



High Precision Proton Charge Radius Experiments at Jefferson Lab

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- 山东大学
- 第六十九届强子物理在线论坛
- May 19th 2023



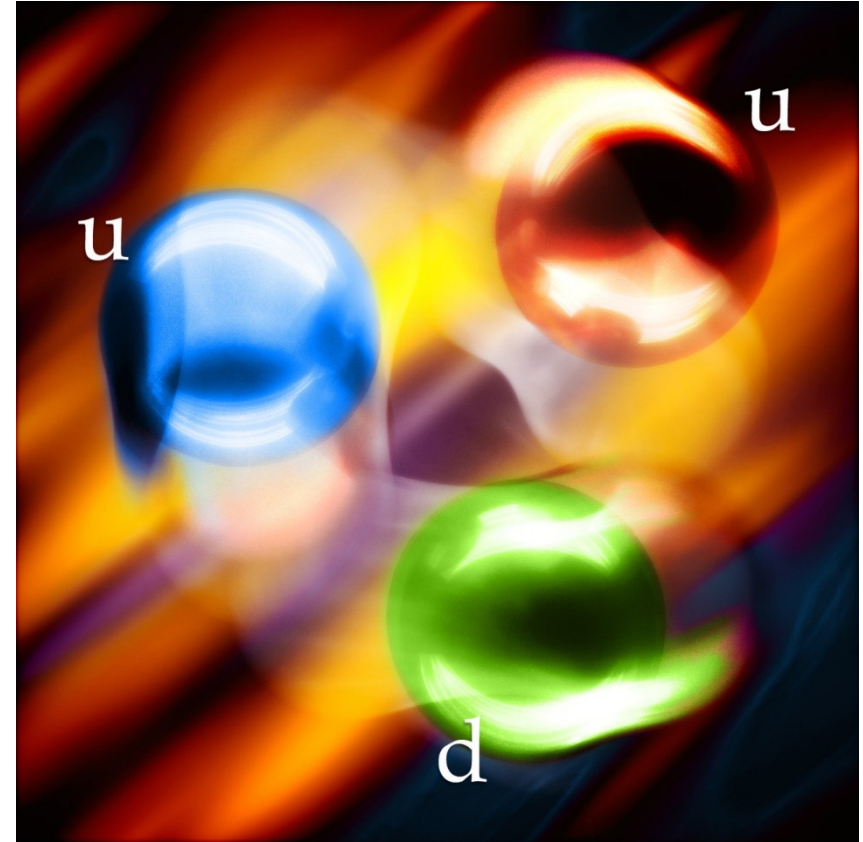
Outline

- Intro to Proton Charge Radius Puzzle
- Recent Progress from ep Scattering Experiments
- Remaining Issues for Lepton Scattering
- Future Lepton Scattering Experiments
- Summary



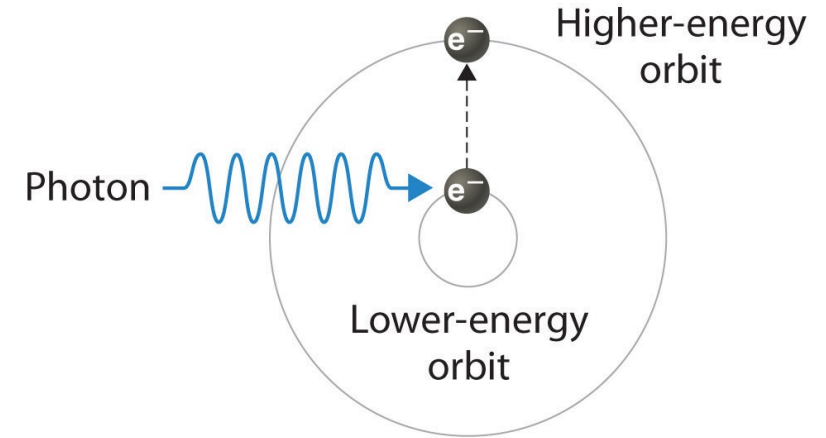
General Info on r_p

- Nucleons (protons and neutrons) make up over **99%** of the mass of visible universe
- Proton charge radius (r_p):
 1. Related to spacial distribution of proton's charge
 2. Important for understanding how QCD works
 3. Critical in determining Rydberg constant (R_∞)
 4. Input to the bound state QED calculation for atomic hydrogen energy levels

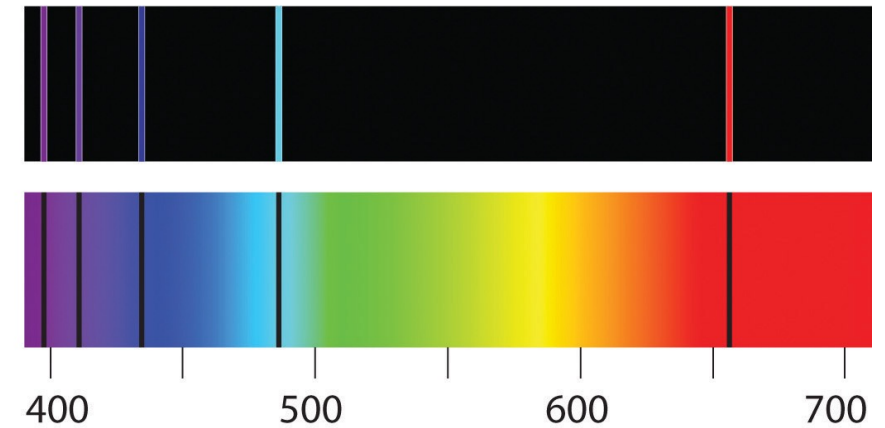


How to Measure

- Two different methods for measuring r_p
 1. Hydrogen spectroscopy ([atomic physics](#))
 - Ordinary hydrogen
 - Muonic hydrogen



(a) Electronic absorption transition



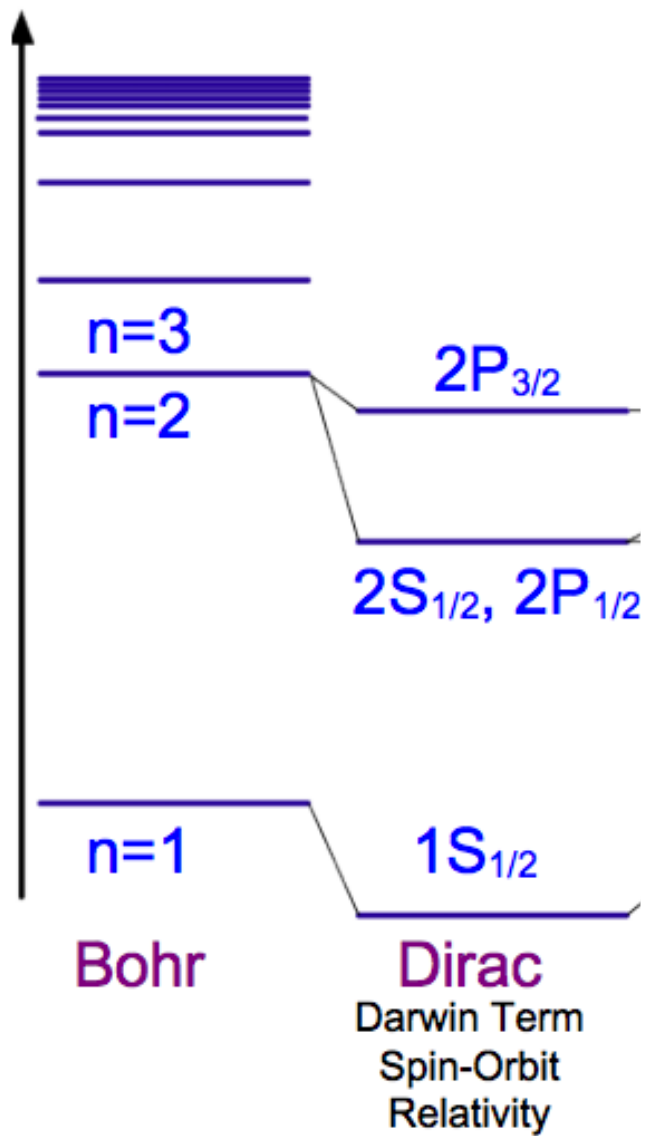
(b) H₂ emission spectrum (top), H₂ absorption spectrum (bottom)

How to Measure

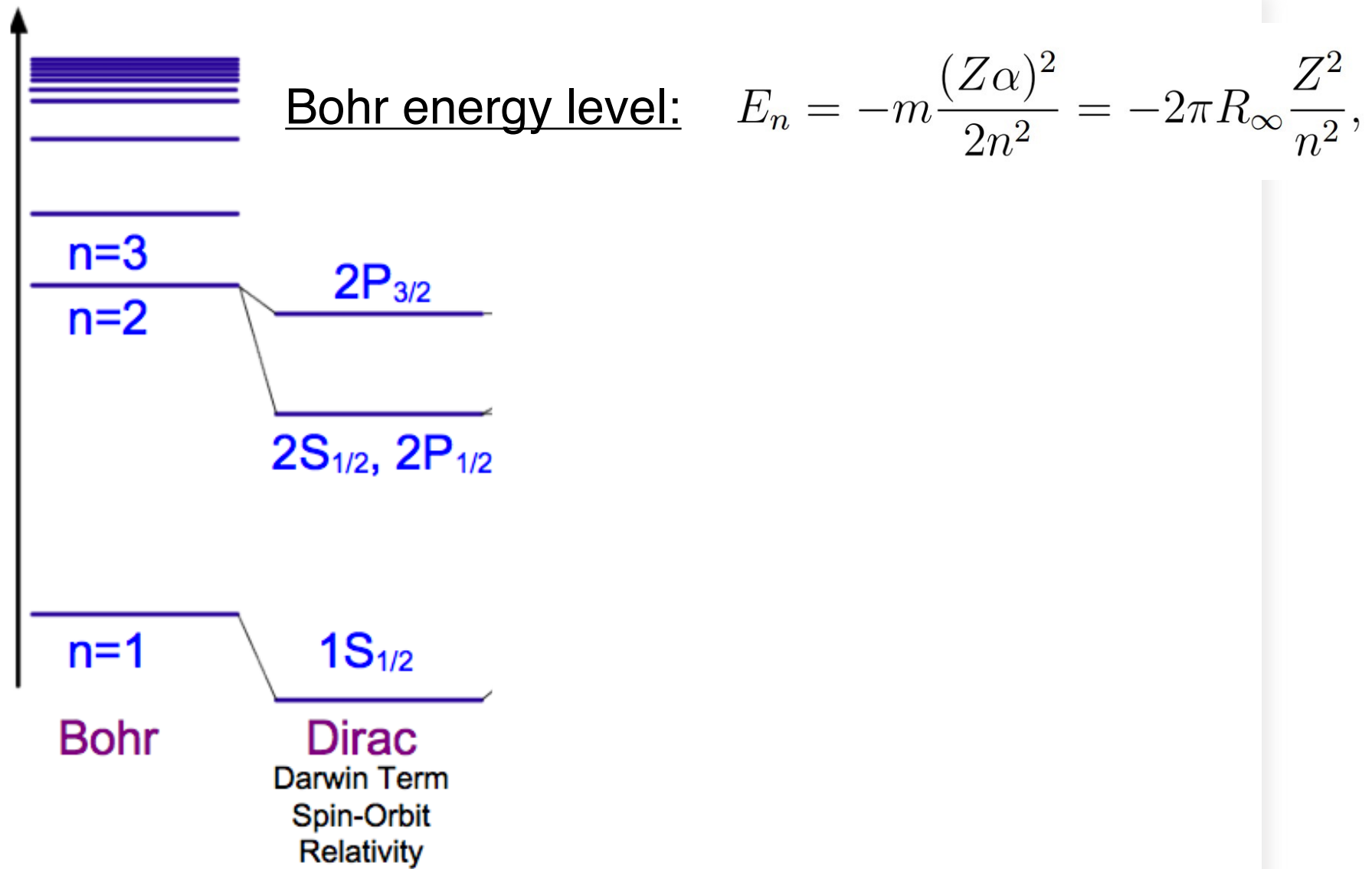
- Two different methods for measuring r_p
 1. Hydrogen spectroscopy ([atomic physics](#))
 - Ordinary hydrogen
 - Muonic hydrogen
 2. Lepton-proton elastic scattering ([nuclear physics](#))
 - ep elastic scattering (like PRad)
 - μp elastic scattering (like MUSE)



Hydrogen Spectroscopy



Hydrogen Spectroscopy

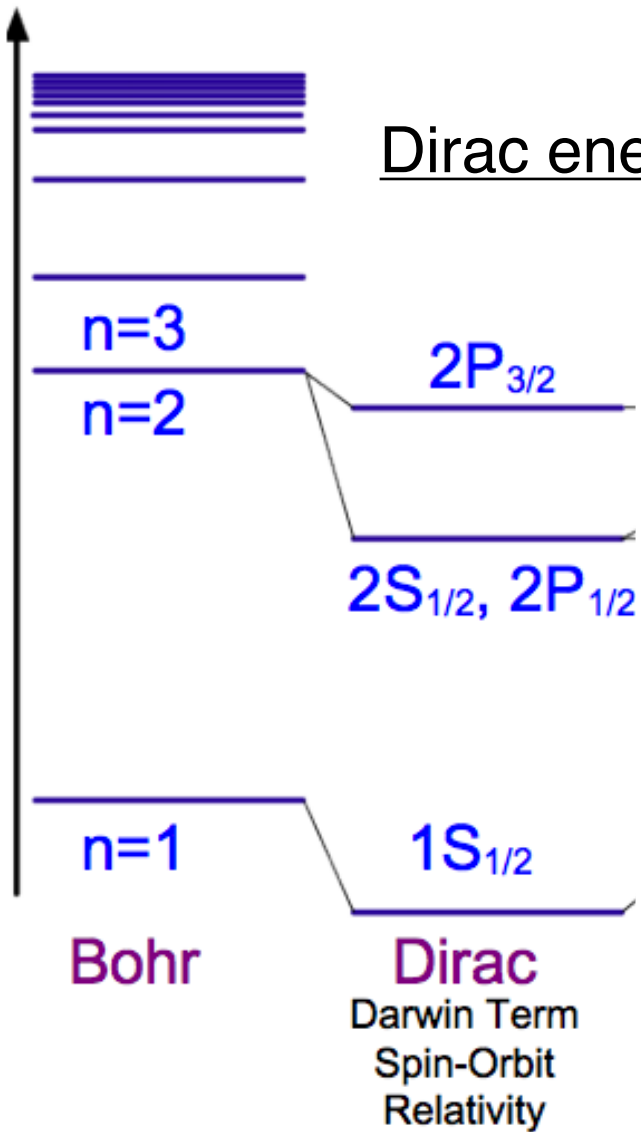


Hydrogen Spectroscopy

$$E_{nj} = mc^2 f(n, j),$$

Dirac energy level:

$$f(n, j) = \left[1 + \frac{(Z\alpha)^2}{\left(n - j - \frac{1}{2} + \sqrt{\left(j + \frac{1}{2} \right)^2 - (Z\alpha)^2} \right)^2} \right]^{-\frac{1}{2}}$$

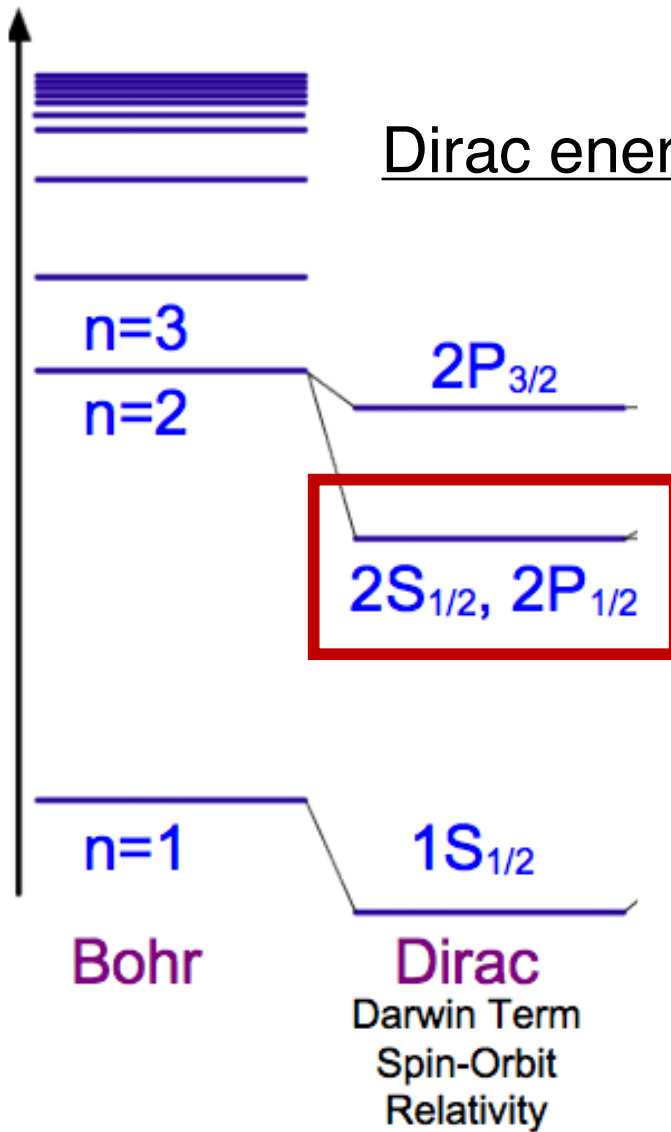


Hydrogen Spectroscopy

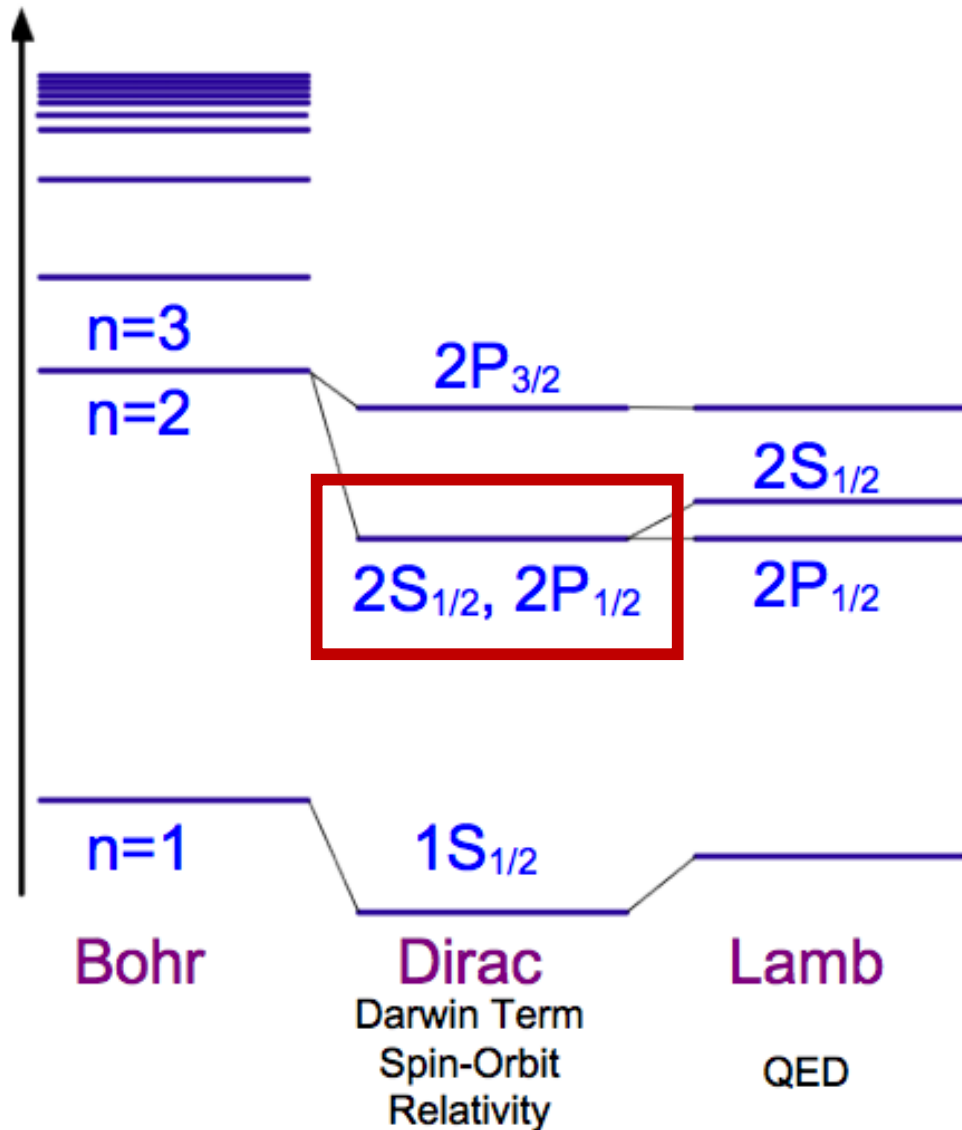
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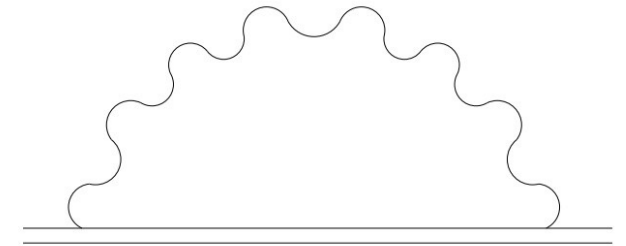
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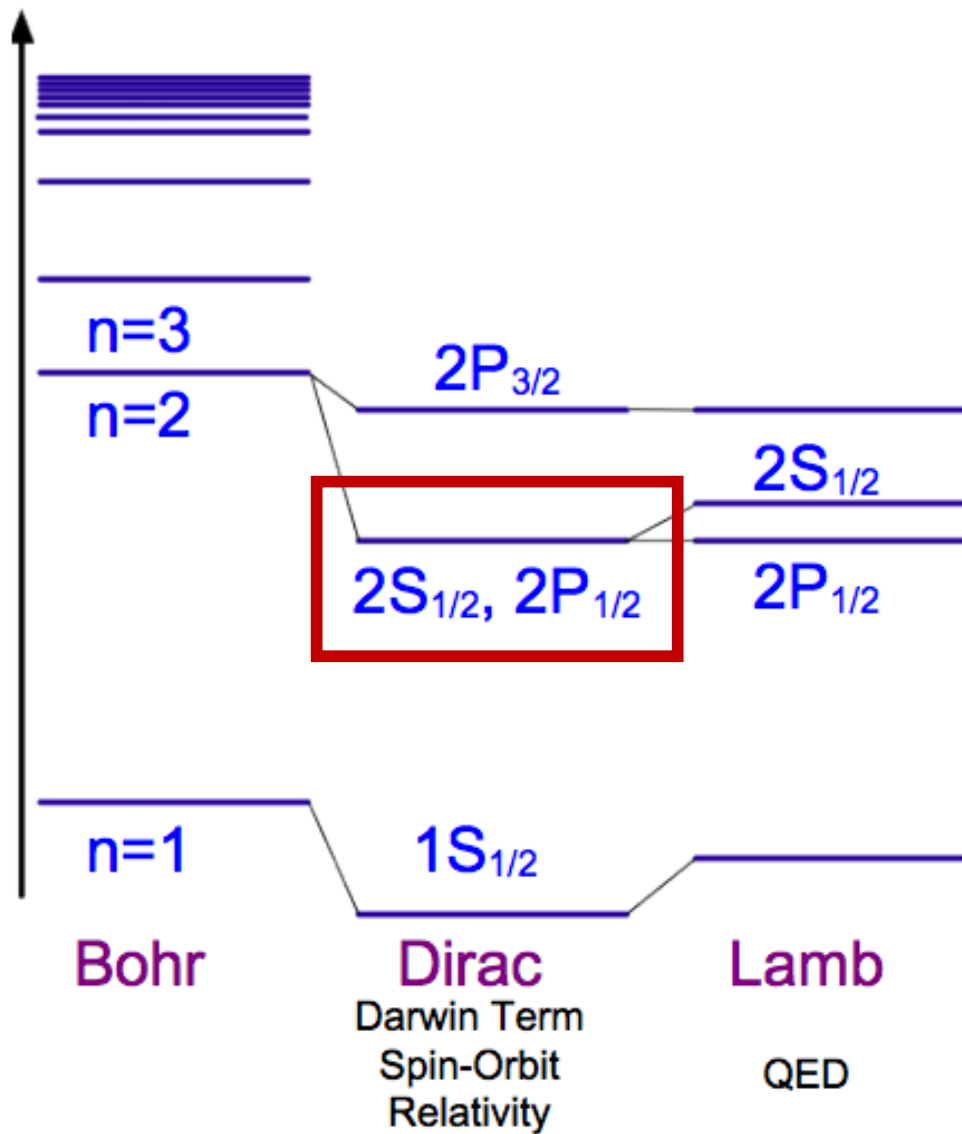
Hydrogen Spectroscopy



- Lamb shift measured in 1947
- Transition frequency about 1 GHz
- Largely dominated by electron self-energy

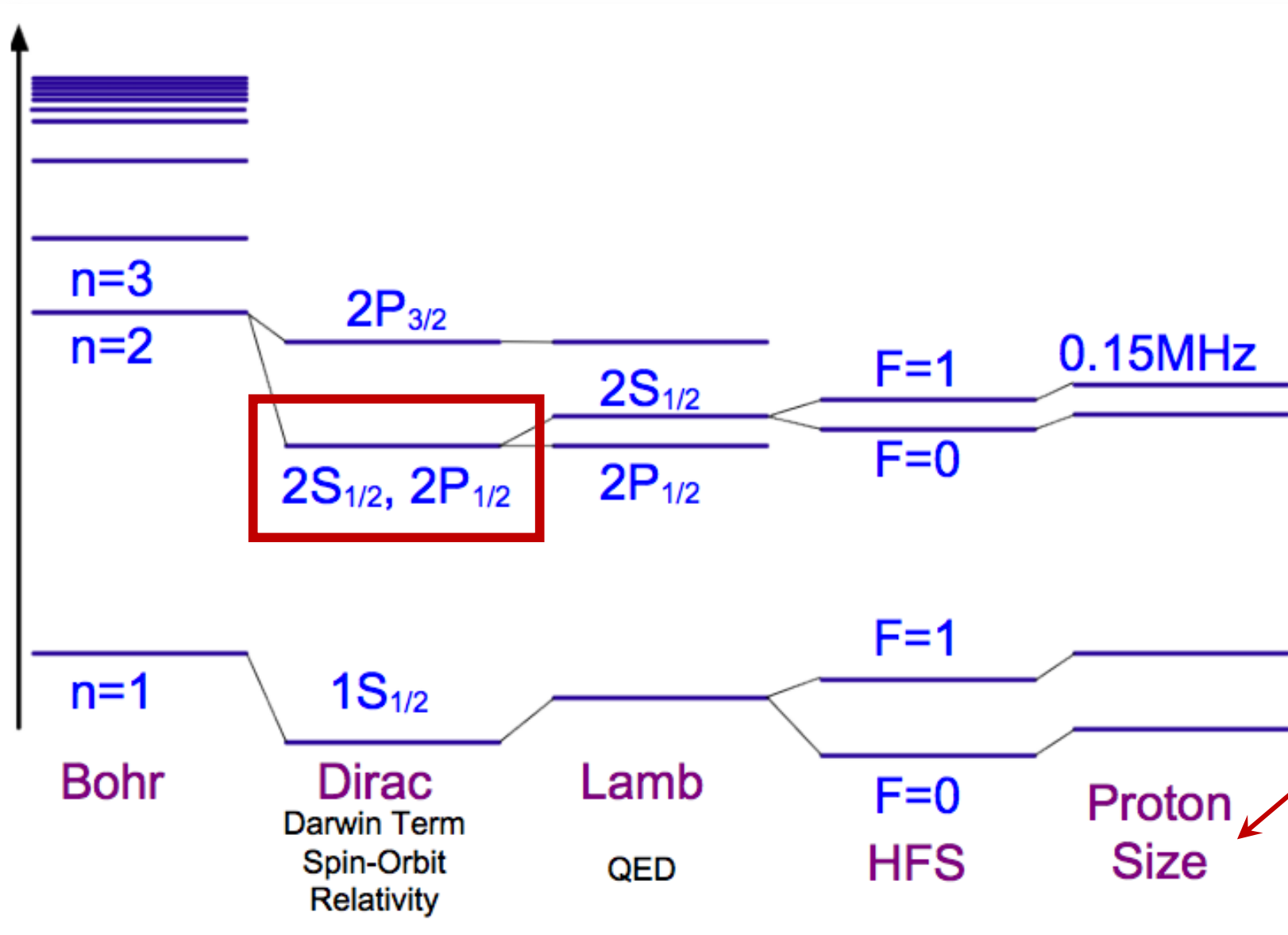


Hydrogen Spectroscopy



- Lamb shift contains (in decreasing order)
 1. Radiative correction (RC)
 2. Recoil term
 3. RC-recoil mixing term

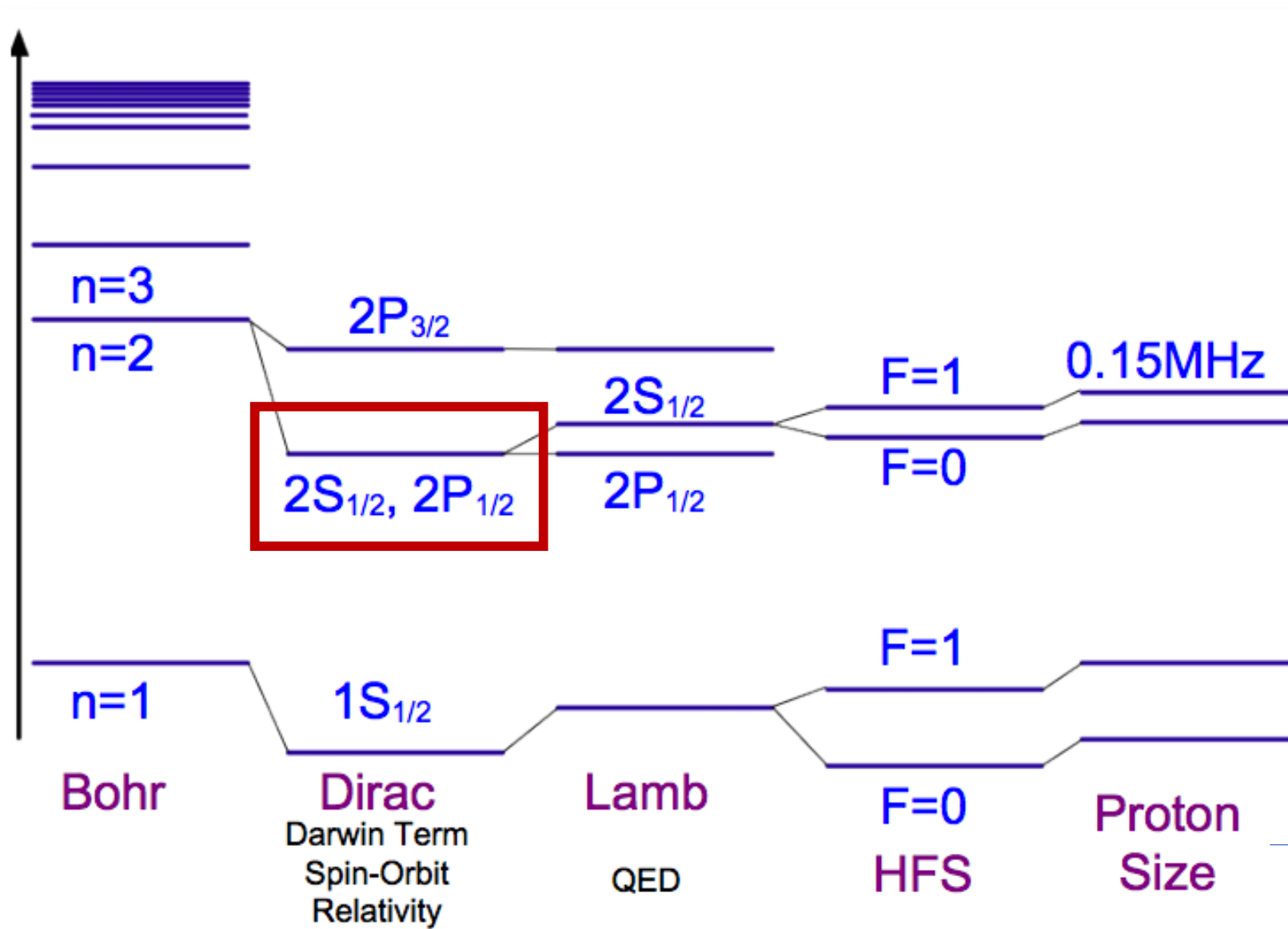
Hydrogen Spectroscopy



- Lamb shift contains (in decreasing order)

1. Radiative correction (RC)
2. Recoil term
3. RC-recoil mixing term
4. Proton finite size term

Hydrogen Spectroscopy



- Physics origin of the proton finite size effect:

- S-state wavefunction has overlap with the proton

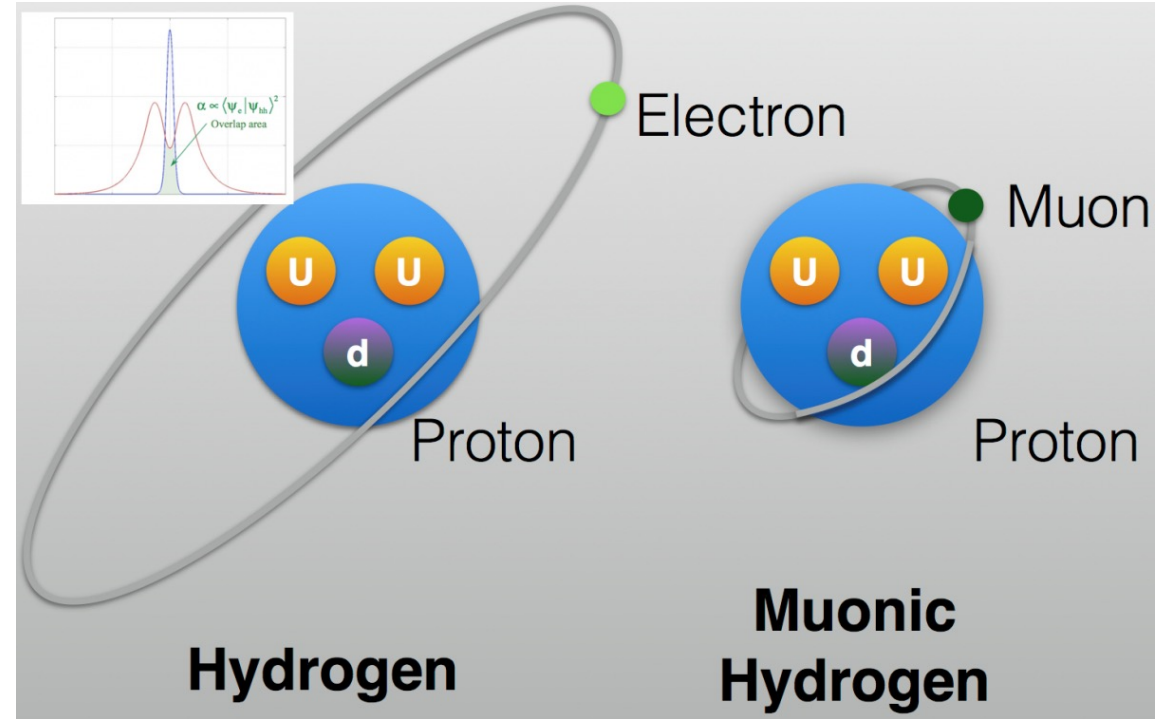
G. Miller PRC 99 035202 (2019)

$$\begin{aligned}\Delta E &= -4\pi\alpha G_E^p(0)|\psi_{n0}(0)|^2\delta_{l0} \\ &= 4\pi\alpha\frac{r_p^2}{6}|\psi_{n0}(0)|^2\delta_{l0}.\end{aligned}$$

Ordinary Hydrogen v.s. Muonic Hydrogen

- One can do this with ordinary hydrogen or muonic hydrogen
- Muon is ~200 times heavier than electron
- Orbit much closer to proton, more sensitive to proton size

$$\langle r^{\text{orbit}} \rangle \simeq \frac{\hbar}{Z\alpha m_r c} n^2$$



Proton finite size effect in 2S-2P: 2% in μH , 0.015% in H

Unpolarized ep Elastic Scattering

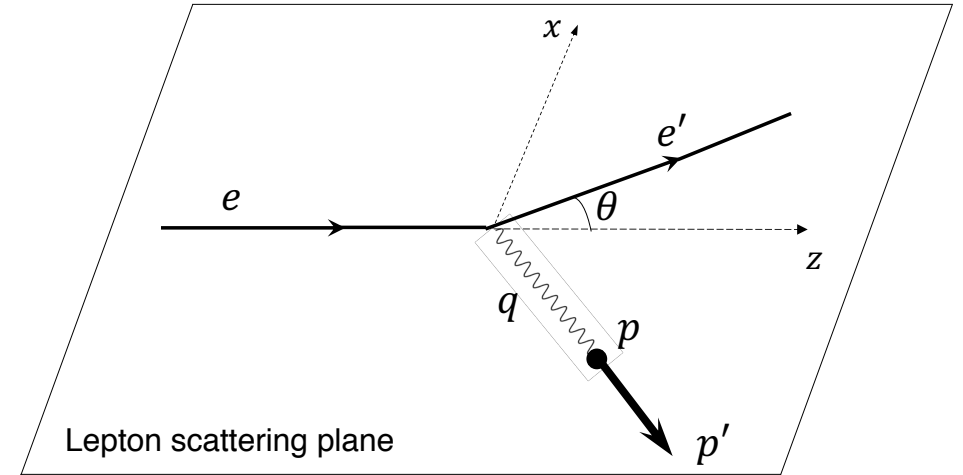
- Elastic ep scattering, in the limit of Born approximation (one photon exchange):

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left(\frac{E'}{E} \right) \frac{1}{1 + \tau} \left(G_E^p{}^2(Q^2) + \frac{\tau}{\varepsilon} G_M^p{}^2(Q^2) \right)$$

$$Q^2 = 4EE' \sin^2 \frac{\theta}{2} \quad \tau = \frac{Q^2}{4M_p^2} \quad \varepsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right]^{-1}$$

- G_E (G_M) is the electric (magnetic) form factor
- Structure-less and spin-less proton:

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \frac{\alpha^2 [1 - \beta^2 \sin^2 \frac{\theta}{2}]}{4k^2 \sin^4 \frac{\theta}{2}}$$



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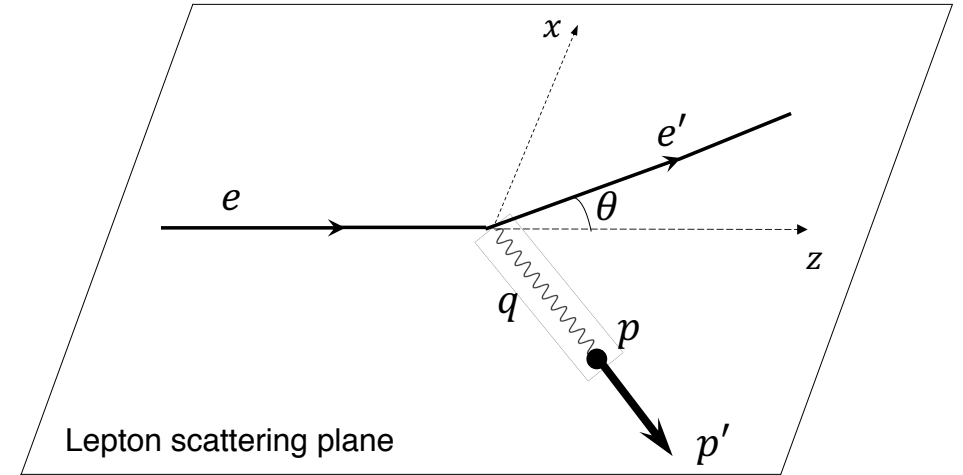
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Taylor expansion of G_E at low Q^2

