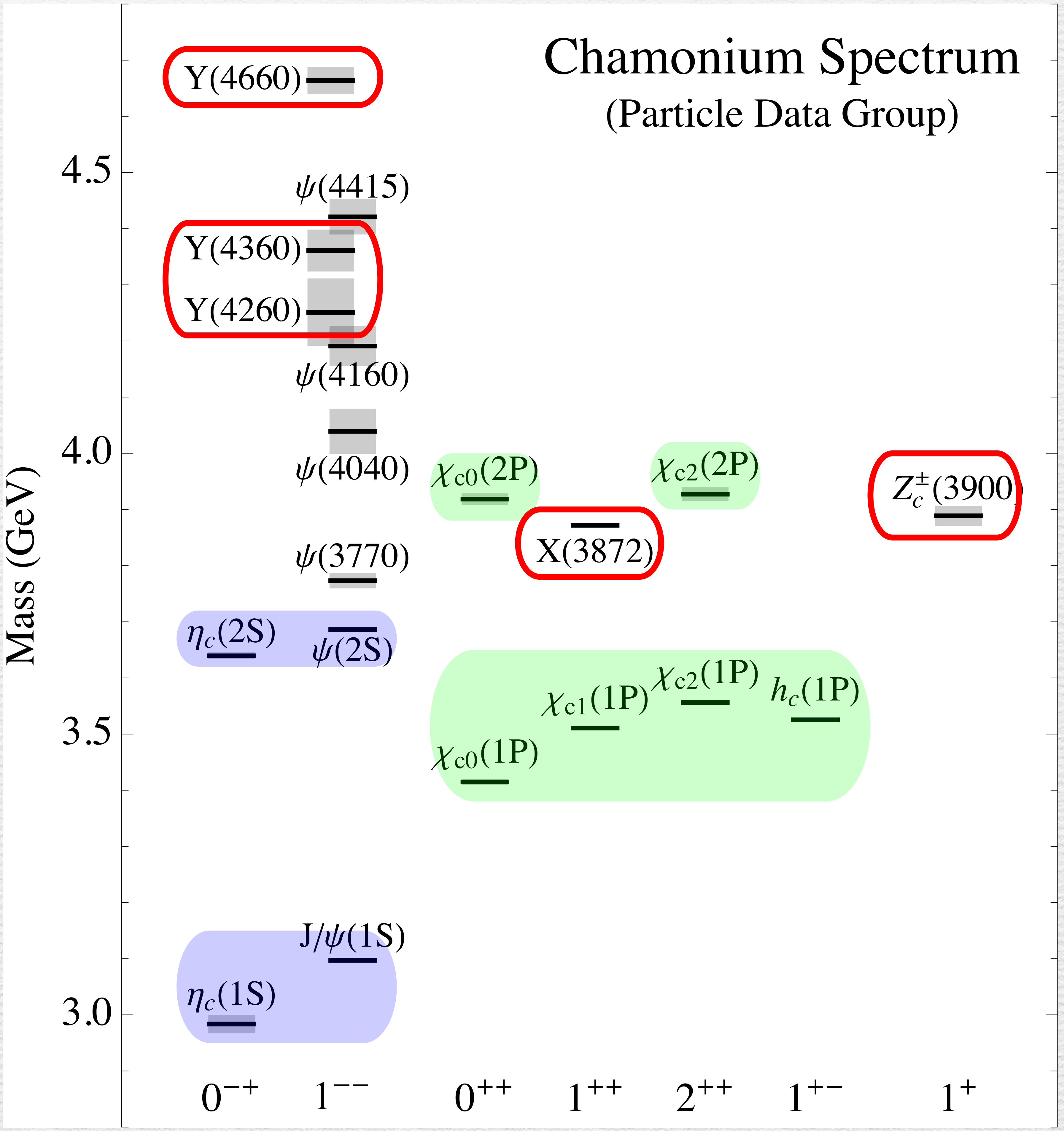
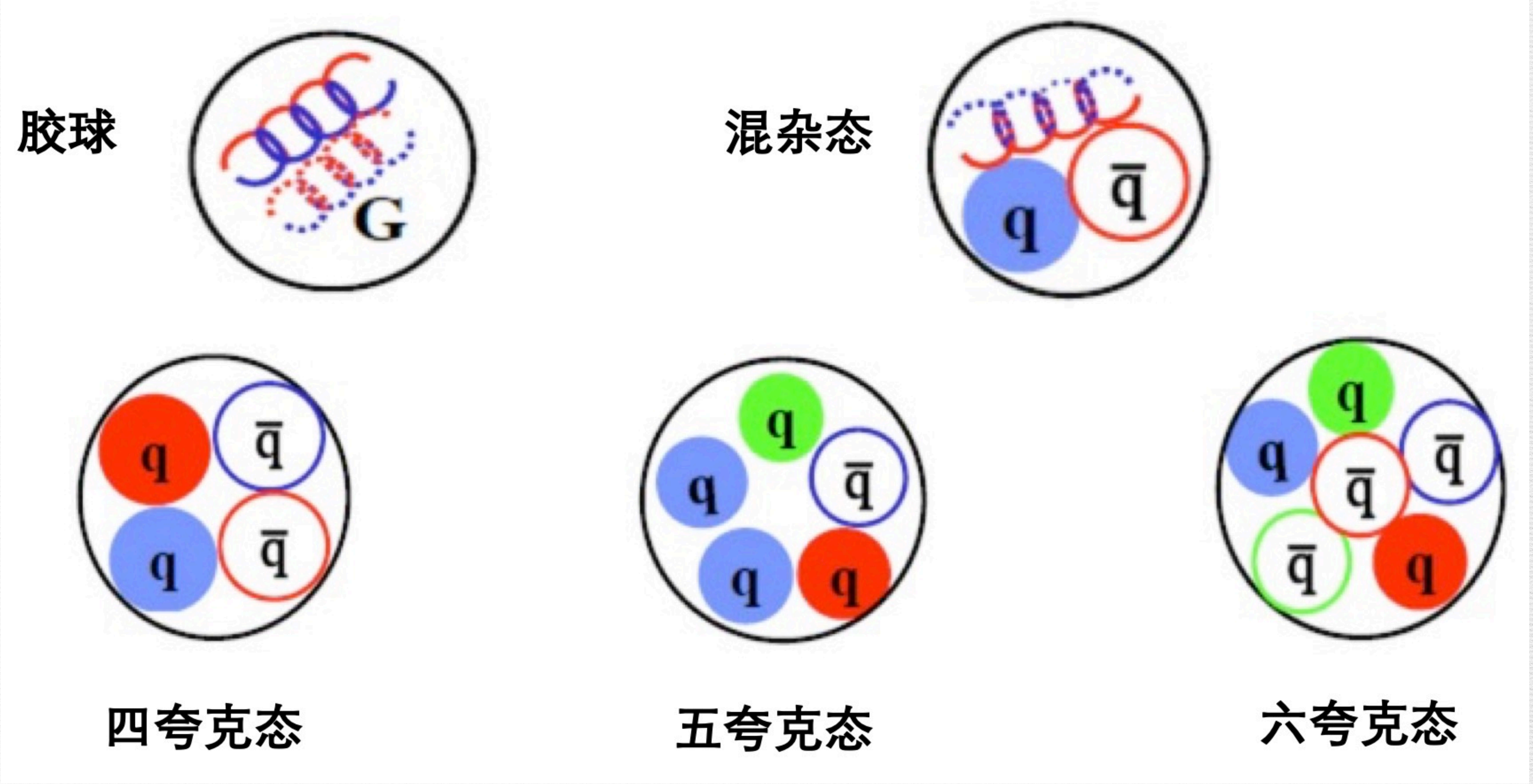


Beyond Quark Model



Spectroscopy on lattice

- Write down an interpolating operator $\mathcal{O}$ with certain quantum number, e.g. pion operator $\bar{u}\gamma_5 d$
- Compute the correlation function

$$< 0|\mathcal{O}(t)\mathcal{O}(0)^{\dagger}|0 > = \sum_n \frac{< 0|\mathcal{O}|n > < n|\mathcal{O}|0 >}{2E_n} e^{-E_n t} \longrightarrow \propto e^{-E_0 t}$$

- At large t, fit the correlation function to an exponential.
- Usually only the ground state can be obtained.

Spectroscopy on lattice

Excited states:

- ✦ build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = < 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 > = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

- ✦ Solve the generalized eigenvalue problem(GEVP):

$$C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$$

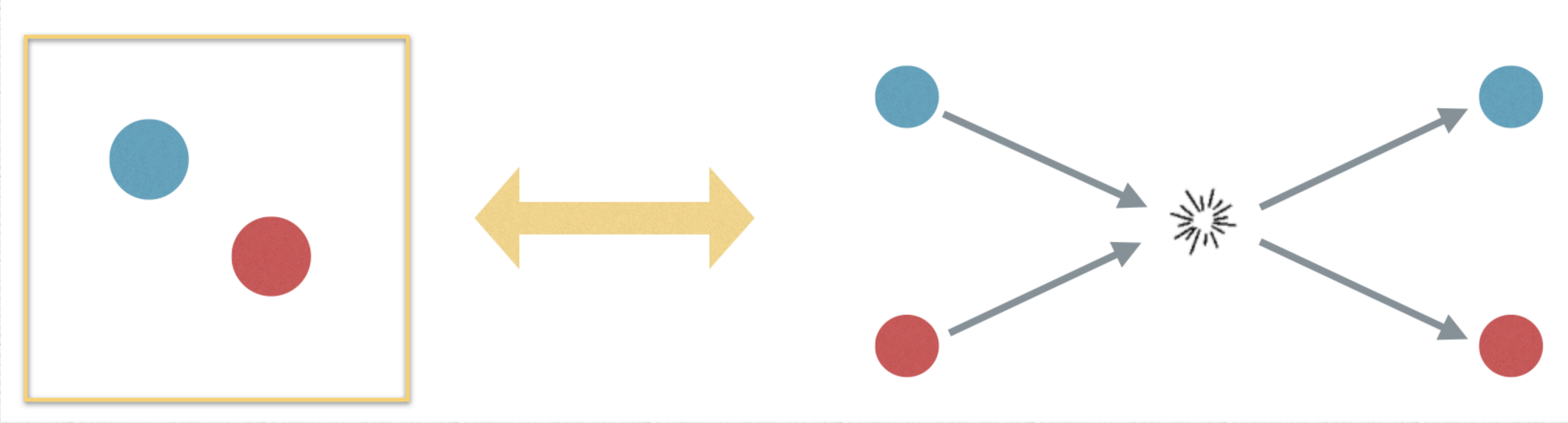
- ✦ Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

- ✦ Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)

