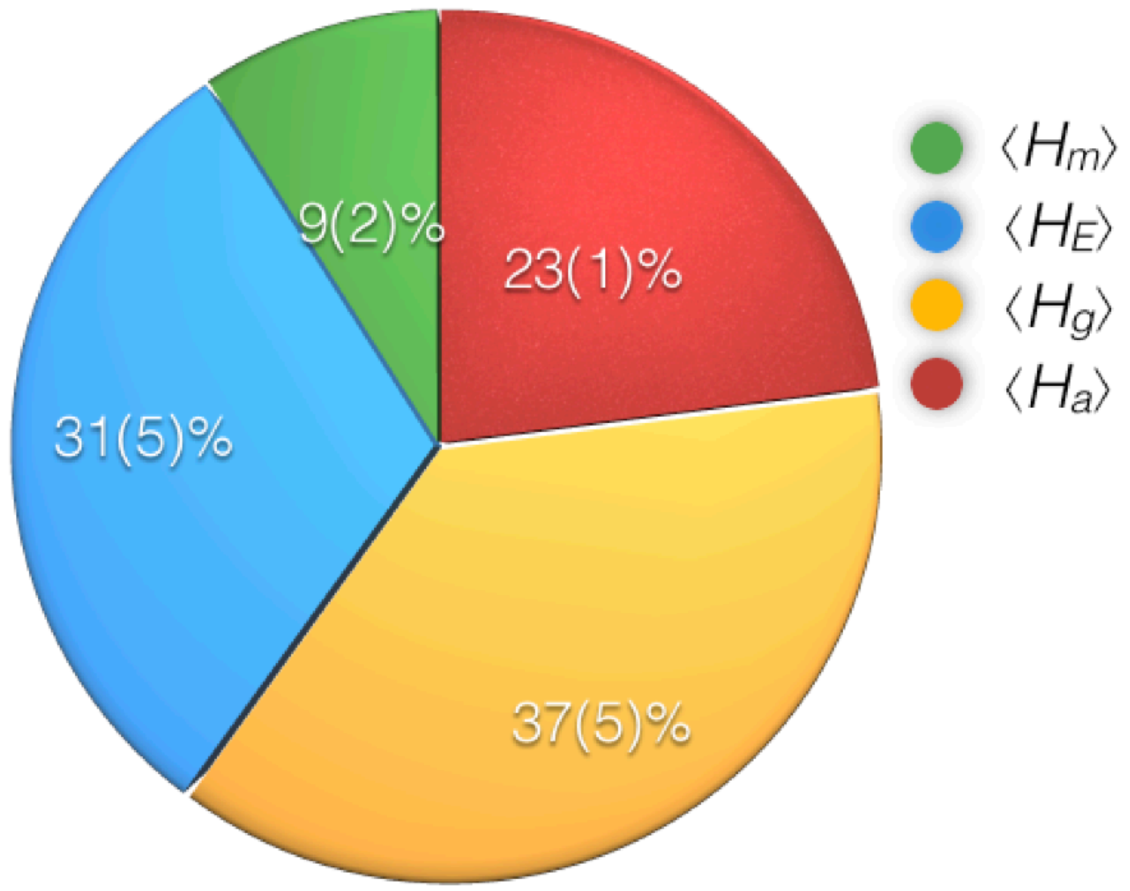
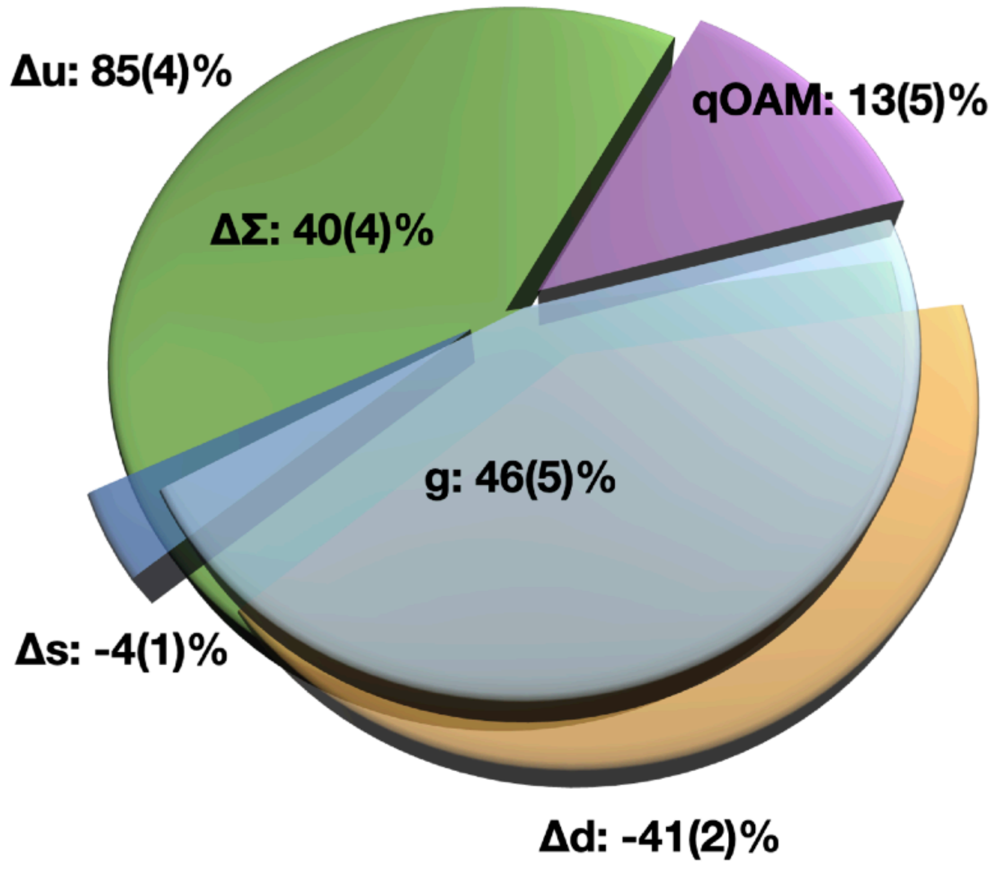
第六届格点量子色动力学夏令营

强子的质量与自旋：从格点QCD看强子结构





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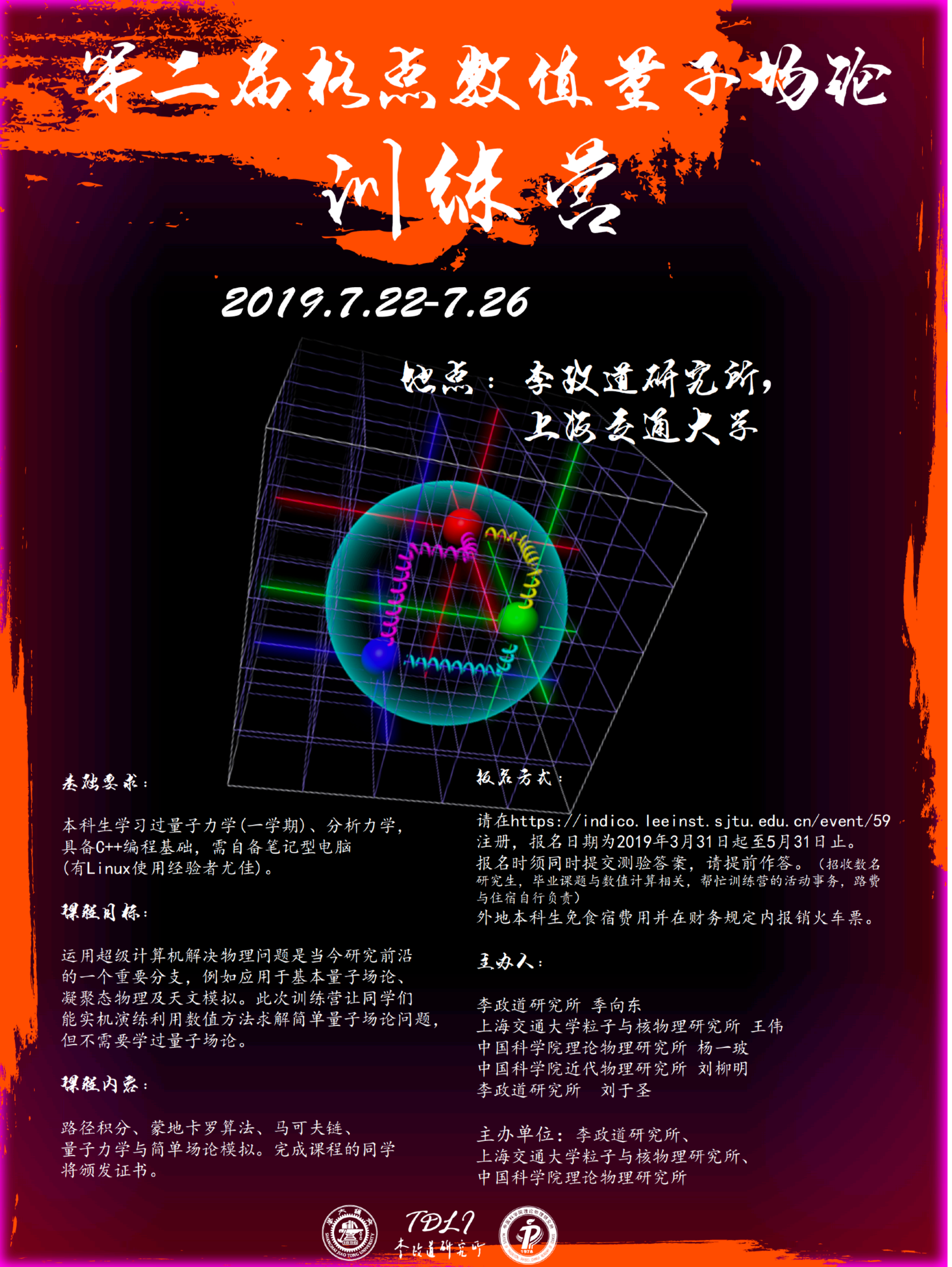
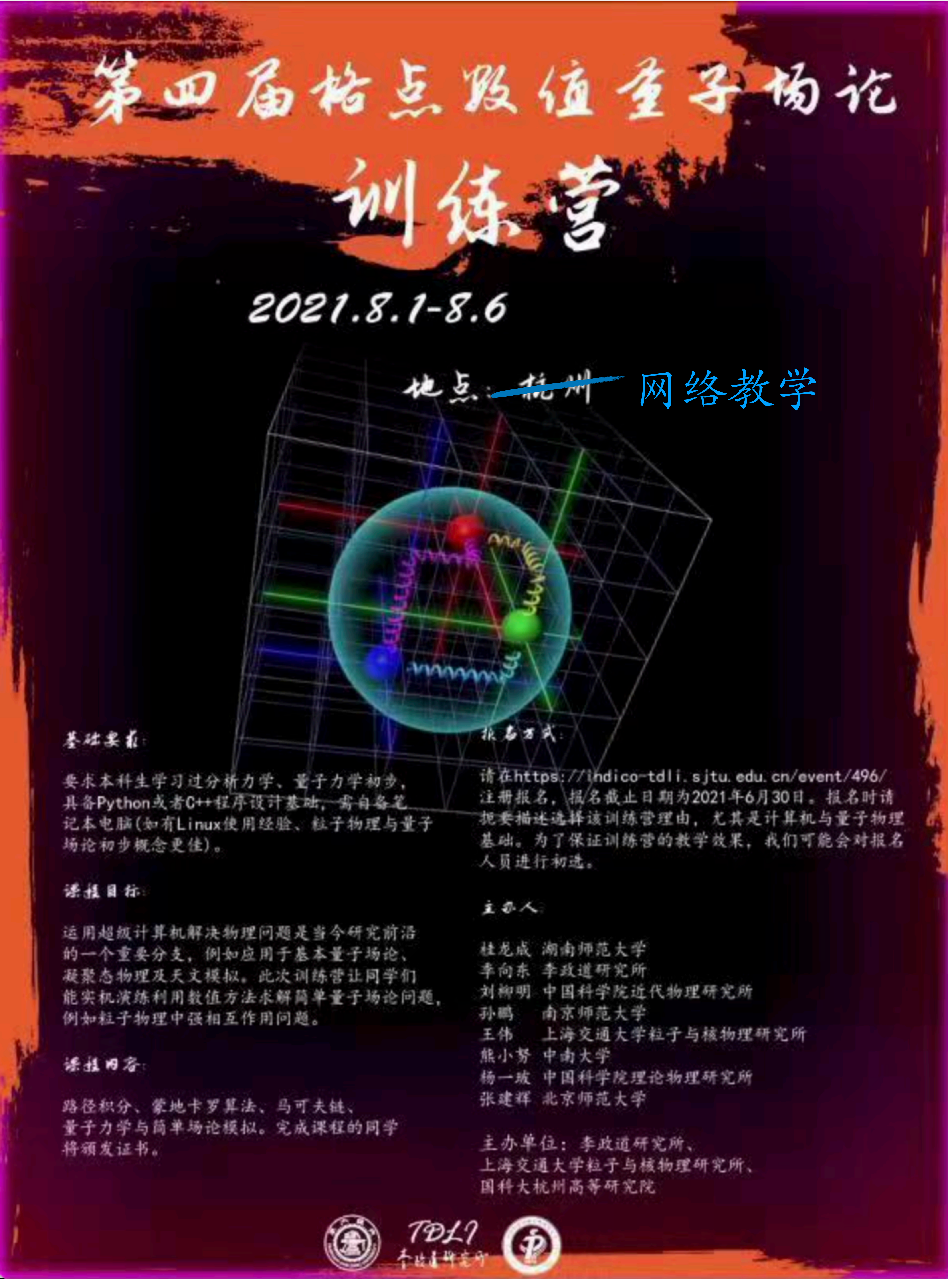
- 2019年1月11-13日
- 主要组织者：季向东
- 内容
  - 数值积分与统计分析基础
  - 作为格点场论的1维谐振子
  - 1+1维标量场论的情形
- 经验
  - 如果和出题人的电波对不上，填空题也非常困难
  - 3天时间只够完成前两项内容



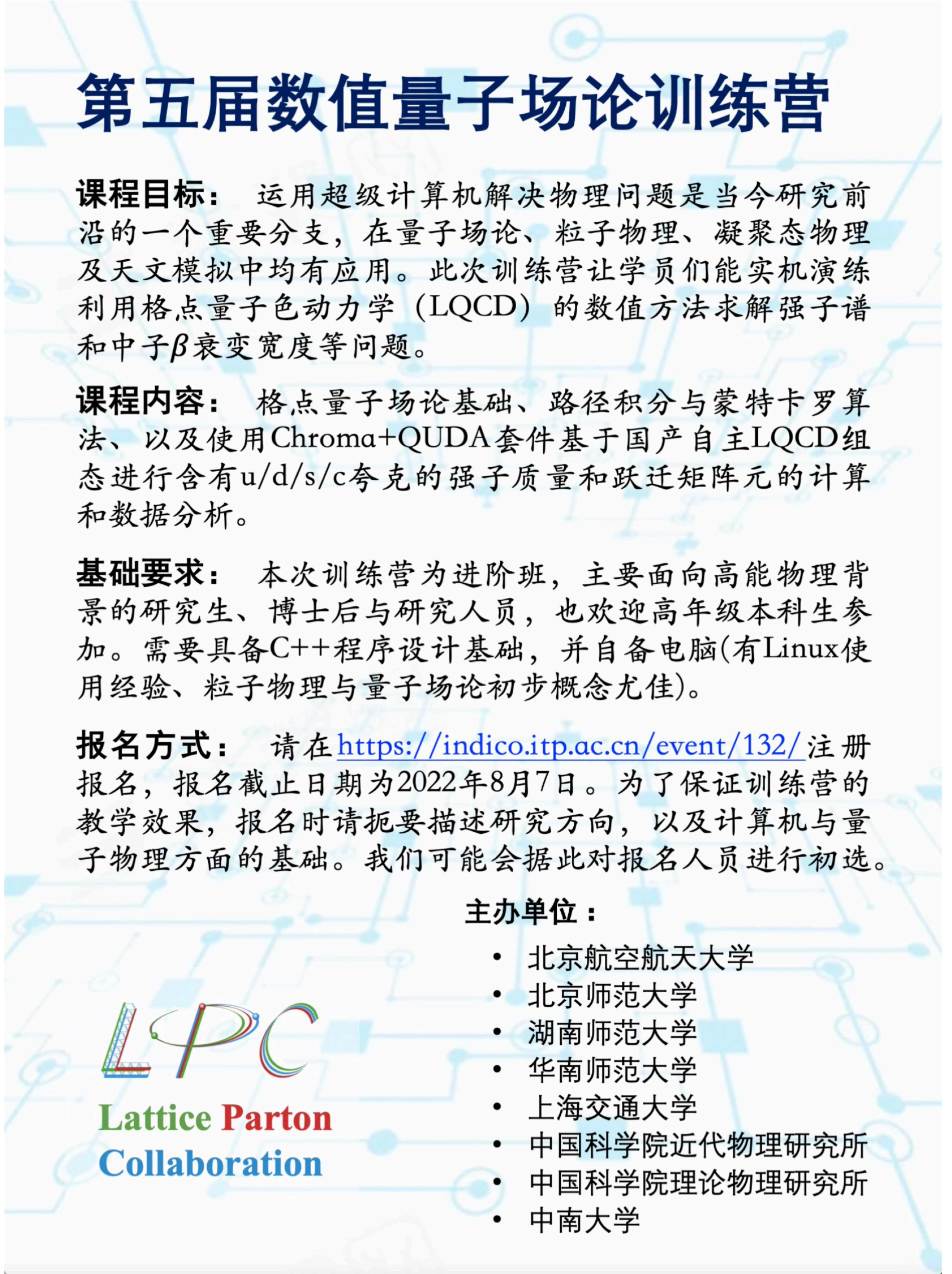
- 2019年7月22-26日
  - 主要组织者：刘于圣
- 内容
  - 数值积分与统计分析基础
  - 作为格点场论的1维谐振子
  - 1+1维标量场论的情形
- 经验
  - 对营员进行分组，并提供专人指导，可以有效地提高学习效率
  - 5天时间在理想状态下可以完成所有内容

- 2020年8月17-22日
- 内容
  - 作为格点场论的1维谐振子
  - LQCD中的两点函数与强子质量
  - 4维SU(3)纯规范场的产生
  - 4维SU(3)纯规范场的静态势
- 经验
  - 在线上活动达到理想效果需要更多努力
  - 课程内容激发了部分营员彻夜奋战克服难题的豪情



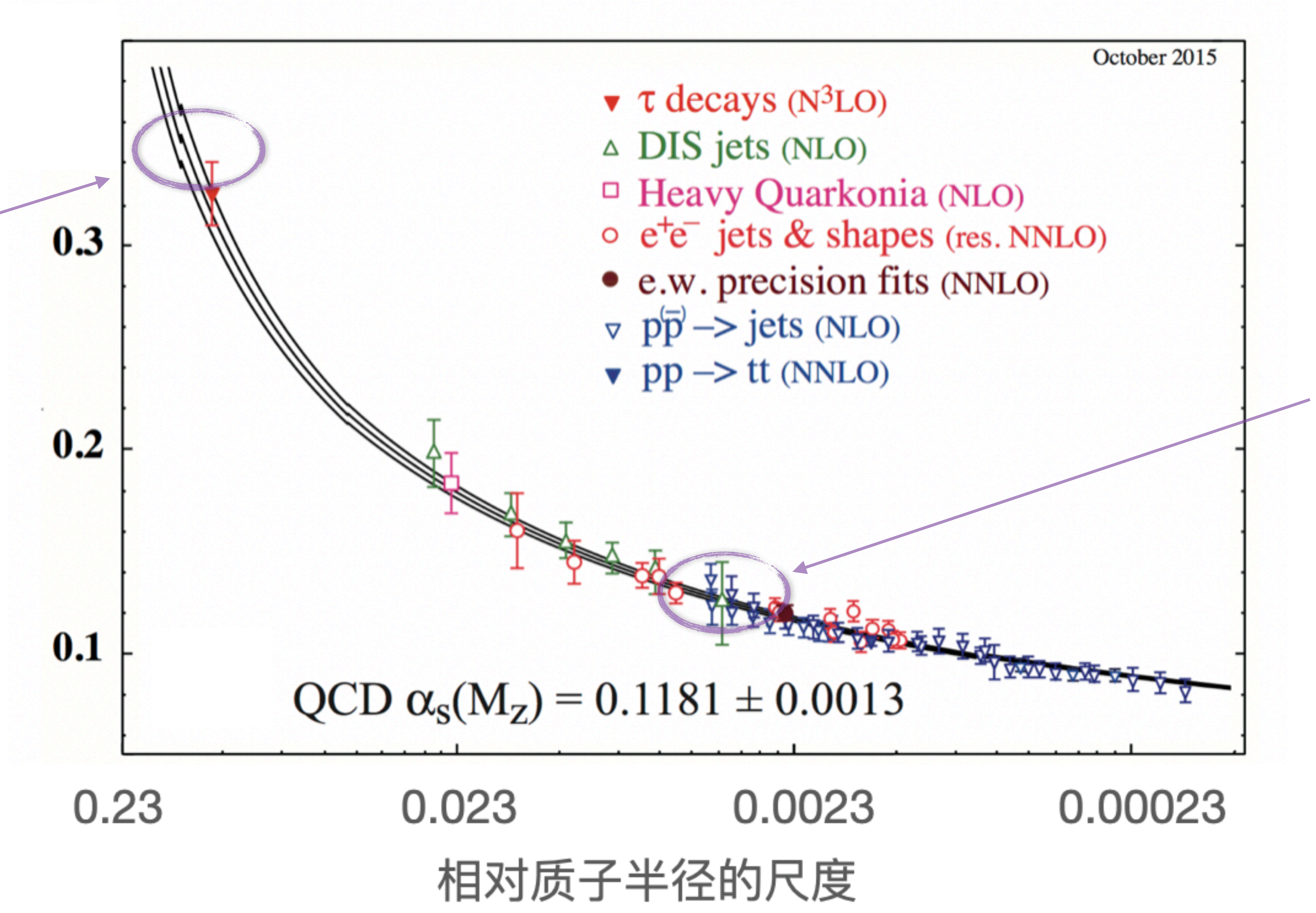
- 2021年8月1-6日
- 内容
  - 数值积分与统计分析基础
  - 作为格点场论的1维谐振子
  - 1+1维U(1)规范场论及电子偶素计算
- 经验
  - 疫情期间进行线下活动具有很大不确定性
  - 1+1维规范场论是很好的教学素材，除了程序要从头写

- 2021年8月22-26日
- 内容
  - 数值积分与统计分析基础
  - LQCD中的两点函数与强子质量
  - LQCD中的三点函数与强子结构
  - LQCD组态产生简介
- 经验
  - 啥时候能办一次线下啊……



微扰量子色动力学

质子半径尺度上(对应1GeV) QCD无法解析计算，在这个尺度上的理论计算是七个**千禧年大奖难题**之一。



- <1%**质子半径的尺度**上QCD已经得到实验和解析计算的**精确检验**；
- 于2004年获得诺贝尔物理学奖。

杨–米尔斯存在性与质量间隙难题：

- 对任意紧致、单的规范群，四维欧几里得空间中的量子杨–米尔斯理论存在一个正的质量间隙。
- 这一间隙对于量子色动力学，就是夸克质量趋于0时的质子质量。

微扰格点量子色动力学

High Energy Physics – Experiment (**hep-ex** new, recent, search)
High Energy Physics – Lattice (**hep-lat** new, recent, search)
High Energy Physics – Phenomenology (**hep-ph** new, recent, search)
High Energy Physics – Theory (**hep-th** new, recent, search)

高能物理中和实验、唯象与理论并列的四个方向之一

如果认为世界定义在离散的格子之上，也可以消除场论中相互作用的无穷大，可惜不是一般地难算

$$G_{\mu\nu}(k) = \frac{1}{k^4}\left(\alpha k_\mu k_\mu + (g_{\mu\nu}k^2 - k_\mu k_\mu)\right) \xrightarrow[\text{格点量子场论}]{\text{从连续量子场论到}}$$

$$G_{\mu\nu}(k) = \frac{1}{(\widehat{k}^2)^2}\left(\alpha\widehat{k}_\mu\widehat{k}_\nu + \sum_\sigma(\widehat{k}_\sigma\delta_{\mu\nu} - \widehat{k}_\nu\delta_{\mu\sigma})\widehat{k}_\sigma A_{\sigma\nu}(k)\right),$$

with

$$\begin{aligned} A_{\mu\nu}(k) \;&=\; A_{\nu\mu}(k) = (1-\delta_{\mu\nu})\,\Delta(k)^{-1}\left[(\widehat{k}^2)^2 - c_1\widehat{k}^2\left(2\sum_\rho \widehat{k}_\rho^4 + \widehat{k}^2\sum_{\rho\neq\mu,\nu}\widehat{k}_\rho^2\right)\right. \\ &\left. + c_1^2\left(\left(\sum_\rho \widehat{k}_\rho^4\right)^2 + \widehat{k}^2\sum_\rho \widehat{k}_\rho^4\sum_{\tau\neq\mu,\nu}\widehat{k}_\tau^2 + (\widehat{k}^2)^2\prod_{\rho\neq\mu,\nu}\widehat{k}_\rho^2\right)\right], \end{aligned}$$

where

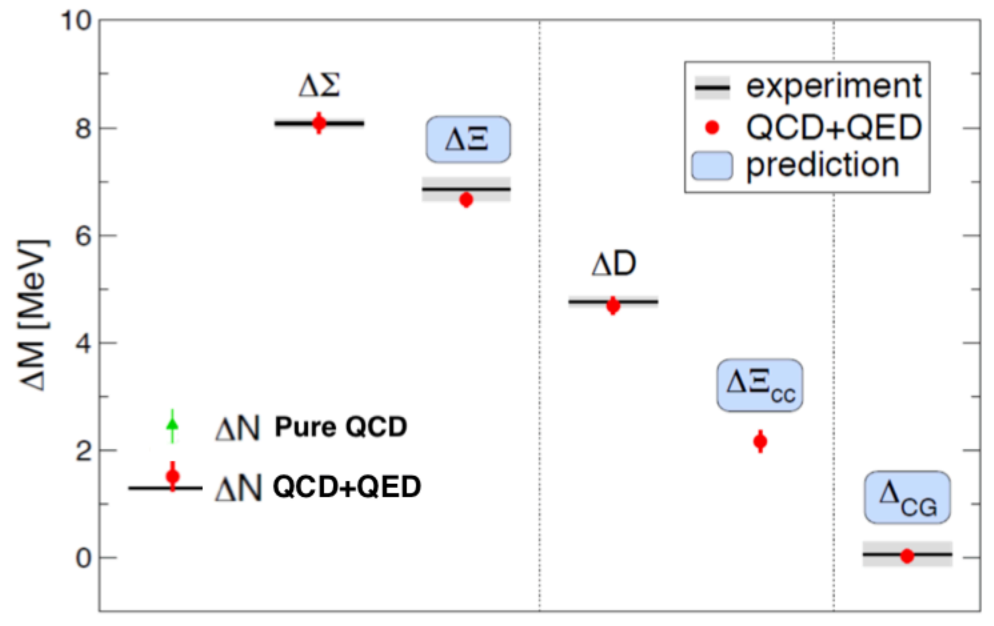
$$\begin{aligned} \Delta(k) \;&=\; \left(\widehat{k}^2 - c_1\sum_\rho \widehat{k}_\rho^4\right)\left[\widehat{k}^2 - c_1\left((\widehat{k}^2)^2 + \sum_\tau \widehat{k}_\tau^4\right) + \frac{1}{2}c_1^2\left((\widehat{k}^2)^3 + 2\sum_\tau \widehat{k}_\tau^6 - \widehat{k}^2\sum_\tau \widehat{k}_\tau^4\right)\right] \\ &- 4c_1^3\sum_\rho \widehat{k}_\rho^4\sum_{\tau\neq\rho}\widehat{k}_\tau^2. \end{aligned}$$

可以算到$\mathcal{O}(g^8)$以上

算到$\mathcal{O}(g^4)$都非常困难

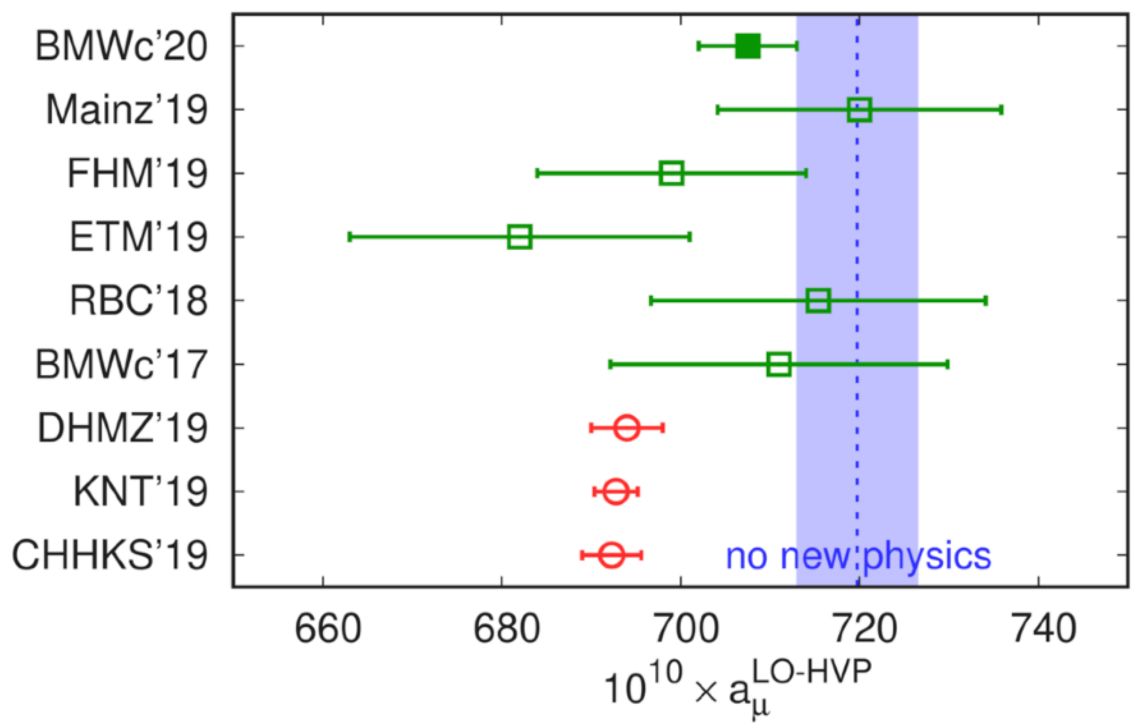
非微扰格点量子色动力学

……但是也不一定非要逐阶用手算



质子与中子的质量差

BMWc, Science 347:1452,2015; 1406.4088



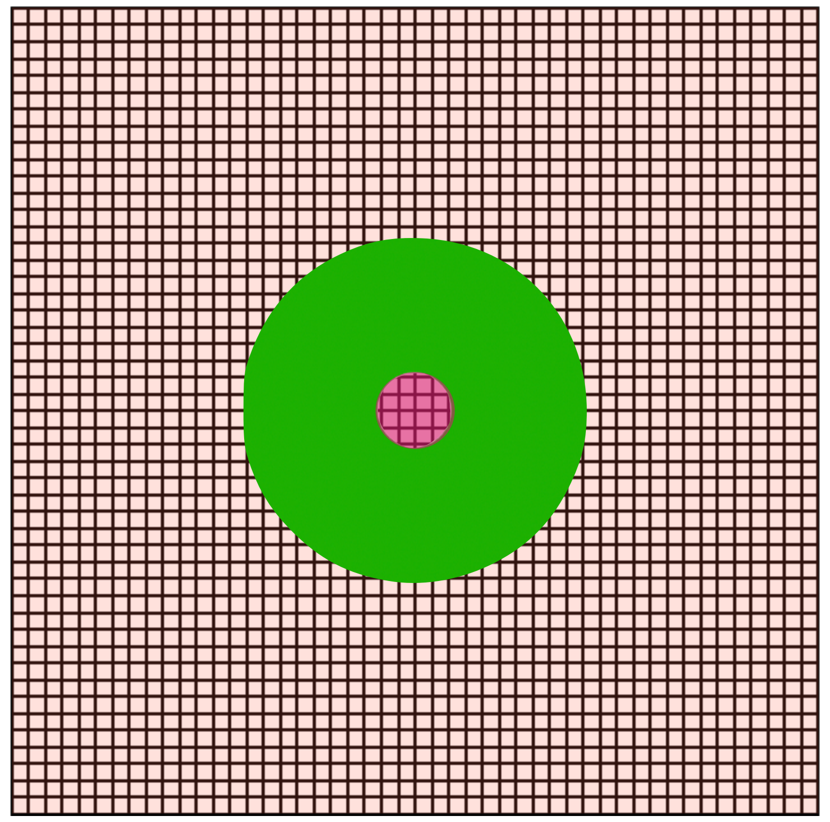
缪子反常磁矩中的真空极化贡献

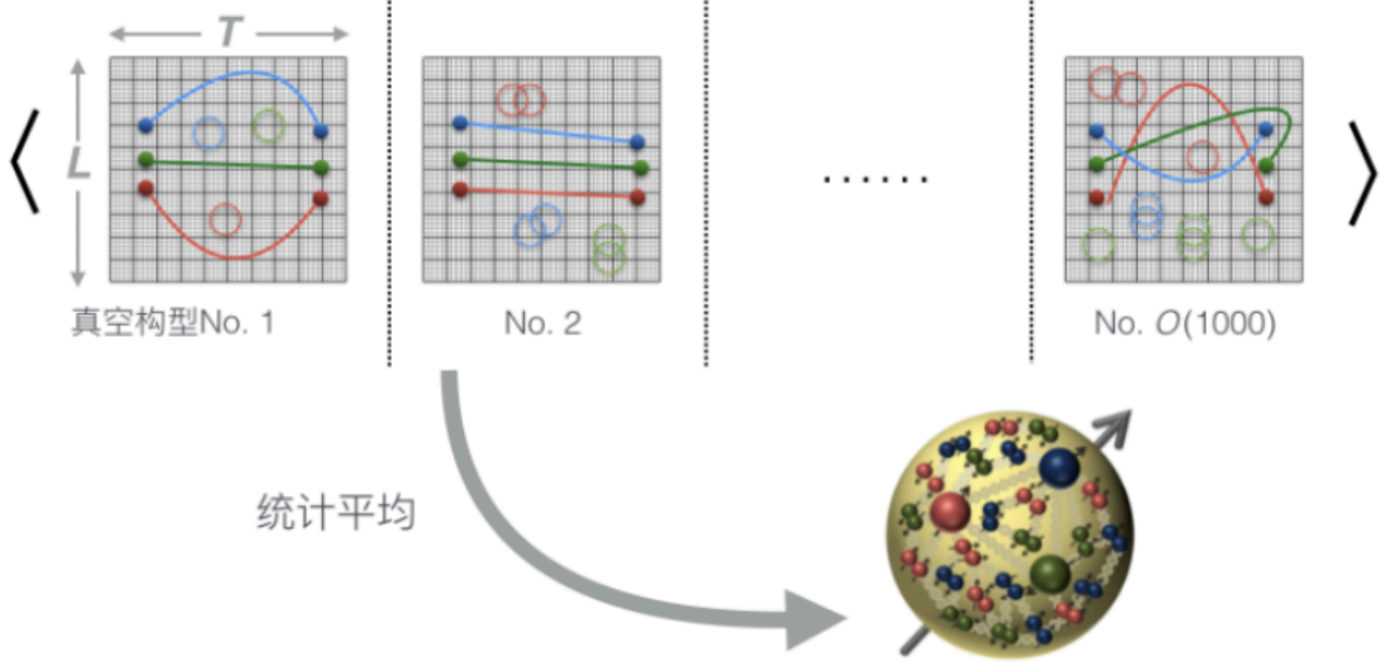
BMWc, Nature 593:7857,2021; 2002.12347

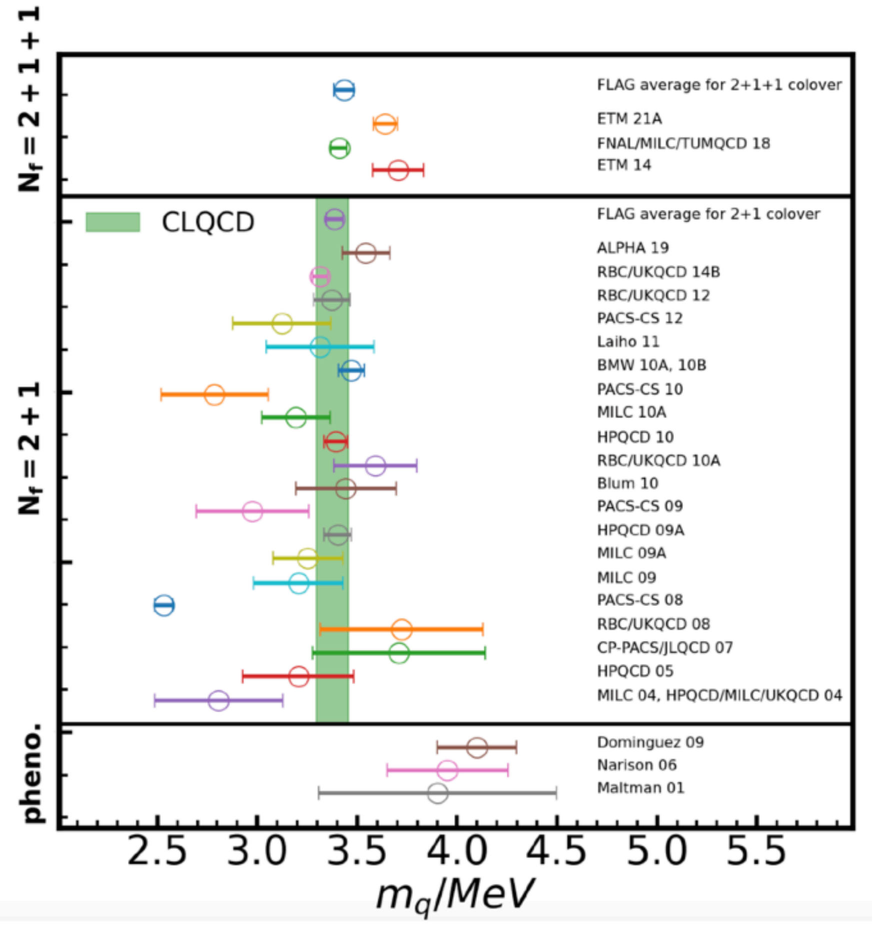
- 格点量子色动力学是目前唯一能精确描述质子半径尺度的强相互作用性质的量子规范场论框架
- 结合巧妙的算法和超级计算机的算力，可以精确计算强子的各种性质。

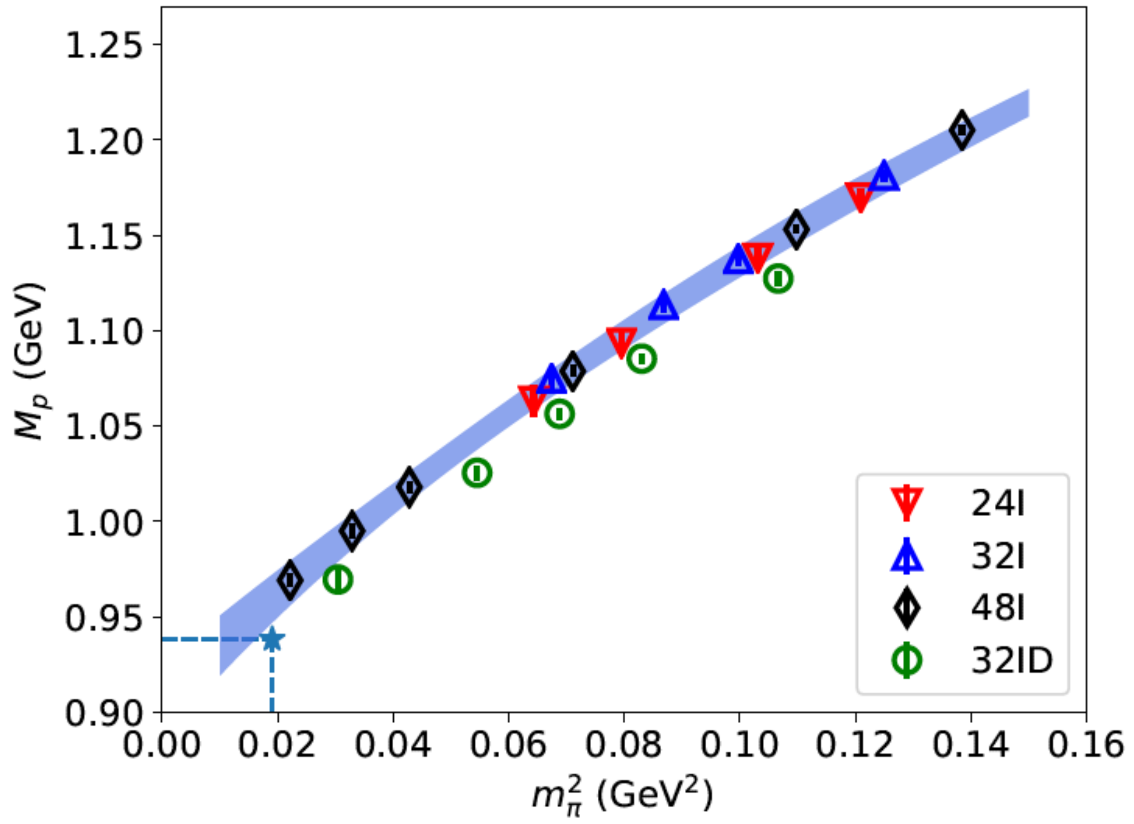
可以算到$\mathcal{O}(g^8)$以上

格点量子色动力学基础



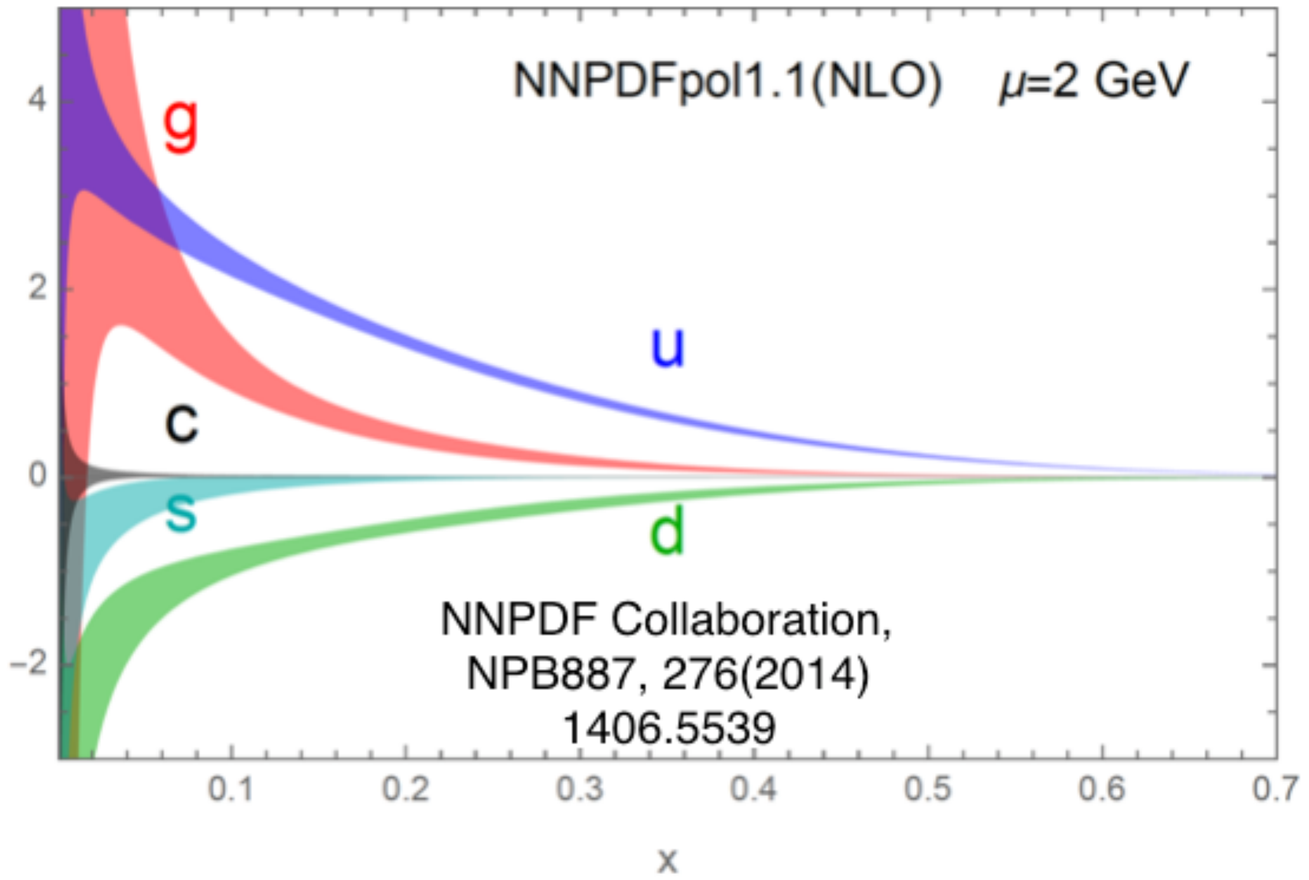


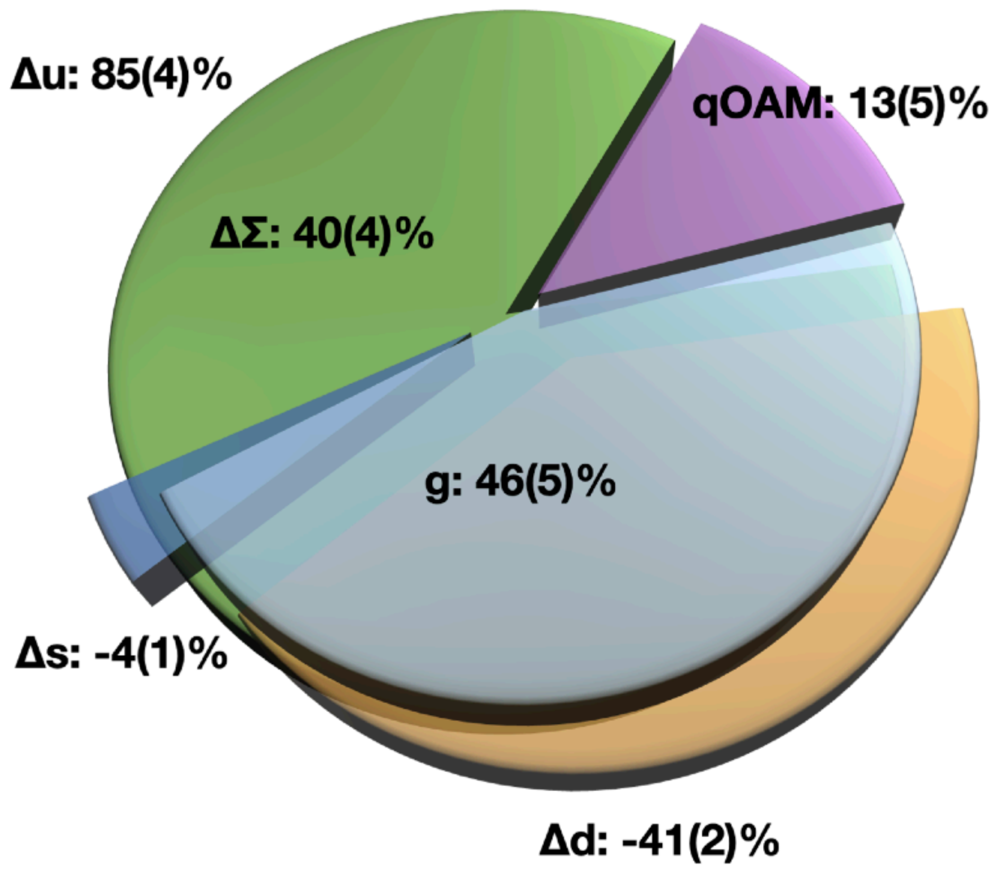




强子质量

强子自旋





短程格点QCD

格点正规化

	Continuum	Lattice
EOM	$(\partial_\mu \partial^\mu + m^2)\phi = 0$	$\sum_\mu \left(\phi(x+\hat{n}_\mu) + \phi(x-\hat{n}_\mu)\right) + (m^2a^2 - 2)\phi(x) = 0$
Propagator	$S = \dfrac{1}{p^2 + m^2}$	$S_L = \dfrac{1}{4\sum_\mu \mathrm{Sin}^2(\frac{ap_\mu}{2})/a^2 + m^2}$
Loop Integral	$\int_{-\infty}^{\infty} \mathrm{d}^4 p$	$\int_{-\pi/a}^{\pi/a} \mathrm{d}^4 p$
	Require additional regulation to be finite	The divergence has been regularized into the $1/a^n$ and log(a) terms

短程格点QCD

格点上的规范场

连续理论里面的QCD作用量有如下形式：

$$\bar{\psi}(\gamma_\mu(\partial_\mu - igA_\mu) - m)\psi + F_{\mu\nu}F_{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

微扰论可以在g很小的时候对这个理论做微扰展开和圈图计算。

Wilson引入的格点正规化把规范场的基本变量从规范势A变成了规范link：

$$A_\mu(x + \frac{1}{2}\hat{n}_\mu) = a^4 \frac{U_\mu(x) - U_\mu^\dagger(x)}{2ig_0 a} + \mathcal{O}(a^2 g^2), U(x) \equiv e^{ig_0 \int_x^{x+a\hat{\mu}} dy A_\mu(y)};$$

$$U(x) \equiv e^{ig_0 \int_x^{x+a\hat{\mu}} dy A_\mu(y)}, U^\dagger(x) \equiv e^{ig_0 \int_{x+a\hat{\mu}}^{x} dy A_\mu(y)}$$

$$x \longrightarrow x + a\hat{\mu}$$

短程格点QCD

格点上的规范变换

$$g^{-1}(x) \xrightarrow{U(x) \equiv e^{ig_0 \int_x^{x+a\hat{\mu}} dy A_\mu(y)}} g(x+a\hat{\mu})$$

- 连续理论中的规范变换：$A'_\mu(x) = g^{-1}(x)A_\mu(x)g(x) - \dfrac{i}{g_0} g^{-1}(x)\partial_\mu g(x)$
- 格点上的规范变换：$U'_\mu(x) = g^{-1}(x)U_\mu(x)g(x+a\hat{\mu})$

$$1 + ig_0 a A'_\mu(x) = g^{-1}(x)\big(1 + ig_0 a A_\mu(x)\big)\big(g(x) + a\partial_\mu g(x)\big) = 1 + ig_0 a\Big(g^{-1}(x)A_\mu(x)g(x) - \frac{ia}{g_0} g^{-1}(x)\partial_\mu g(x)\Big) + \mathcal{O}(a^2)$$

- 格点上的规范协变性表现为在link两端可以乘任何规范转动矩阵；
- 格点上的规范**不变性**表现为若干link连成一个闭合的圈。

短程格点QCD

- $\mathscr{P}_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)\,.$
  也可以被用来定义格点上的胶子作用量：
  $$\mathcal{S}_g = \frac{1}{2g_0^2}\sum_{x,\mu\nu}\mathrm{Re}\left[1-\mathrm{Tr}[\mathscr{P}_{\mu\nu}(x)]\right] = \frac{1}{2}\mathrm{Tr}\int \mathrm{d}^4x F_{\mu\nu}F_{\mu\nu} + \mathcal{O}(a^2)$$

- 这个作用量具有$\mathcal{O}(a^2)$的离散误差。

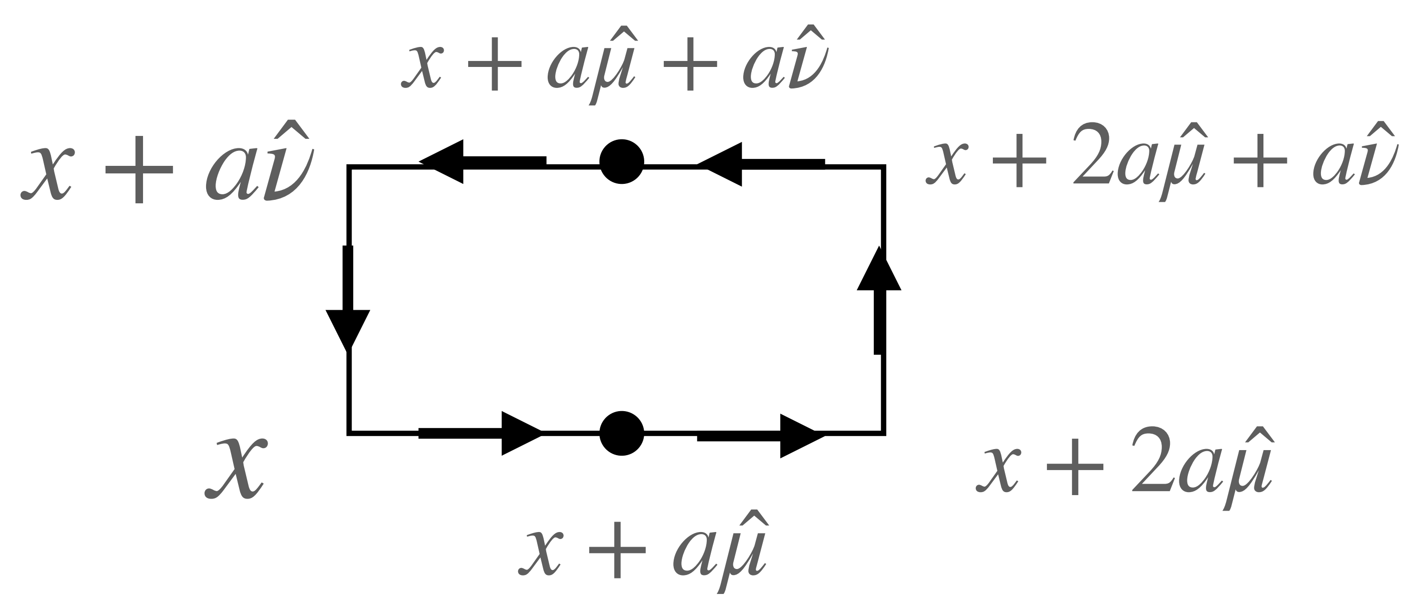
- 可以进一步与1x2的loop
  $$\mathscr{P}_{\mu\nu}^{Rect}(x) = U_\mu(x)U_\mu(x+a\hat{\mu})U_\nu(x+2a\hat{\mu})U_\mu^\dagger(x+a\hat{\mu}+a\hat{\nu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)$$
  组合，构造Symanzik或者Iwasaki作用量, 把离散误差压低到$\mathcal{O}(a^4)$。

$$S_g^{\mathrm{Symanzik}} = \frac{5}{3}S_g^{1x1} - \frac{1}{12}S_g^{1x2}$$

$$S_g^{\mathrm{Iwasaki}} = (1+8\times 0.331)S_g^{1x1} - 0.331S_g^{1x2}$$

规范场作用量

$$\mathscr{P}_{\mu\nu}(x) = U_\mu(x)U_\nu(x+a\hat{\mu})U_\mu^\dagger(x+a\hat{\nu})U_\nu^\dagger(x)\,.$$
$$= 1 + ig_0a^2F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu}))$$
$$-\frac{1}{2}a^4g_0^2F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu}))F_{\mu\nu}(x+\frac{a}{2}(\hat{\mu}+\hat{\nu})) + i\mathcal{O}(a^4)$$



Discretized QCD

Naive and Staggered actions

- The naive discretization suffers from the doubling problem：
  - $\mathcal{S}_q^{Naive}(m) = \sum_{x,y} \bar{\psi}(x) D_{Naive}(m;x,y)\psi(y),\ D_{Naive}(m;x,y) = \frac{1}{2a}\sum_{\mu} \gamma_\mu \left(U_\mu(x)\delta_{y,x+a\hat{\mu}} - U^\dagger_\mu(x-a\hat{\mu})\delta_{y,x-a\hat{\mu}}\right) + m\delta_{y,x}$
  - The propagator has $1/m$ IR poles at $pa = (0/\pi, 0/\pi, 0/\pi, 0/\pi)$, which is different from the continuum theory.
- Staggered fermion:
  - $\psi^{\mathrm{st}}(x) = \gamma_4^{x_4}\gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\psi(x),\ \{\gamma_1^{\mathrm{st}}, \gamma_2^{\mathrm{st}}, \gamma_3^{\mathrm{st}}, \gamma_4^{\mathrm{st}}\} = \{(-1)^{x_4}, (-1)^{x_1+x_4}, (-1)^{x_1+x_2+x_4}, 1\};$
  - 16 IR poles $\rightarrow$ 4 IR poles.
  - Mixing between IR poles can be suppressed with kinds of the improvement, likes the so-call highly-improved staggered quark (HISQ).