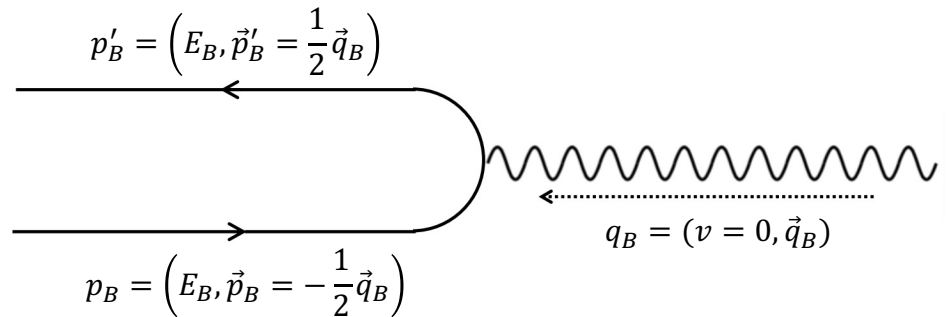
$$G_E^p(Q^2) = 1 - \frac{Q^2}{6} \langle r^2 \rangle + \frac{Q^4}{120} \langle r^4 \rangle + \dots$$

Derivative at low Q^2 limit

$$\langle r^2 \rangle = -6 \left. \frac{dG_E^p(Q^2)}{dQ^2} \right|_{Q^2=0}$$

Physical Interpretation of G_E

- Classical interpretation: in Breit frame and non-relativistic static system

$$\begin{aligned}
 G_{E,M}(Q^2) &= \int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r} \\
 &= \int \rho(\vec{r}) d^3\vec{r} - \frac{\vec{q}^2}{6} \int \rho(\vec{r}) \vec{r}^2 d^3\vec{r} + \dots
 \end{aligned}$$


- Not rigorous, lots of research activities to refine the interpretation:

- Y. Chen (陈毅) and C. Lorcé, *PRD* 106 (2022) 11, 116024
- E. Epelbaum *et al.* *PRL* 129 (2022) 1, 012001
- R. L. Jaffe *PRD* 103 (2021) 1, 016017
- ...
- C. Lorcé *PRL* 125 (2020) 23, 232002
- G. A. Miller, *PRC* 99 (2019) 3, 035202
- Y. Li (李阳) *et al.* *PLB* 838 (2013) 137676

- Charge radius is defined for FF slope
- For both scattering and hydrogen spectroscopy, we are going after $G'_E(0)$

Unpolarized ep Elastic Scattering

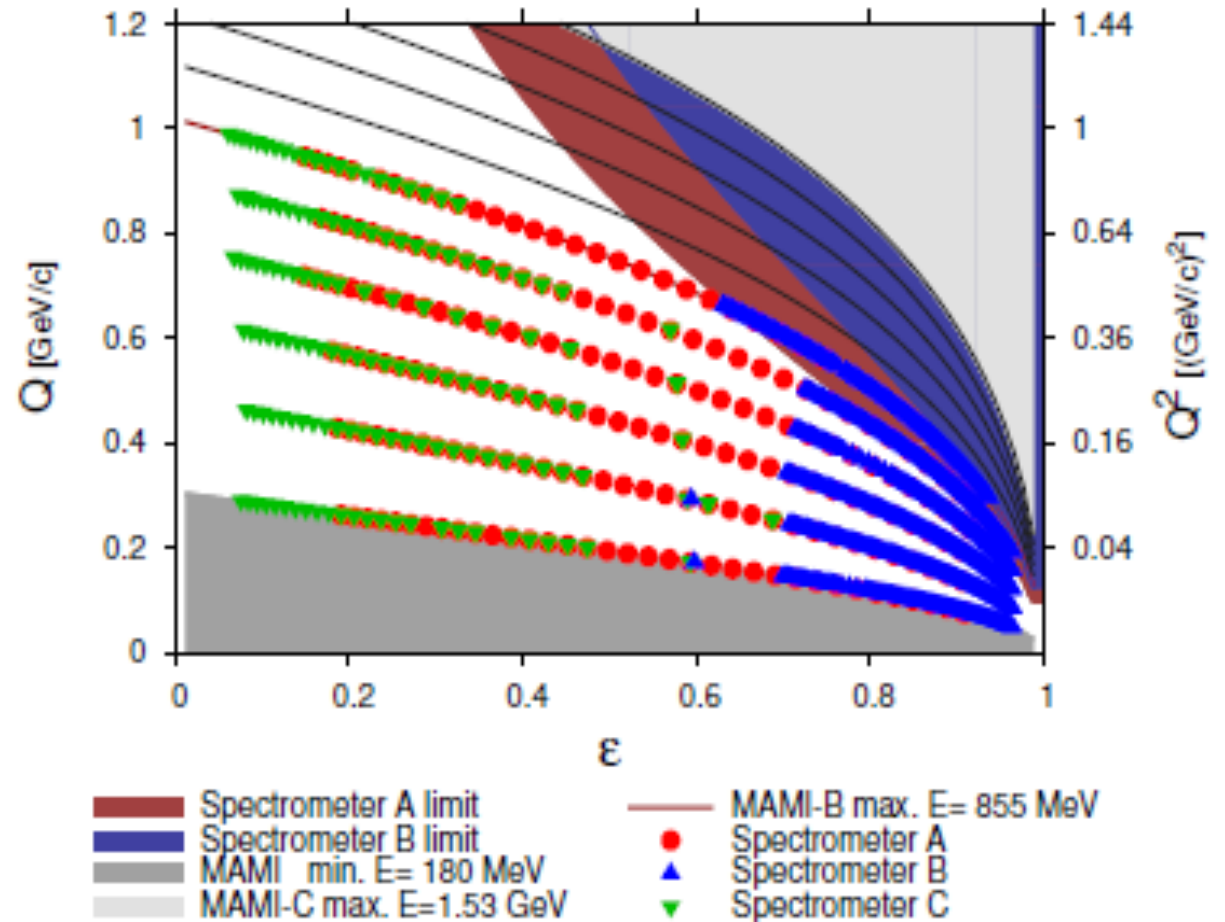
Three spectrometer facility of the A1 collaboration:



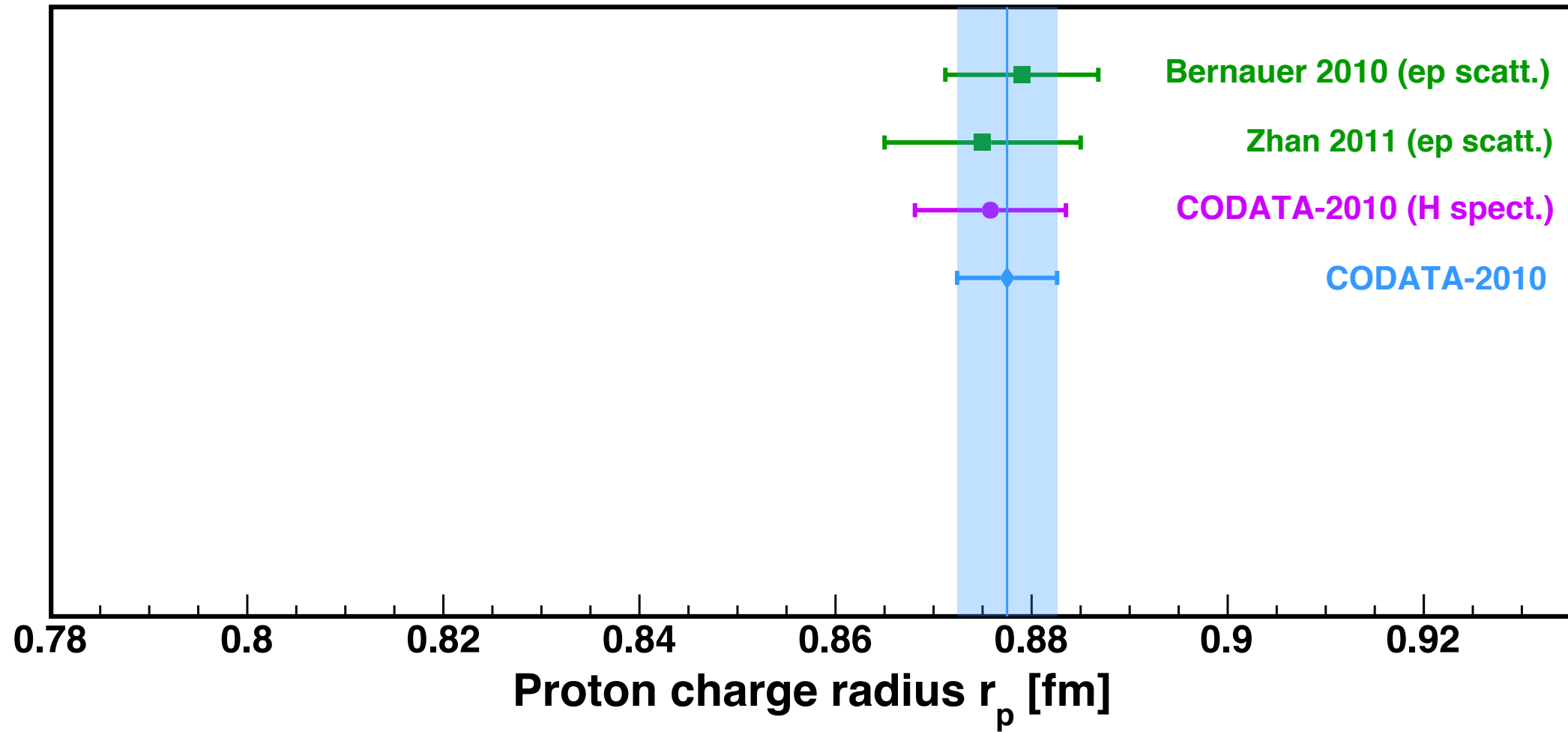
- Large amount of overlapping data sets
- Statistical error $\leq 0.2\%$
- Luminosity monitoring with spectrometer
- $Q^2 = 0.004 - 1.0 \text{ (GeV/c)}^2$
- result: $r_p = 0.8791(79) \text{ fm}$

J.C. Bernauer *et al.* PRL. 105 (2010) 242001

Measurements @ Mainz

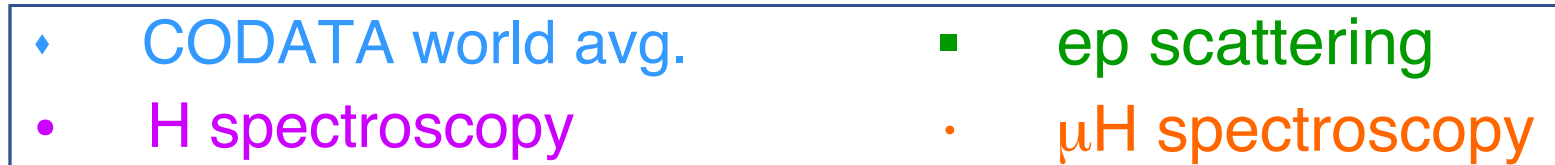
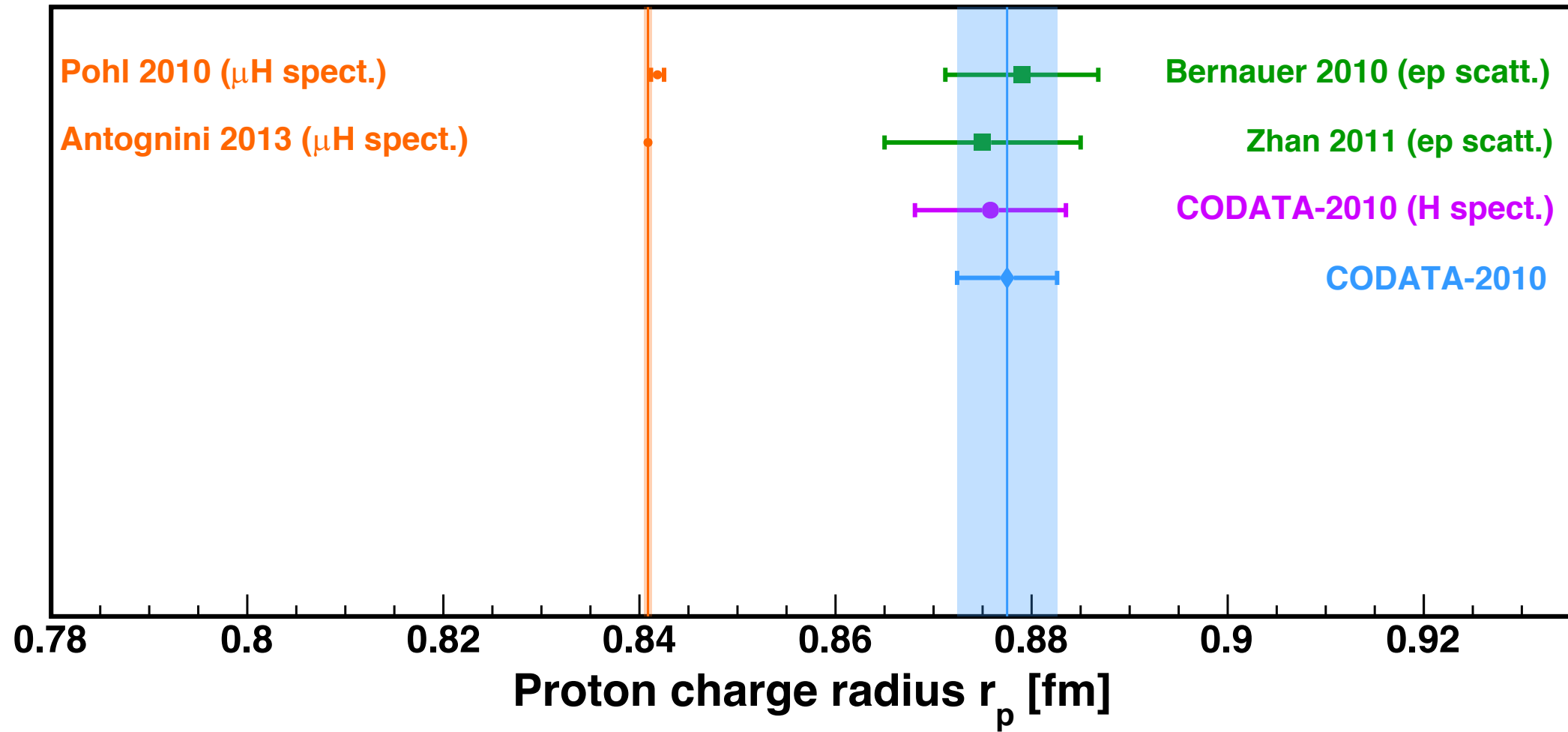


Proton Charge Radius

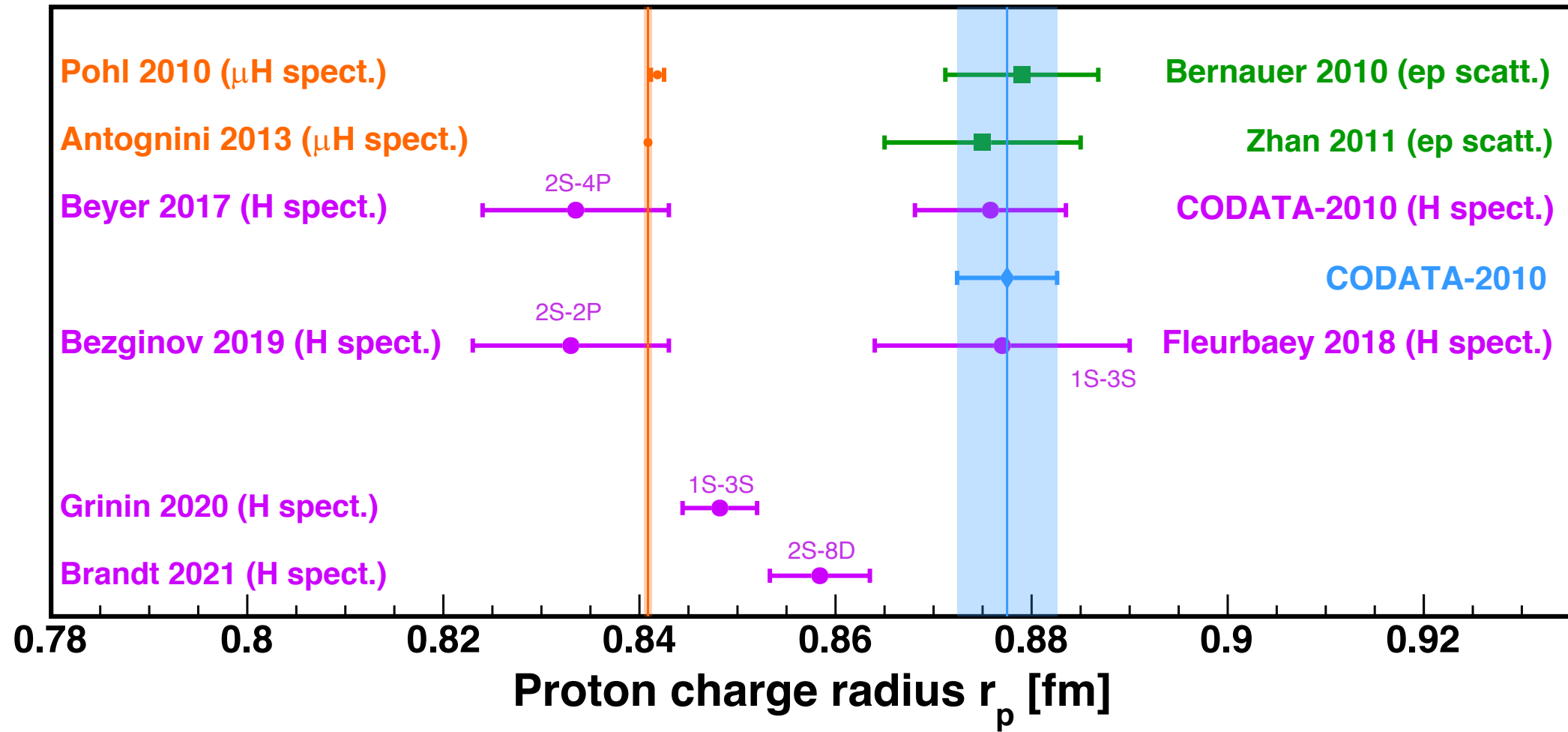


- ◆ CODATA world avg.
- H spectroscopy
- ep scattering

Proton Charge Radius Puzzle



Proton Charge Radius Puzzle



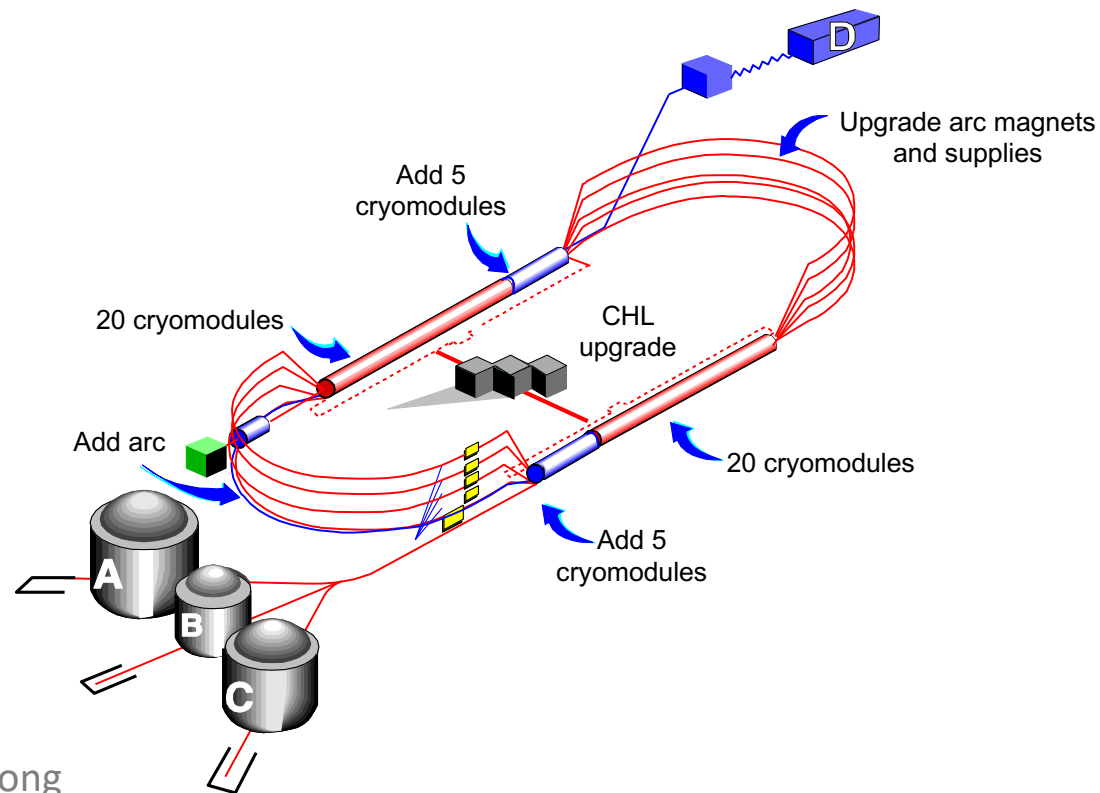
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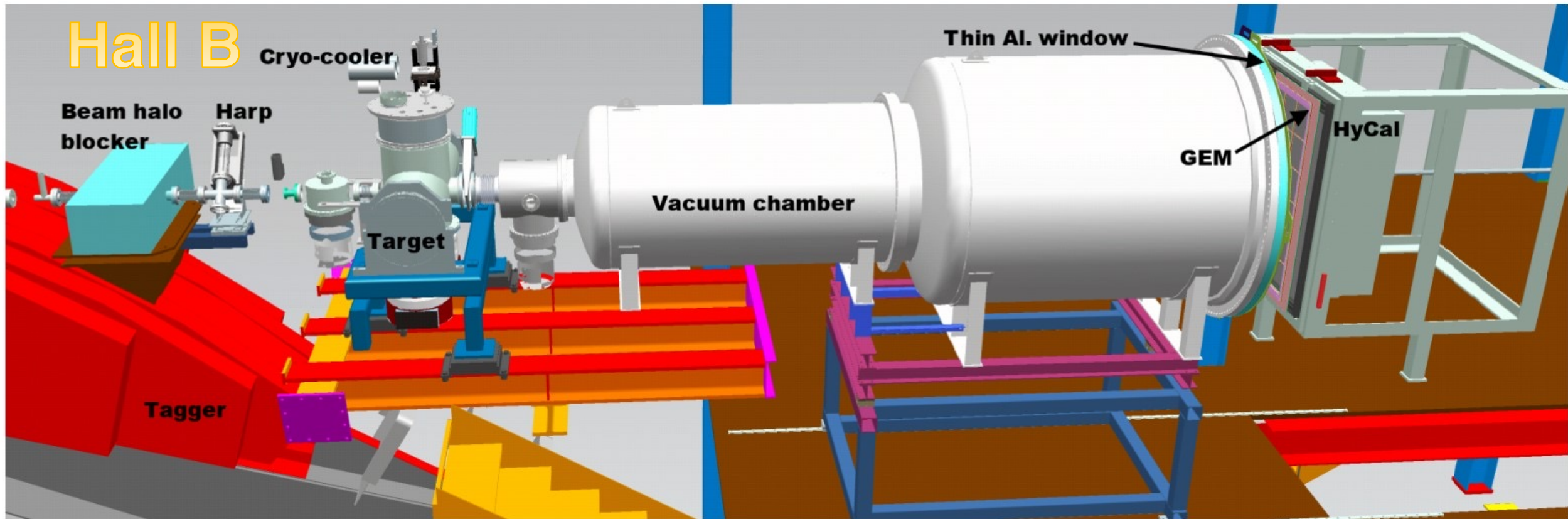
Jefferson Lab

- Thomas Jefferson National Accelerator Facility (JLab), Newport News, VA
- Completed 6 GeV to 12 GeV upgrade in 2015
- 4 experimental Halls
- PRad data taking May/June 2016, with 1.1 GeV and 2.2 GeV electron beams



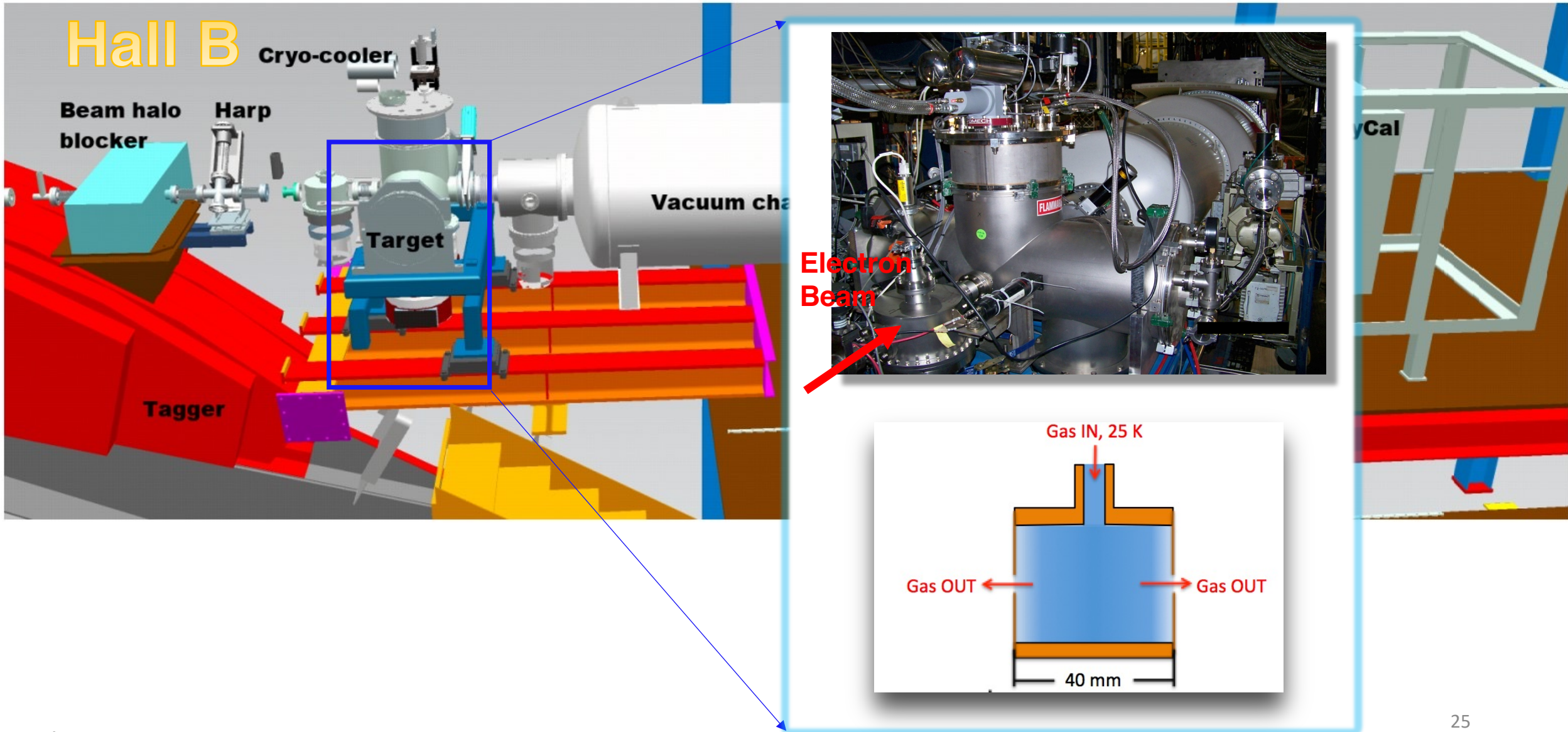
PRad Experimental Apparatus

Large acceptance, small angle and non-spectrometer apparatus



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Large acceptance, small angle and non-spectrometer apparatus



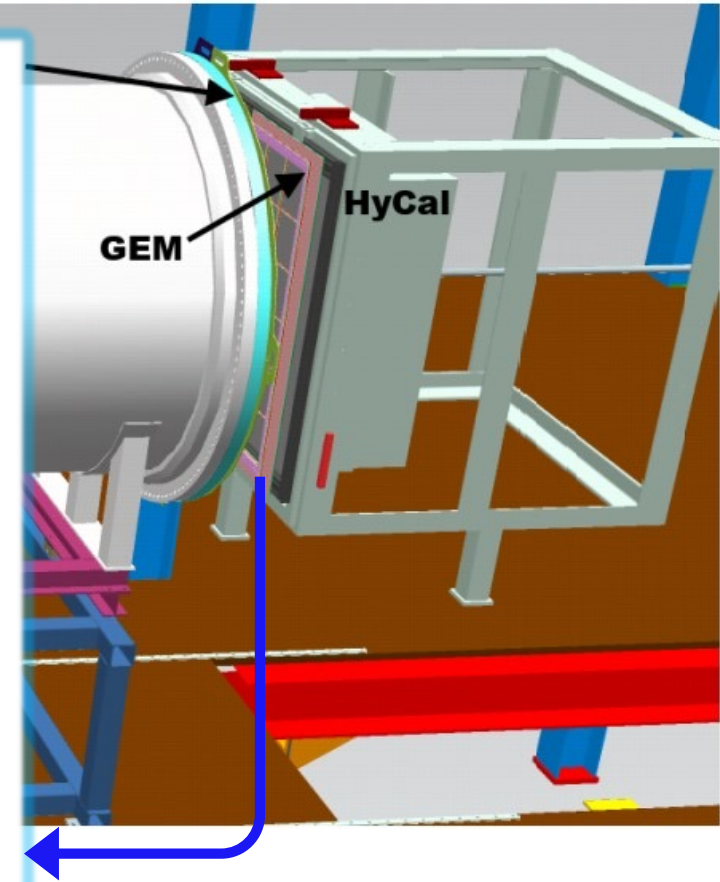
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Hall B



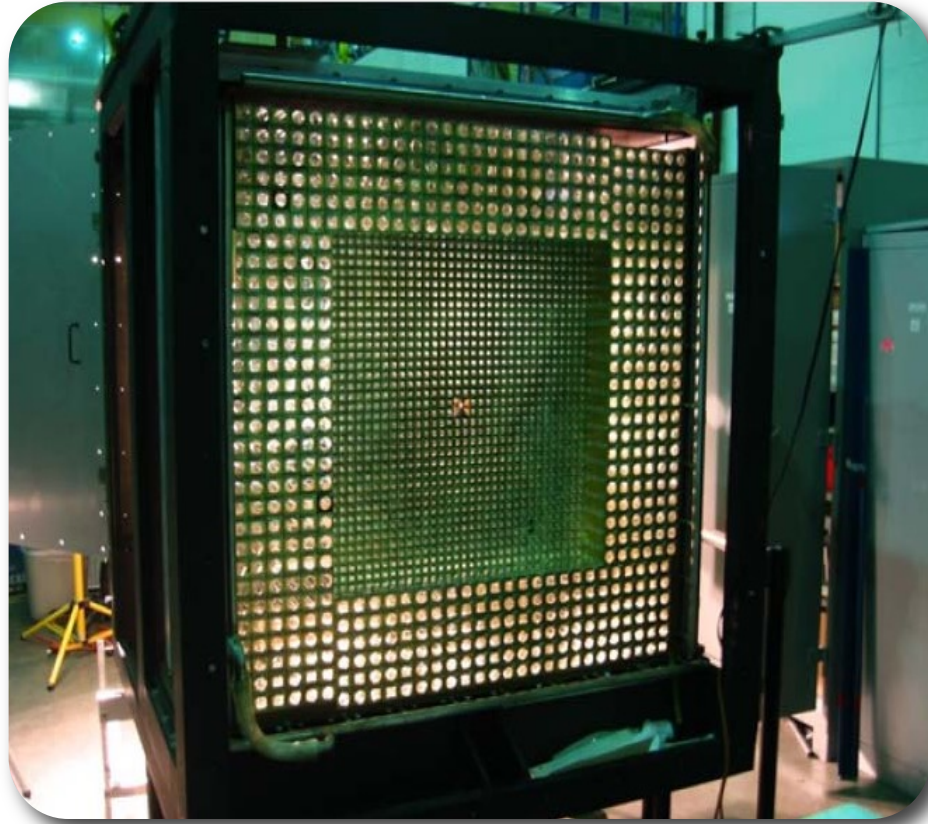
- Two large area GEM detectors
- Small overlap region in the middle
- Excellent position resolution ($72\ \mu\text{m}$)



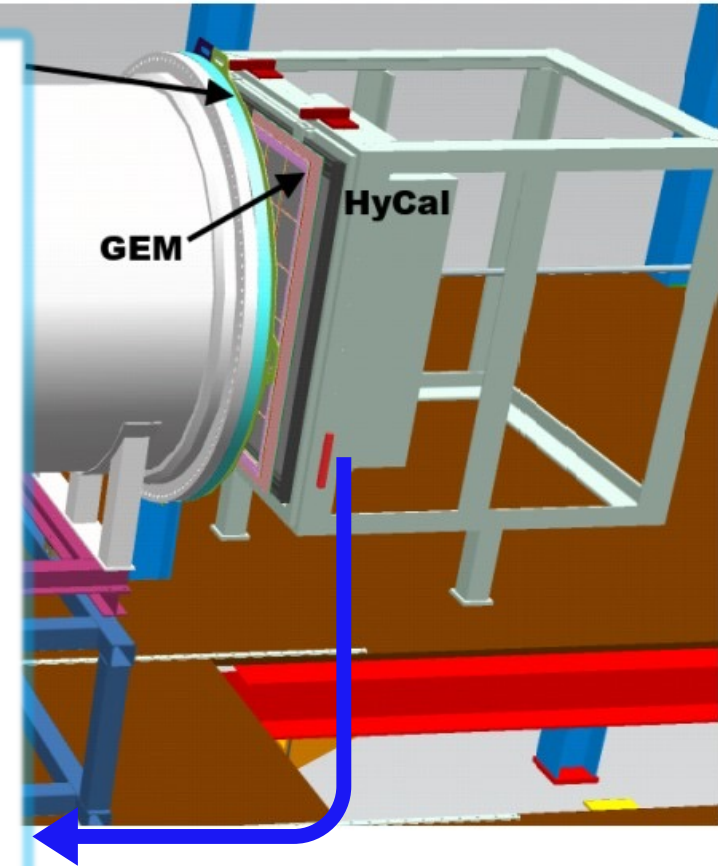
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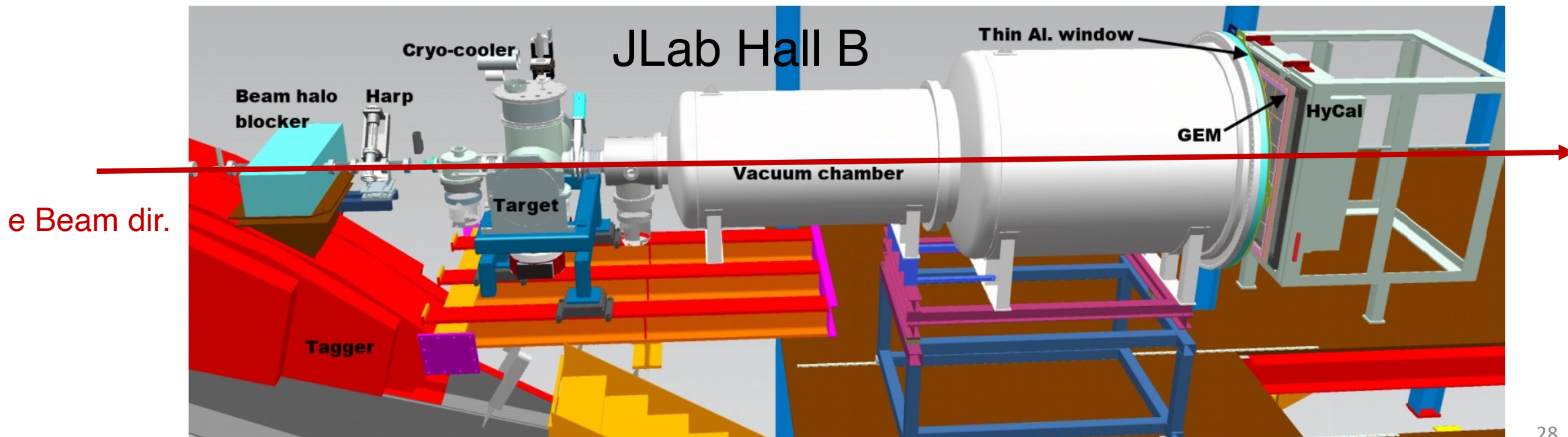
- Hybrid EM calorimeter (HyCal)
 - Inner 1156 PbWO_4 modules
 - Outer 576 lead glass modules
- Scattering angle coverage: $\sim 0.7^\circ$ to 7.0°
- Full azimuthal angle coverage
- High resolution and efficiency



PRad Experiment Overview

Large acceptance, small angle and non-spectrometer apparatus

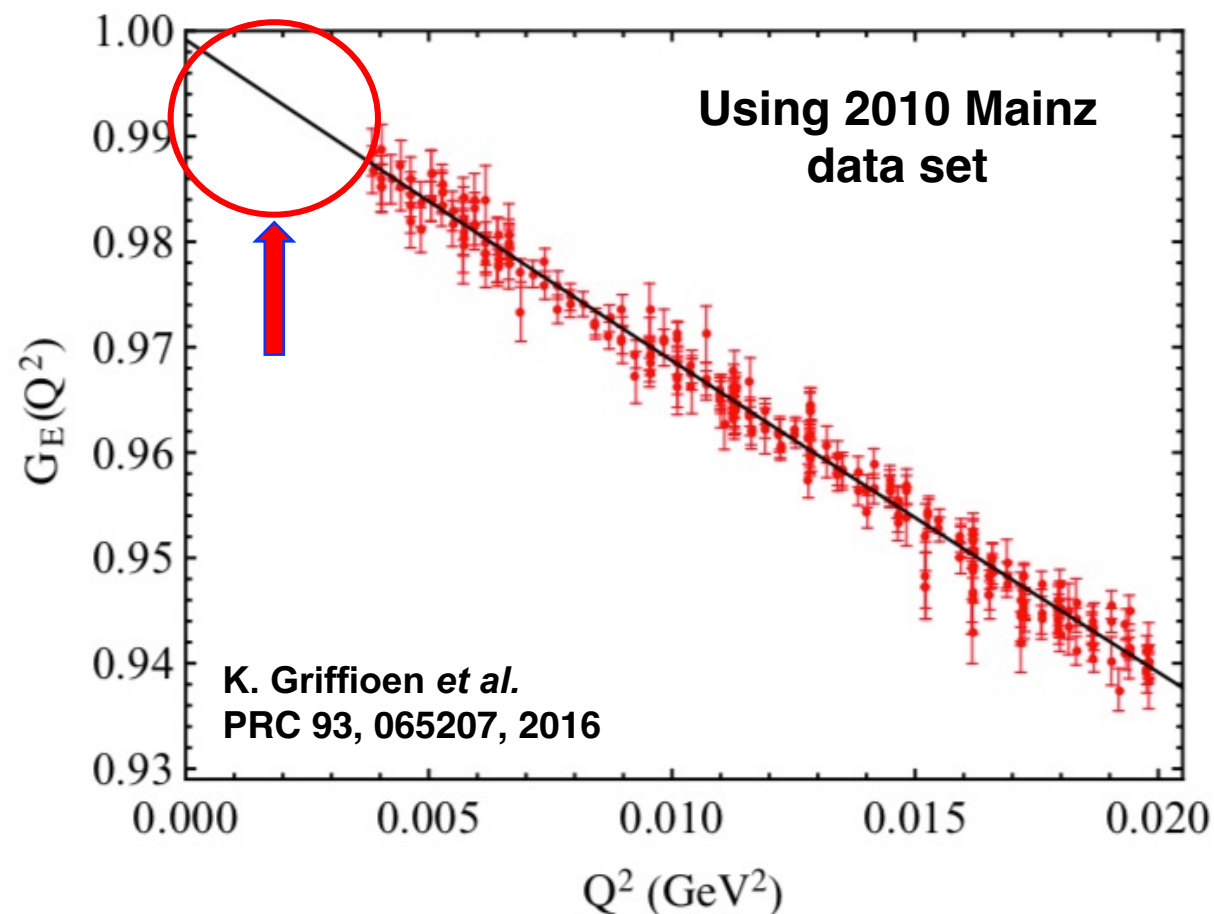
- At each beam energy, different Q^2 data collected **at the same time**
- Covers **two orders** of magnitude in low Q^2 with the **same detector setting**
 - $\sim 2 \times 10^{-4} - 6 \times 10^{-2} \text{ GeV}^2$