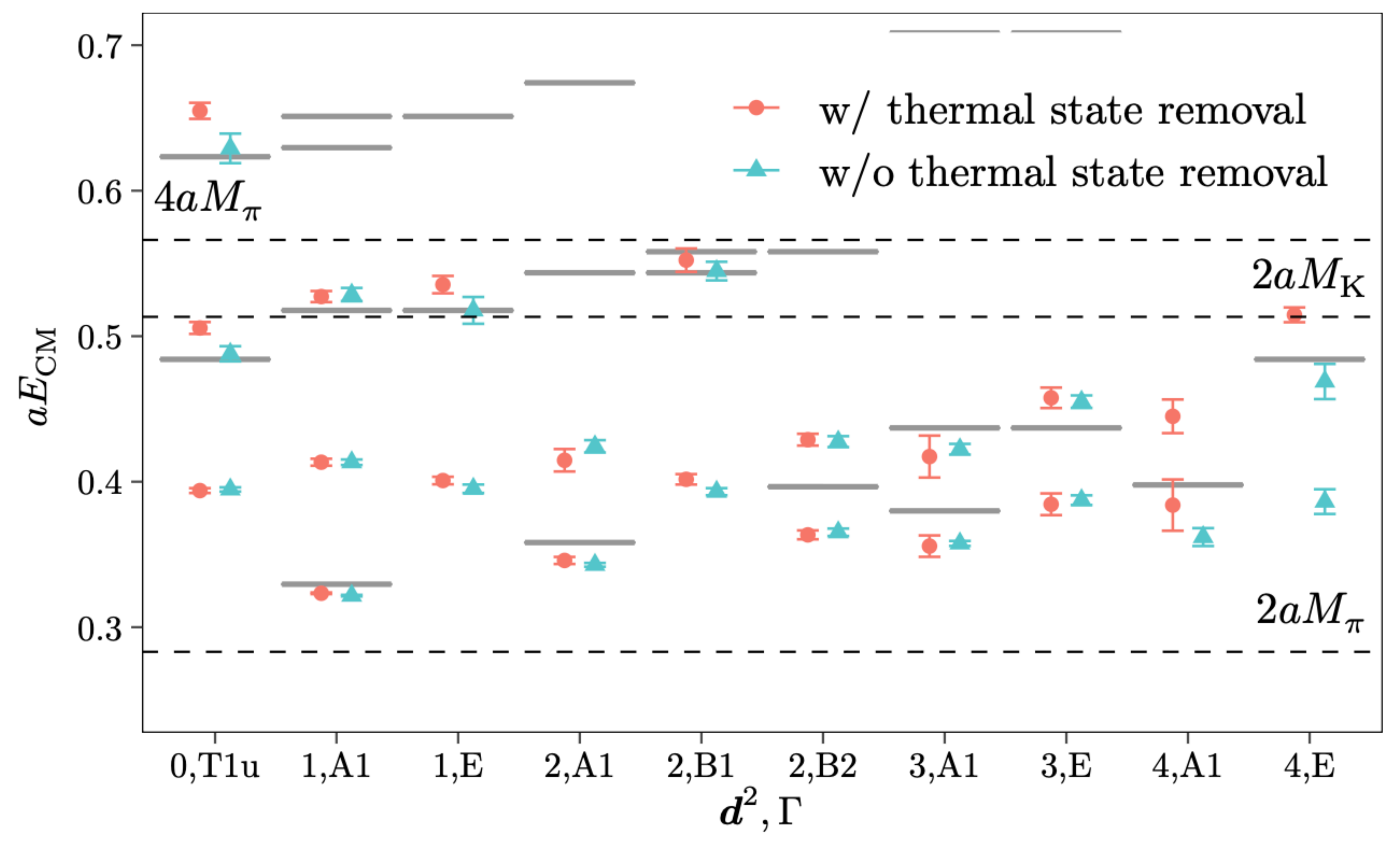


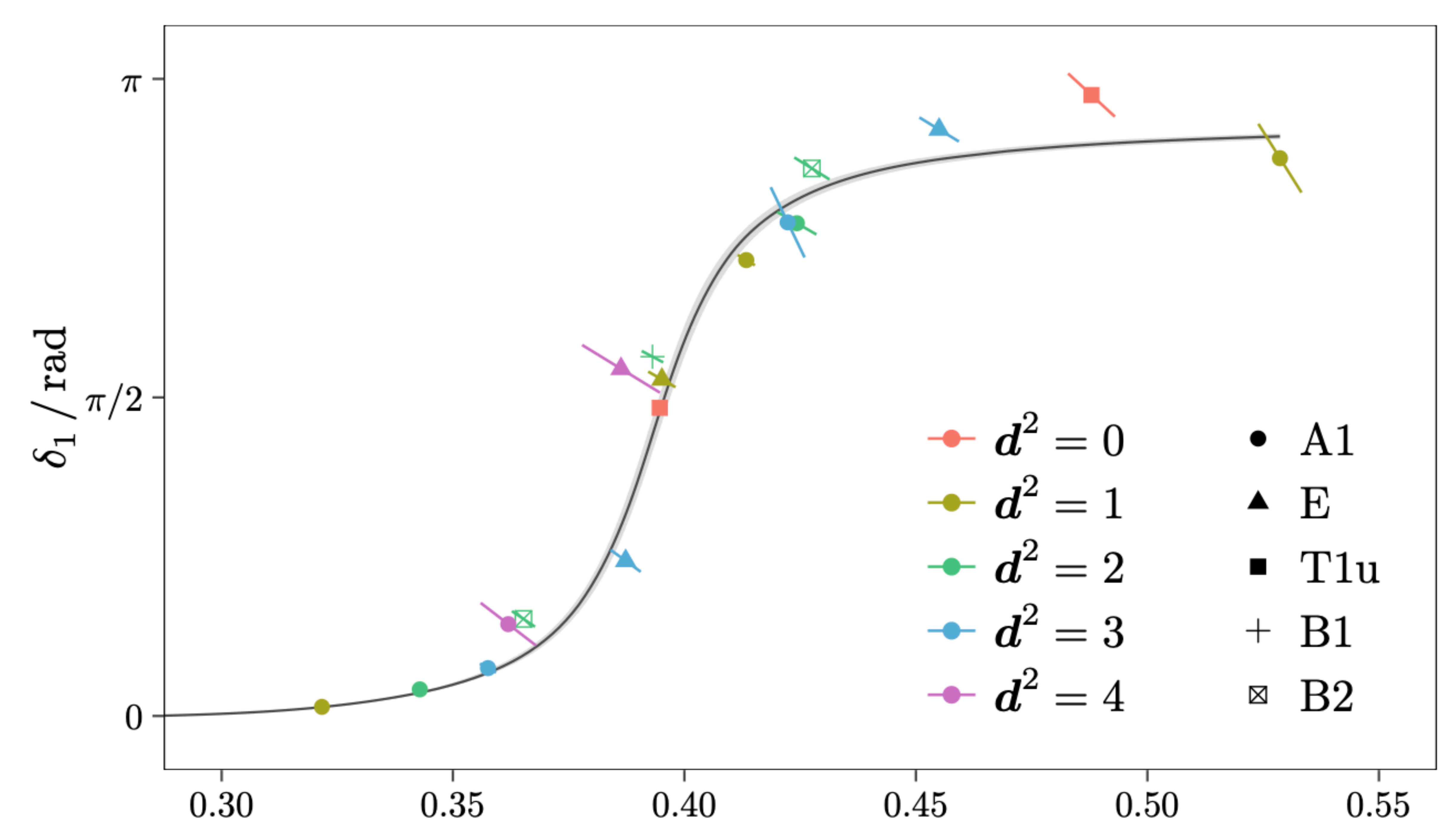
Finite volume spectrum

Infinite volume scattering parameters

Scattering on lattice

An example: ρ resonance $\rightarrow \pi\pi$ scattering





M. Werner et. al., Eur.Phys.J.A 56 (2020) 2, 61

Scattering on lattice

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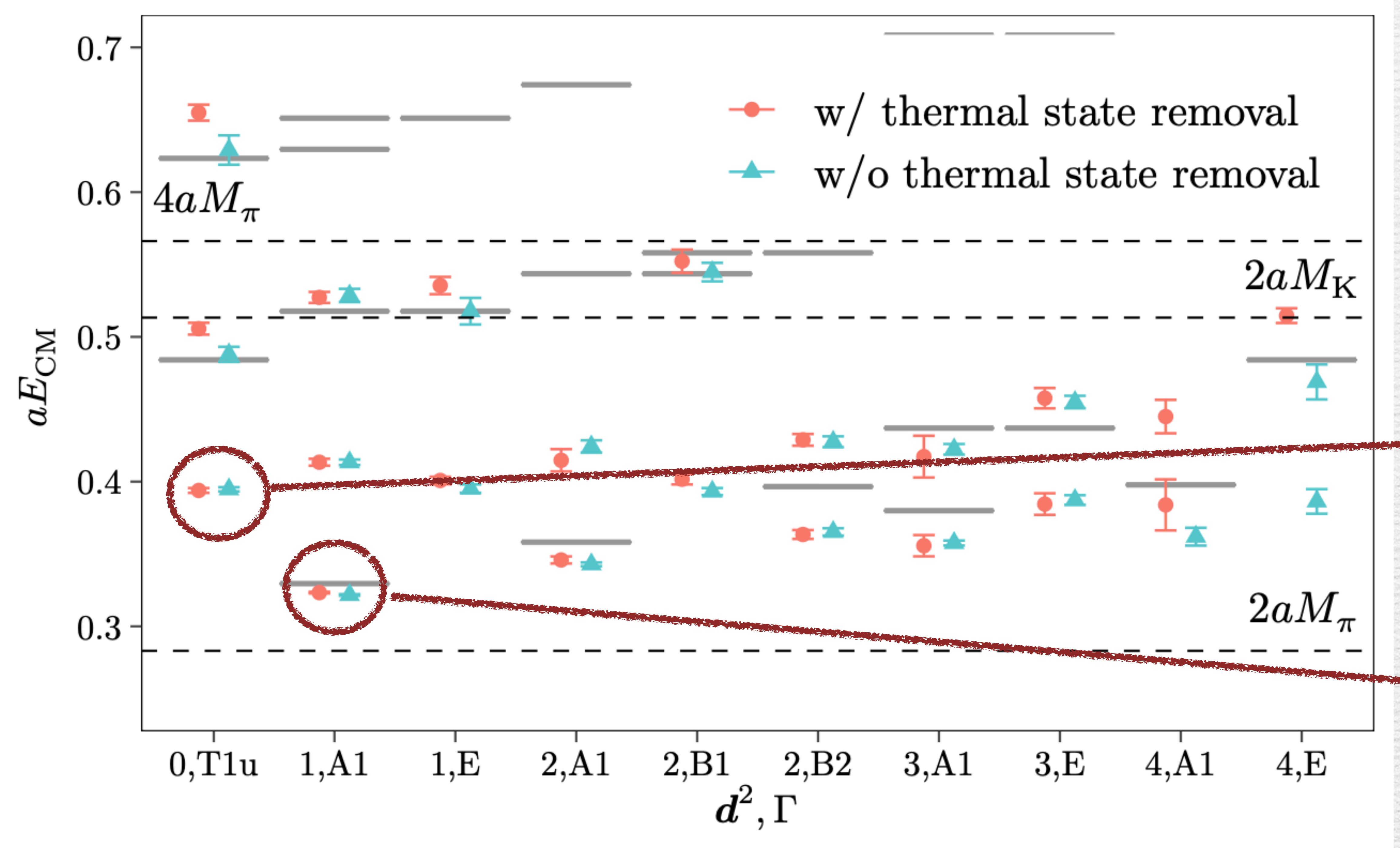
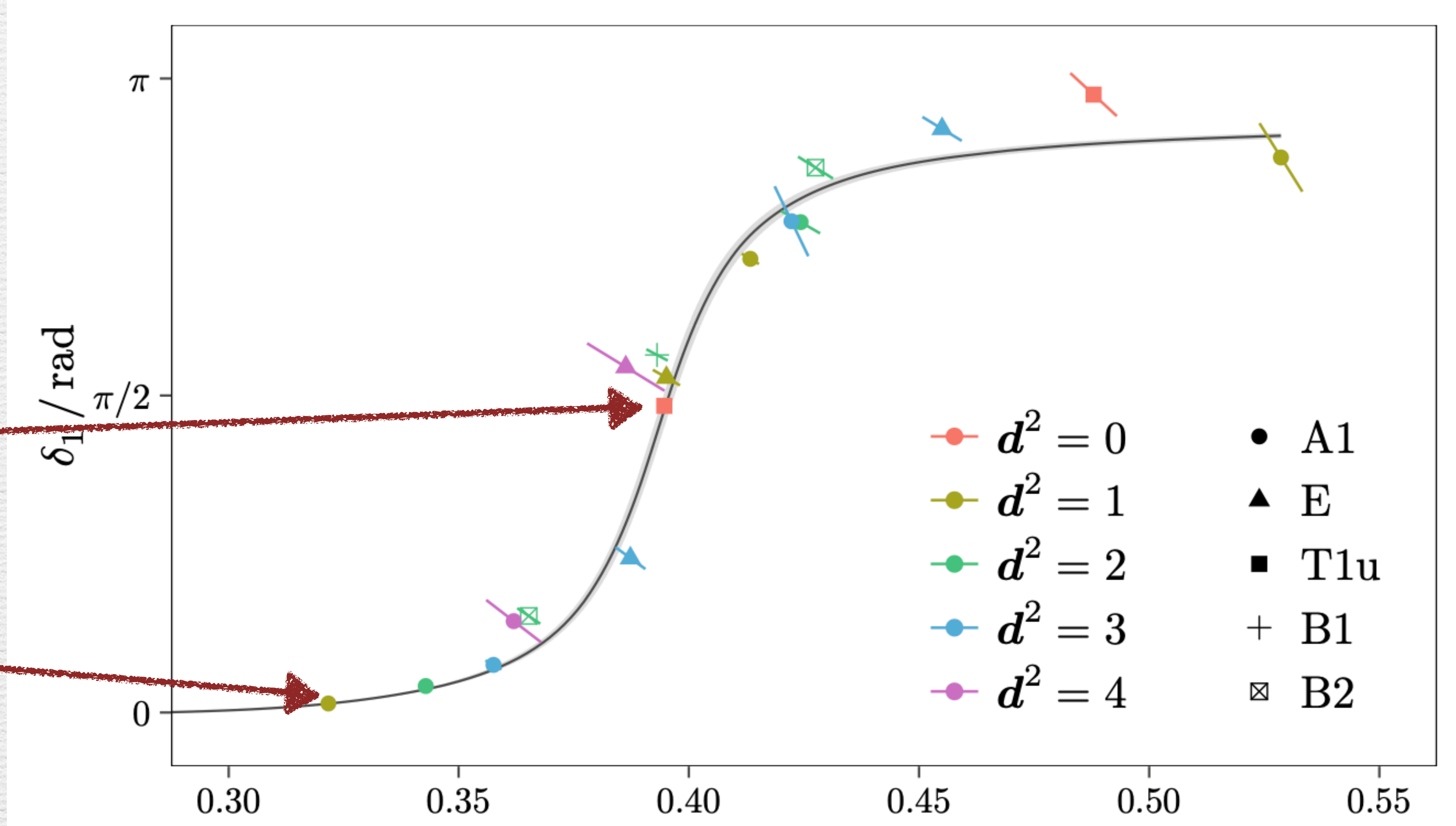