Staggered/HISQ $\xrightarrow{\text{Cost x10}}$ Wilson/Clover $\xrightarrow{\text{Cost x10}}$ Domain wall $\xrightarrow{\text{Cost x10}}$ Overlap

Discretized QCD

Wilson and clover actions

- Wilson fermion action：
  - $D + m \to D + aD^2 + m$
  - It removes the unphysical IR pole at $p_i = \pi/a$, while introduce the additional chiral symmetry breaking at $\mathcal{O}(\alpha_s/a)$.
- Clover fermion action:
  - $D + m \to D + aD^2 + m + ac_{\rm sw}\sigma_{\mu\nu}F^{\mu\nu}$
  - Suppress the additional chiral symmetry breaking at $\mathcal{O}(\alpha_s^2/a)$.
- The cost of either Wilson or Clover action is $\mathcal{O}(10)$ of the Staggered fermion.

Staggered/HISQ $\xrightarrow{\text{Cost x10}}$ Wilson/Clover $\xrightarrow{\text{Cost x10}}$ Domain wall $\xrightarrow{\text{Cost x10}}$ Overlap

Discretized QCD

Ginsparg-Wilson action

- Ginsparg-Wilson relation: $\gamma_5 D_{\mathrm{GW}} + D_{\mathrm{GW}}\gamma_5 = \frac{1}{\rho} D_{\mathrm{GW}}\gamma_5 D_{\mathrm{GW}}$.
- Overlap fermion as a possible solution:
$$\mathcal{S}_q^{ov}(m) = \sum_{x,y} \bar{\psi}(x)\big(\delta_{xy} m + \sum_z D_{ov}(\rho; x, z)\frac{\rho/a}{\delta_{zy} - D_{ov}(\rho; z, y)/2}\big)\psi(y),\ D_{ov}(\rho) = 1 + \frac{D_w(-\rho)}{\sqrt{D_w(-\rho)D_W^\dagger(-\rho)}}$$

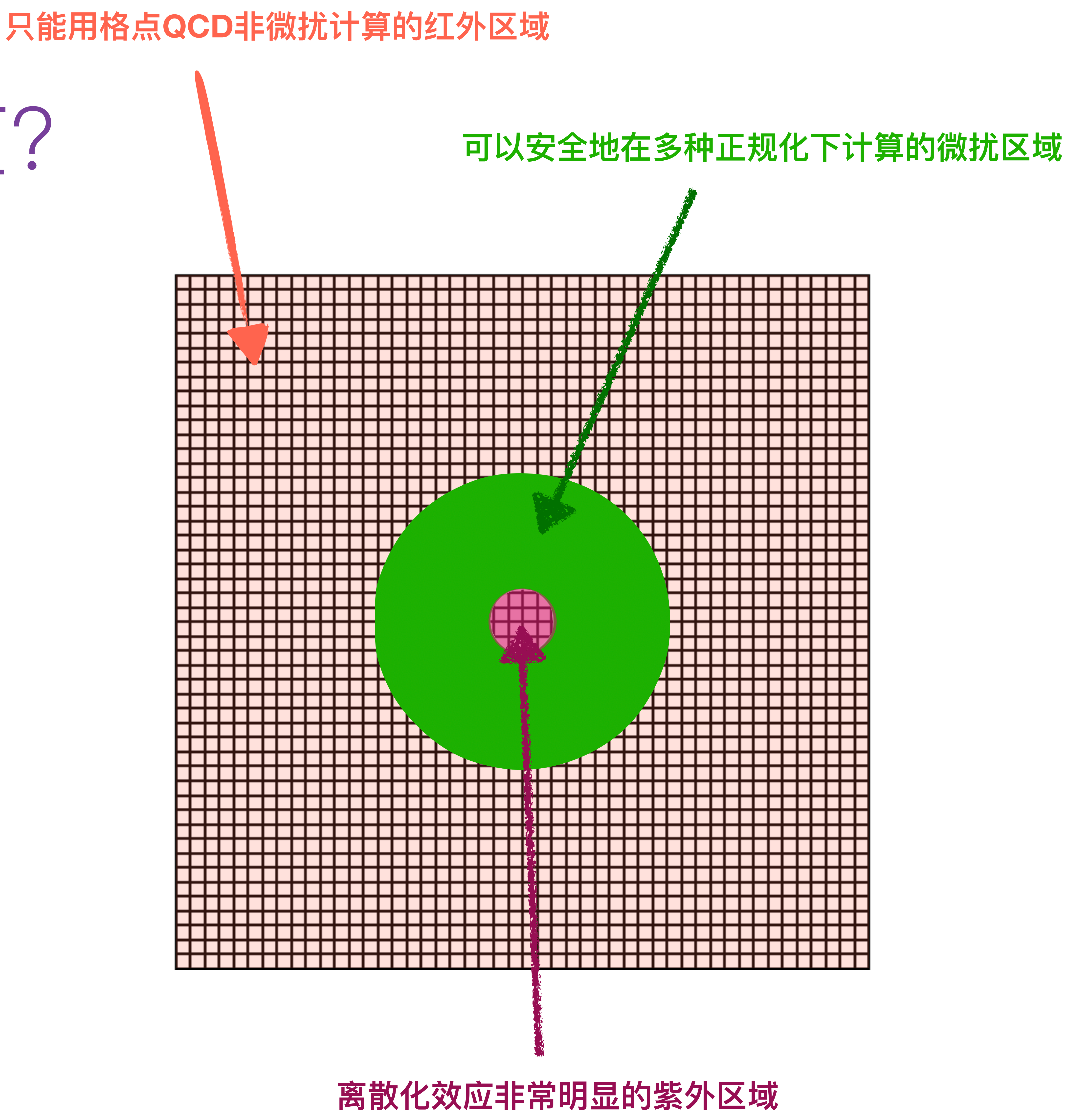
- In $p \to 0$ region, $D_{ov} \to a\gamma_\mu p_\mu$;
- In $p \to \pi/a$ region, $D_{ov} \to \mathcal{O}(1)$.
- But approximate the sign function $\frac{\gamma_5 D_w(-\rho)}{\sqrt{D_w(-\rho)D_W^\dagger(-\rho)}} = \frac{\gamma_5 D_w(-\rho)}{|\gamma_5 D_w(-\rho)|}$ need $\mathcal{O}(100)$ cost of the Wilson/Clover action.
- Domain wall fermion action is an approximation of overlap fermion with $\mathcal{O}(10)$ cost of the Wilson/Clover action.

Staggered/HISQ $\xrightarrow{\text{Cost x10}}$ Wilson/Clover $\xrightarrow{\text{Cost x10}}$ Domain wall $\xrightarrow{\text{Cost x10}}$ Overlap

短程格点QCD

如何消除离散化效应?

- 格点QCD的短程行为显著地依赖于离散化。
- 而通常的实验唯象分析是在修改维度的MS-bar方案下进行的。
- 不改变维度的格点正规化无法直接引入这个维度，从而无法直接得到MS-bar下的结果。
- 所以我们必须在“安全的区域”内，把格点计算的短程部分替换成维数正规化下的结果，同时消除离散化效应的短程影响。

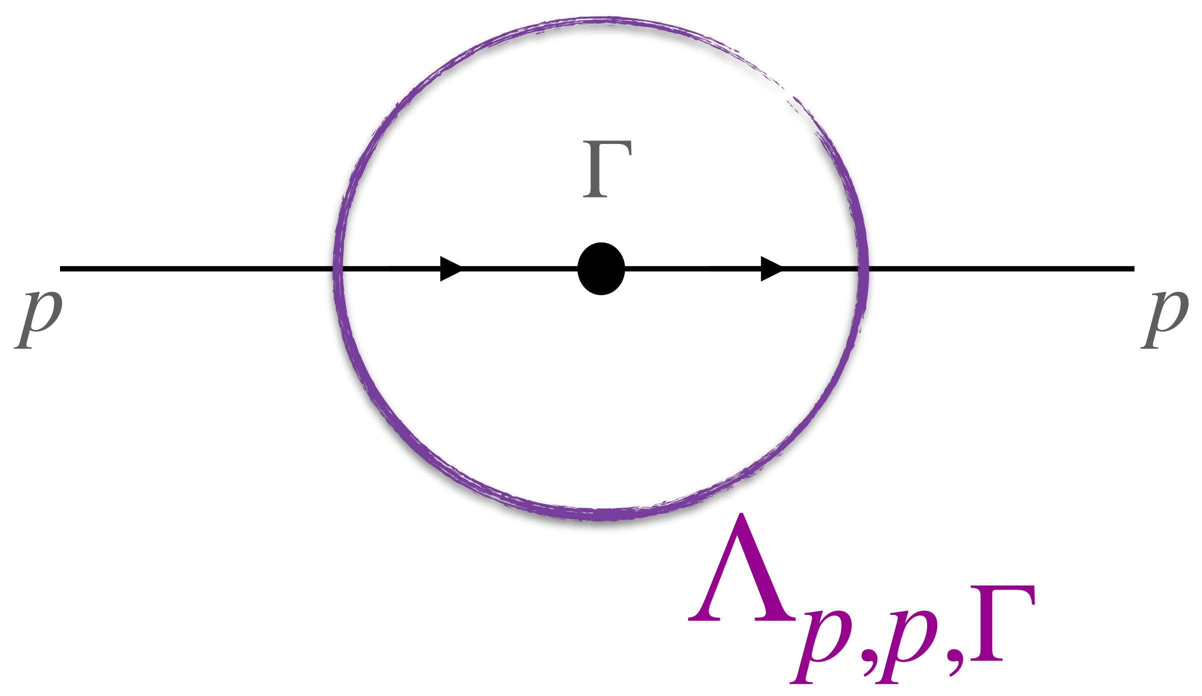


短程格点QCD

RI/MOM 方案

首先我们需要引入一个在微扰区域的新能标$Q^2 = -p^2$，

- 然后在这个能标下计算夸克传播子 $S(p) = \sum_x e^{-i(p\cdot x)} \langle \psi(x)\bar{\psi}(0) \rangle$，



- 以及顶角函数

$$\Lambda(p,p,\Gamma) = S^{-1}(p) \sum_{x,y} e^{-i\,p\cdot(x-y)} \langle \psi(x)\bar{\psi}(0)\Gamma\psi(0)\bar{\psi}(y) \rangle S^{-1}(p);$$

- 最后就可以定义一个“与正规化无关的动量减除方案（RI/MOM）”下的短程顶点（重整化常数）

$$\frac{12 Z_q^{\mathrm{RI}}(Q)}{\mathrm{Tr}[\Gamma^\dagger \Lambda(p,p,\Gamma)]}\bigg|_{p^2=-Q^2} = \frac{C_0}{m_q^n} + Z_S^{\mathrm{MOM}}(Q) + \mathcal{O}(m_q)。$$

- 这个方案可以在任意正规化下计算。

短程格点QCD

从微扰论看RI/MOM

第二个阶段是在维数正规化下微扰计算相同能标下的矩阵元，得到上述减除方案与 $\overline{\mathrm{MS}}$ 方案的差异，从而等效地将上述重整化常数转换到 $\overline{\mathrm{MS}}$ 方案。以标量流的重整化为例，第一个阶段的 RI/MOM 方案在格点正规化下单圈水平得到的重整化常数具有如下形式：

$$Z_S^{\mathrm{MOM,Lat}}(Q,a) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2Q^2) - \xi + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2),$$

其中 ξ 是胶子传播子 $D_{\mu\nu} = \frac{1}{k^2}(\delta_{\mu\nu} - \xi\frac{k_\mu k_\nu}{k^2})$ 中的规范依赖参数。b_S 是依赖于夸克与胶子离散化方案的有限项，不同的离散化方案下的值和符号都会有差异（具体的数值可以参考 hep-lat/0404007）。那么对应地，在维数正规化下计算 RI/MOM 方案的重整化常数，可以得到

$$Z_S^{\mathrm{MOM,Dim}}(Q,\mu) = 1 - \frac{\alpha_s C_F}{4\pi}[\frac{3}{\tilde{\epsilon}} - 3\log(\frac{Q^2}{\mu^2}) - \xi + 5] + \mathcal{O}(\alpha_s^2),$$
$$Z_S^{\overline{\mathrm{MS}},\mathrm{Dim}}(Q,\mu) = 1 - \frac{\alpha_s C_F}{4\pi}\frac{3}{\tilde{\epsilon}} + \mathcal{O}(\alpha_s^2).$$

所以格点正规化下等效的 $\overline{\mathrm{MS}}$ 方案重整化常数就可以定义为：

$$Z_S^{\overline{\mathrm{MS}},\mathrm{Lat}}(\mu,a) \equiv \frac{Z_S^{\mathrm{MOM,Lat}}(Q,a)}{Z_S^{\mathrm{MOM,Dim}}(Q,\mu)} Z_S^{\overline{\mathrm{MS}},\mathrm{Dim}}(Q,\mu) = 1 - \frac{\alpha_s C_F}{4\pi}[-3\log(a^2\mu^2) - 5 + b_S] + \mathcal{O}(\alpha_s^2, a^2Q^2).$$

短程格点QCD

非微扰RI/MOM

由于 RI/MOM 方案包括了所有有限项，所以同等能标下在确定的 α_s^n 阶修正远大于 $\overline{\mathrm{MS}}$ 方案。所以实践上说，第二阶段通常会分成两步操作：首先在比较高的能标 Q 把 $Z_S^{\mathrm{MOM,Lat}}(Q,a)$ 转换为 $Z_S^{\overline{\mathrm{MS}},\mathrm{Lat}}(Q,a)$，

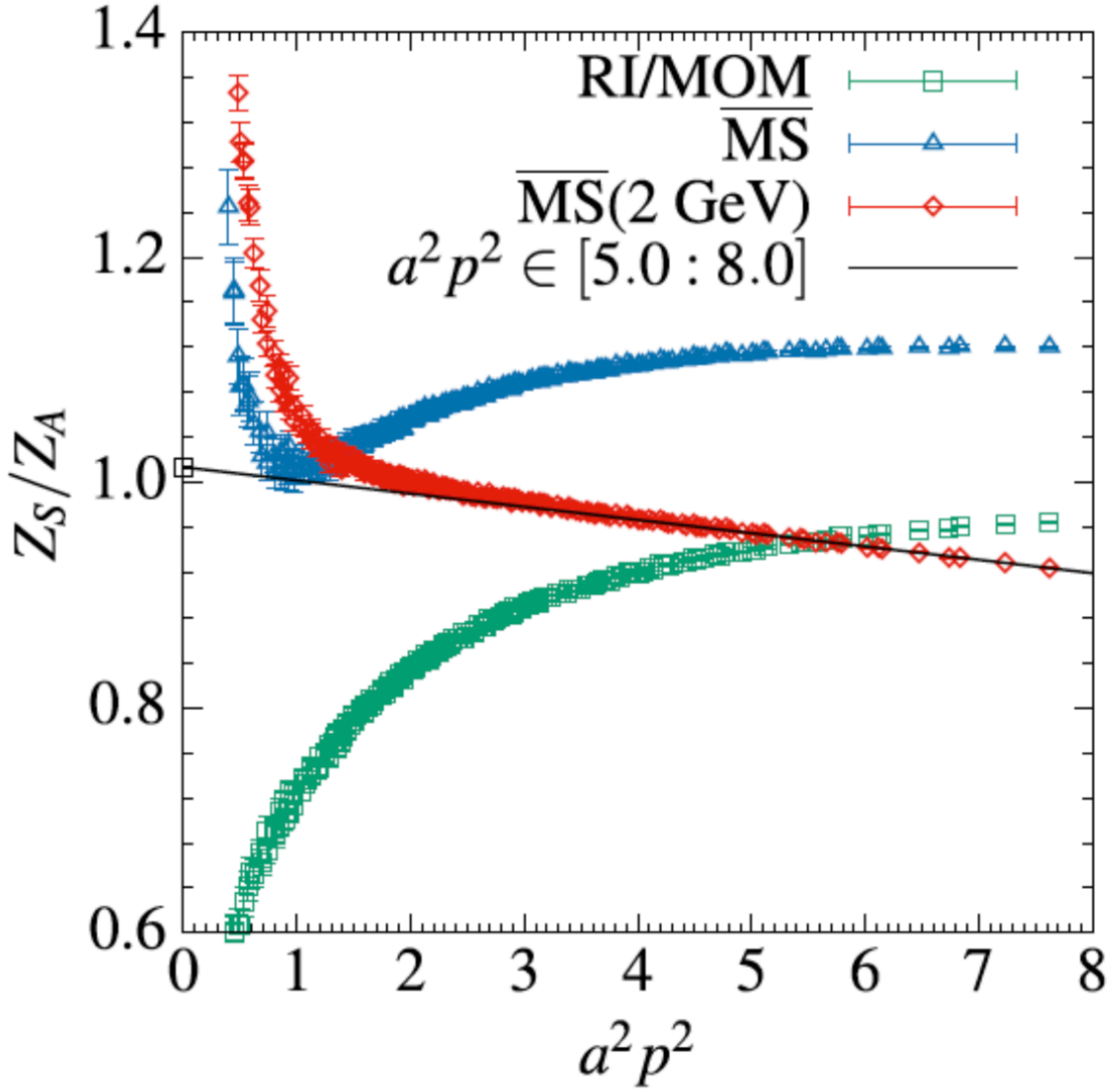
$$Z_S^{\overline{\mathrm{MS}},\mathrm{Lat}}(Q,a) = \frac{Z_S^{\mathrm{MOM,Lat}}(Q,a)}{1-\frac{\alpha_s^{\overline{\mathrm{MS}}}(Q)C_F}{4\pi}[5-\xi]} + \mathcal{O}(\alpha_s^2, a^2Q^2),$$

来转换重整化方案而避免在低能标下使用 RI/MOM 方案进行计算，然后再利用重整化群方法在 $\overline{\mathrm{MS}}$ 方案下把能标依赖重求和并演化到目标能标 μ：

$$Z_S^{\overline{\mathrm{MS}},\mathrm{Lat}}(\mu,a) = \left(\frac{\alpha_s^{\overline{\mathrm{MS}}}(\mu)}{\alpha_s^{\overline{\mathrm{MS}}}(Q)}\right)^{\frac{4}{11-\frac{2n_f}{3}}} Z_S^{\overline{\mathrm{MS}},\mathrm{Lat}}(Q,a) + \mathcal{O}(\alpha_s^2, a^2Q^2).$$

第一步：从$Z_S^{\mathrm{MOM,Lat}}(Q)$到$Z_S^{\overline{\mathrm{MS}},\ \mathrm{Lat}}(Q)$；

第二步：从$Z_S^{\overline{\mathrm{MS}},\ \mathrm{Lat}}(Q)$到$Z_S^{\overline{\mathrm{MS}},\ \mathrm{Lat}}(\mu)$。

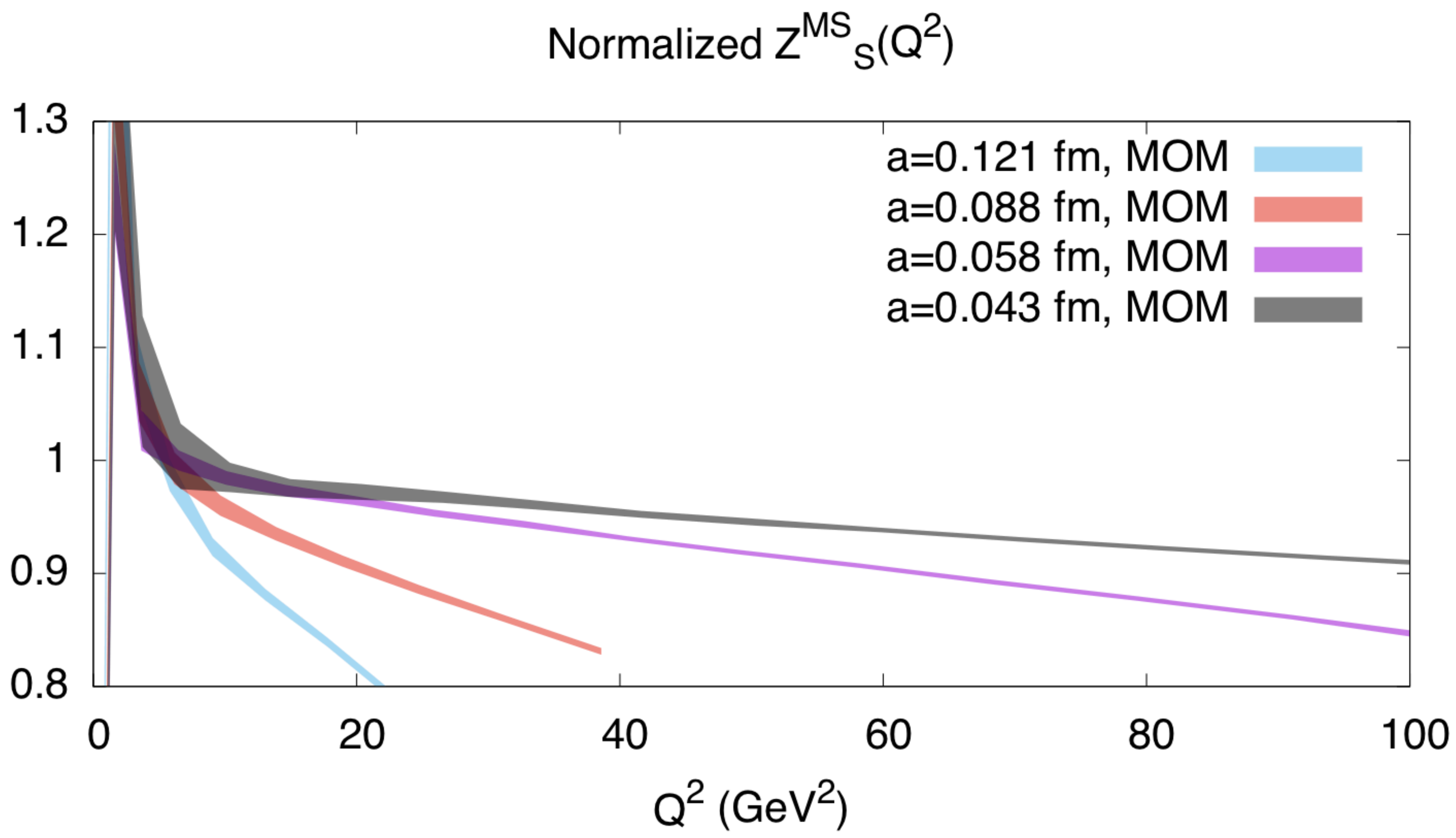


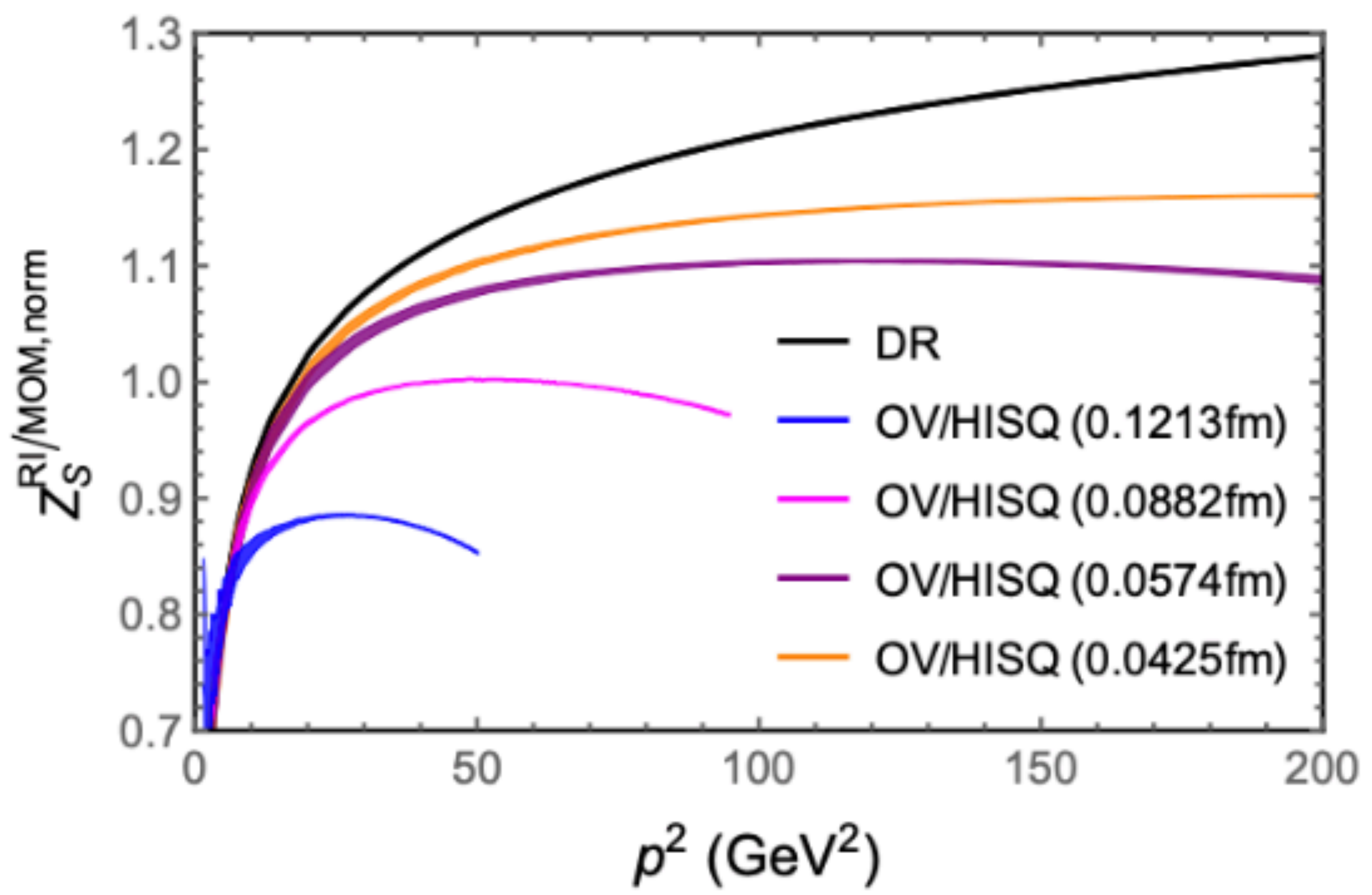
短程格点QCD

残余的Q依赖

$Z_S^{\overline{\mathrm{MS}},\,\mathrm{Lat}}(\mu)$依然对RI/MOM能标Q有剩余的依赖。这些剩余依赖的性质可以通过在不同格距上重复计算来分辨其来源。

- 在小Q上弯的行为不依赖于格距，来自于高圈修正或非微扰效应；
- 在大Q下斜的行为随格距减小而减小，是一个离散误差。





长程格点QCD

组态的马尔可夫链蒙特卡罗模拟

- 路径积分在作用量中对空间点积分，同时路径积分本身也需要对规范场的各种可能性积分。

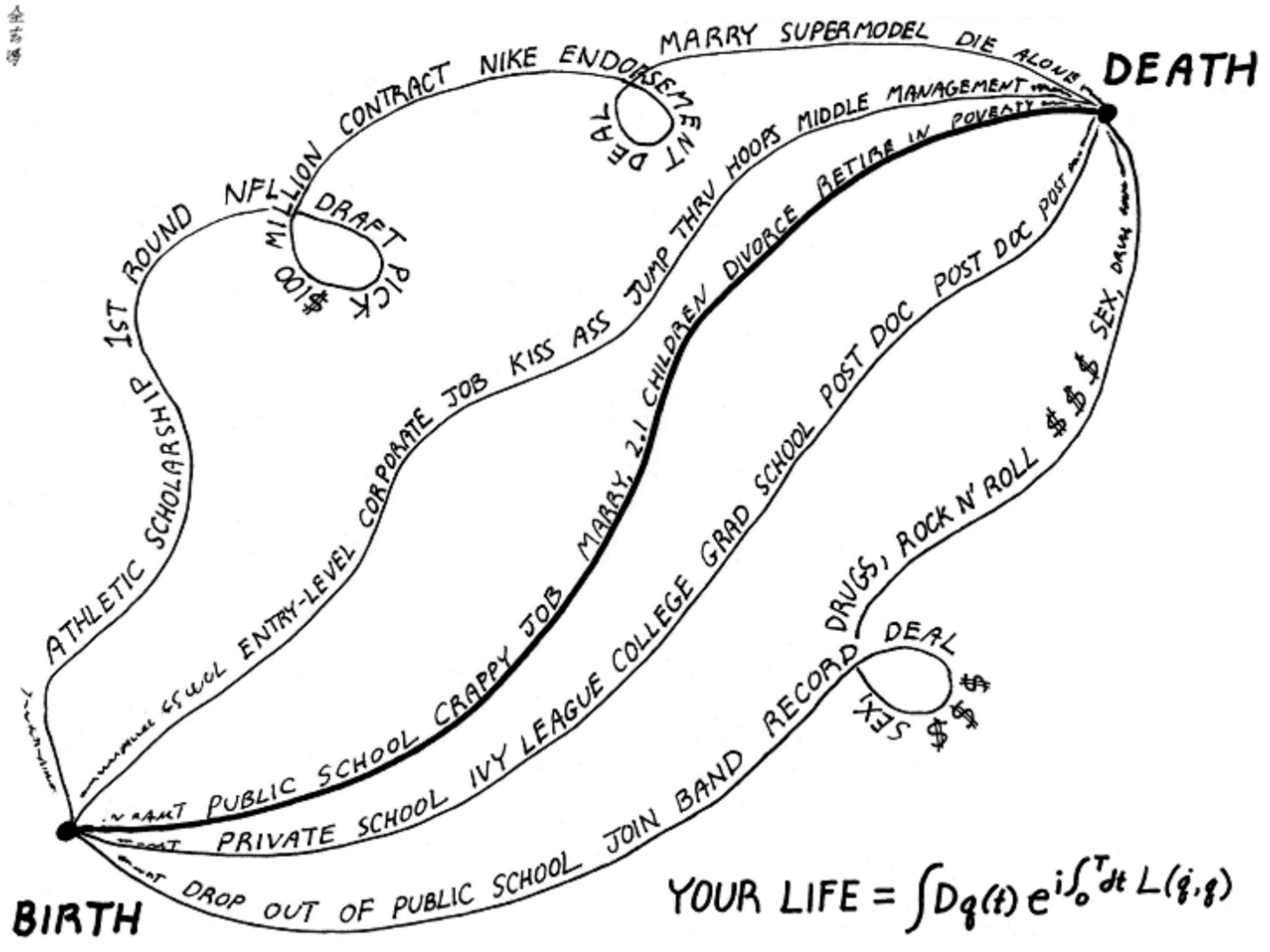
- 以标量场为例，就是要对每个空间点上的场强积分。

- 考虑一个$10^4 = 10000$的格子，每个空间点取10种可能的值，那么就有10^{10000}种可能性。

- 但是这些可能性并不是平权的，出现概率正比于$e^{-\mathcal{S}_g}$.

- 所以实践层面上说，我们可以也只能使用蒙特拉罗方法，对这些可能性做重点抽样。

$$\langle O[\phi]\rangle = \frac{\int[\Pi_y d\phi(y)]O[\phi]e^{-S[\phi]}}{\int[\Pi_y d\phi(y)]e^{-S[\phi]}} = \frac{1}{n}\sum_i O[\phi_i] + \mathcal{O}(\frac{1}{\sqrt{n}})$$

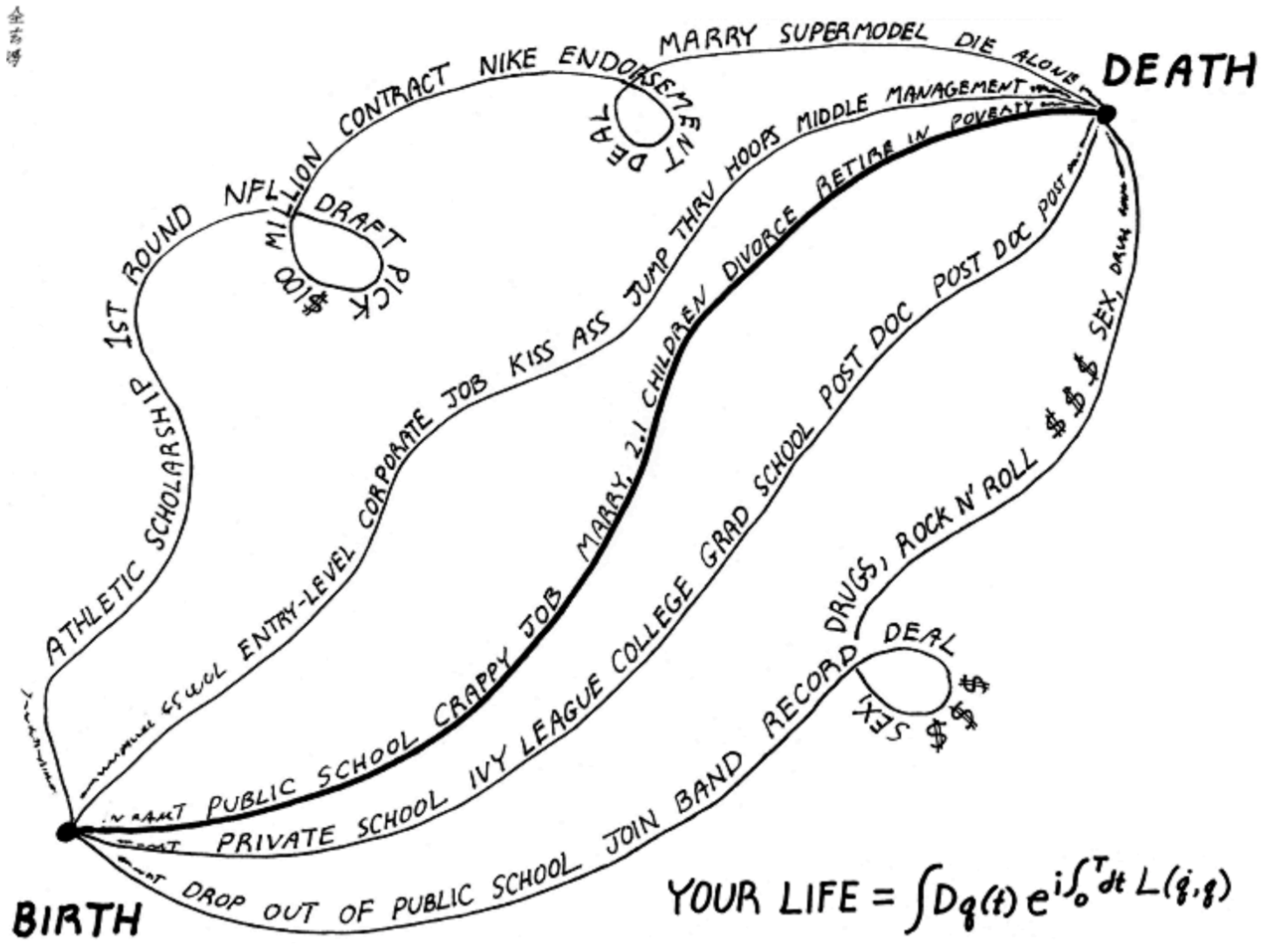


长程格点QCD

强耦合体系

$$\langle U_p \rangle = \frac{1}{Z}\int [\Pi_y \mathrm{d}U(y)] U_p[U_y] e^{-\frac{6}{g^2}\sum_z (1-\mathrm{Re}\{U_p[U_z]\})}$$

- 我们考虑一个有趣的特例：
- 对于U(1)规范场，1x1Wilson圈U_p的取值在-1到1之间；
- 当且仅当$U_p = 1$时，作用量取极小；
- 所以如果只考虑作用量取极小的贡献，$\langle U_p \rangle = 1$。这也是耦合强度无限小时的极限。
- 但是当耦合强度增大时，偏离作用量极小的$U_p < 1$的可能性就越来越多，从而使$\langle U_p \rangle$随之减小；
- 当耦合强度无穷大时，所有可能性近似于平权，这时$\langle U_p \rangle = 0$。
- 强耦合场论中，偏离“最短路径”的路径也非常重要！

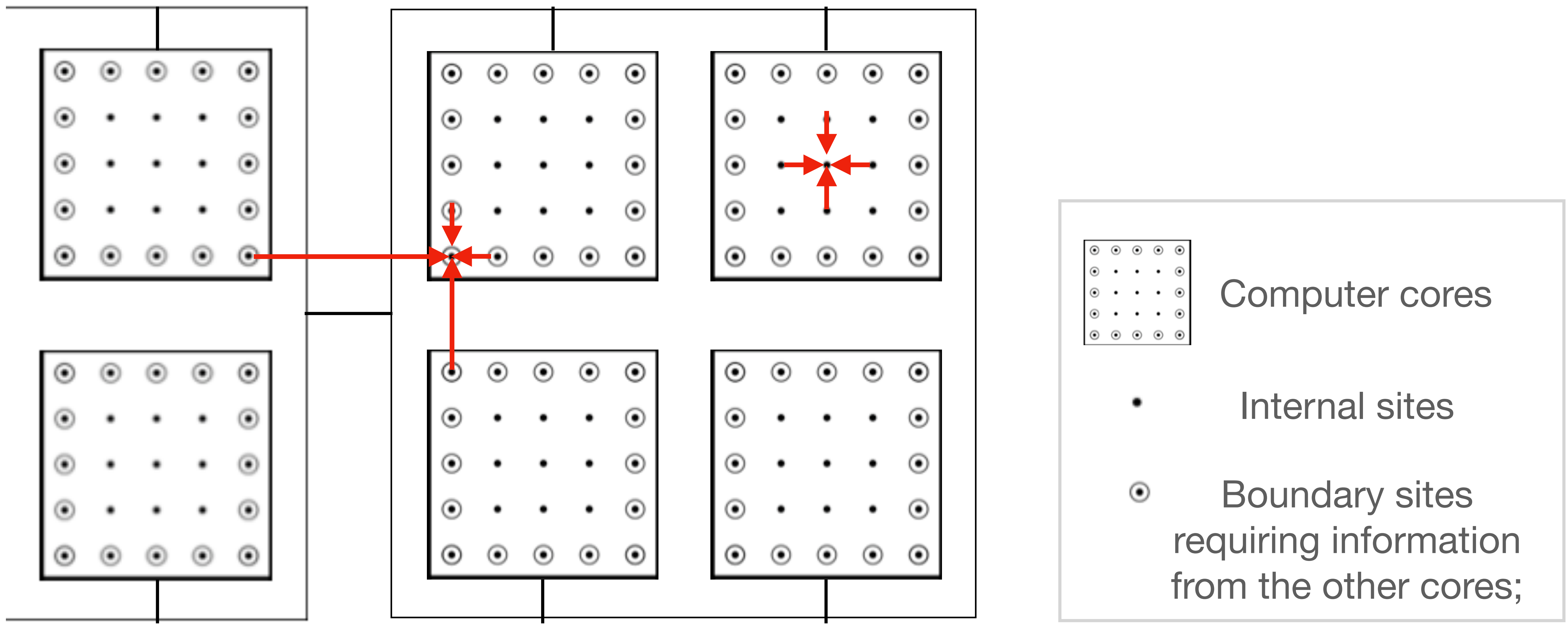


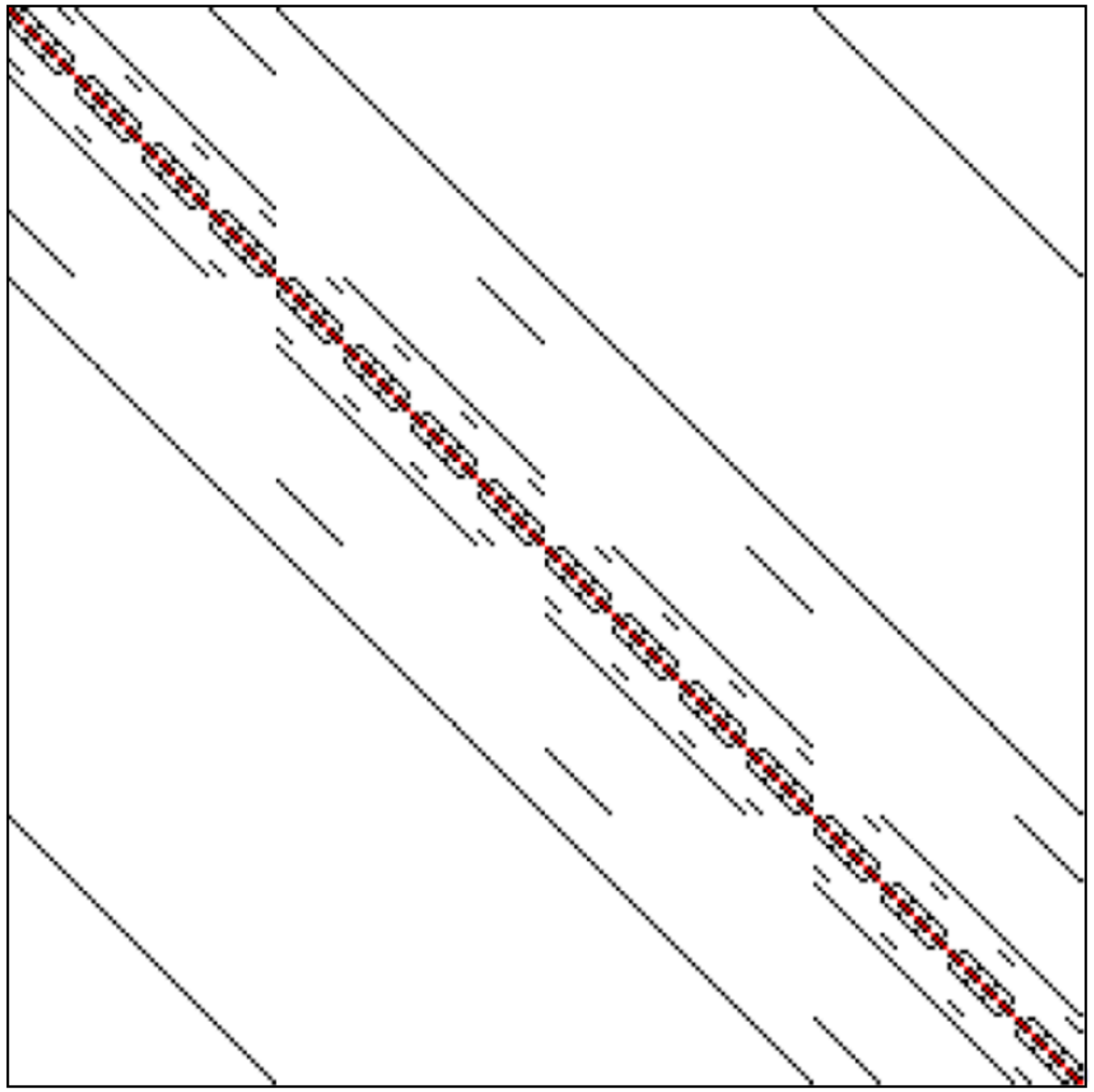
Basic unit of Lattice QCD

$$\Big(\gamma_4(\partial_\tau - igA_4)\psi + \sum(\partial_i - igA_i)\gamma_i - m\Big)\psi = 0$$

The discretized Dirac equation with the coupling with the non-abelian SU(3) gauge field:

- $\gamma_{1,2,3,4}$ are 4×4 complex matrices, $A_{1,2,3,4}$ are space-time dependent 3x3 complex matrices;
- Can be converted to a problem of sparse matrix inversion.





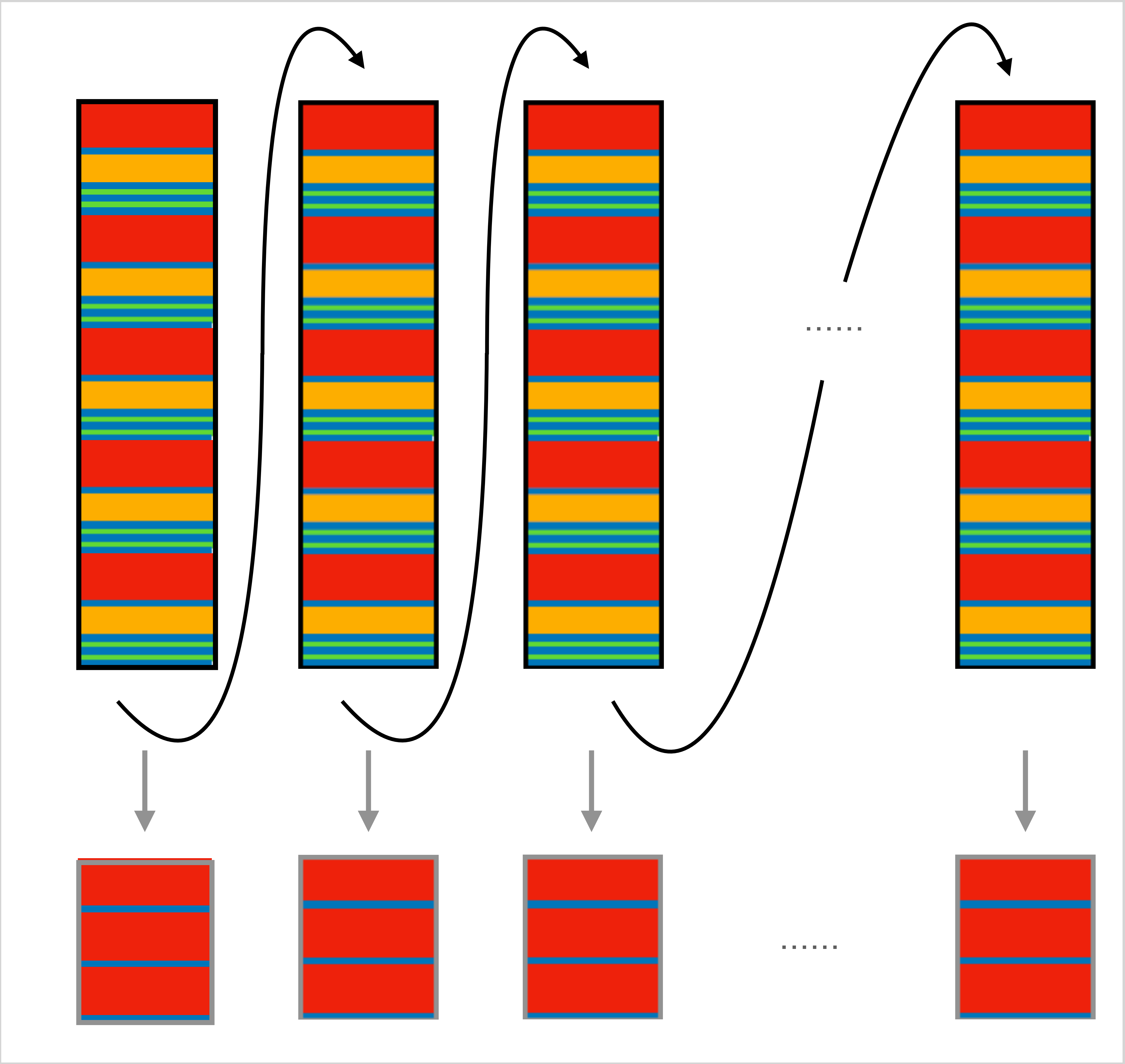
$L^3\times T = 4^3\times 4$ lattice:

- Red point: 12×12 diagonal matrix
- Black point: 12×12 sparse matrix

Basic flow of Lattice QCD

hotspot： sparse linear operation,

$$\xi = D\eta$$



Generate **configurations** using important sampling

Analysis the **configurations** to get the physical results

Linear system solver

$$\xi = D^{-1}\eta = \sum_i c_i D^i \eta + \mathcal{O}(10^{-12})$$

Linear algebra operation,

$$\xi = c_1\eta_1 + (\eta_2^\dagger \cdot \eta_3)\eta_4$$

Square root of sparse matrix,

$$\xi = (D+m_0)^{-1/2}\eta = \sum_i \frac{d_i}{D+m_0+e_i}\eta + \mathcal{O}(10^{-12})$$

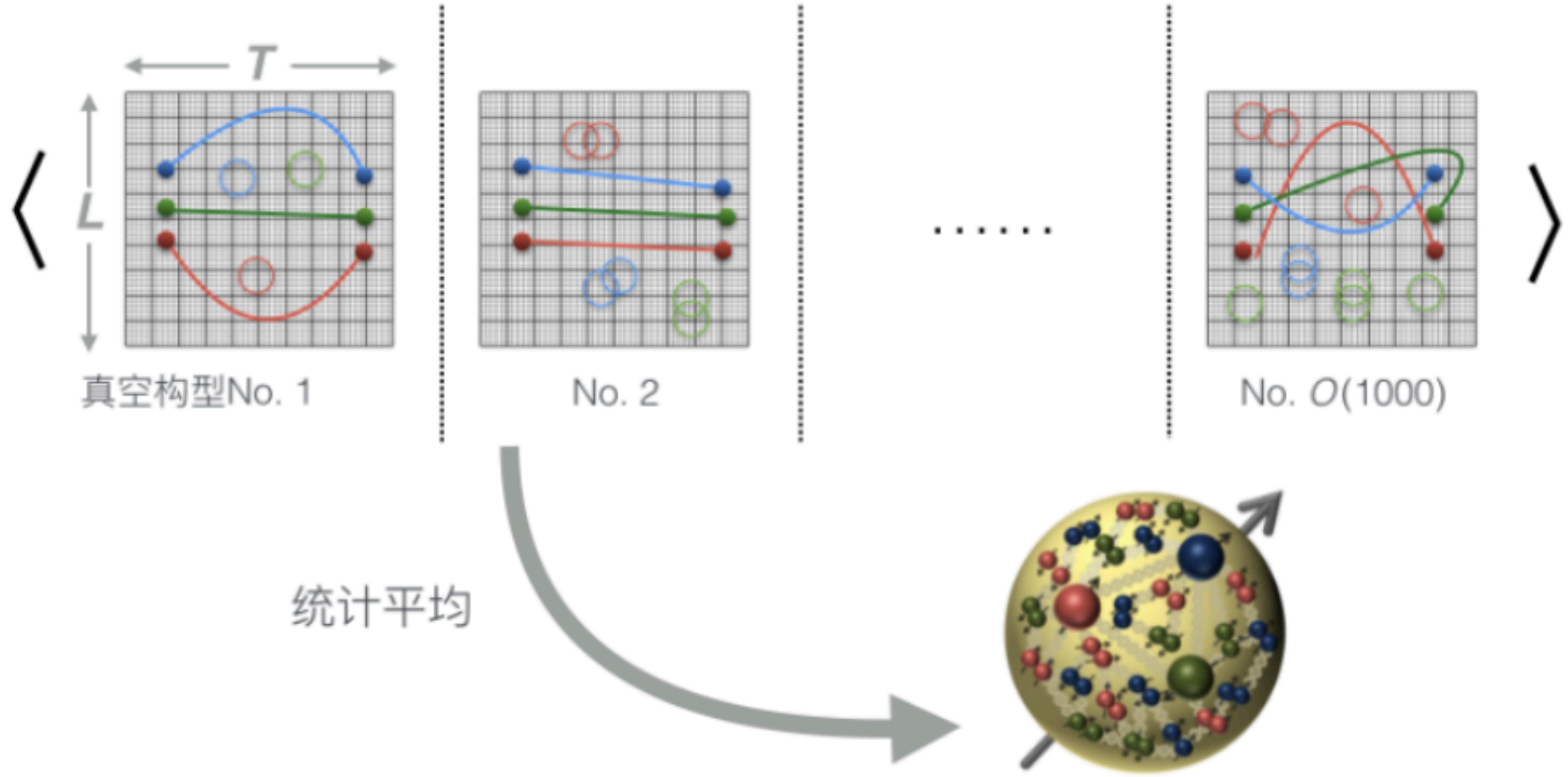
Derivative of sparse matrix,

$$\xi = \frac{\partial D}{\partial U}\eta = D_1 D_2 D_3 \eta$$

- The major hotspot is linear system solver；
- But after the acceleration of this hotspot, linear algebra operation, square root and derivative of sparse matrix will be the bottlenecks of the performance.
- Configurations are the foundation of all the physical analysis!