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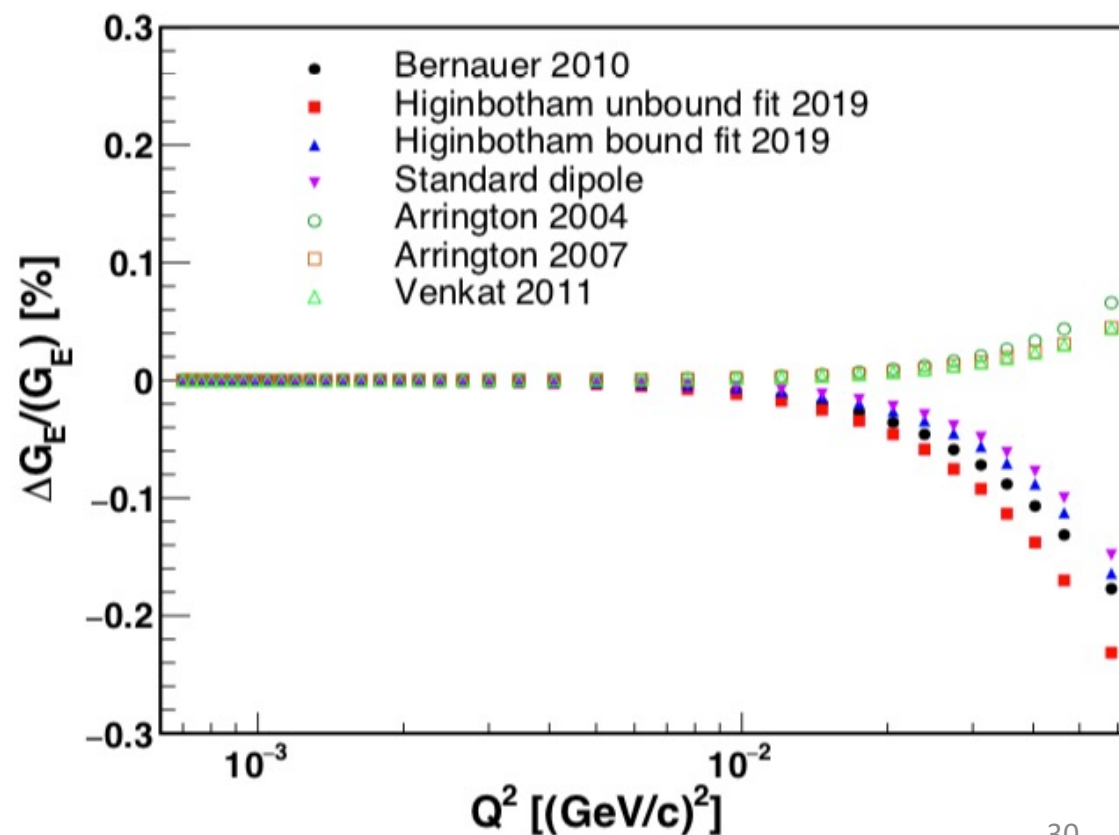


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- Extreme forward angles (0.7° to 7°), **minimize** G_M contribution

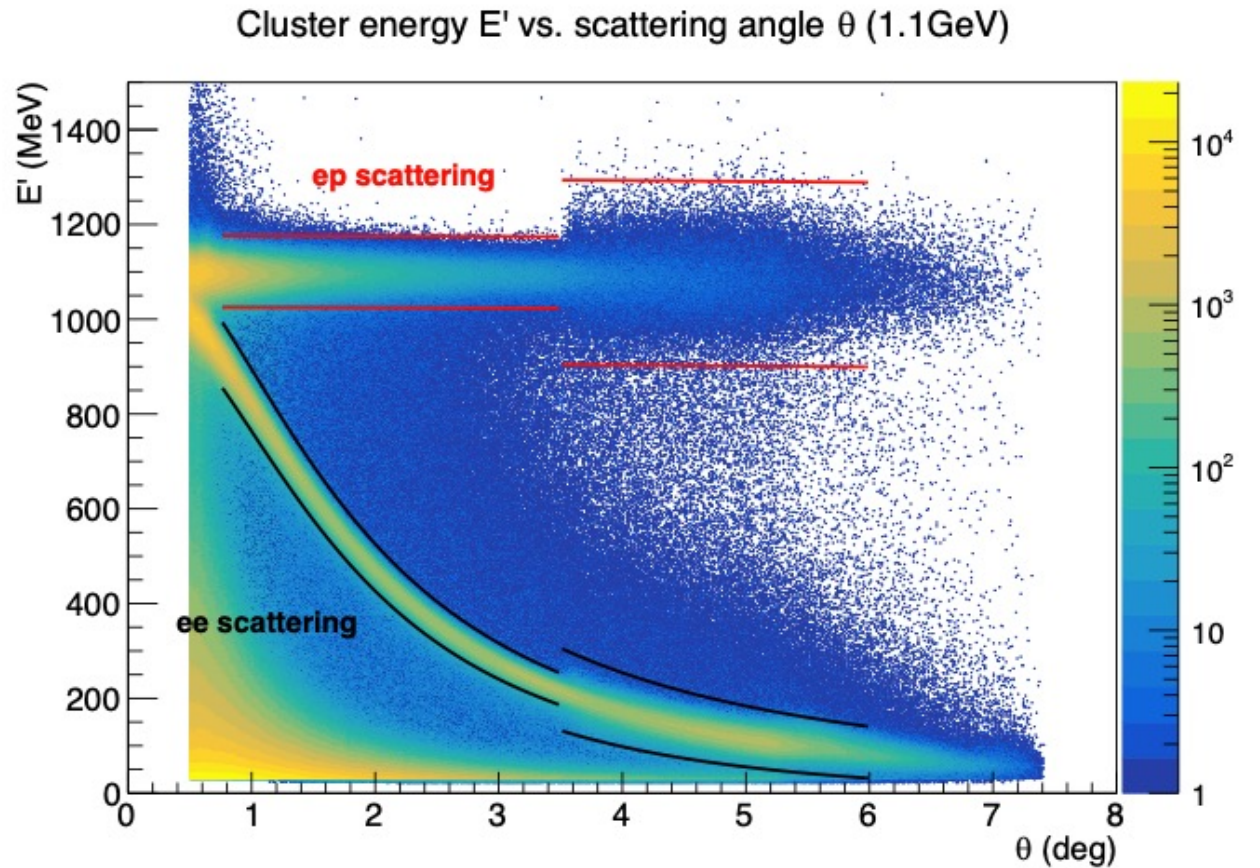
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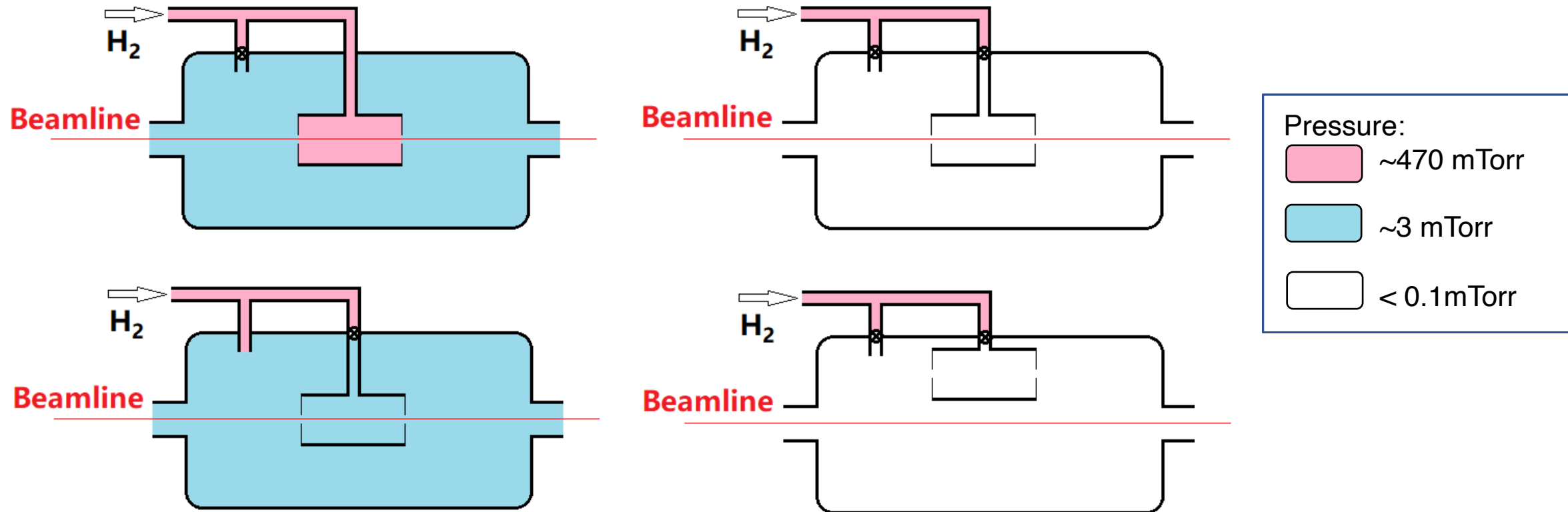
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- Normalize to the simultaneously measured **Møller** scattering process



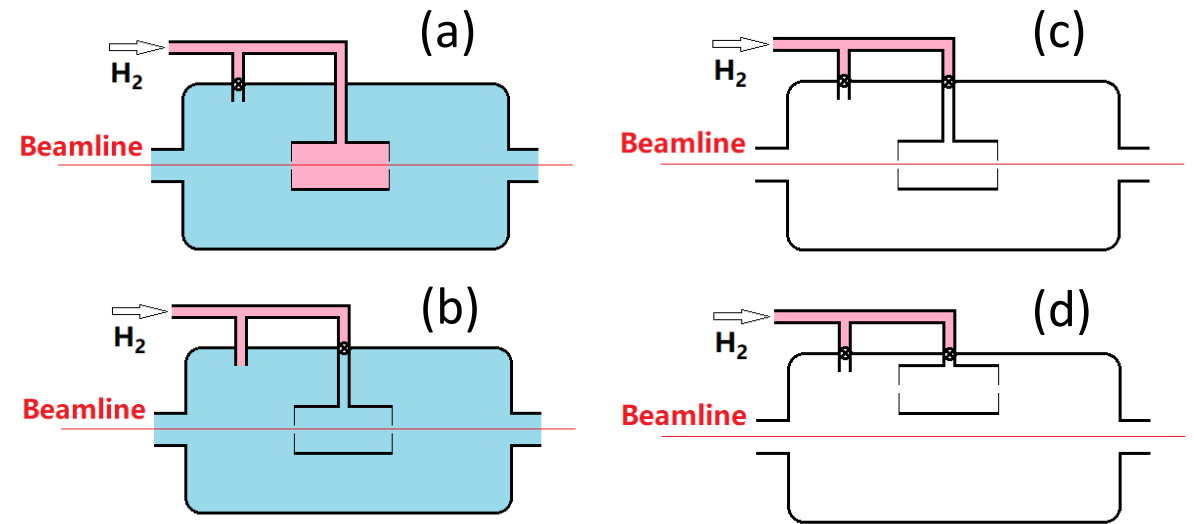
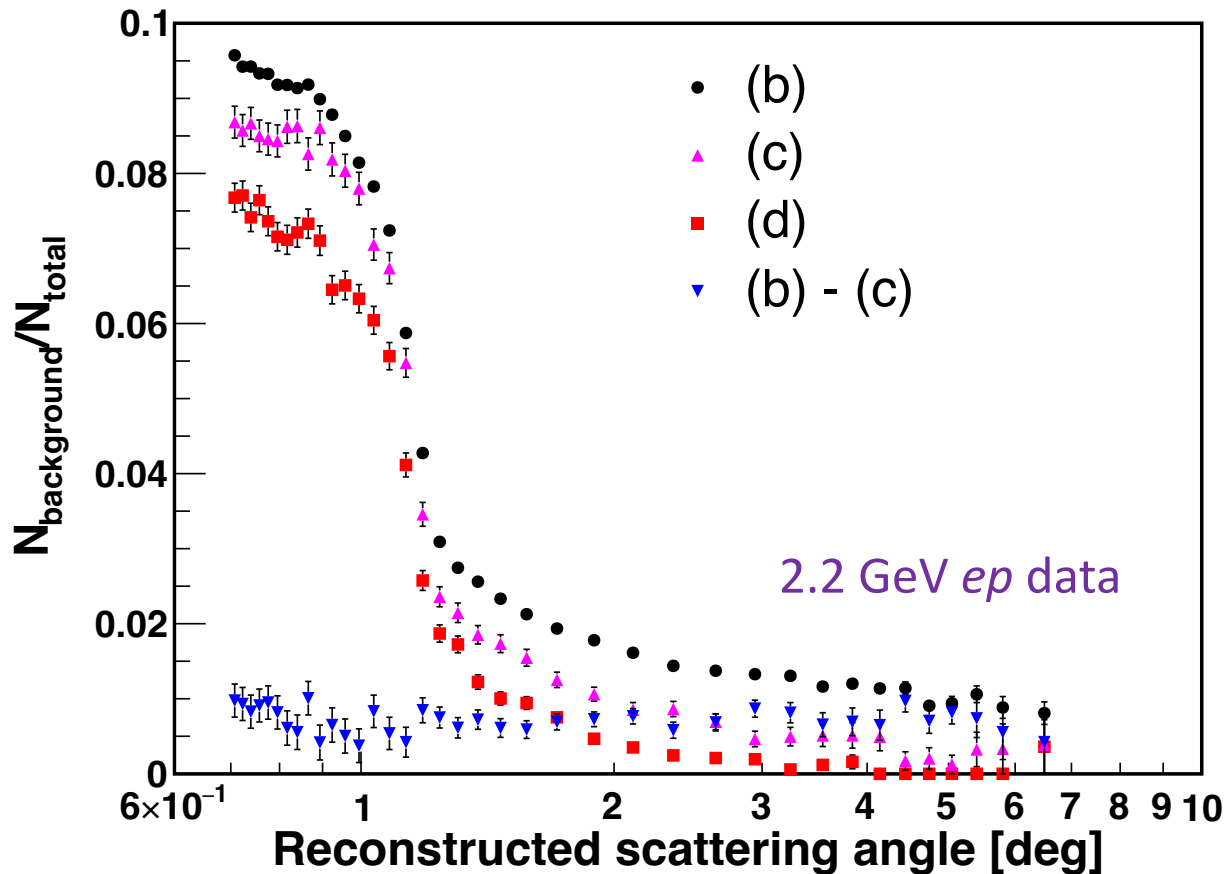
Analysis – Background Subtraction

- Runs with different target condition taken for background subtraction and studies for the systematic uncertainty
- Developed simulation program for target density (COMSOL finite element analysis)



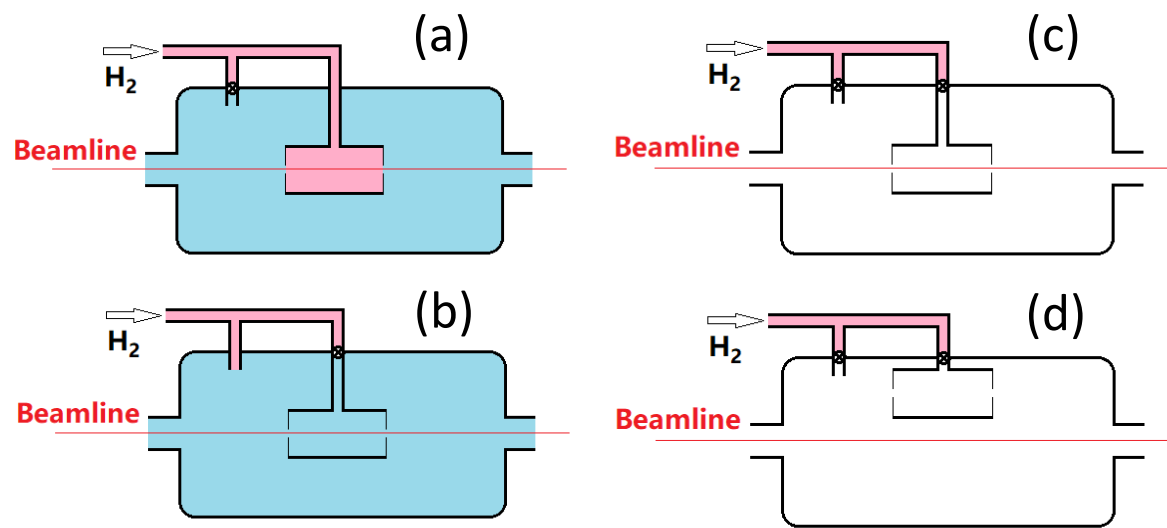
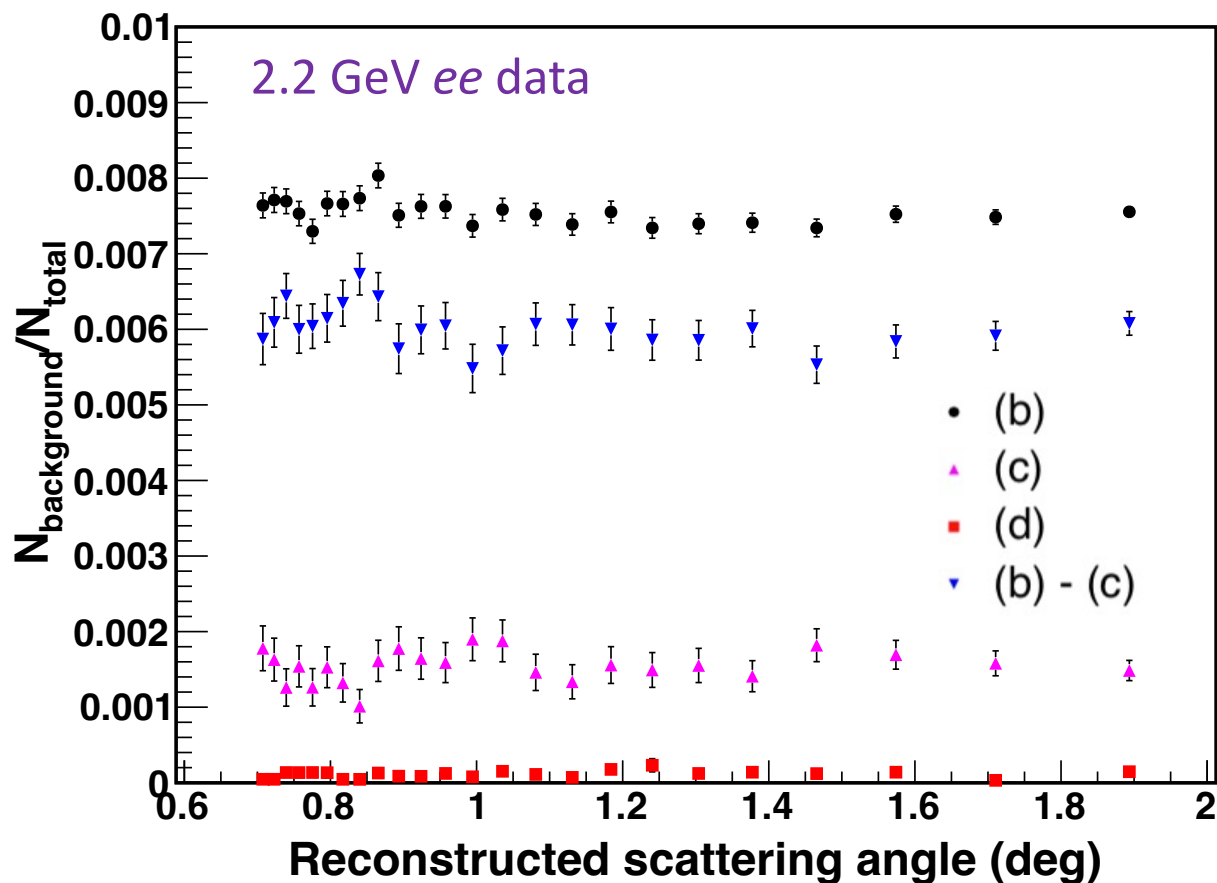
Analysis – Background Subtraction (2.2 GeV)

- ep background rate $\sim 10\%$ at forward angle (<1.1 deg, dominated by upstream beam halo blocker), less than 2% otherwise
- ee background rate $\sim 0.8\%$ at all angles



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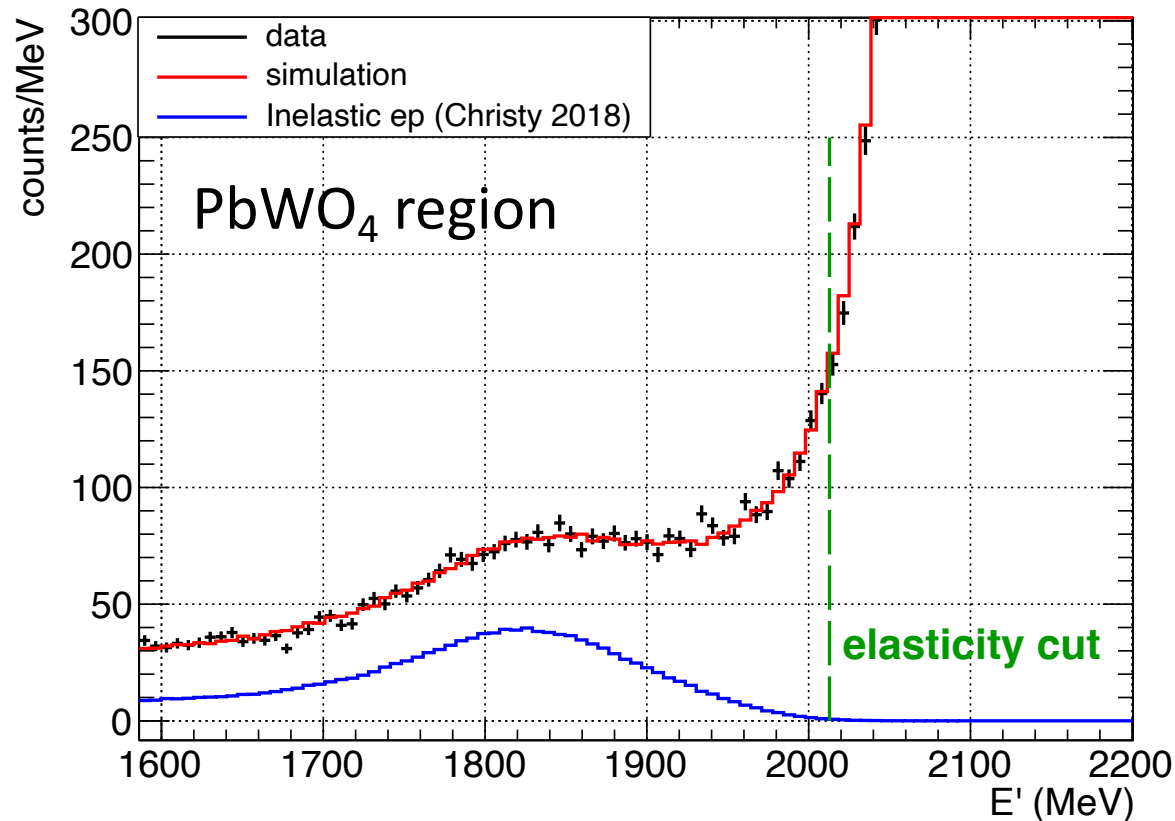
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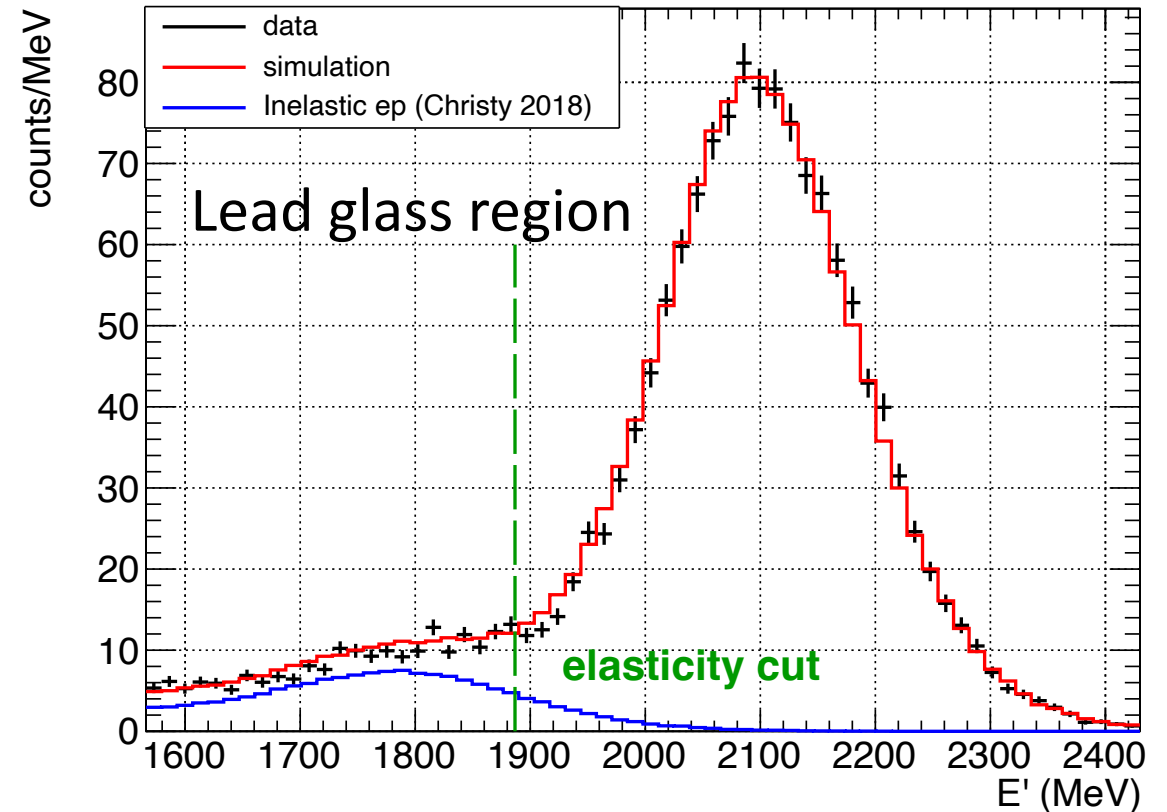
Analysis – Inelastic ep Contribution

- Using Christy 2018 empirical fit to study inelastic ep contribution
- Good agreement between data and simulation
- Negligible for the PbWO_4 region ($<3.5^\circ$), less than 0.2%(2.0%) for 1.1GeV(2.2GeV) in the Lead glass region

spectrum for $3.0^\circ < \theta < 3.3^\circ$ ($Q^2 \sim 0.014 \text{ GeV}^2$)



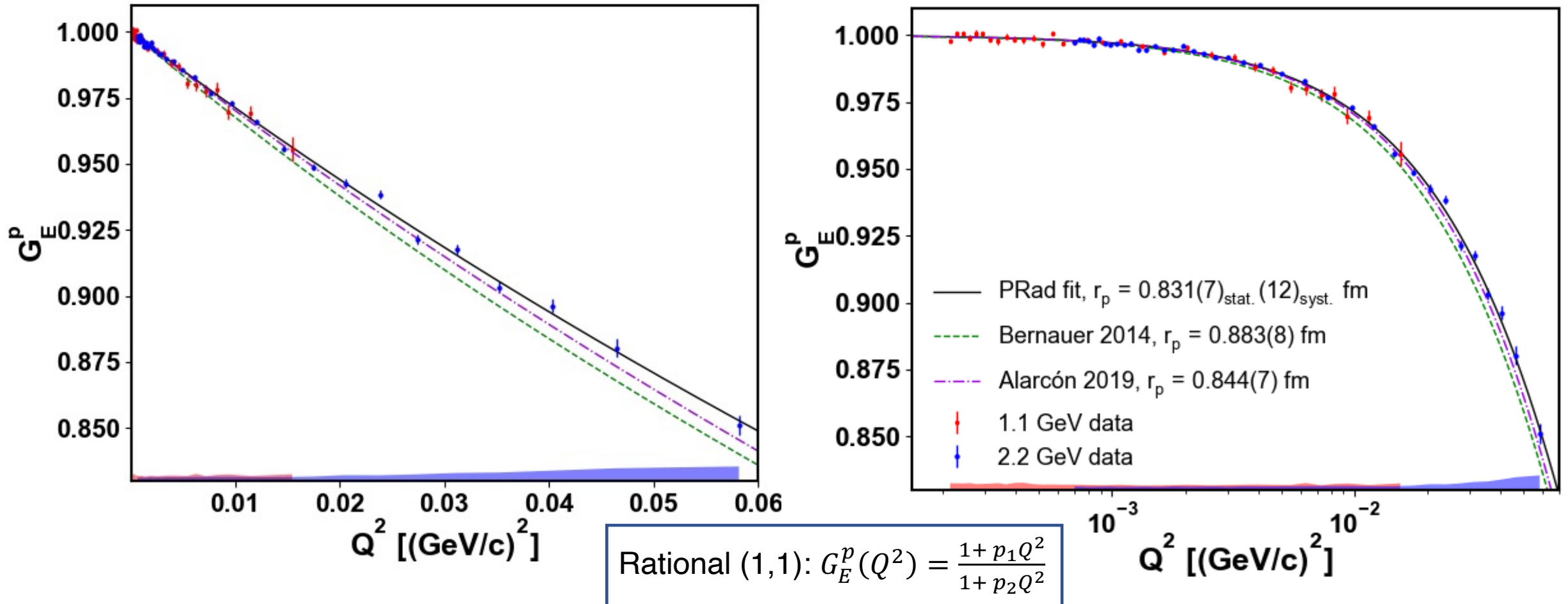
spectrum for $6.0^\circ < \theta < 7.0^\circ$ ($Q^2 \sim 0.059 \text{ GeV}^2$)



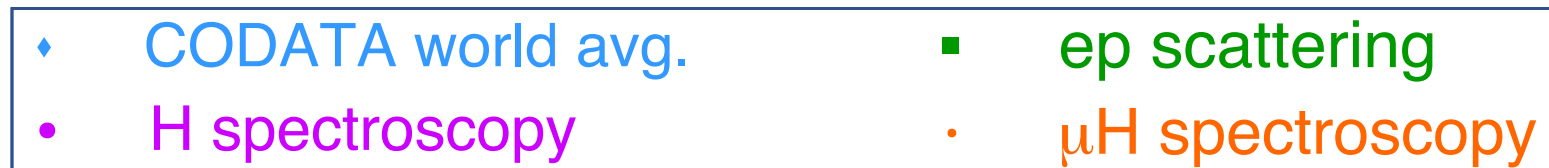
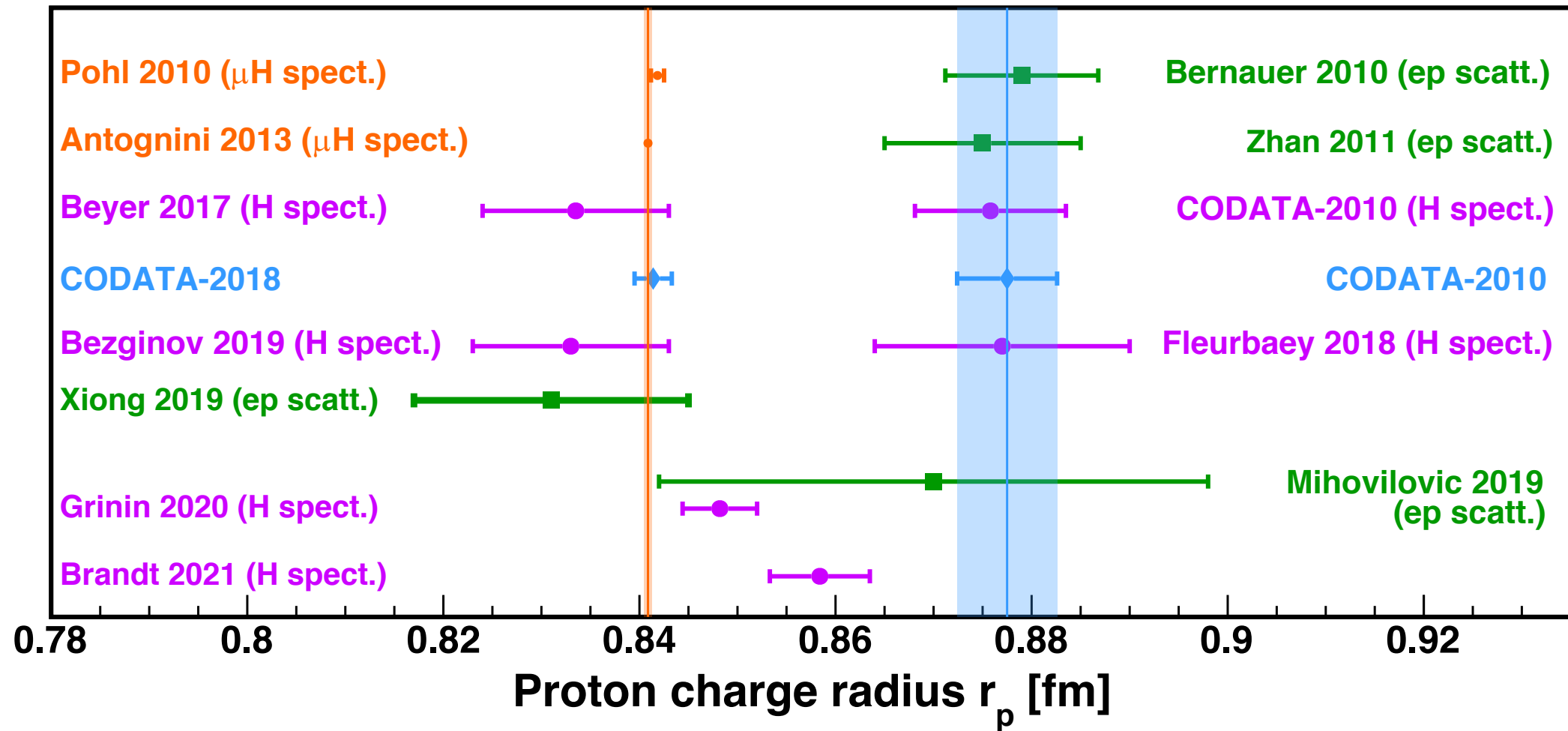
Proton Electric Form Factor G_E^p

$$r_p = 0.831 \pm 0.007 \text{ (stat.)} \pm 0.012 \text{ (syst.) fm}$$

W. Xiong *et al.* *Nature* 575 (2019) 7781

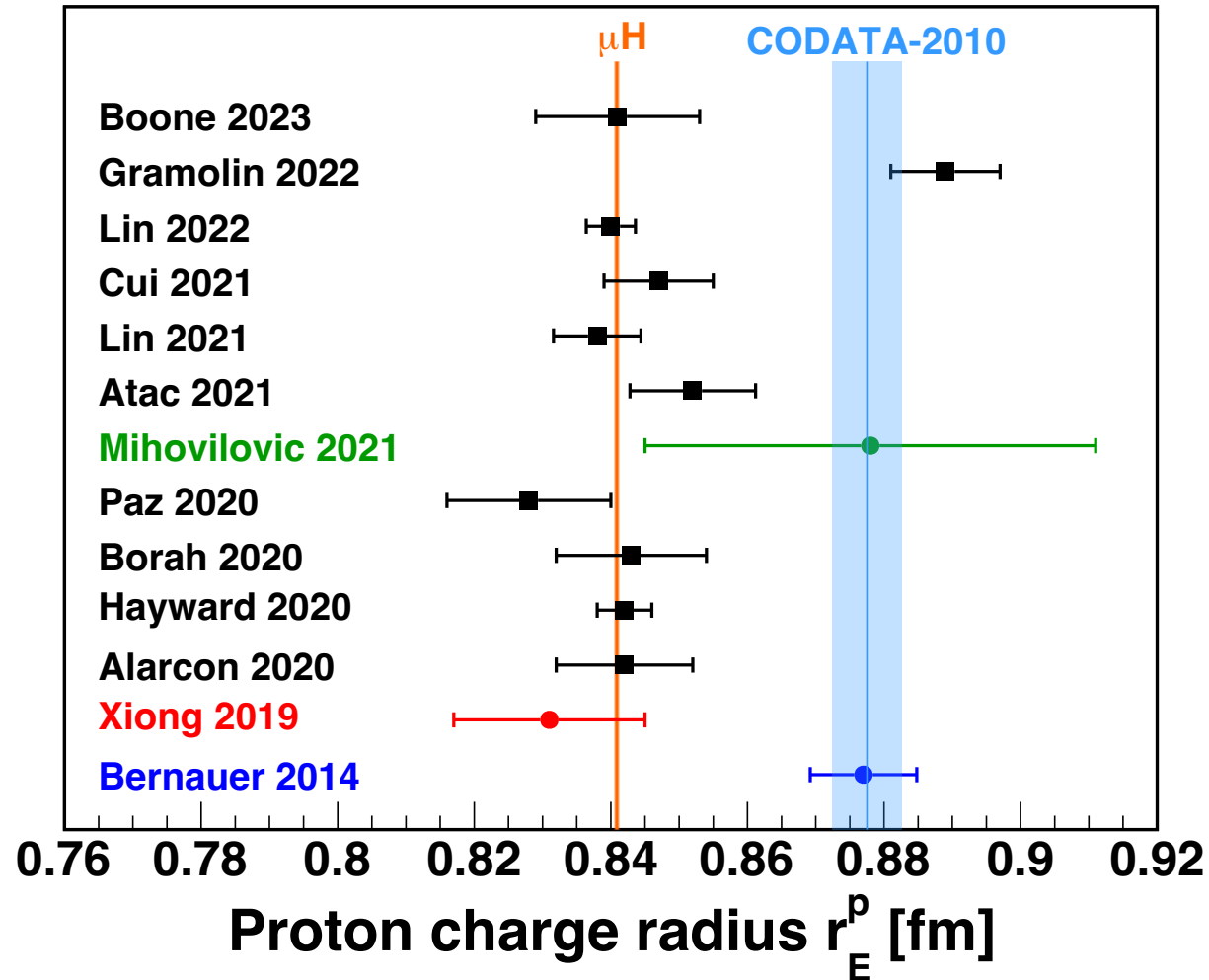


Current Status on Proton Charge Radius

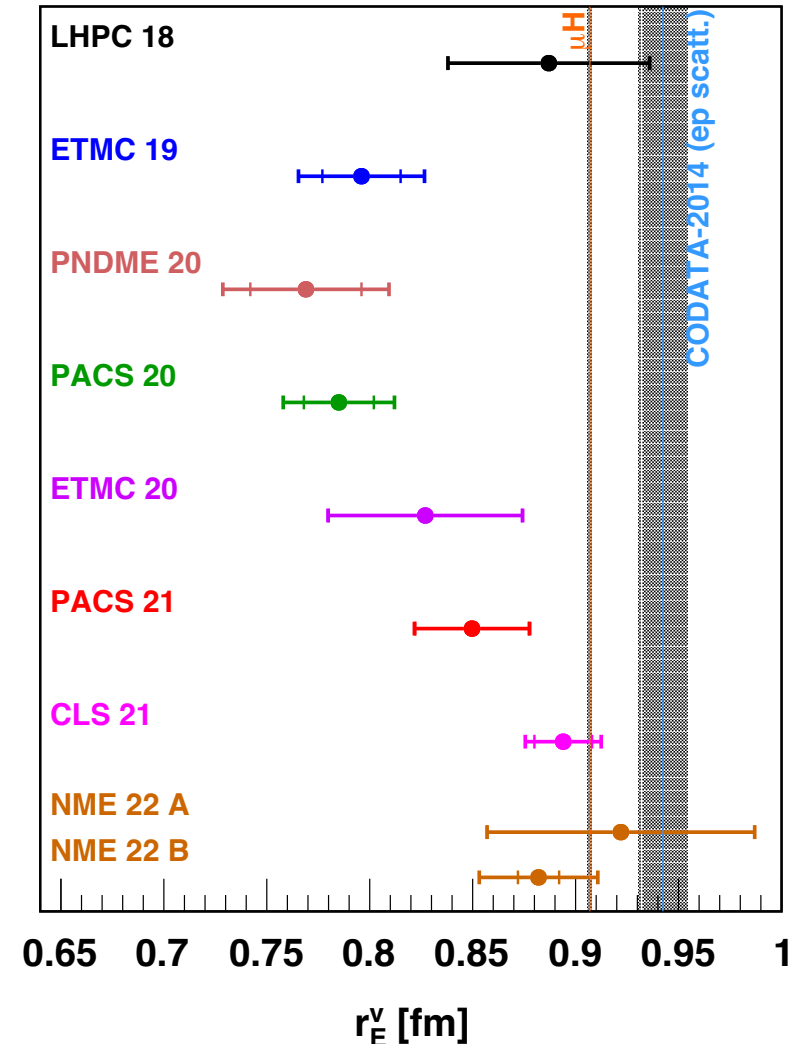


Other Results from Global Analysis and Lattice QCD

Global Analysis



Lattice QCD



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Remaining Issues for Lepton Scattering