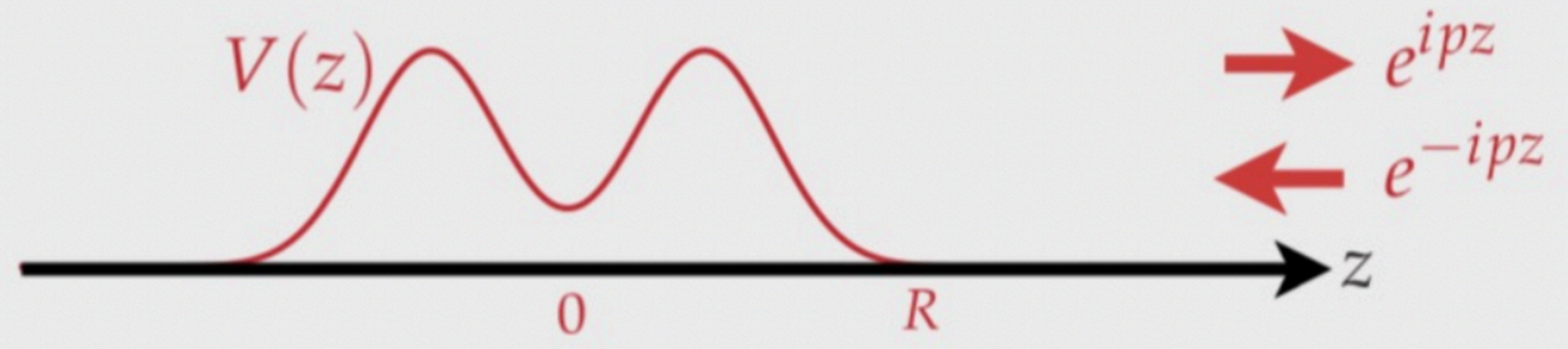
M. Werner et. al., Eur.Phys.J.A 56 (2020) 2, 61

Scattering on lattice

- Two identical bosons interacting through a finite-range potential in 1-d space.

$$\psi(z) \sim \cos\left[\, p|z| + \delta(p) \right]$$ outside the range of the potential, $|z| > R$

$$\psi(z > 0) \sim e^{-ipz} + e^{2i\delta(p)} \cdot e^{ipz}$$



– periodic boundary conditions

$$\left.\begin{aligned} \psi\left(-\tfrac{L}{2}\right) &= \psi\left(\tfrac{L}{2}\right) \\ \frac{d\psi}{dz}\left(-\tfrac{L}{2}\right) &= \frac{d\psi}{dz}\left(\tfrac{L}{2}\right) \end{aligned}\right\} \Longrightarrow 0 = \sin\left[\tfrac{pL}{2} + \delta(p)\right]$$

$$\tfrac{pL}{2} + \delta(p) = n\pi$$

$$p = \frac{2\pi}{L}n - \frac{2}{L}\delta(p)$$

discrete energy spectrum

Scattering on lattice

- ✦ Lüscher's formula was originally derived for two identical scalar particles in rest frame.
- ✦ Generalization of Lüscher's method: moving frames, particles with arbitrary spin, coupled channels, three-body interaction…

- K. Rummukainen and S. A. Gottlieb, Nucl. Phys. B 450, 397 (1995).
- C. h. Kim, C. T. Sachrajda and S. R. Sharpe, Nucl. Phys. B 727, 218 (2005)
- M. Gockeler et al., Phys. Rev. D 86, 094513 (2012)
- R. A. Briceno, Phys. Rev. D 89, 074507 (2014)
- K. Polejaeva and A. Rusetsky, Eur. Phys. J. A48, 67 (2012)
- M. T. Hansen and S. R. Sharpe, Phys. Rev. D 92, 114509 (2015)
- H. W. Hammer, J. Y. Pang and A. Rusetsky, JHEP 10, 115 (2017)
- M. Mai and M. D¨oring, Phys. Rev. Lett. 122, 062503 (2019)
- ……

Scattering on lattice

- General Lüscher's formula for two-body scattering:

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

Diagonal matrix of phase-space factors $\rho_{ij} = \delta_{ij}\frac{2k_i}{E_{cm}}$

Infinite-volume scattering matrix

Finite volume information $M(E_{cm}, L)$

- Resonances/bound states are formally defined as poles in scattering amplitudes.

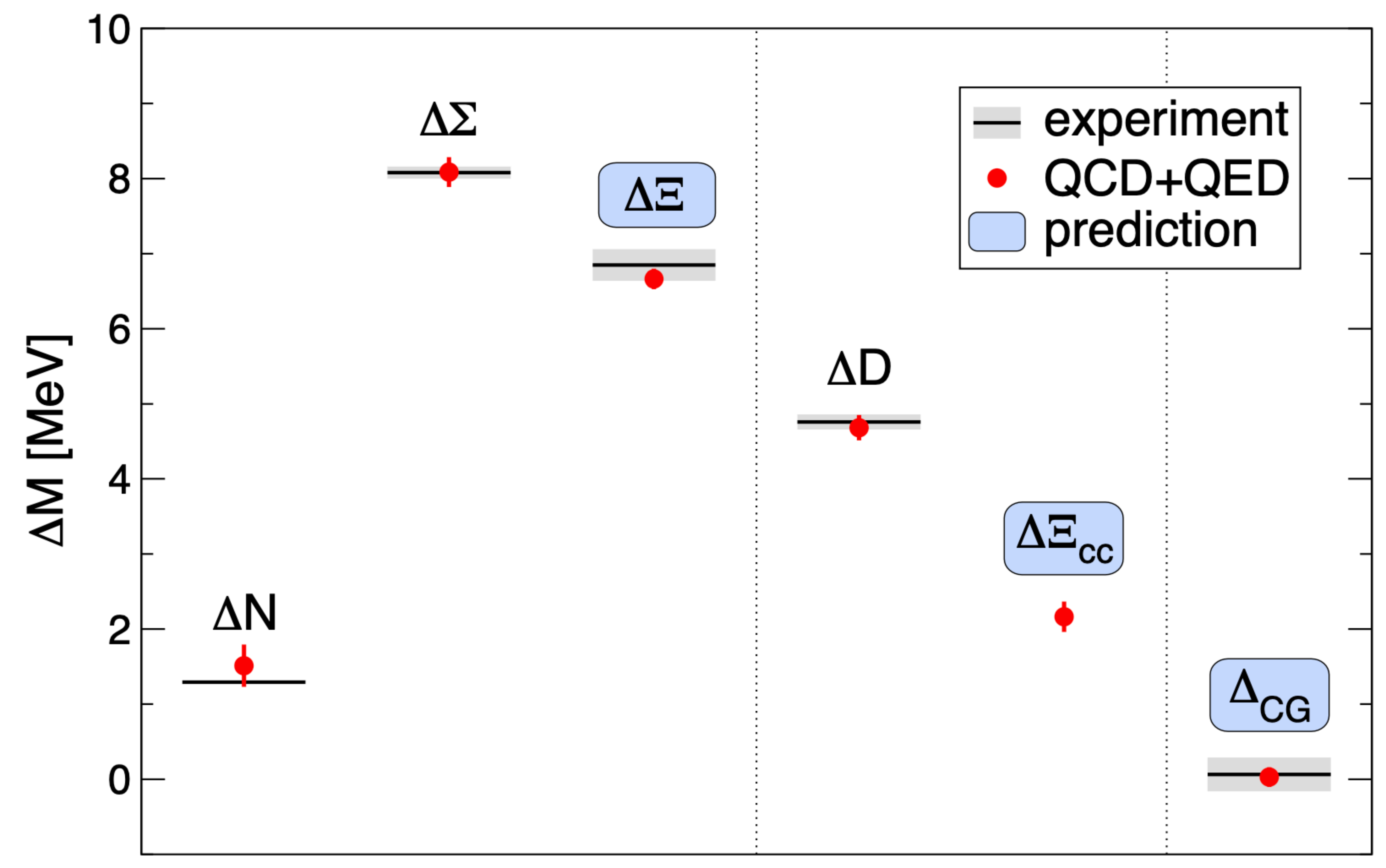
Systematics in lattice QCD

- Light quark mass (pion mass) heavier than physical value. Up and down quark are usually degenerate.
- Finite lattice spacing.
- Finite volume.