长程格点QCD

组态平均

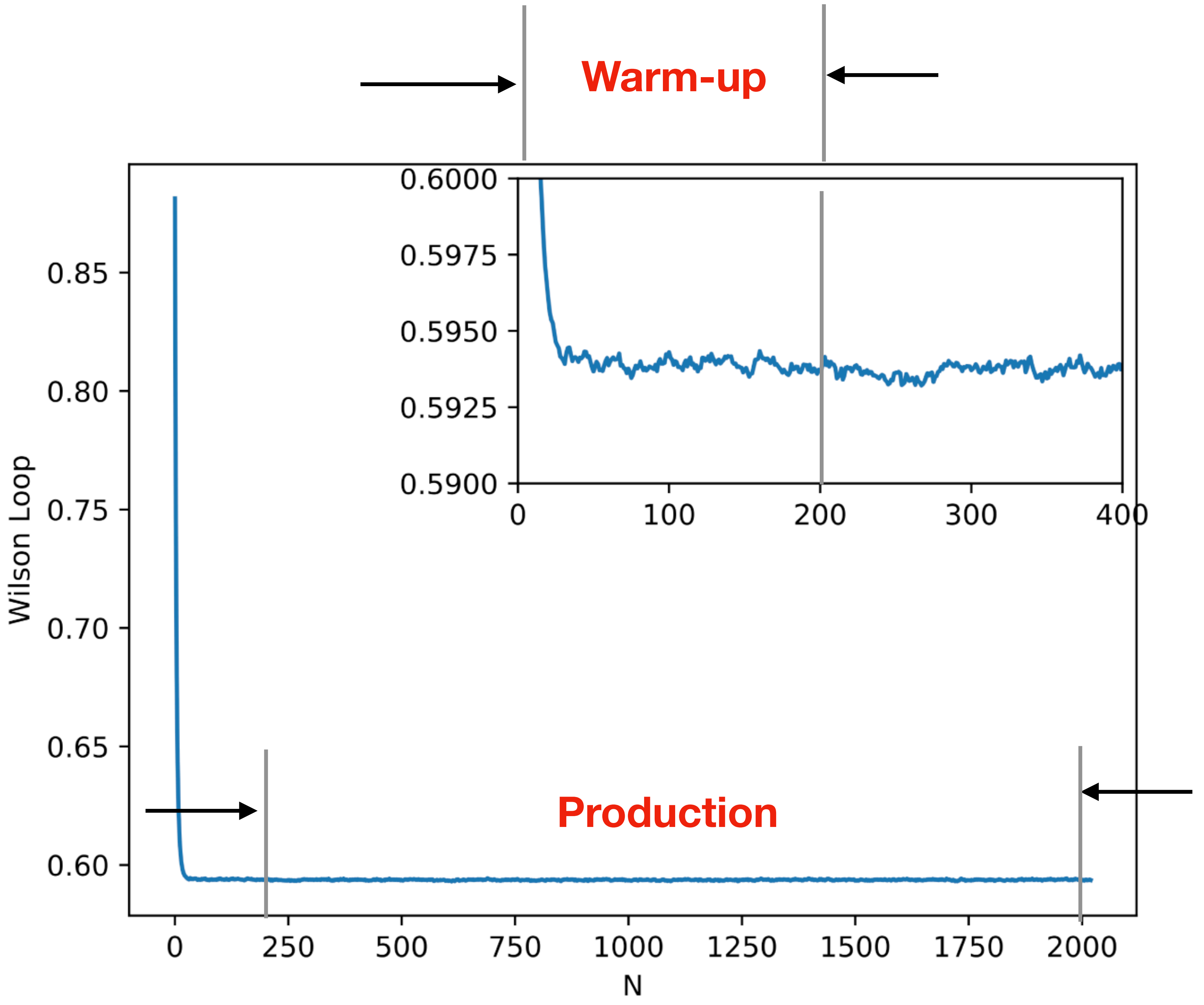


$$\langle O[U]\rangle = \frac{\int [\Pi_y \mathrm{d}U(y)] O[U] e^{-S[U]}}{\int [\Pi_y \mathrm{d}U(y)] e^{-S[U]}} = \frac{1}{n}\sum_i O[U] + \mathcal{O}(\frac{1}{\sqrt{n}})$$

在每个组态上重复计算并统计平均，就可以对完整QCD路径积分的估计值。

长程格点QCD

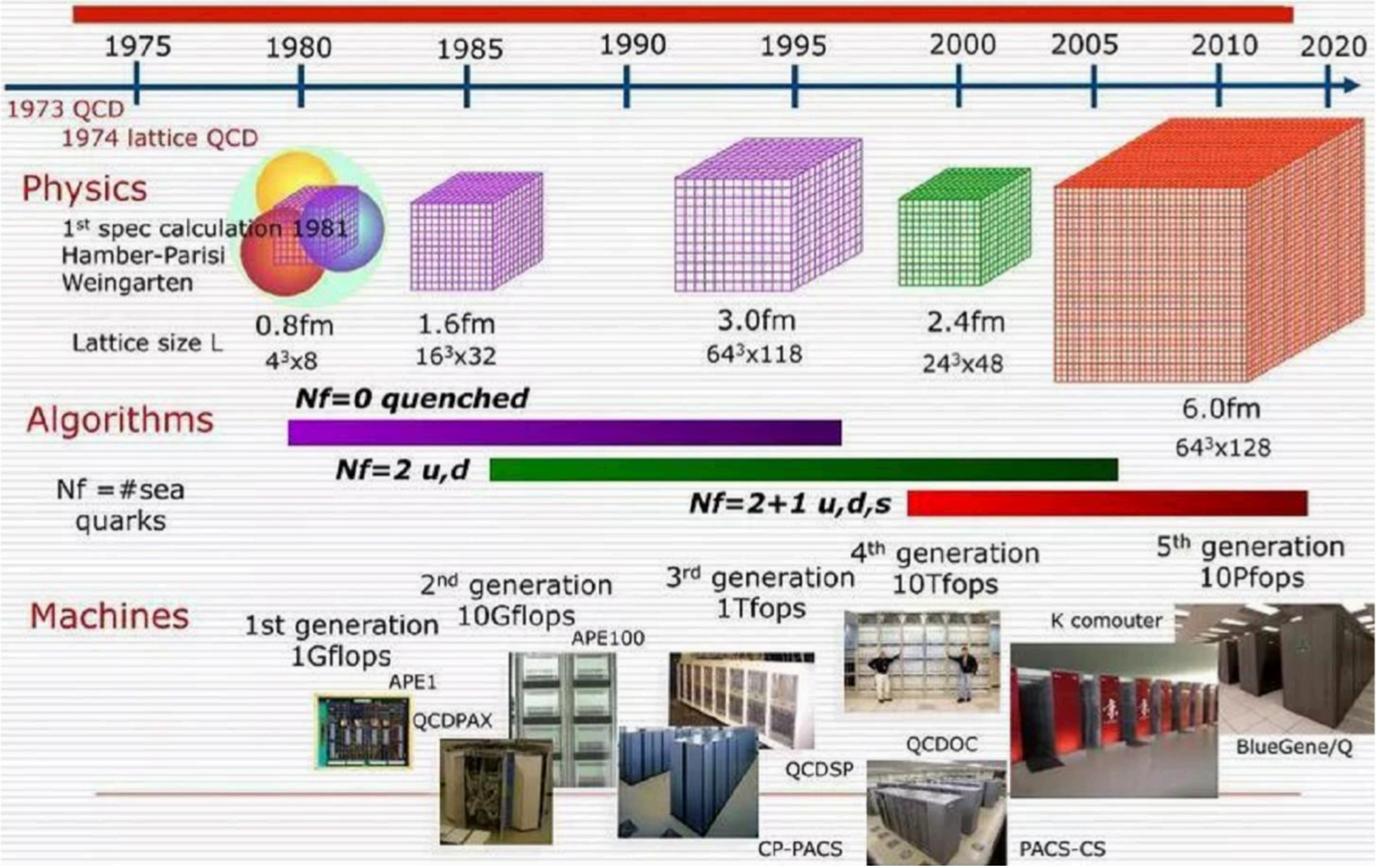
生成真实组态的成本



- Case 1:
  - Clover+Symanzik,
  - 24^3x72, a=0.108 fm, m_π=300 MeV,
  - 8 V100 GPUs:
    - One week for warn-up;
    - Another week for 200 configurations (5 traj. per conf.)
    - And 13 GB storage.

- Case 2:
  - Mobius DWF+Iwasaki,
  - 96^3x192, a=0.071 fm, m_π=140 MeV,
  - 512 V100 GPUs:
    - One year for warn-up;
    - Another year for 200 configurations (5 traj. per conf.)
    - And 2,278 GB storage.

Four decades of Lattice QCD



附表：美国目前的高能物理高性能计算应用情况及未来的E级计算需求（以CPU核小时为单位做数量级估计）（ ASCR/HEP Exascale requirement review report， http://arxiv.org/pdf/1603.09303.pdf)

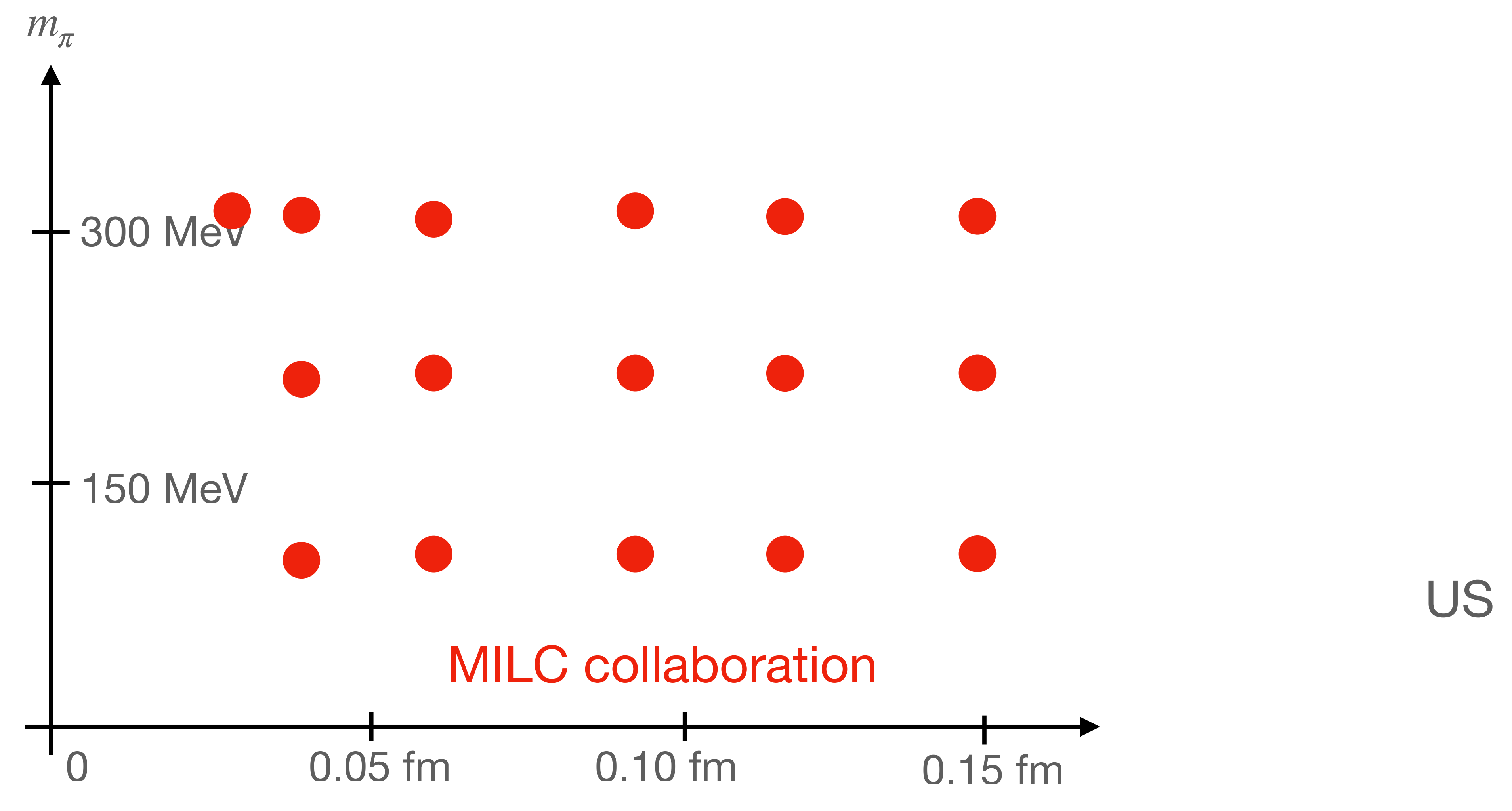
Computational Task	Current Usage	2025 Usage	Current Storage (Disk)	2025 Storage (Disk)	2025 Network Requirements (WAN)
Accelerator Modeling	$\sim$ 10M – 100M core-hrs/yr	$\sim$ 10G – 100G core-hrs/yr			
Computational Cosmology	$\sim$ 100M – 1G core-hrs/yr	$\sim$ 100G – 1000G core-hrs/yr	~10PB	>100PB	300Gb/s (burst)
Lattice QCD	~1G core-hrs/yr	$\sim$ 100G – 1000G core-hrs/yr	~1PB	>10PB	
Theory	$\sim$ 1M – 10M core-hrs/yr	$\sim$ 100M – 1G core-hrs/yr			
Cosmic Frontier Experiments	$\sim$ 10M – 100M core-hrs/yr	$\sim$ 1G – 10G core-hrs/yr	~1PB	10 – 100PB	
Energy Frontier Experiments	$\sim$ 100M core-hrs/yr	$\sim$ 10G – 100G core-hrs/yr	~1PB	>100PB	300Gb/s
Intensity Frontier Experiments	$\sim$ 10M core-hrs/yr	$\sim$ 100M – 1G core-hrs/yr	~1PB	10 – 100PB	300Gb/s

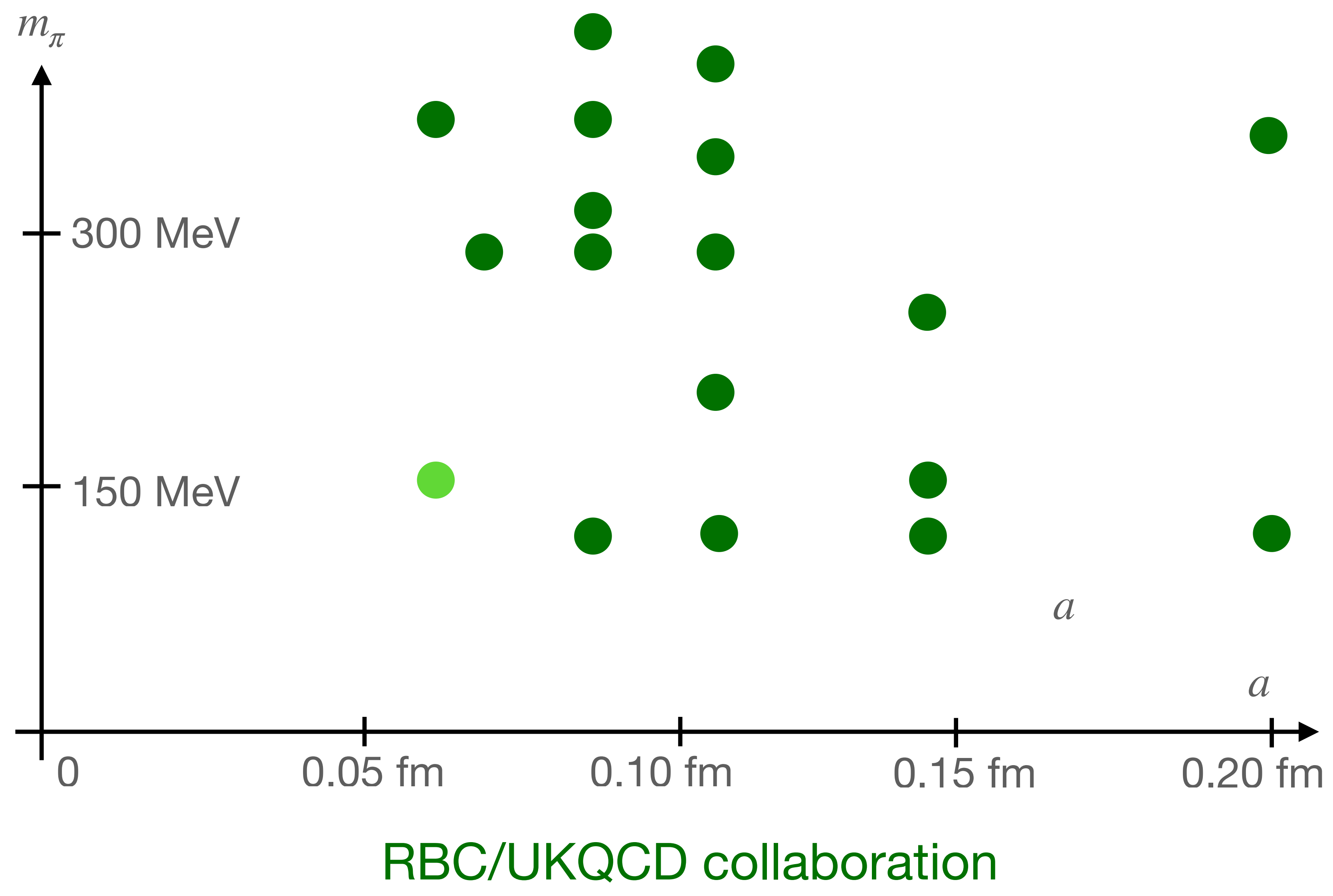
现在： 1G core-hrs/yr 意味着 **10亿CPU核小时/年=百万核机器运行1000小时**

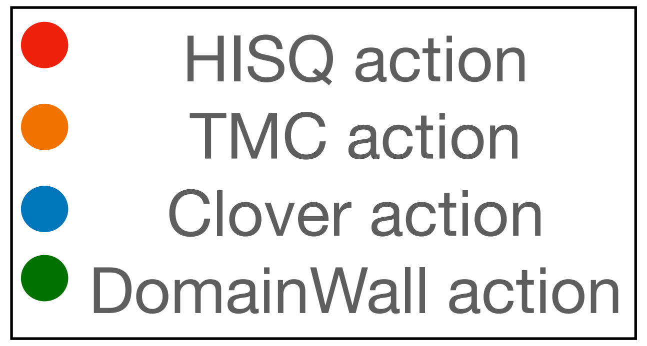
2025年：再乘10-100倍！！！

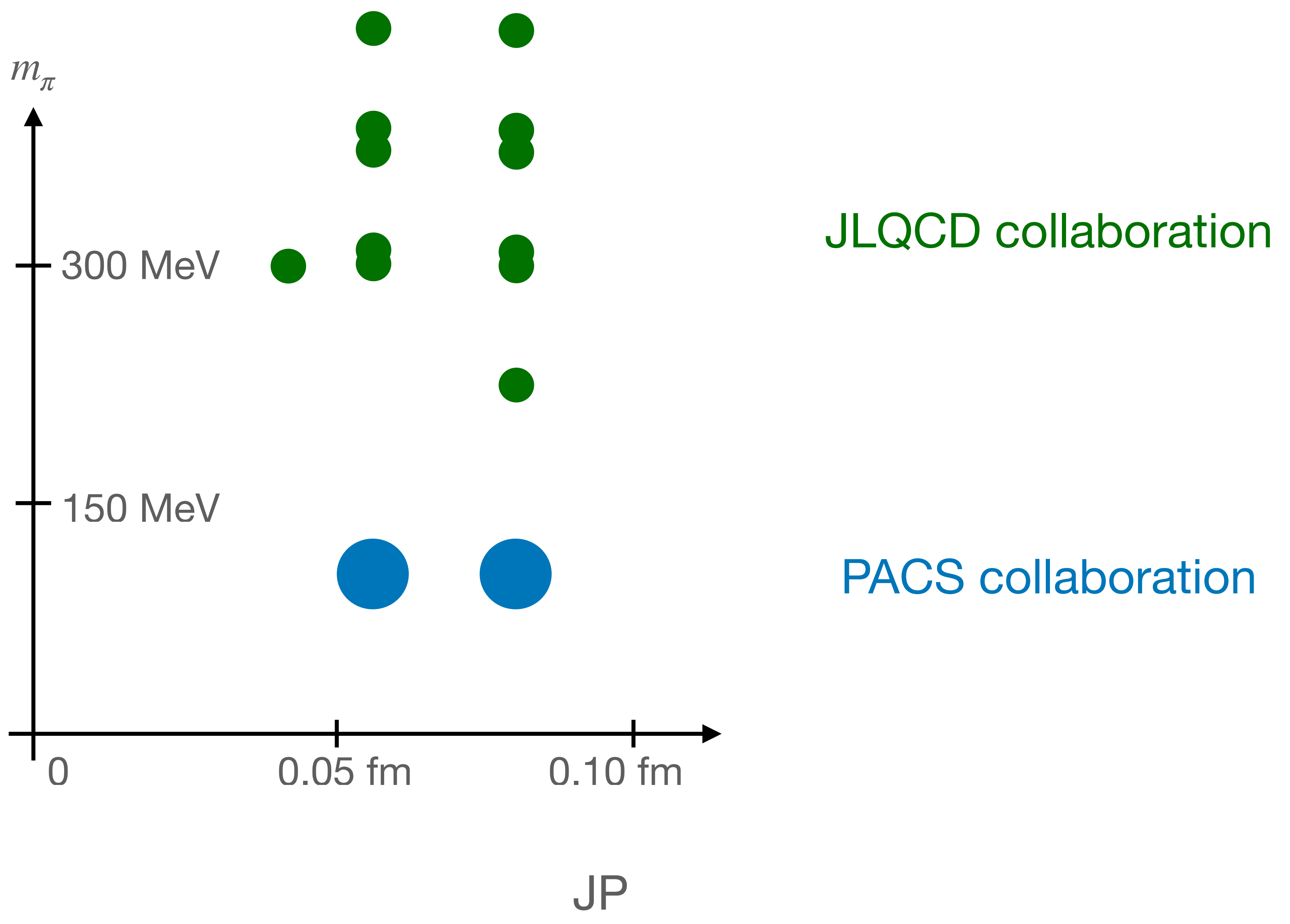
长程格点QCD

世界范围内的组态

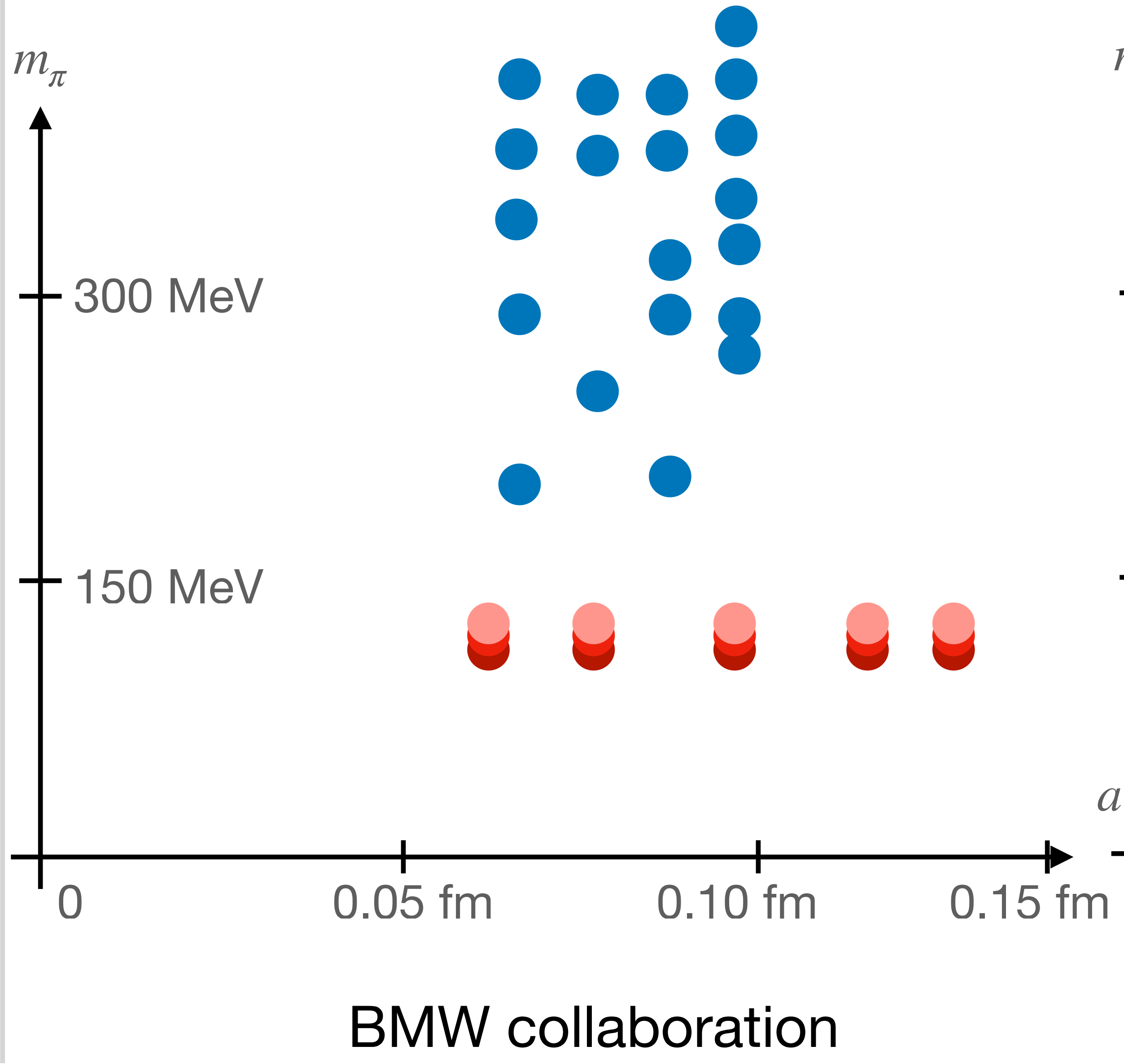


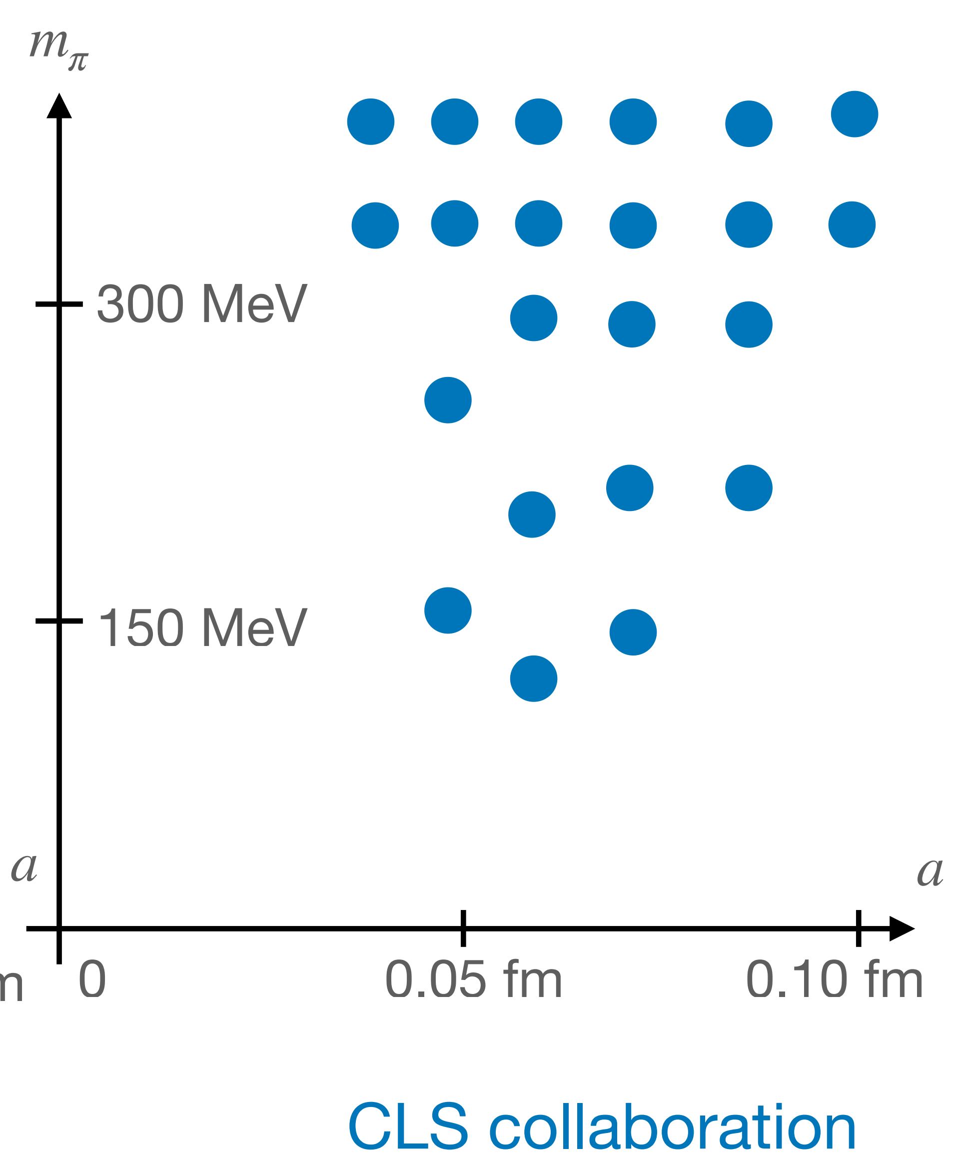


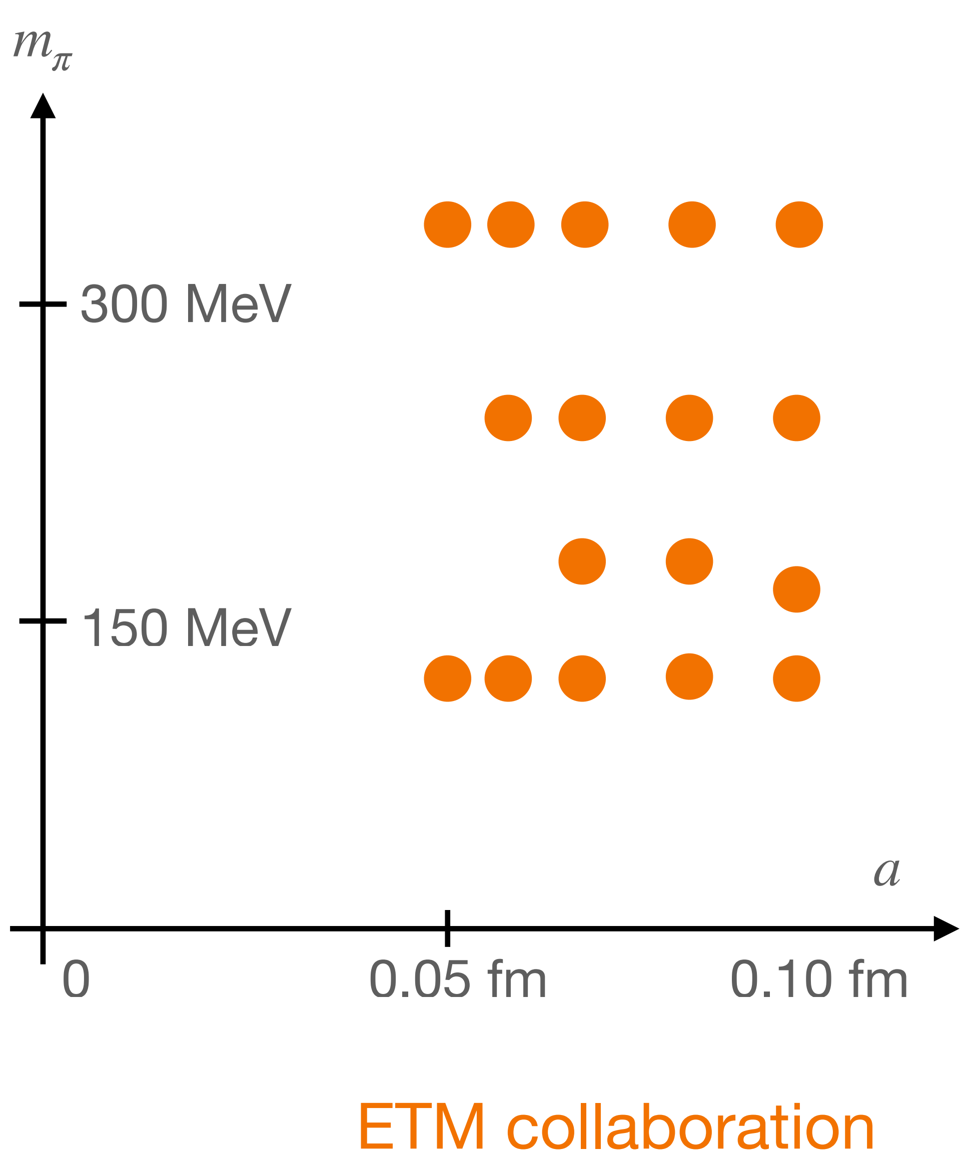




EU



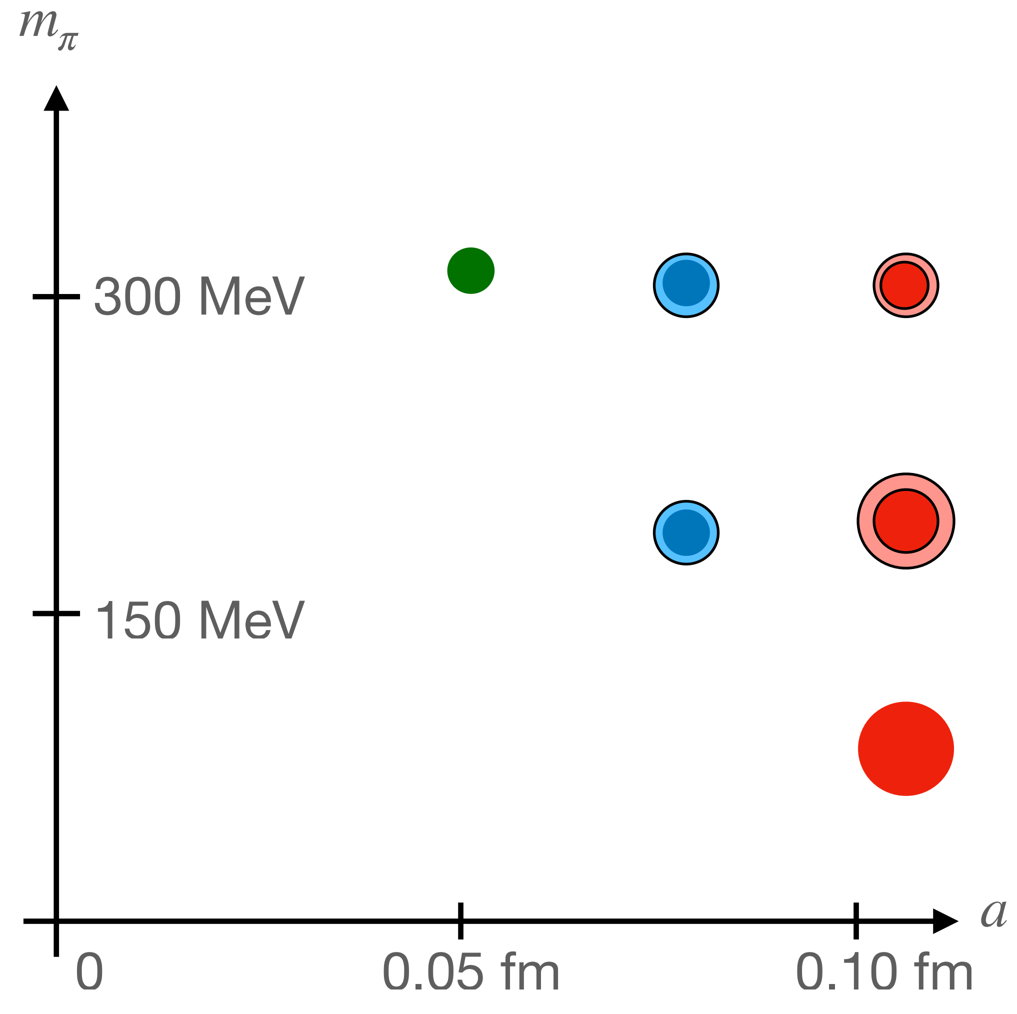


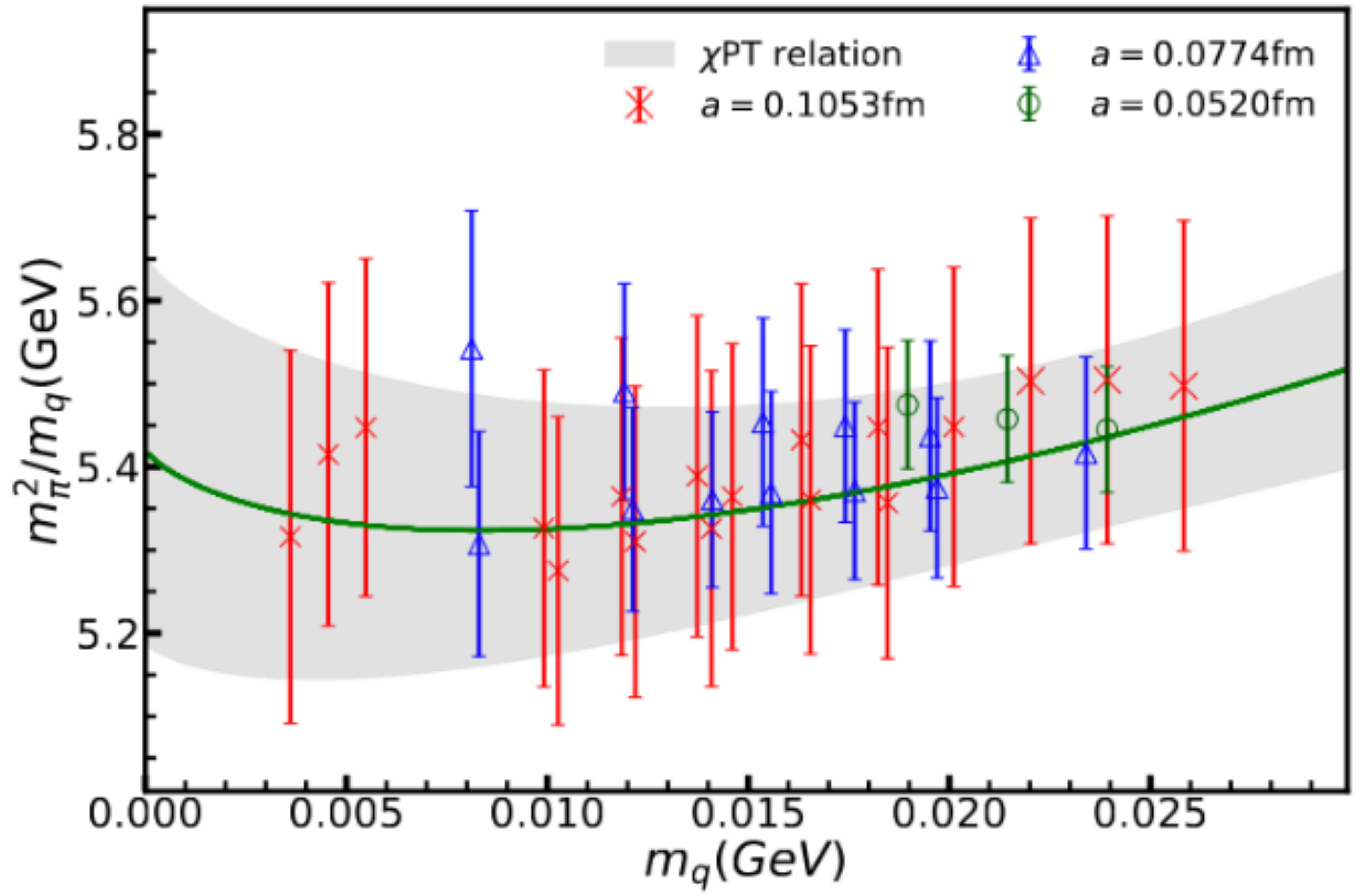


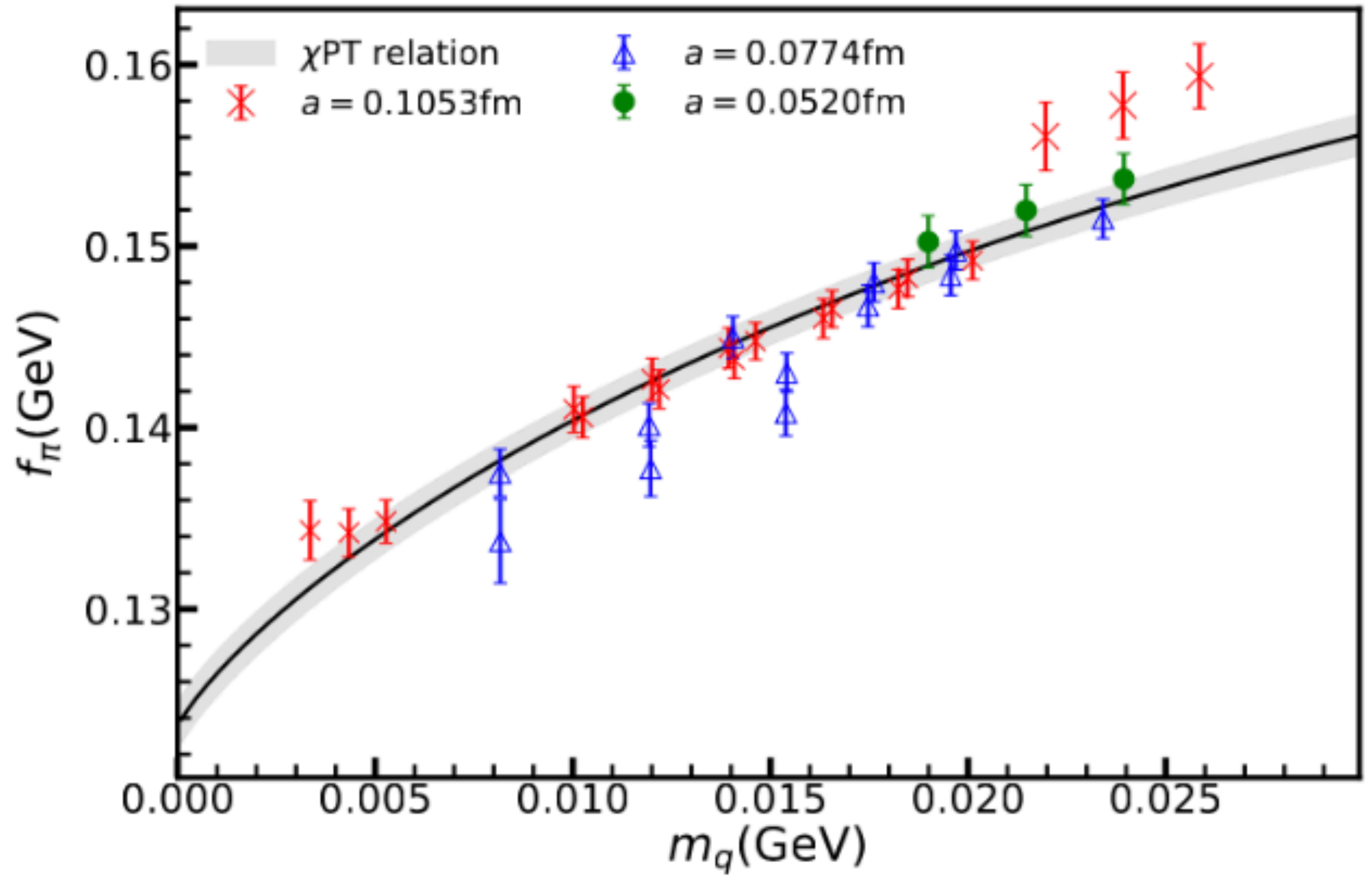
长程格点QCD

中国的泛用性组态

CLQCD

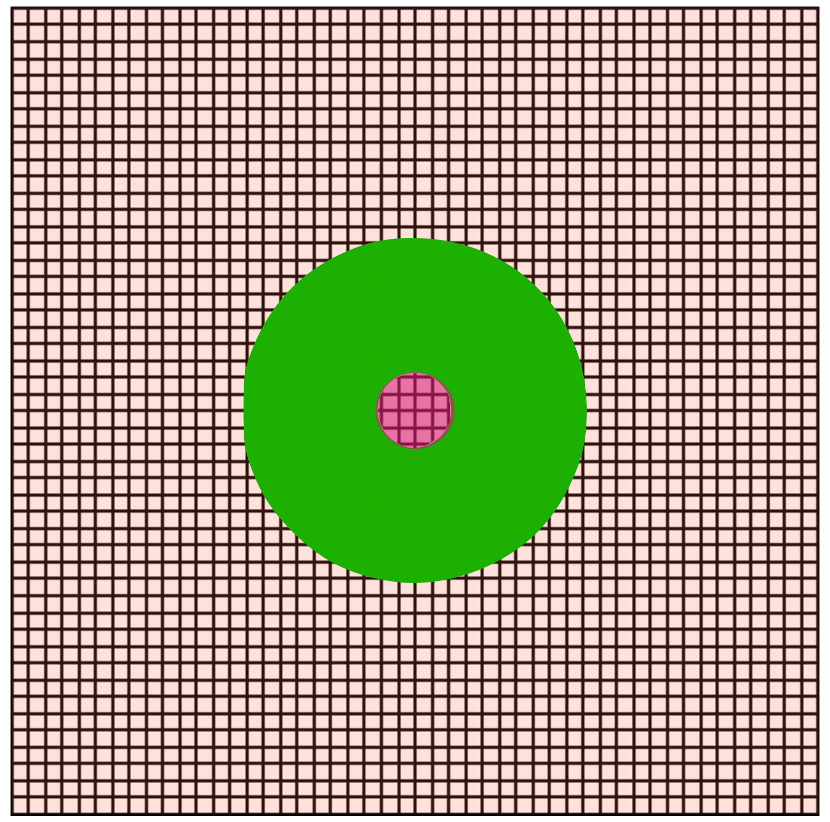


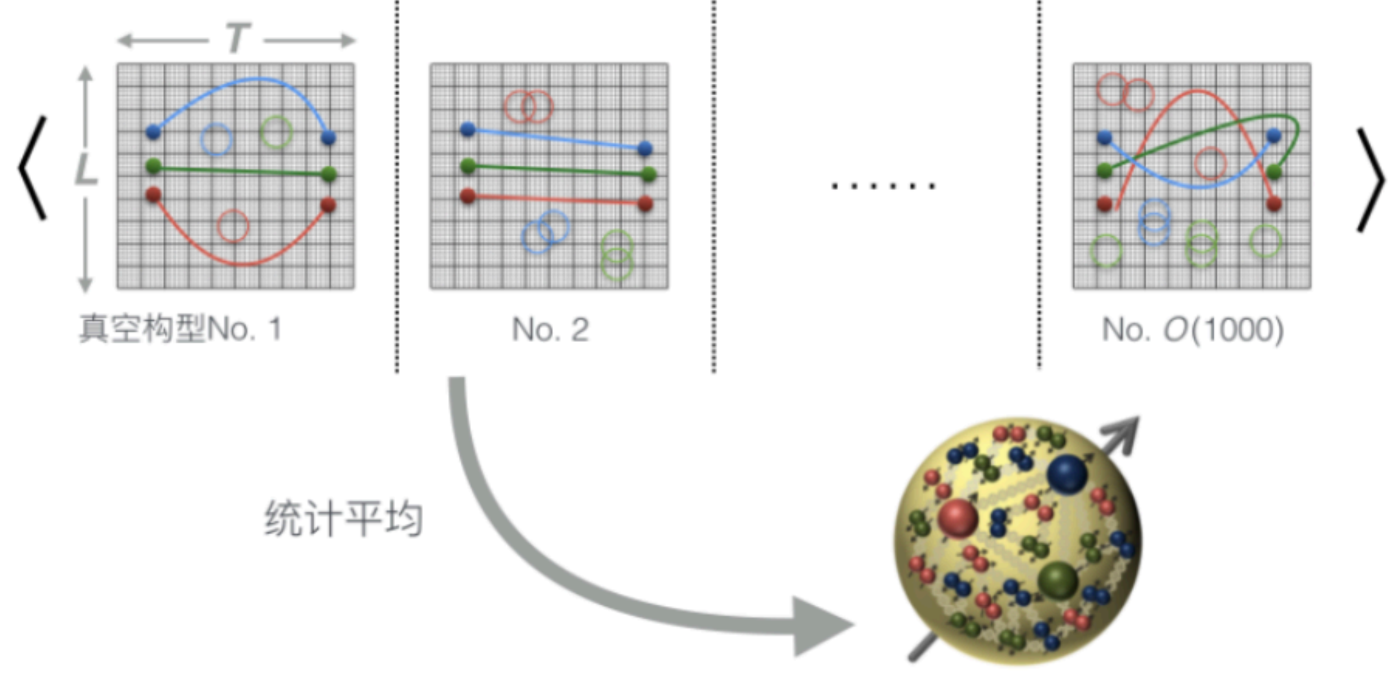




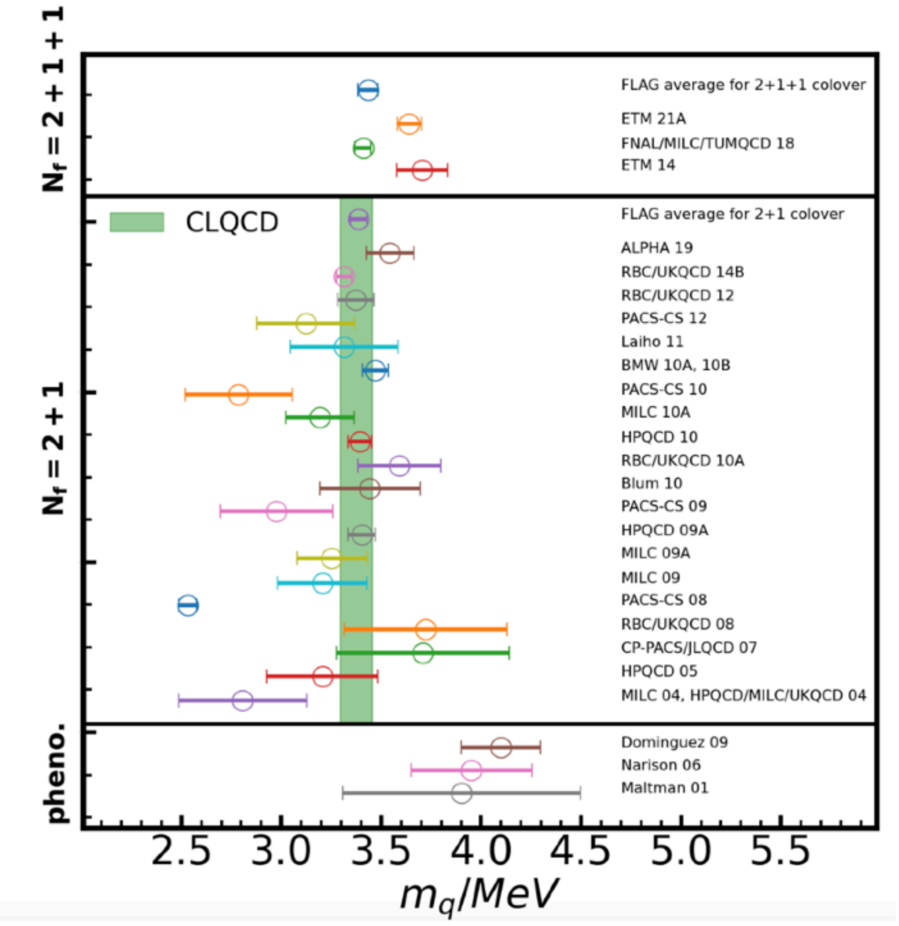
	C24P34	C24P31	C32P31	C32P23	C48P23	C48P14	F32P31	F48P31	F32P21	F48P21	H48P33
$\tilde{L}^3 \times \tilde{T}$	$24^3 \times 64$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 96$	$32^3 \times 96$	$48^3 \times 96$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 144$
$\hat{\beta}$			6.20					6.41			6.72
a (fm)			0.10530(18)					0.07746(18)			0.05187(26)
$\tilde{m}_l^b$	-0.2770	-0.2770	-0.2770	-0.2790	-0.2790	-0.2825	-0.2295	-0.2295	-0.2320	-0.2320	-0.1850
$\tilde{m}_s^b$	-0.2310	-0.2400	-0.2400	-0.2400	-0.2400	-0.2310	-0.2050	-0.2050	-0.2050	-0.2050	-0.1700
m_l^R (MeV)	22.90(19)	16.94(12)	17.35(11)	10.55(11)	10.27(10)	3.638(83)	18.54(12)	18.511(92)	8.58(16)	8.587(89)	19.667(49)
m_s^R (MeV)	109.39(16)	86.37(09)	87.01(10)	84.69(07)	84.36(08)	101.41(05)	90.19(10)	90.45(08)	87.84(09)	87.88(07)	94.55(04)
m_π (MeV)	340.5(1.7)	292.7(1.2)	292.4(1.1)	228.0(1.2)	225.61(86)	135.5(1.6)	303.2(1.3)	303.44(86)	210.9(2.2)	207.2(1.1)	321.44(79)
m_{η_s} (MeV)	748.7(0.9)	657.4(0.6)	658.0(0.7)	643.5(0.5)	643.2(0.5)	706.3(0.3)	681.6(0.9)	679.9(0.6)	665.6(0.7)	667.7(0.7)	709.0(0.5)