- Need other experiments to confirm/reject PRad result
- Is r_p the same in lepton scattering and spectroscopy?
 - C. Peset *et al.* *Prog. Part. Nucl. Phys.* 121 (2021) 103901

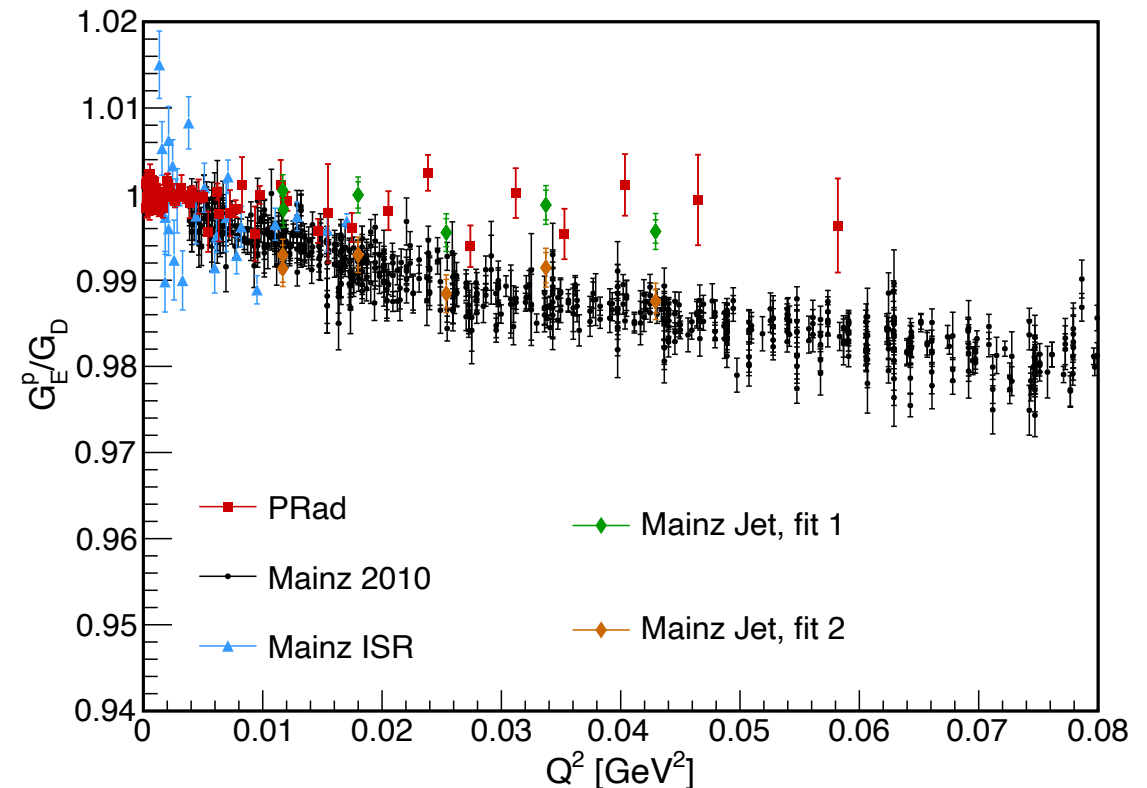
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➤ C. Peset *et al. Prog. Part. Nucl. Phys.* 121 (2021) 103901

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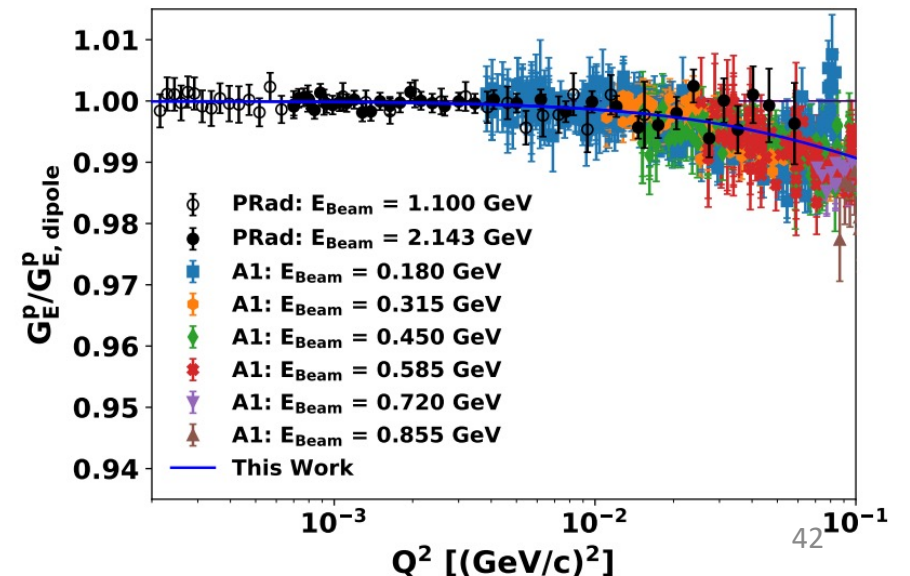
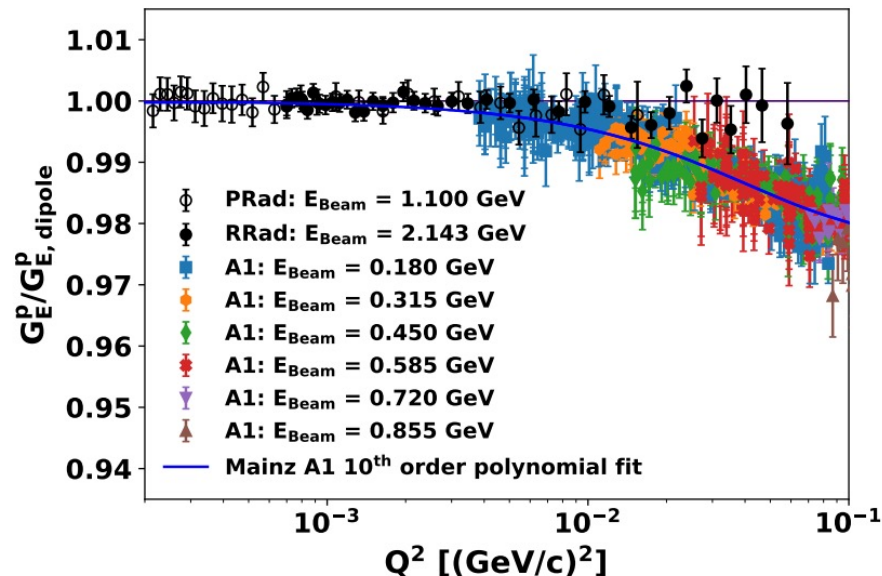
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 3. Problem with fitting procedure?

- J. Zhou (周璟怡) *et al. PRC* 106 (2022) 6, 065505
- Use rational (1, 1) to fit Mainz A1 10th order polynomial fit



Remaining Issues for Lepton Scattering

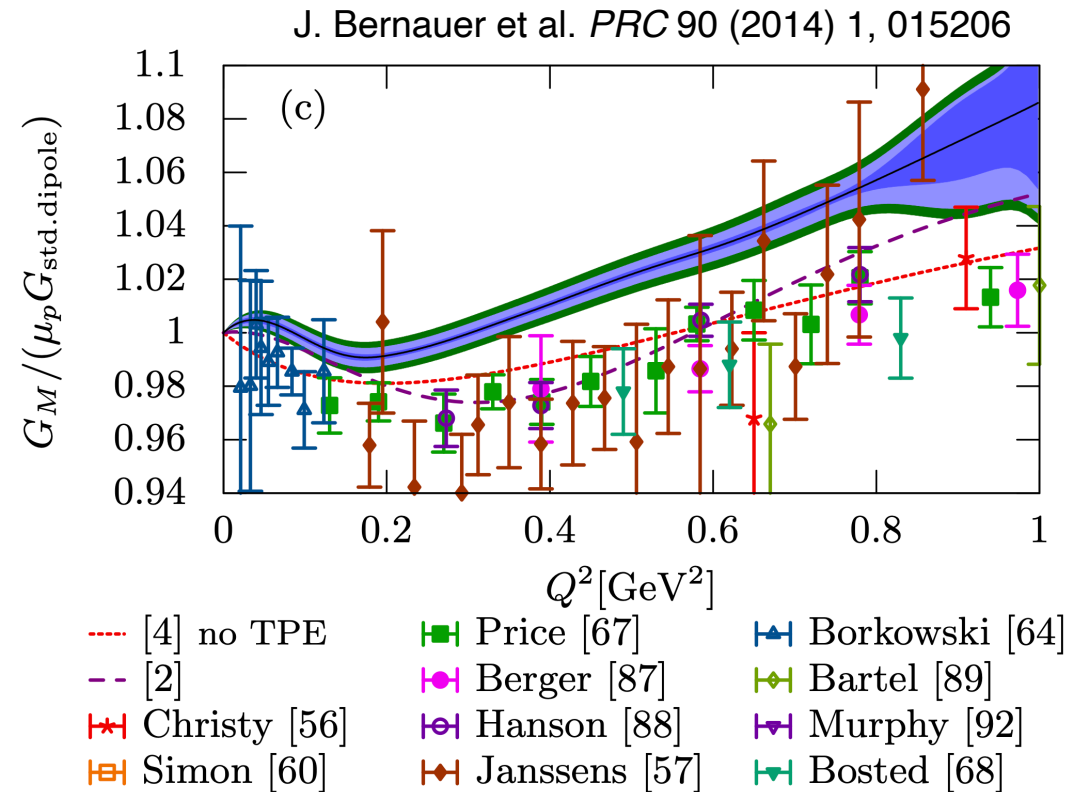
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3. Problem with fitting procedure?
4. Problem with G_M and r_M ?
5. ...

- G. Lee *et al. PRD* 92 013013:
- 0.776(38) fm for Mainz data
 - 0.914(35) fm for world data excluding Mainz



Remaining Issues for Lepton Scattering

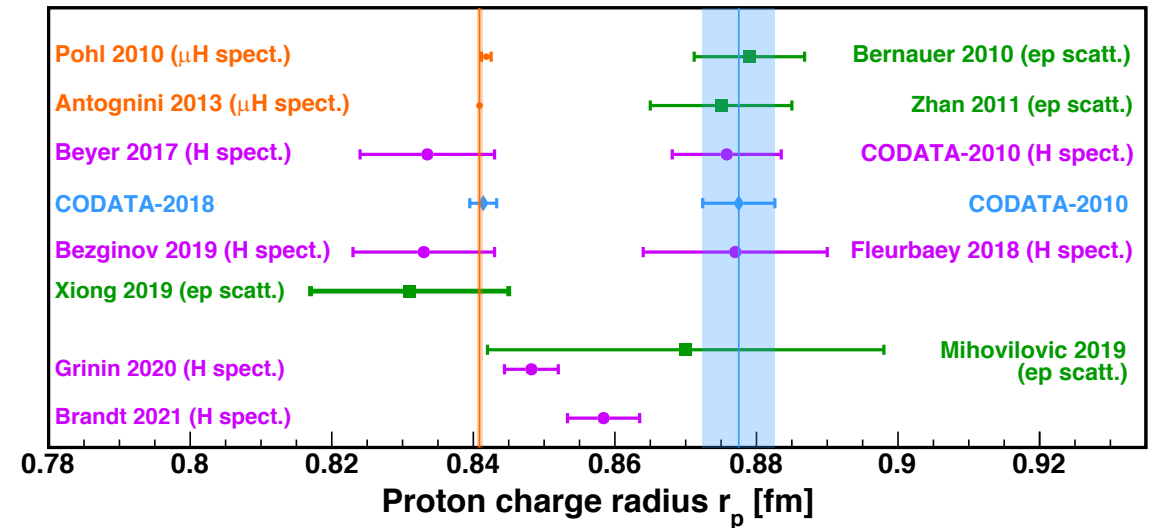
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➤ C. Peset *et al. Prog. Part. Nucl. Phys.* 121 (2021) 103901

- Why G_E data are different?

1. Problem with RC?
2. Unknown systematics?
3. Problem with fitting procedure?
4. Problem with G_M and r_M ?
5. ...

- Maybe new physics is still there...



Remaining Issues for Lepton Scattering

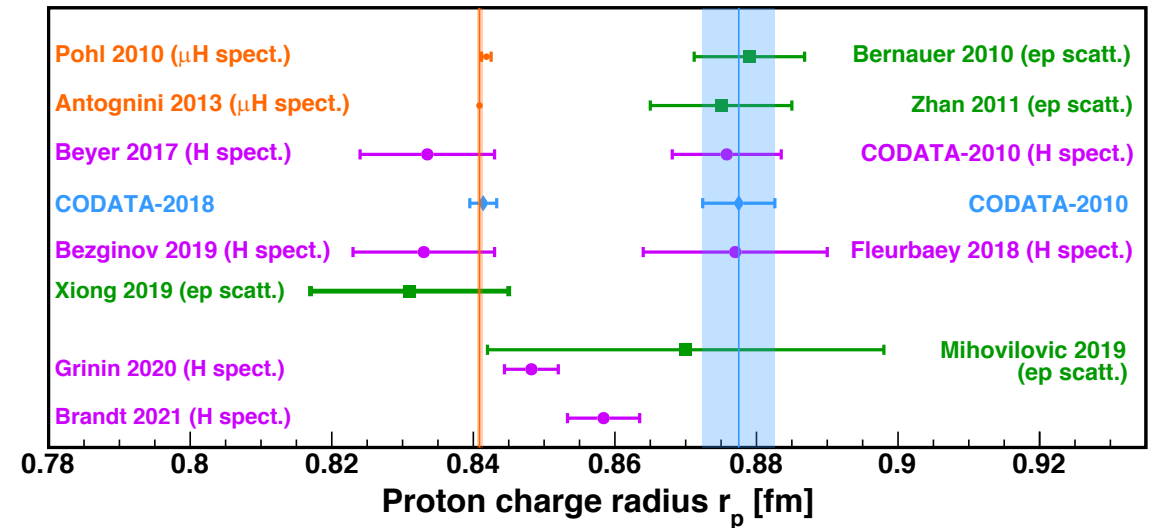
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➤ C. Peset *et al. Prog. Part. Nucl. Phys.* 121 (2021) 103901

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5. ...

- Maybe new physics is still there...



Need future lepton scattering experiments with higher precision!

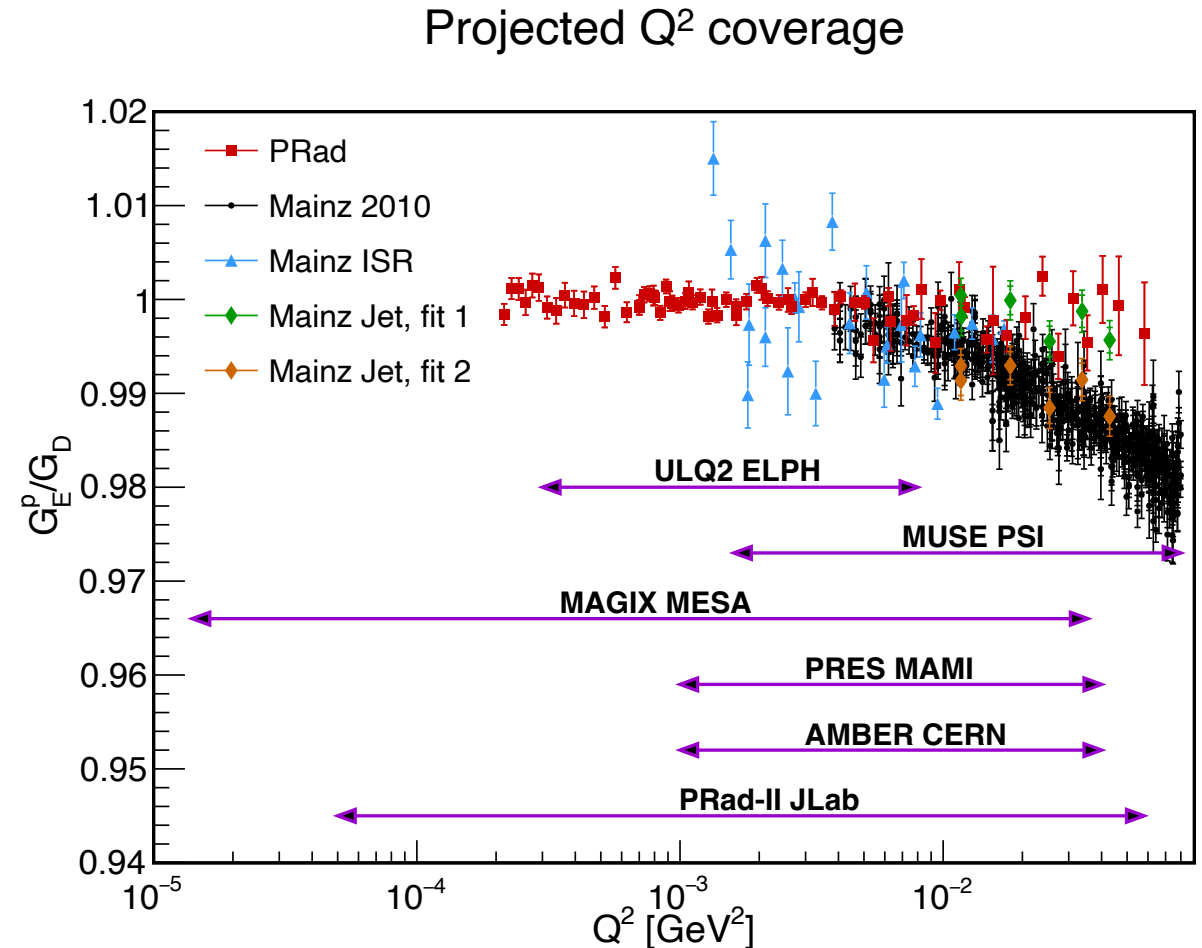
Outline

- Intro to Proton Charge Radius Puzzle
- Recent Progress from ep Scattering Experiments
- Remaining Issues for Lepton Scattering
- Future Lepton Scattering Experiments
- Summary



Highlights of Future Lepton Scattering Experiments

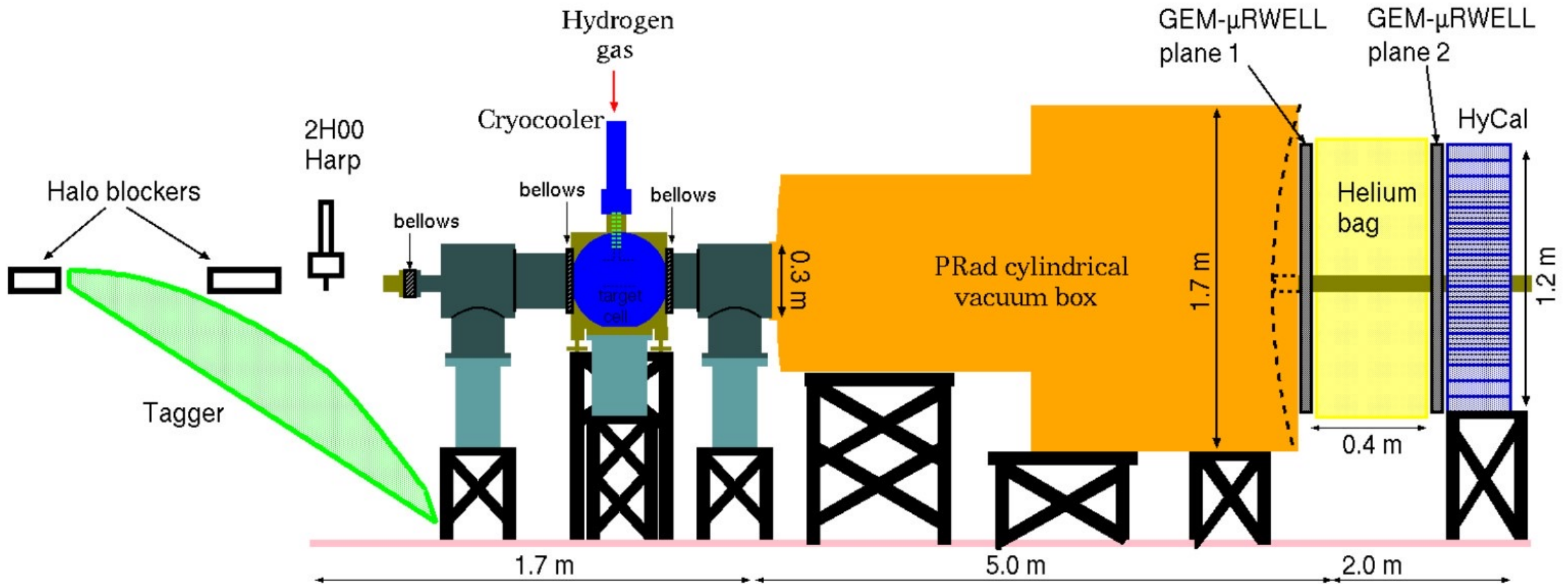
- MUSE experiment at PSI
 - First r_p measurement using muon
 - 4 types of incident leptons: $e^\pm$ and $\mu^\pm$
- AMBER experiment at CERN
 - 100 GeV muon beam, detecting scattered muon and recoiled proton
 - Ultra-small scattering angle, minimize G_M
 - Smaller RC for muon
- PRES experiment at Mainz
 - detecting both scattered electron and recoiled proton
 - Q^2 reconstructed using proton, suppress RC
- MAGIX experiment at Mainz
 - Using jet target
 - Strong sensitivity on both G_E and G_M
- ULQ2 experiment at Tohoku University, Japan
 - Normalize to the well-known e - ^{12}C cross section
 - Strong sensitivity on both G_E and G_M



WX and Chao Peng (彭潮) arXiv:2302.13818

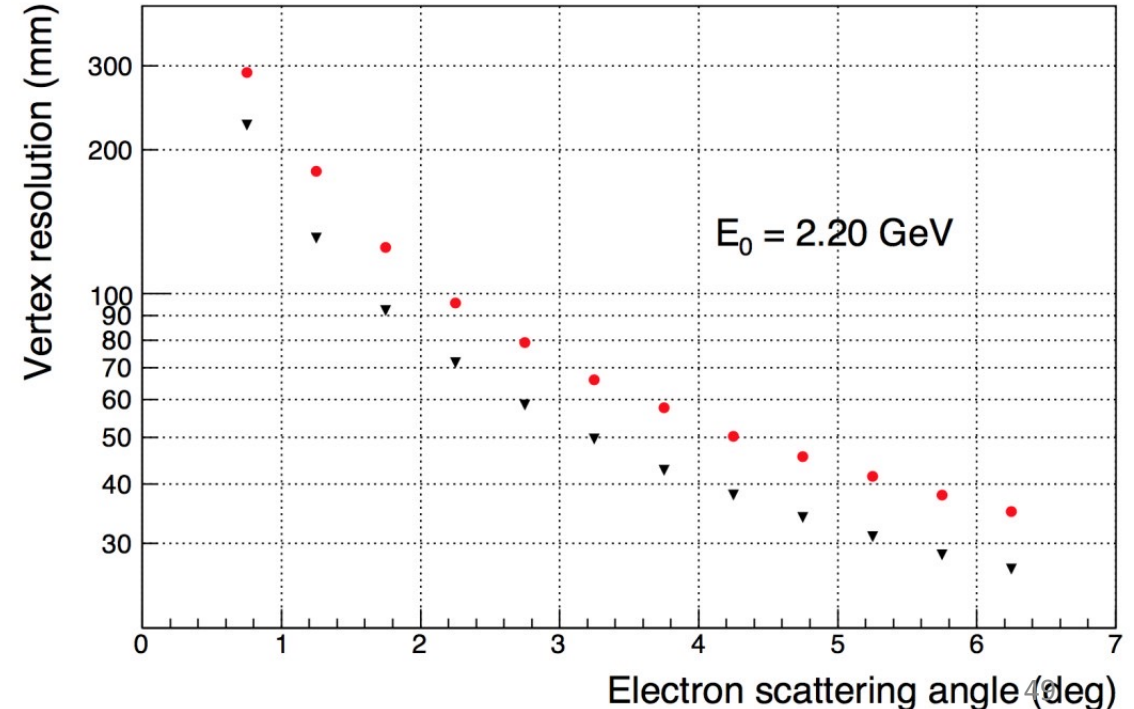
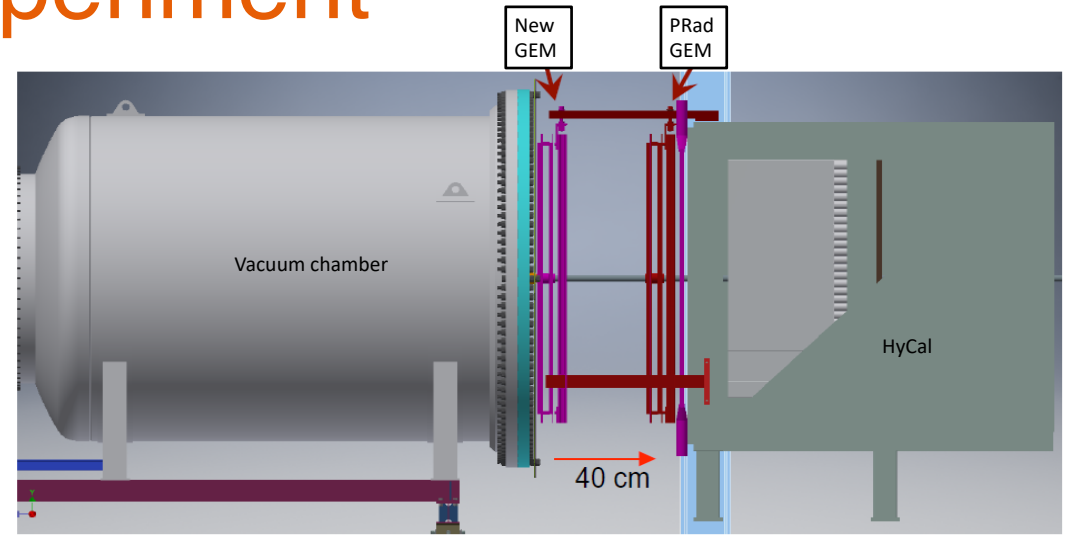
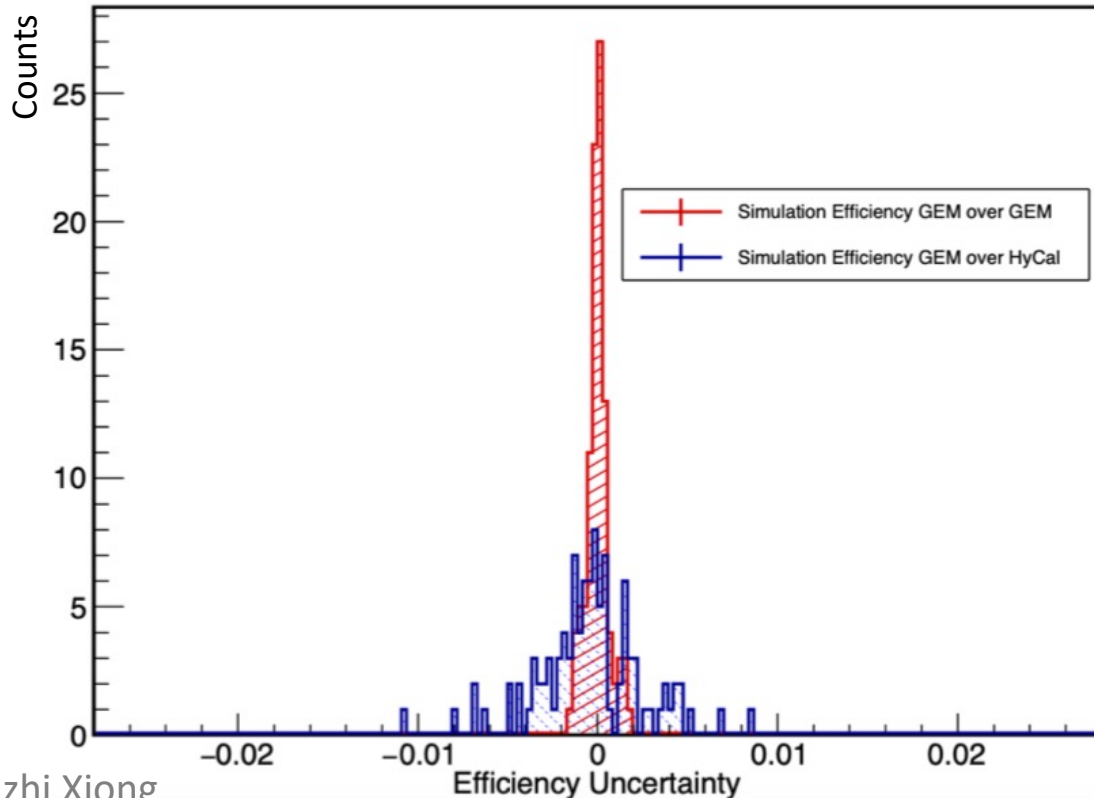
PRad-II Experiment

- JLab PAC 48 approved **PRad-II** (PR12-20-004) with the highest scientific rating “**A**”
- Goal: reach ultra-high precision (~ 4 times smaller total uncertainty), resolve tension between modern $e-p$ scattering results



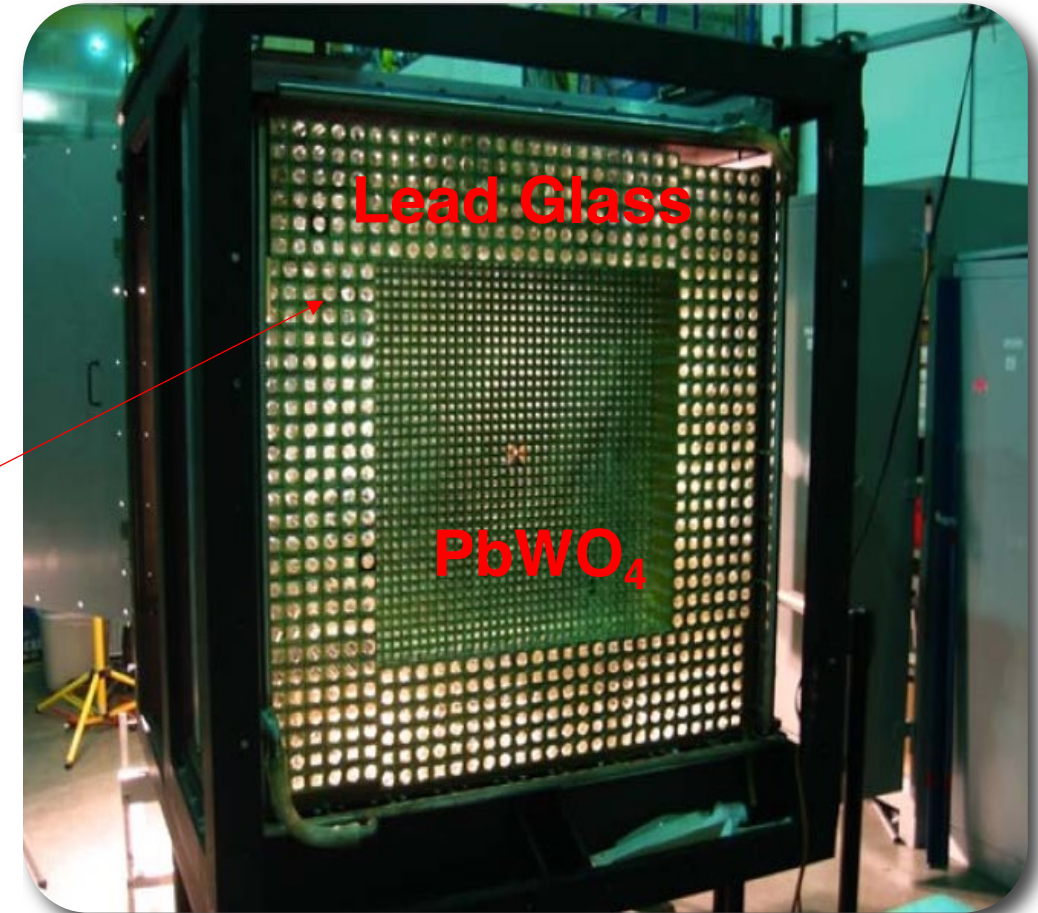
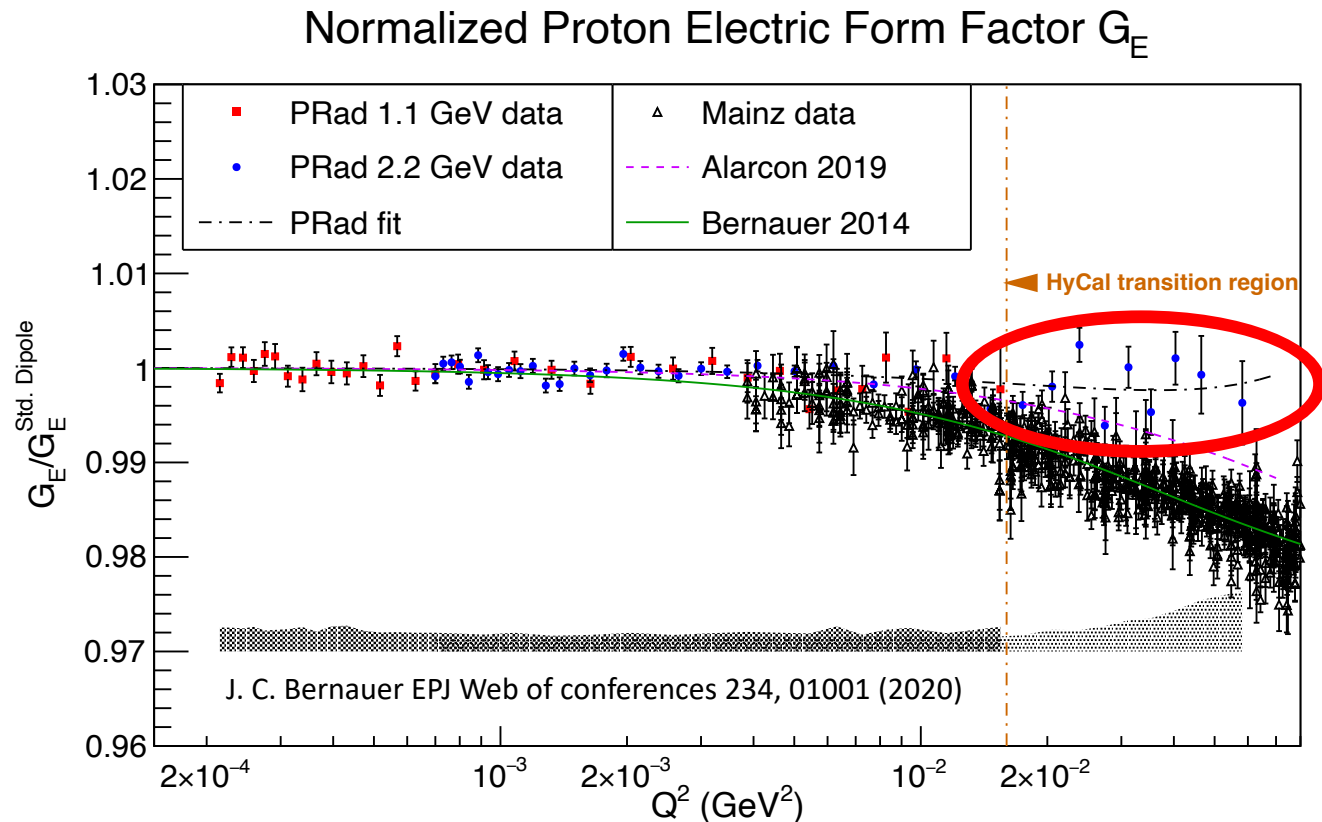
PRad-II Experiment

- Adding tracking capacity (**second GEM plane**)
 - Improve GEM efficiency measurement
 - Vertex-z reconstruction for *ep* to reject upstream background



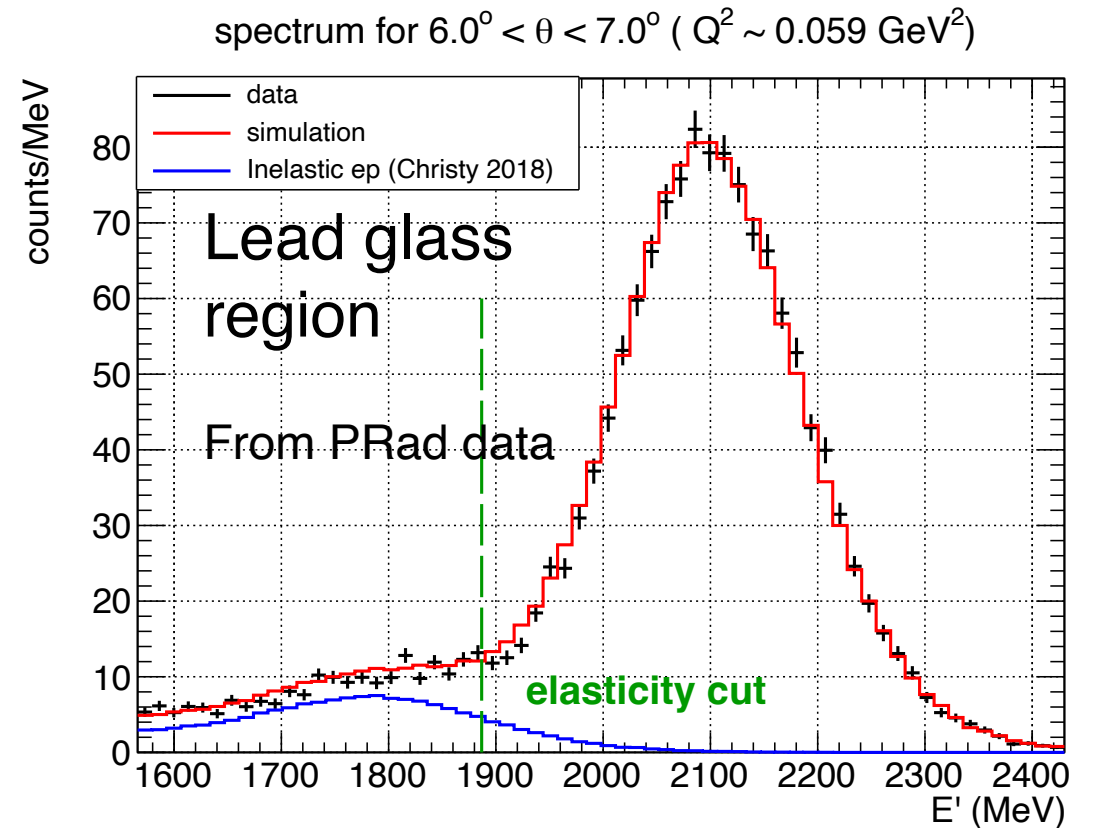
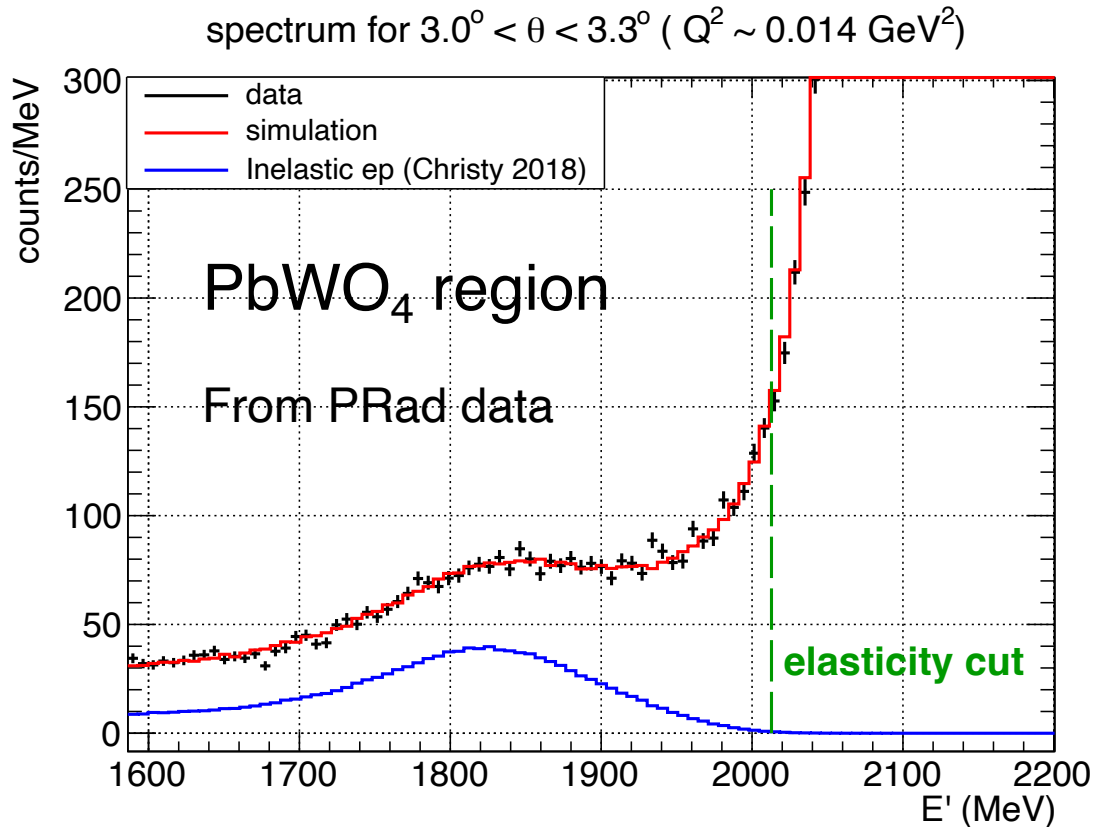
PRad-II Experiment – Cont.