Roadmap

- Ground-state spectrum
  - High precision, physical point, isospin-breaking effects, QED corrections…
- Exited and exotic states
  - Large basis of operators, highly exited states, exotic states…
- Two-particle scattering
  - Single channel
  - Multi coupled channels
  - Exotic candidates near threshold
- Three-particle scattering
  - Formalism are developing
  - Simple 3-body systems are studied: $\pi\pi\pi$, KKK…
- ……

Ground state spectrum

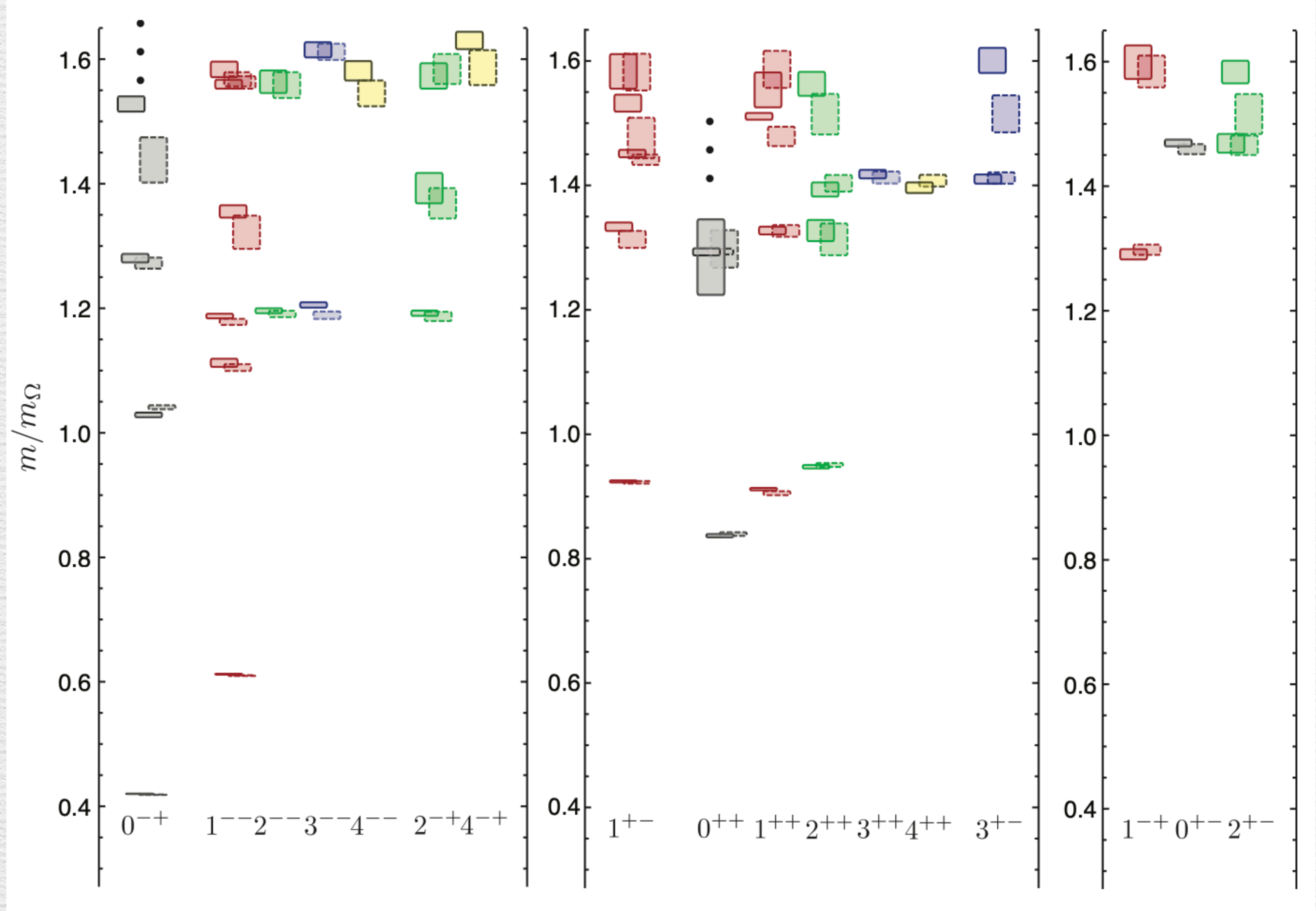


- Proton and neutron mass difference is a key quantity that explains the physical world as we know it today.
- Per mille precision level.
- Isospin breaking effects, QED corrections.
- Large number of ensembles at different values of light quark mass, lattice spacing and volume to fully control systematics.

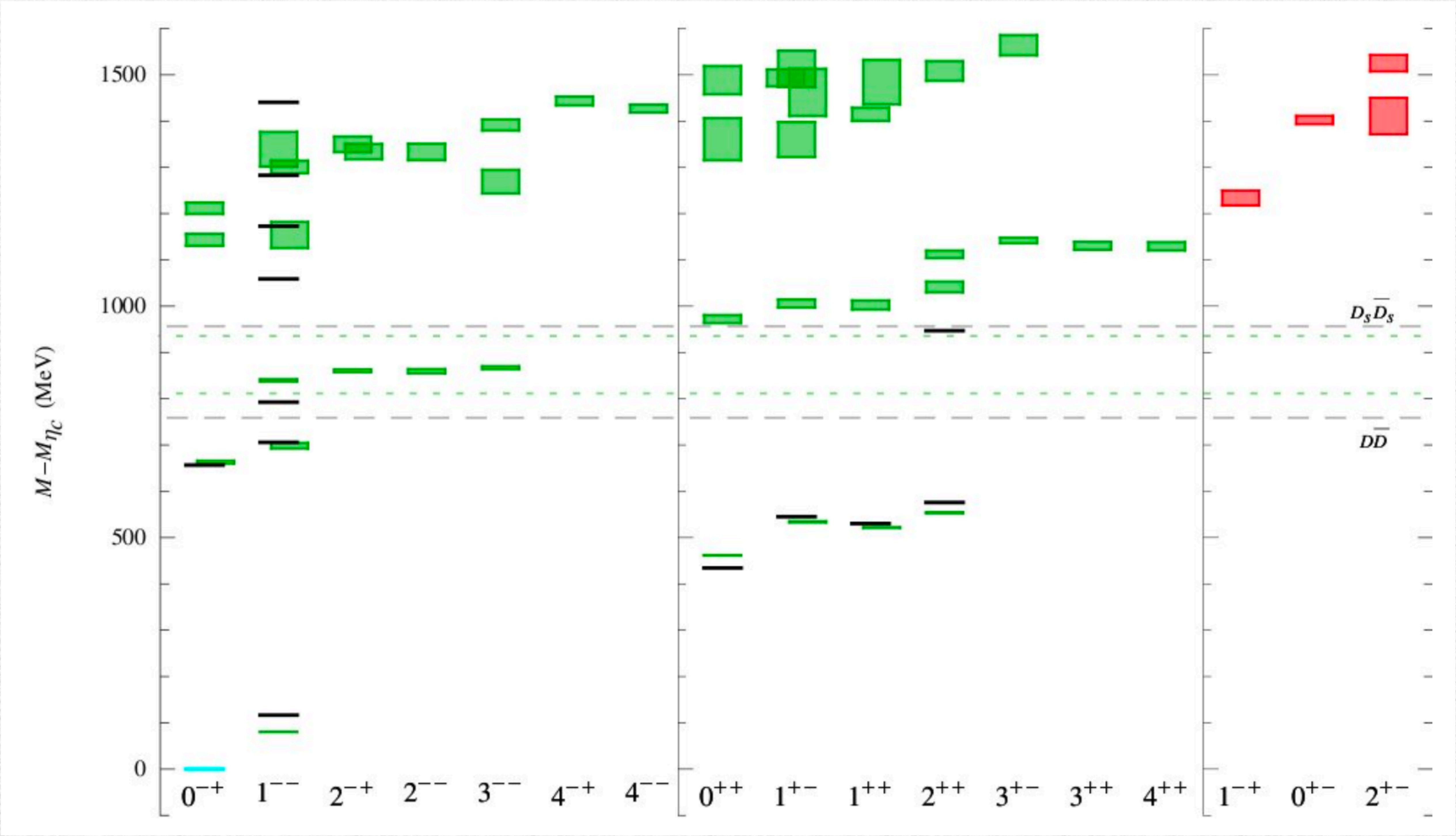
S. Durr et. al., Science 322:1224-1227,2008

Excited and exotic states

Light meson spectrum



Charmonium spectrum

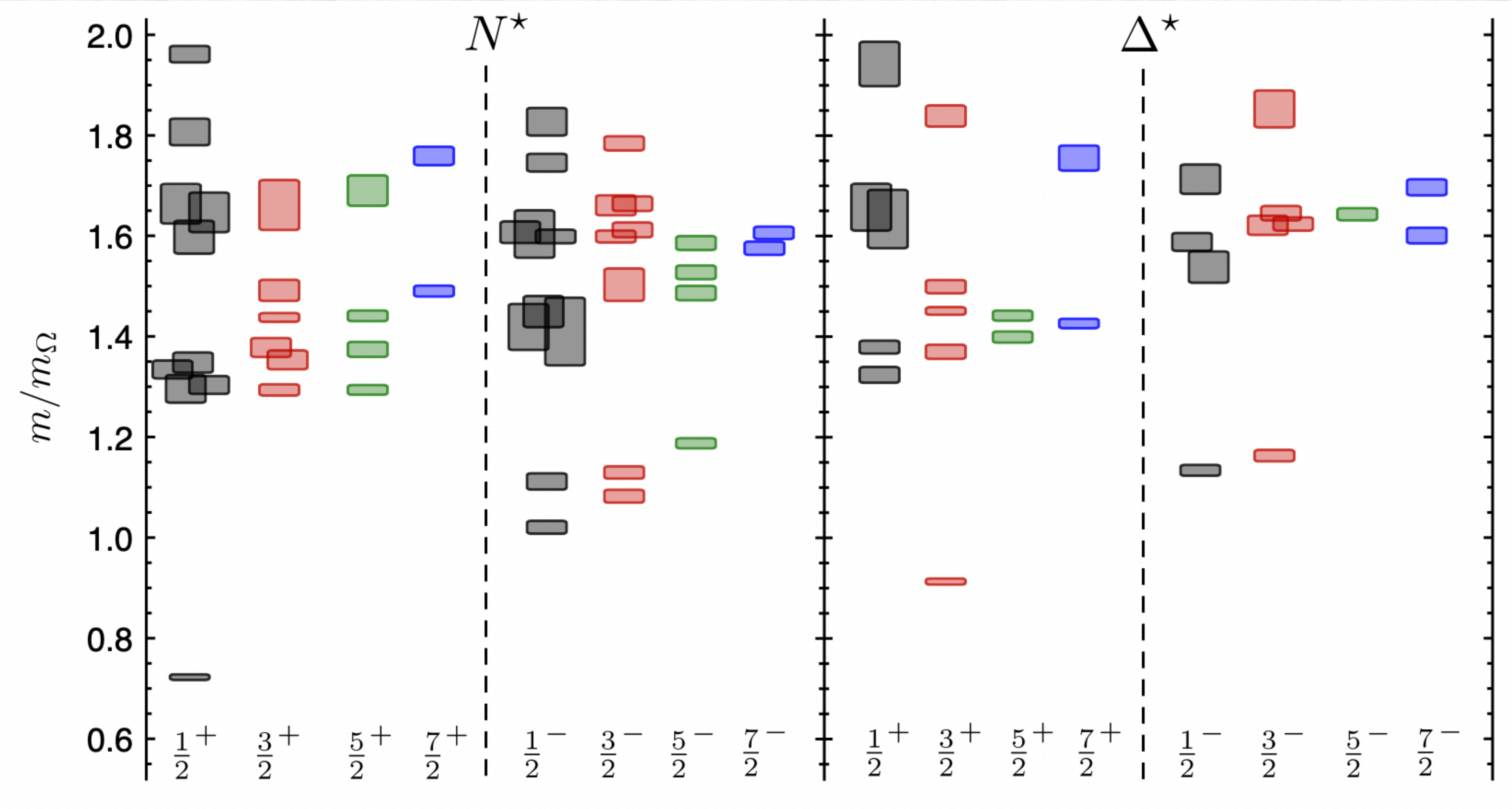


J. Dudek et. al., Phys.Rev.D 82 (2010) 034508

L. Liu et al. JHEP 1207 (2012) 126

Excited and exotic states

Baryon spectrum



R. Edwards et. al., Phys.Rev.D 84 (2011) 074508

Operator construction

- Simplest meson interpolating operators: local quark bilinears

$$\mathcal{O}_M(x) \sim \bar{q}^i_\alpha(x)\Gamma_{\alpha\beta}q^i_\beta(x) \quad , \quad J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{-+}$$