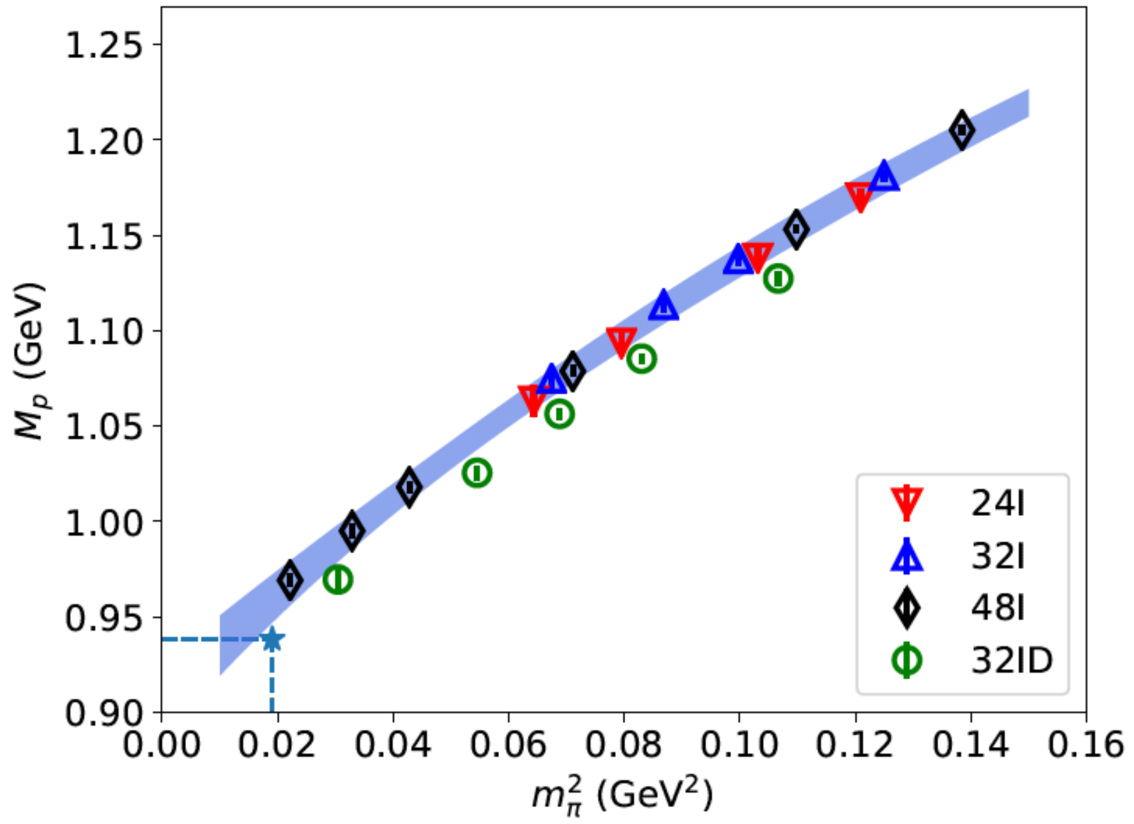
格点量子色动力学基础



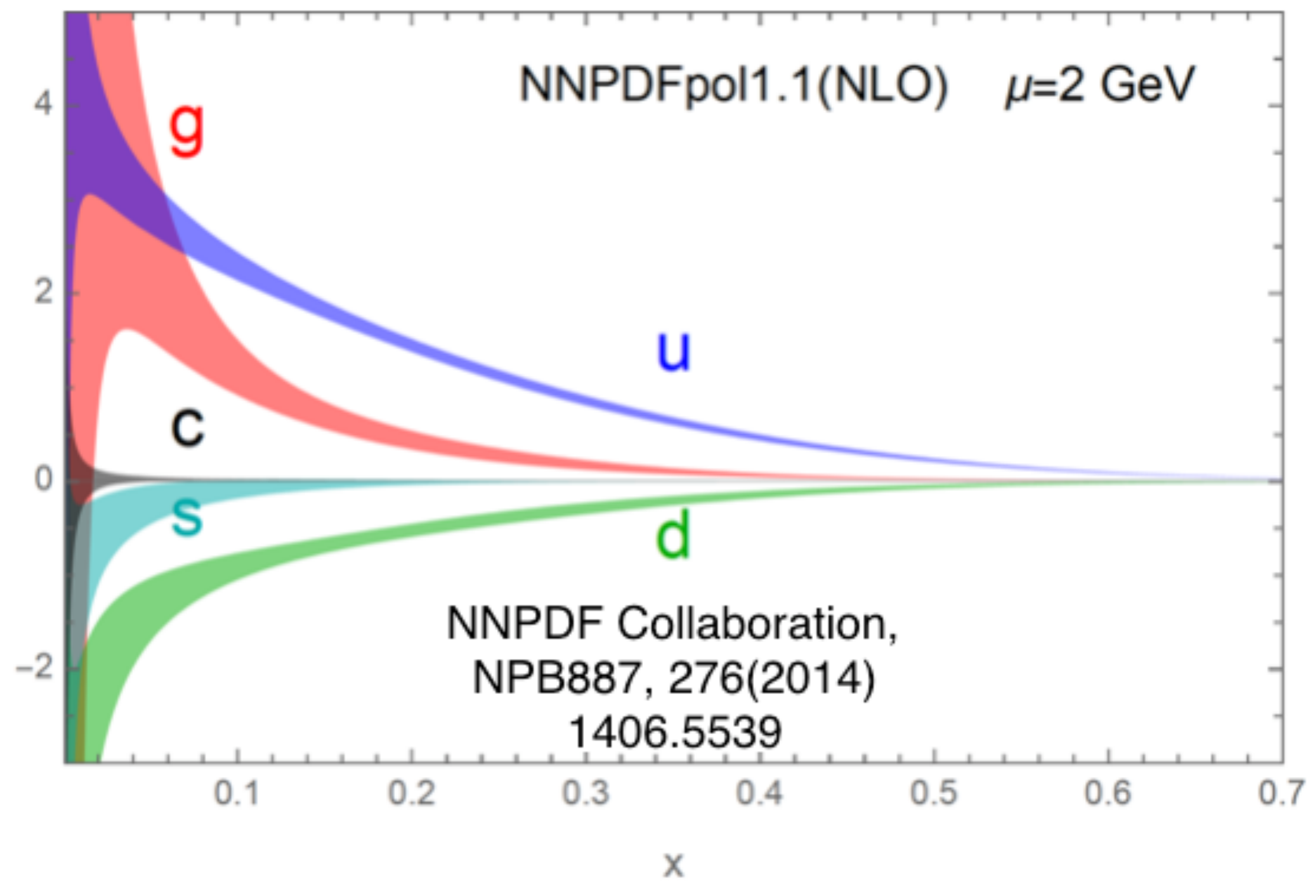


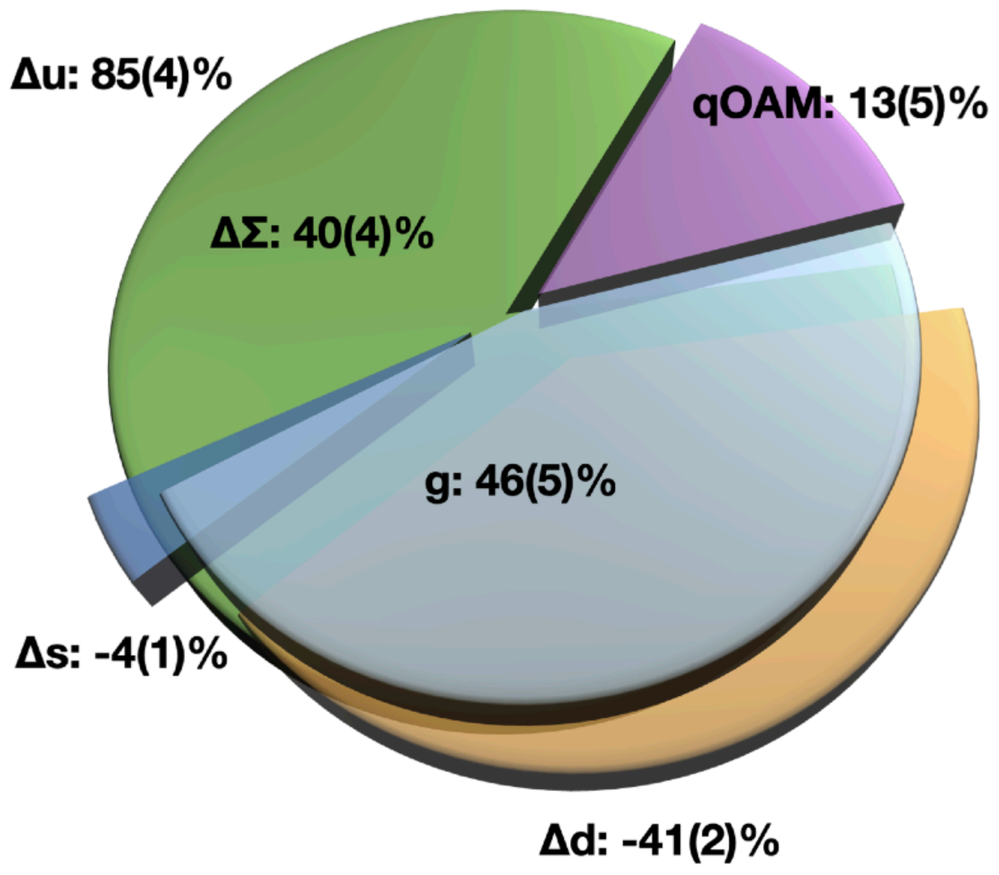
强子质量





强子自旋





Hadron mass from Lattice QCD

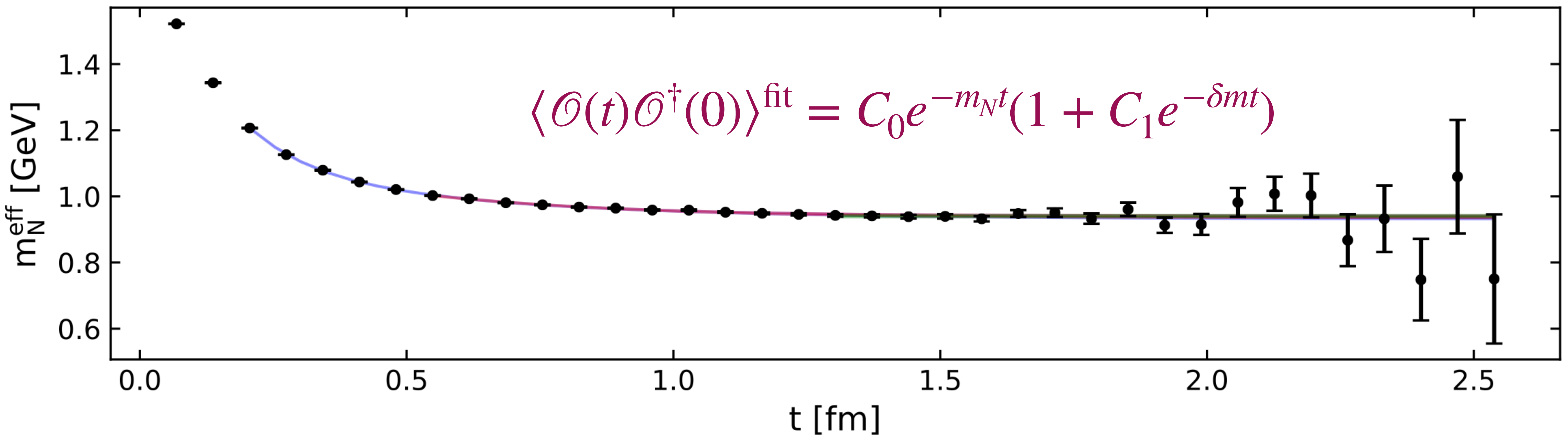
- From the time order product ($\mathcal{O} = \bar{\psi}\gamma_5\psi$):

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_n \langle \mathcal{O}(t)\,|\,n\rangle \frac{e^{-E_n t}}{2E_n}\langle n\,|\,\mathcal{O}^\dagger(0)\rangle \xrightarrow[t\to\infty]{} \frac{|\langle \mathcal{O}(t)\,|\,0\rangle|^2}{2E_0} e^{-E_0 t}$$

- From the path integral ($S(x;y) = (D(x;y)+m)^{-1}$):

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_{\vec{x}} \langle \mathrm{Tr}[S(\vec{0},0;\vec{x},t)\gamma_5 S(\vec{x},t;\vec{0},0)\gamma_5]\rangle = \sum_{\vec{x}} \langle \mathrm{Tr}[S^\dagger(\vec{x},t;\vec{0},0)S(\vec{x},t;\vec{0},0)]\rangle$$

- All the ground state hadron masses can be obtained with different $\mathcal{O}$ and m.



$$m_N^{\mathrm{eff}} = \frac{1}{a}\log\frac{\langle \mathcal{O}(t-a)\mathcal{O}^\dagger(0)\rangle}{\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle}$$

C. Alexandrou, et,al. ETMC, PRD104(2021)074515

The light quark masses

From lattice QCD

P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27 \text{ MeV} = m_{p,\text{QCD}} + 1.00(16) \text{ MeV} + \ldots;$

Sz. Borsanyi, et,al. Science347(2015)1452

- $m_n = 939.57 \text{ MeV};$
- $m_\pi^0 = 134.98 \text{ MeV};$

$(m_{p,\text{QCD}} + m_n)/2 = 938.4(1) \text{ MeV}$

- $m_\pi^+ = 139.57 \text{ MeV} = m_\pi^0 + 4.53(6) \text{ MeV} + \ldots;$

X. Feng, et,al. PRL128(2022)062003

- $m_K^0 = 497.61(1) \text{ MeV} = m_{K,\text{QCD}}^0 + 0.17(02) \text{ MeV} + \ldots;$
- $m_K^+ = 493.68(2) \text{ MeV} = m_{K,\text{QCD}}^+ + 2.07(15) \text{ MeV} + \ldots$

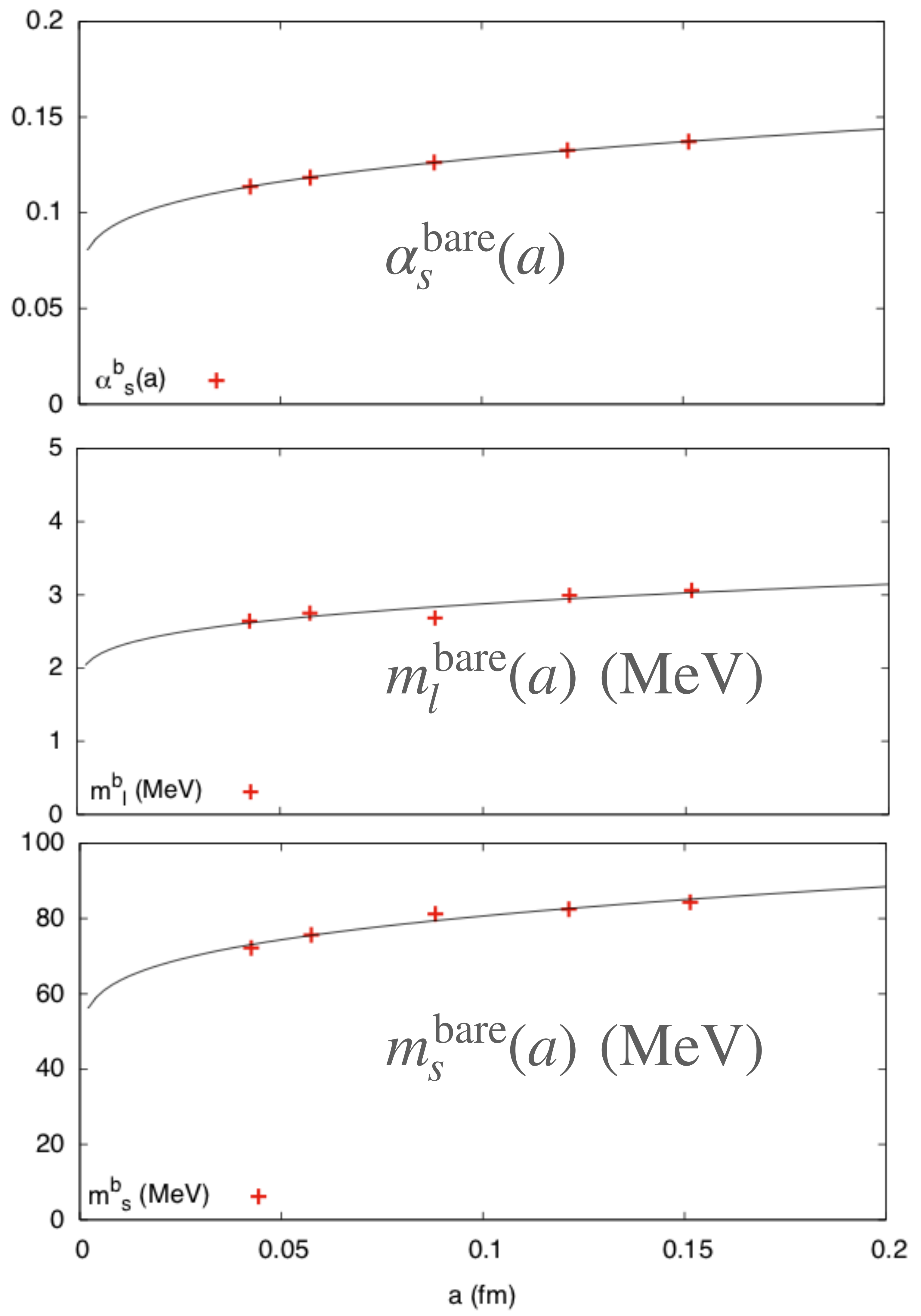
D. Giusti, et,al. PRD95(2017)114504

α_s^{bare}

$(m_u^{\text{bare}} + m_d^{\text{bare}})/2$ and a (lattice spacing)

m_u^{bare}, m_d^{bare} and m_s^{bare}

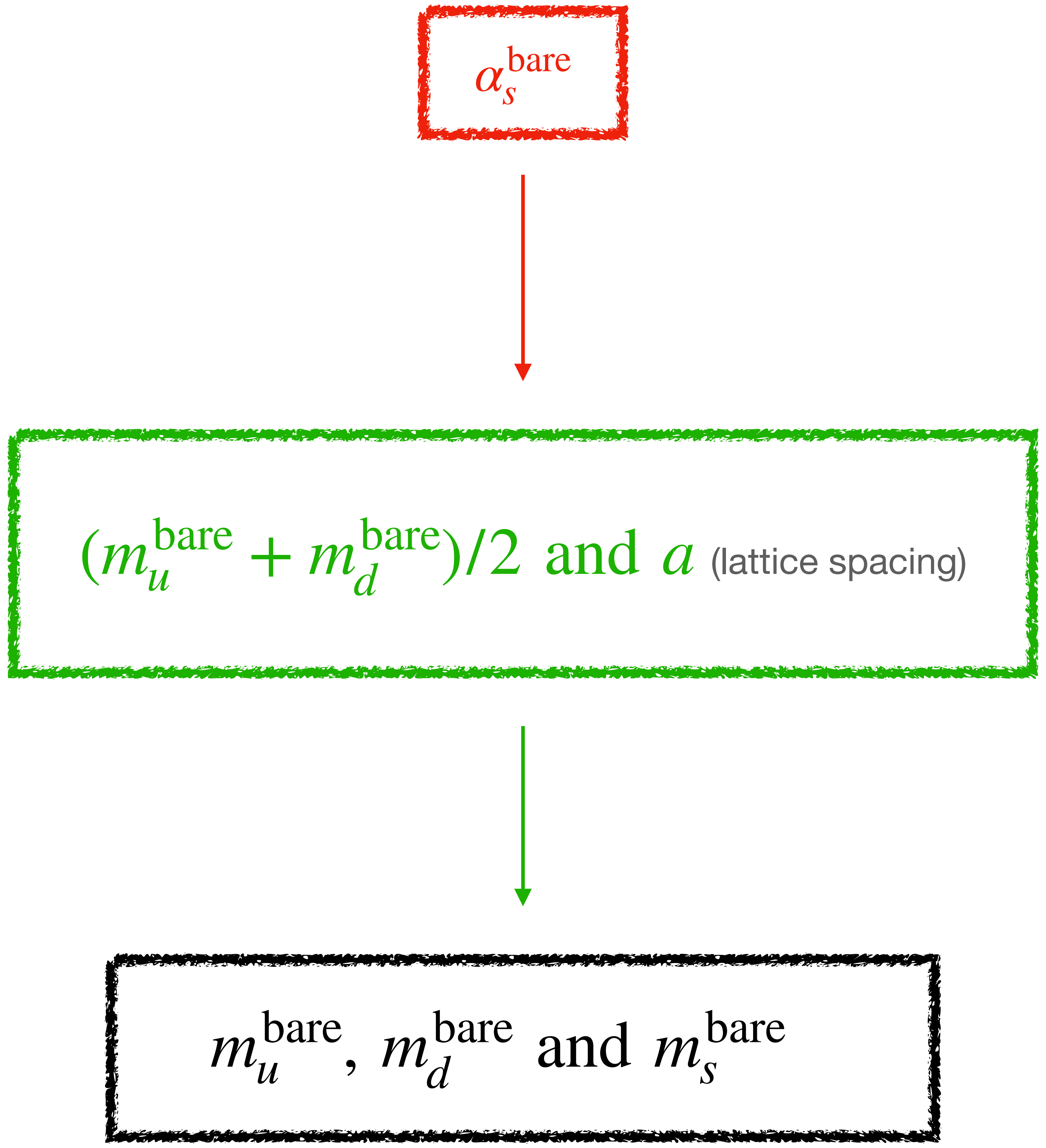
The light quark masses



A. Bazavov, et,al., MILC, PRD87(2013)054505

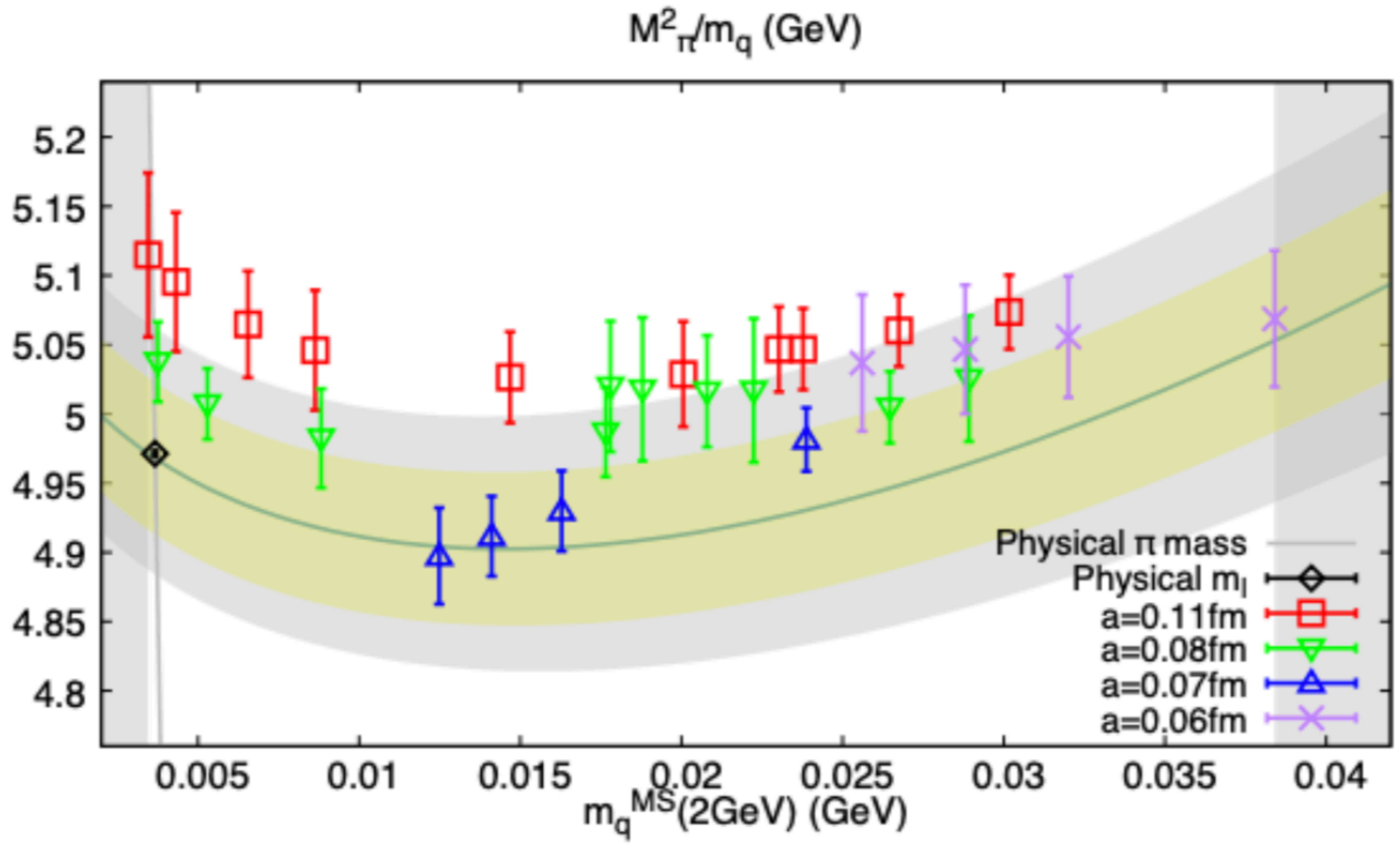
Lattice spacing dependence

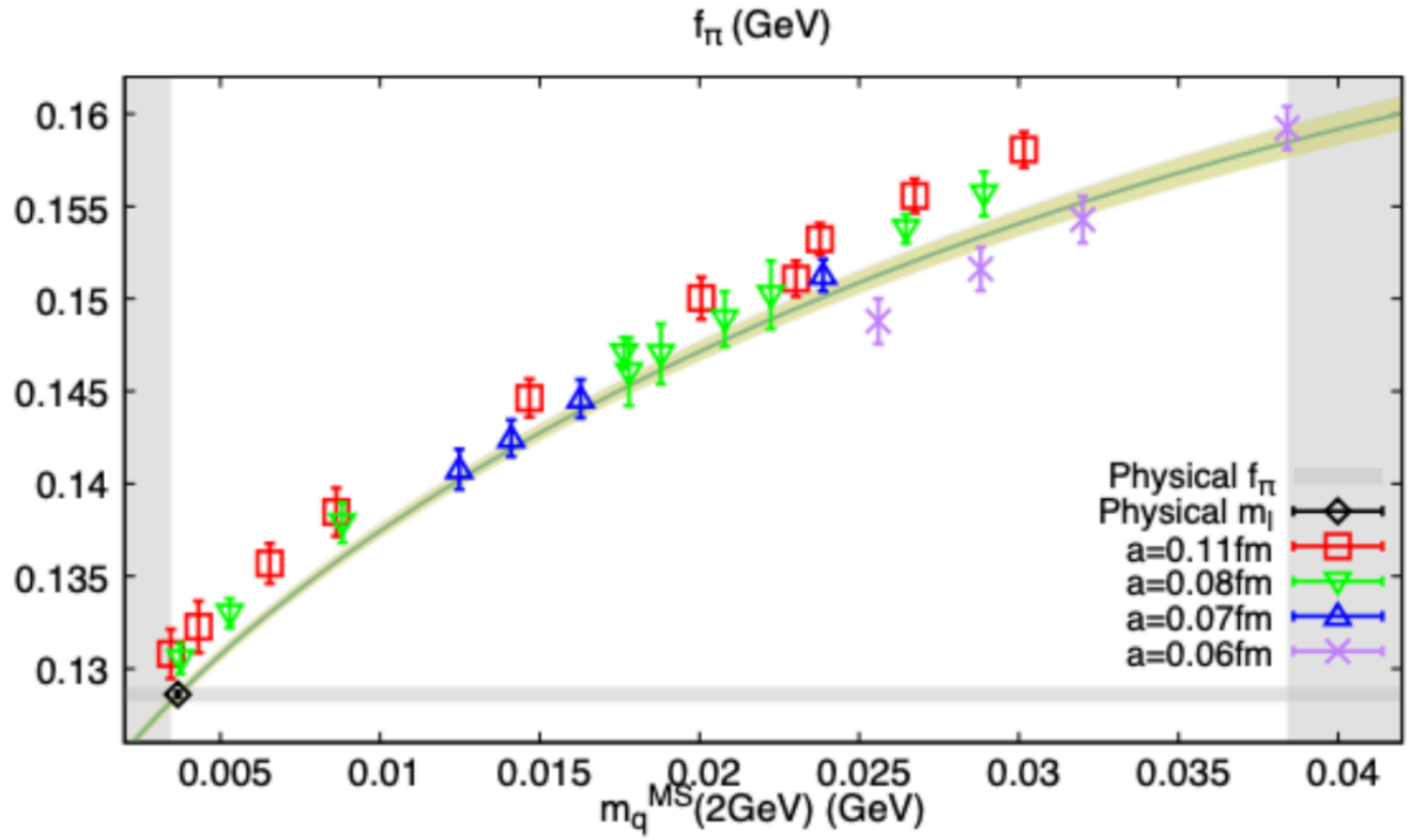
- The lattice spacing a is very sensitive to the bare coupling;
- The light quark mass to satisfy the condition is very small;
- Renormalization is needed to convert the result to MS-bar.



The light quark masses

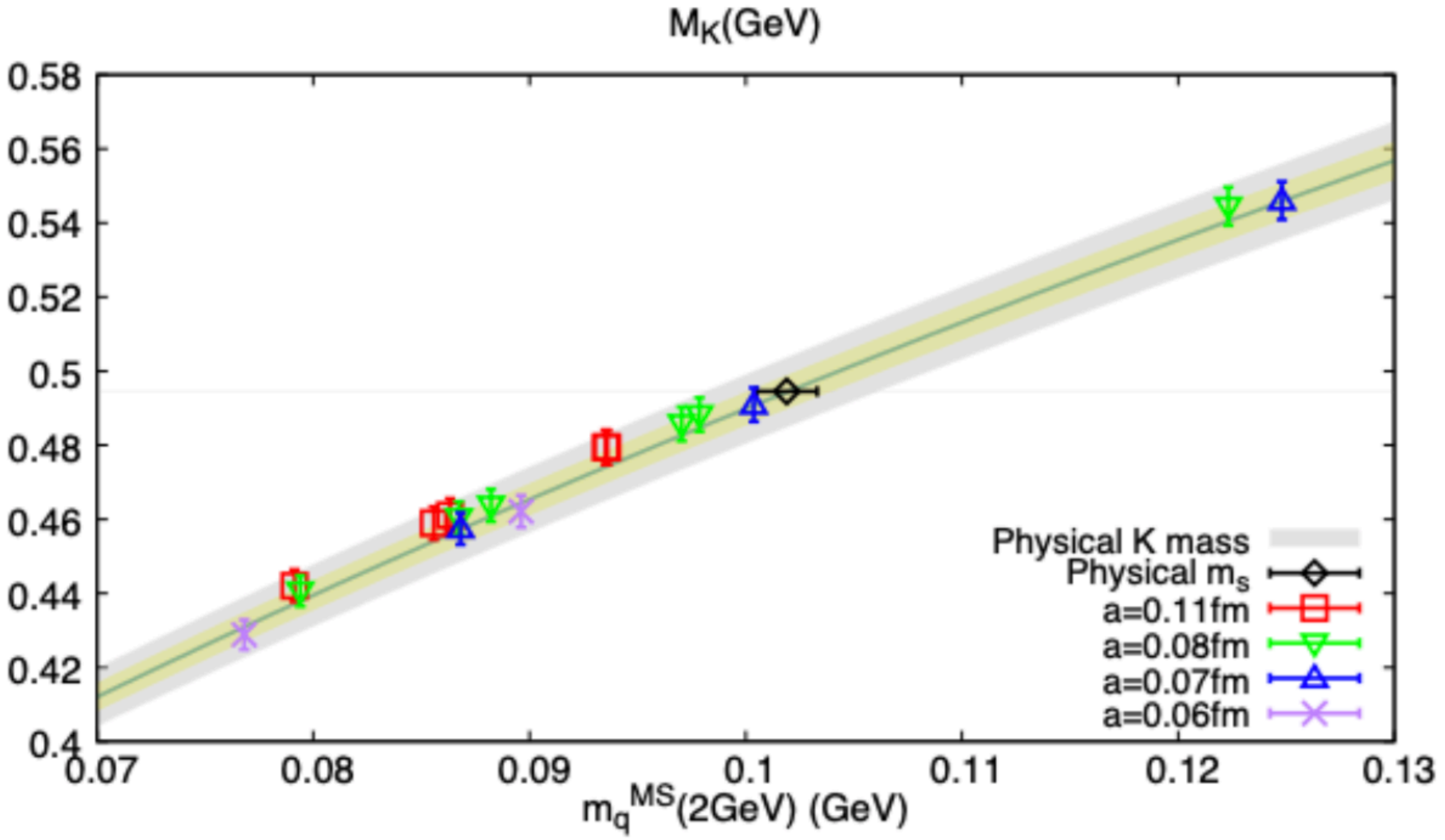
m_π and f_π

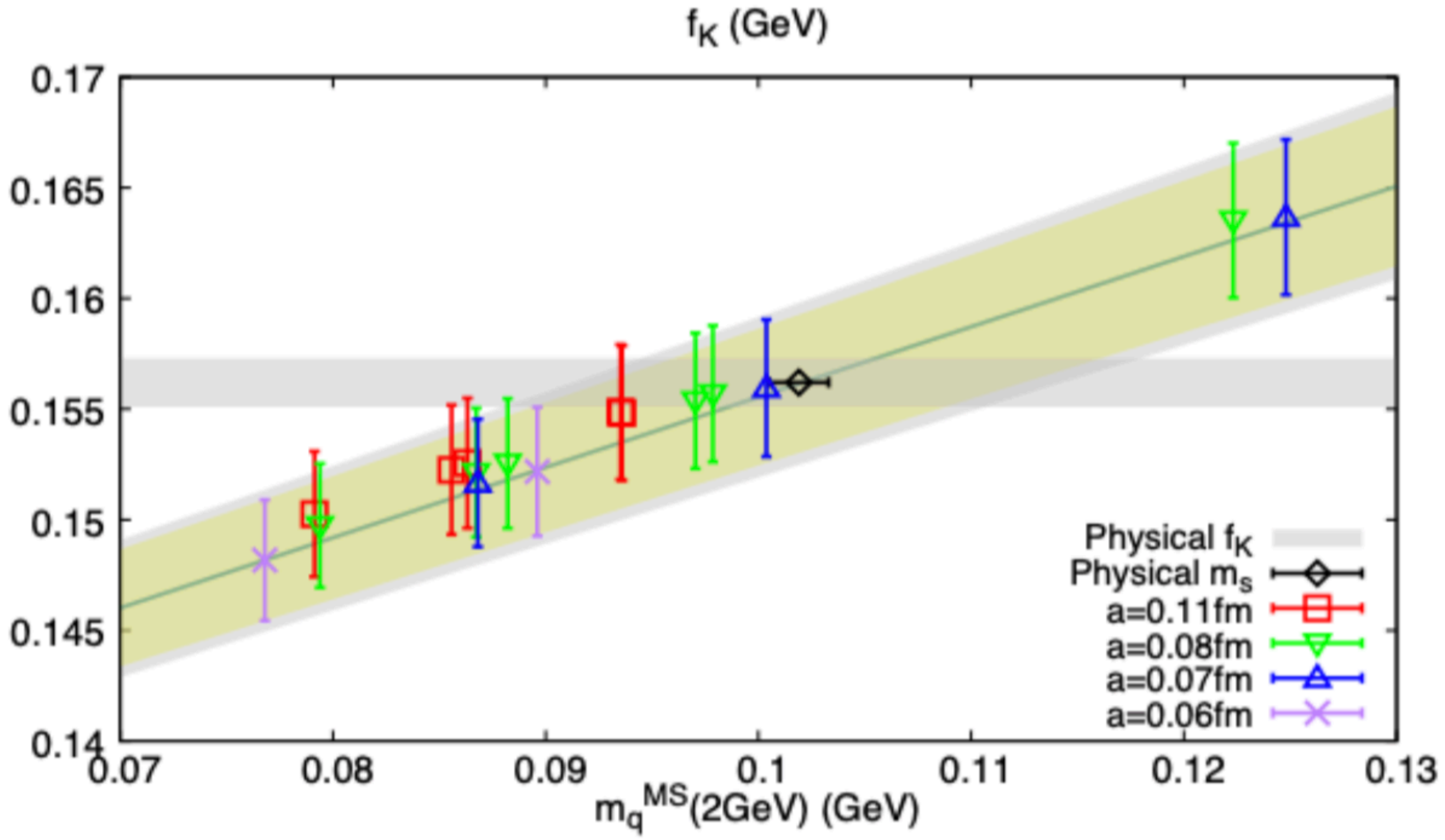




The light quark masses

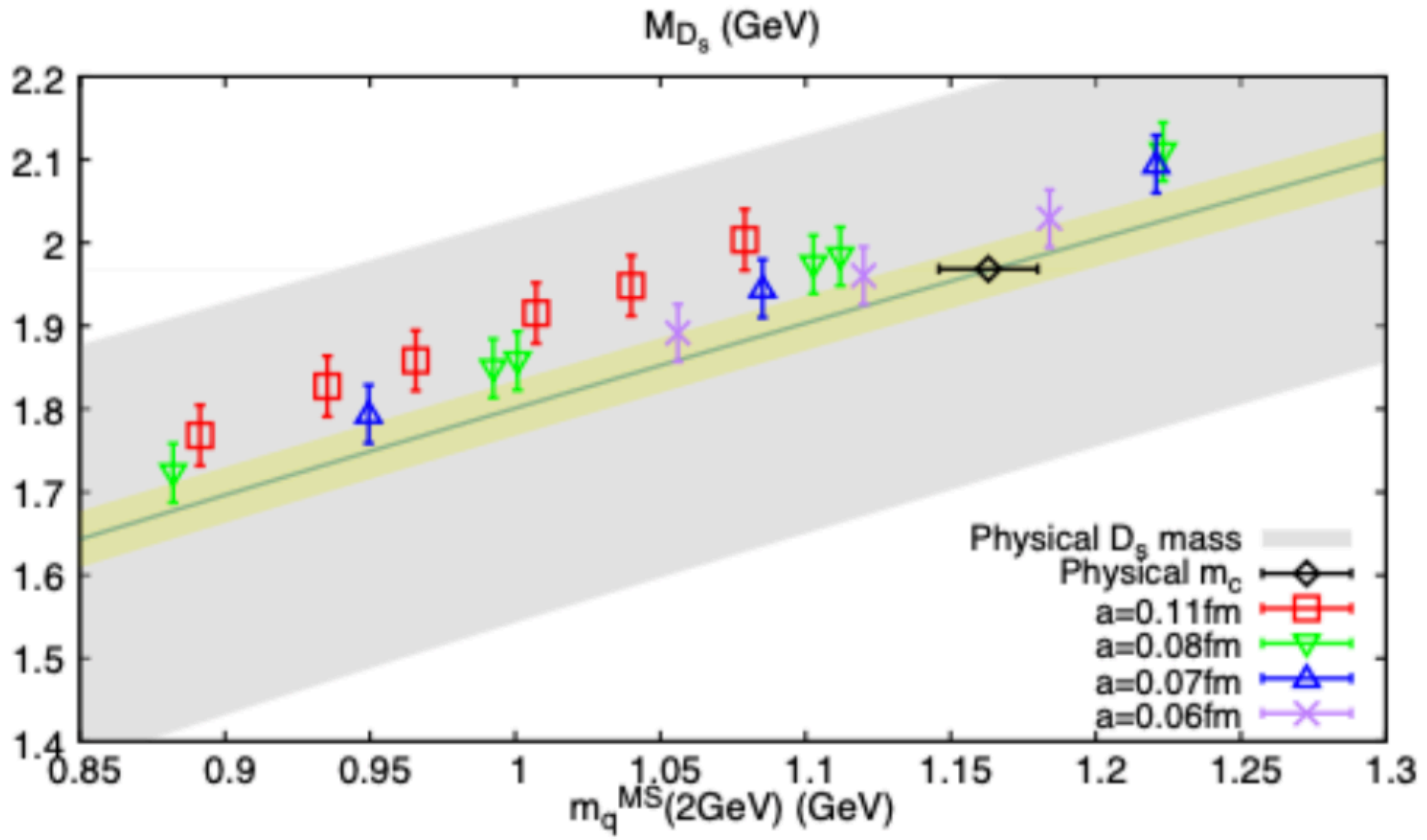
m_K and f_K

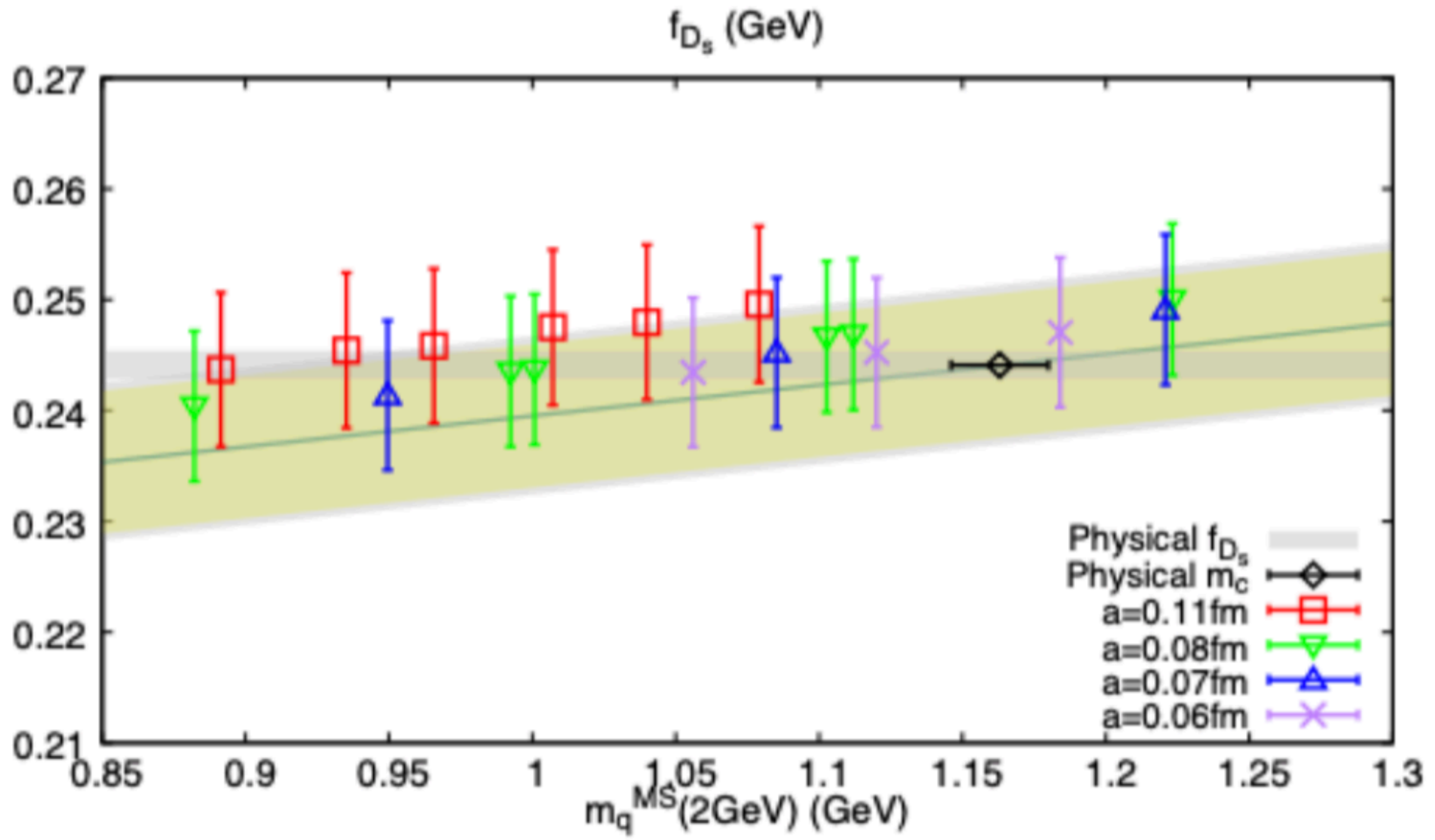




The light quark masses

m_{D_s} and f_{D_s}

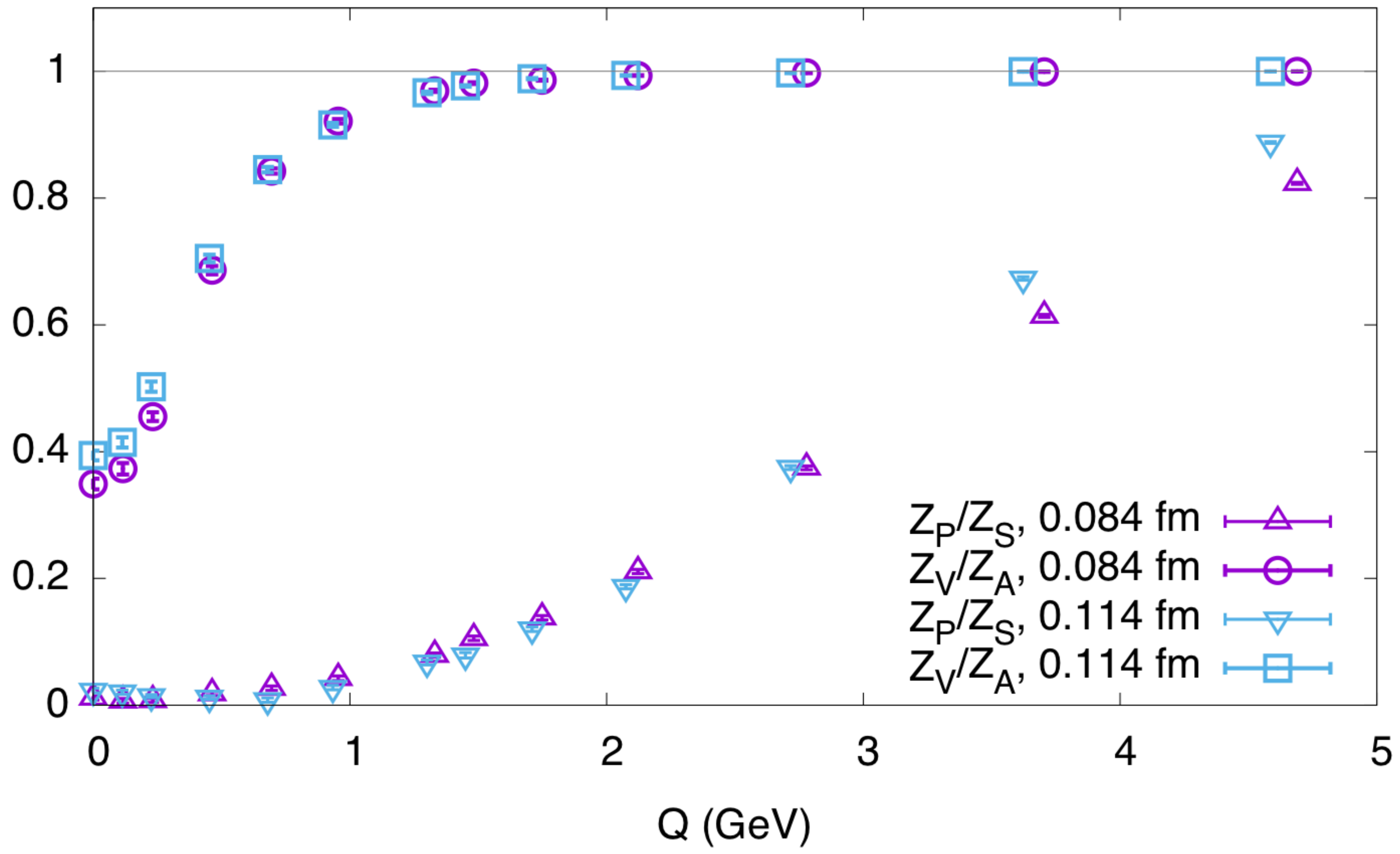




长程格点QCD

手征对称性自发破缺

- 在Q小于～2 GeV时，矢量流和轴矢流就会有很明显的破缺；
- 同时赝标流由于$Z_p^{-1}m = m^{RI}$的恒等式，会有反比与夸克质量的巨大红外修正，从而使赝标流与标量流在更高的能标就变得非常不同。

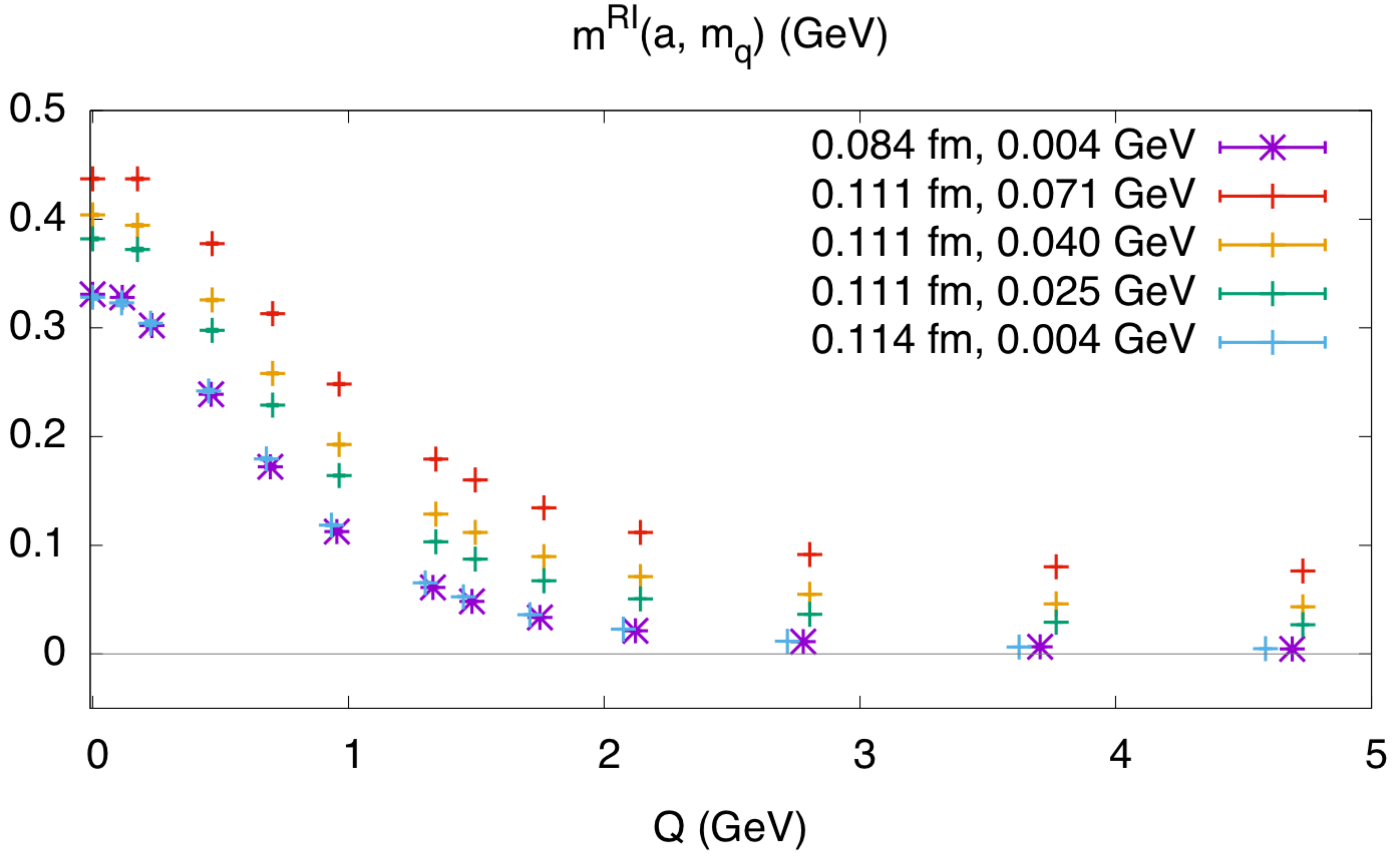


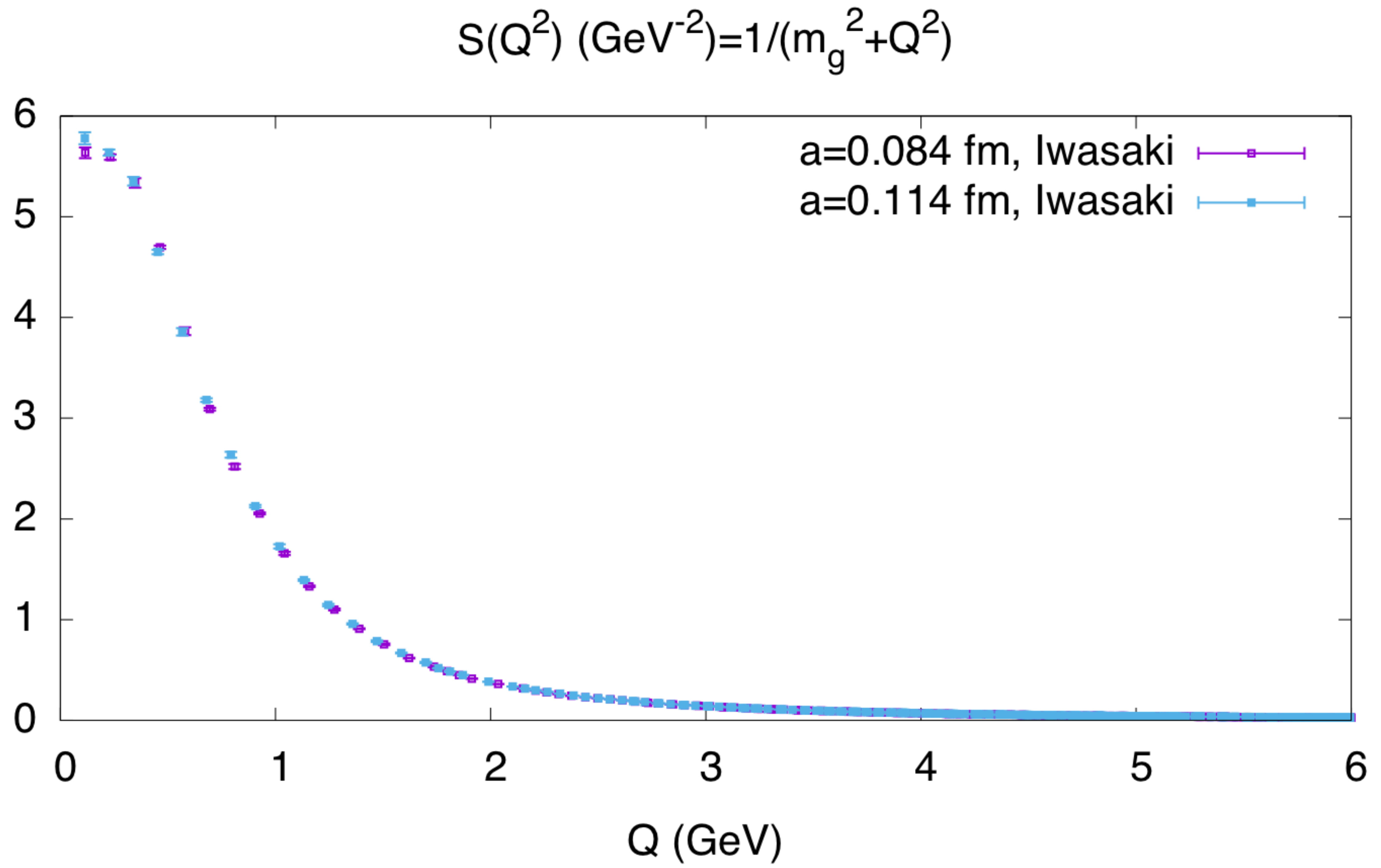
长程格点QCD

“动力学质量”