- Upgraded HyCal with all high resolution PbWO_4 modules
 - Better energy resolution: **2.4% v.s. 6.0%** from LG
 - Better position resolution
 - Better non-linearity response



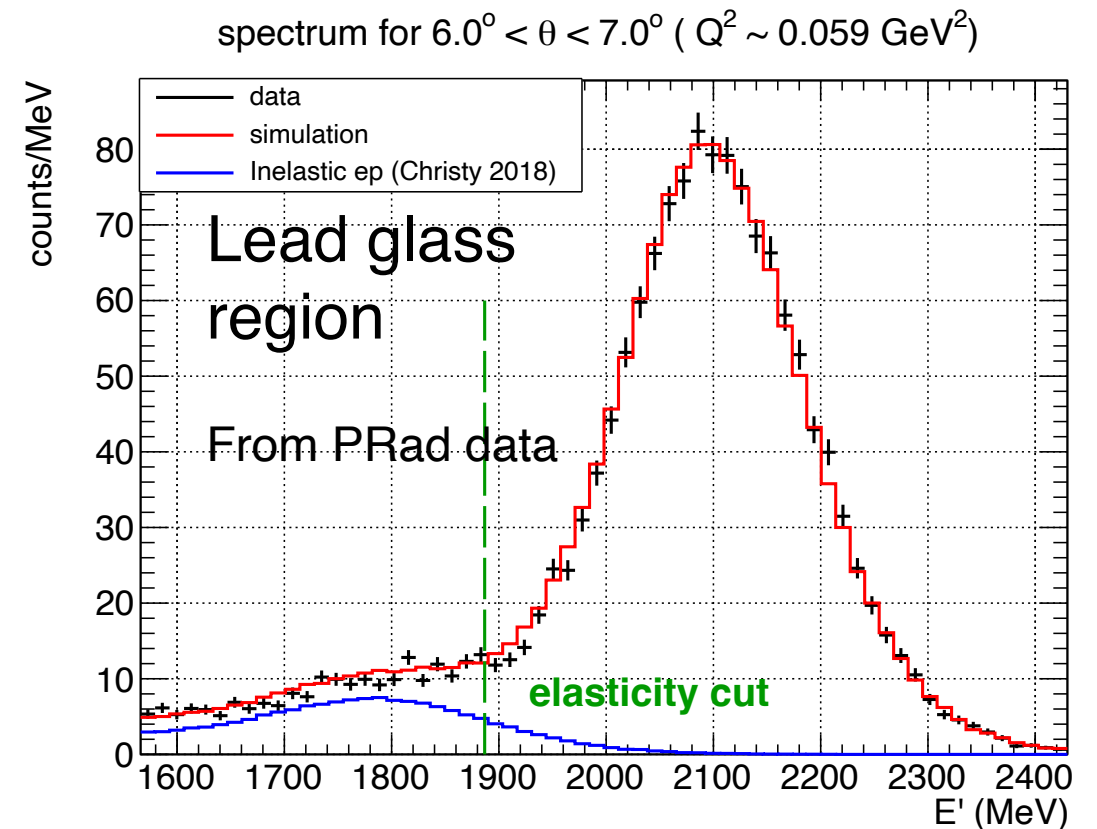
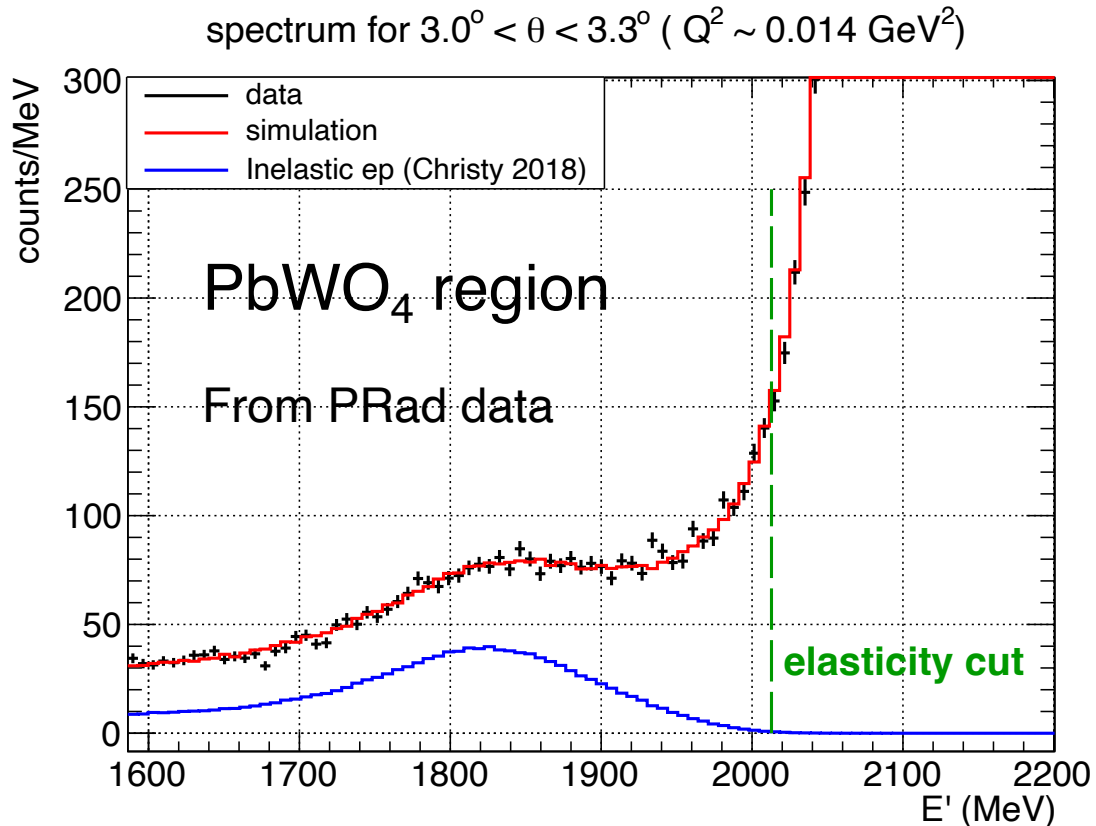
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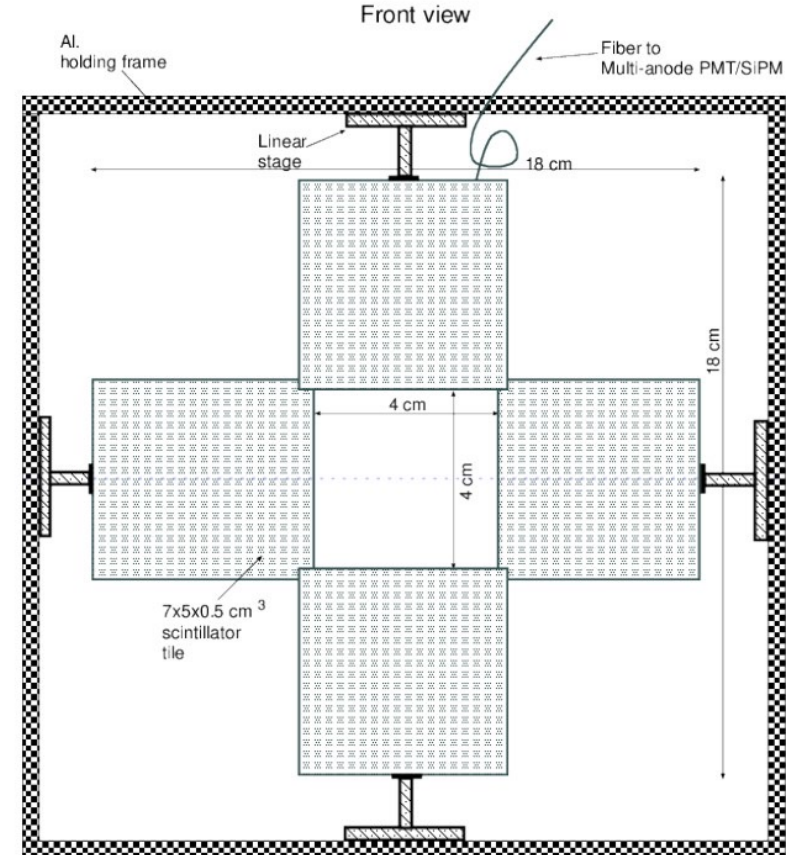
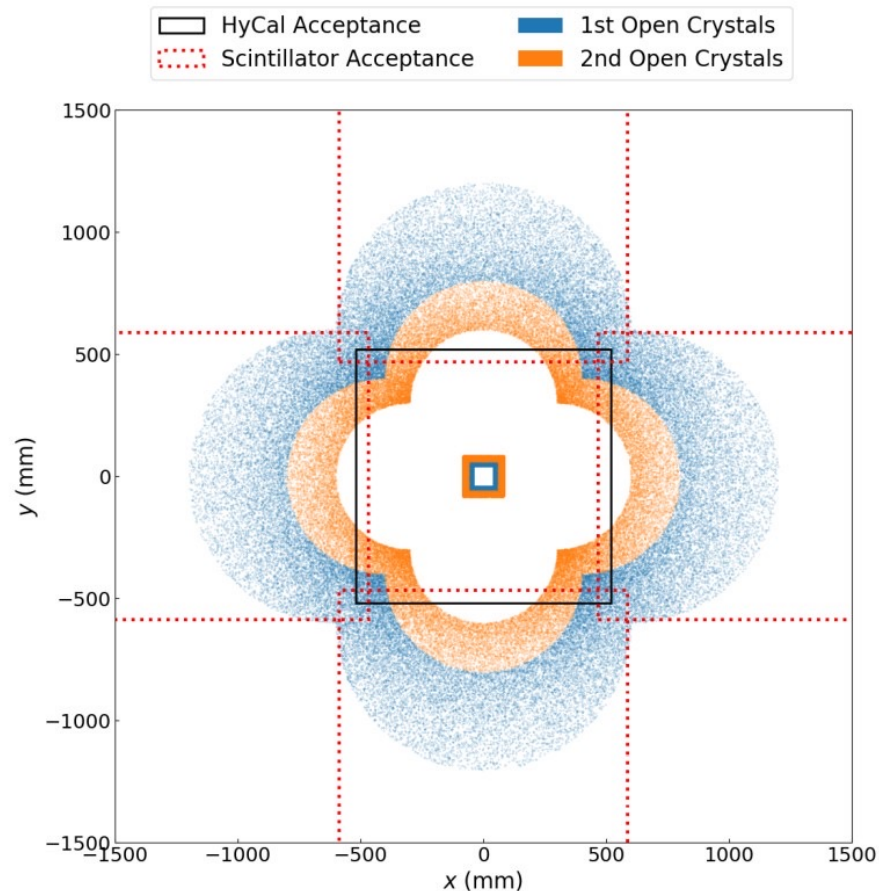
PRad-II Experiment – Cont.

- Upgraded HyCal with all high resolution PbWO_4 modules
 - Better energy resolution: **2.4% v.s. 6.0%** from LG
 - Better position resolution
 - Better non-linearity response
- **Currently seeking funding or used modules for the upgrade**

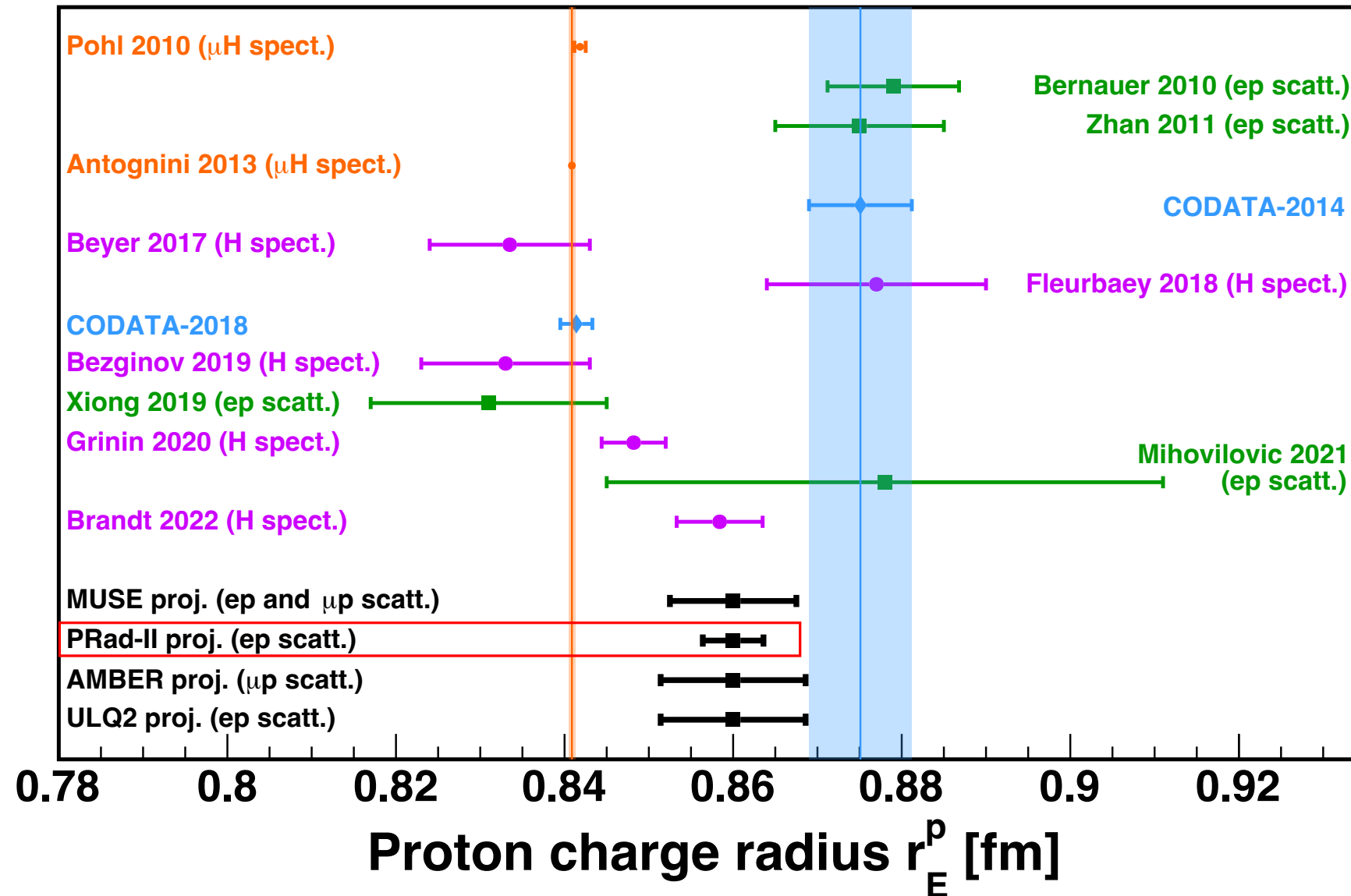


PRad-II Experiment – Cont.

- Convert to FADC based readout for HyCal
- **Four times** smaller stat. uncertainty
- Better RC calculating including NNLO diagrams
- New scintillating detectors, help reaching $Q^2 \sim 10^{-5} \text{ GeV}^2$

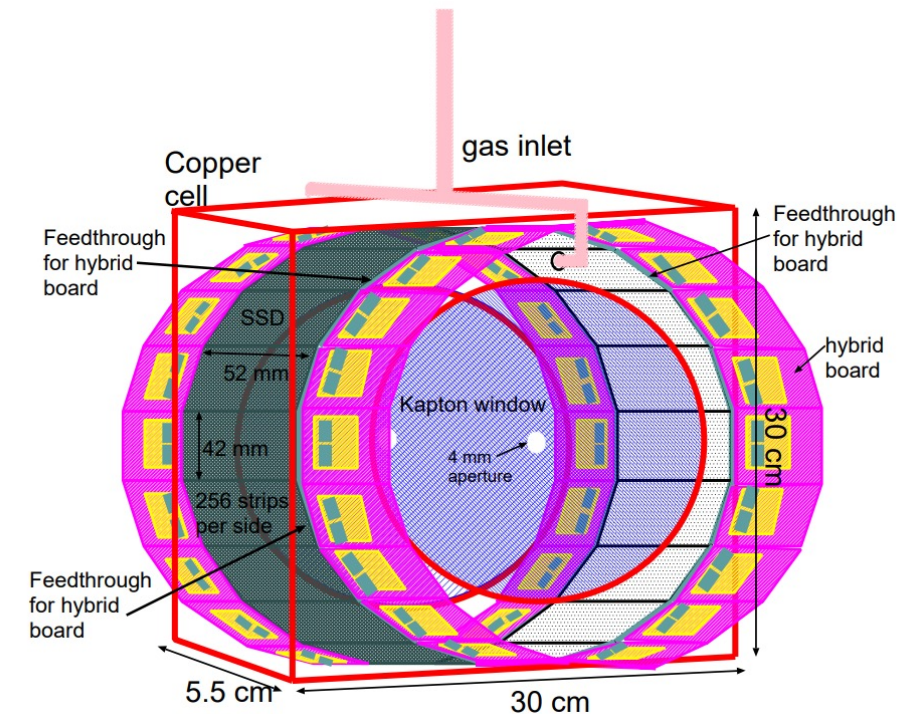
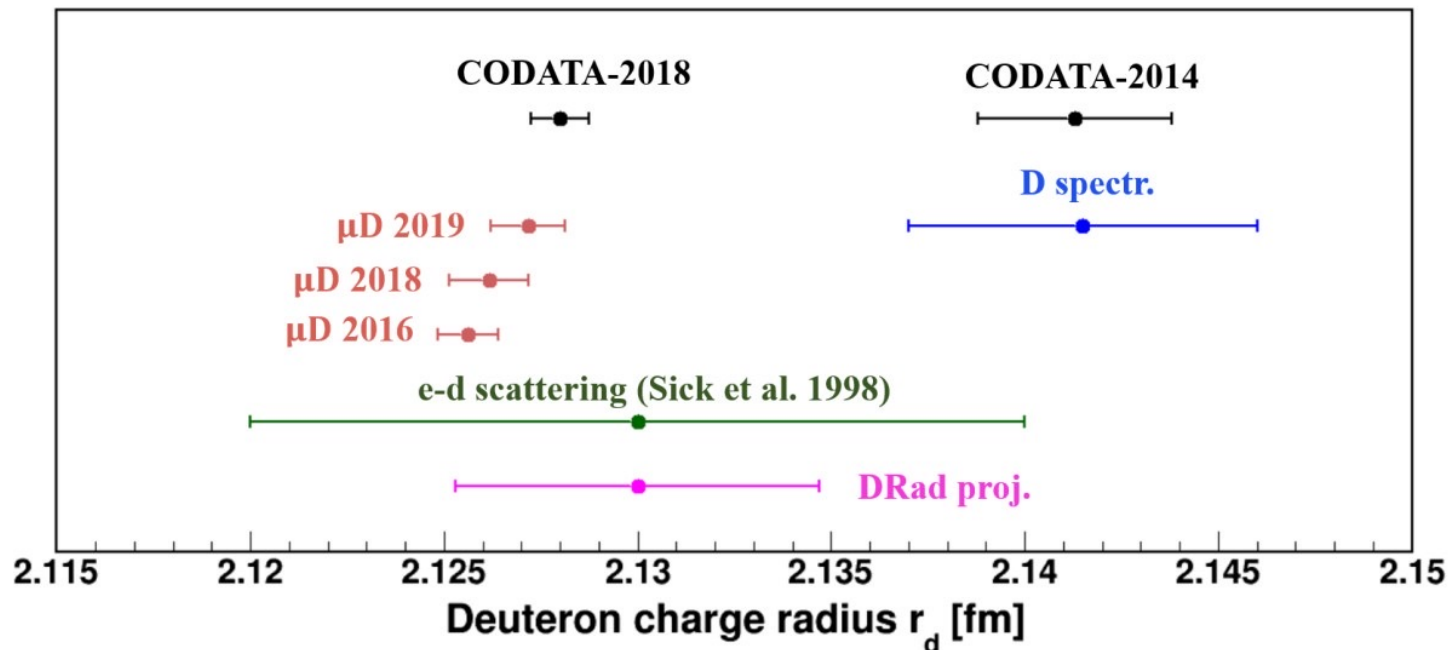


Projected Results



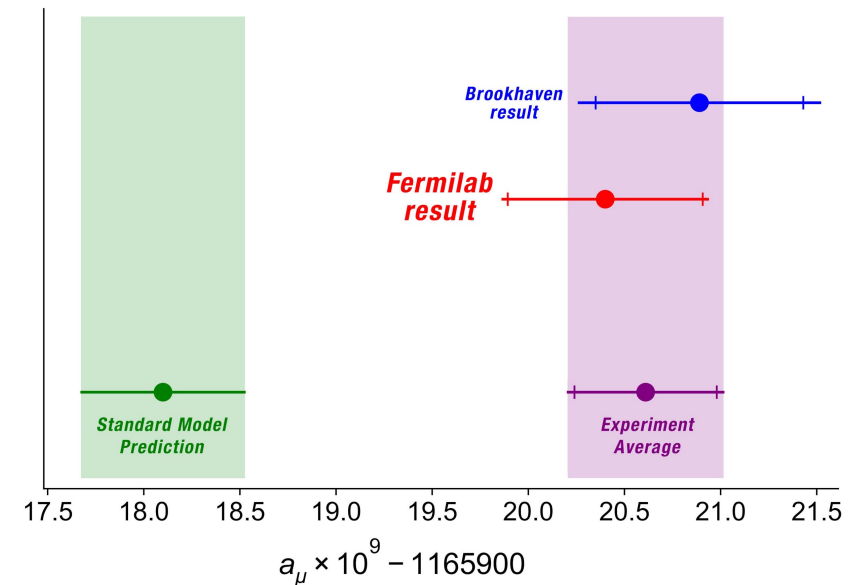
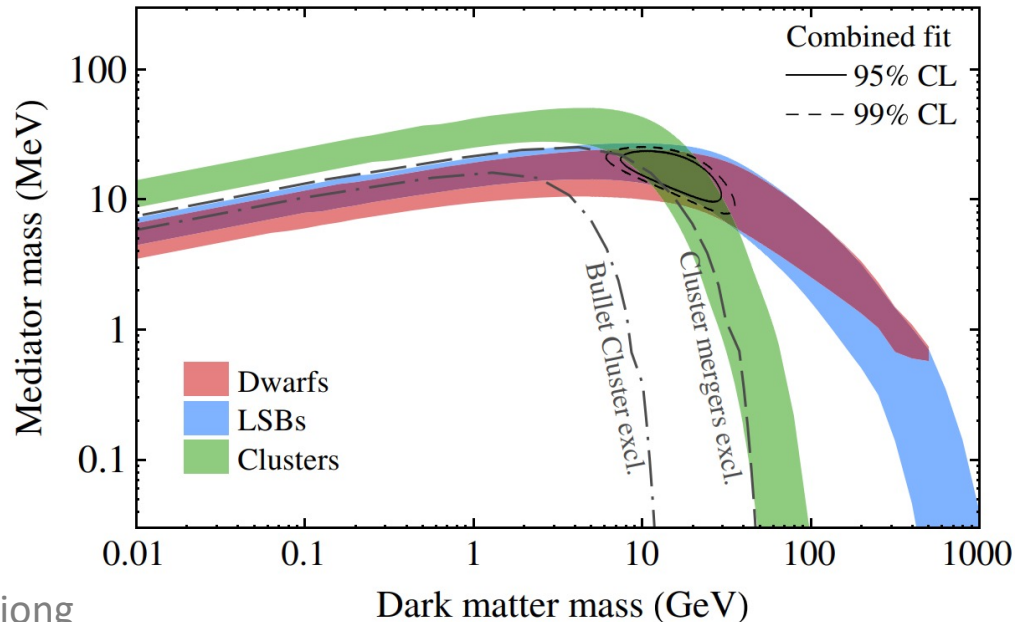
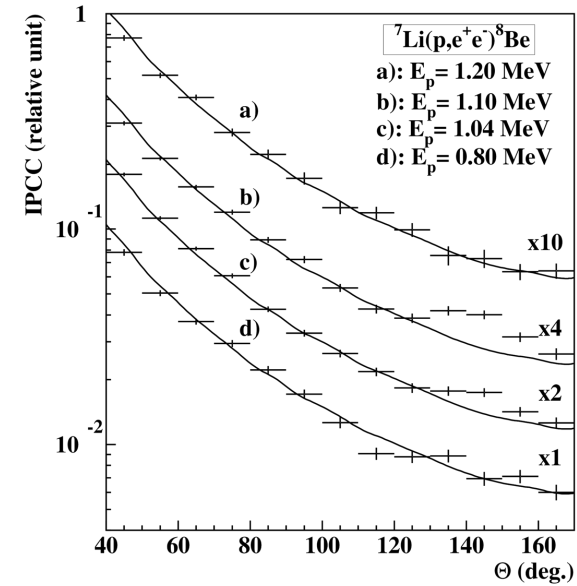
Deuteron Charge Radius Puzzle and DRad Experiment

- Similar 7σ discrepancy exists between μ D and D spectroscopy (“deuteron charge radius puzzle”)
- Previous ed scattering precision not good enough, need better data
- Use mostly PRad setup with **additional recoil detector** for deuteron detection
- Plan to submit proposal this year to JLab

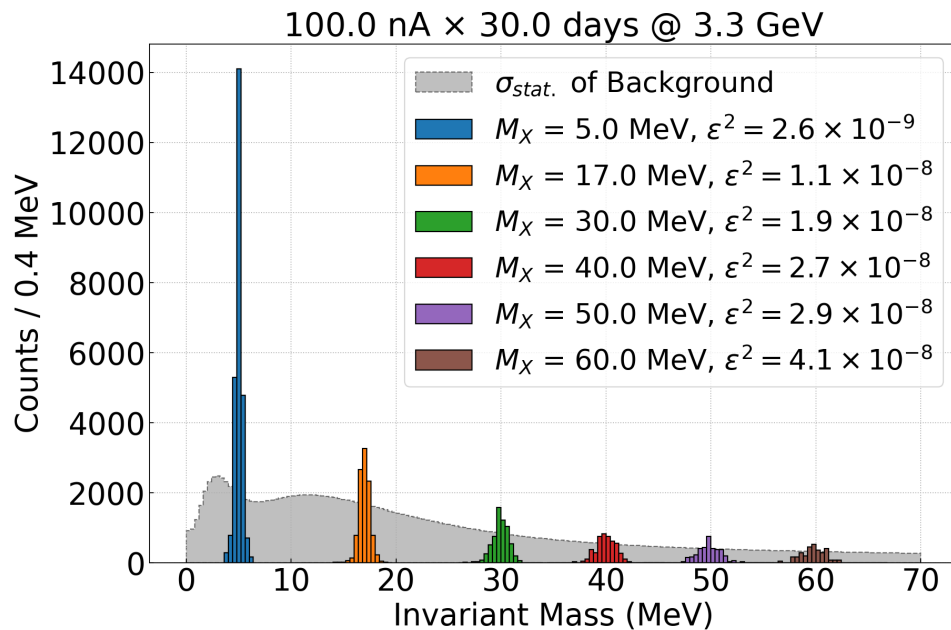
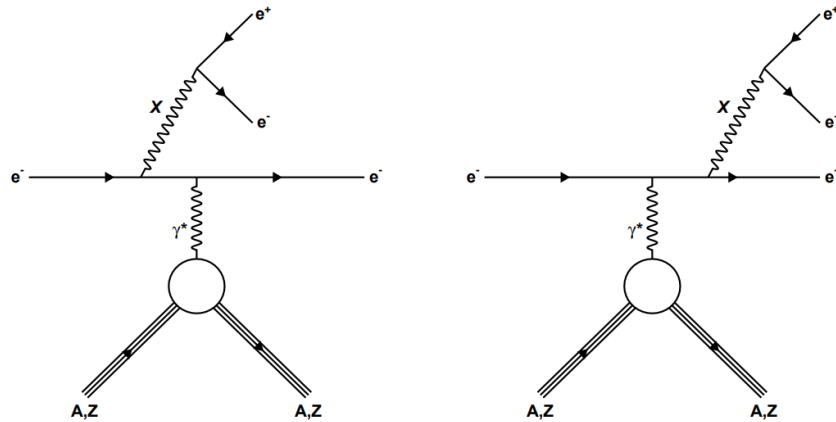


X17 Particle Search Experiment

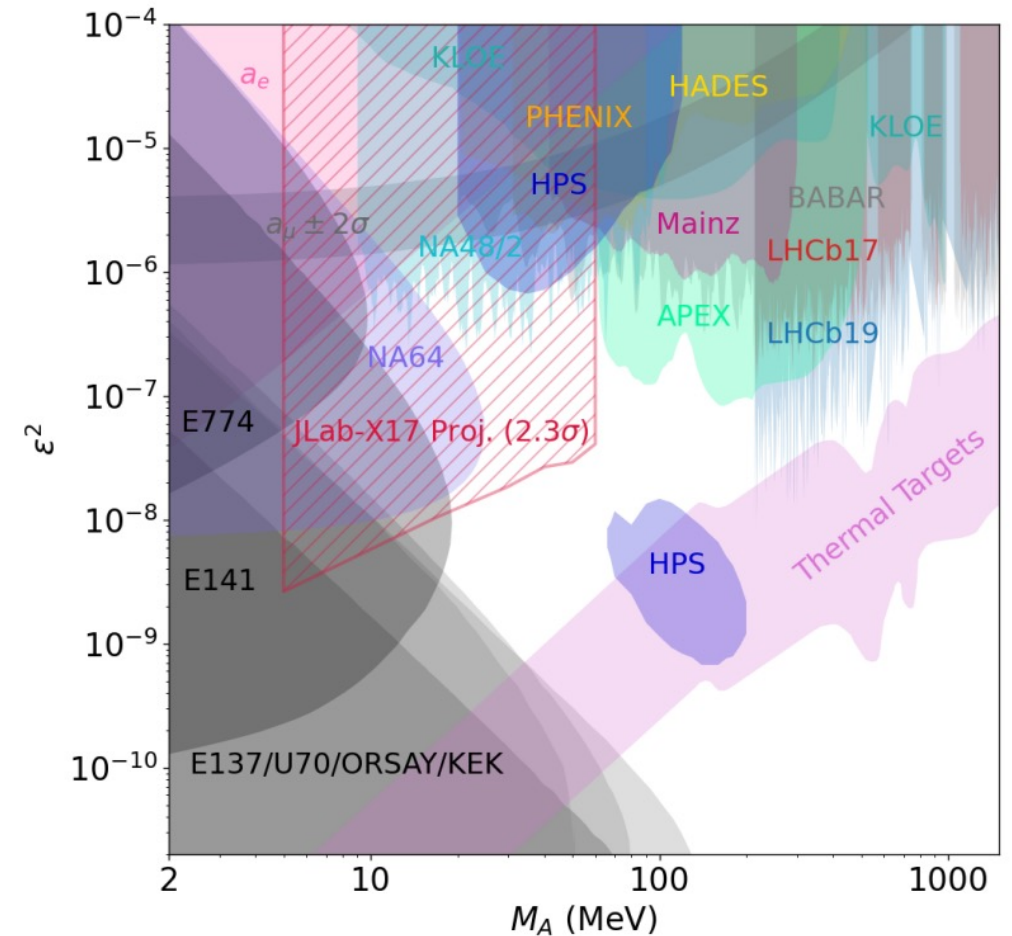
- Search for new 3-60 MeV dark hidden sector particle using PRad setup
- Currently approved with highest scientific rating “A”
- Motivated by multiple recent anomalies:
 1. ^8Be anomaly and the X17 particle
 2. Astronomical small structure
 3. muon g-2



X17 Particle Search Experiment



Projected Search Range



Summary

- PRad measured r_p using novel scattering technique:
 - $r_p = 0.831 \pm 0.007$ (stat.) ± 0.012 (syst.) fm (*Nature* 575 (2019) 7781)
- Puzzle considered partially resolved, but many problems remain, particularly in lepton scattering
 - r_p definition between scattering and spectroscopy
 - Form factor difference between PRad and Mainz data
- Many future lepton scattering experiments will help address these issues, and push precision frontier
 - PRad-II experiment with $\delta_r \sim 0.0036$ fm, will be **most precise** scattering result, new search for lepton-universality violation

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Thank you for your attention and we welcome all physics ideas

Backup

PRad-II Uncertainty Budget

Item	PRad δr_p [fm]	PRad-II δr_p [fm]	Reason
Stat. uncertainty	0.0075	0.0017	more beam time
GEM efficiency	0.0042	0.0008	2nd GEM detector
Acceptance	0.0026	0.0002	2nd GEM detector
Beam energy related	0.0022	0.0002	2nd GEM detector
Event selection	0.0070	0.0027	2nd GEM + HyCal upgrade
HyCal response	0.0029	negligible	HyCal upgrade
Beam background	0.0039	0.0016	better vacuum 2nd halo blocker vertex res. (2nd GEM)
Radiative correction	0.0069	0.0004	improved calc.
Inelastic ep	0.0009	negligible	-
G_M^p parameterization	0.0006	0.0005	HyCal upgrade
Total syst. uncertainty	0.0115	0.0032	
Total uncertainty	0.0137	0.0036	

Extracting Form Factors

- One of the methods for form factor extraction is the well known Rosenbluth separation:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left(\frac{E'}{E} \right) \frac{1}{1+\tau} \left(G_E^{p^2}(Q^2) + \frac{\tau}{\epsilon} G_M^{p^2}(Q^2) \right)$$



$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{reduced}} = G_M^{p^2}(Q^2) + \frac{\epsilon}{\tau} G_E^{p^2}(Q^2)$$

$$\tau = \frac{Q^2}{4M_p^2} \quad \epsilon = \left[1 + 2(1+\tau) \tan^2 \frac{\theta}{2} \right]^{-1}$$

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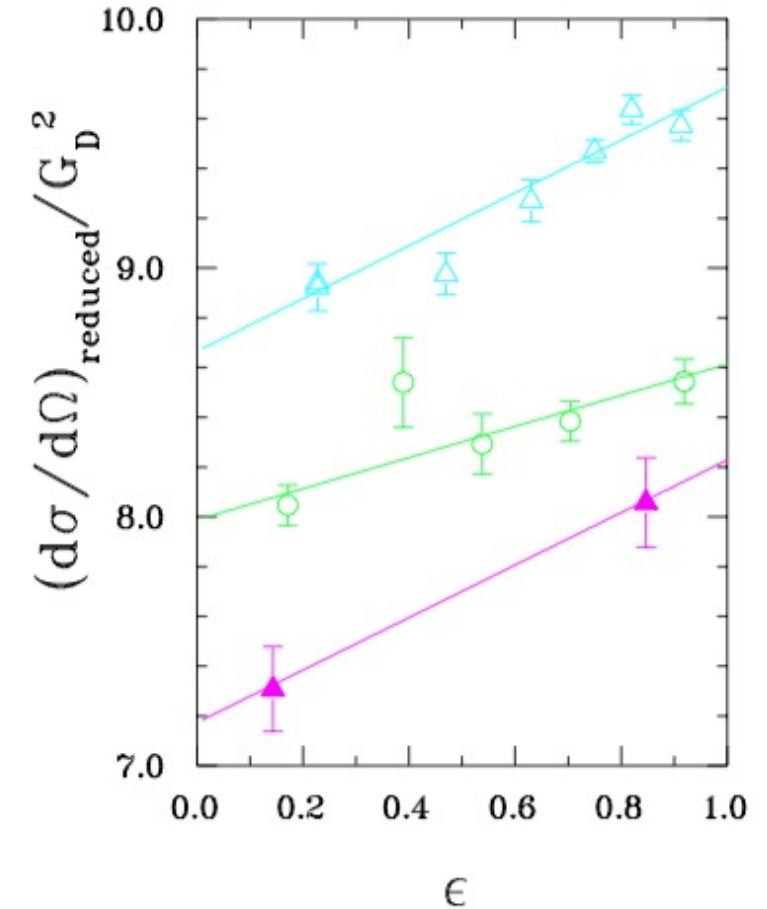