- Non-local operators:

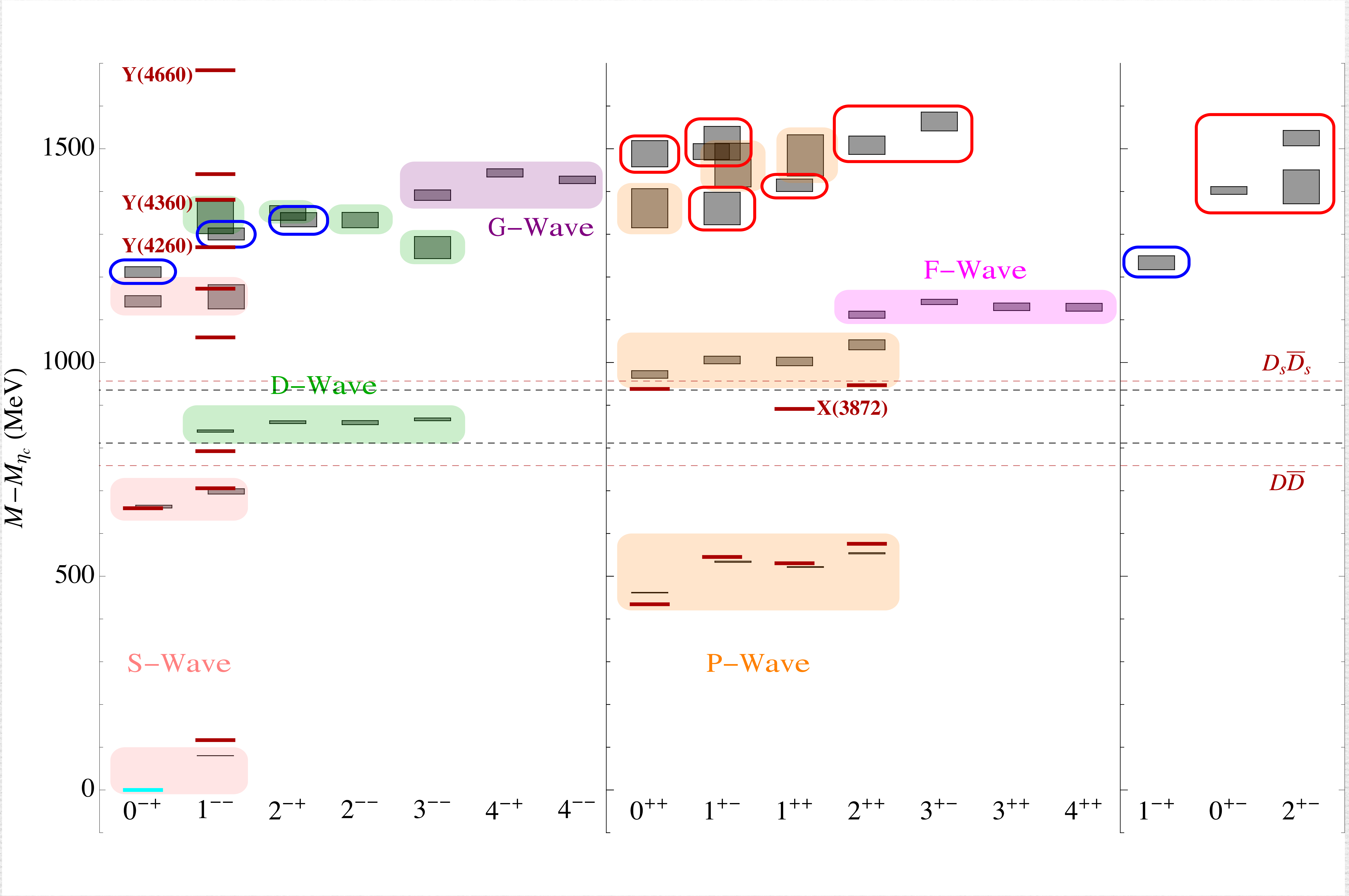
$$\mathcal{O}_M(x) \sim \bar{q}(x)\Gamma \overrightarrow{D}_i \overrightarrow{D}_j \cdots q(x) \quad \rightarrow \quad \text{Can have any quantum numbers.}$$

- Two-meson operators:

$$\mathcal{O}^1_M(p_1)\mathcal{O}^2_M(p_2) \quad , \quad (q^i\Gamma_1 q^i)(\bar{q}^j\Gamma_2\bar{q}^j)$$

Continuum space $SO(3)$ group (J, M)	⟷	Discretized space Cubic group (Γ, r)

Excited and exotic states



Excited and exotic states

$\Rightarrow$ $\bar{\tilde{\psi}}\gamma_i \tilde{\psi}$ $\longrightarrow$ $^3S_1$

$\wedge$ $\frac{1}{2}[1-\gamma_0]$

upper component projector
"non-relativistic"

two-derivative constructions : $D^{[2]}_{J,m} = \langle 1m_1; 1m_2 | Jm \rangle \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2}$

$\Rightarrow$ $\langle 1m_1; 2m_2 | 1m \rangle \, \bar{\tilde{\psi}}\gamma_{m_1} D^{[2]}_{J=2,m_2} \tilde{\psi}$ $\xrightarrow[\text{ignoring the gauge-field}]{}$ $Y_2^m(\overleftrightarrow{\partial})$ $\longrightarrow$ $^3D_1$

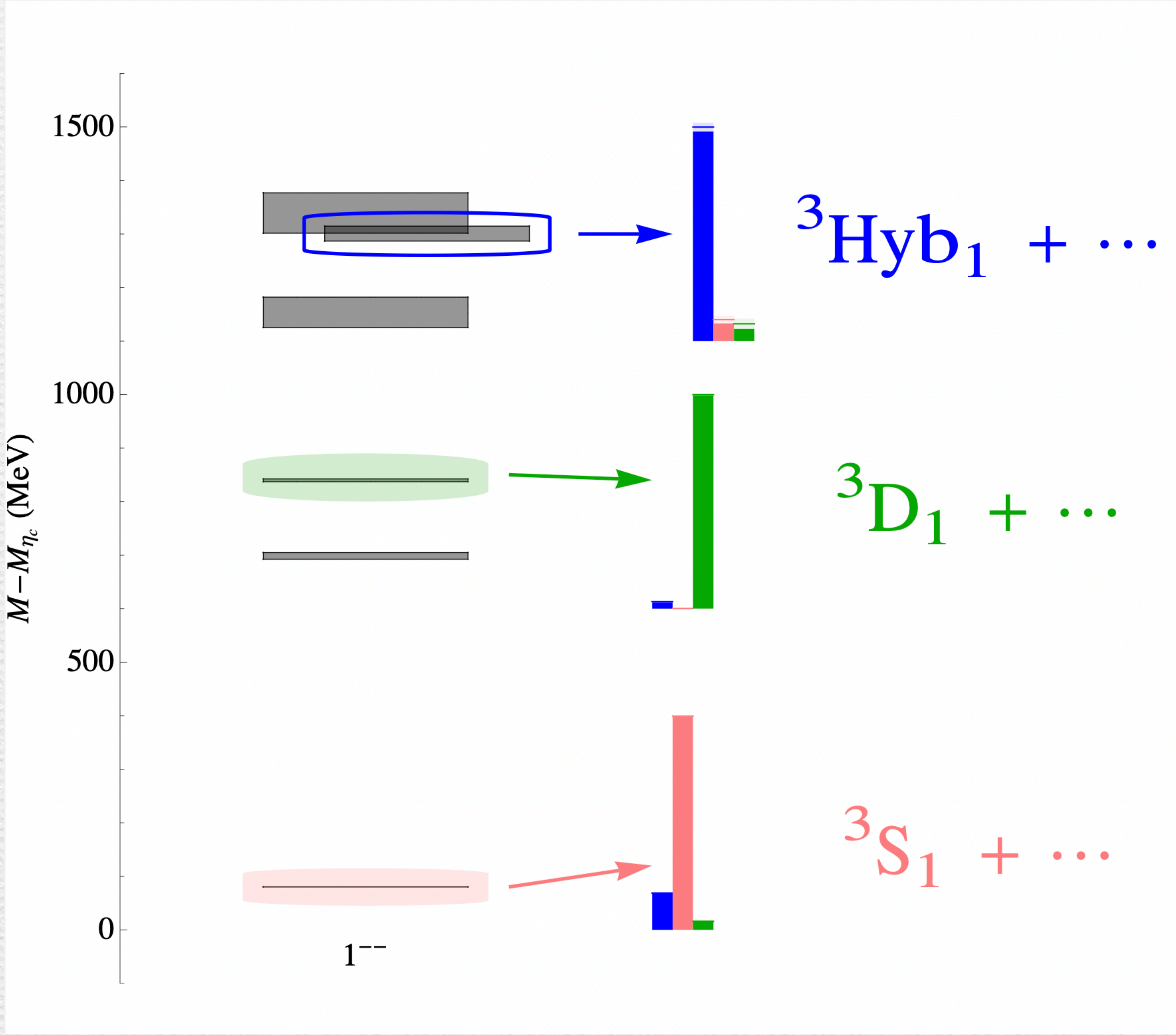
$\wedge$ $\frac{1}{2}[1-\gamma_0]$

gauge-invariant
version of a D-wave ?