在$Q \sim \Lambda_{QCD}$ 区域，RI/MOM方案Landau规范下的手征极限下夸克会像是有一个320 MeV左右的mass，而胶子像是有一个400 MeV左右的mass。

这个值对规范的依赖，尚不清楚；与色禁闭是否有关也有争议。



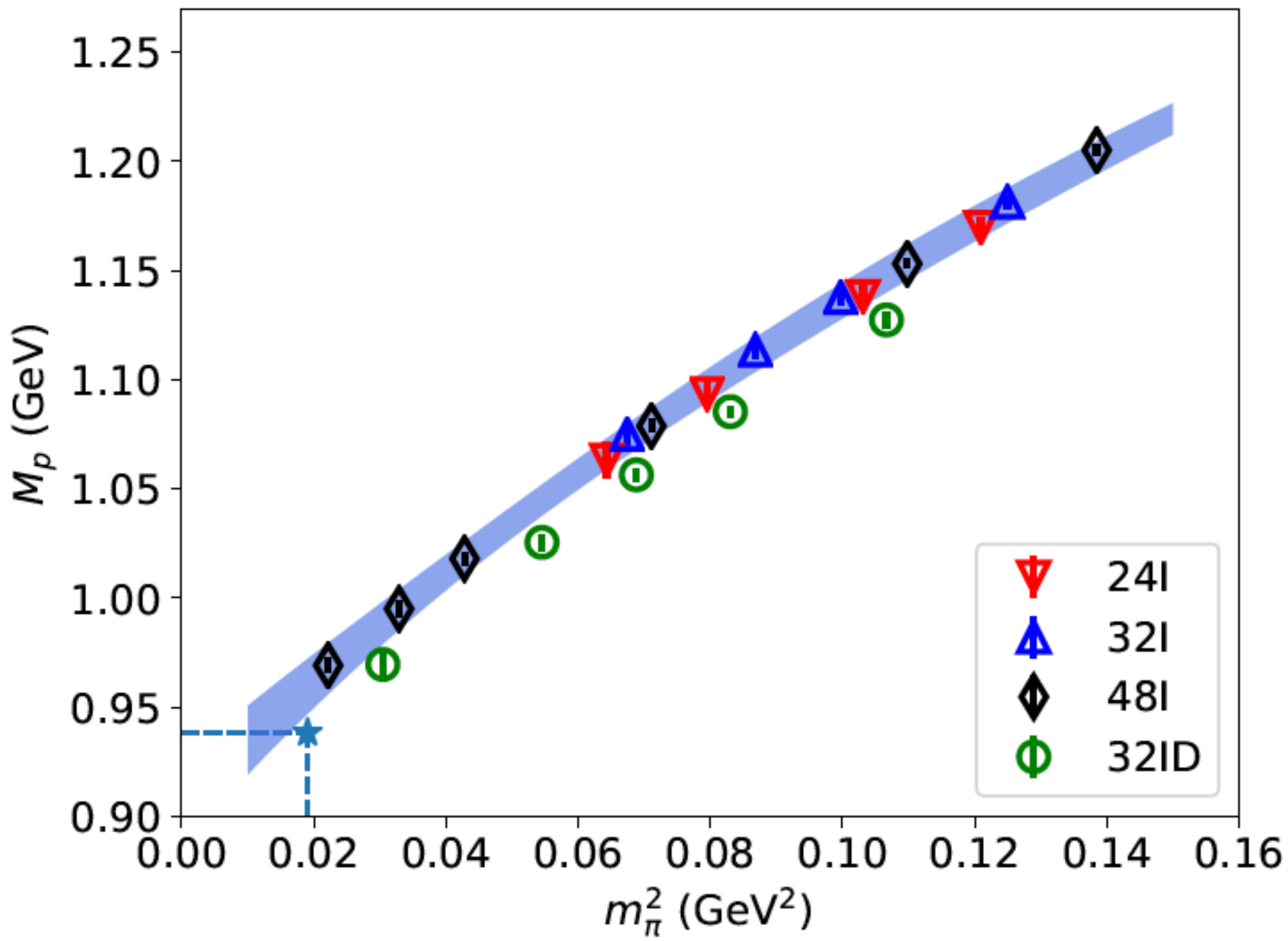


长程格点QCD

质子质量

- 格点QCD预言轻夸克质量在MS-bar 2 GeV为大约3-4 MeV;
- 但是质子质量是938 MeV,
- 而且对夸克质量并不敏感:

1. 在手征极限附近约为常数+15 $m_q^{\overline{\mathrm{MS}}(2\mathrm{GeV})}$;
2. 在重夸克极限下约为~3 $m_q^{\overline{\mathrm{MS}}(m_q)}$ 。



YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001
ViewPoint and Editor's suggestion

长程格点QCD

标度反常

- QCD能动量张量 (EMT),

$$T_{\mu\nu} = \frac{i}{2}\bar{\psi}\overleftrightarrow{D}_{(\mu}\gamma_{\nu)}\psi + \frac{1}{4}g_{\mu\nu}F^2 - F_{\mu\rho}F_{\rho\nu}$$

的迹在经典水平上只有夸克质量项 $m\bar{\psi}\psi$ 。

- 但是圈图修正的量子效应，会引入标度反常项：

$$T^{\mu}_{\mu} = m\bar{\psi}\psi - 2\epsilon\frac{F^2}{4} + \mathcal{O}(\epsilon^2) = \left[1 + \frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right]m\bar{\psi}\psi + \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right]F^2,$$

J.Collins et,al. PRD16(1977) 438

其中正比于 α_s 的两项都是QCD的量子修正。

- 因而质子质量可以分解为如下形式：

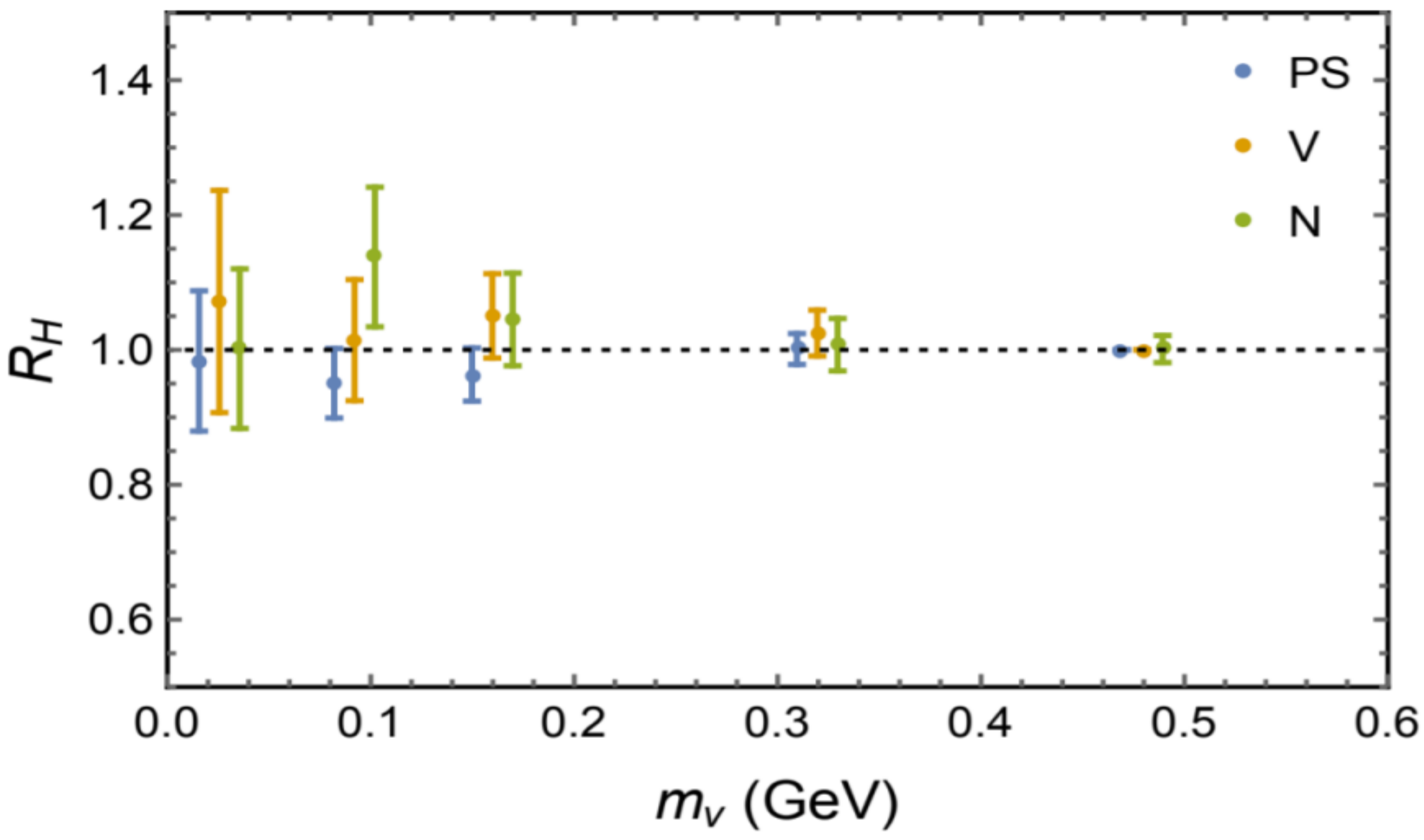
M.A. SHIFMAN et,al. PLB78(1978)

$$m_N = m\langle\bar{\psi}\psi\rangle_N + \left[\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)\right]m\langle\bar{\psi}\psi\rangle_N + \left[(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)\right]\langle F^2\rangle_N.$$

长程格点QCD

验证求和规则

- 通过计算可以得到 $\gamma_m = 0.38(3)$ 以及 $\frac{\beta}{g^3} = -0.056(6)$.
- 和MS-bar方案下的值大体相当:
  - $\gamma_m(1/a) = 0.325(10)$,.
  - $\frac{\beta}{g^3} = -0.057 + \mathcal{O}(\alpha_s)$
- 同时求和规则 $\frac{(1+\gamma_m)m\langle\bar{\psi}\psi\rangle_H + \frac{\beta}{2g}\langle F^2\rangle_H}{m_H}$ 对于所有强子态都满足。



F. He, P. Sun, **YBY**, χQCD Collaboration, arXiv:2101.04942

Supported by Strategic Priority Research Program of Chinese Academy of Sciences, Grant No. XDC01040100

长程格点QCD

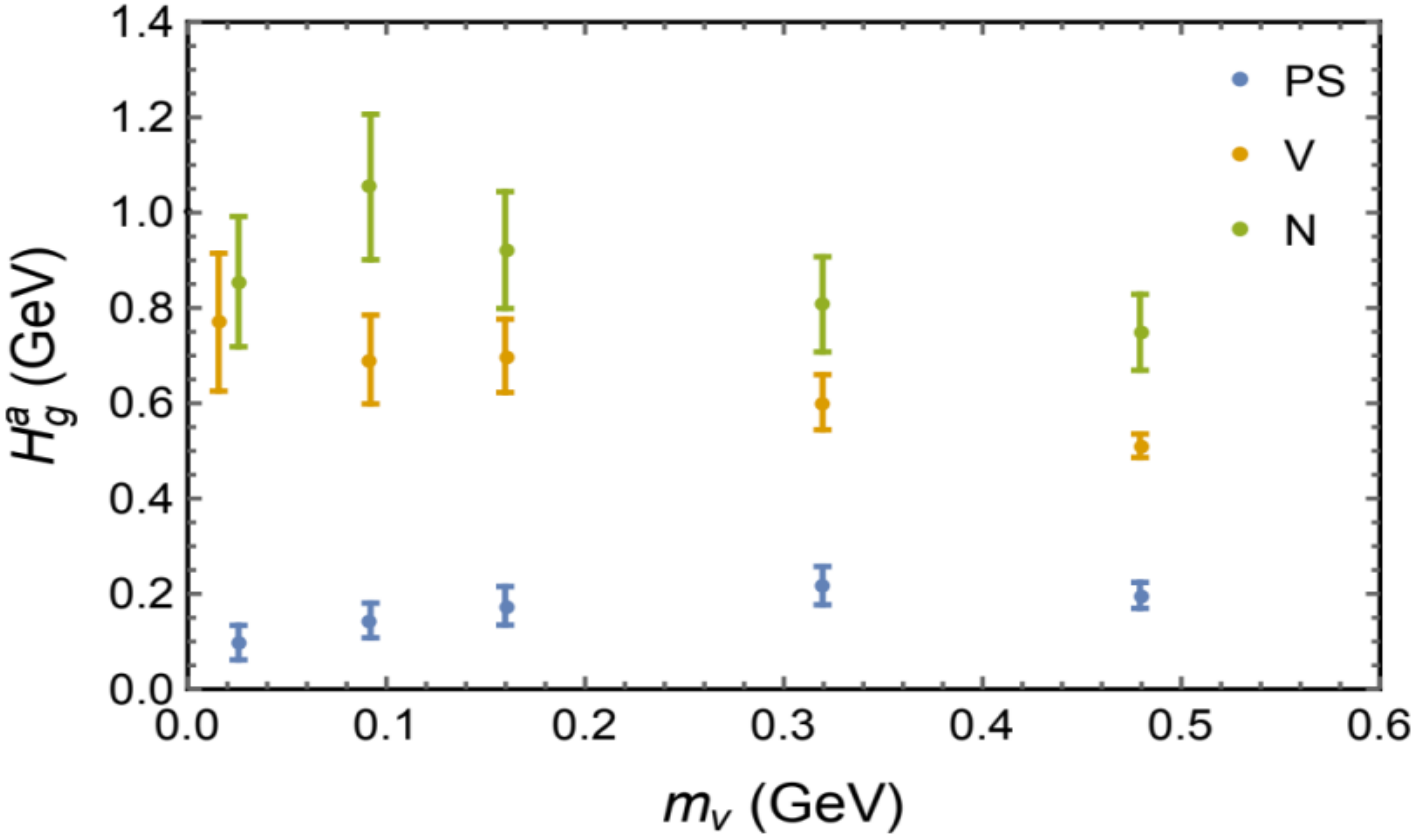
$$m_N = m\langle\bar\psi\psi\rangle_N + [\frac{2}{\pi}\alpha_s + \mathcal{O}(\alpha_s^2)]m\langle\bar\psi\psi\rangle_N + [(-\frac{11}{8\pi} + \frac{N_f}{12\pi})\alpha_s + \mathcal{O}(\alpha_s^2)]\langle F^2\rangle_N.$$

胶子贡献

- 对各种强子质量的胶子贡献 $\frac{\beta}{2g}\langle F^2\rangle_H$:

- $\frac{\beta}{2g}\langle F^2\rangle_N$**在手征极限下约为800 MeV;**
- $\frac{\beta}{2g}\langle F^2\rangle_\pi$ **在手征极限下小于100 MeV。**

- 与理论预期完全符合。



F. He, P. Sun, **YBY**, χQCD Collaboration, arXiv:2101.04942

Supported by Strategic Priority Research Program of Chinese Academy of Sciences, Grant No. XDC01040100

The trace less part of EMT

Let us go back to the ME of the traceless EMT:

$$\frac{\langle P|\bar{T}^{q,g}_{\mu\nu}|P\rangle}{\langle P|P\rangle} = A^{q,g}\frac{P_\mu P_\nu + \frac{1}{d}g_{\mu\nu}m_N^2}{P_0},$$

where $\bar{T}^q_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi - \frac{1}{d}g_{\mu\nu}m\bar{\psi}\psi,\ \bar{T}^g_{\mu\nu} = F_{\mu\rho}F_\nu{}^\rho - \frac{1}{d}g_{\mu\nu}F^2.$

The Lorentz quark/gluon momentum fraction *A* can be obtained in any frame, likes the rest frame:

$$\left.\frac{\langle P|\bar{T}^{q,g}_{44}|P\rangle}{\langle P|P\rangle}\right|_{P_{x,y,z}=0,} = -\frac{d-1}{d}A^{q,g}m_N,$$

or on the light-cone as:

$$\left.\frac{\langle P|\bar{T}^{q,g}_{++}|P\rangle}{\langle P|P\rangle}\right|_{P_{x,y,z}=0,} = A^{q,g}P_+,$$ where $\bar{T}^q_{++} = \frac{1}{2}\bar{\psi}\gamma_+\overleftrightarrow{D}_+\psi,\ \bar{T}^g_{++} = F_{+\rho}F_+{}^\rho.$

The trace terms are omitted as $P_+ >> m_N$

Momentum fractions

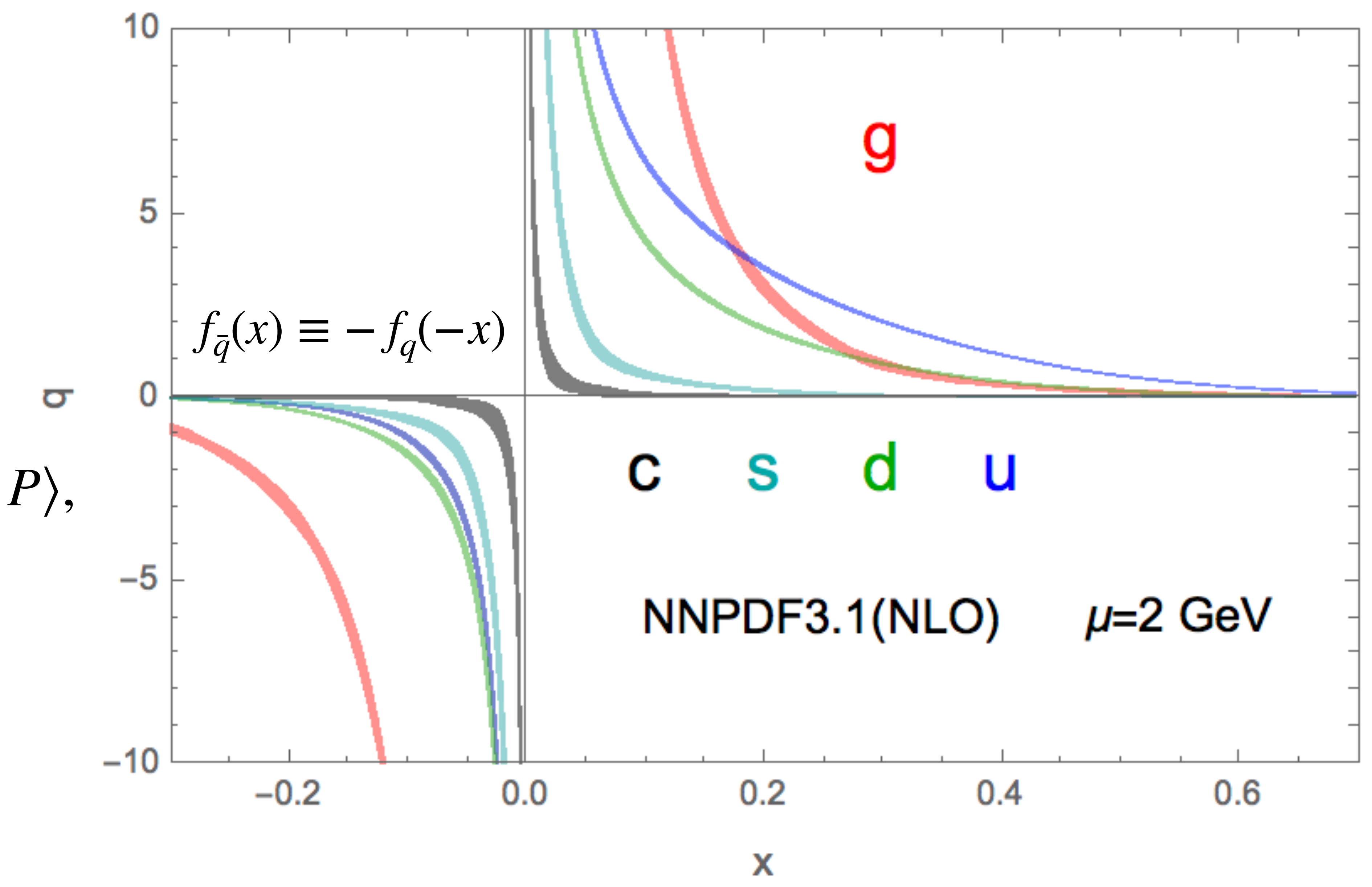
as the moments of PDF

On the light-cone, the quark and gluon unpolarized parton distribution function (PDF) are defined by:

$$q(x) = \int \frac{\mathrm{d}\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P | \bar{\psi}(\xi^-)\gamma_+ U(\xi^-,0)\psi(0) | P \rangle,$$

$$g(x) = \int \frac{\mathrm{d}\xi^-}{2x\pi} e^{-ix\xi^- P^+} \langle P | \mathrm{Tr}\left[F_{+\rho}(\xi^-)U(\xi^-,0)F_+^{\rho}(0)U(0,\xi^-)\right] | P \rangle,$$

NNPDF Collaboration, NPB887(2014)276



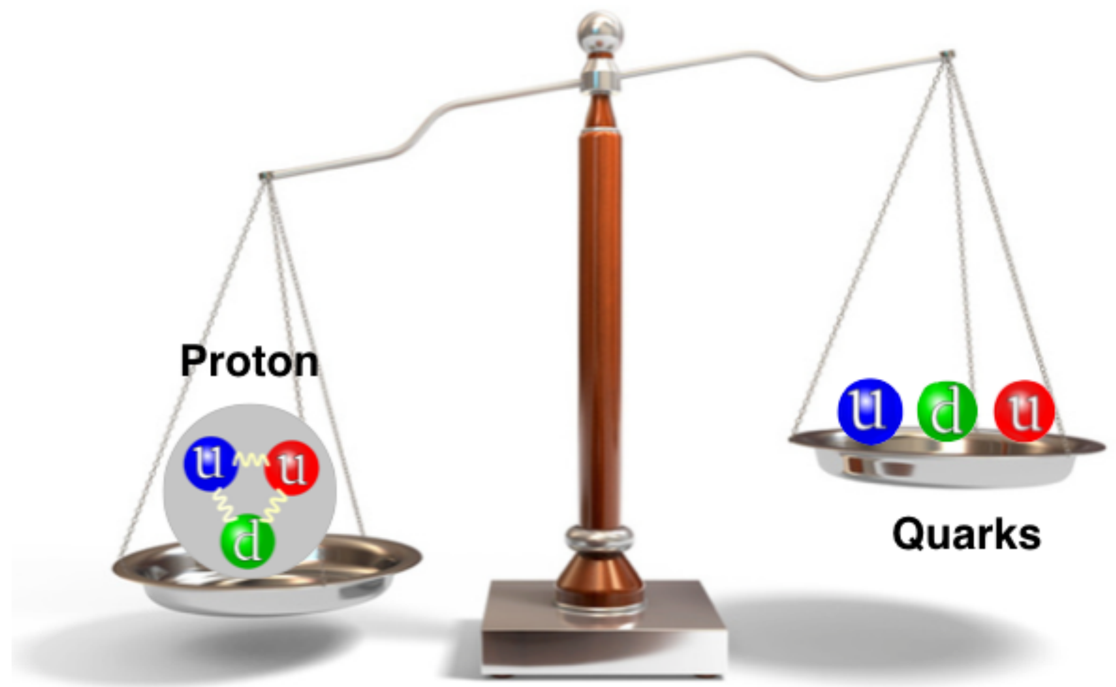
and it is easy to obtain that,

$$\int_{-1}^{1} xq(x)\mathrm{d}x = \frac{\langle P | \bar{T}^q_{++} | P \rangle}{P_+ \langle P | P \rangle} = A^q, \quad \int_{-1}^{1} xg(x)\mathrm{d}x = \frac{\langle P | \bar{T}^g_{++} | P \rangle}{P_+ \langle P | P \rangle} = A^g,$$

with $\bar{T}^q_{++} = \frac{1}{2}\bar{\psi}\gamma_+ \overleftrightarrow{D}_+ \psi, \; \bar{T}^g_{++} = F_{+\rho}F_+{}^{\rho}$.

Thus the momentum fractions we obtained in the rest frame is directly the moments of the unpolarized PDF.

The decompositions of the QCD EMT



Thus one can have the following Ji's decomposition of the nucleon mass (the energy in the rest frame) :

$$
\begin{aligned}
m_N = -\langle T_{44}\rangle_{P_{x,y,z}=0,d\to 4} &= -\langle \bar{T}^q_{44}\rangle - \langle \bar{T}^g_{44}\rangle + \frac{1}{4}(1+\gamma_m)\langle H_m\rangle + \frac{\beta}{8g}\langle F^2\rangle \\
&= -\langle \bar{\psi}\gamma_4 \overrightarrow{D}_4 \psi\rangle + \langle -\bar{T}^g_{44}\rangle + \frac{1}{4}\gamma_m\langle H_m\rangle + \frac{\beta}{8g}\langle F^2\rangle \\
&= \langle \sum_i \bar{\psi}\gamma_i \overrightarrow{D}_i \psi\rangle + \langle -\bar{T}^g_{44}\rangle + \langle H_m\rangle + \frac{1}{4}\gamma_m\langle H_m\rangle + \frac{\beta}{8g}\langle F^2\rangle
\end{aligned}
$$

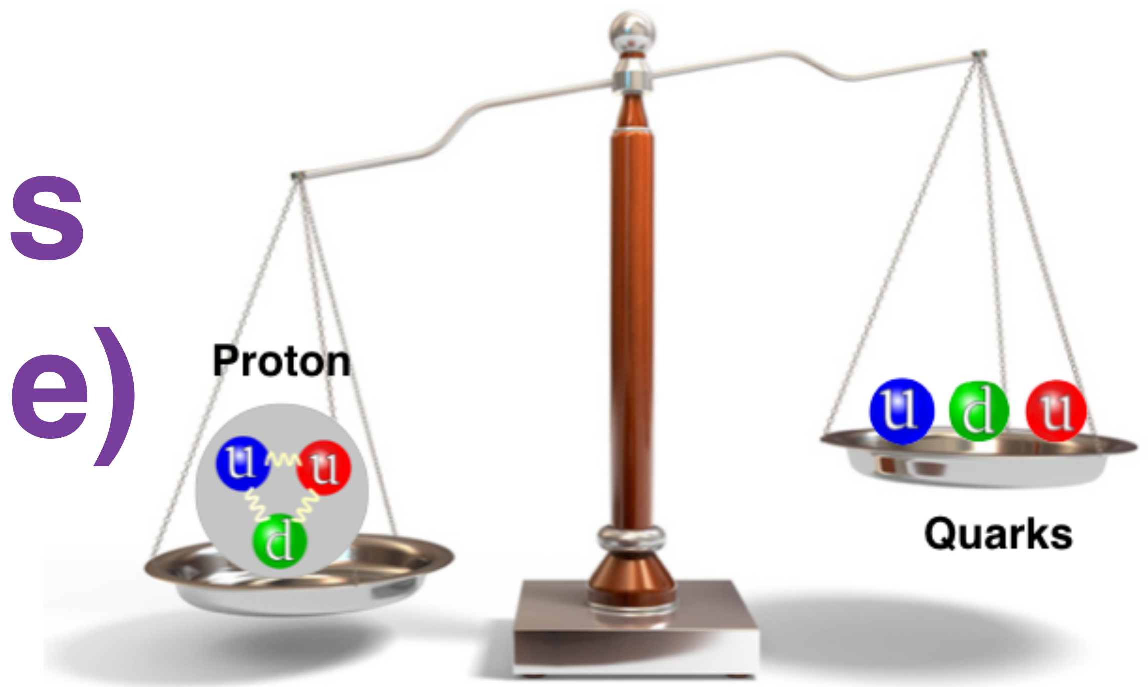
Xiangdong Ji, PRL 74(1995)1071

Or the following decomposition of EMT following the structure of perfect fluid,

$$
\langle P|T_{i,\mu,\nu}|P\rangle = \frac{\langle P|P\rangle}{2E}\left(2P_\mu P_\nu \langle x\rangle_i - 2m_N g_{\mu\nu}\bar{p}_i\right),\ \bar{p}_i = (-\langle x\rangle_i + \langle H_{m,i}\rangle)/4.
$$

C. Lorce, EPJC78(2018)120

Ji's decomposition of proton mass (the proton energy in the rest frame)



$$M = -\langle T_{44}\rangle = \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \frac{1}{4}\langle H_a\rangle,$$

$$M = -\langle \hat{T}_{44}\rangle = \langle H_m\rangle + \langle H_a\rangle$$

Xiangdong Ji, PRL 74(1995)1071

With

$$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\bar{\psi}\psi,$$

The quark mass

Gauge Invariant and scale independent combinations.

The QCD anomaly

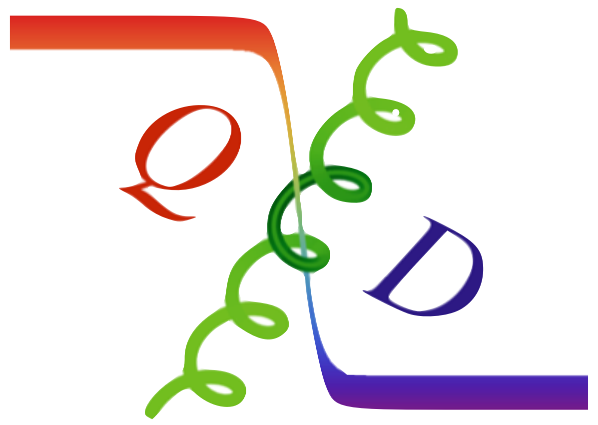
$$H_a = H_g^a + H_m^\gamma,$$

The glue anomaly

$$H_g^a = \int d^3x \frac{-\beta(g)}{g}(E^2 + B^2),$$

$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x\, \gamma_m m \bar{\psi}\psi.$$

The quark mass anomaly



The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \bar{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

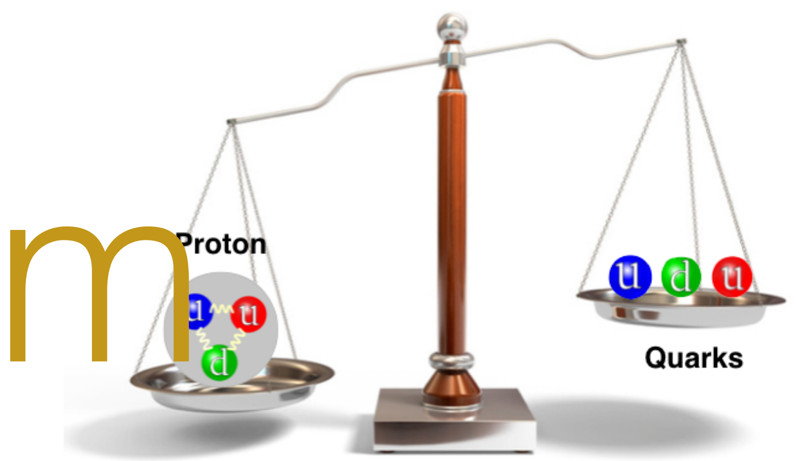
$$H_g = \int d^3x \frac{1}{2}(B^2 - E^2),$$

The glue field energy

YBY, et. al., **χ**QCD collaboration**,** Phys. Rev. D 91(2015)074516

Proton mass decomposition

The quark mass term



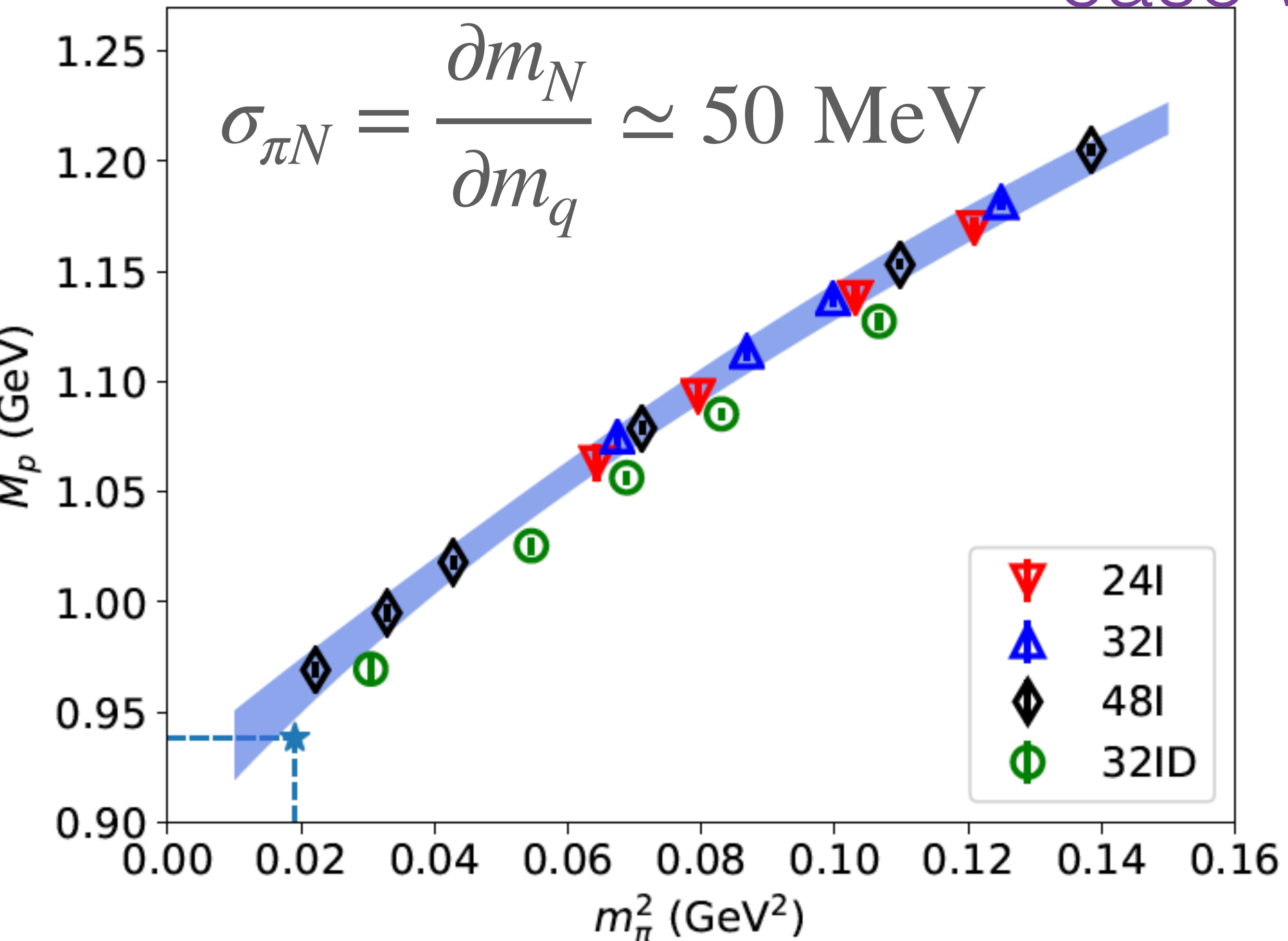
Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_a^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \tfrac{1}{4}\langle H_a\rangle,$$

$$M = -\langle \hat{T}_{44}\rangle = \langle H_m\rangle + \langle H_a\rangle,$$ *in the rest frame.*

$$H_m = \sum_{u,d,s\cdots}\int d^3x\, m\bar{\psi}\psi,$$ ***The quark mass***

- Renormalization scheme/scale independent in continuum; also in discrete case when the chiral fermion is used.



$\sigma_{\pi N} = \langle H_m(u)+H_m(d)\rangle = 45.9(7.4)(2.8)$ MeV

$f_s^N M_N = \langle H_m(s)\rangle = 40.2(11.7)(3.5)$ MeV

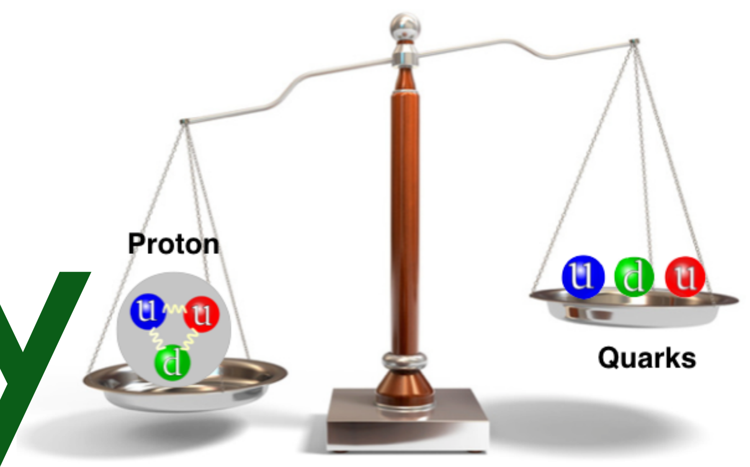
$\langle H_m(u,d,s)\rangle / M_N = 9(2)\%$

The best lattice result free of the systematic uncertainty from the explicit chiral symmetry breaking

YBY, et.al. χQCD Collaboration, PRD94(2016)054503

Proton mass decomposition

The QCD anomaly



Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_g^a\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \tfrac{1}{4}\langle H_a\rangle,$$

$$M = -\langle \hat{T}_{44}\rangle = \langle H_m\rangle + \langle H_a\rangle,$$

The QCD anomaly

$$H_a = H_g^a + H_m^\gamma,$$

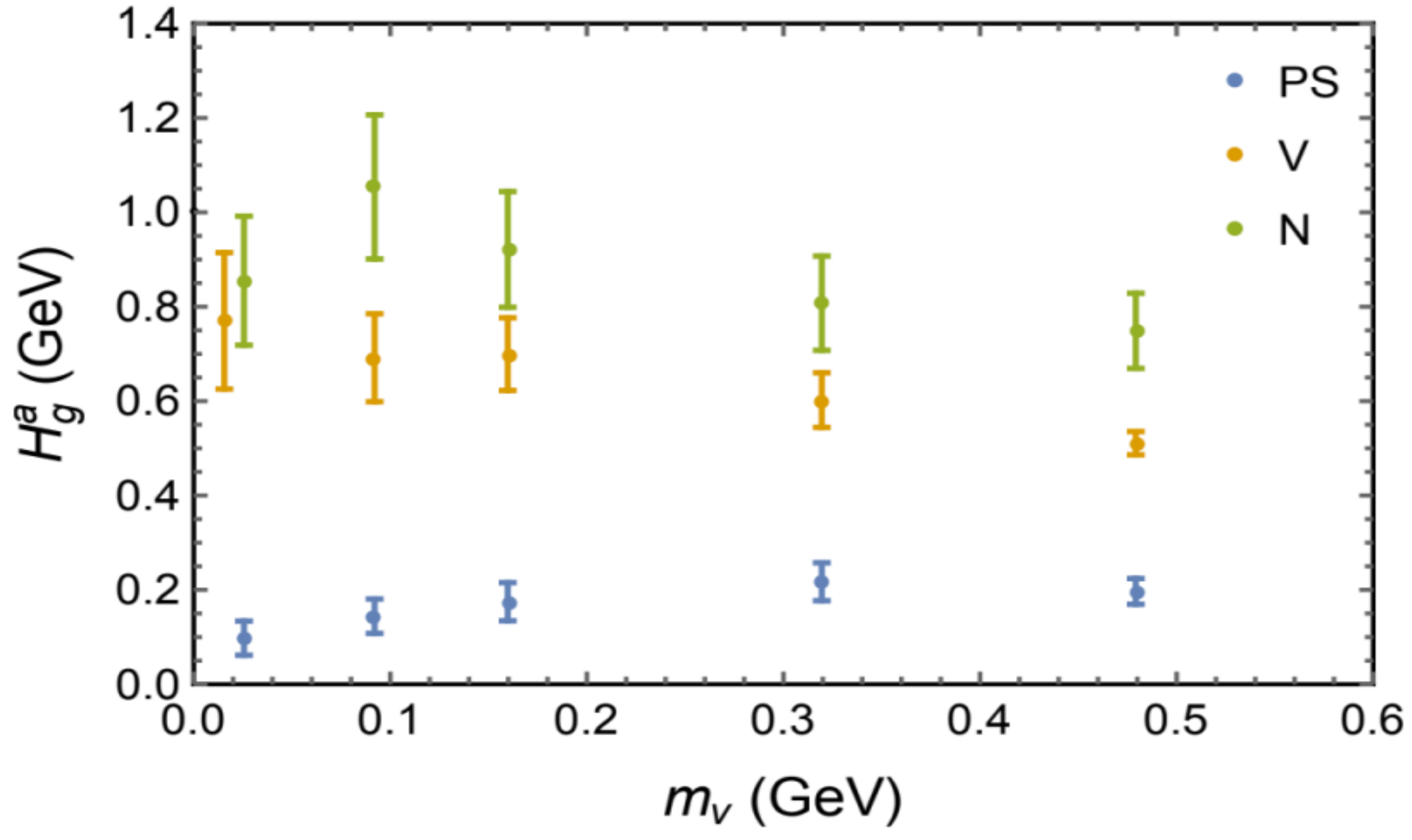
The glue anomaly

$$H_g^a = \int d^3x \, \frac{-\beta(g)}{4g}(E^2 + B^2),$$

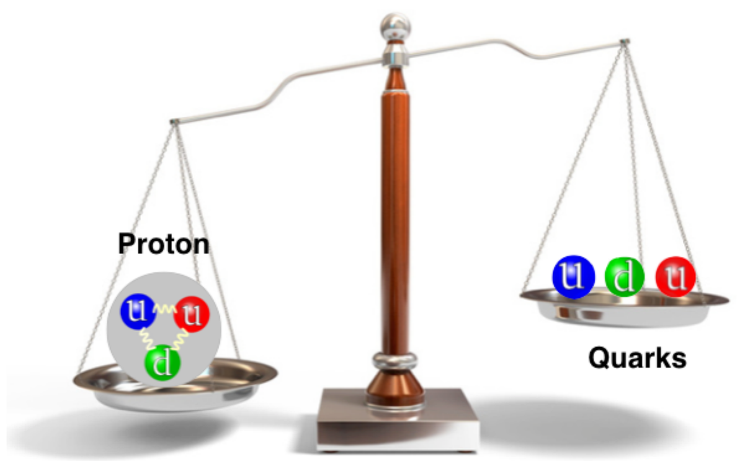
$$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x \, \frac{1}{4}\gamma_m m \overline{\psi}\psi.$$

The quark mass anomaly

- **The joint contribution of the QCD anomaly can be deduced from the quark mass term, with the sum rule above.**
- The total QCD anomaly is renormalization scheme/scale independent.
- ***$H_a/(4M_N)$= 23(1)%***



The quark/gluon energy



Then we have

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_a^g\rangle + \langle H_m^\gamma\rangle$$
$$= \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \tfrac{1}{4}\langle H_a\rangle,$$

$$M = -\langle \hat{T}_{44}\rangle = \langle H_m\rangle + \langle H_a\rangle,$$

- The quark/glue energy can be deduced from the momentum fraction,

$$\langle H_E\rangle = \frac{3}{4}\langle x\rangle_q M - \frac{3}{4}\langle H_m\rangle$$
$$\langle H_q\rangle = \frac{3}{4}\langle x\rangle_q M + \frac{1}{4}\langle H_m\rangle$$

$$\langle H_g\rangle = \frac{3}{4}\langle x\rangle_g M.$$

- The renormalization of the quark momentum fraction is much more trivial, which is just mixed with the glue one.

- **It is more straightforward to obtain the quark/glue momentum fraction first, and convert it to the quark/glue energy.**

The total energy

$$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$$

The quark energy

$$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2),$$

The glue field energy

Renormalization

of the momentum fractions

The MS-bar renormalization matrix can be obtained through the RI/MOM scheme,

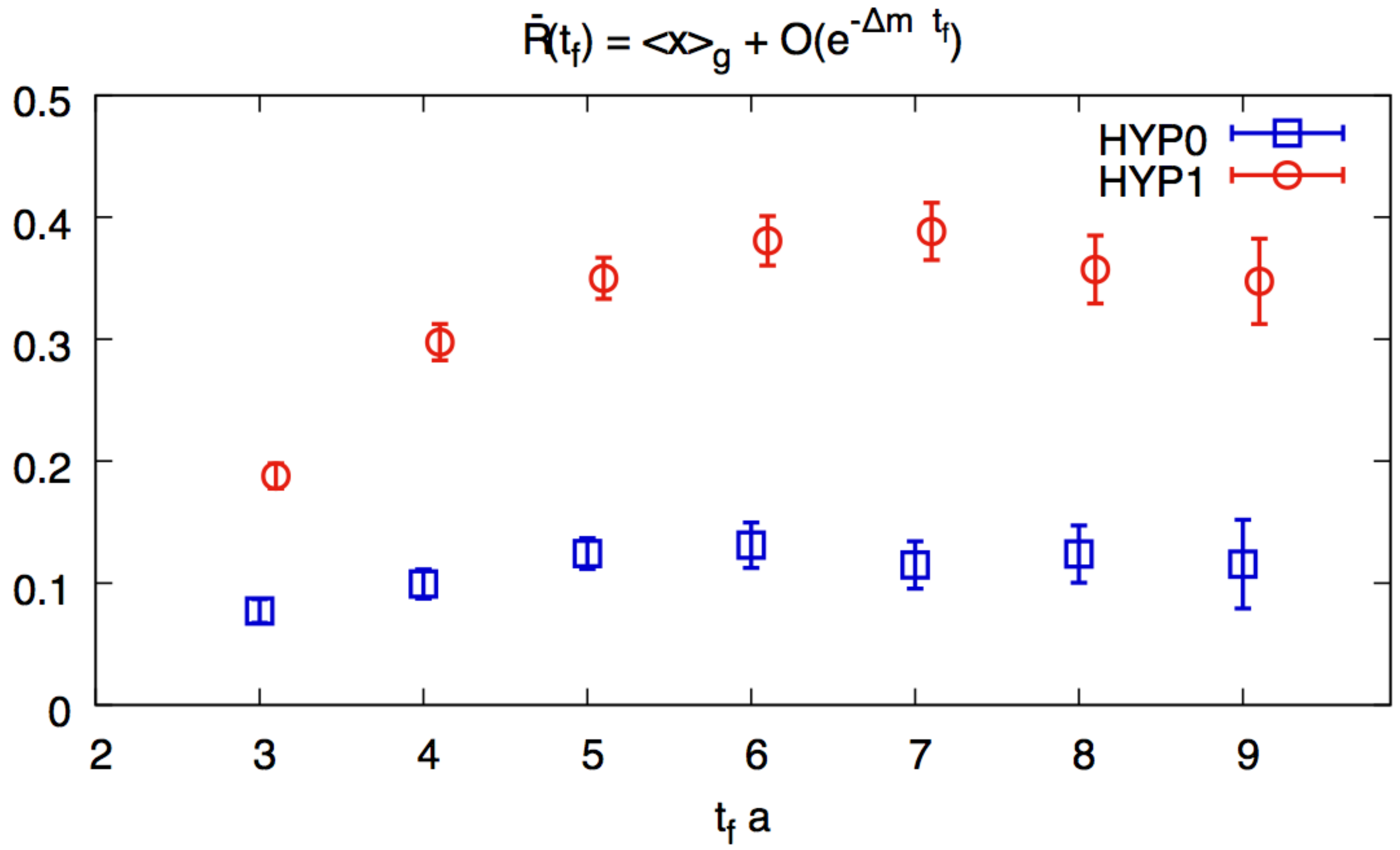
$$\begin{pmatrix} Z_{QQ}^{\overline{\rm MS}}(\mu)+N_f\delta Z_{QQ}^{\overline{\rm MS}}(\mu) & N_f Z_{QG}^{\overline{\rm MS}}(\mu) \\ Z_{GQ}^{\overline{\rm MS}}(\mu) & Z_{GG}^{\overline{\rm MS}}(\mu) \end{pmatrix} \equiv \left\{\left[\begin{pmatrix} Z_{QQ}(\mu_R)+N_f\delta Z_{QQ} & N_f Z_{QG}(\mu_R) \\ Z_{GQ}(\mu_R) & Z_{GG}(\mu_R) \end{pmatrix}\right.\right.$$
$$\left.\left.\begin{pmatrix} R_{QQ}(\frac{\mu}{\mu_R})+\mathcal{O}(N_f\alpha_s^2) & N_f R_{QG}(\frac{\mu}{\mu_R}) \\ R_{GQ}(\frac{\mu}{\mu_R}) & R_{GG}(\frac{\mu}{\mu_R}) \end{pmatrix}\right]|_{a^2\mu_R^2\to 0}\right\}^{-1}$$

$$= \left\{\begin{pmatrix} (Z_{QQ}R_{QQ})+N_f\left(\delta Z_{QQ}R_{QQ}+Z_{QG}R_{GQ}\right) & N_f\left((Z_{QQ}+N_f\delta R_{QQ})R_{QG}+Z_{QG}R_{GG}\right) \\ (Z_{GQ}R_{QQ}+Z_{GG}R_{GQ}) & (N_f Z_{GQ}R_{QG}+Z_{GG}R_{GG}) \end{pmatrix}(\mu_R,\frac{\mu}{\mu_R})|_{a^2\mu_R^2\to 0}\right\}^{-1}$$

- *All the Z's can be calculated non-perturbatively with Lattice QCD simulation;*
- *Perturbative calculation is necessary to get the R's.*

YBY, J. Liang, et. al., χQCD Collaboration,
PRL121(2018)212001,
ViewPoint and Editor's suggestion

Renormalized **glue** momentum fraction?