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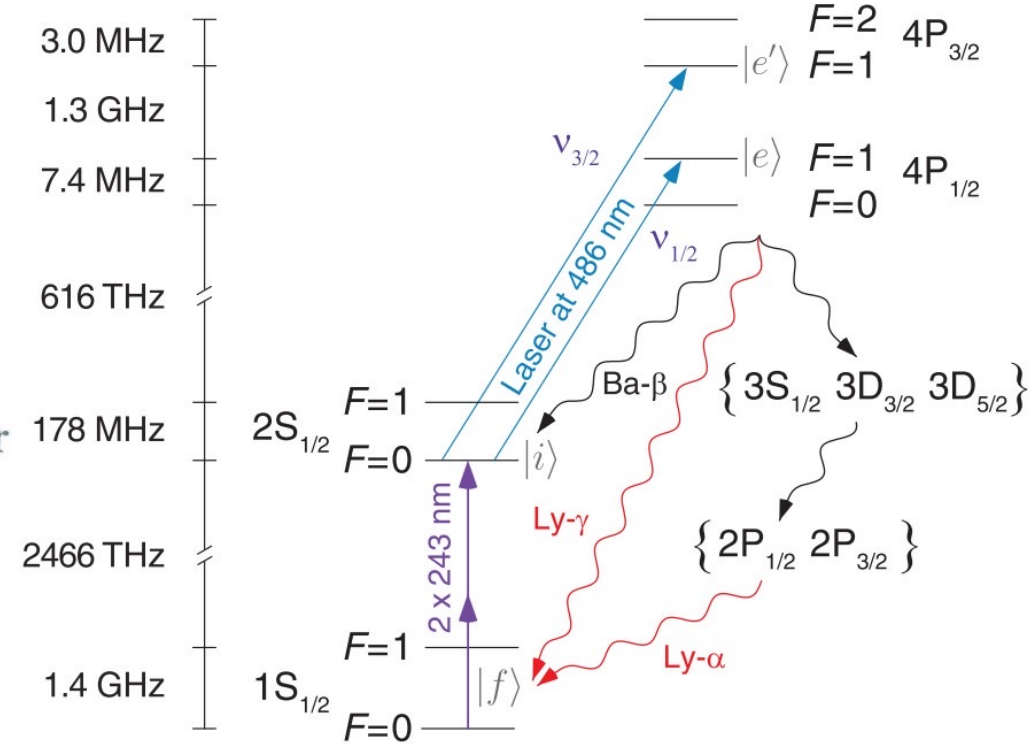
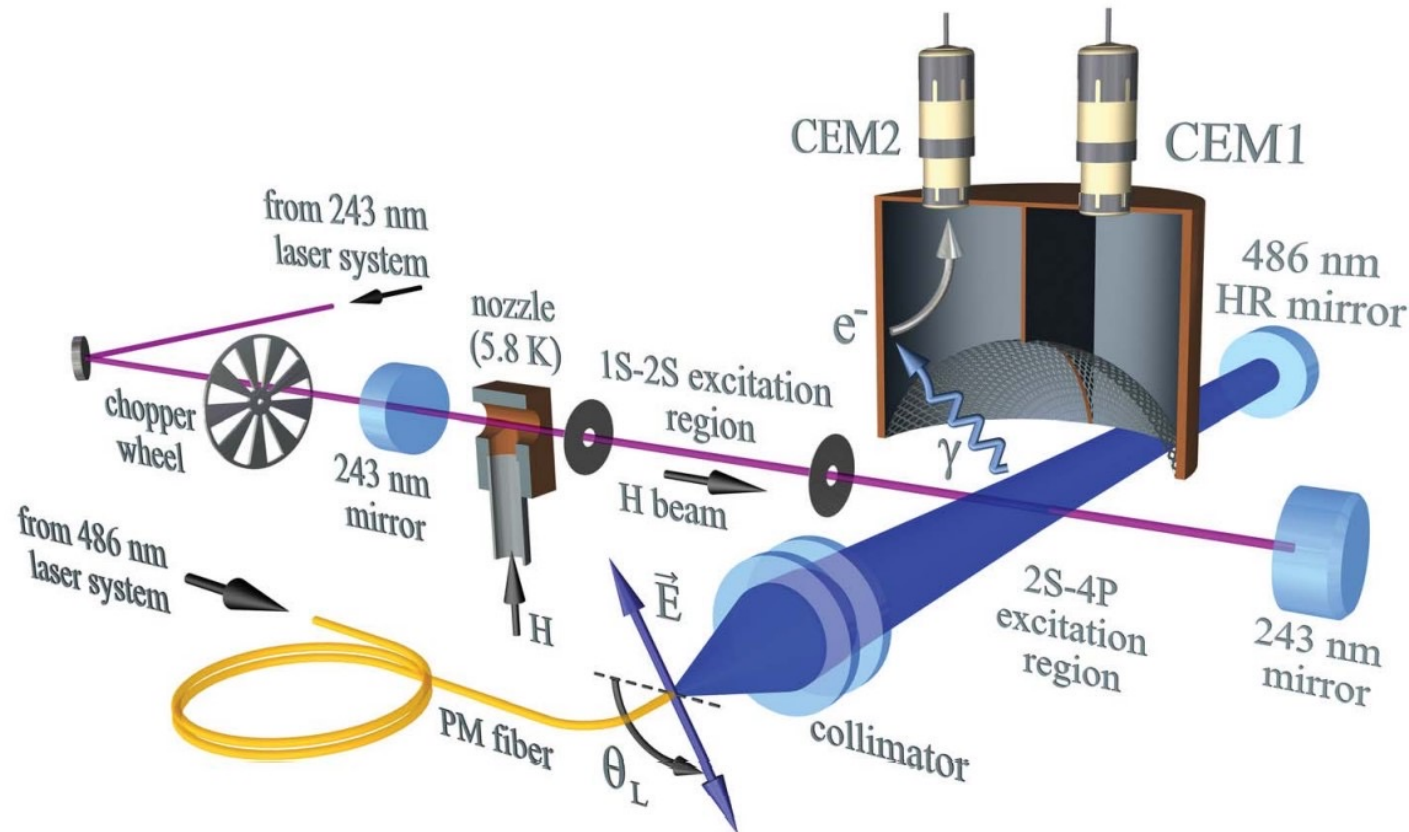
- Measure σ_{reduced} at same Q^2 but different values of ϵ
- G_E^p and G_M^p determined as slope and intersection from fits

$$G_D = \frac{1}{\left(1 + \frac{Q^2}{0.71 \text{ GeV}^2}\right)^2}$$

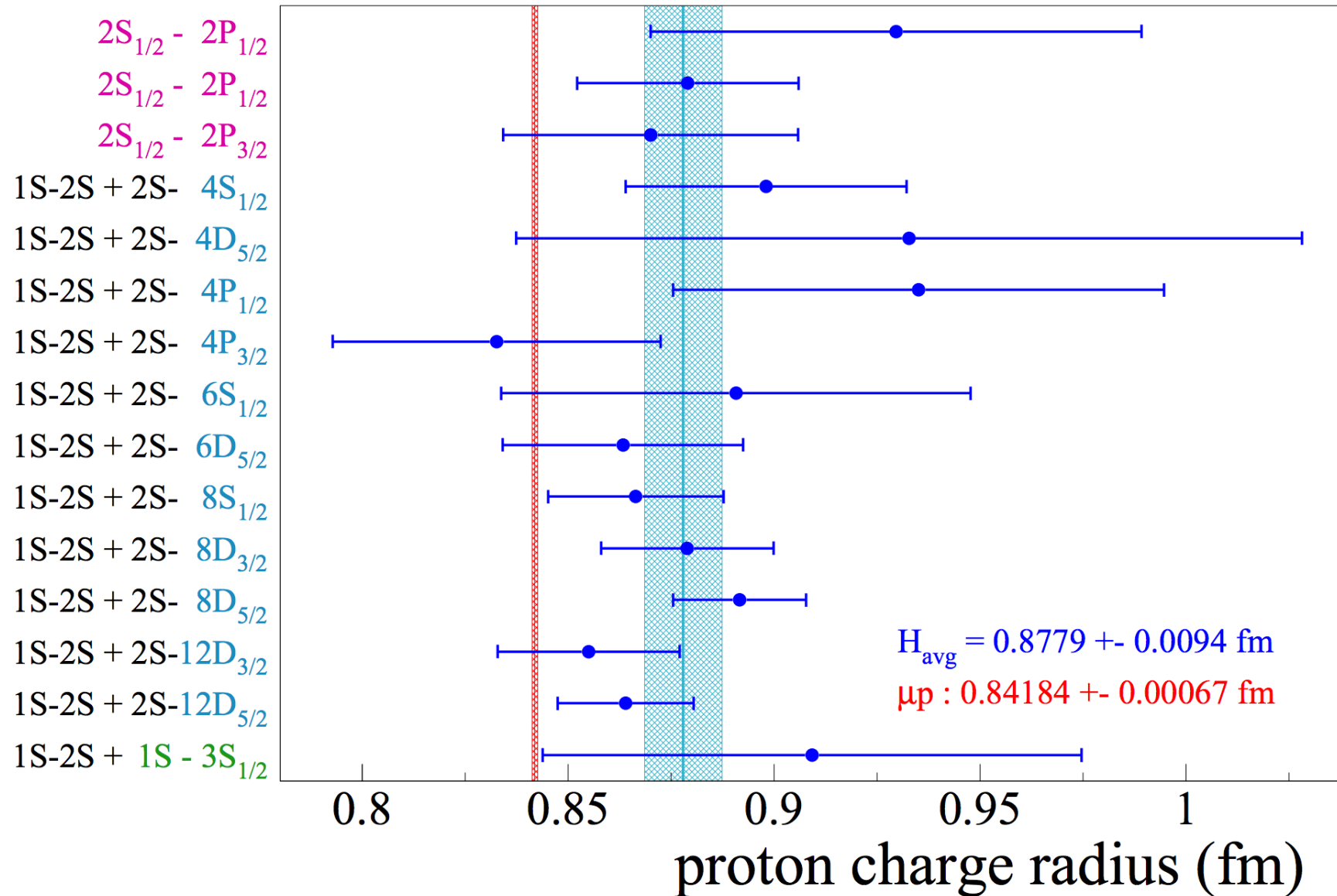


Ordinary Hydrogen Measurement

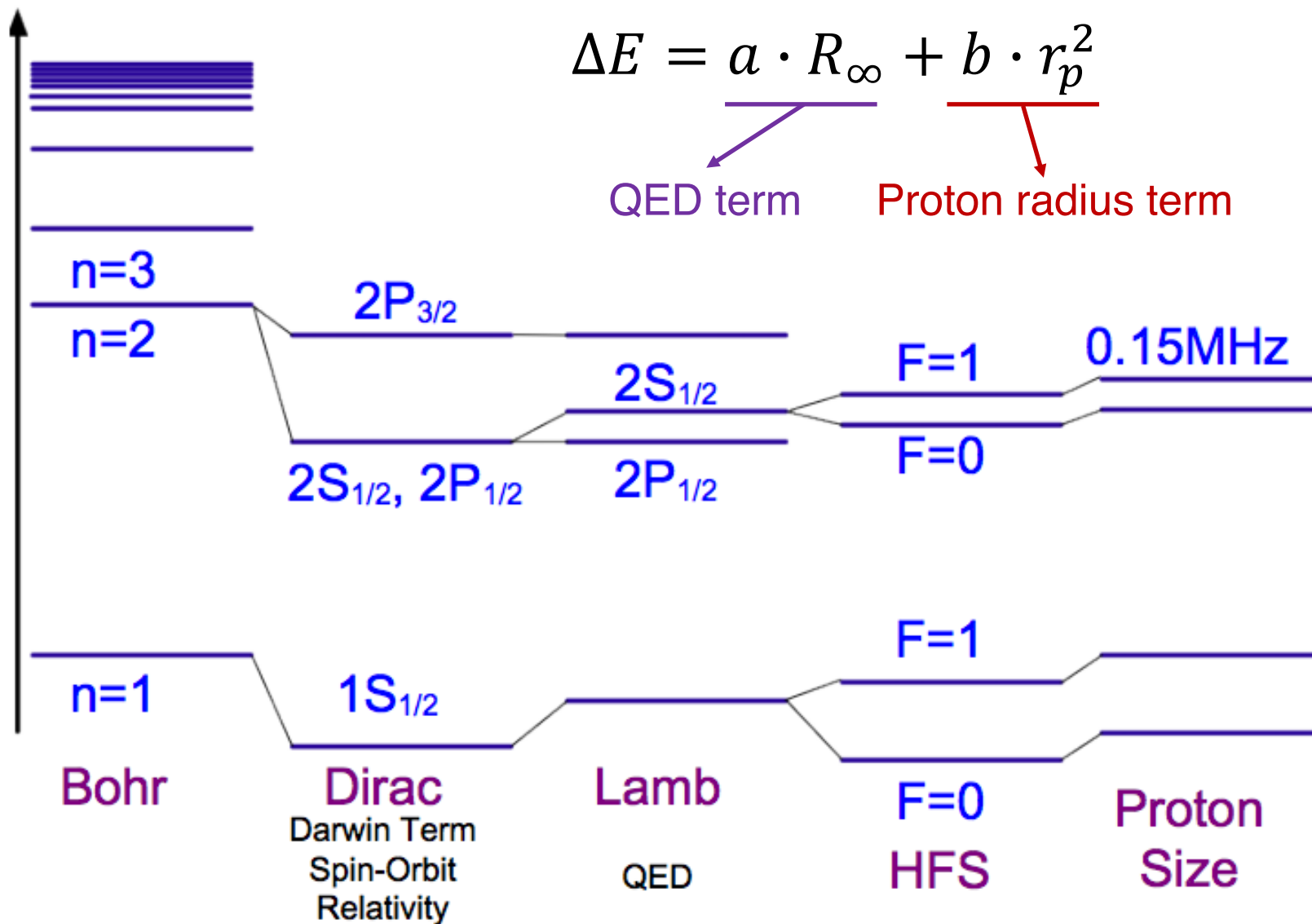
- Ordinary hydrogen 2S-4P transition measurement (A. Beyer *et al. Science* 358, 79-86 (2017))



Ordinary Hydrogen v.s. Muonic Hydrogen



Hydrogen Spectroscopy



1. Small splitting measurements:
 - States with **the same** n
 - Precise knowledge of R_{∞} **not required**
 - Independent measurement on r_p
2. Large splitting measurements:
 - States with **different** n
 - Precision on R_{∞} not good enough
 - At least need **two** different transition
 - Solve for r_p and R_{∞} at the same time

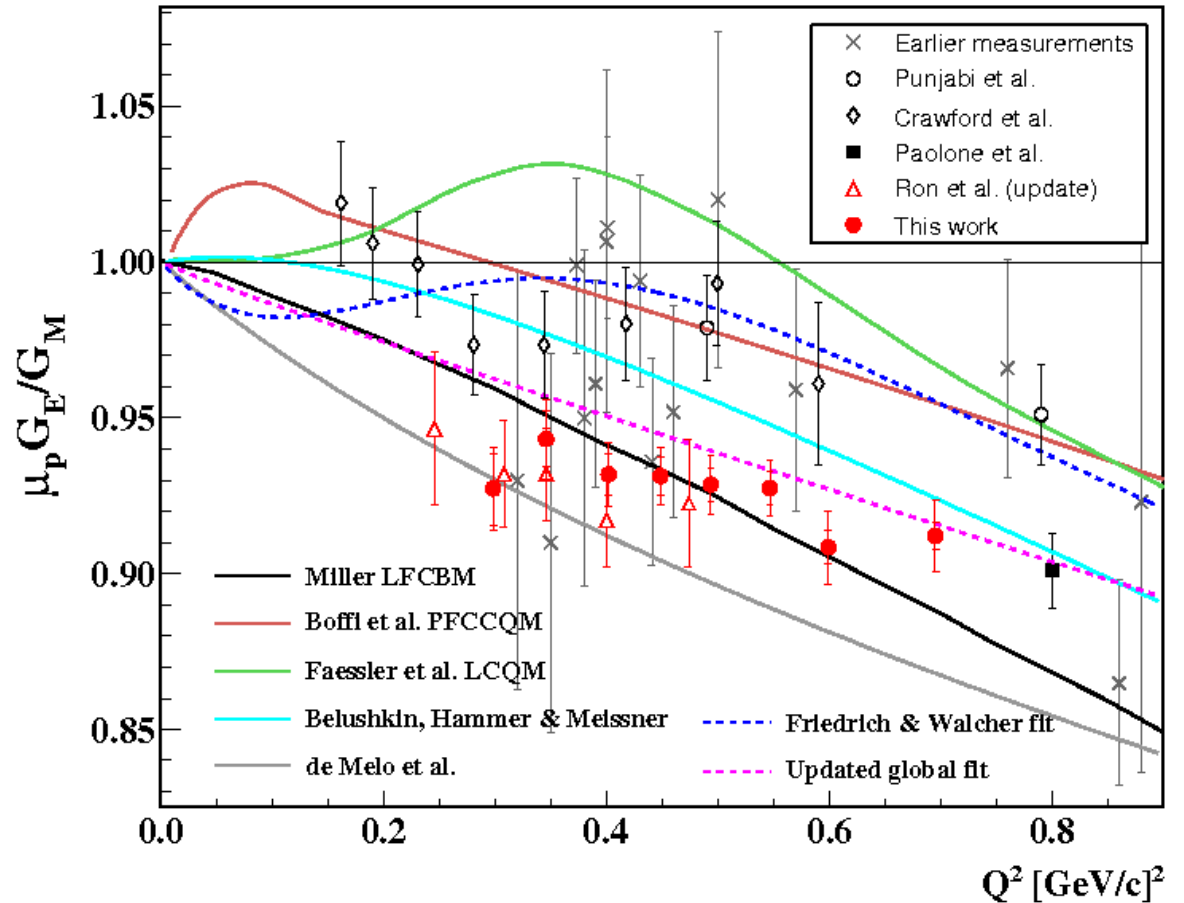
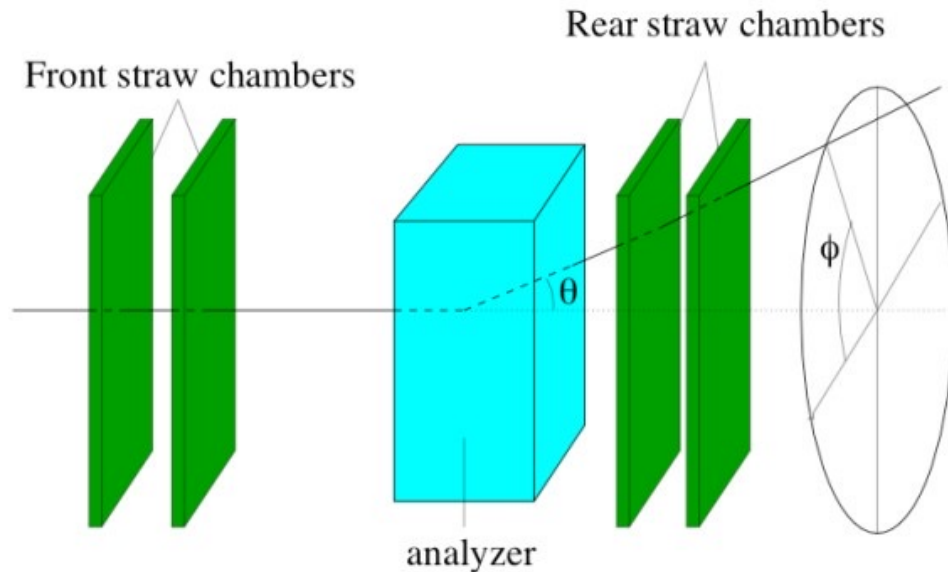
Polarized ep Elastic Scattering

(longitudinally polarized electron beam and recoil proton polarization measurement)

- Extract form factor ratio by measuring polarization of recoil proton:

$$\frac{G_E}{G_M} = -\frac{P_y}{P_z} \frac{E + E'}{2m} \tan \frac{\theta_e}{2}.$$

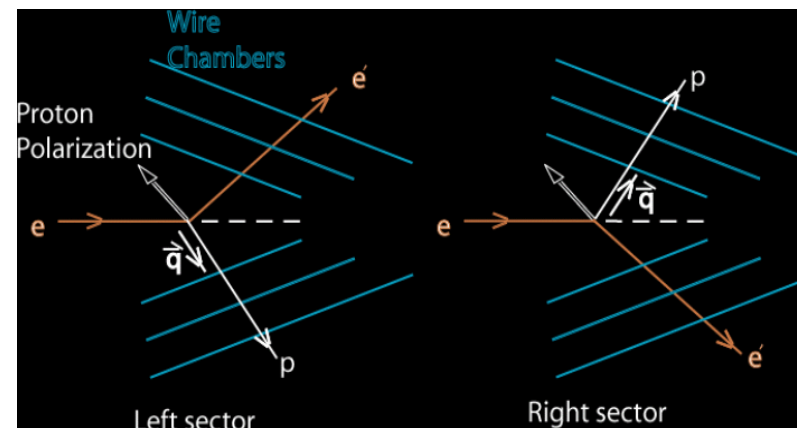
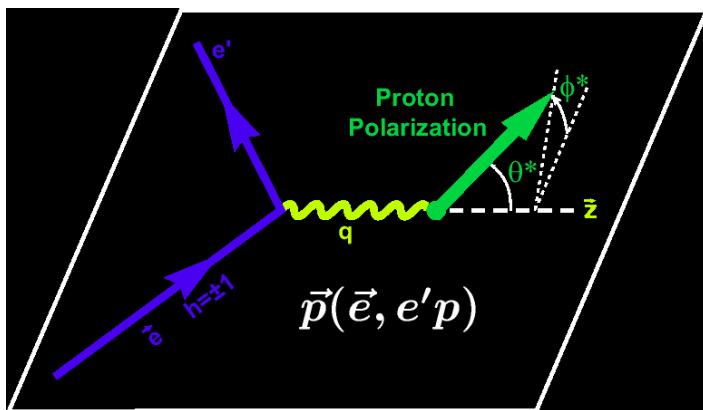
- Couple with cross section measurement to separate form factors
- Reduce many typical systematics for RS



X. Zhan et al. Phys. Lett. B 705 (2011) 59-64

Polarized ep Elastic Scattering

(Polarized electron – polarized proton measurement)

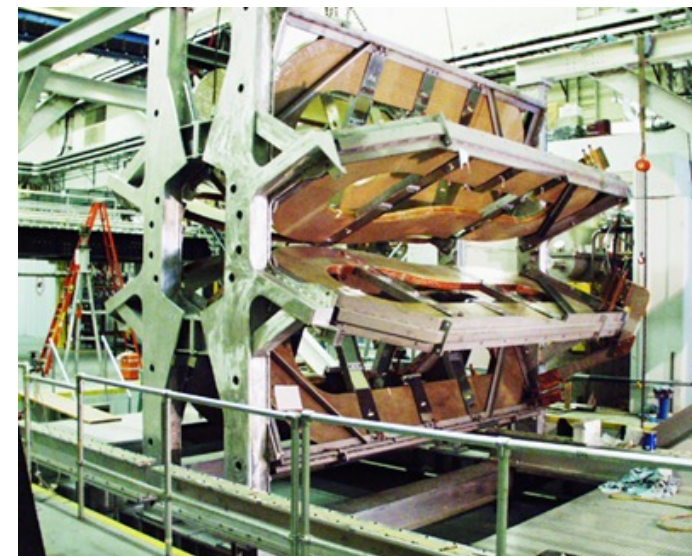


- Elastic scattering asymmetry (longitudinally polarized beam, polarized target):

$$A_{phys} = \frac{v_z \cos \theta^* G_M^p{}^2 + v_x \sin \theta^* \cos \phi^* G_M^p G_E^p}{(\tau G_M^p{}^2 + \epsilon G_E^p{}^2) / [\epsilon(1 + \tau)]},$$

$$A_{exp} = P_b P_t A_{phys}$$

- Form factor ratio can be obtained from two experimental asymmetries (A_l and A_r), at same Q^2 but with different target spin orientations

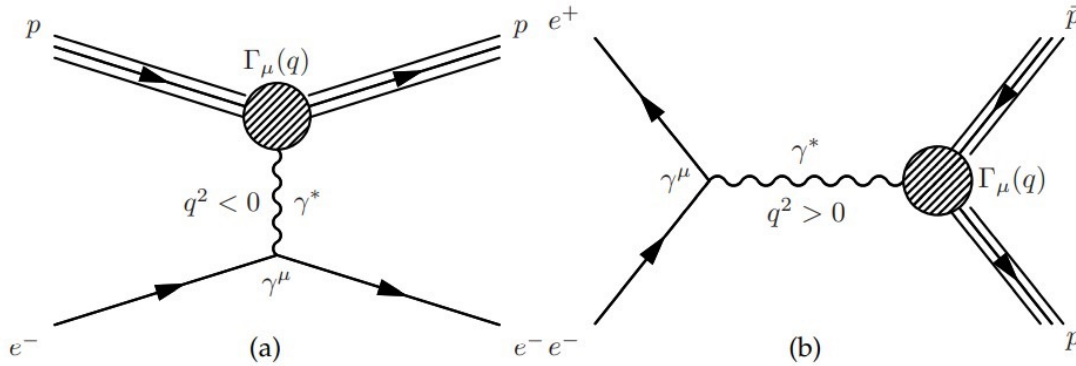


BLAST pioneered the technique, later also used in Jlab Hall A experiment

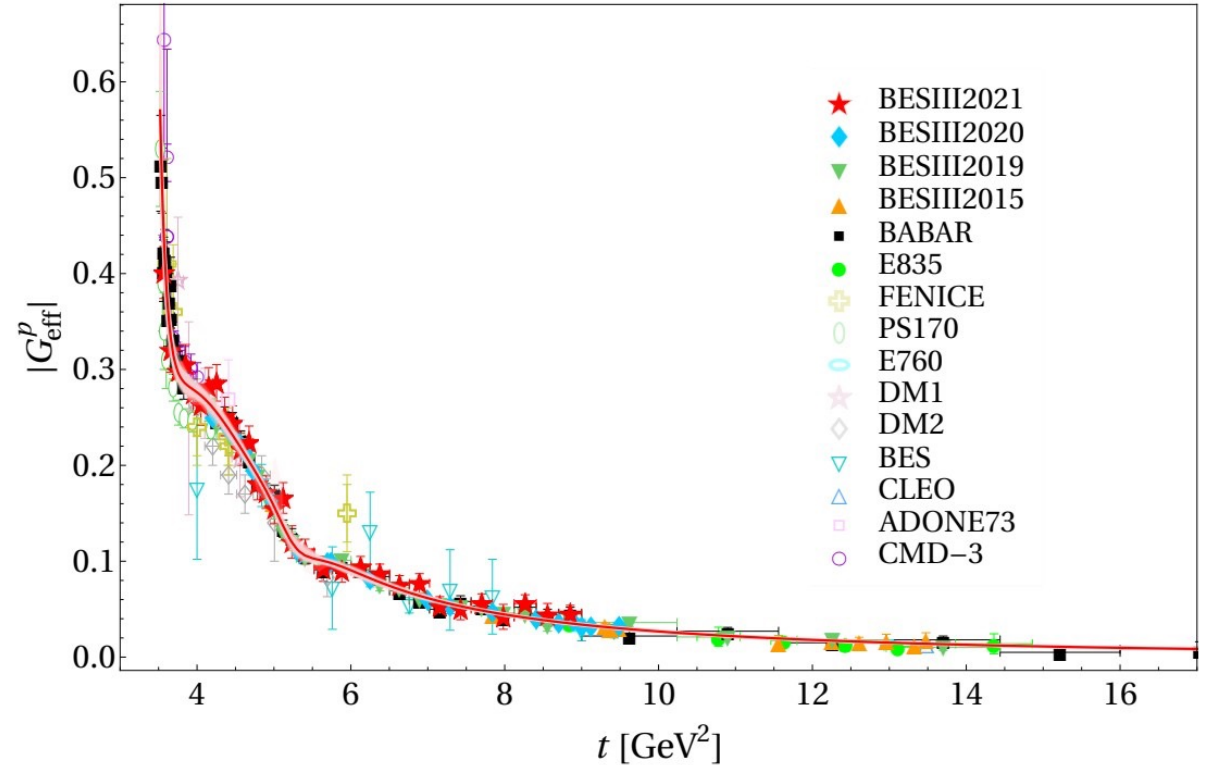
Timelike Form Factor

arXiv.2211.05419

- EM form factor can also be measured in e^+e^- annihilation:



$$\begin{aligned}\sigma_{e^+e^- \rightarrow p\bar{p}}(t) &= \frac{4\pi\alpha^2\beta}{3t} C(t) \left[|G_M(t)|^2 + \frac{2m_p^2}{t} |G_E(t)|^2 \right] \\ &\equiv \frac{4\pi\alpha^2\beta}{3t} C(q^2) \left(1 + \frac{2m_p^2}{t} \right) |G_{\text{eff}}^p(t)|^2.\end{aligned}$$



$$|G_{\text{eff}}| \equiv \sqrt{\frac{|G_E|^2 + \xi |G_M|^2}{1 + \xi}}$$

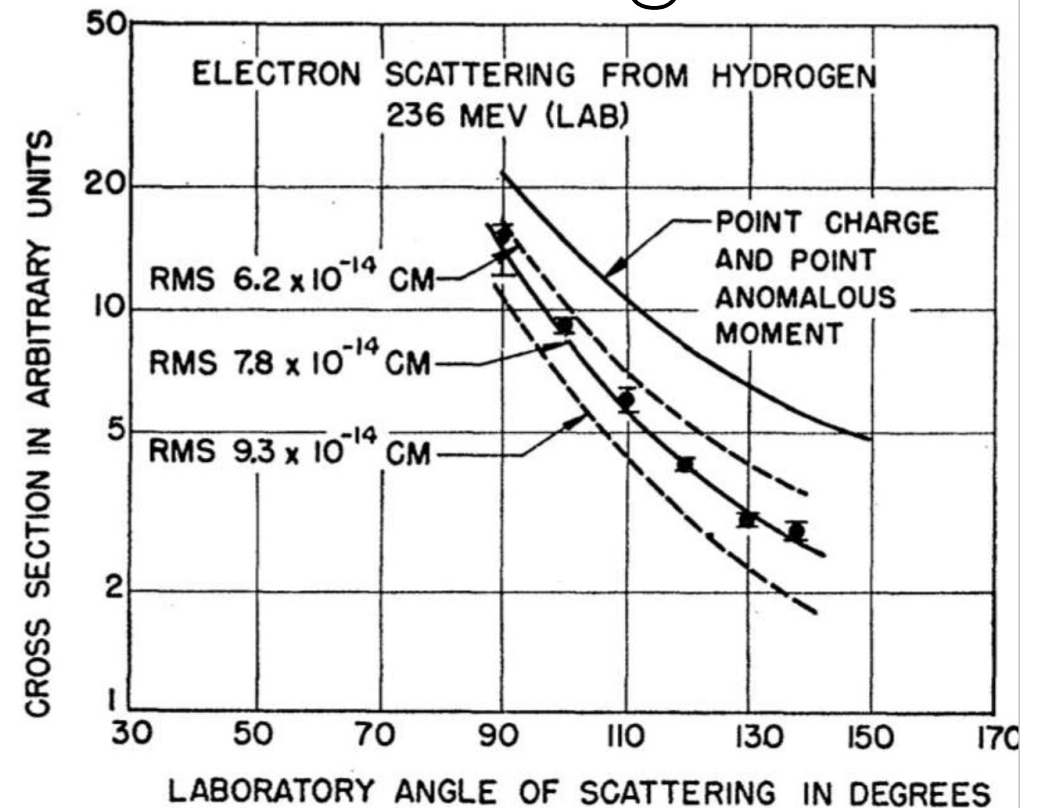
Unpolarized *ep* Elastic Scattering

(First measurement of proton charge radius)



- started with Robert Hofstadter
 - Nobel prize in physics (1961): ... for his pioneering studies of **electron scattering** in atomic nuclei and for his consequent discoveries concerning the **structure of nucleons** ...”

Measurements @ Stanford

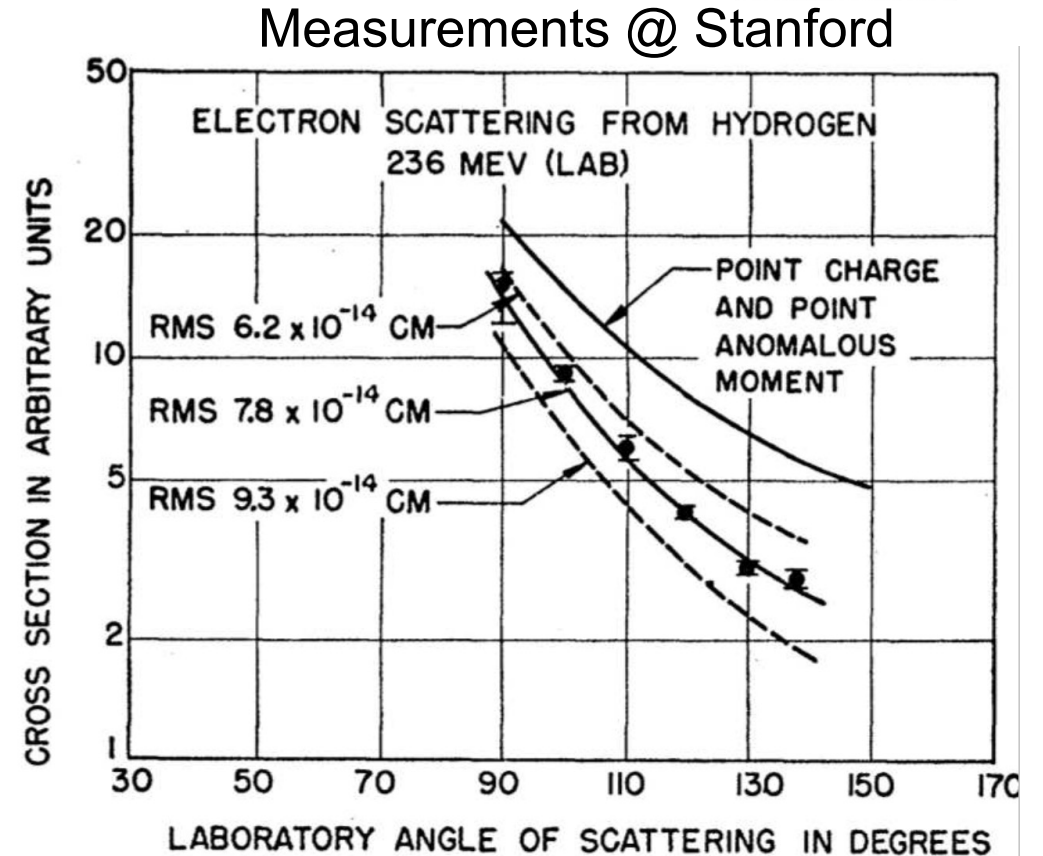


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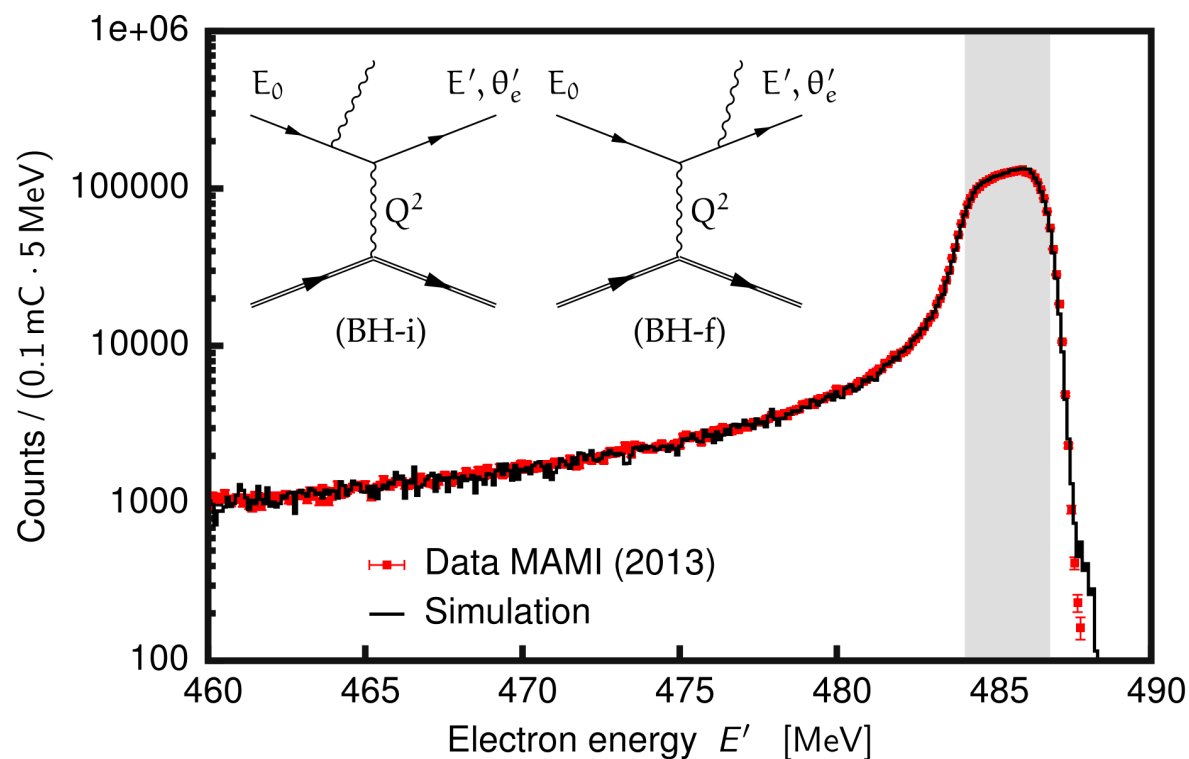


- started with Robert Hofstadter
 - Nobel prize in physics (1961): ... for his pioneering studies of **electron scattering** in atomic nuclei and for his consequent discoveries concerning the **structure of nucleons ...**
- The Proton rms charge radius in 1956 was measured to be:
 - **7.8×10^{-14} cm (0.78 fm)**
Hofstadter, McAllister, Phys. Rev. 102, 851 (1956).
- Over 60 years of experimentation

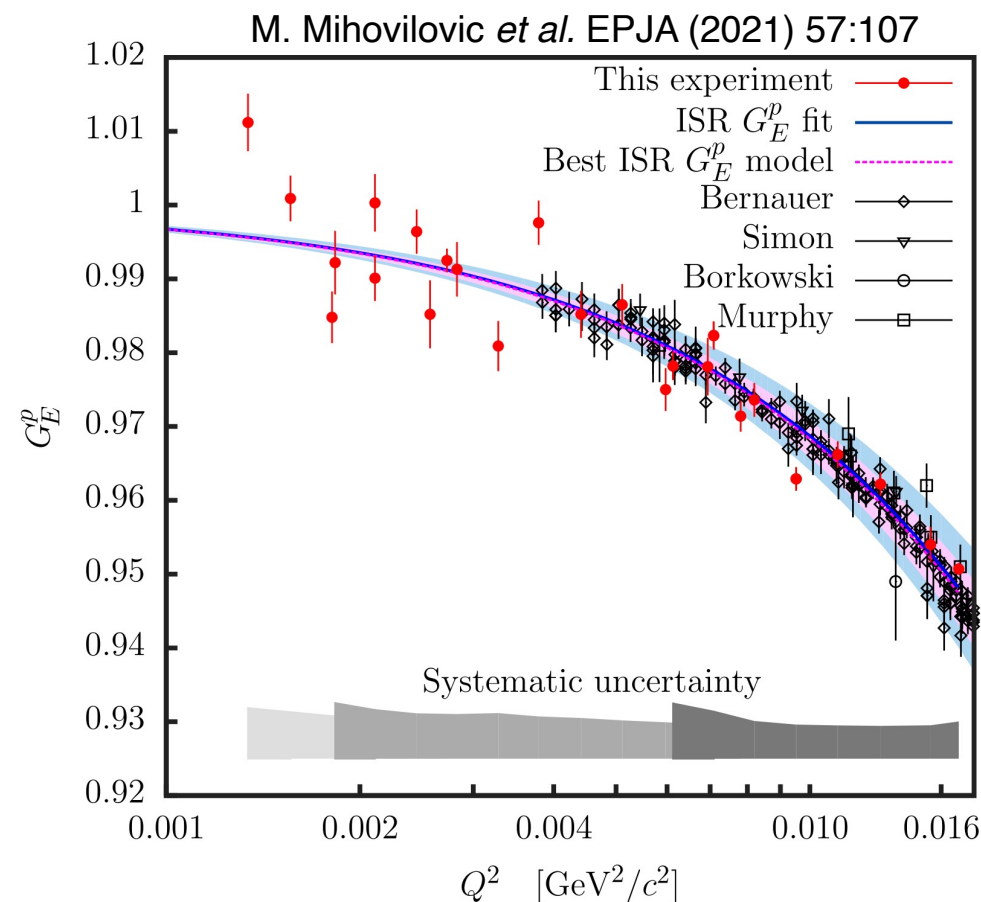


Mainz Initial State Radiation (ISR) Experiment

- Using ISR technique to reach lower Q^2 : 0.001 to 0.016 GeV^2
- Final result: $r_p = 0.878 \pm 0.011_{\text{stat.}} \pm 0.031_{\text{syst.}} \pm 0.002_{\text{mod.}} \text{ fm}$