$\Rightarrow$ $\bar{\tilde{\psi}}\gamma_5 D^{[2]}_{J=1,m} \tilde{\psi}$ $\xrightarrow[\text{ignoring the gauge-field}]{}$ $^1\mathrm{hyb}_1$

$\wedge$ $\frac{1}{2}[1-\gamma_0]$

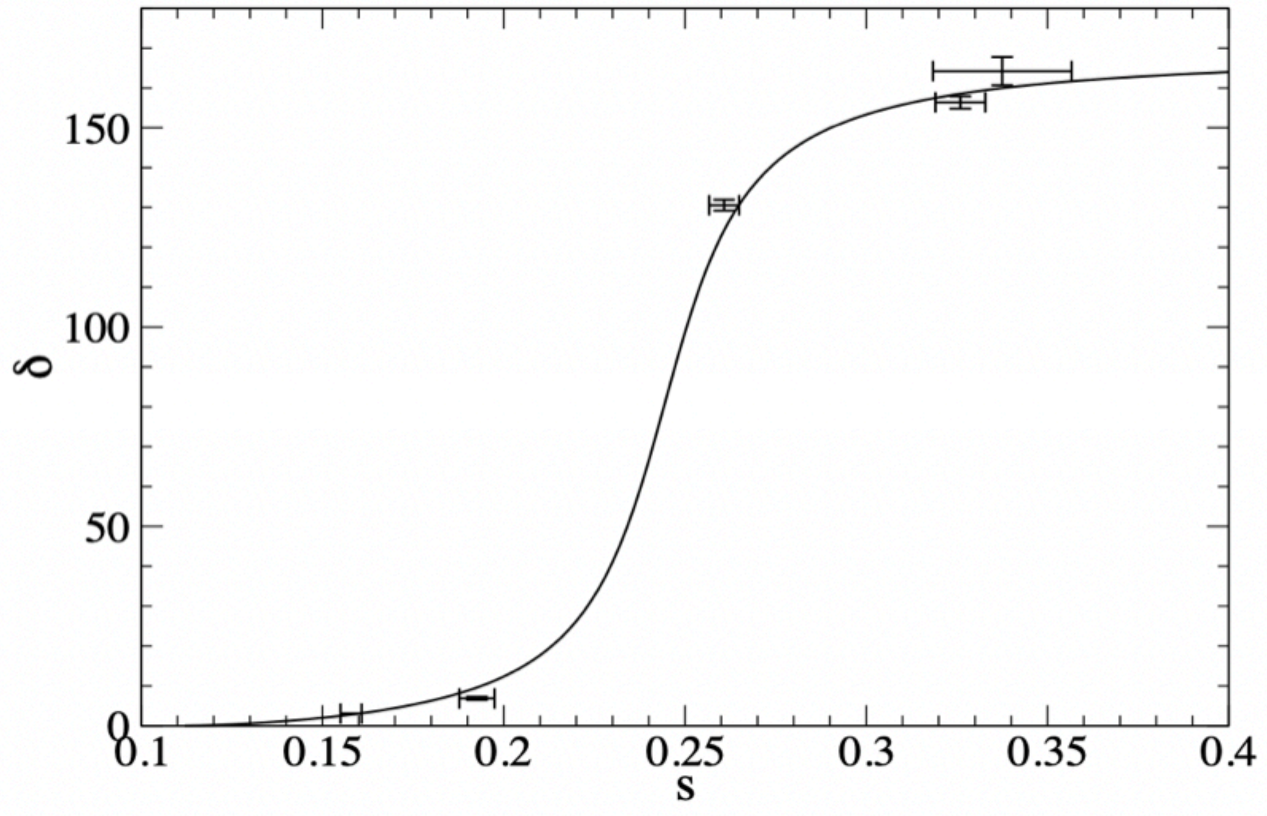
$[D, D] \to [\partial, \partial] = 0$



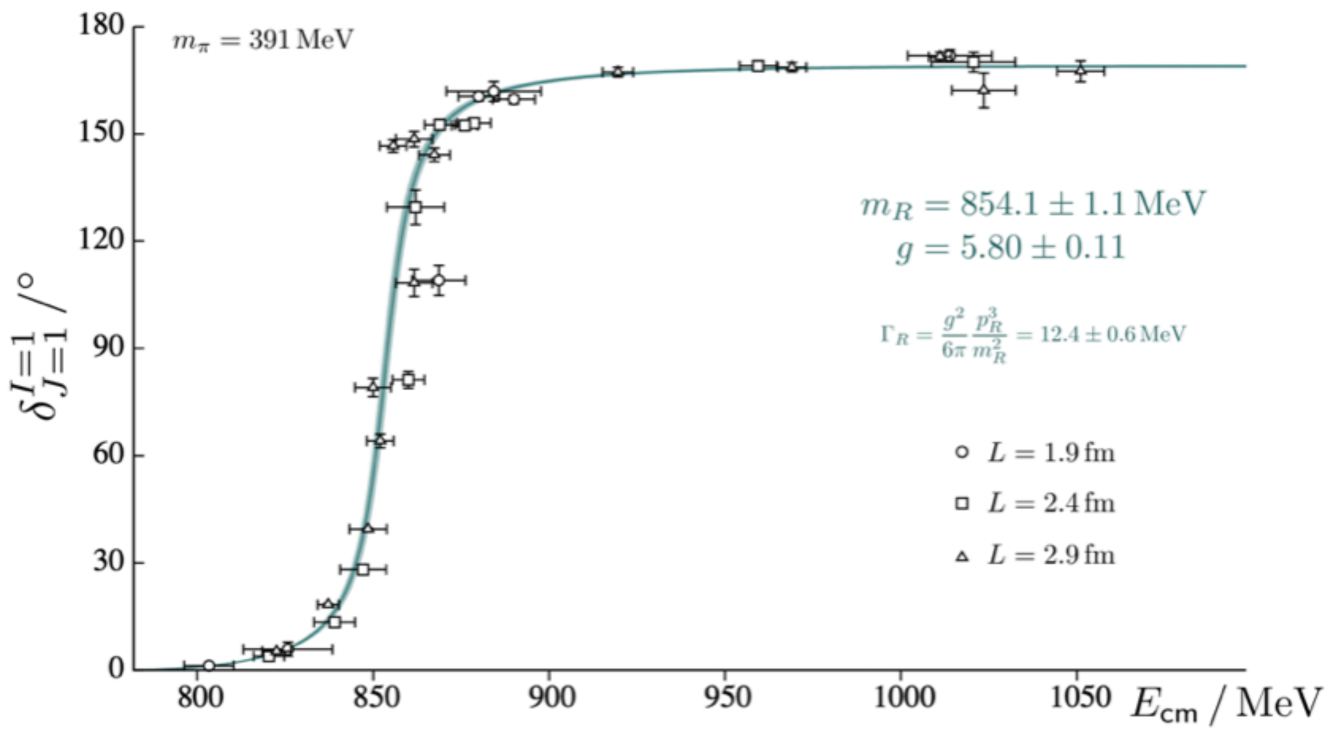
Two-particle scattering

$\pi\pi$ scattering and ρ resonance

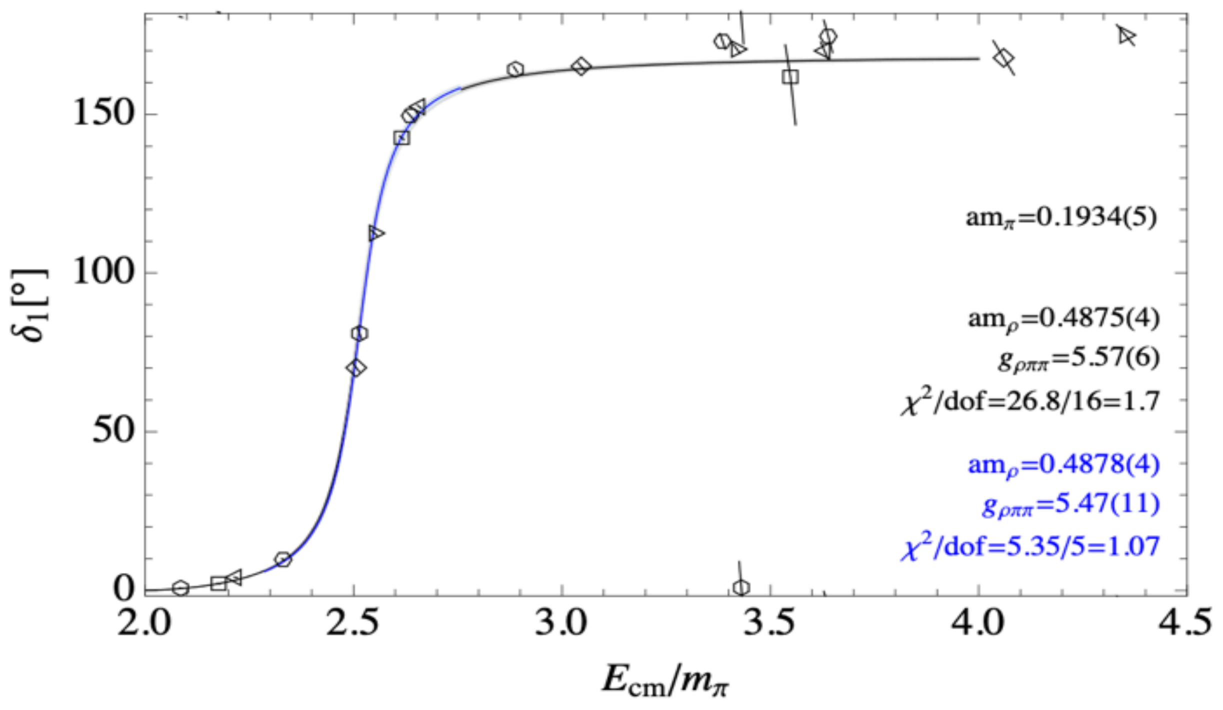
C. B. Lang et, al., Phys. Rev. D 84, 054503 (2011)



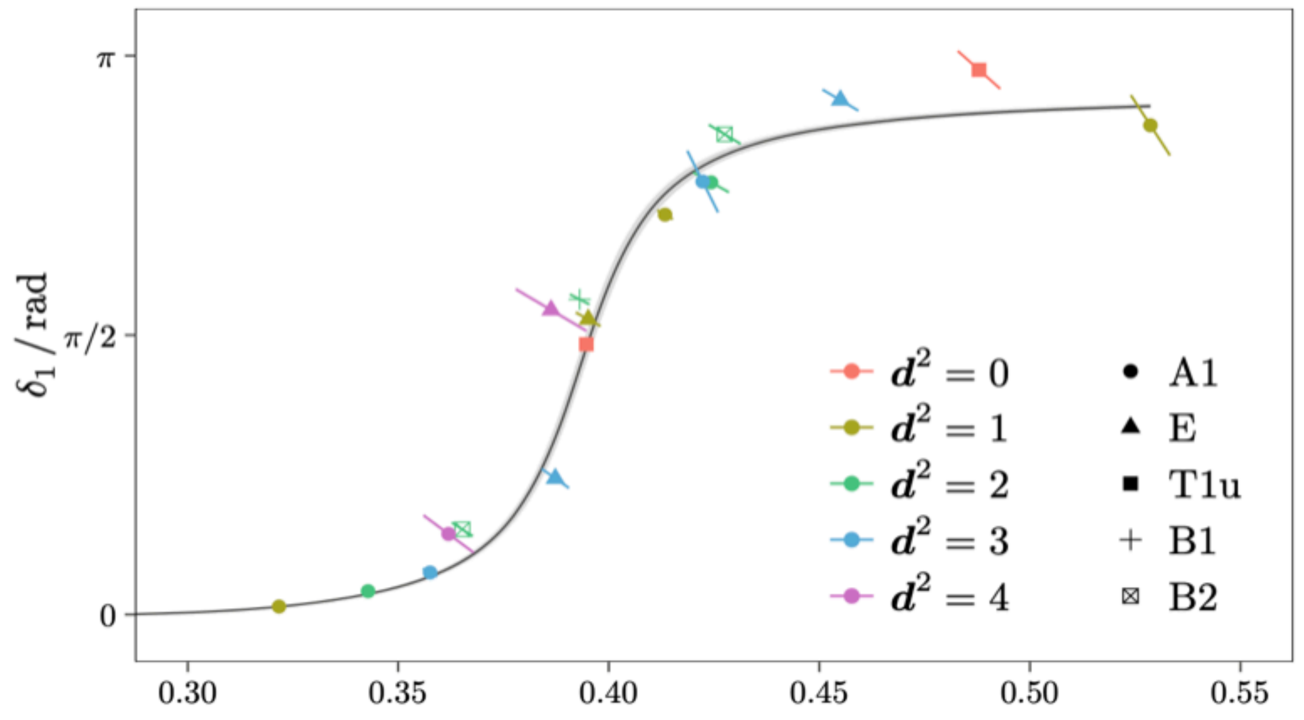
J. Dudek, el. al.,Phys. Rev. D 87, 034505 (2013)