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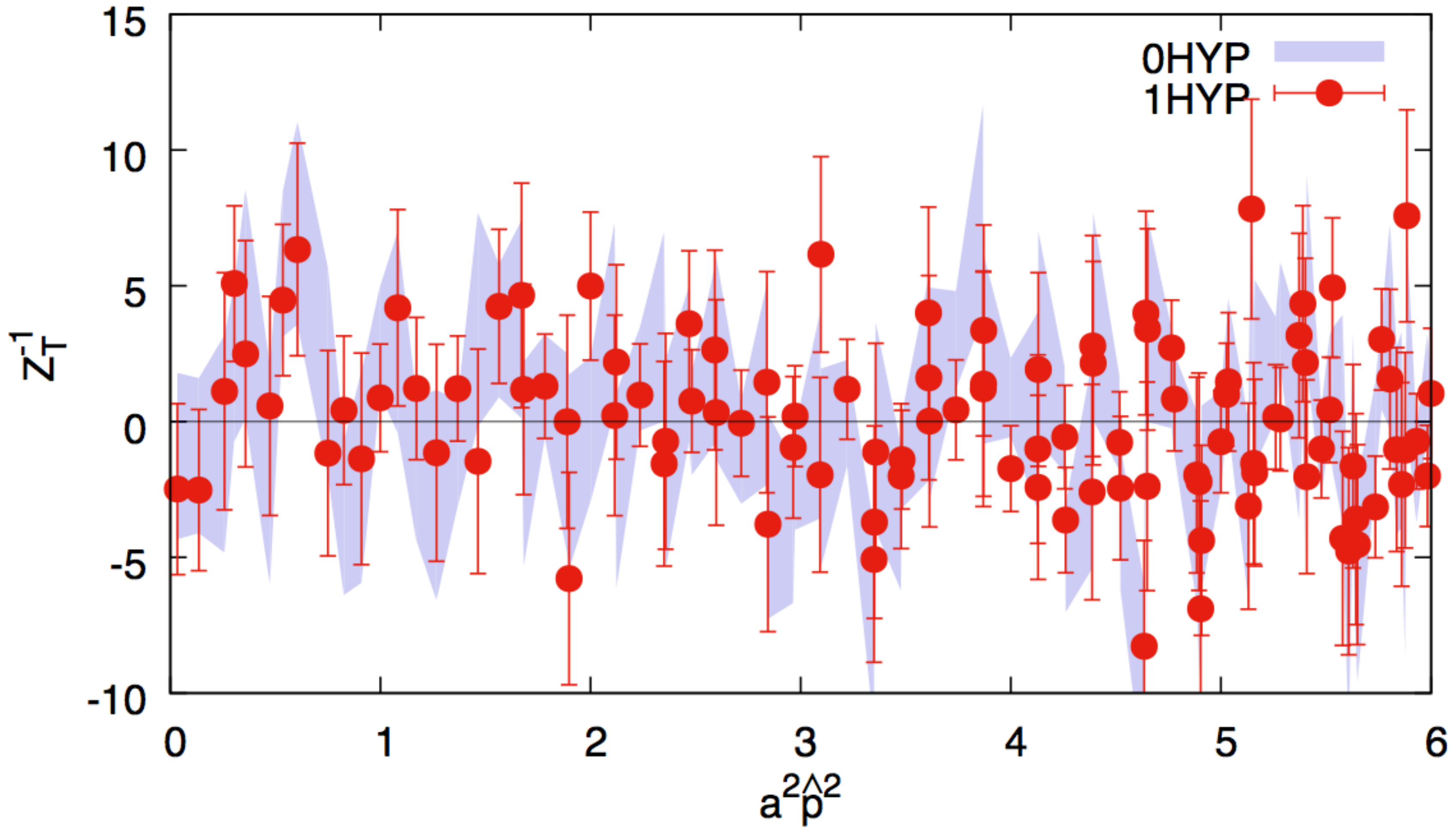

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?

Seem to be impossible to obtain the renormalization of the glue operators ***non-perturbatively?***

Gluon renormalization with CDER

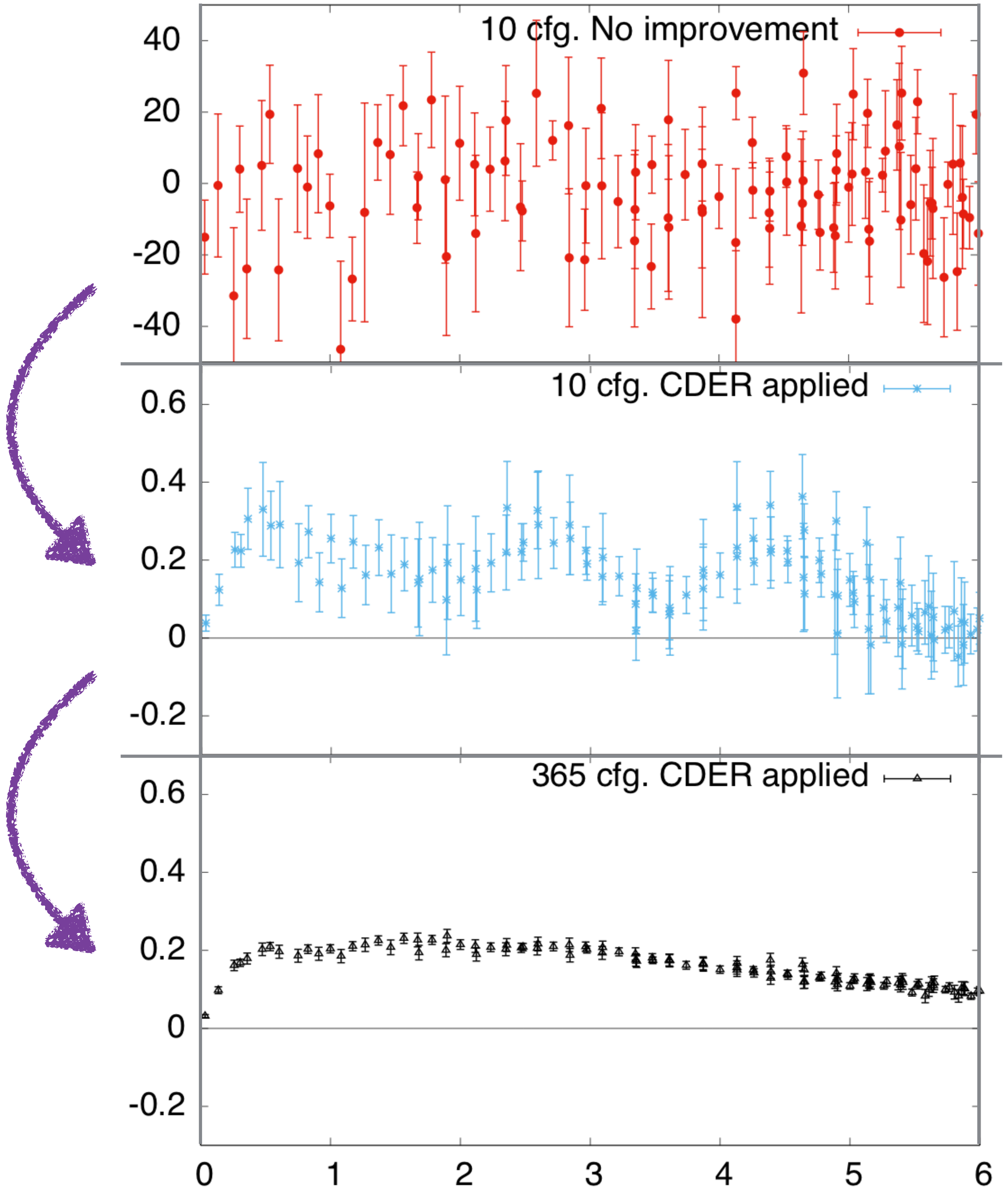
W. Sun, et.al, χQCD collaboration, CPC42, 063102(2018), 1507.02541
K. Liu, J. Liang, **YBY**, PRD96, 114504(2017), 1805.00531

- Calculate the renormalization factor of the glue EMT non-perturbatively on a ~5 fm box will require ~30,000,000 configurations to make the uncertainty to be ~0.01;

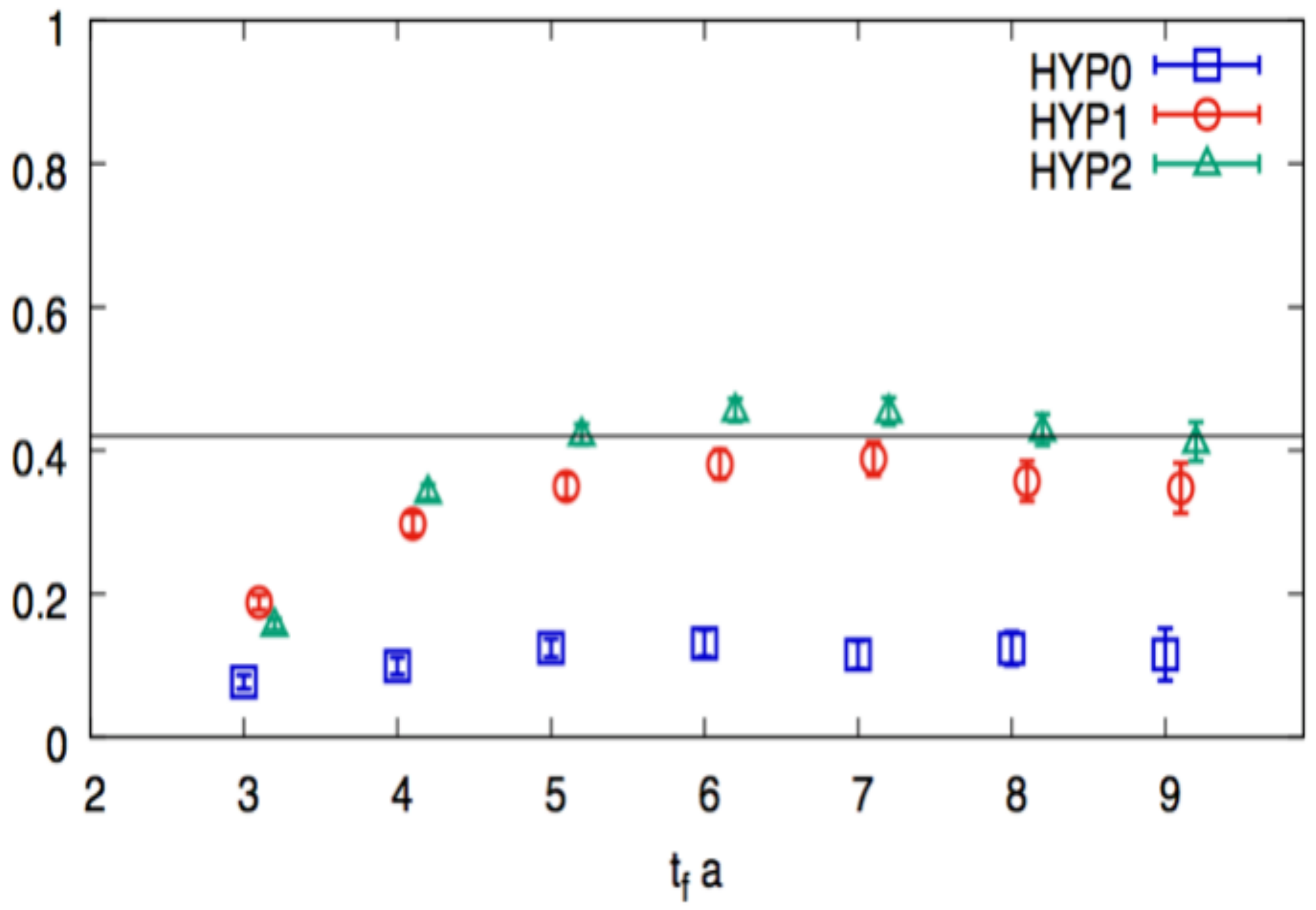
- Taking the localization of the correlations between the glue fields/ operators into account, the uncertainty can be reduced by a factor ~200;

- Use reasonable computer resource (~1M CPU hours) to increase the statistics, the ~0.01 uncertainty goal can be obtained with 365 configurations.

YBY, et. al., χQCD collaboration, PRD98(2018) 074506

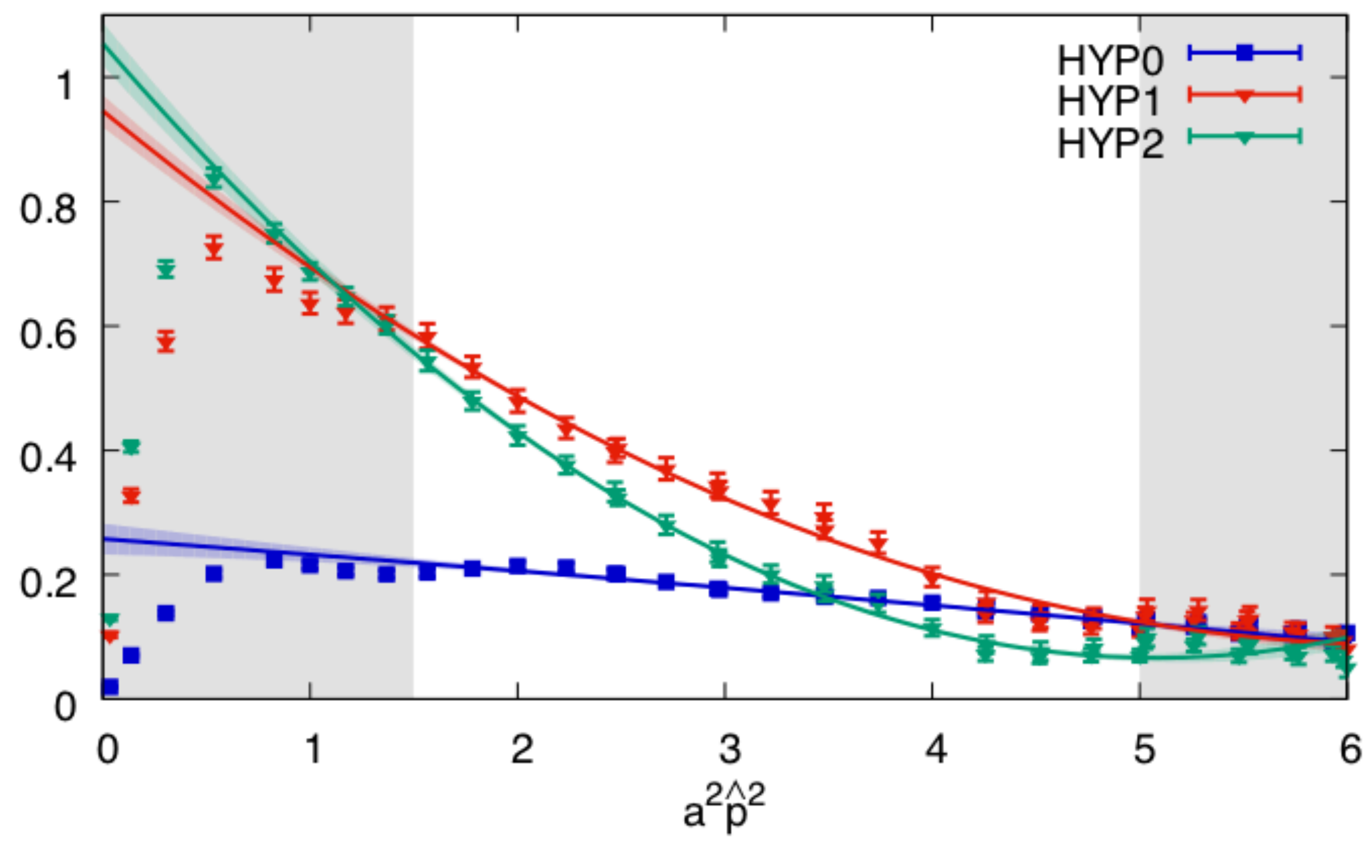


Non-perturbative renormalized glue momentum fraction



$\langle x \rangle_g^{bare}$

Z^{-1}



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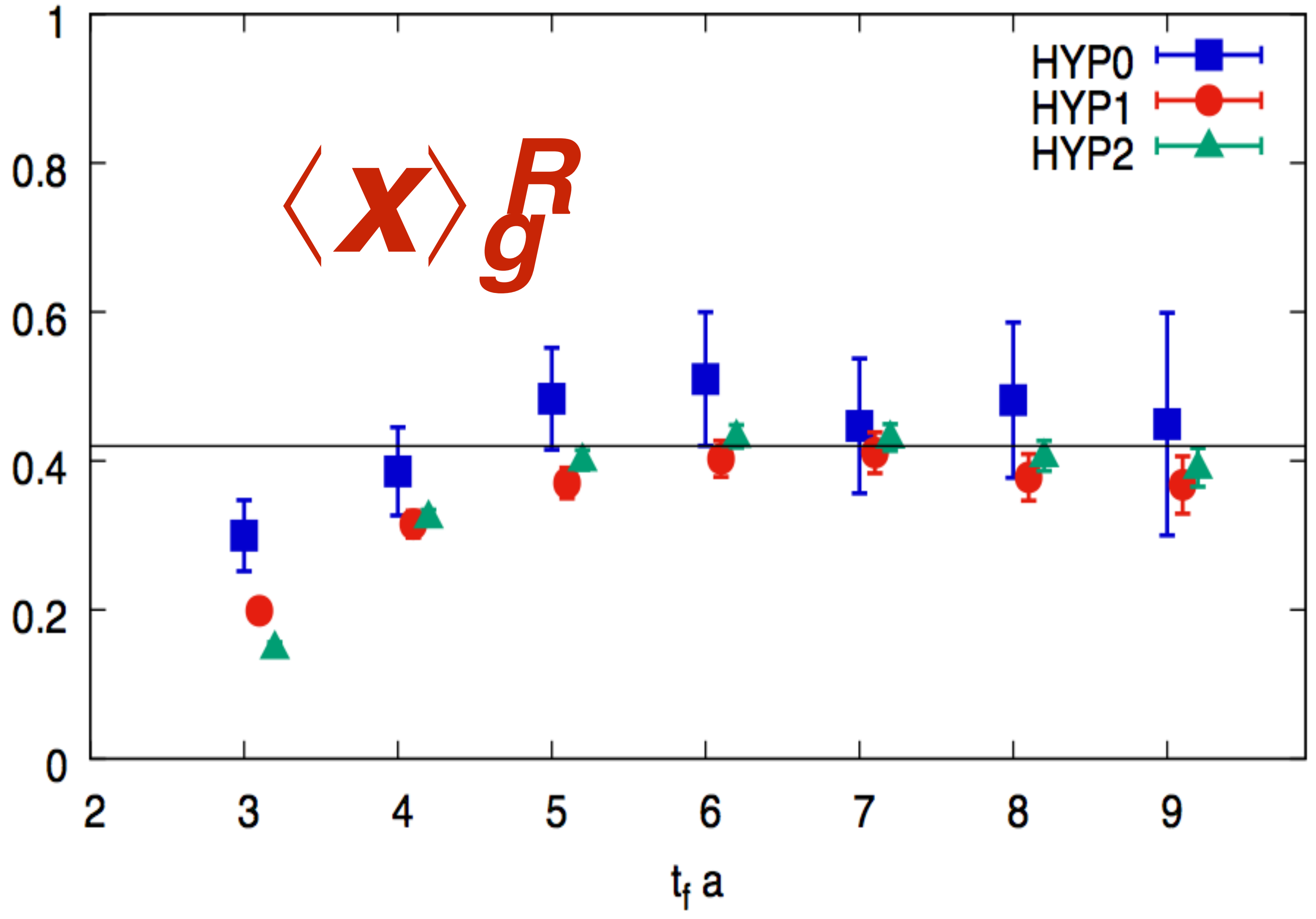
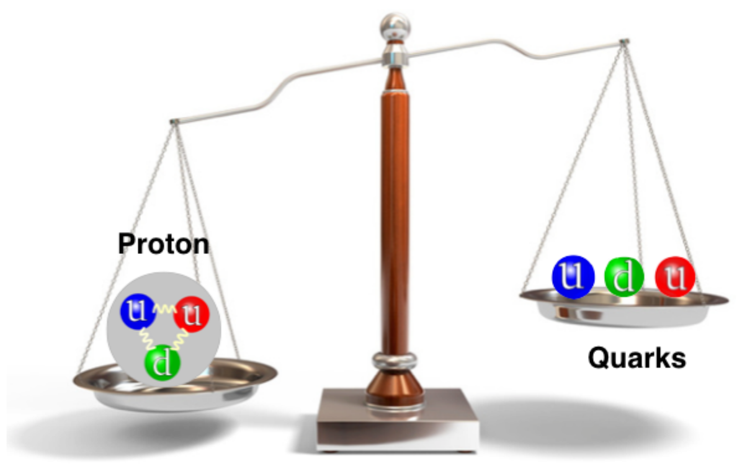


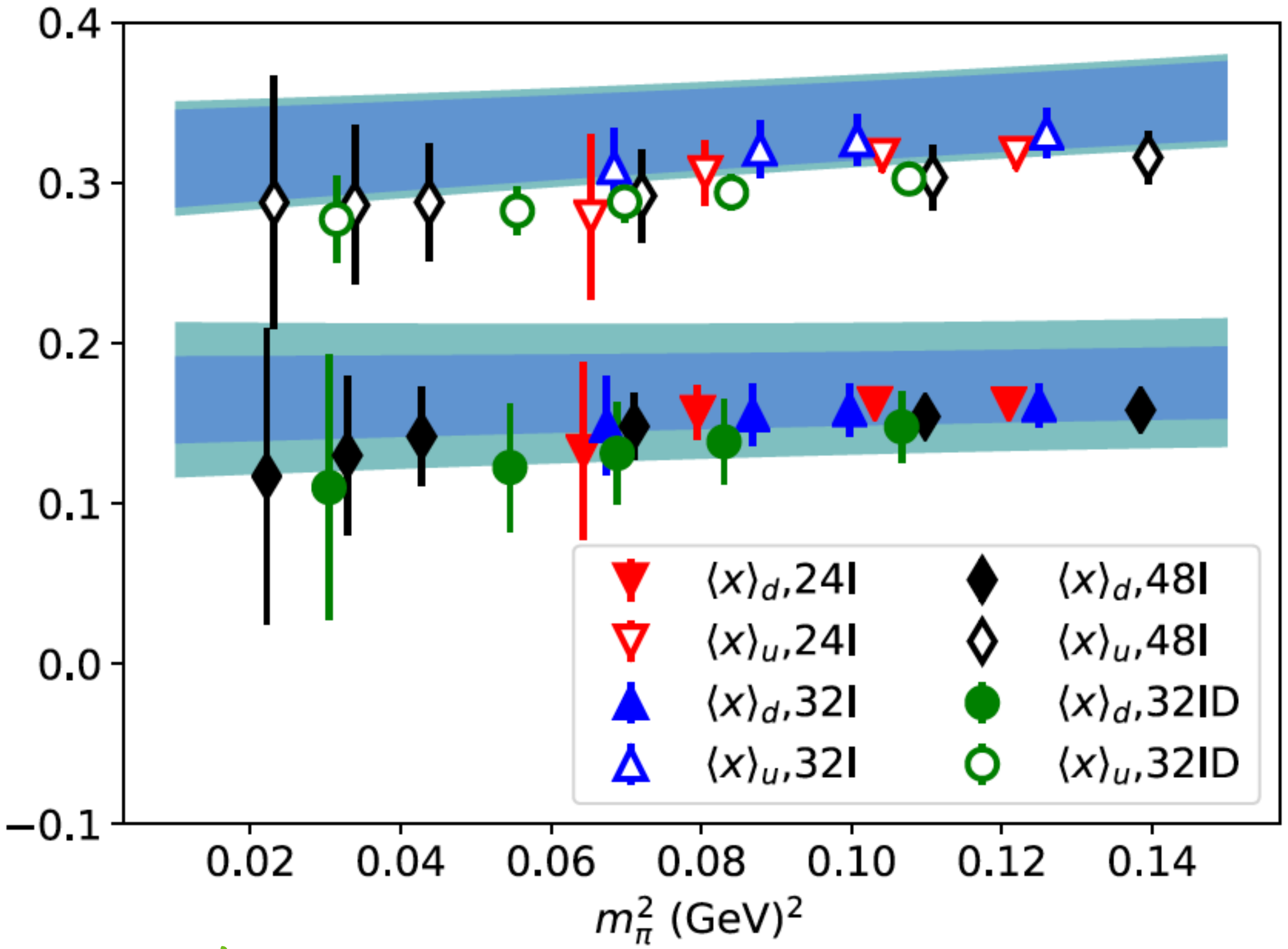
- The lattice regularization effects are **fully cancelled** within the statistical uncertainties;
- The **non-perturbative renormalized quark/glue AM results** will come out soon.

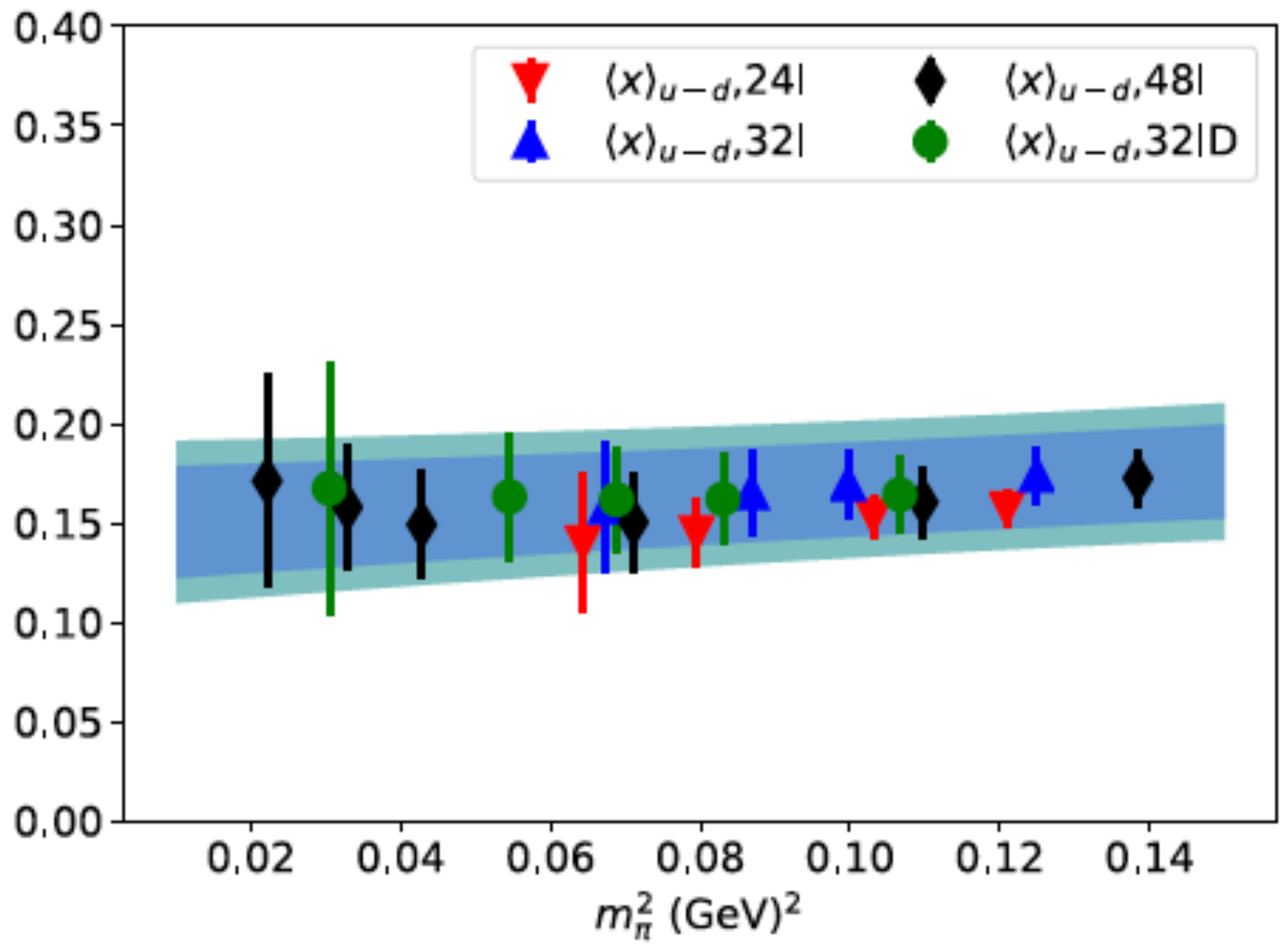
YBY, et. al., χQCD collaboration, PRD98(2018) 074506

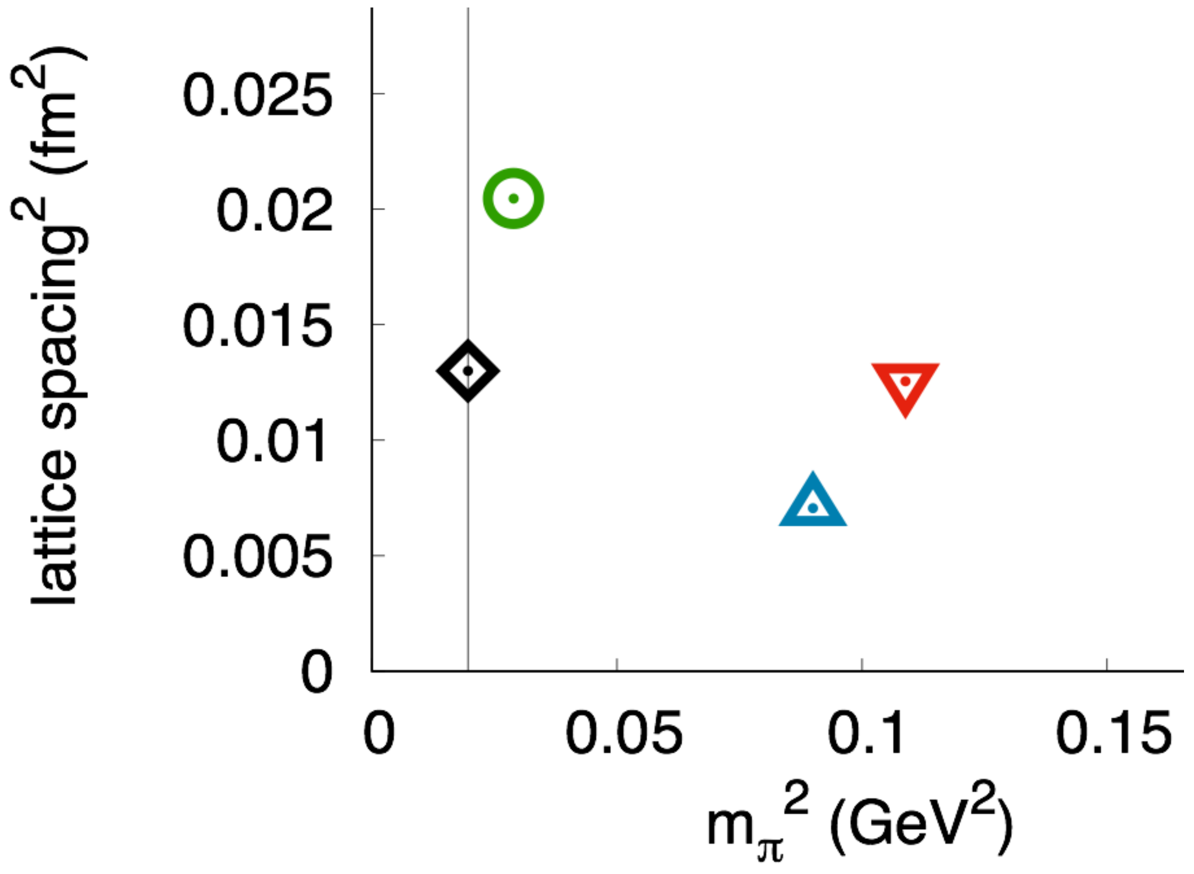
Proton mass decomposition

Momentum fractions of u and d quarks







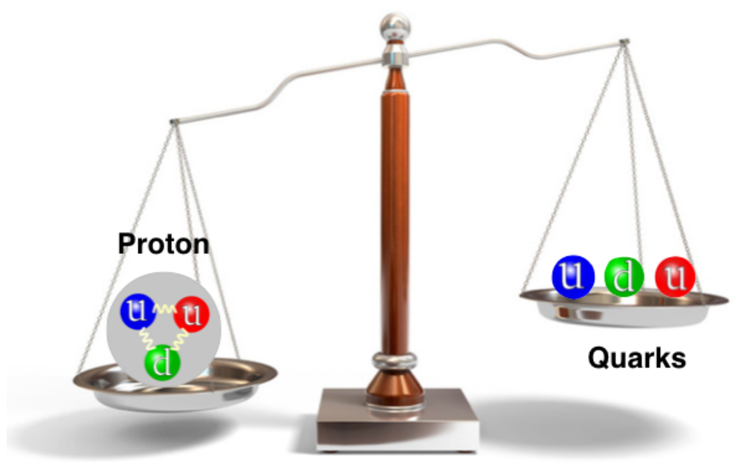


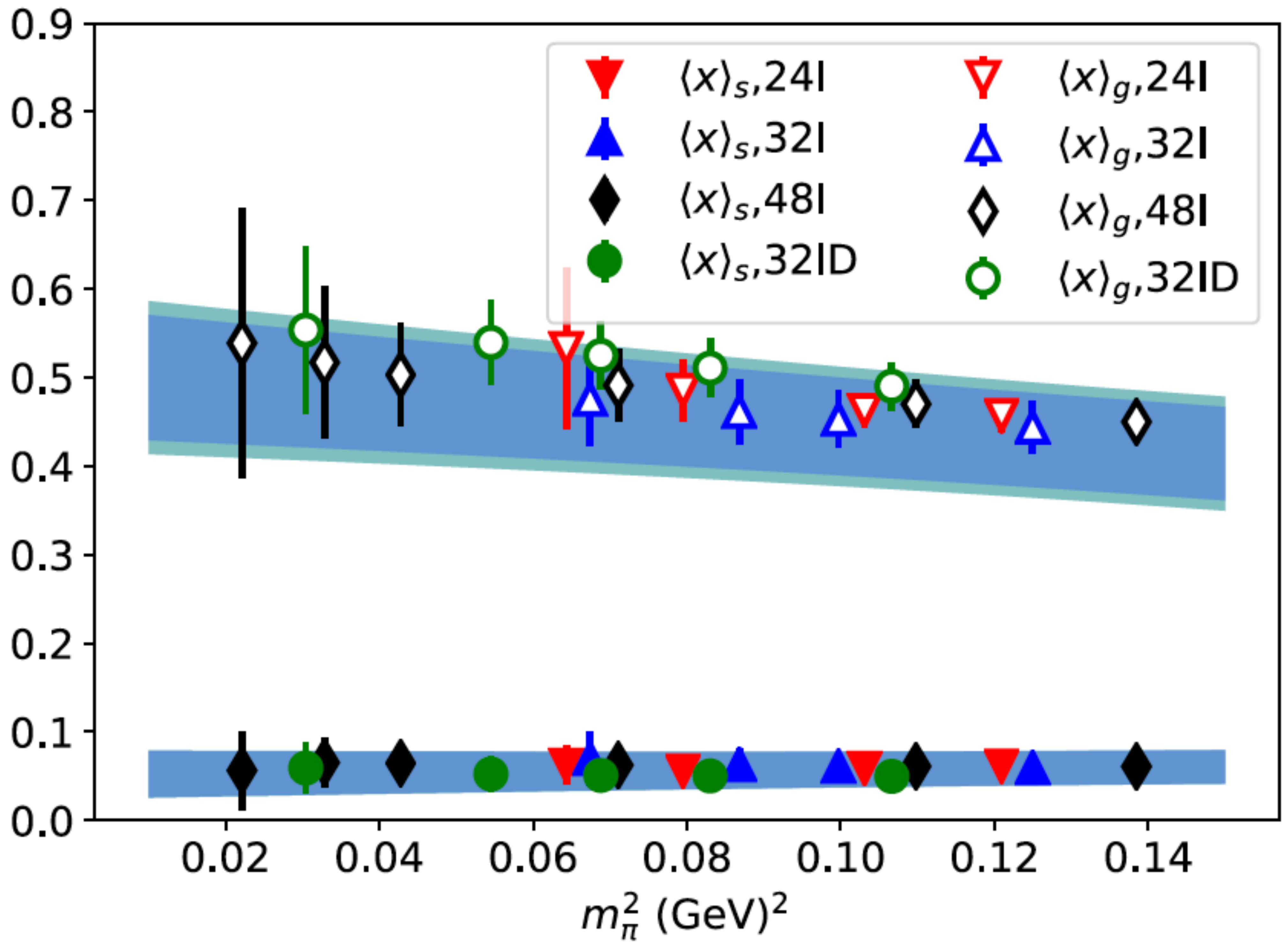
YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001,
ViewPoint and Editor's suggestion

Proton mass decomposition

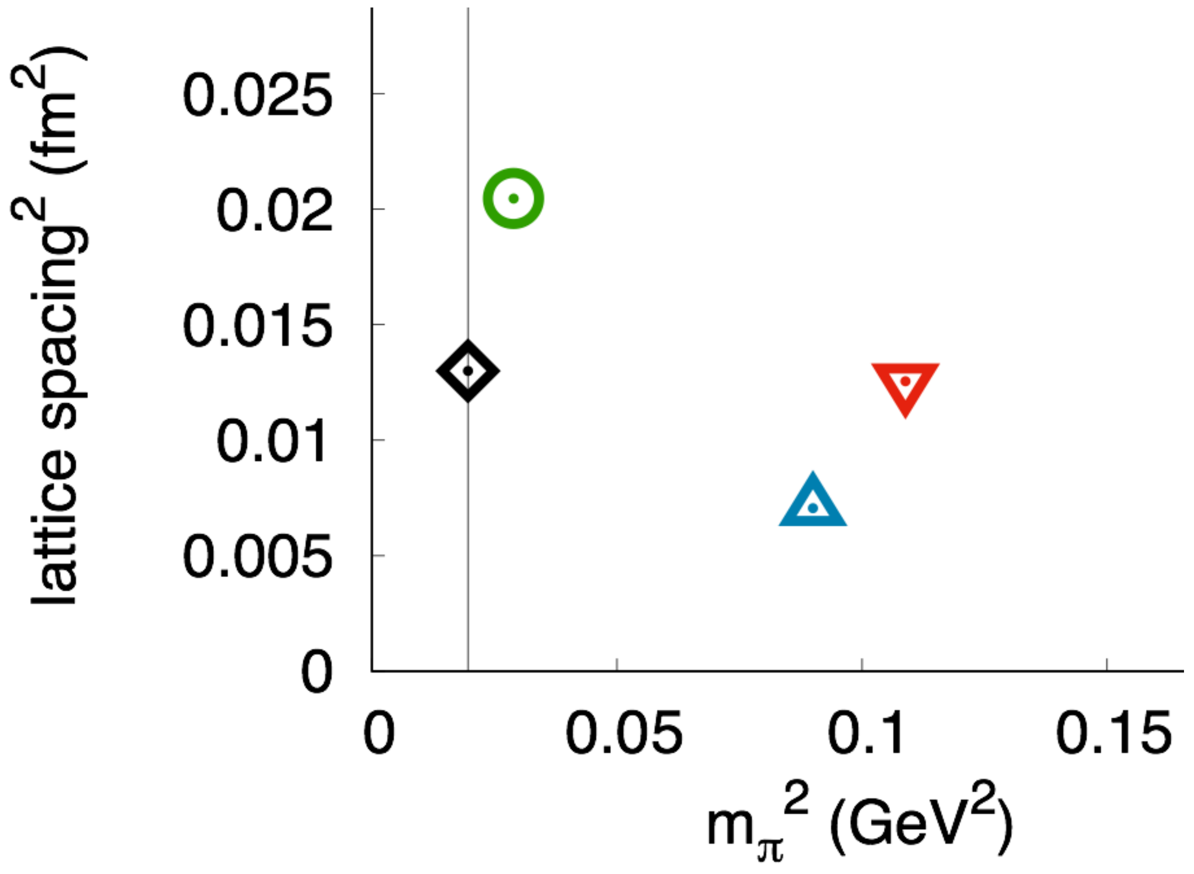
Pure DI momentum fractions:

strange quark and glue ones



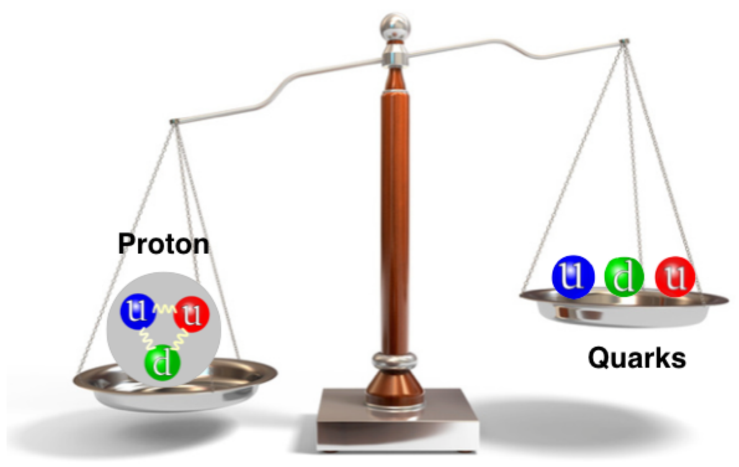


- The glue momentum fraction become **larger** when the quark mass is **lighter**;
- The strange one is small as expected.



YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001,
ViewPoint and Editor's suggestion

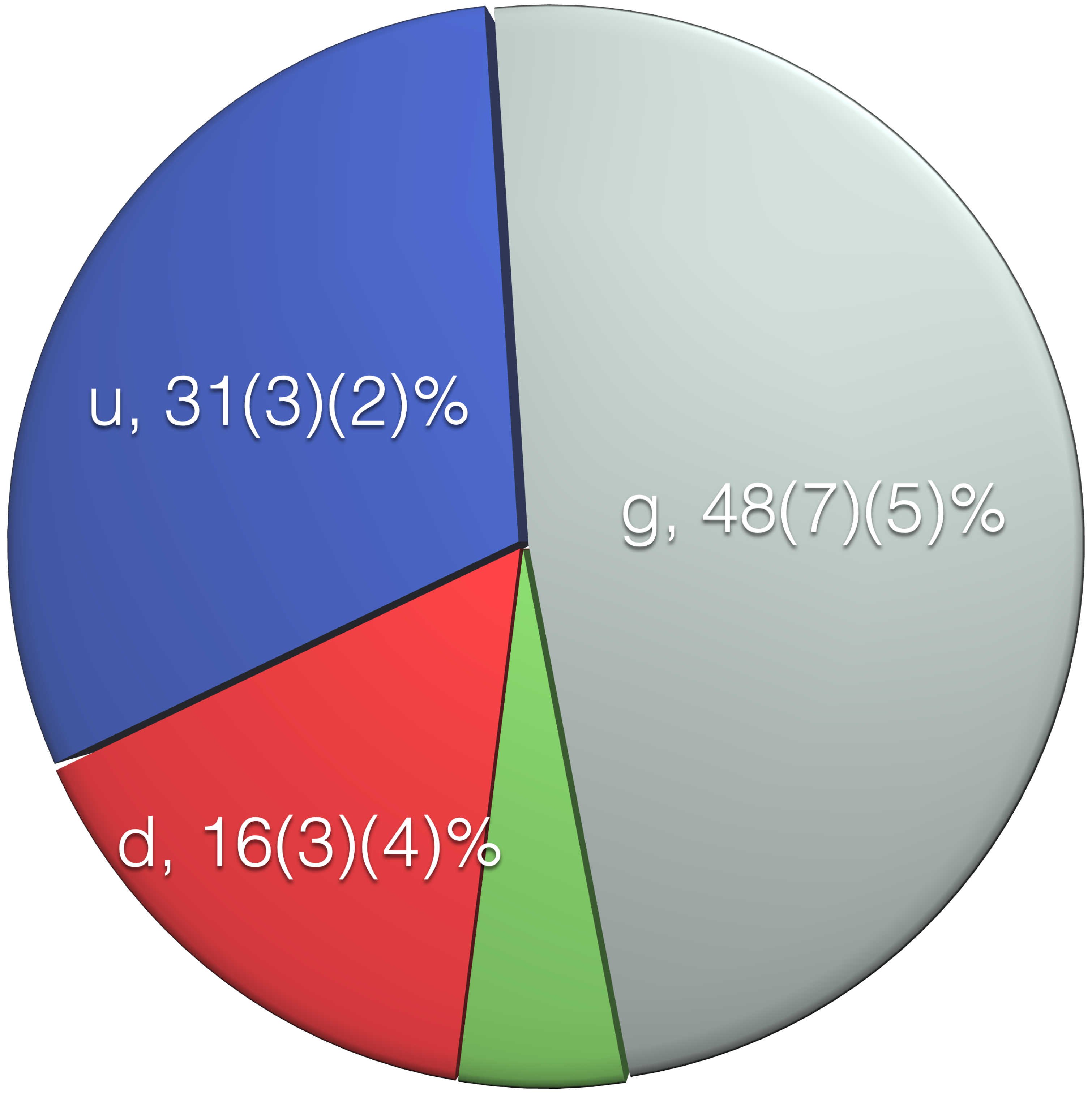
Proton mass decomposition

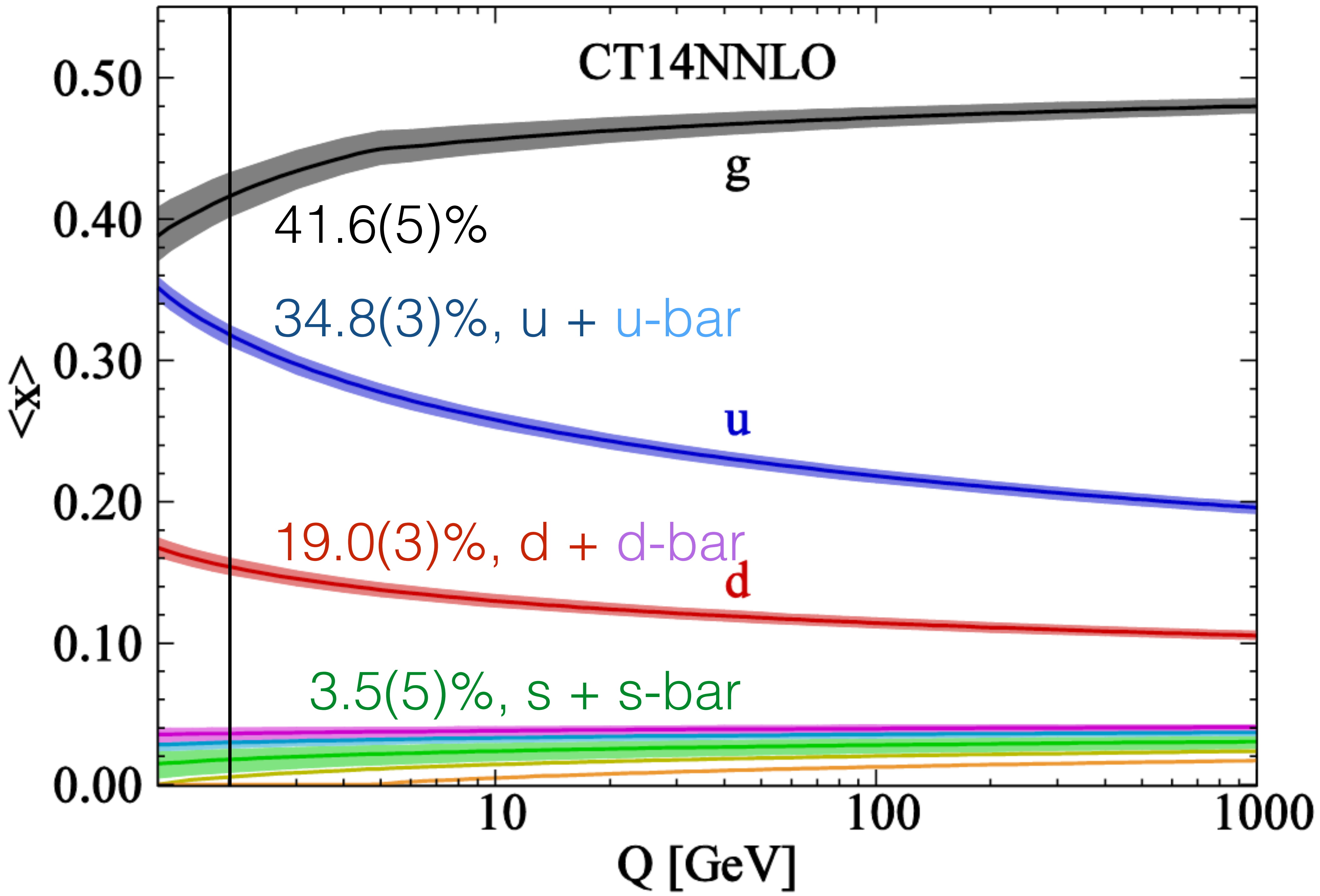


Comparing the momentum fractions

from the experiment

YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001, **ViewPoint and Editor's suggestion**

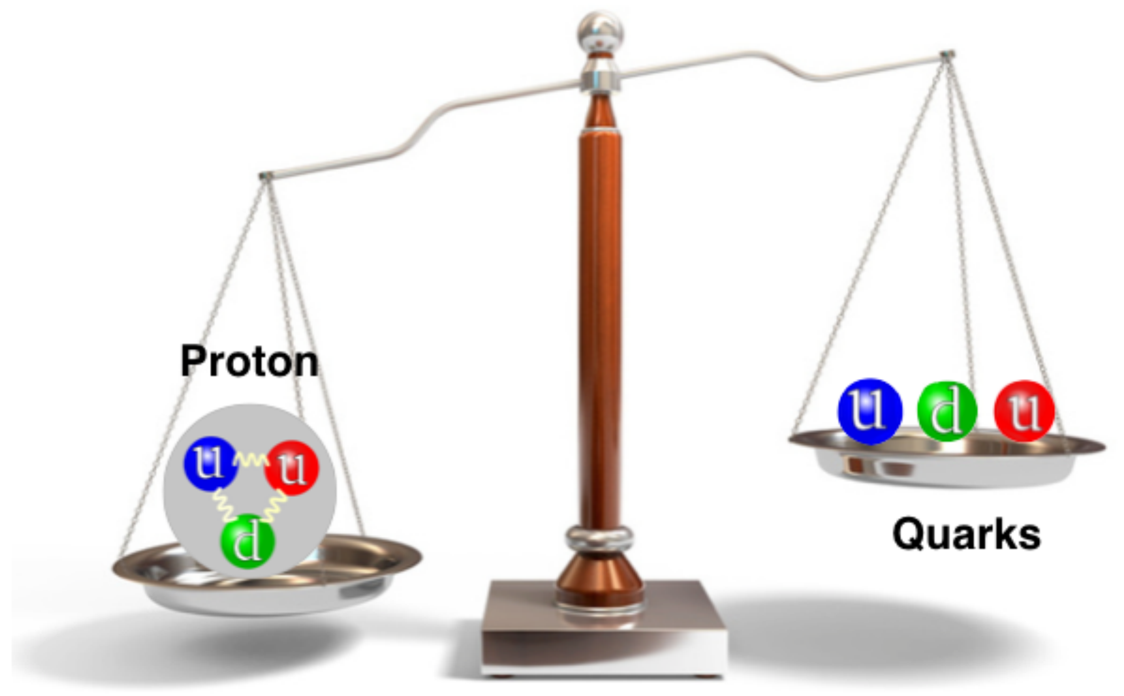




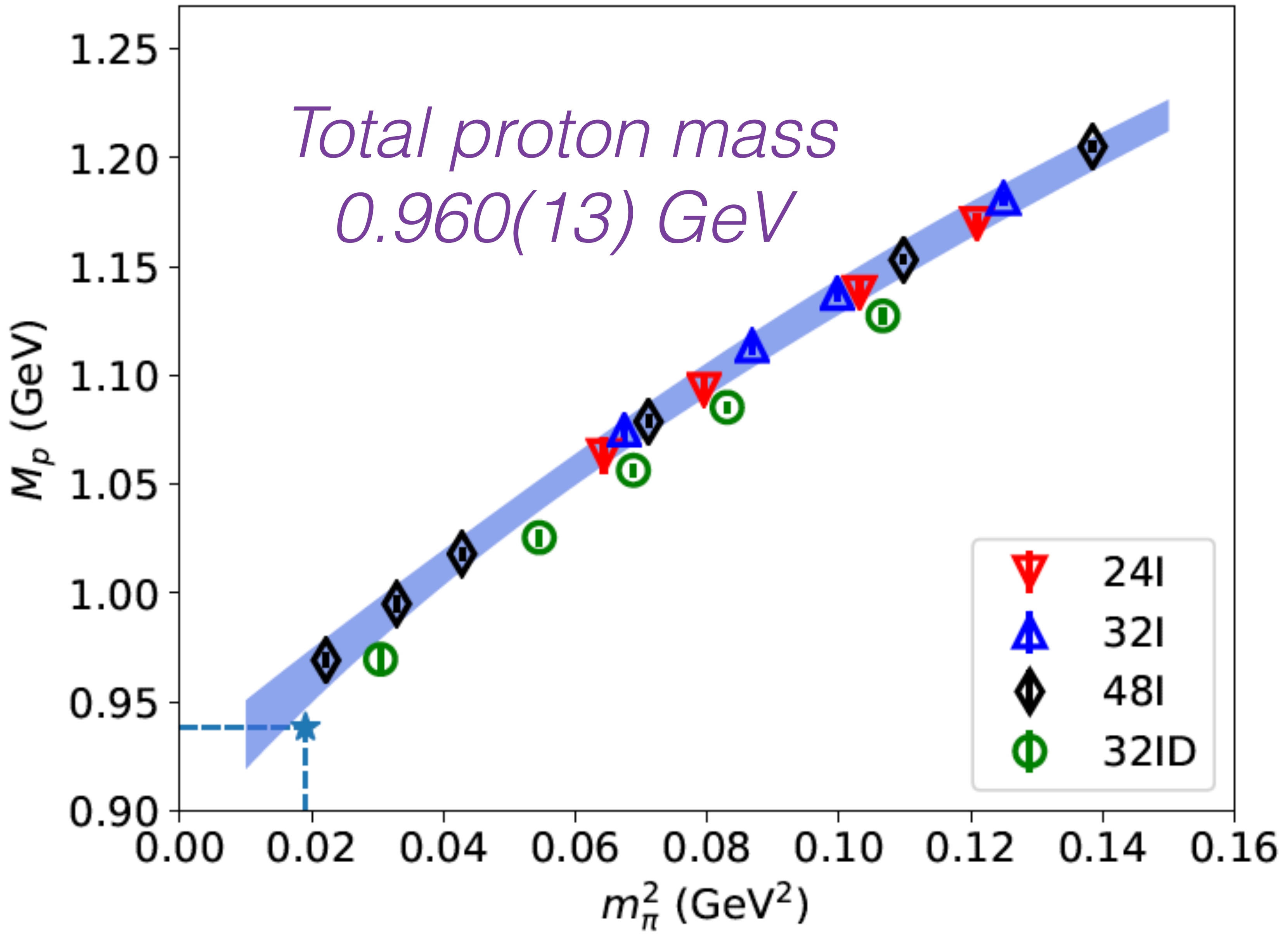
S. Dulat et al, PRD93(2016)033006

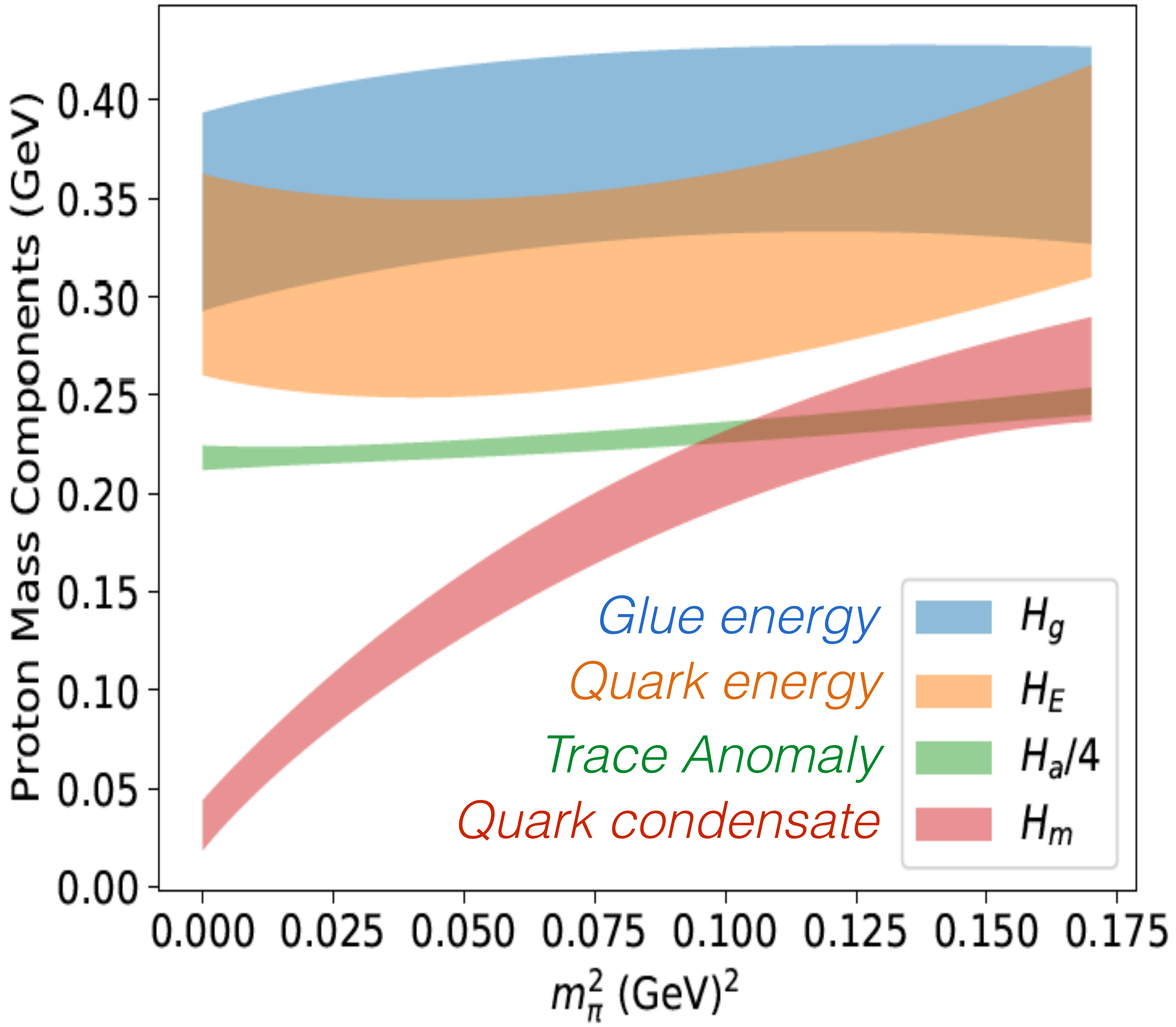
- Direct calculation of the quark/glue momentum fraction with non-perturbative renormalization and normalization.