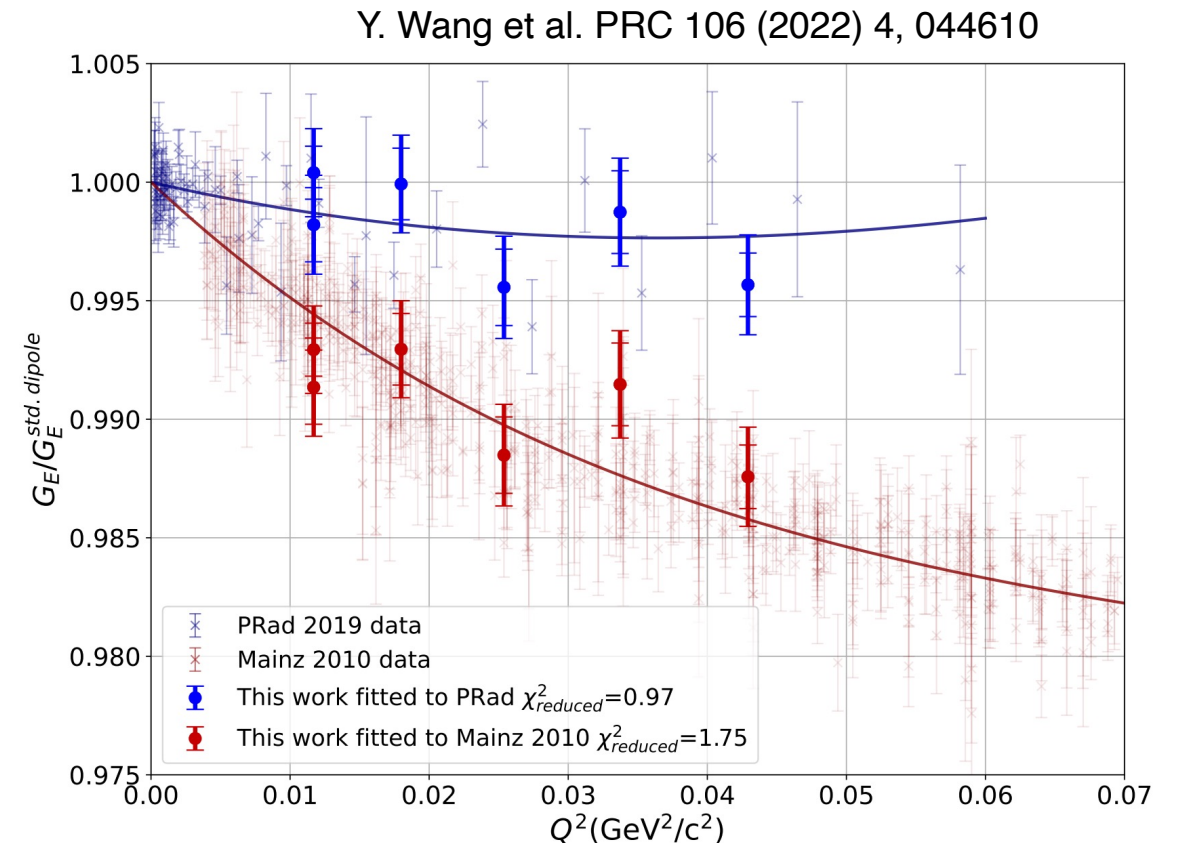
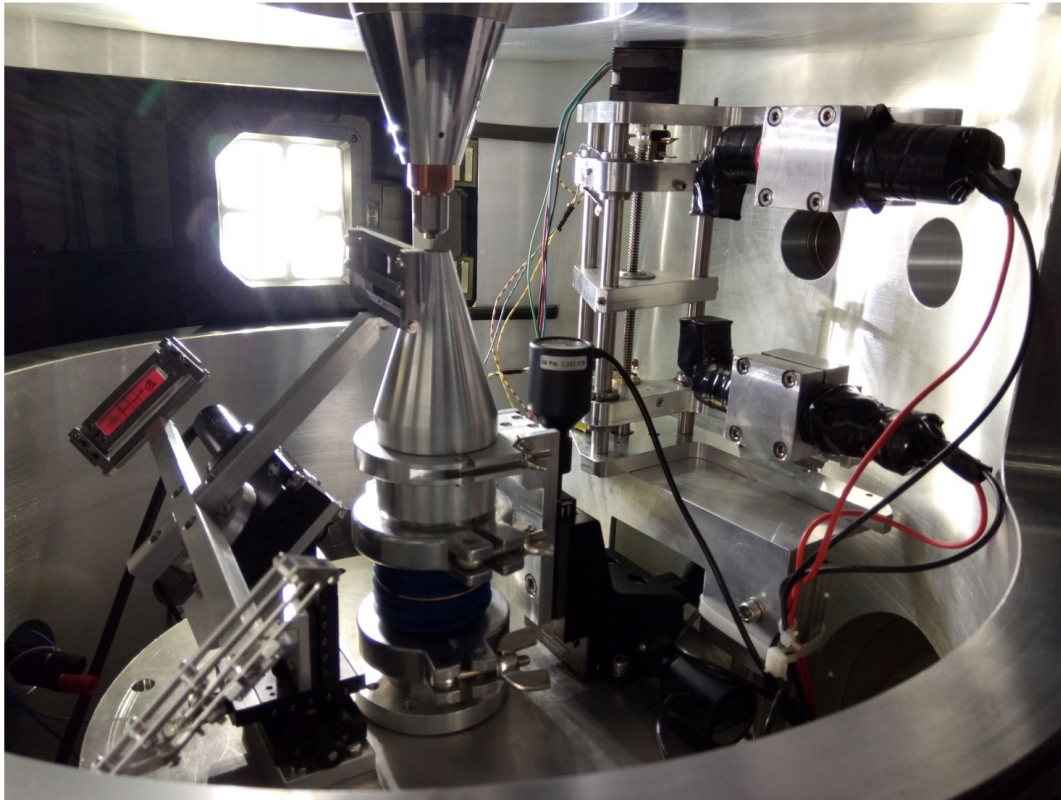


M. Mihovilovic *et al.* arXiv.1905.11182



Mainz Jet Target Experiment

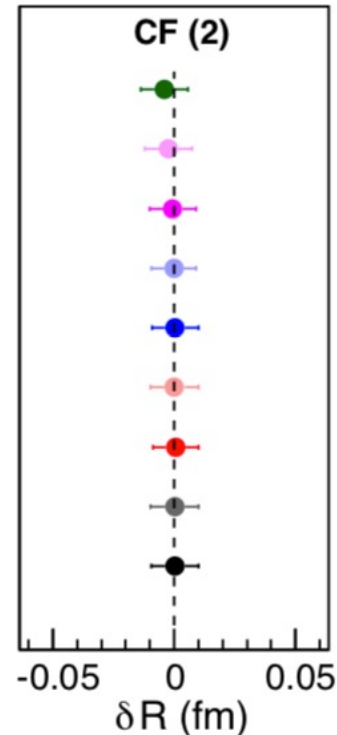
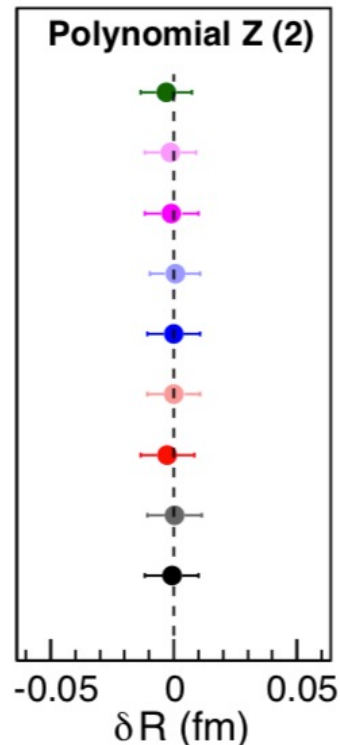
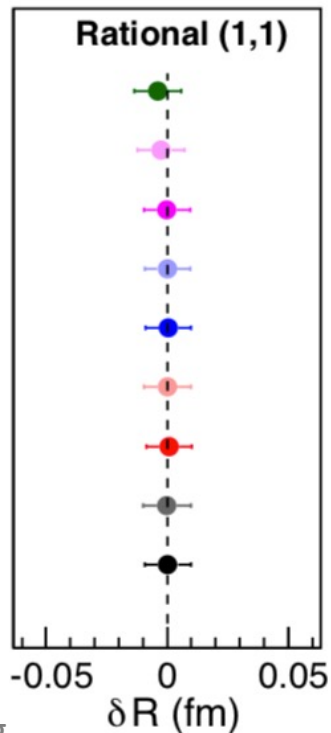
- Using novel gas-jet target, but limited by statistics
- Fit to PRad: $\chi^2_{reduced} = 0.97$, fit to Mainz $\chi^2_{reduced} = 1.75$



Searching the Robust Fitters

- Various fitters tested with a wide range of G_E parameterizations, using PRad kinematic range and uncertainties (X. Yan *et al.* PRC 98, 025204 (2018))
- Rational (1,1), 2nd order z transformation and 2nd order continuous fraction are identified as robust fitters with also reasonable uncertainties
- Typically a floating parameter n is included to take care normalization uncertainties

$$f(Q^2) = n G_E^p(Q^2)$$



Ye-2018
Bernauer-2014
Alarcón-2017
Arrington-2007
Arrington-2004
Kelly-2004
Gaussian
Monopole
Dipole

Rational (1,1)

$$\frac{1 + p_1 Q^2}{1 + p_2 Q^2}$$

2nd order z transformation

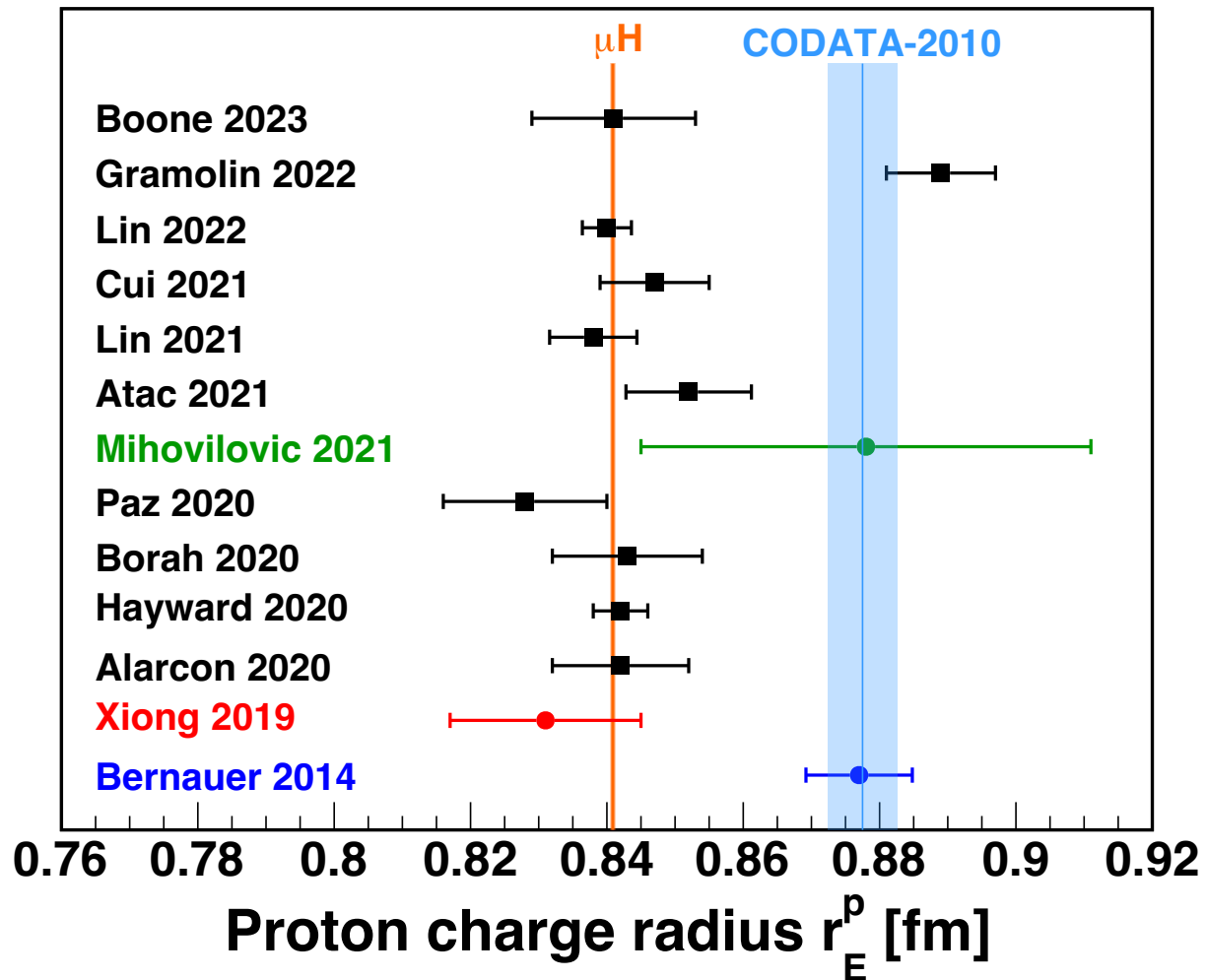
$$1 + p_1 z + p_2 z^2, \\ z = \frac{\sqrt{T_c + Q^2} - \sqrt{T_c - T_0}}{\sqrt{T_c + Q^2} + \sqrt{T_c - T_0}}$$

2nd order continuous fraction

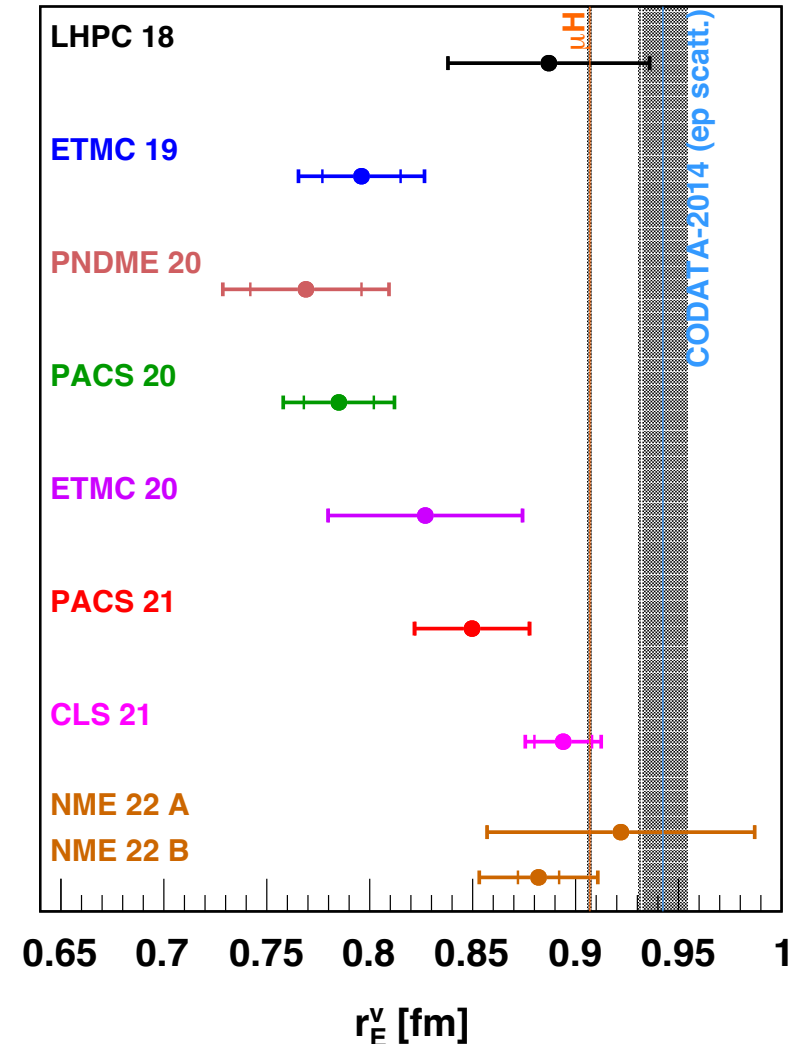
$$\frac{1}{1 + \frac{p_1 Q^2}{1 + p_2 Q^2}}$$

Other Results from Global Analysis and Lattice QCD

Global Analysis

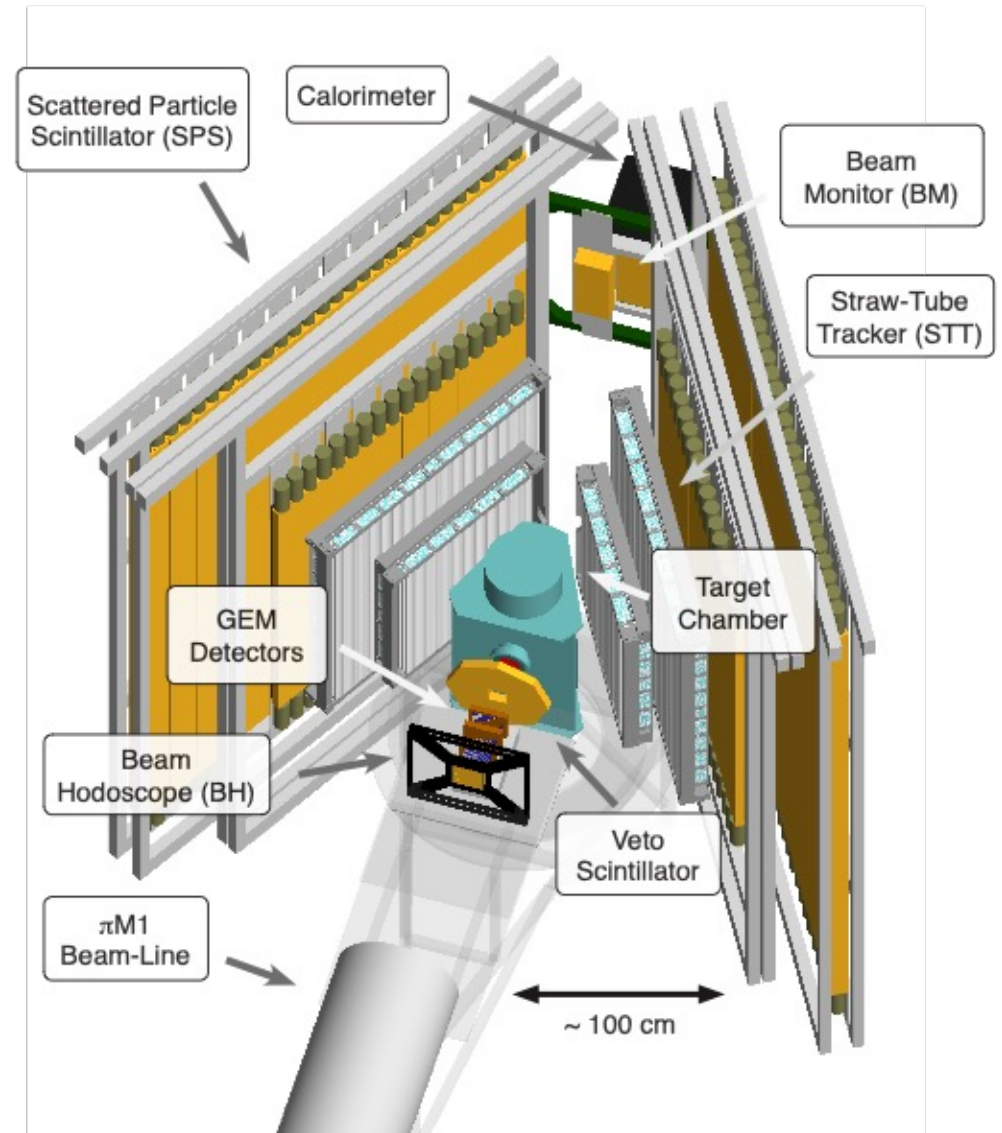


Lattice QCD



MUSE Experiment

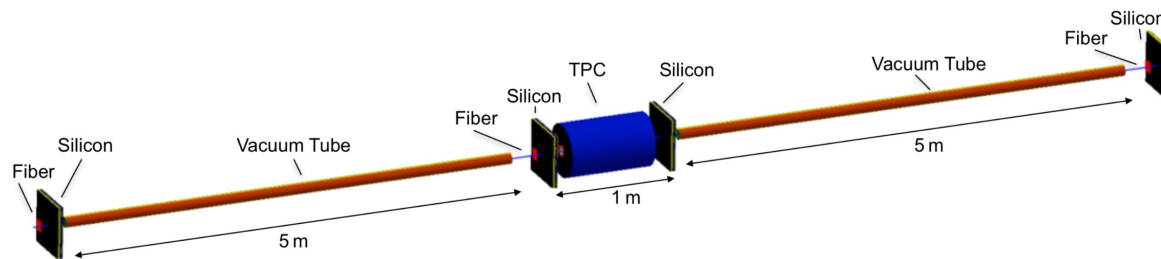
- **First** r_p measurement using muon
 - 4 types of incident leptons: $e^\pm$ and $\mu^\pm$
- **Direct test** for lepton-universality violation
- Different beam polarity can constrain two-photon exchange
- Currently taking data at PSI



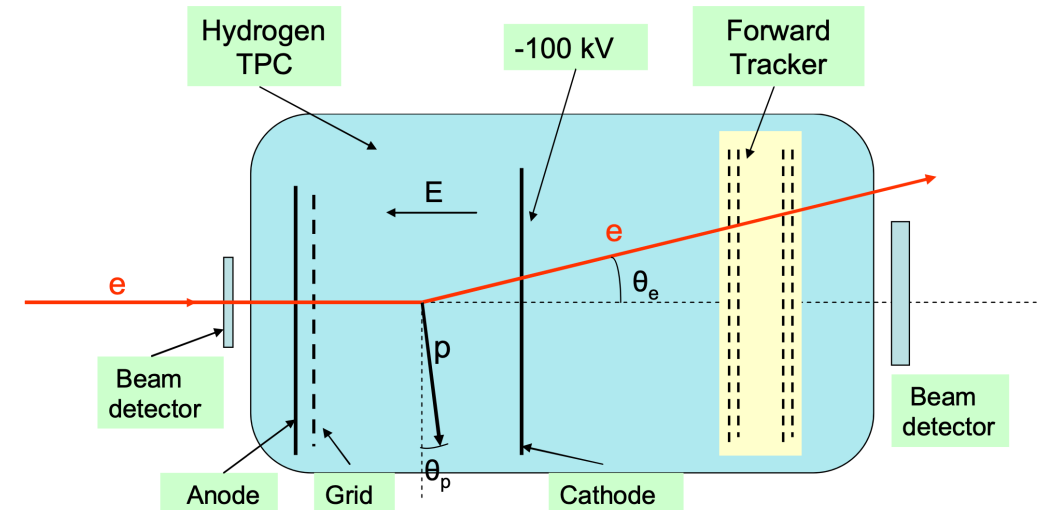
AMBER and PRES Experiments

- **AMBER@CERN** uses high energy (~ 100 GeV) **muon** beam
- **PRES@Mainz** uses 720 MeV **electron** beam
- Both use time-projection chamber as active target, detecting both scattered electron and recoil proton
- Q^2 can be reconstructed by recoil proton, **largely suppress radiative effect**

AMBER@CERN

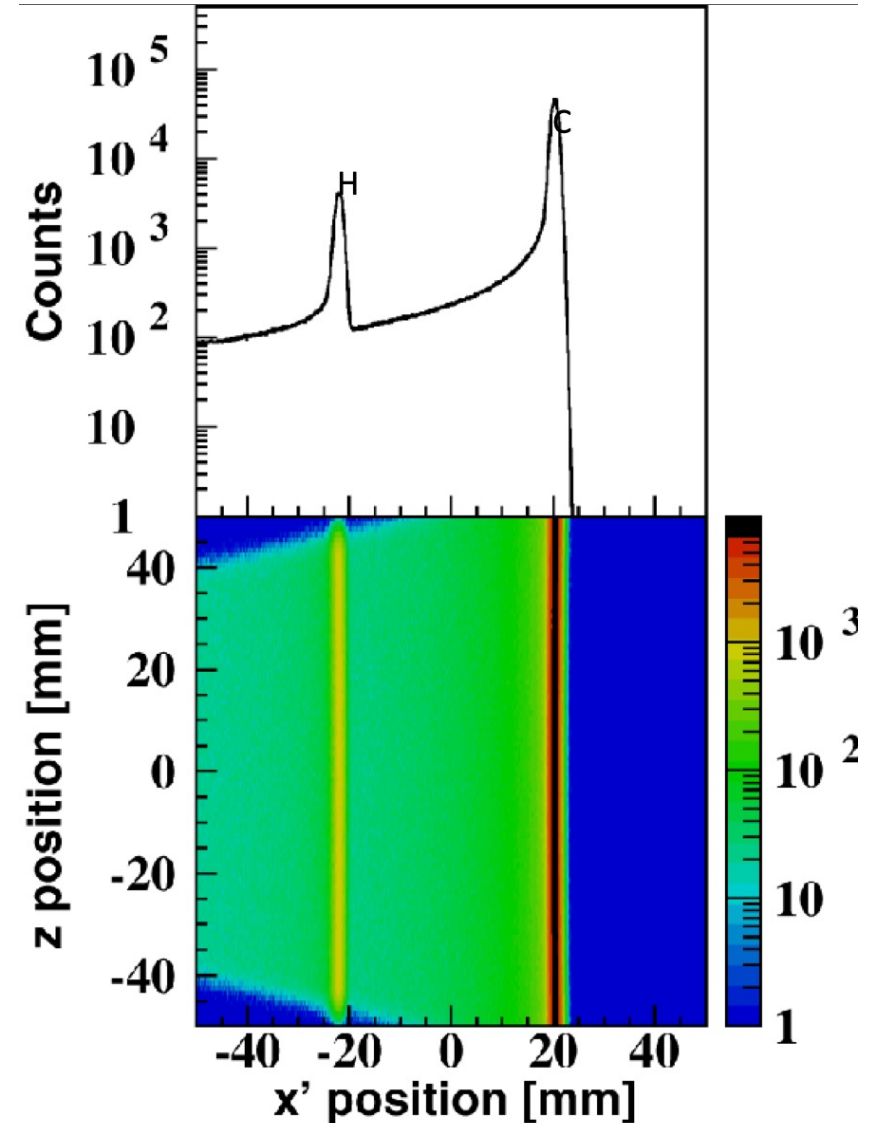
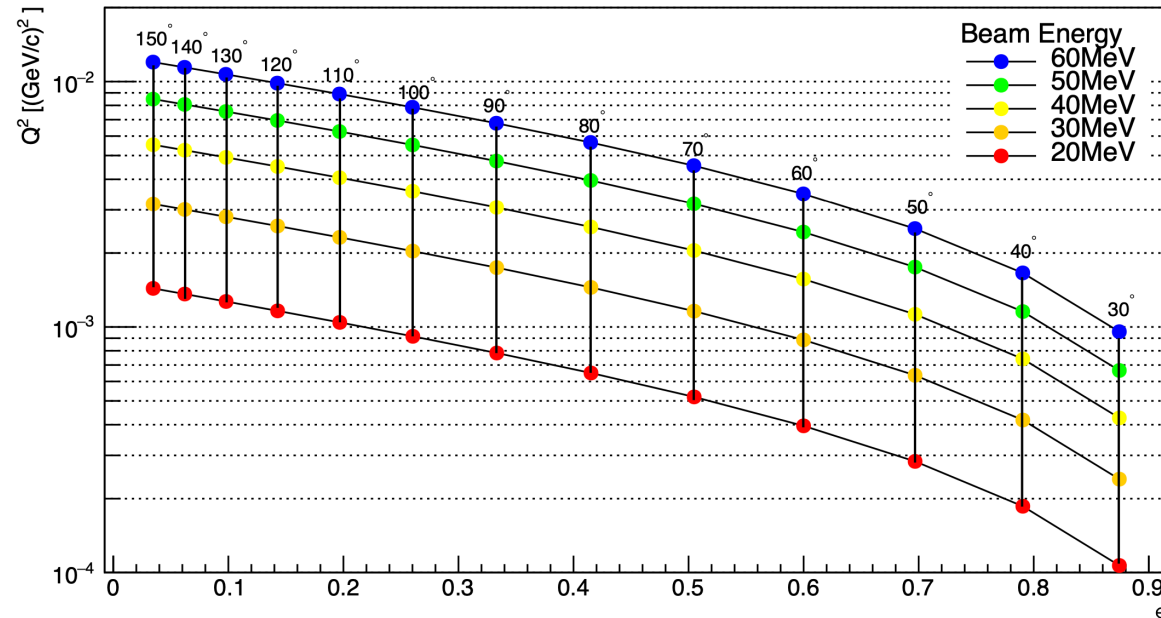


PRES@Mainz



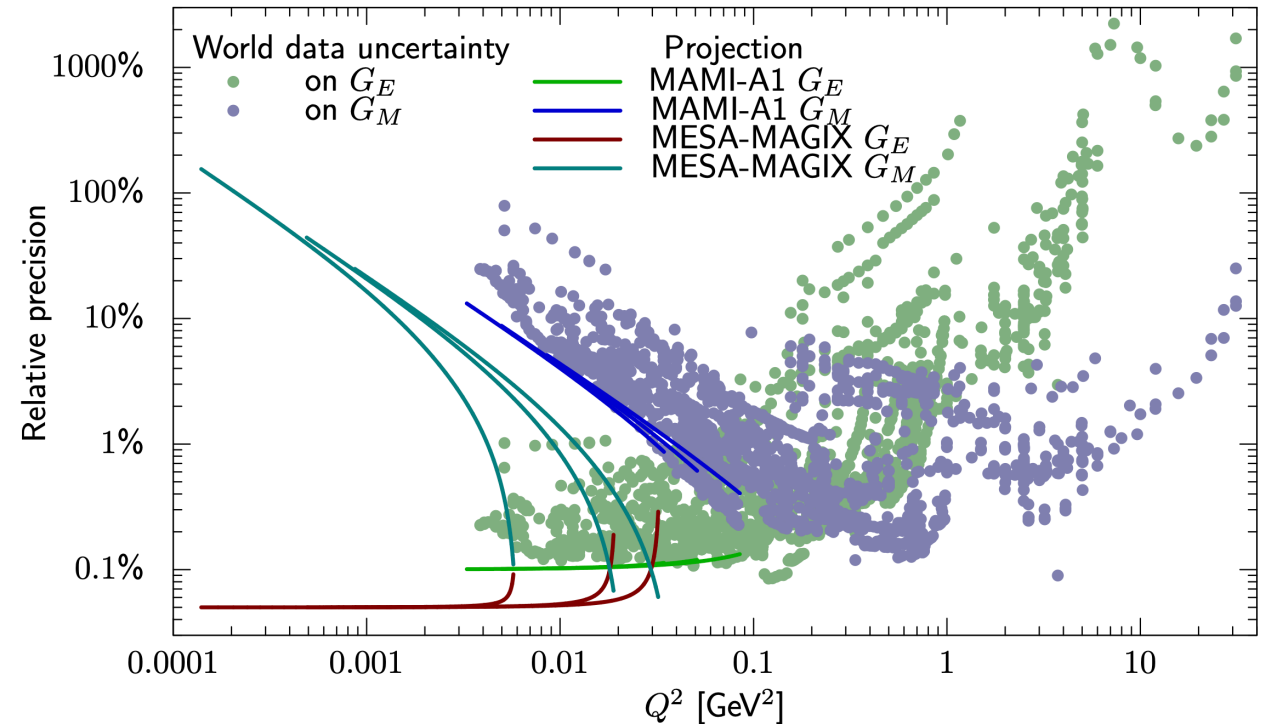
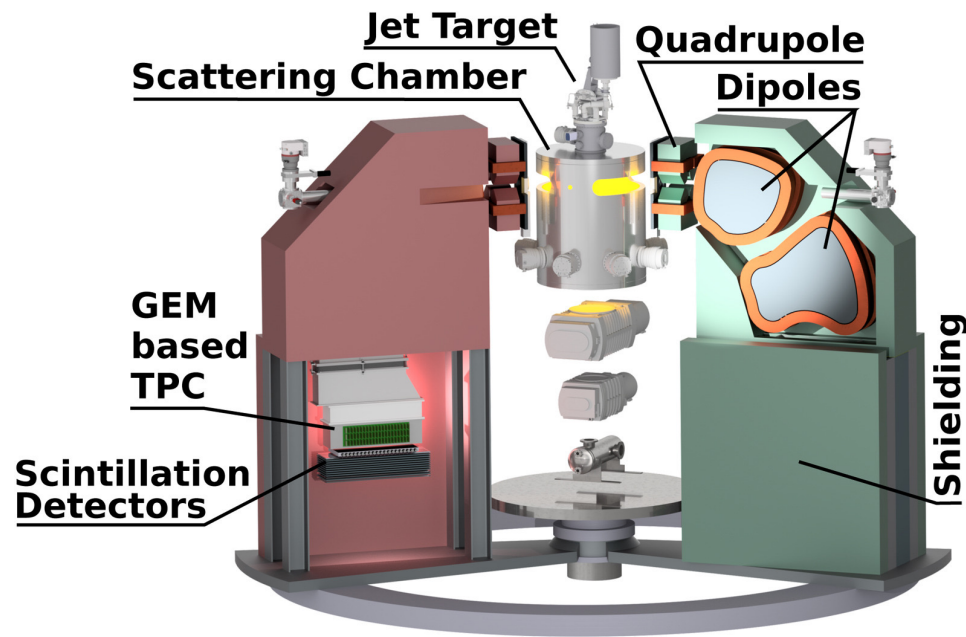
ULQ2 Experiment

- ULQ2 experiment at Tohoku University, Japan
- 20-60 MeV electron beam
- Normalize to the well-established $e^{-12}\text{C}$ cross section
- Rosenbluth separation to measure both G_E and G_M
- Projected uncertainty for $G_E \sim 0.1\%$
- Q^2 : $3 \times 10^{-4} \sim 8 \times 10^{-3} \text{ GeV}^2$



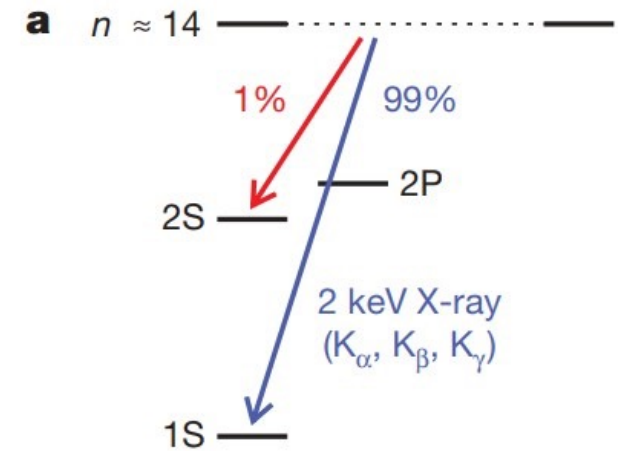
MAGIX Experiment

- Will use the new MESA accelerator at Mainz (under construction), 20-105 MeV electron beam up to 1 mA
- Will use the fully tested jet target and two new multi-purpose spectrometers
- Strong sensitivity on both G_E and G_M , can achieve an order of magnitude better precision for low Q^2 G_M



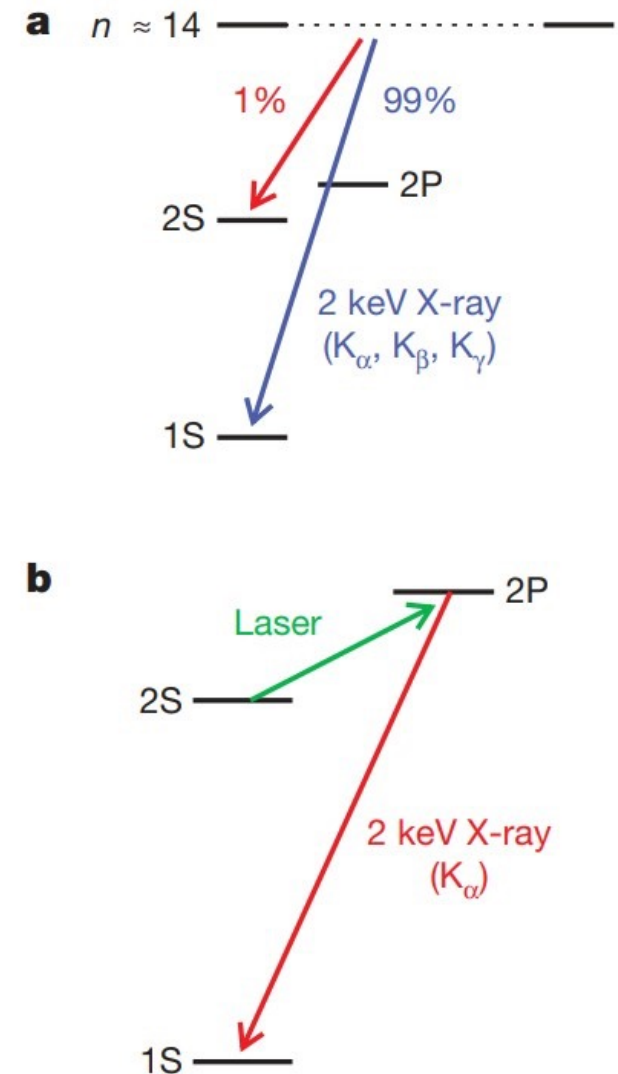
Muonic Hydrogen Measurement

- Steps in muonic hydrogen 2S-2P Lamb shift measurement (R. Pohl, *et al. Nature* 466 213-216 (2010))
 1. Slow muon ($\sim\text{keV}$) captured and replace a electron
 2. 99% muons decay to ground state right away, 1% decay to meta-stable 2S state ($\tau\sim 1\mu\text{s}$)



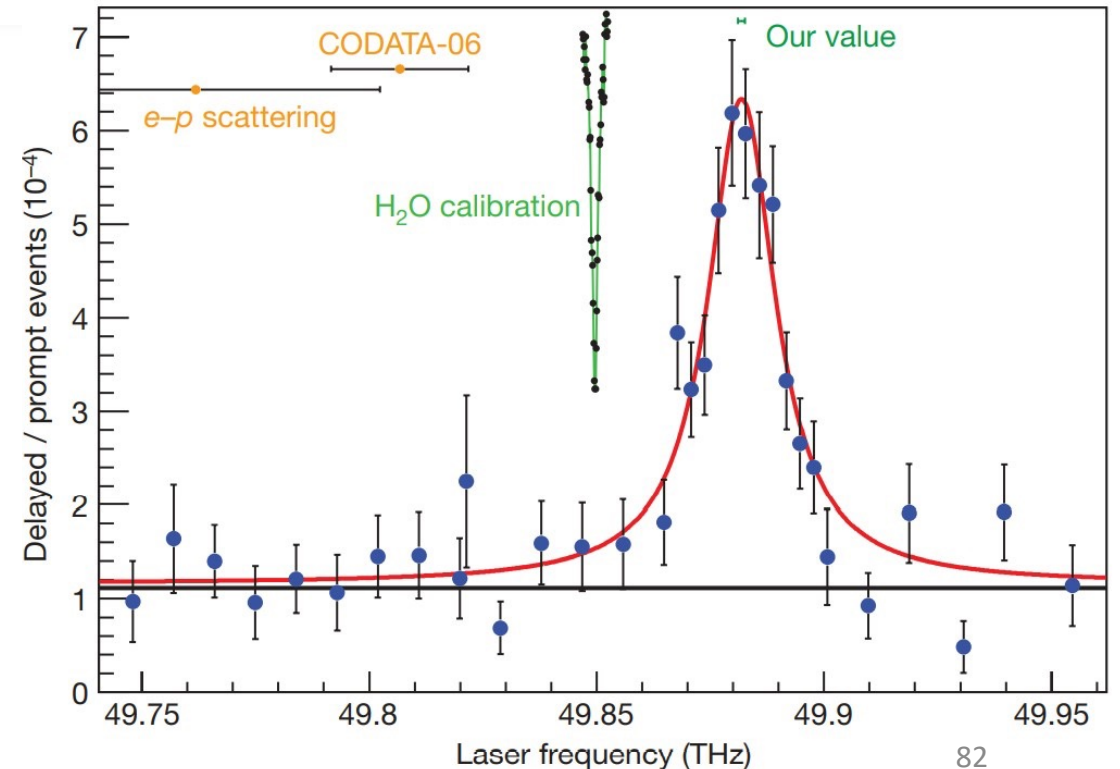
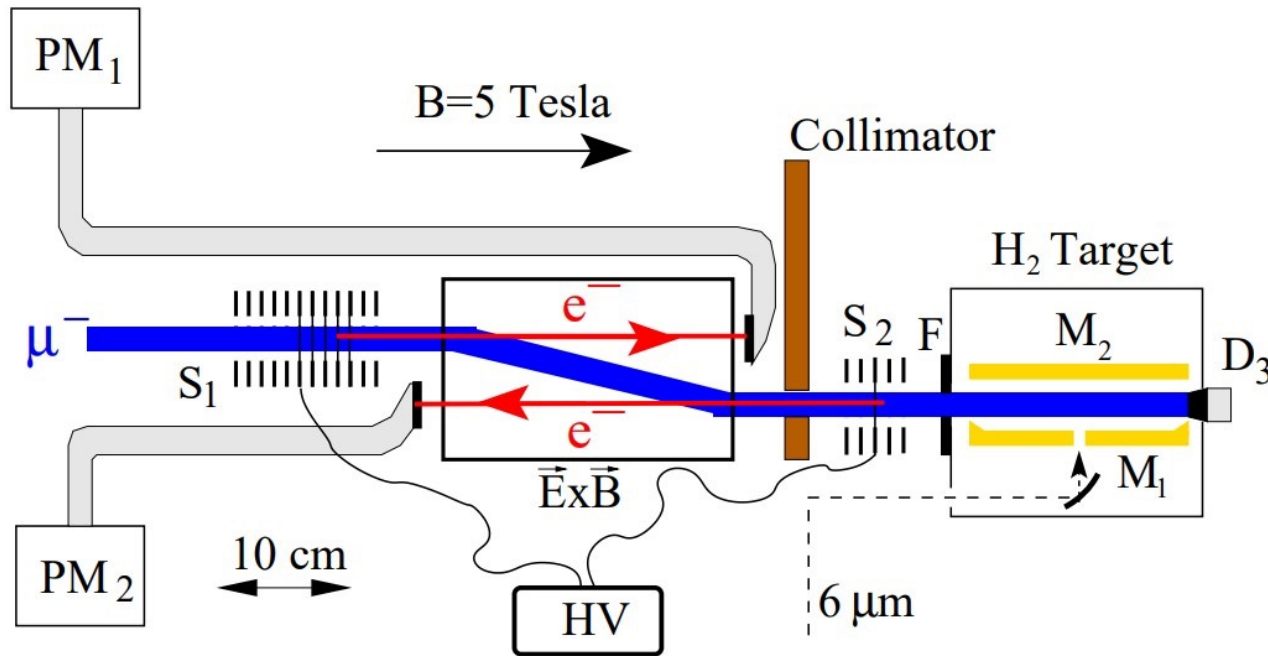
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 3. Use tunable laser to drives muons from 2S to 2P
 4. 2P muons decay right away ($\sim\text{ps}$) to 1S, emitting a 2keV X-ray
 5. Events selected by the 2keV X-ray signal and a delayed electron signal from muon decay