D. Guo, el. al., Phys. Rev. D 94, 034501 (2016)

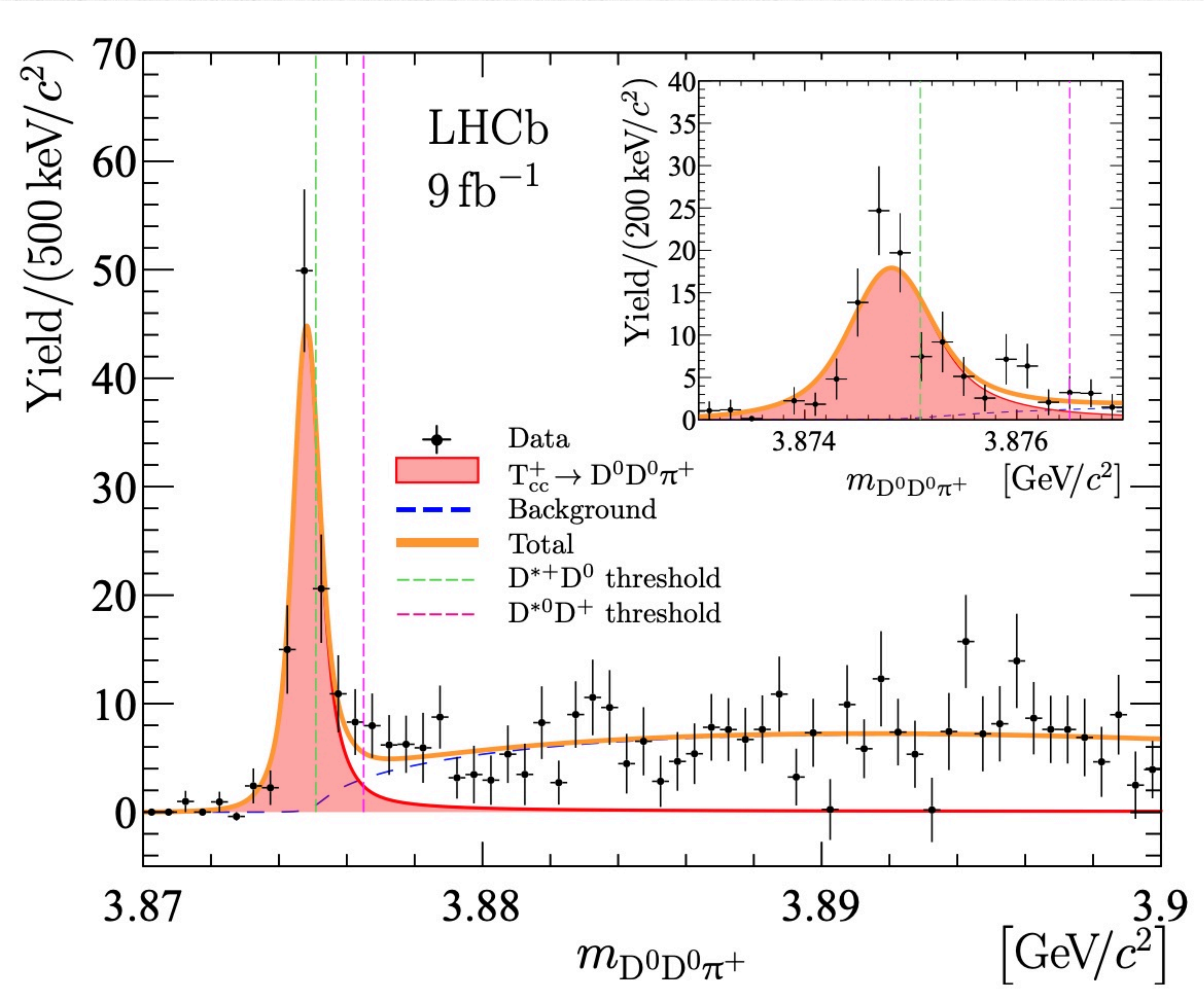


M. Werner, el.al., Eur. Phys. J. A (2020) 56:61



Two-particle scattering

DD^* scattering and T_{cc}



$$\delta m = M_{T_{cc}^+} - (M_{D^{*+}} + M_{D^0})$$

$$= -361 \pm 40(\text{keV})$$

$$\Gamma = 47.8 \pm 1.9(\text{keV})$$

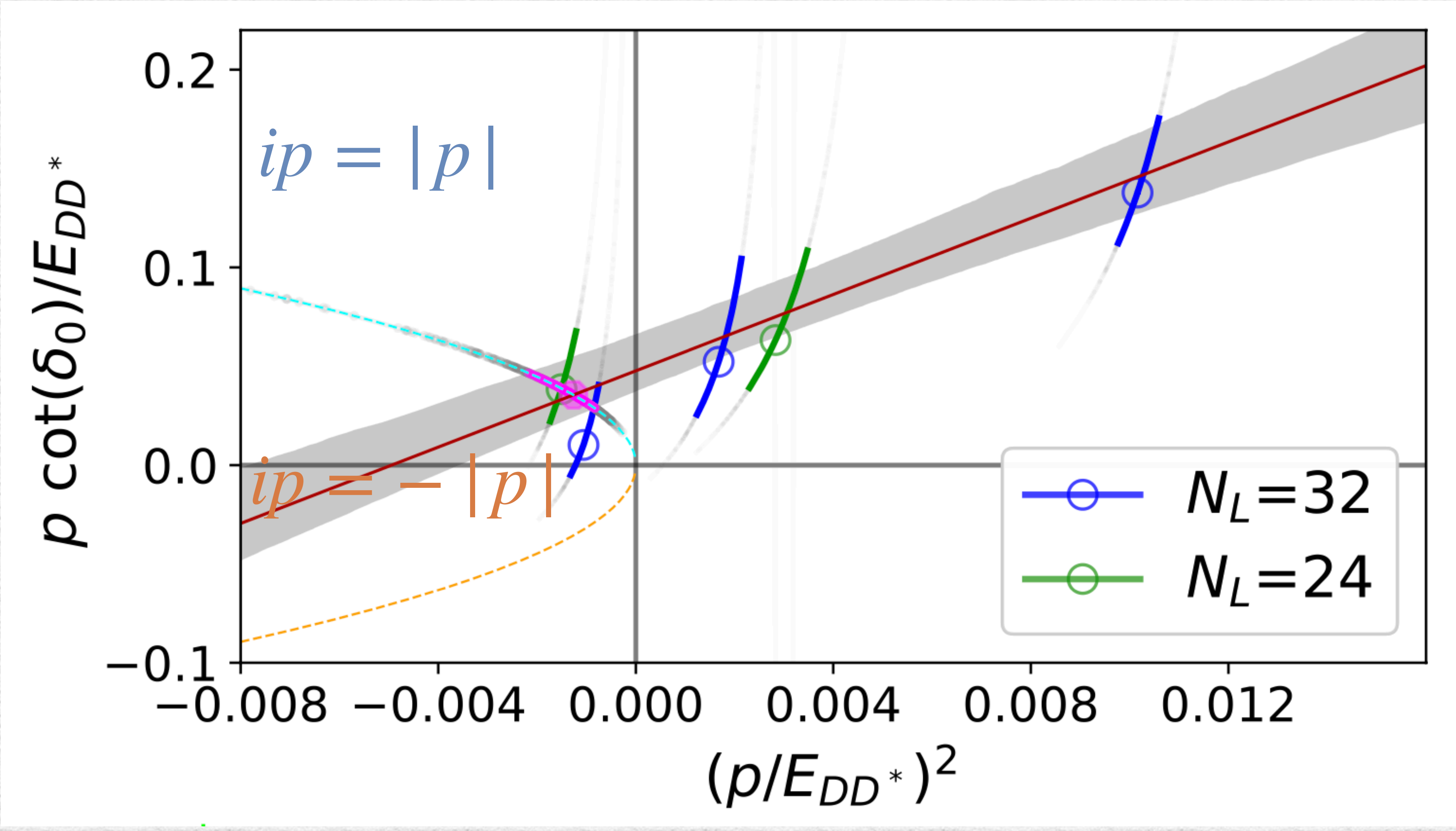
LHCb collaboration, R. Aaij et al., *Nature Phys*. 18 (2022) 7, 751-754.

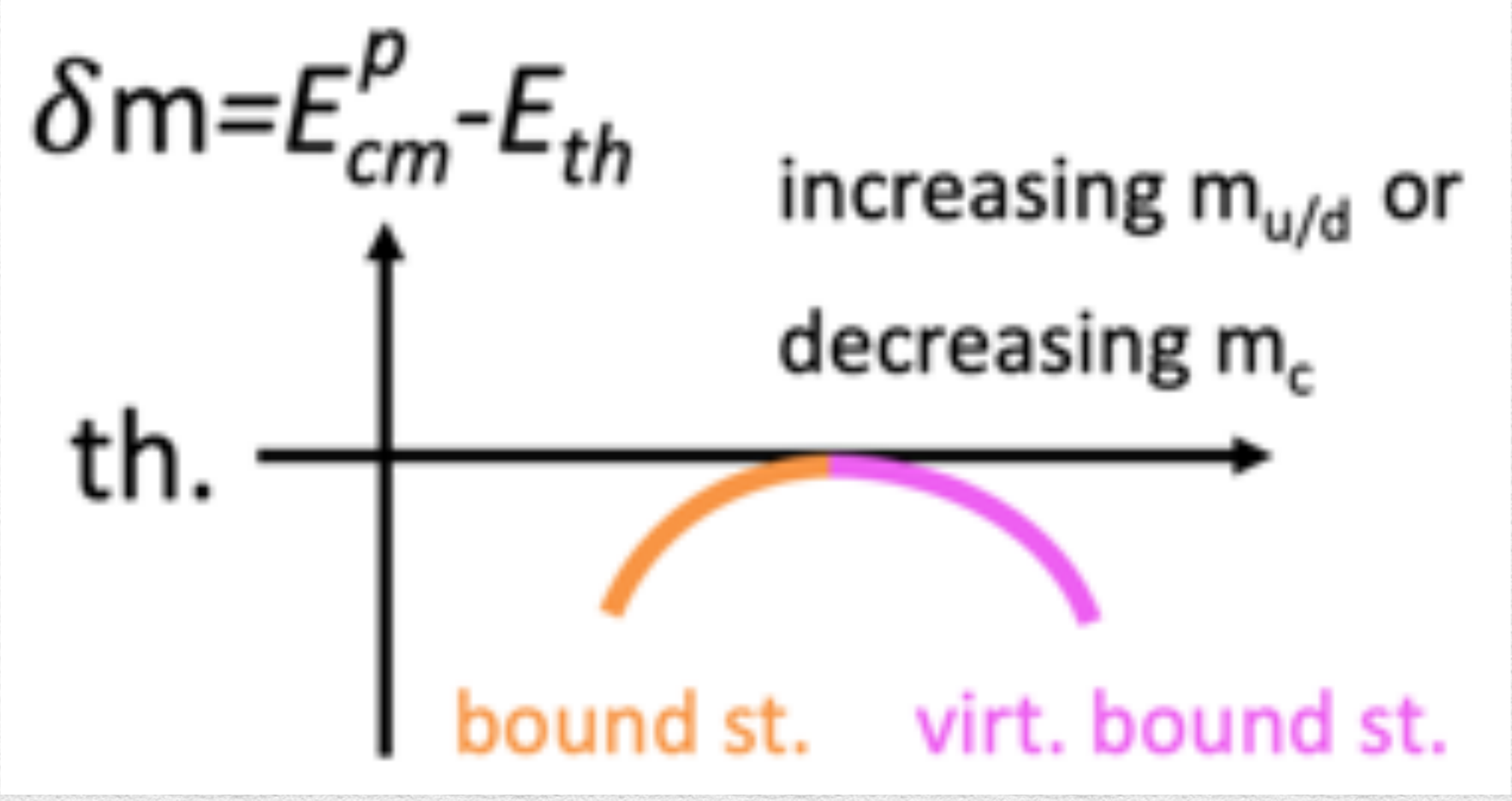
LHCb collaboration, R. Aaij et al., Nature Communicatitions, 13, 3351 (2022)

Two-particle scattering

DD^* scattering and T_{cc}

M. Padmanath and S. Prelovsek, Phys.Rev.Lett. 129 (2022) 3, 032002





Scattering amplitude: $T \sim \dfrac{1}{p\cot\delta - ip}$

Effective range expansion:

$$p\cot\delta = \frac{1}{a} + \frac{1}{2} r_0 p^2 + \mathcal{O}(p^4)$$

- Finite-volume energy eigenvalues are extracted using DD^* interpolating operatros in rest and moving frames.
- A virtual bound state pole is found in the scattering amplitude.
- Quark mass dependence of the binding energy and pole position.