- Trace anomaly contribution deduced by the direct calculation of the quark scalar condensate in nucleon, based on the sum rule



$$\frac{1}{4}M = -\langle \hat{T}_{44} \rangle = \frac{1}{4}\langle H_m \rangle + \frac{1}{4}\langle H_a \rangle$$





YBY, J. Liang, et. al., χQCD Collaboration, PRL121(2018)212001,
ViewPoint and Editor's suggestion

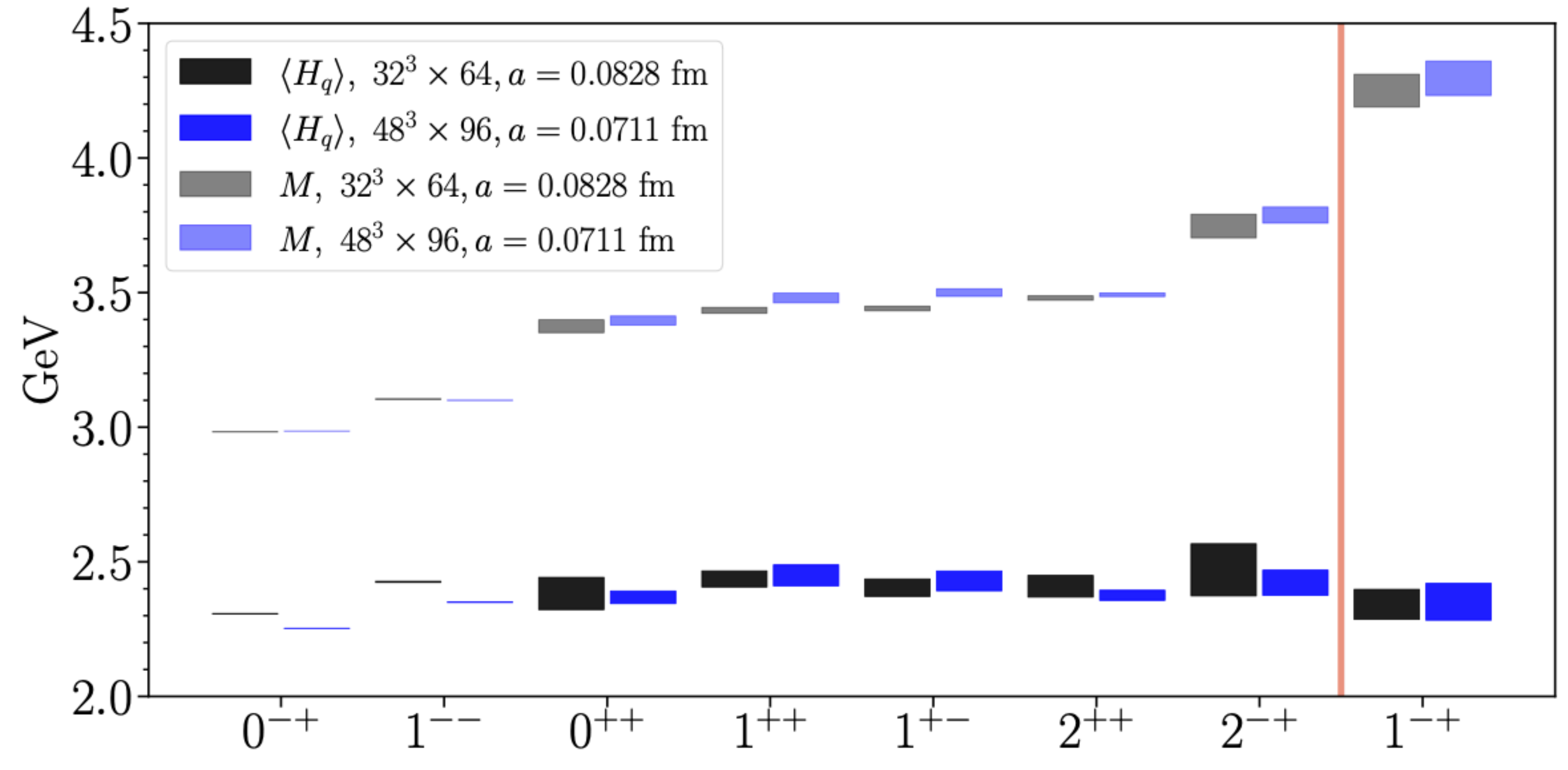
基于中科院战略先导专项C类No. XDC01040100的支持

粲偶素质量分解

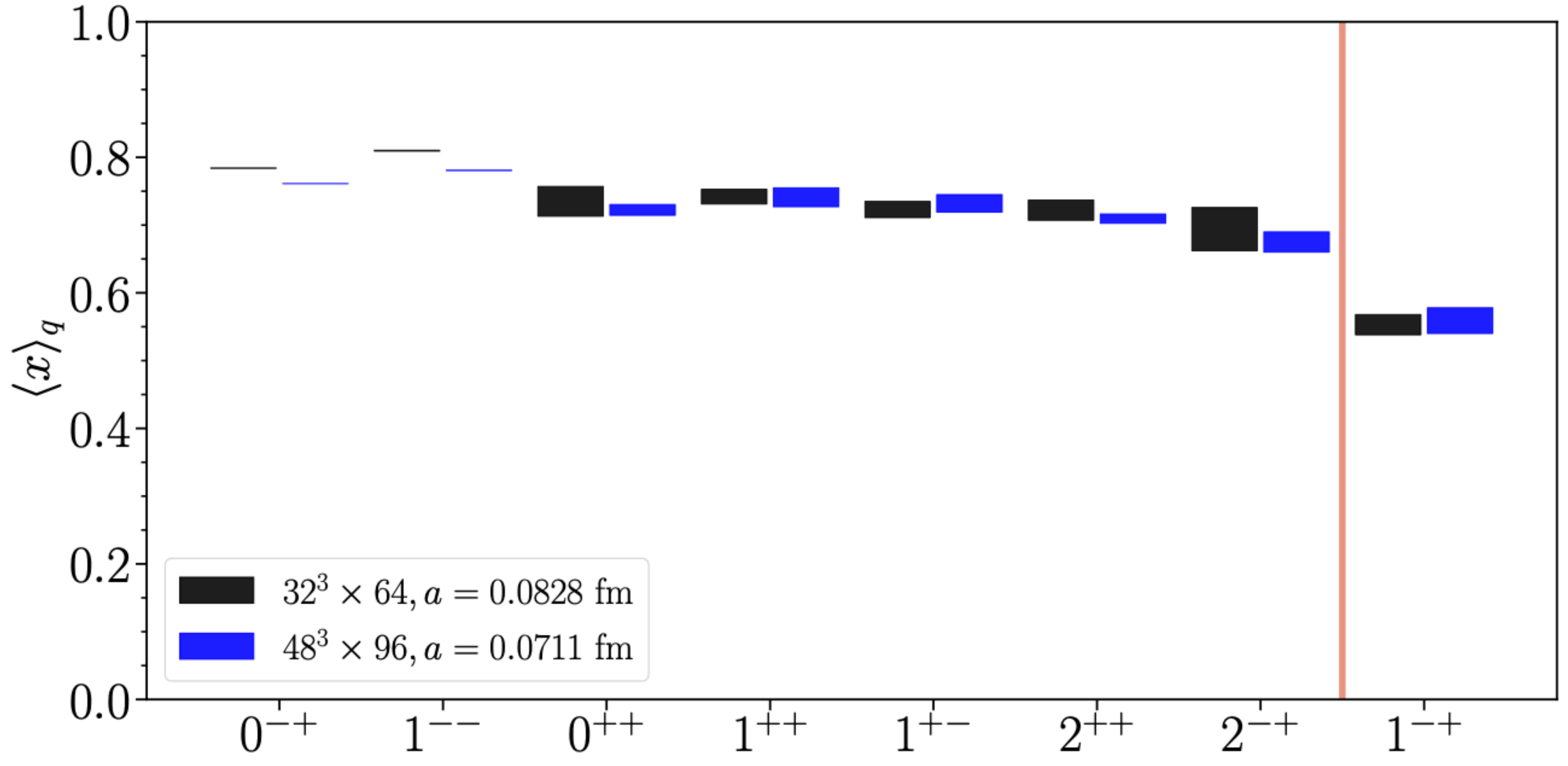
进一步的研究表明：

- 在各种粲偶素中，夸克总能量 $\langle H_q \rangle_H = \langle H_m \rangle_H + \langle H_E \rangle_H$ 几乎不依赖于强子态；

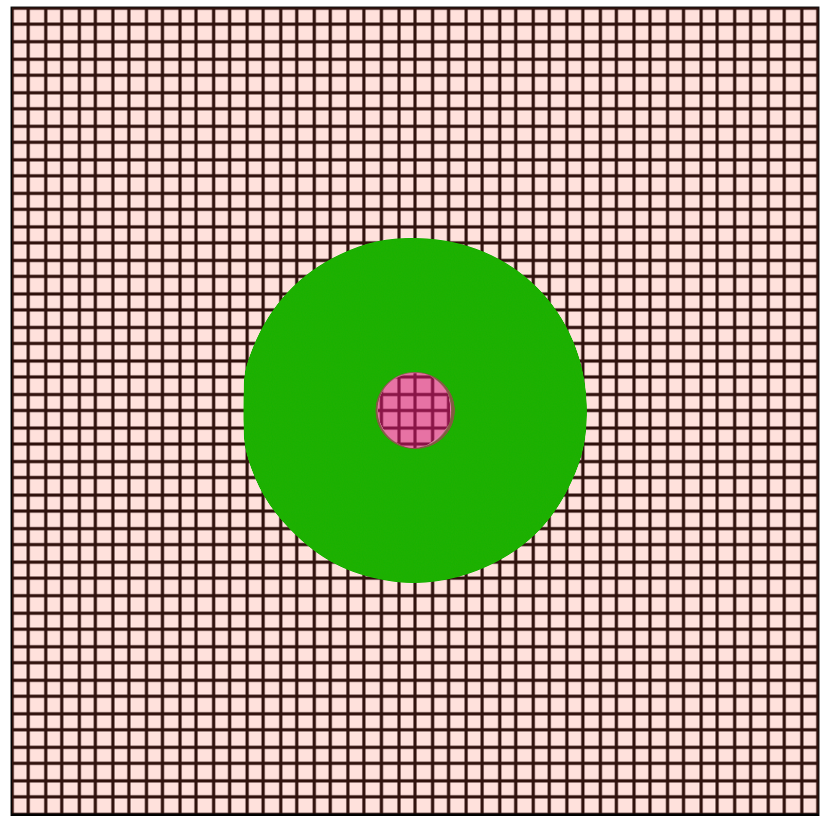
- 粲夸克动量分数 $\langle x_c \rangle_H$ 在各种正常量子数的粲偶素中均较大，而在反常量子数的疑似混杂态中较小。

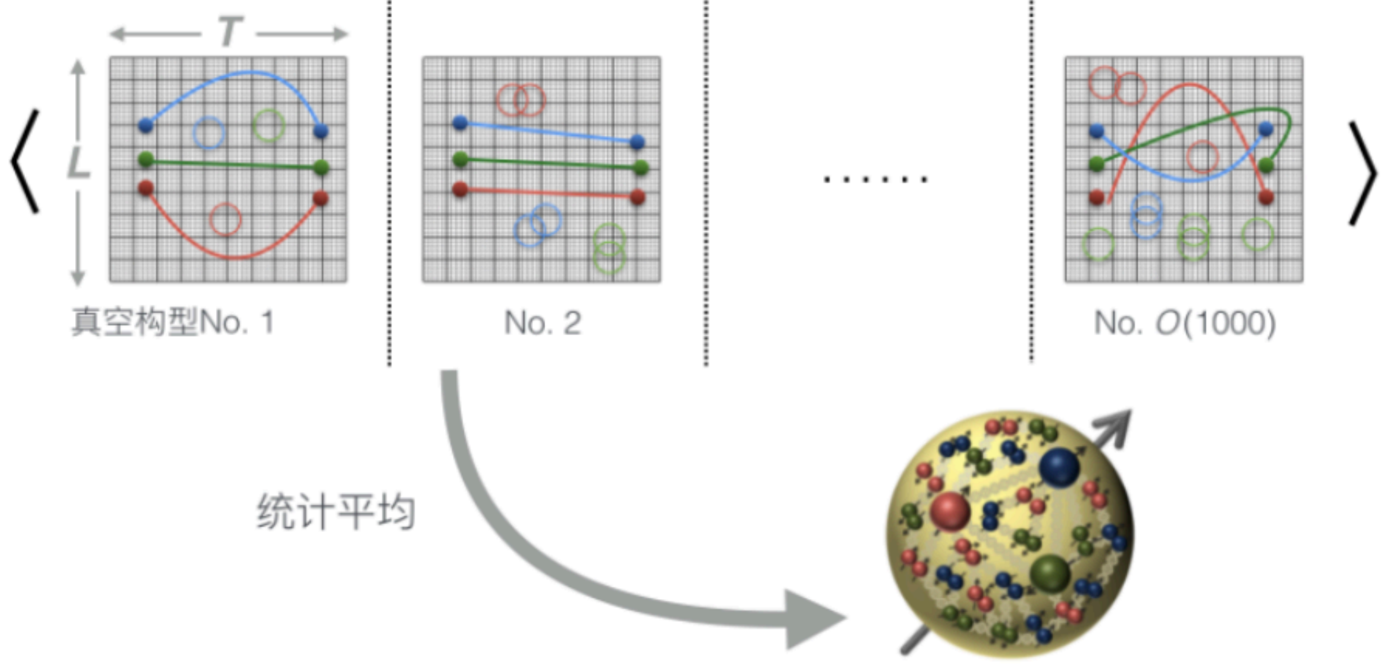


W. Sun et.al., χQCD, PRD 103 (2021) 094503

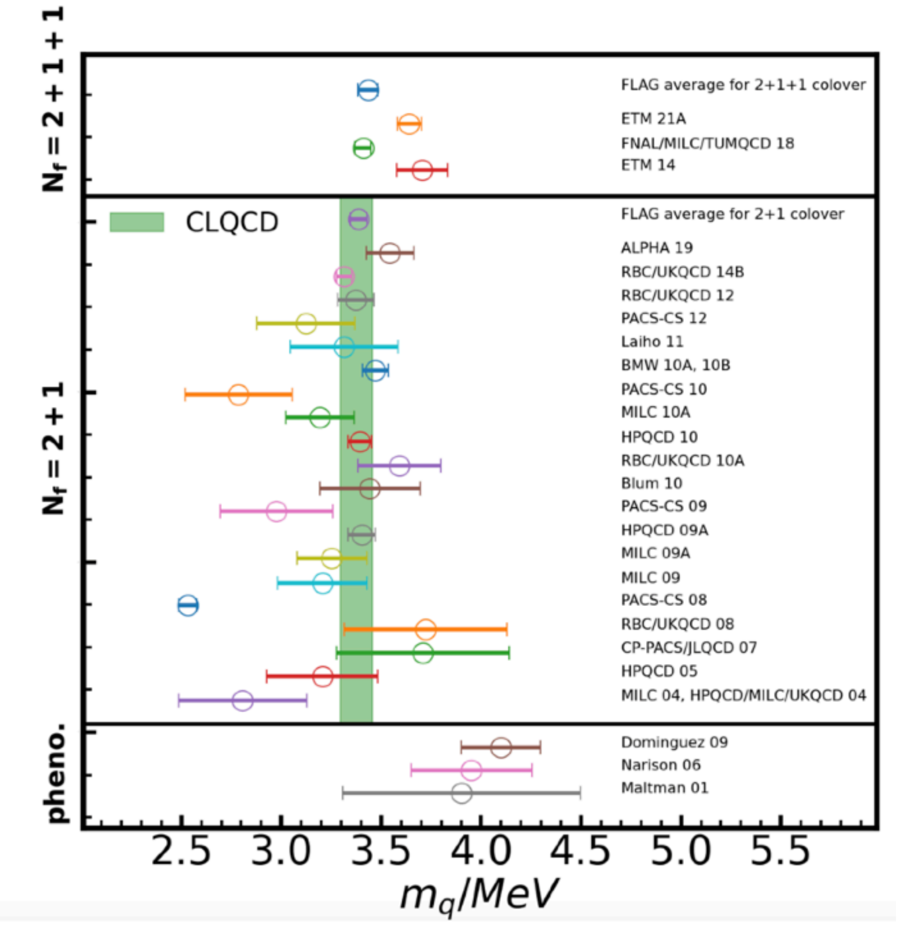


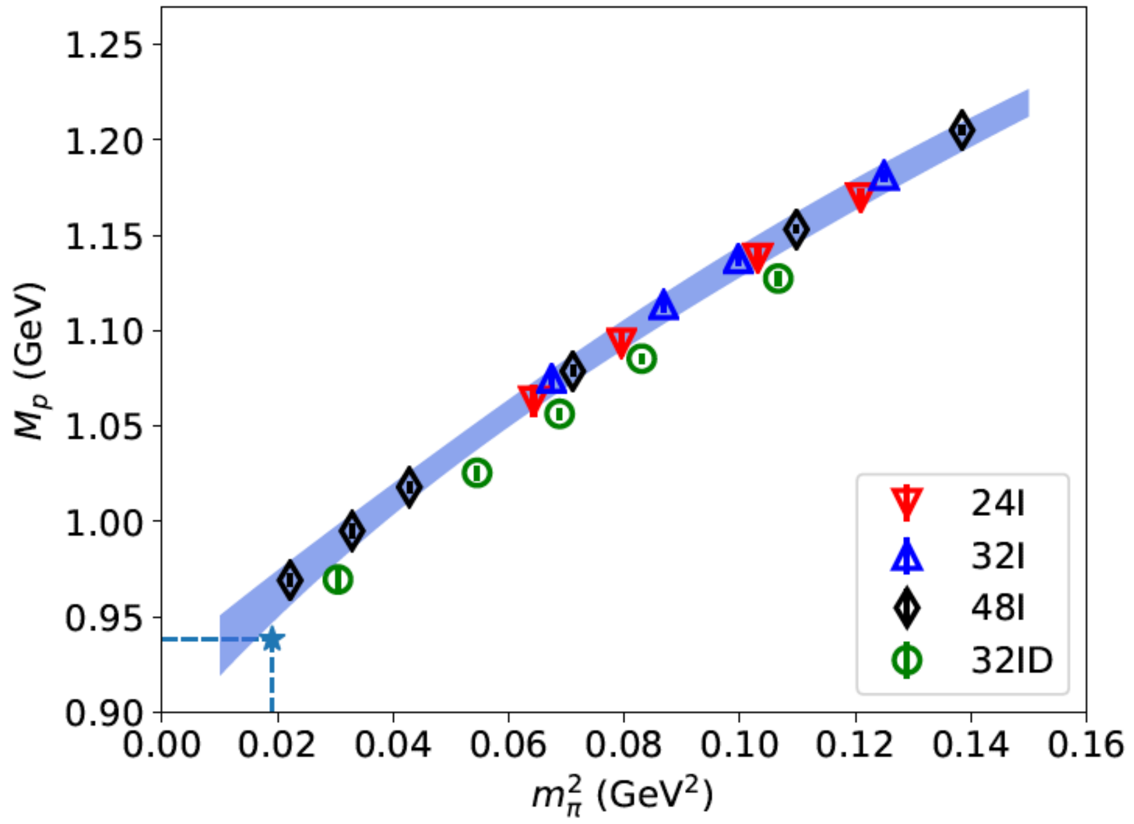
格点量子色动力学基础



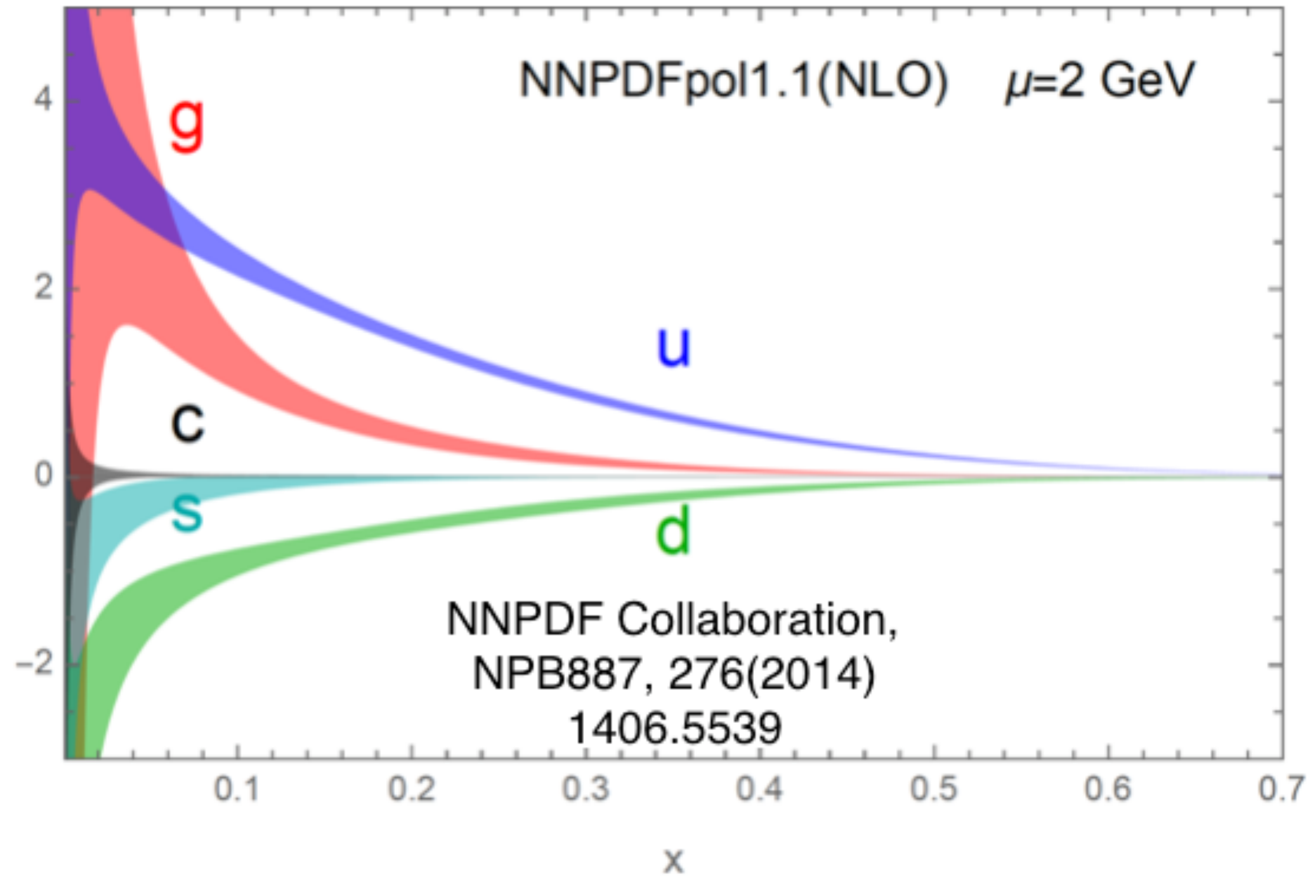


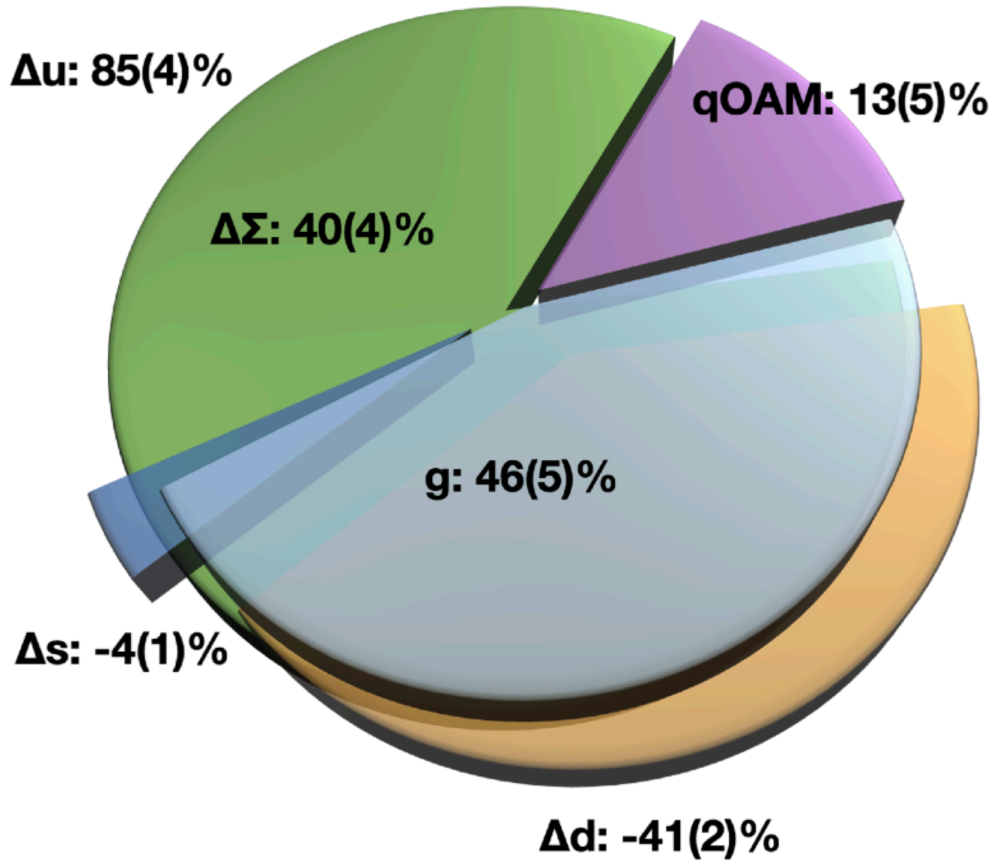
强子质量





强子自旋





长程格点QCD

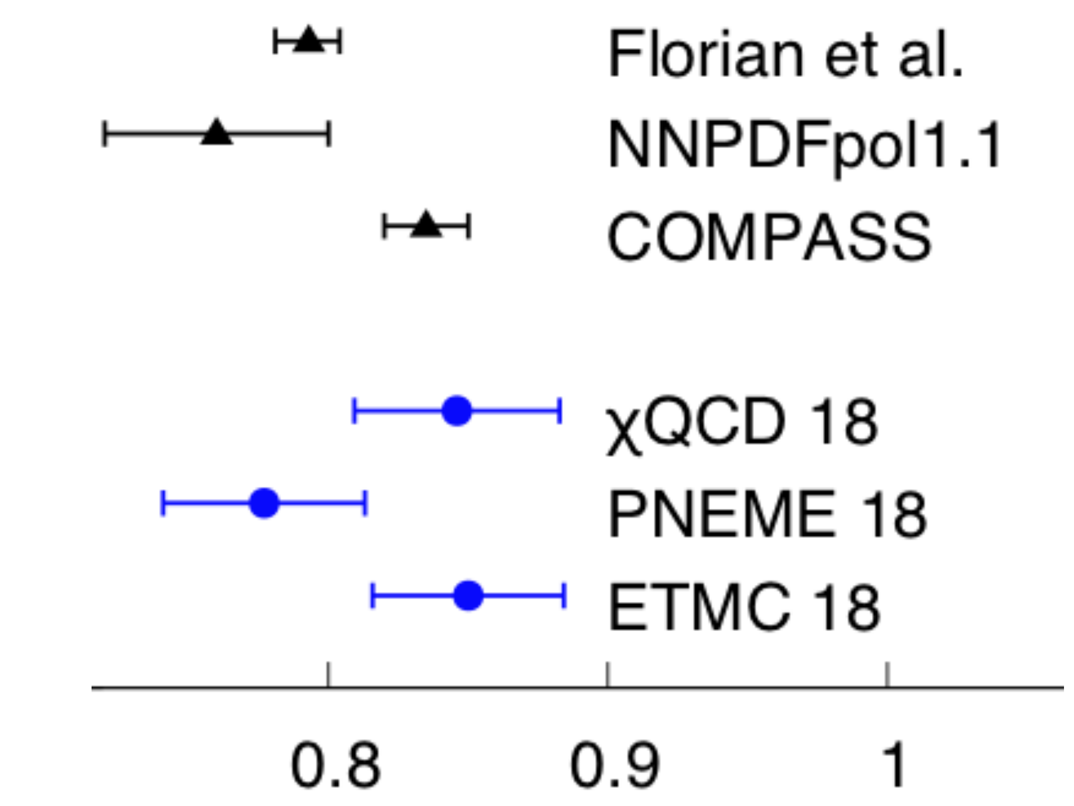
夸克自旋

	N_f	Disc	m_π	FV	Ren	ESC
χQCD 18	2+1	o	o	★	★	★
PNDME 18	2+1+1	o	★	★	■	★
ETMC 18	2+1+1	■	★	★	★	★

As the result at just one lattice spacing

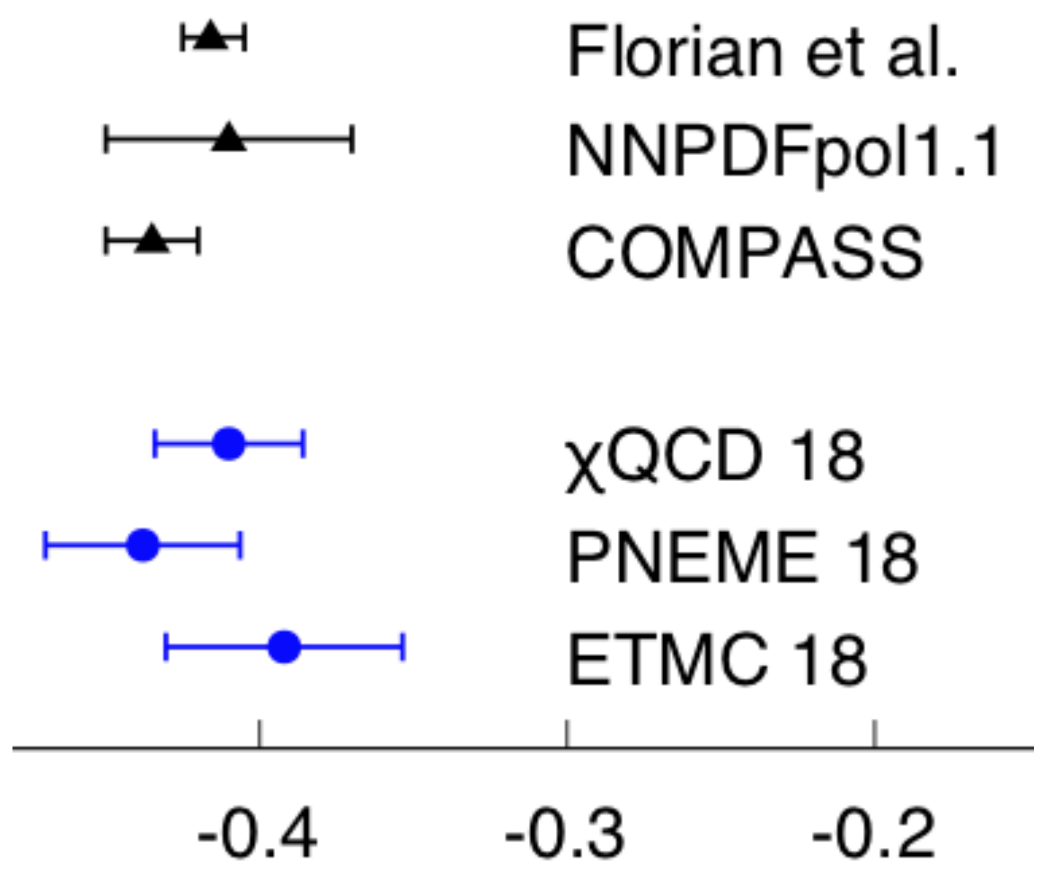
As the non-perturbative normalization only

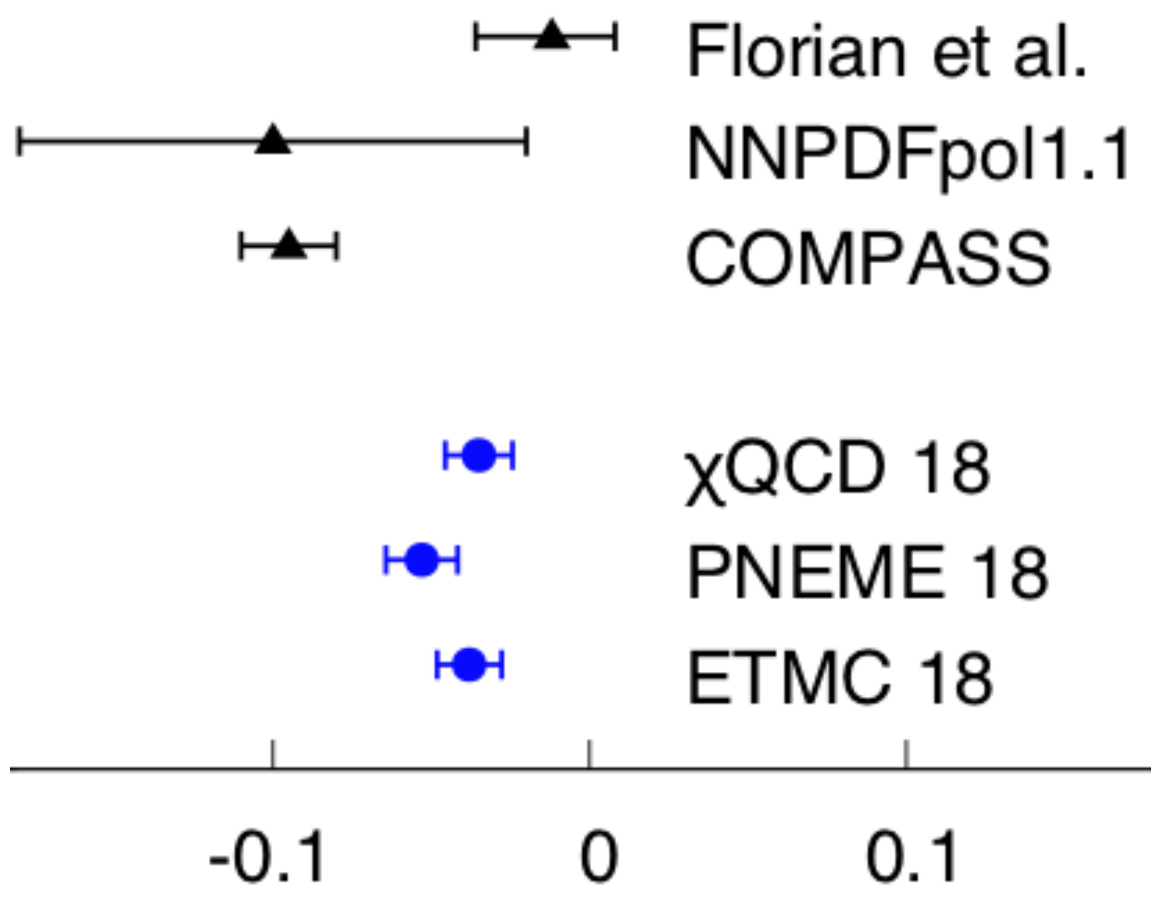
★ for NP renormalization + (A)WI normalization



Δu

Δd





Δs

长程格点QCD

核子矩阵元

http://flag.unibe.ch/2021/

	V	S	A	T
u	2	6.9(7)	0.77(4)	0.78(3)
d	1	5.8(7)	-0.44(4)	-0.20(2)
s	0	0.53(7)	-0.05(1)	-0.003(2)
u-d	1	1.1(1)	1.27(1)	0.98(3)

电荷数 质子-中子质量差 中子弱衰变

- $\overline{\text{MS}}$ 2GeV, $N^{+}_{1/2}$:
- 标量荷远大于矢量荷；
- 轴矢和张量荷的结果相似（d夸克贡献反号）
- s中贡献相对较小。

长程格点QCD

核子矩阵元的Dirac基分解

$$S=\bar{q}q=\bar{q}^{+}_{1/2}q^{+}_{1/2}+\bar{q}^{+}_{-1/2}q^{+}_{-1/2}+\bar{q}^{-}_{1/2}q^{-}_{1/2}+\bar{q}^{-}_{-1/2}q^{-}_{-1/2};$$

$$V_4=\bar{q}\gamma_4 q=\bar{q}^{+}_{1/2}q^{+}_{1/2}+\bar{q}^{+}_{-1/2}q^{+}_{-1/2}-\bar{q}^{-}_{1/2}q^{-}_{1/2}-\bar{q}^{-}_{-1/2}q^{-}_{-1/2};$$

$$\frac{1+\gamma_4}{2}=\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$ 非极化投影

$$\mathrm{Tr}[\frac{1+\gamma_4}{2}\langle N|\bar{q}\Gamma q|N\rangle]$$

$$A_3=\bar{q}\mathrm{i}\gamma_5\gamma_3 q=\bar{q}^{+}_{1/2}q^{+}_{1/2}-\bar{q}^{+}_{-1/2}q^{+}_{-1/2}-\bar{q}^{-}_{1/2}q^{-}_{1/2}+\bar{q}^{-}_{-1/2}q^{-}_{-1/2};$$

$$T_3=\bar{q}\mathrm{i}\gamma_5\gamma_4\gamma_3 q=\bar{q}^{+}_{1/2}q^{+}_{1/2}-\bar{q}^{+}_{-1/2}q^{+}_{-1/2}+\bar{q}^{-}_{1/2}q^{-}_{1/2}-\bar{q}^{-}_{-1/2}q^{-}_{-1/2};$$

$$\mathrm{i}\gamma_3\gamma_5=\begin{bmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{bmatrix}$$ 极化投影

$$\mathrm{Tr}[\frac{1+\gamma_4}{2}\mathrm{i}\gamma_5\gamma_3\langle N|\bar{q}\Gamma q|N\rangle]$$

自旋向上的quark　自旋向下的quark　自旋向上的反quark　自旋向下的反quark

$$=2\mathrm{Tr}[\langle N^{+}_{1/2}|\bar{q}\Gamma q|N^{+}_{1/2}\rangle]$$

长程格点QCD

核子矩阵元的Dirac基分解

	V	S	A	T	$\bar{q}^+_{1/2}q^+_{1/2}$	$\bar{q}^+_{-1/2}q^+_{-1/2}$	$\bar{q}^-_{1/2}q^-_{1/2}$	$\bar{q}^-_{-1/2}q^-_{-1/2}$
u	2	6.9(7)	0.77(4)	0.78(3)	~2.6 ~0.6	~1.8	~1.2	~1.2
d	1	5.8(7)	-0.44(4)	-0.20(2)	~1.5 ~0.5	~1.9	~1.3	~1.1
s	0	0.53(7)	-0.05(1)	-0.003(2)	~0.12	~0.15	~0.15	~0.12
u-d	1	1.1(1)	1.27(1)	0.98(3)	~1.1 ~0.1	~-0.04	~-0.05	~0.1

- $\overline{\text{MS}}$ 2GeV, $N^+_{1/2}$:
- 反夸克中自旋向上/向下的量差不多；
- s中各种分量贡献都接近；
- u-d中主要的贡献是来自于自旋向上的quark。
- 也许可以定义出一个对u和d差不多的“sea”贡献

长程格点QCD

核子矩阵元的Dirac基分解

	V	S	A	T	$\bar{q}^{+}_{1/2}q^{+}_{1/2}$	$\bar{q}^{+}_{-1/2}q^{+}_{-1/2}$	$\bar{q}^{-}_{1/2}q^{-}_{1/2}$	$\bar{q}^{-}_{-1/2}q^{-}_{-1/2}$
u	2	6.9(7)	0.77(4)	0.78(3)	~2.6 ~0.6	~1.8	~1.2	~1.2
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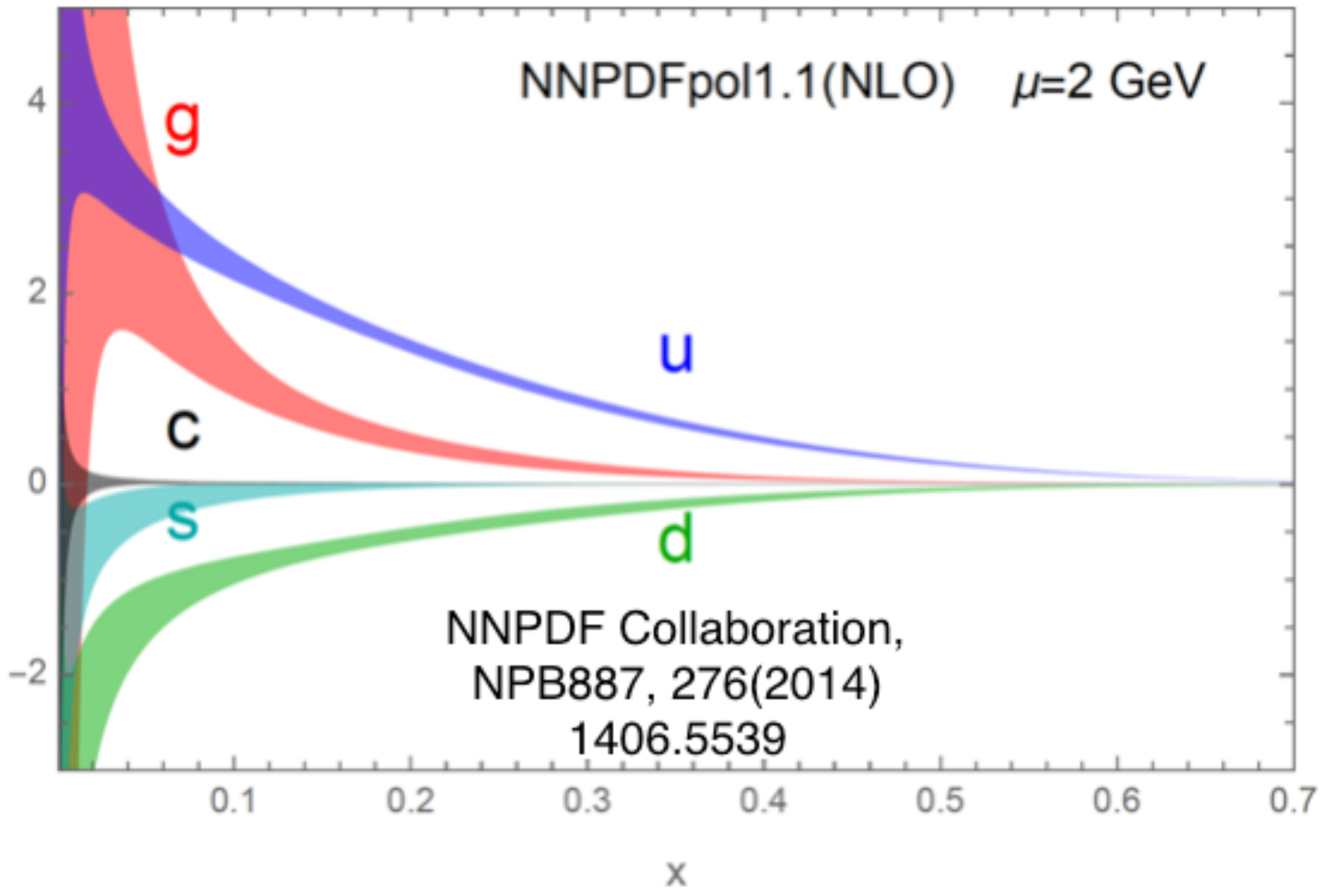
s	0	0.53(7)	-0.05(1)	-0.003(2)	~0.12	~0.15	~0.15	~0.12
u-d	1	1.1(1)	1.27(1)	0.98(3)	~1.1 ~0.1	~-0.04	~-0.05	~0.1

- 上述结果是标度依赖的；
- 重夸克极限下的形式并不清楚；
- 如何与光锥上的部分子分布函数联系也不完全清楚。

长程格点QCD

自旋与极化的部分子分布函数



夸克极化 (*u*,*d*,*s*...): 夸克极化分布函数 (PDF) 的积分：

$$\Delta q = \int_0^1 dx \Delta q(x)$$

胶子极化 (*g*): 胶子极化分布函数的积分

$$\Delta G = \int_0^1 dx \Delta g(x)$$

- 夸克模型预言Δu→4/3, Δd→-1/3, Δs→0, Δg→0;

PDG, CPC40, 100001 (2016)

- 但是中子衰变常数要求Δu-Δd ≈ **1.2723(23),** 而夸克极化分布函数的唯象分析给出 Δu~0.8, Δd~-0.4, Δs~-0.1, Δg~0.4;

EPJA52, 268 (2016), 1212.1701

D. Florian, PRL 113, 012001 (2014), 1404.4293

- 核心在于夸克质量很轻，在质子中可以高速运动！

The decomposition

of the proton spin

Quark spin/helicity: the integration of the quark helicity distribution

$$\Delta q \;=\; \int_0^1 dx \Delta q(x) = \int_0^1 dx \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|\psi_q(\xi^-)\gamma_5\gamma^+\mathcal{L}(\xi^-,0)\psi_q(0)|PS\rangle$$

Glue helicity: that of the quark helicity distribution

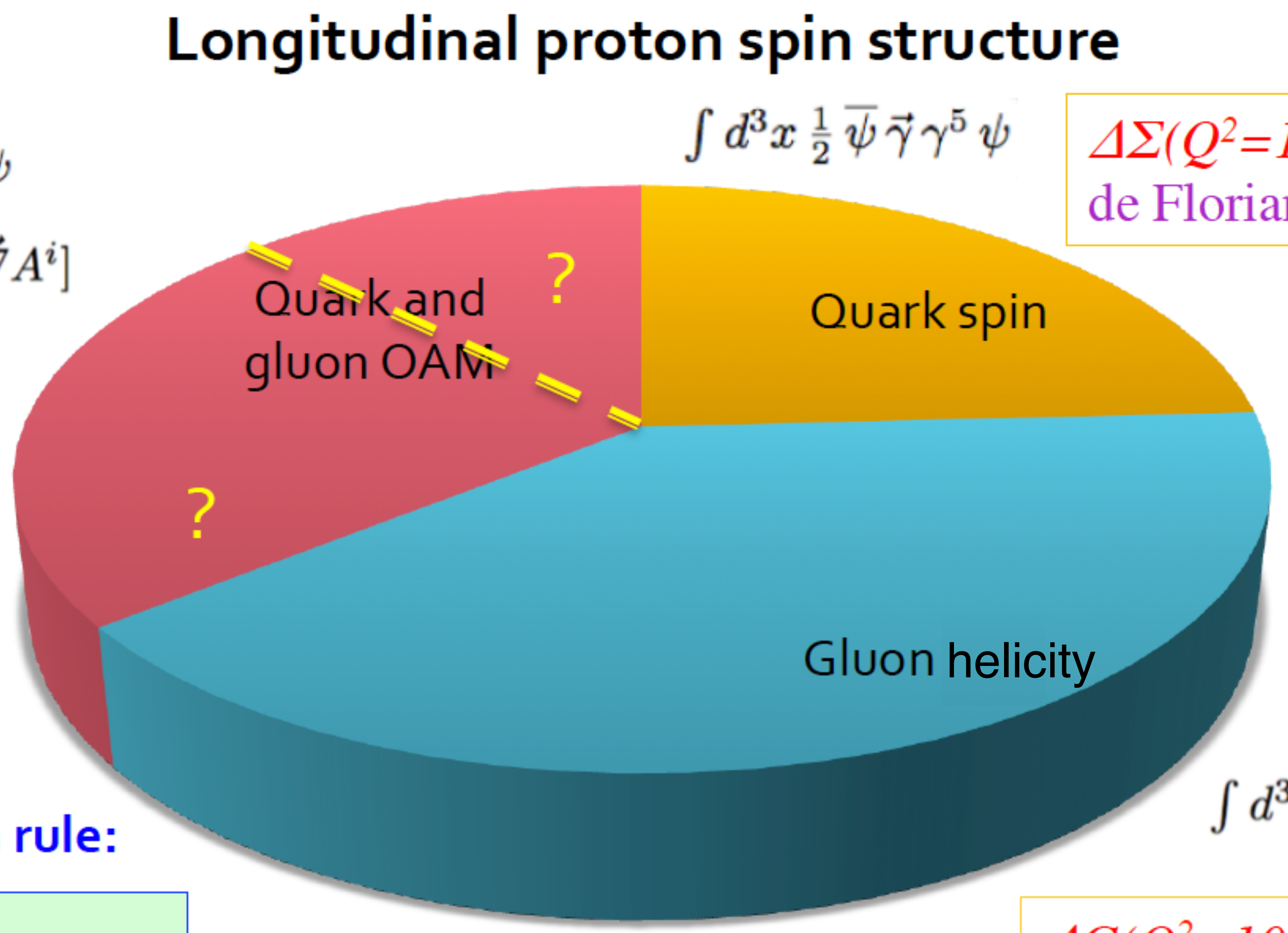
$$\Delta G \;=\; \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle$$

The rest parts should be the orbital angular momentums,

$$L_q + L_g = \frac{1}{2} - \frac{1}{2}\sum_{q=u,\bar{u},d,\bar{d},s..} \Delta q - \Delta G$$

Proton Spin decomposition

From the experiments



$\int d^3x \psi^\dagger \{\vec{x} \times (i\vec{\nabla})\} \psi$

$+ \quad \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]$

$\int d^3x \frac{1}{2} \overline{\psi} \vec{\gamma} \gamma^5 \psi$

$\Delta\Sigma(Q^2{=}10\ \mathrm{GeV}^2) = 0.242$, de Florian et al., 2009

SLAC
HERMES (DESY)
COMPASS (CERN)
JLab
RHIC

$\int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}]$

$\Delta G(Q^2{=}10\ \mathrm{GeV}^2) \sim 0.2$, de Florian et al., 2014

Naïve spin sum rule:

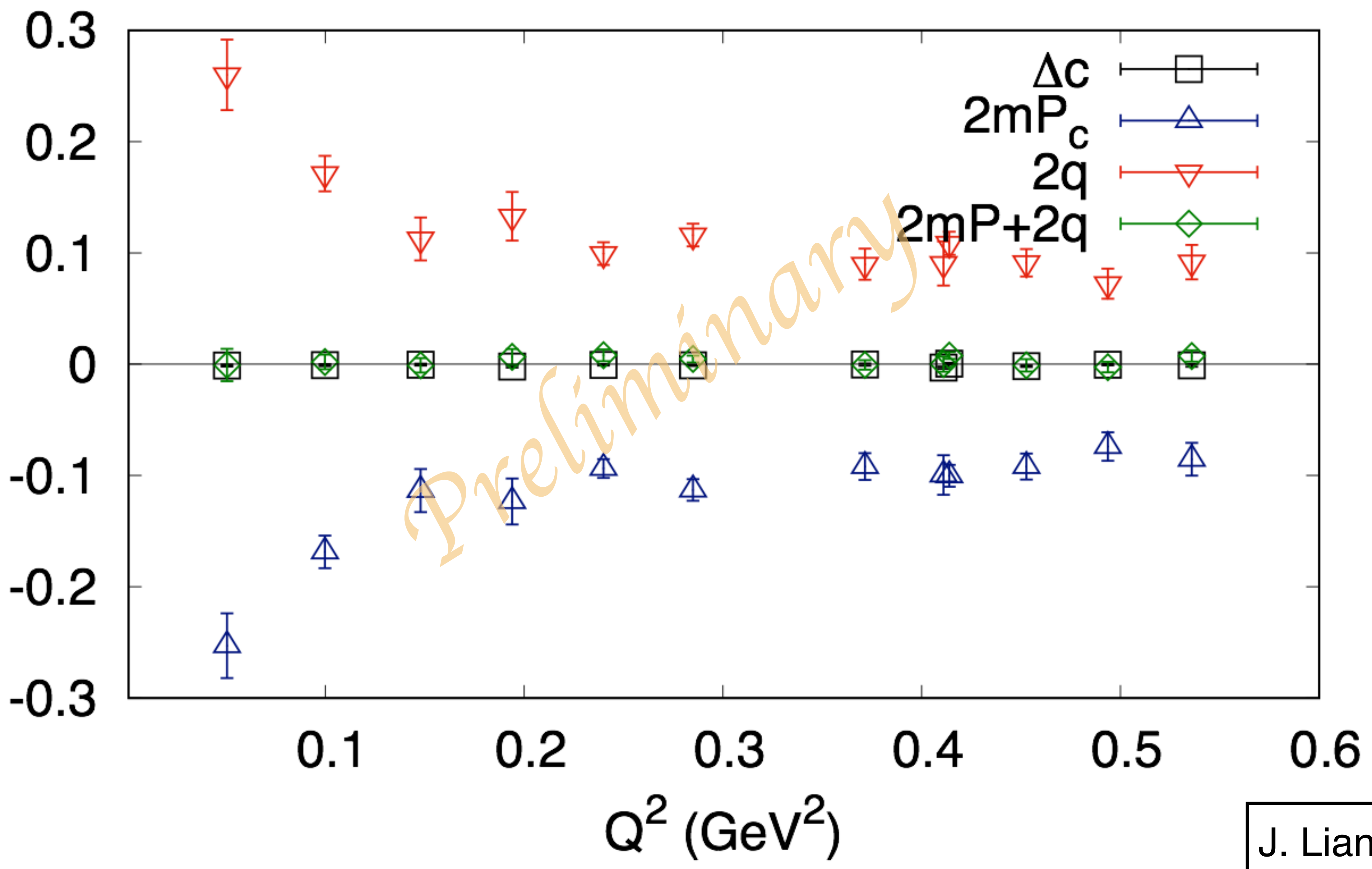
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + l_q^z + l_q^z$$

Quark spin

a story behind Δc

$$\langle ps \left| \vec{A}_\mu \cdot \vec{s} \right| ps \rangle = \lim_{p'-p\to 0} \frac{i|\vec{s}|}{(\vec{p'}-\vec{p})\cdot\vec{s}} \langle p', s| 2m_f \mathcal{P} - 2i\frac{\alpha_s}{4\pi} F\tilde{F} \, |p, s\rangle$$

$$= 2m_f \langle p, s| \int d^3x \; \vec{x}\cdot\vec{s} \; \mathcal{P}(x) |p, s\rangle - 2i \langle p, s| \int d^3x \; \vec{x}\cdot\vec{s} \; \frac{\alpha_s}{4\pi} F(x)\tilde{F}(x) \, |p, s\rangle$$



- The contribution from the glue anomaly turns out to be **very large** (~0.2-0.3 per flavor)!
- Accurate calculation of the light flavor cases is ongoing.

J. Liang, **YBY**, et. al., χQCD collaboration, in preparation

M. Gong, **YBY**, et. al., χQCD collaboration, PRD95, 114509 (2017), 1511.03671

Glue helicity

A possible simplification

The glue helicity is defined as,

A. V. Manohar, Phys. Lett. B255, 579 (1991)

$$\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS | F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) | PS \rangle$$

and the glue helicity operator itself can be rewritten into,

Y. Hatta, Phys. Rev. D84, 041701 (2011),
X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

$$\begin{aligned}
\tilde{S}_g &= \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) \\
&= \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} 2\mathrm{Tr}[F^{+\alpha}(\xi^-) \mathcal{L}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) \mathcal{L}(0, \xi^-)] \\
&= 2\mathrm{Tr}\left[\vec{E}(0) \times (\vec{A}(0) - \int \frac{d\xi^-}{2\pi} \int dx\, i \frac{e^{-ixP^+\xi^-}}{2xP^+} \mathcal{L}(0, \xi^-) \vec{\nabla} A^+(\xi^-) \mathcal{L}(\xi^-, 0))\right] \\
&\xrightarrow[A^+=0]{} 2\mathrm{Tr}[\vec{E}(0) \times \vec{A}(0)] = \vec{E}^a(0) \times \vec{A}^a(0)
\end{aligned}$$

But it can not be calculated on the lattice directly.

LaMET

The available gauge conditions

- The **glue helicity** operator equivalent to ExA under the light cone gauge.

$$O_{\Delta_G} = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-,0))\right]^z = \vec{E}_{LC} \times \vec{A}_{LC},\ A^+_{LC} = 0$$

- We have to use the Lorentz in-covariant **glue spin (quasi-glue helicity)** operators to reach it in the large momentum limit:

Hatta, Ji and Zhao, Phys. Lett. B743, 180 (2015)

$\boxed{O_{S^c_G} = \vec{E}^c \times \vec{A}^c,\ \partial_i A^c_i = 0}$ or $\boxed{O_{S^a_G} = \vec{E}^a \times \vec{A}^a,\ A^a_z = 0}$ or $\boxed{O_{S^t_G} = \vec{E}^t \times \vec{A}^t,\ A^t_0 = 0}$

Coulomb gauge ***Axial gauge*** ***Temporal gauge***

or something else…

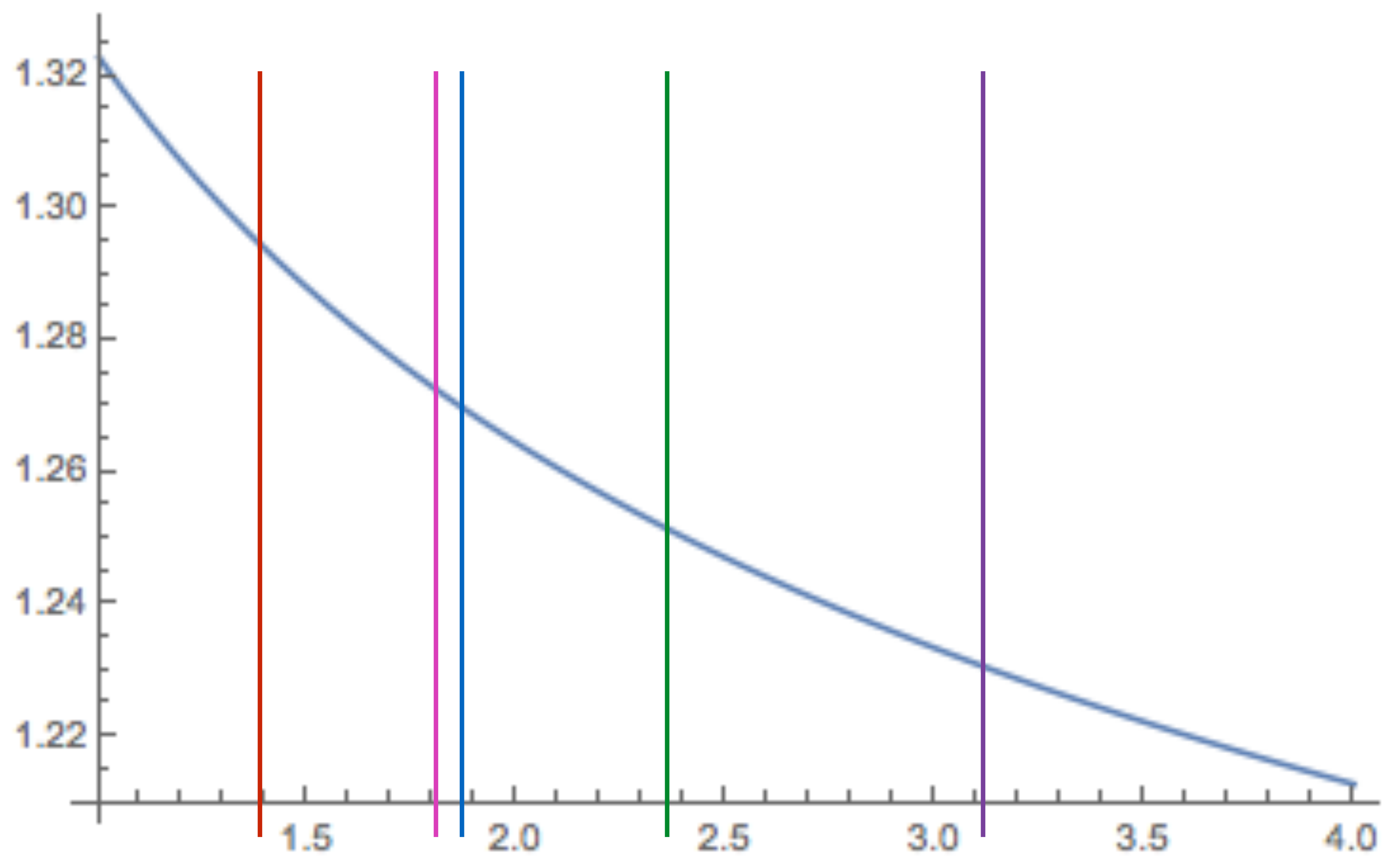
Glue spin

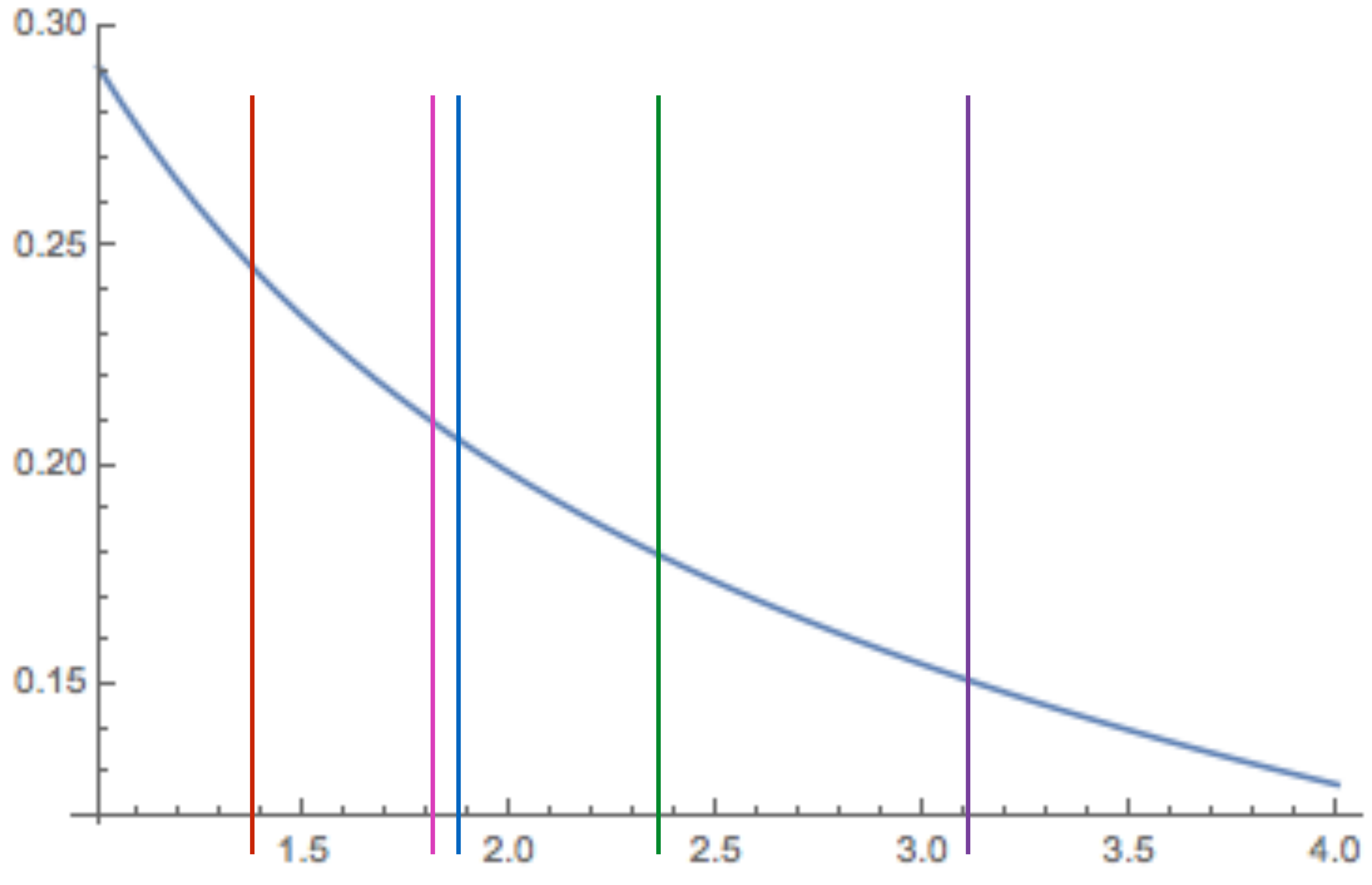
The renormalization and mixing

$$
\begin{aligned}
S^{\overline{MS}}_{G,(1)} &= \Big(1 - \frac{g^2 N_f}{16\pi^2}\Big[\frac{2}{3}\log(\mu^2 a^2) + 2.41\Big] + \frac{g^2 C_A}{16\pi^2}\Big[-\frac{4}{3}\log(\mu^2 a^2) + \sim 2\Big]\Big) S^{L}_{G,(1)} \\
&+ \frac{g^2 C_F}{16\pi^2}\Big[\frac{5}{3}\log(\mu^2 a^2) + 6.99\Big] \sum_{q=u,d,s\ldots} \Delta_q^{L,(1)} + O(a^2 p^2) + O(g^4)
\end{aligned}
$$

The overlap fermion and Iwasaki gluon with HYP smearing

The scale used by the experiment for the glue helicity is ***μ^2=10 GeV^2***





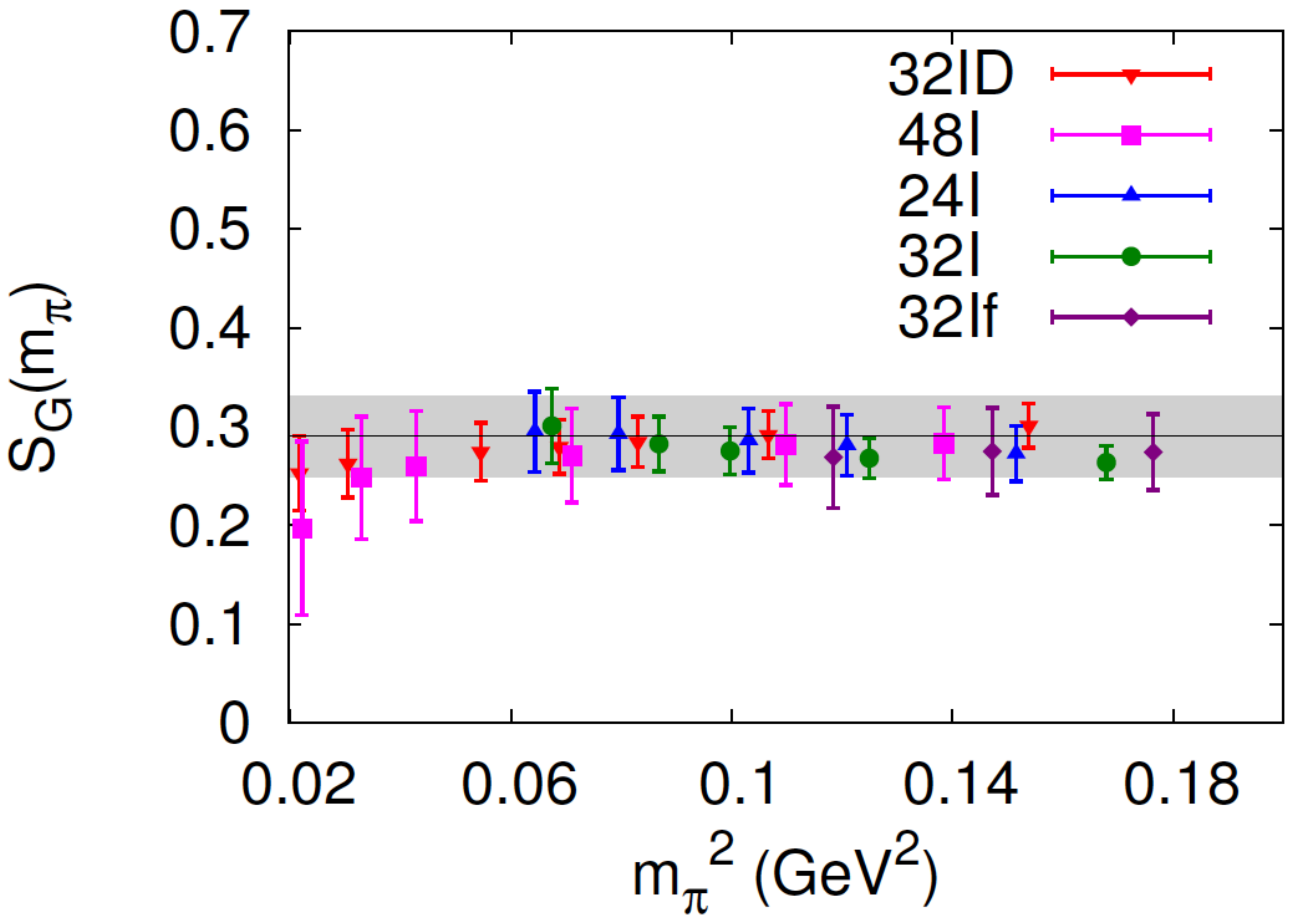
renormalization a^{-1} (GeV), $\alpha_s^{Lat}(a^{-1}) \sim \alpha_s^{MS}(2a^{-1})$ *mixing*

The dependence

Y. Yang, R. S. Sufian, et al, χQCD Collaboration, arXiv 1609.05937.

of m_π, a, and V

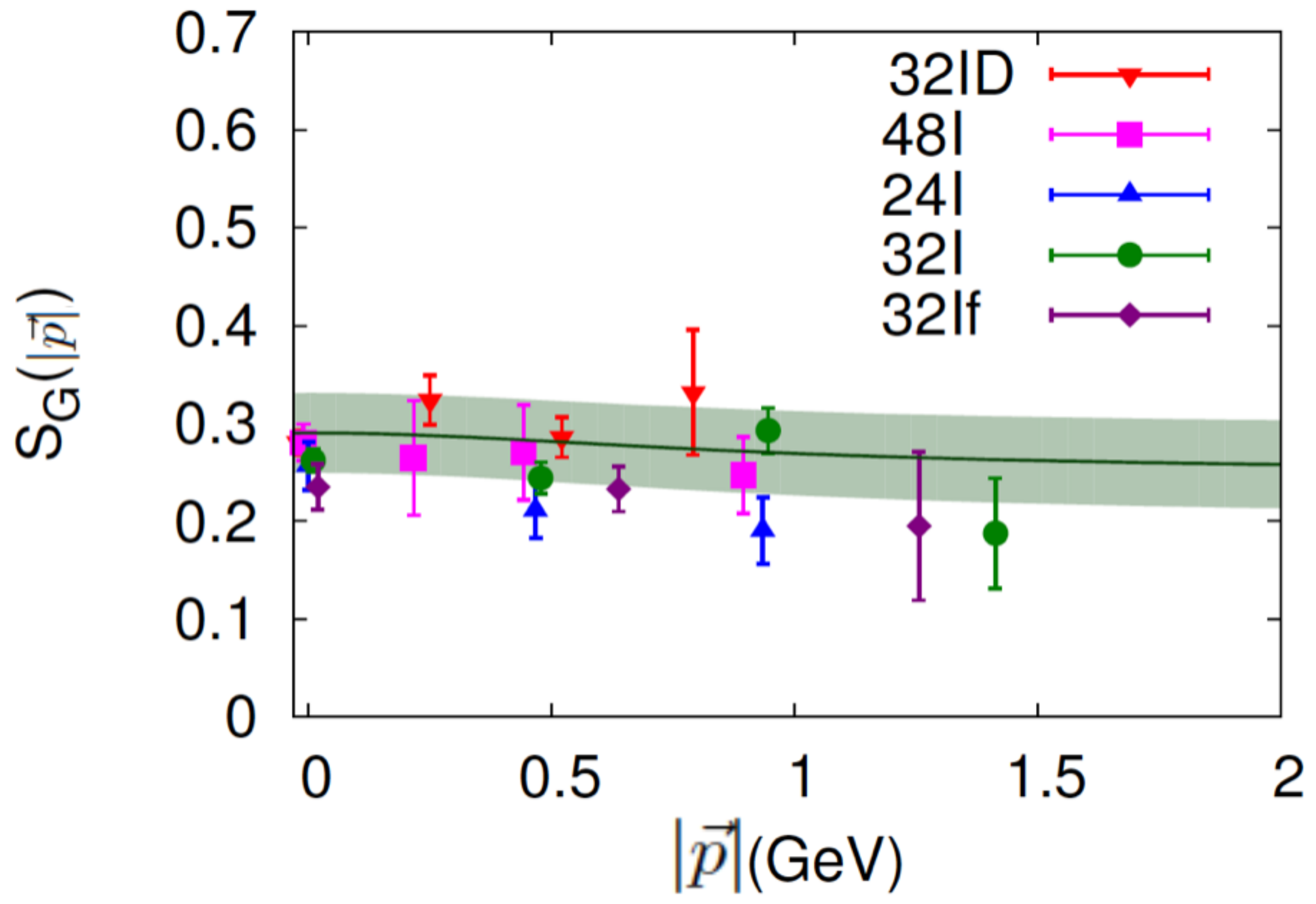
μ^2=10 GeV^2



In the rest frame,
the pion mass (both valence and sea), lattice spacing and volume dependences are mild.

From glue spin to helicity

with *Large-momentum effective field theory*



X. Ji, J.-H. Zhang, and Y. Zhao, Phys. Lett. B743, 180 (2015)

$$S_G(|\vec{p}|,\mu) = \left[1 + \frac{g^2 C_A}{16\pi^2}\left(\frac{7}{3}\log\frac{(\vec{p})^2}{\mu^2} - 10.2098\right)\right]\Delta G(\mu)$$
$$+\frac{g^2 C_F}{16\pi^2}\left(\frac{4}{3}\log\frac{(\vec{p})^2}{\mu^2} - 5.2627\right)\Delta\Sigma(\mu)$$
$$+O(g^4) + O(\frac{1}{(\vec{p})^2}) \ .$$

- **The large finite pieces indicates a convergence problem**
- **Large frame dependence need re-summation.**

At $\mu^2 = 10$ GeV^2 and $|\vec{p}| = 1.5$ GeV,

the factor before ΔG is 0.22,

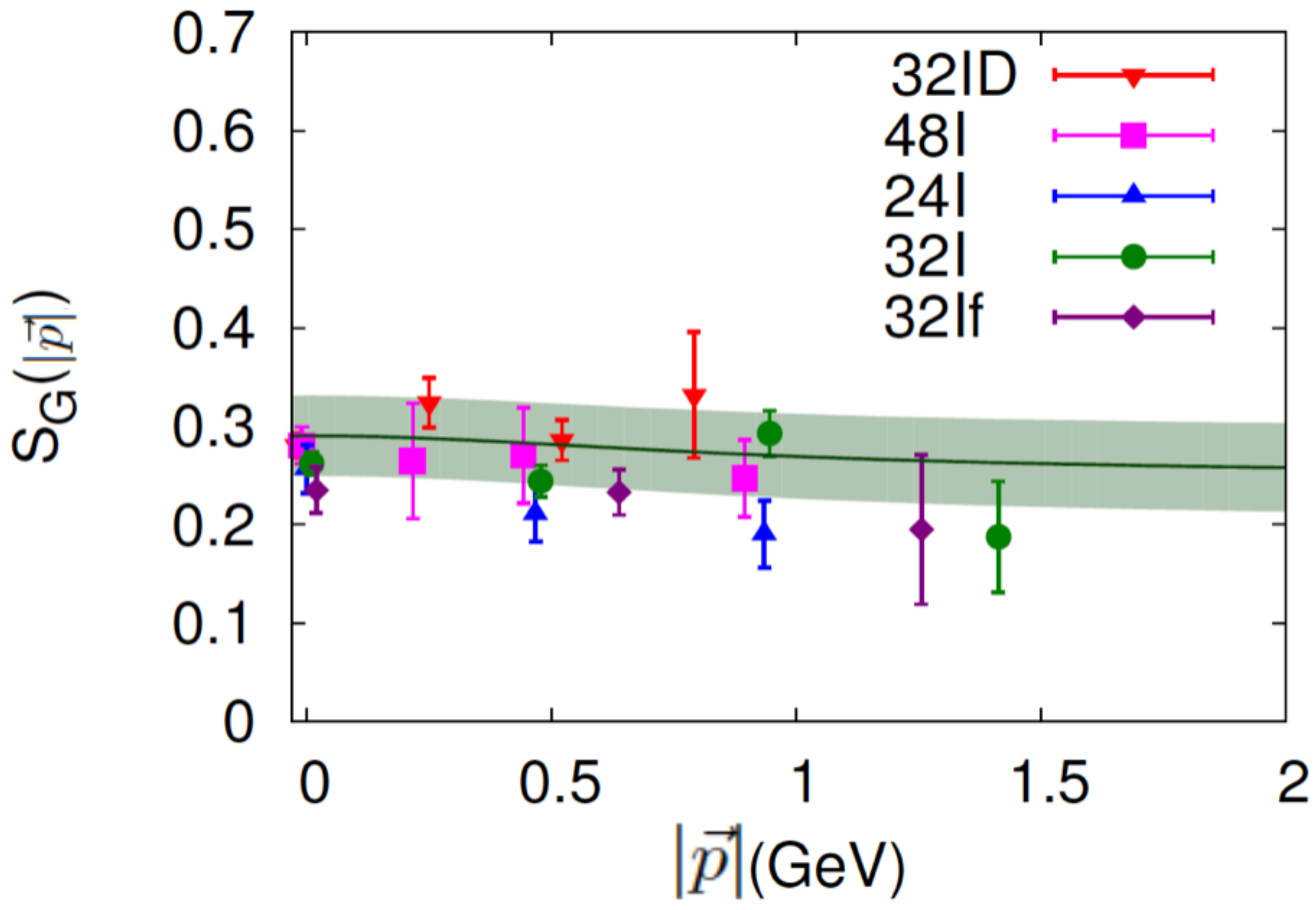
It means that ΔG will be ~ 3

if we apply this matching.

长程格点QCD

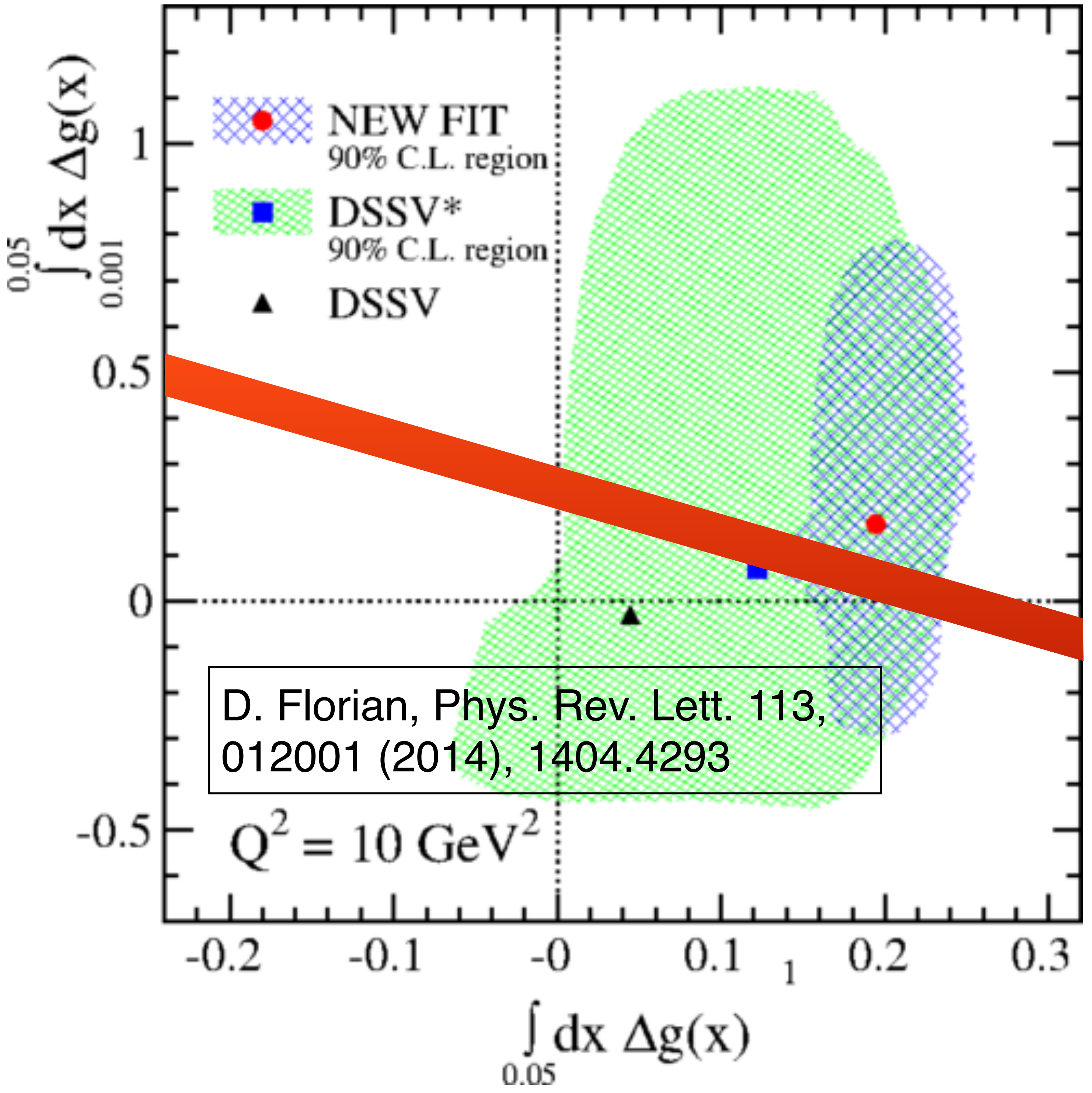
YBY, R. Sufian, et al., χQCD collaboration, PRL118, 042001(2017), 1609.05937
ViewPoint and Editor's suggestion

胶子自旋



拟合格点数据给出（忽略紫外圈图修正）：

$$\int_{0.001}^{0.05} \mathrm{d}x\Delta g(x) + \int_{0.05}^{1} \mathrm{d}x\Delta g(x) \simeq S_g$$



据此可以预言胶子极化在 μ^2=10GeV^2 的值为

S_G=0.251(47)(16).

Proton spin

Connections between decompositions

X. Ji, PRL78 (1997) 610, hep-ph/9603249

$$\vec{J} = \int d^3x\, \tfrac{1}{2}\, \overline{\psi}\, \vec{\gamma}\gamma^5\, \psi + \int d^3x \psi^\dagger \{\vec{x} \times (i\vec{D})\}\, \psi + \int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}$$

$$\int d^3x \psi^\dagger \{\vec{x} \times \overrightarrow{A}\}\psi$$

quark spin

Different definitions of the quark OAM

Further decomposition of the glue AM

$$\vec{J} = \int d^3x\, \tfrac{1}{2}\, \overline{\psi}\, \vec{\gamma}\gamma^5\, \psi + \int d^3x \psi^\dagger \{\vec{x} \times (i\vec{\nabla})\}\, \psi$$

$$\int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}] + \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]$$

glue spin

glue OAM

R. L. Jaffe and A. V. Manohar, NPB337(1990)509

Proton Spin decomposition

Calculation through the EMT form factors

X. Ji, PRL78 (1997) 610, hep-ph/9603249

Ji's angular momentum (AM) can be written in terms of the symmetrized energy momentum tensor (EMT) as,

$$J^{q,g} = \langle p, s| \int d^3x\, x \times \mathcal{T}^{\{0i\}q,g} |p, s\rangle, \qquad \mathcal{T}^{\{0i\}q} = \frac{1}{4}\bar{\psi}\gamma^{(0}\overleftrightarrow{D}^{i)}, \; \mathcal{T}^{\{0i\}g} = \vec{E} \times \vec{B}.$$

, with the form factors of the off-diagonal part of EMT defined by,

$$\begin{aligned}\langle p', s'|\mathcal{T}^{\{0i\}q,g}|p, s\rangle = \left(\frac{1}{2}\right) \bar{u}(p', s') \Big[& T_1(q^2)(\gamma^0 \bar{p}^i + \gamma^i \bar{p}^0) + \frac{1}{2m} T_2(q^2) \left(\bar{p}^0 (i\sigma^{i\alpha}) + \bar{p}^i (i\sigma^{0\alpha})\right) q_\alpha \\ & + \frac{1}{m} T_3(q^2) q^0 q^i \Big]^{q,g} u(p, s),\end{aligned}$$

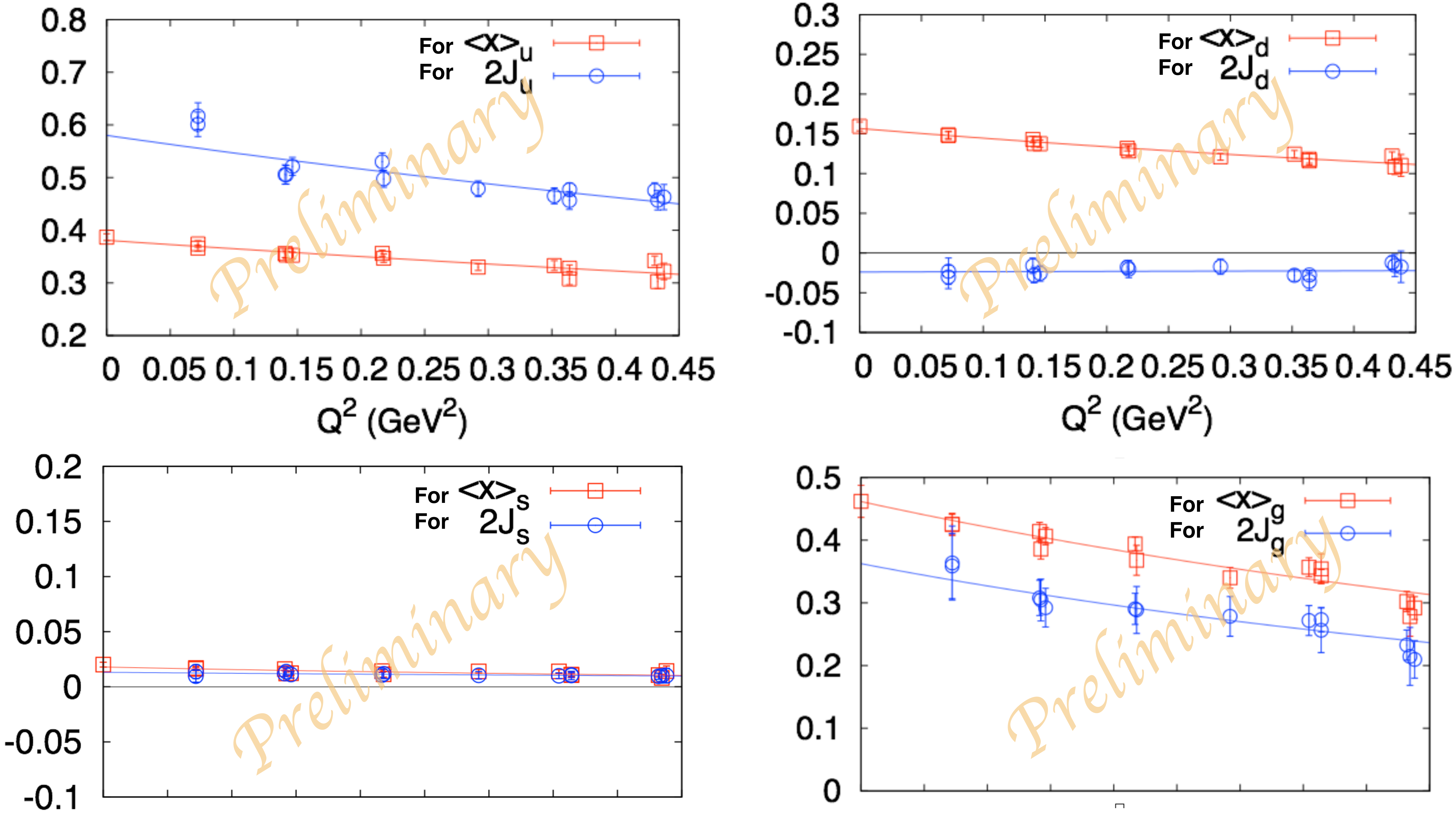
Ji's quark and glue AM correspond to the forward limit of the form factor combination,

$$\boxed{J^{q,g} = \frac{1}{2}\left[T_1(0) + T_2(0)\right]^{q,g}}$$

Angular momenta

YBY, χQCD collaboration, 1904.04138

as the second moment of GPDs



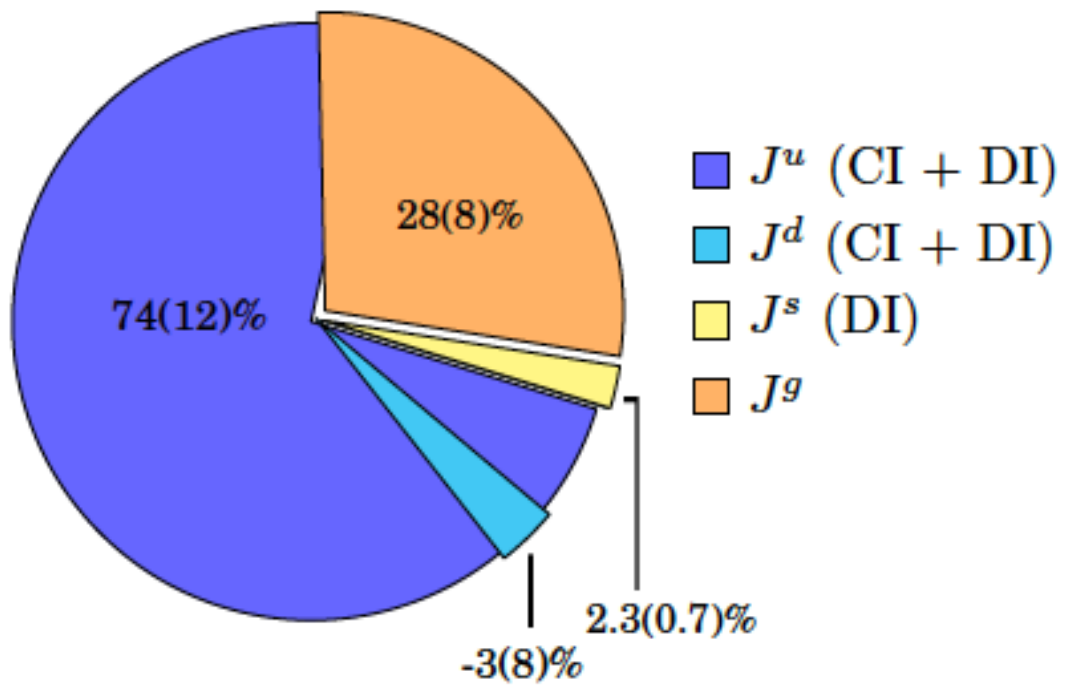
Proton spin

Lattice result of Ji AM

Quark ***AM*** Glue ***AM***

$$\vec{J} = \underbrace{\int d^3x \, \tfrac{1}{2} \, \overline{\psi} \vec{\gamma} \gamma^5 \psi + \int d^3x \psi^\dagger \{\vec{x} \times (i\vec{D})\} \psi}_{} + \underbrace{\int d^3x 2\{\vec{x} \times \mathrm{Tr}[\vec{E} \times \vec{B}]\}}_{}$$

Quenched result

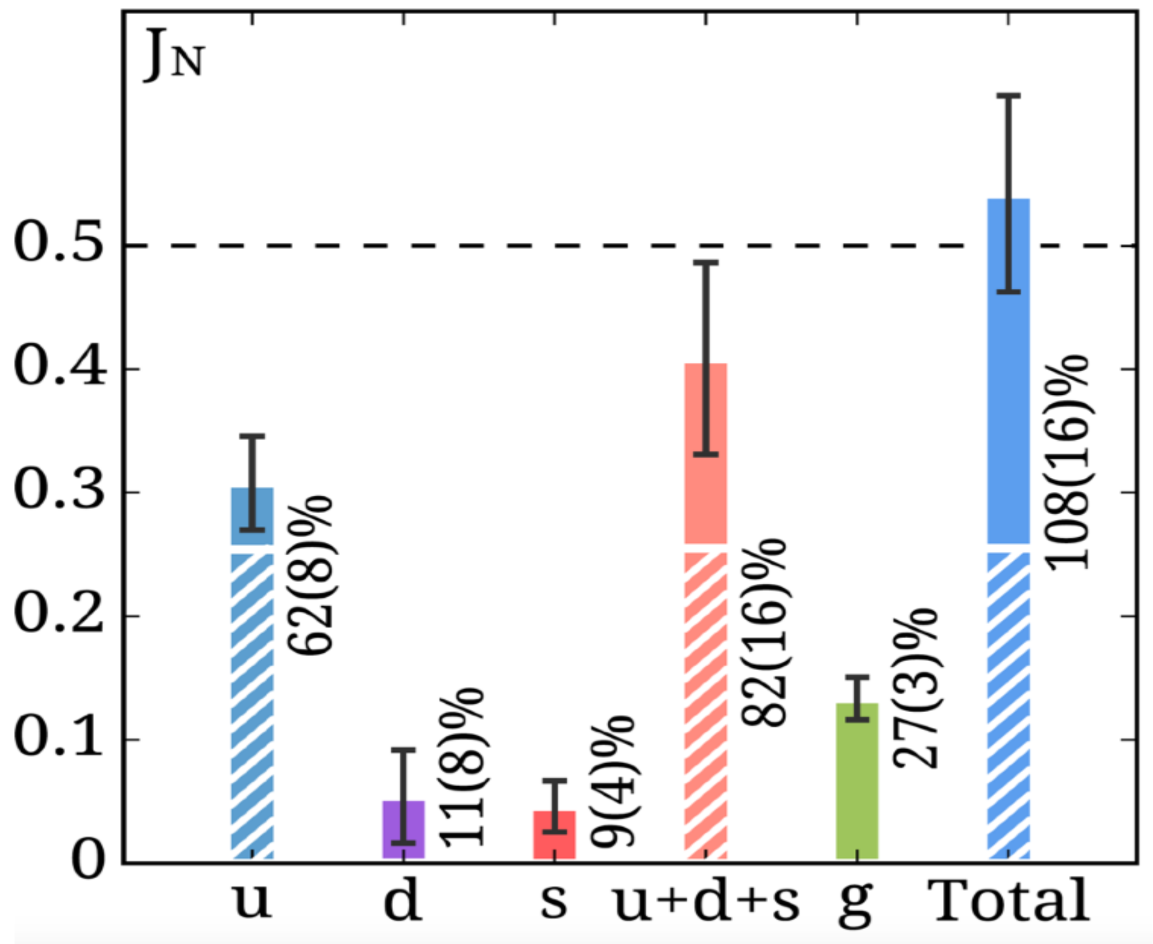


M. Deka, T. Doi, **YBY**, et. al., χQCD collaboration, PRD91, 014505 (2015), 1312.4816

2-flavor result

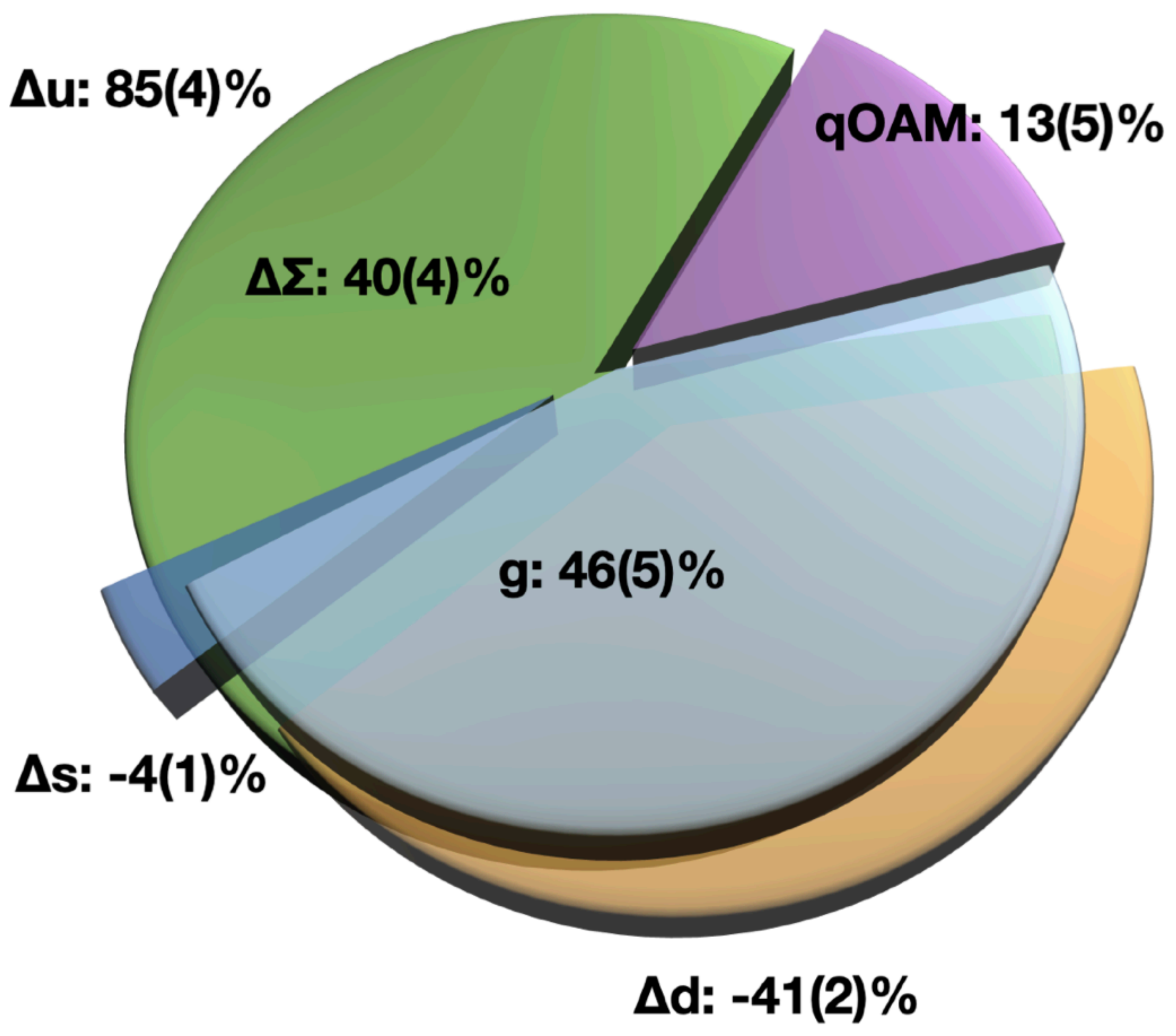
(neglecting the difference between the glue momentum and AM fractions)

EMTC 17: PRL119(2017), 1706.02973



1-loop perturbative renormalized

2+*1 flavors result*



G. Wang, χQCD collaboration, PRD106(2022) 014512

Non-perturbative renormalized

Proton spin

Lattice result of Ji AM

	u(CI)	d(CI)	u/d(DI)	s(DI)	Sum^q	glue	Sum
$\langle x \rangle$	0.233(12)(26)	0.085(5)(3)	0.065(6)(2)	0.043(6)(4)	0.491(20)(23)	0.509(20)(23)	1.0
$2J$	0.319(22)(63)	0.017(9)(23)	0.075(7)(16)	0.052(6)(10)	0.539(22)(44)	0.461(22)(44)	1.0
T_2	0.086(22)(37)	-0.067(9)(26)	0.010(7)(14)	0.010(6)(14)	0.048(22)(21)	-0.048(22)(21)	0.0
g_A [13]	0.917(13)(28)	-0.337(10)(10)	-0.070(12)(15)	-0.035(6)(7)	0.405(25)(37)	$\cdots$	$\cdots$
$2L$	-0.598(22)(63)	0.354(9)(23)	0.145(7)(16)	0.087(6)(10)	0.134(22)(44)	$\cdots$	$\cdots$

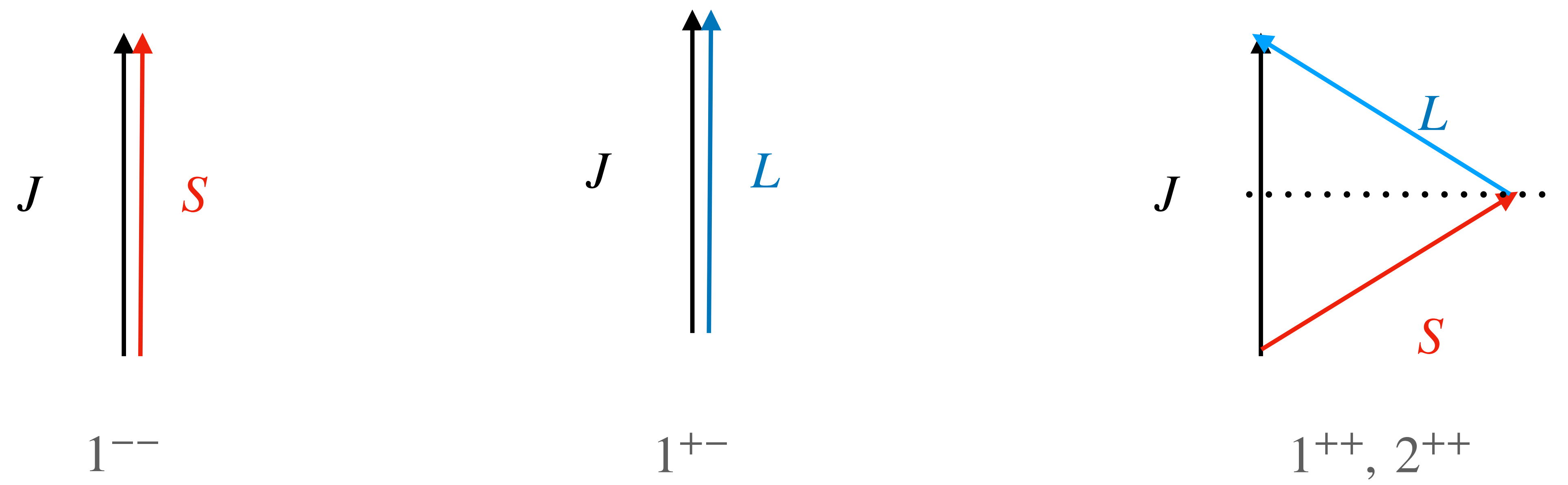
G. Wang, χQCD collaboration, PRD106(2022) 014512

Charmonia spin

Lattice result of Ji AM

- 粲偶素中的夸克更重，从而更接近传统的夸克模型；
- 自旋的Ji分解中的夸克自旋与轨道角动量，在粲偶素中应该具有更清晰的物理含义。

	J^{PC}	quark spin (S_q^R)	quark orbital angular momentum (L_q^R)	gluon angular momentum (J_G^R)
$S=1,\ L=0$	1^{--}	0.893(2)	-0.11(04)	0.21(04)
$S=1,\ L=1$	1^{++}	0.448(25)	0.36(15)	0.19(15)
$S=0,\ L=1$	1^{+-}	0.12(12)	0.70(19)	0.18(19)
$S=1,\ L=1$	$2^{++}(J_z=1)$	0.436(11)	0.37(15)	0.19(15)
$S=1,\ L=0,\ L_g=?$	1^{-+}	0.44(22)	—	—



- 初步结果显示，夸克自旋与轨道角动量在粲偶素中满足极化投影定则。

总结

- 格点量子色动力学(格点QCD)依托超级计算机，能够提供与实验独立的窗口，揭示强相互作用的奥秘：
- 在包括质子与粲偶素在内的各种强子的质量中，胶子都有很大的贡献；
- 在质子自旋中非微扰效应非常明显，但是在基态粲偶素中已经不太重要。