Two-particle scattering

NN scattering and deuteron

NN scattering has a long history of controversy …

- NPLQCD, S. R. Beane *et al*., Phys. Rev. C88, 024003 (2013)
- NPLQCD, S. R. Beane *et al*., Phys. Rev. D87, 034506 (2013)
- NPLQCD, M. Wagman et al., Phys. Rev. D96, 114510(2017)
- CalLat, E. Berkowitz et al., PLB765,285(2017)

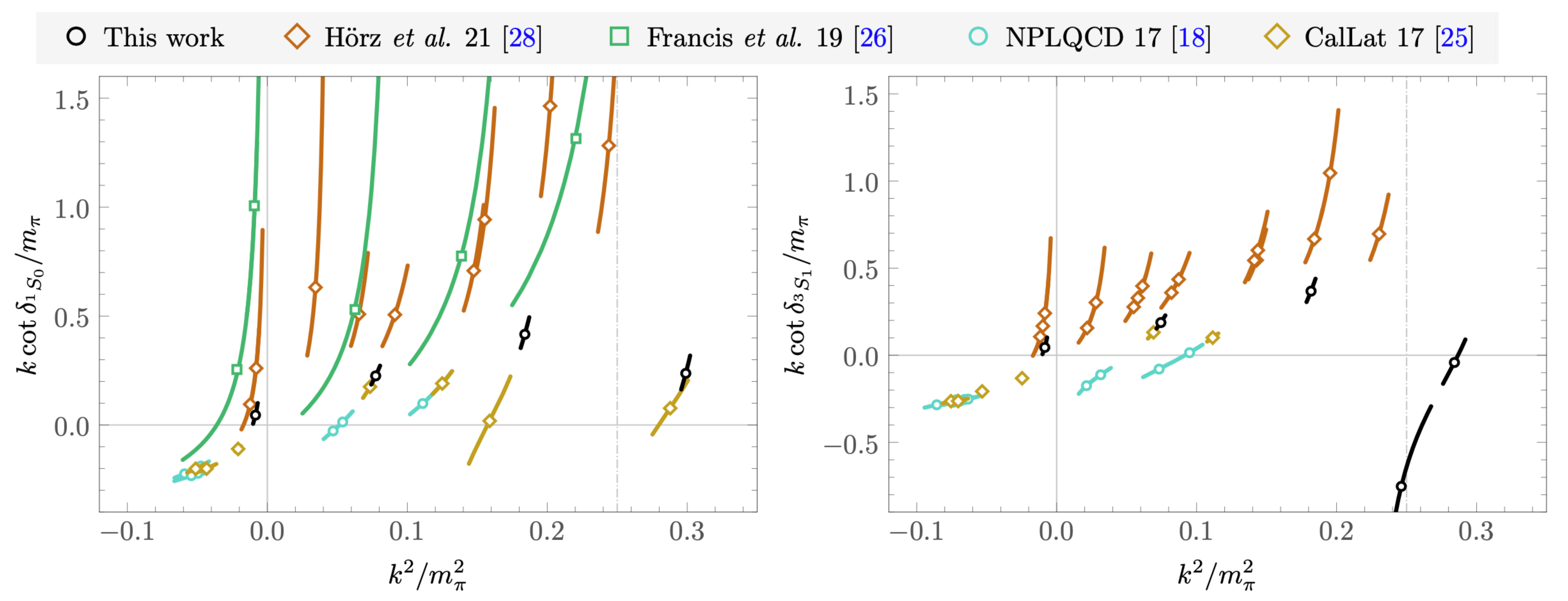
→ Deeply bound

- J. R. Green, A. D. Hanlon, P. M. Junnarkar, and H. Wittig, (2021), 2103.01054.
- B. Hörz *et al*., Phys. Rev. C **103**, 014003 (2021), 2009.11825.
- A. Francis *et al*., Phys. Rev. D **99**, 074505 (2019), 1805.03966.
- S. Amarasinghe et. al.,(NPLQCD), arXiv:2108.10835

→ Do not support the existence of a bound state at the studied pion mass.

Two-particle scattering

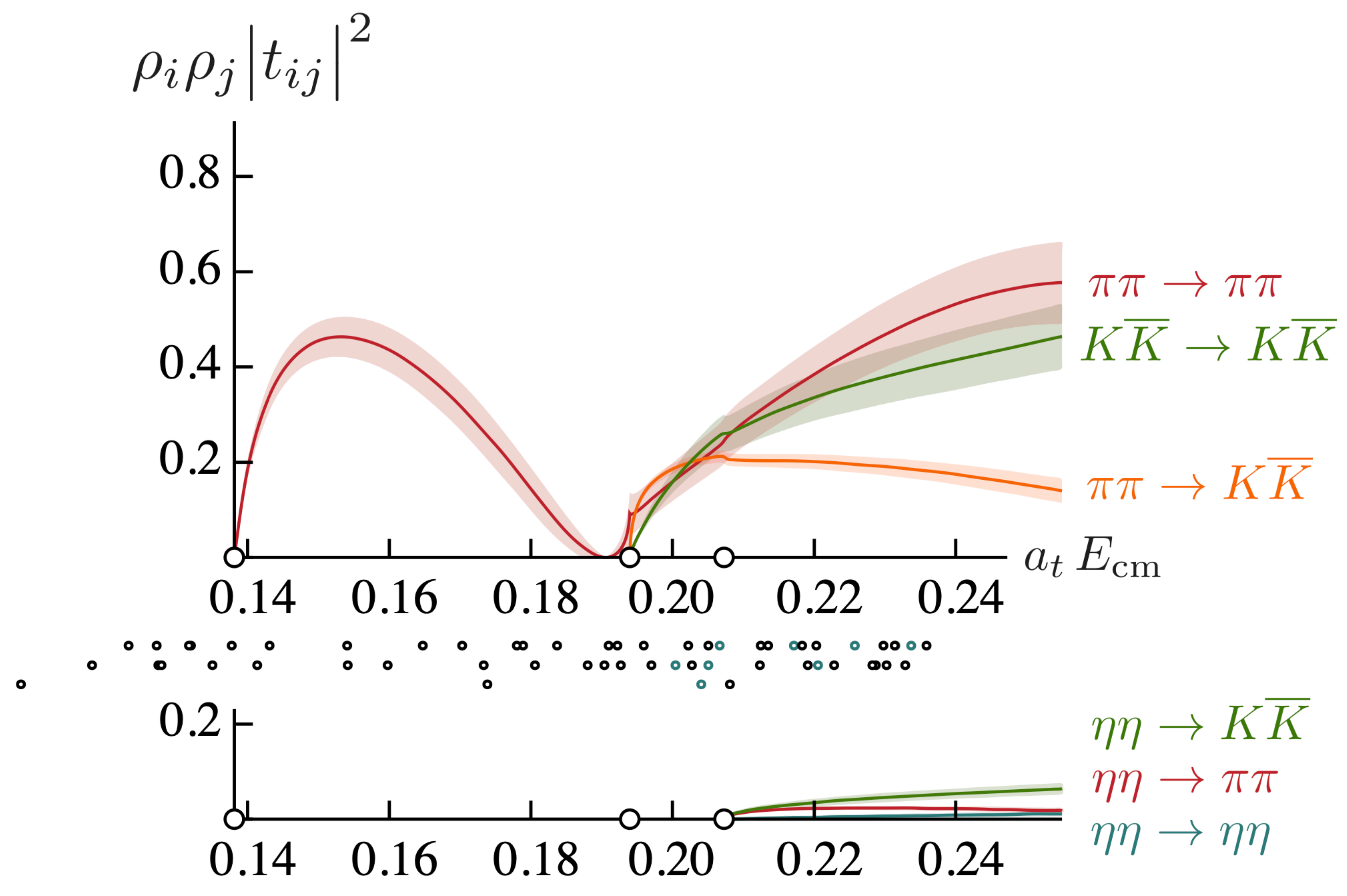
NN scattering and deuteron



S. Amarasinghe et. al.,(NPLQCD), arXiv:2108.10835

Two-particle scattering

Coupled channels: $\pi\pi, K\bar{K}, \eta\eta$ scattering and σ, f_0, f_2 resonances



- More than 50 energy levels were used to determine the scattering amplitudes of the three coupled channels.
- Bound state pole is found below the $\pi\pi$ threshold, which is related to σ.
- Resonance pole is found near the $K\bar{K}$ threshold in both $J = 0$ and $J = 2$ channel, which correspond to f_0 and f_2 .

R. A. Briceno et. al., Phys. Rev. D 97, 054513 (2018)

Three-particle scattering

Why three-particle?

- Many interesting physical processes involve three or more particle interaction.
  - The ρ resonance couples to four pions.
  - Roper resonance: $N(1440) \to N\pi \to N\pi\pi$
  - Nucleus: many-body nuclear physics.
  - ……

Three-particle scattering

- Finite volume formalism has been developed independently by three groups.
  - Relativistic Field Theory
    - M. T. Hansen and S. R. Sharpe, Phys. Rev. D 90, 116003 (2014)
    - M. T. Hansen and S. R. Sharpe, Phys. Rev. D 92, 114509 (2015)
  - Effective Field Theory
    - H.-W. Hammer, J.-Y. Pang and A. Rusetsky, JHEP 09, 109 (2017)
    - H.-W. Hammer, J.-Y. Pang and A. Rusetsky, JHEP 10, 115 (2017)
    - F. Müller, J.-Y. Pang, A. Rusetsky and J.-J. Wu, JHEP 02, 158 (2022)
  - Finite-Volume Unitarity
    - M. Mai and M. Döring, Eur. Phys. J. A 53, 240 (2017)
- Three-pion and three-kaon systems at maximal isospin have been explored.
  - T. D. Blanton et al., JHEP 10, 023 (2021).
  - M. T. Hansen et al., Phys. Rev. Lett. 126, 012001(2021).
  - B. Hörz and A. Hanlon, Phys. Rev. Lett. 123, 142002 (2019)

Discussions

- Precision frontier:
  - Control of the systematics: continuum extrapolation, physical pion mass, finite volume effects, iso-spin breaking effects, QED corrections etc.
  - Depends on the available configurations.

- Multi coupled channels :
  - Light meson-meson coupled channel have been investigated. Coupled channel heavy meson scattering and baryon scattering is still rare.

Discussions

✦ Multi-particle scattering:
- Formalism is developing, simple systems has been investigated.

✦ Nucleon and nuclei system with near physical pion mass:
- Signal to noise ratio, more open multi-particle channels…
- Probably will remain a challenge in the near future.
- Revolutionary computation techniques may help? — quantum computing