Muonic Hydrogen Measurement

1. DAQ and laser system triggered by muon signal from PM1 and PM2
2. Anti-trigger provided by muons hitting D3
3. Laser activated to drive muon from 2S to 2P
4. 2keV photon and delayed signal from muon decay detected



Proton Charge Radius Puzzle

- Possible solution to the puzzle:
 1. The ep scattering results are wrong
 - Fit procedure not good enough
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 - Proton magnetic radius



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 3. μH spectroscopy results are wrong
 - Laser calibration, two photon exchange...?
 4. New physics beyond the standard model
 - Lepton universality violation



Extraction of ep Elastic Scattering Cross Section

- To reduce the systematic uncertainty, the ep cross section is normalized to the Møller cross section:

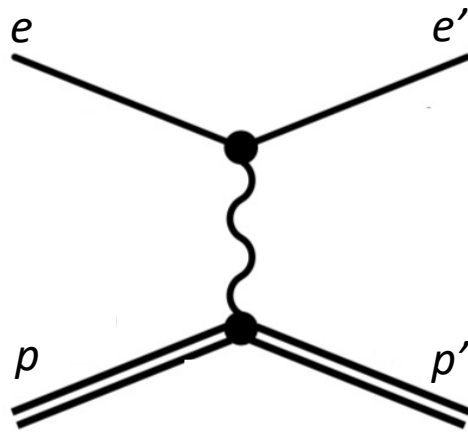
$$\left(\frac{d\sigma}{d\Omega}\right)_{ep} = \left[\frac{N_{\text{exp}}(ep \rightarrow ep \text{ in } \theta_i \pm \Delta\theta_i)}{N_{\text{exp}}(ee \rightarrow ee)} \cdot \frac{\varepsilon_{\text{geom}}^{ee}}{\varepsilon_{\text{geom}}^{ep}} \cdot \frac{\varepsilon_{\text{det}}^{ee}}{\varepsilon_{\text{det}}^{ep}} \right] \left(\frac{d\sigma}{d\Omega}\right)_{ee}$$

- Method 1: bin-by-bin method** – taking ep/ee counts from the same angular bin
 - Cancellation of energy independent part of the efficiency and acceptance
 - Limited converge due to double arm Møller acceptance
- Method 2: integrated Møller method** – integrate Møller in a fixed angular range and use it as common normalization for all angular bins
- Luminosity cancelled from both methods
- Bin-by-bin range: 0.7° to 1.6° for 2.2 GeV, 0.75° to 3.0° for 1.1 GeV. Larger angles use integrated Møller method

Radiative Correction

Leading order

- From experiment, we look for $ep \rightarrow ep$ process and want to measure the Born level ep elastic scattering cross section



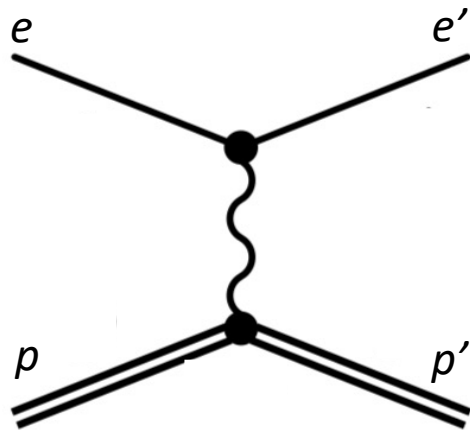
- Described by the Rosenbluth formula:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} \left(\frac{E'}{E} \right) \frac{1}{1 + \tau} \left(G_E^p{}^2(Q^2) + \frac{\tau}{\epsilon} G_M^p{}^2(Q^2) \right)$$

Radiative Correction

Leading order

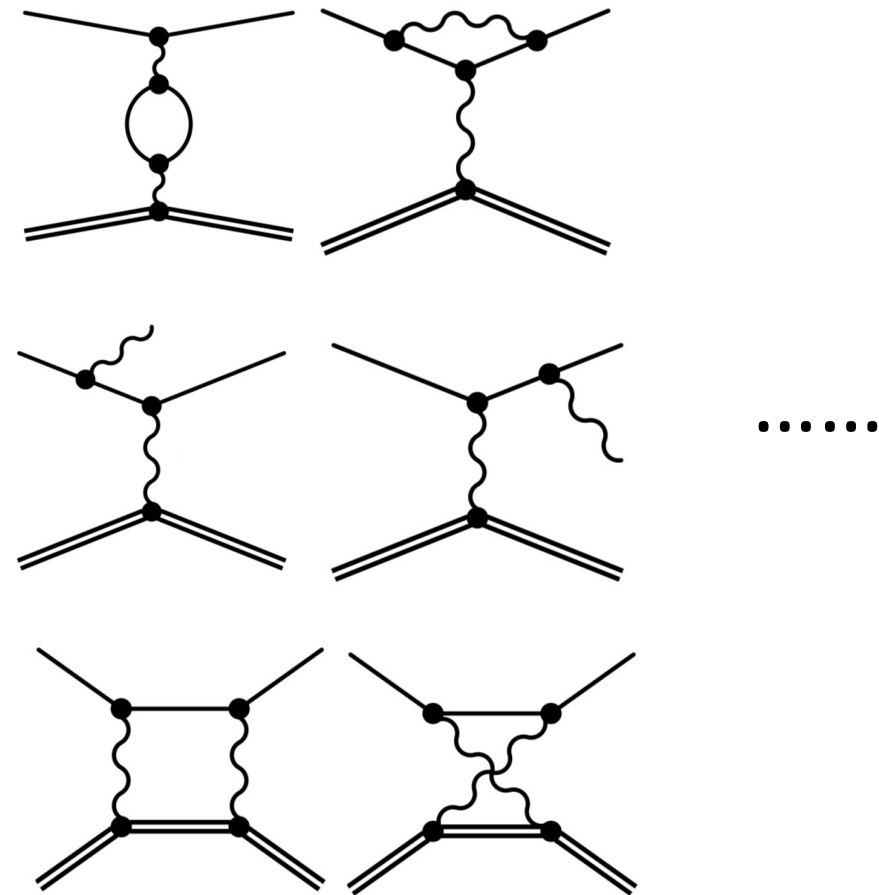
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Next-to-Leading order



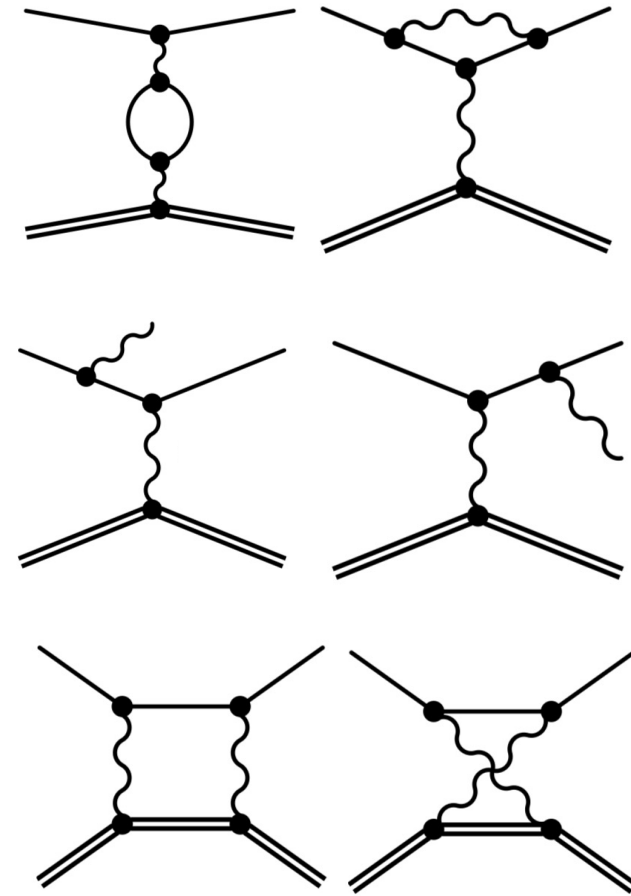
Radiative Correction

- Radiative effects corrected by Monte-Carlo method:

1. Geant4 simulation package with full geometry setup
2. event generators with complete calculations of radiative corrections^{1,2} **beyond ultra relativistic approximation**
3. Include emission of radiative photons **beyond peaking approximation**
4. Include Two Photon Exchange effect³, less than **0.2%** for *ep* in PRad kinematic range
5. Iterative procedure applied for radiative correction

$$\sigma_{ep}^{Born(exp)} = \left(\frac{\sigma_{ep}}{\sigma_{ee}} \right)^{exp} / \left(\frac{\sigma_{ep}}{\sigma_{ee}} \right)^{sim} \cdot \left(\frac{\sigma_{ep}}{\sigma_{ee}} \right)^{Born(model)} \cdot \sigma_{ee}^{Born(model)}$$

Diagrams for *ep* elastic scattering



1. I. Akushevich et al., Eur. Phys. J. A 51(2015)1 (fully beyond ultra relativistic approximation)
2. A. V. Gramolin et al., J. Phys. G Nucl. Part. Phys. 41(2014)115001
3. O. Tomalak, Few Body Syst. **59**, no. 5, 87 (2018)

Radiative Correction

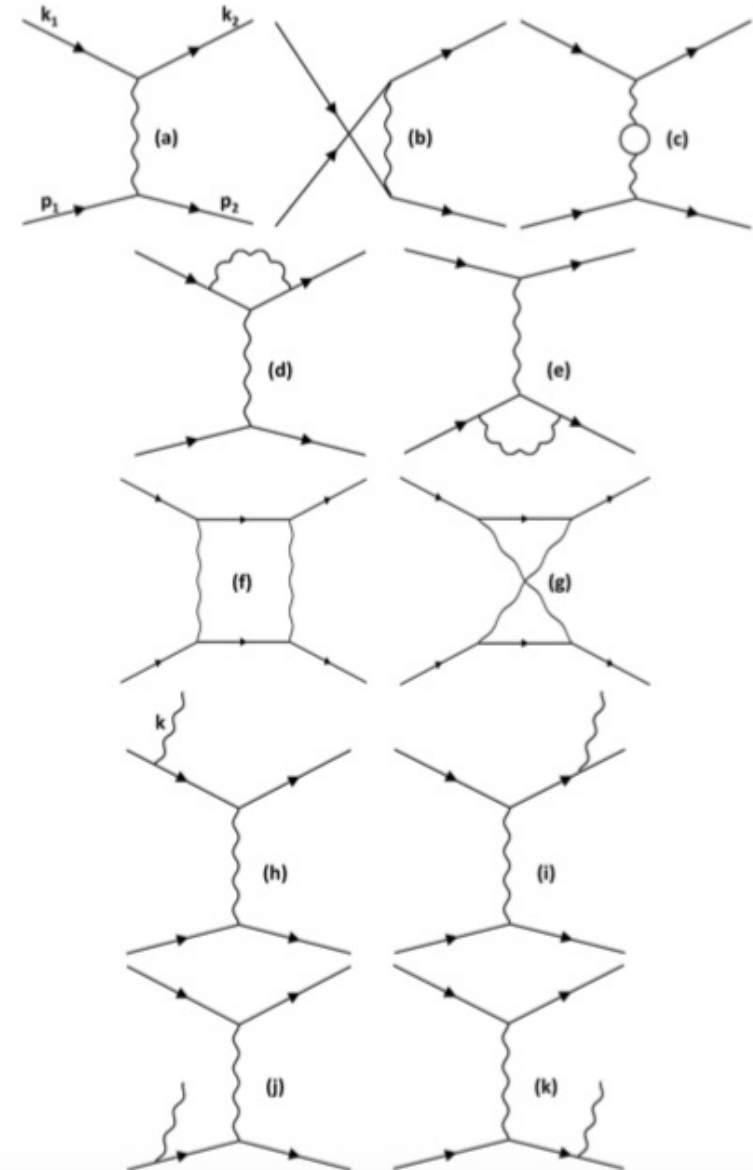
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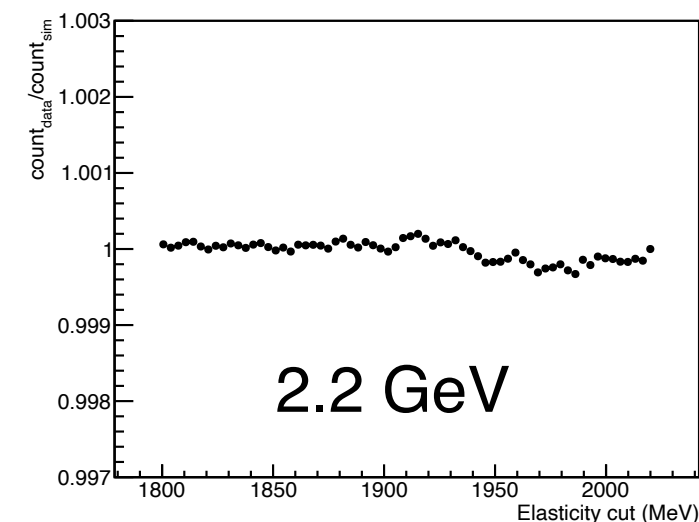
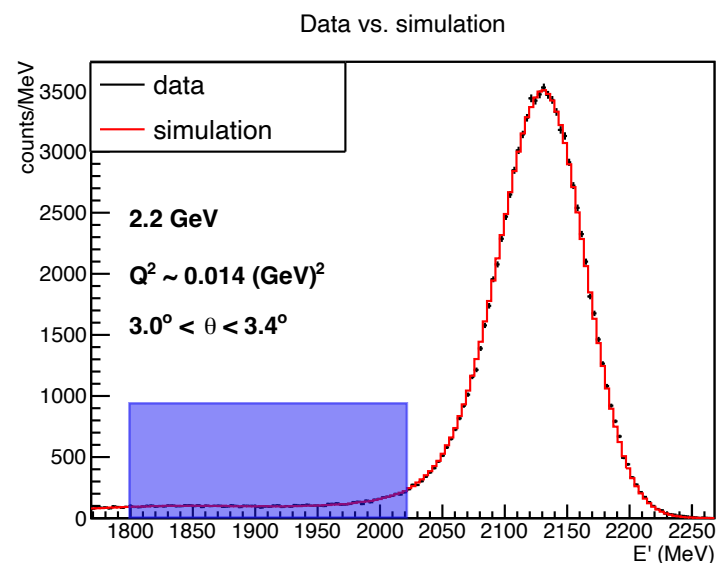
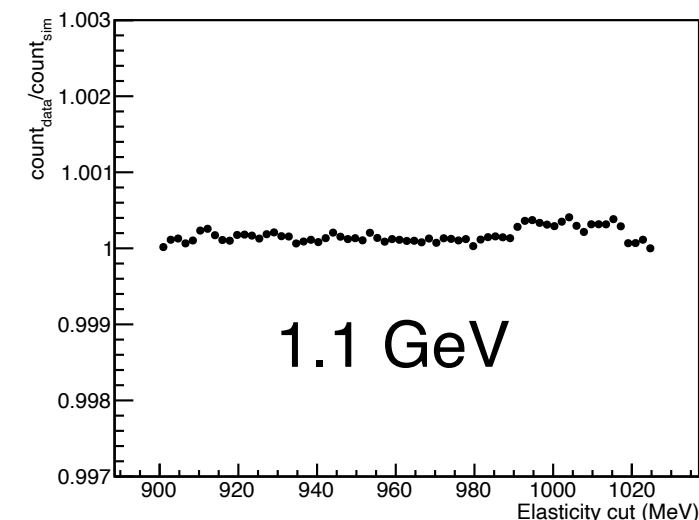
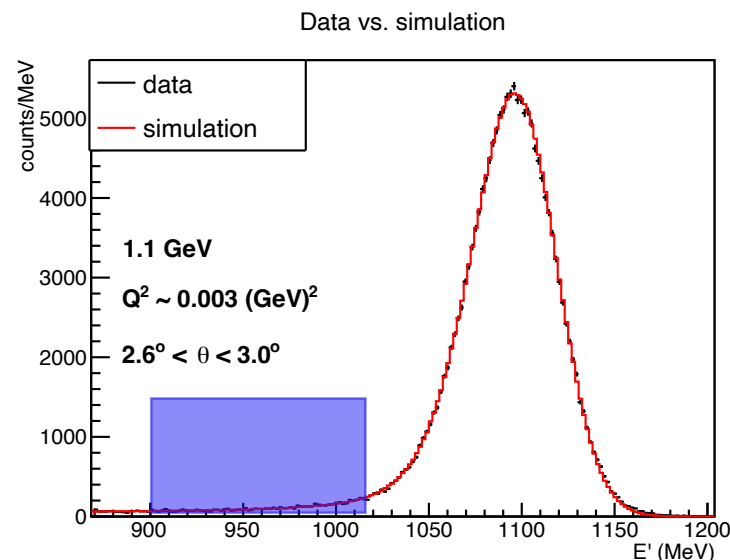
Diagrams for *ee* elastic scattering



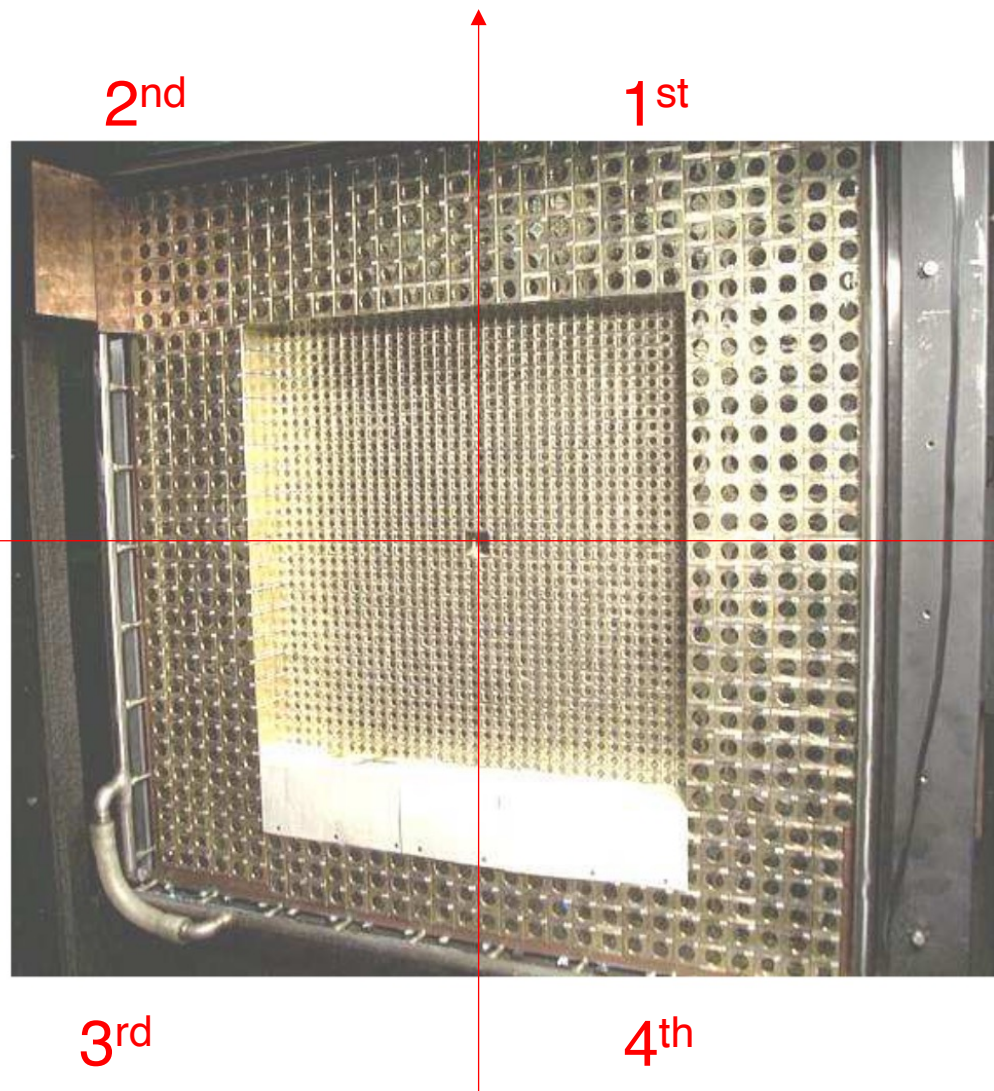
Systematic Uncertainties

(Example of Event Selection)

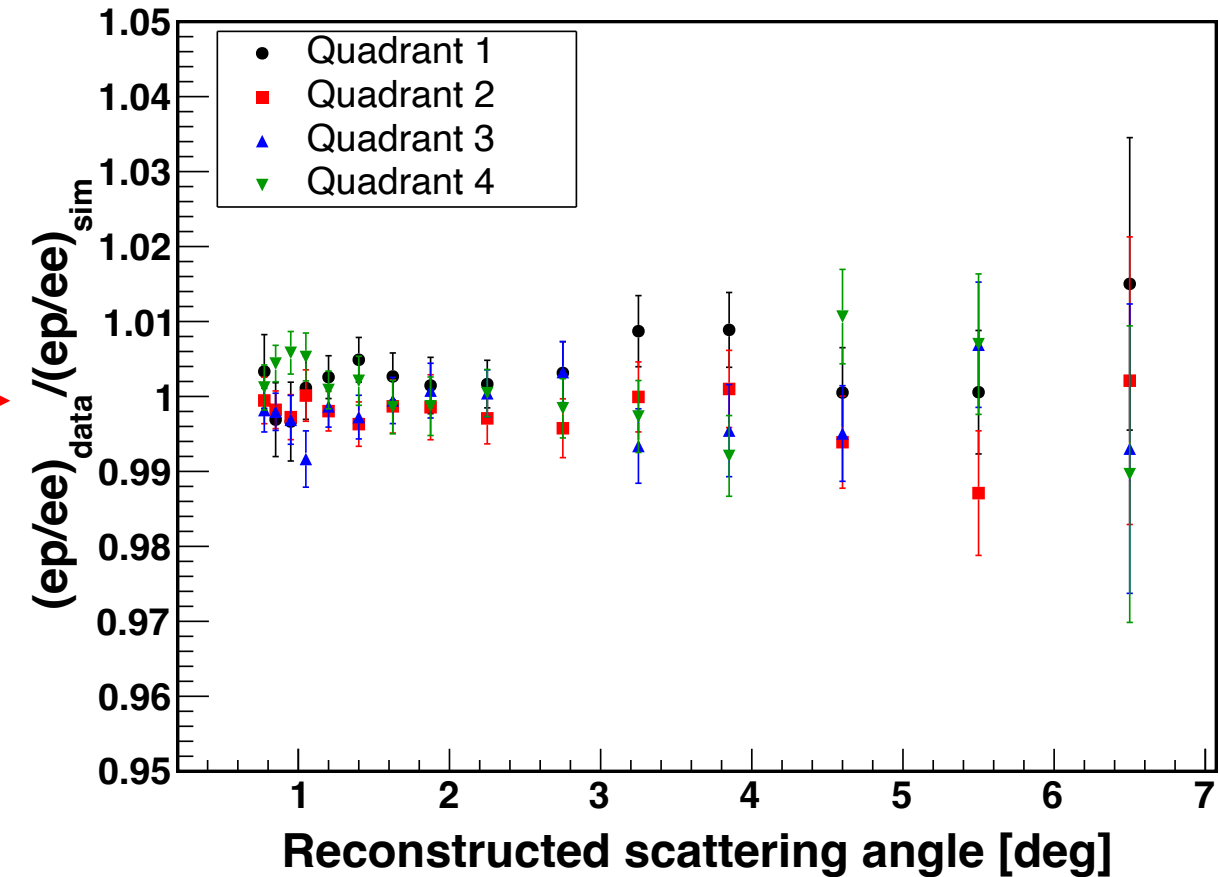
- Changing elasticity cut at the radiative tail and obtain different sets of cross section results
- Sensitivity on cross section: typically within $\pm 0.15\%$
- Mostly due to non-uniformity of HyCal modules



Checking Systematics – Azimuthal Symmetry

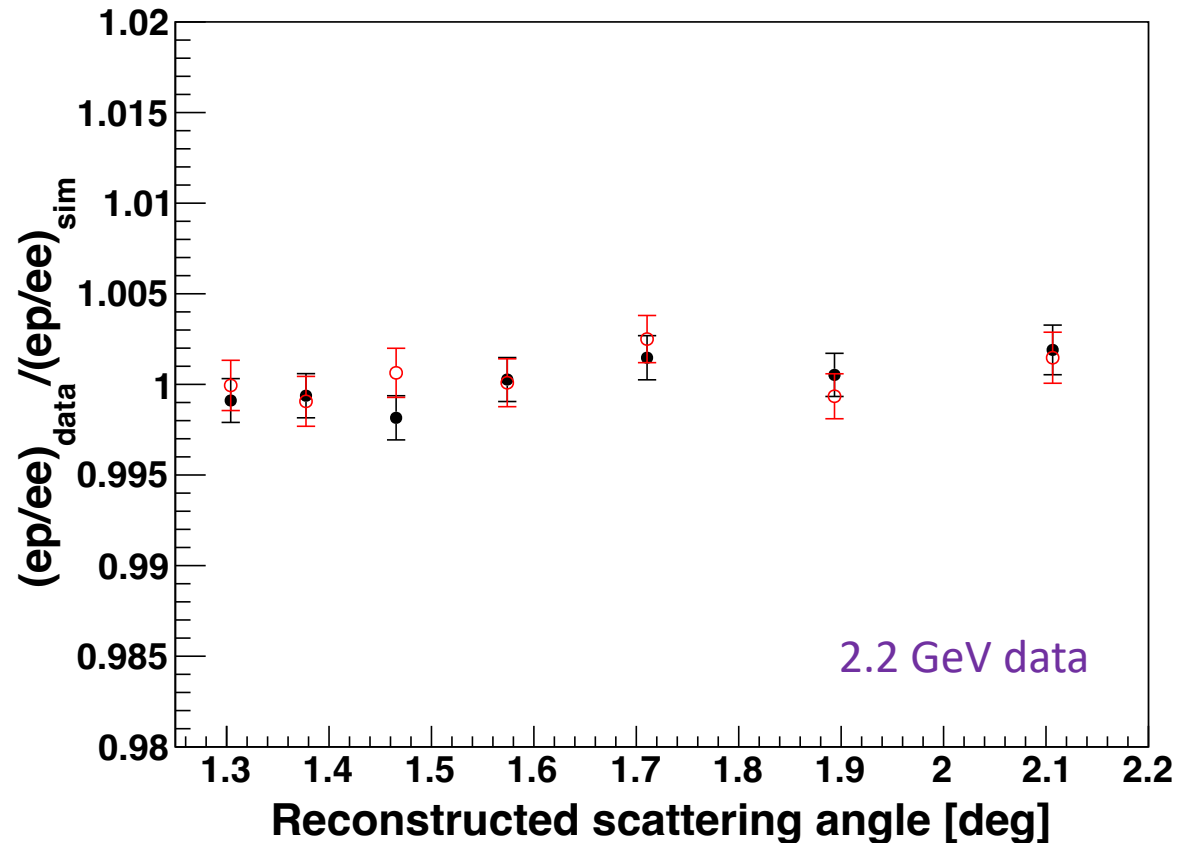
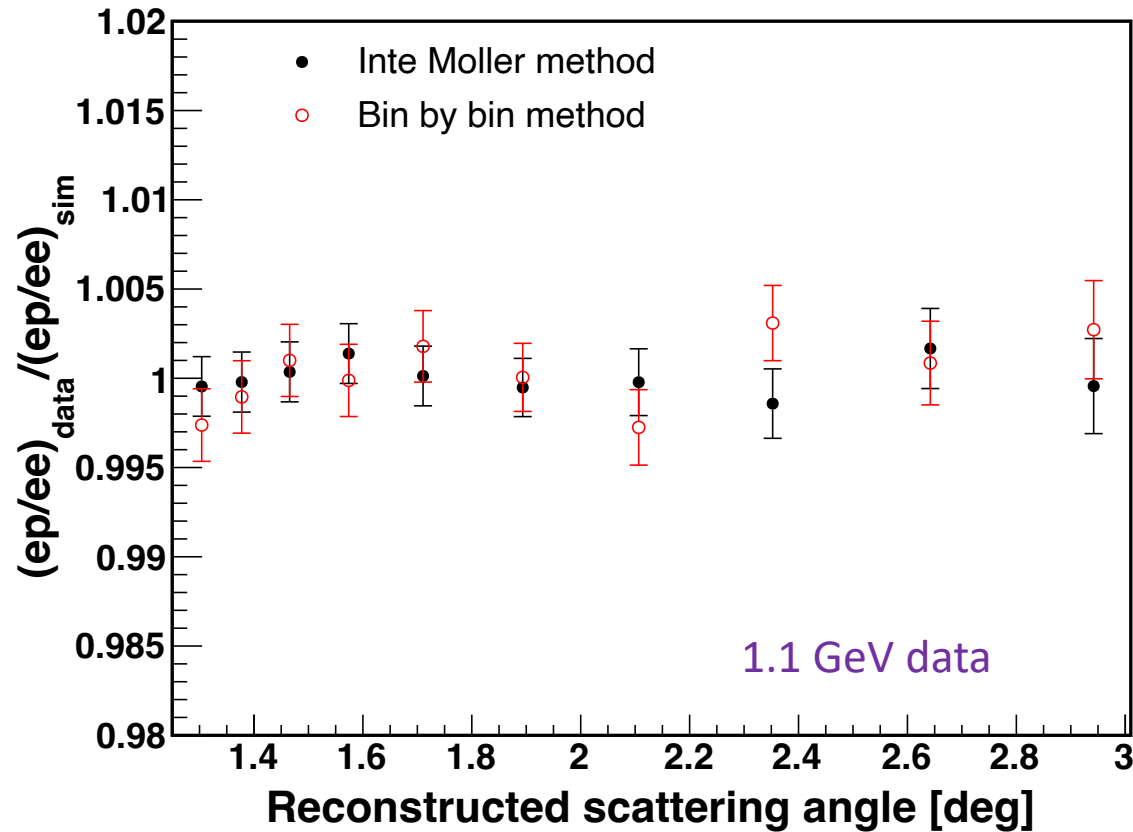


1.1 GeV data



Checking Systematics – Different methods of Forming ep/ee ratio

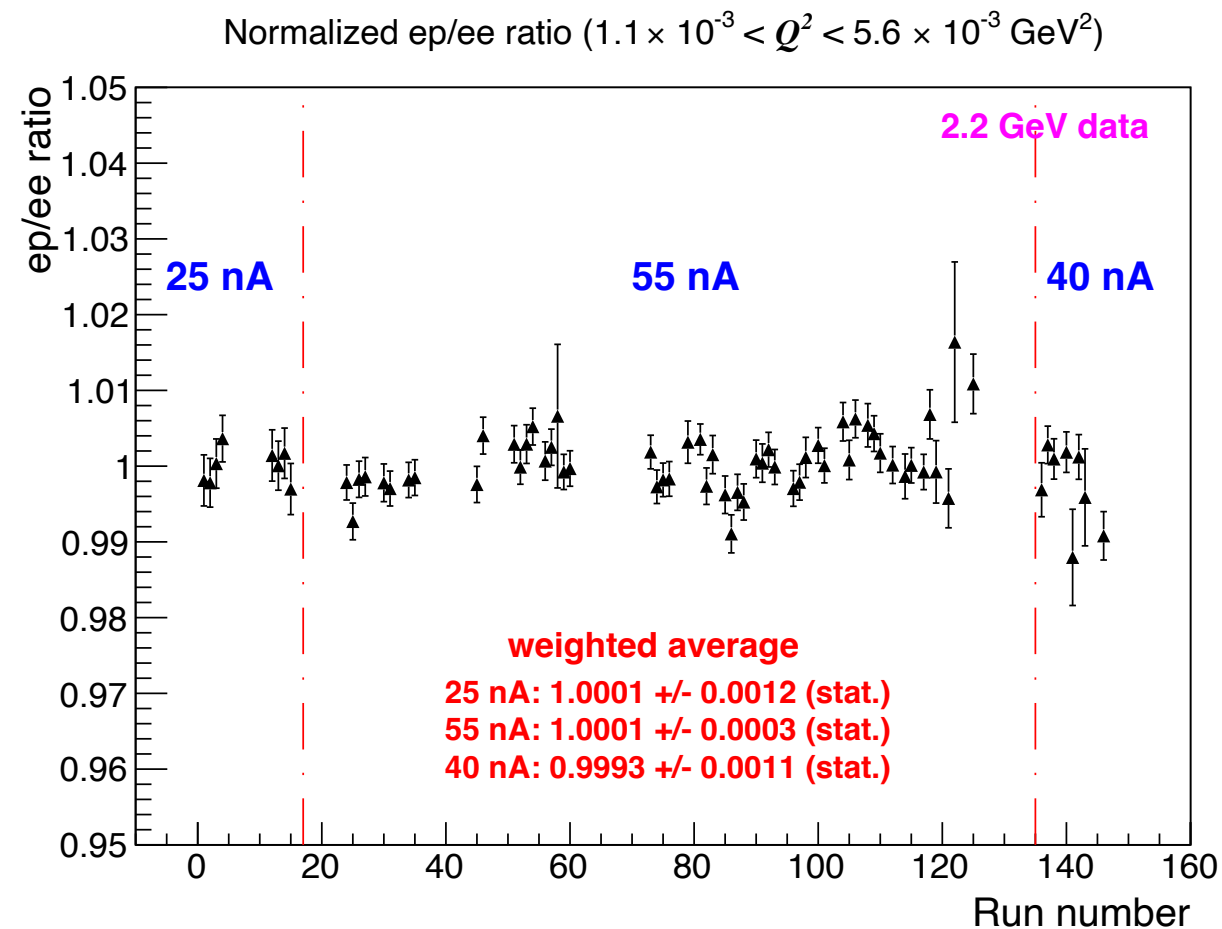
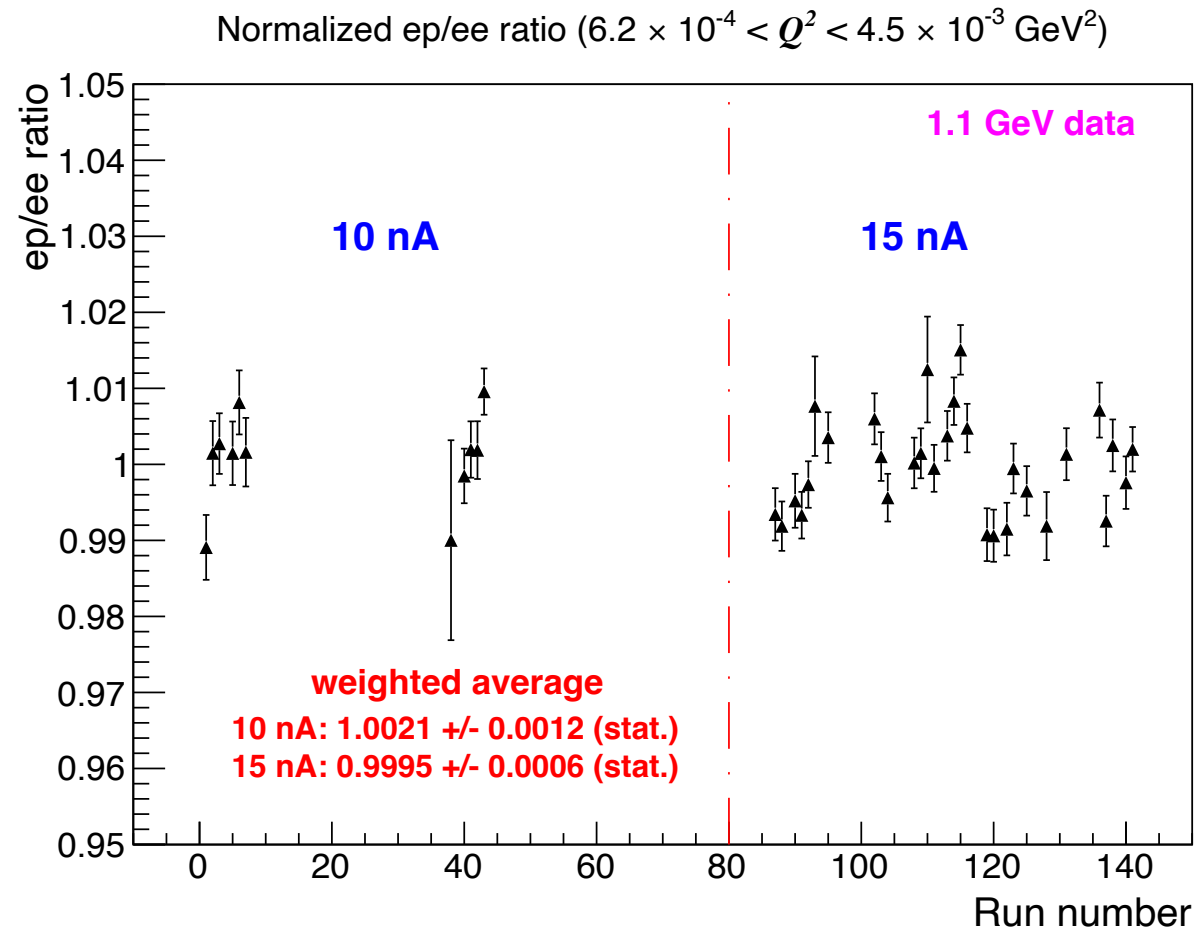
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Systematic Uncertainties

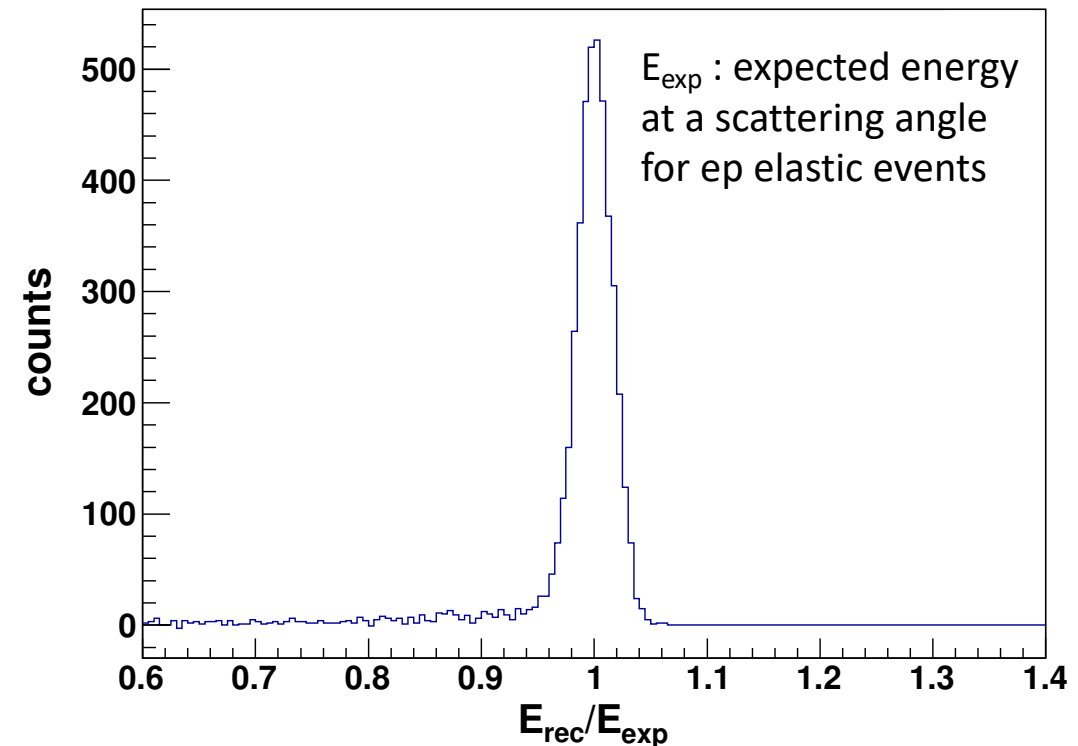
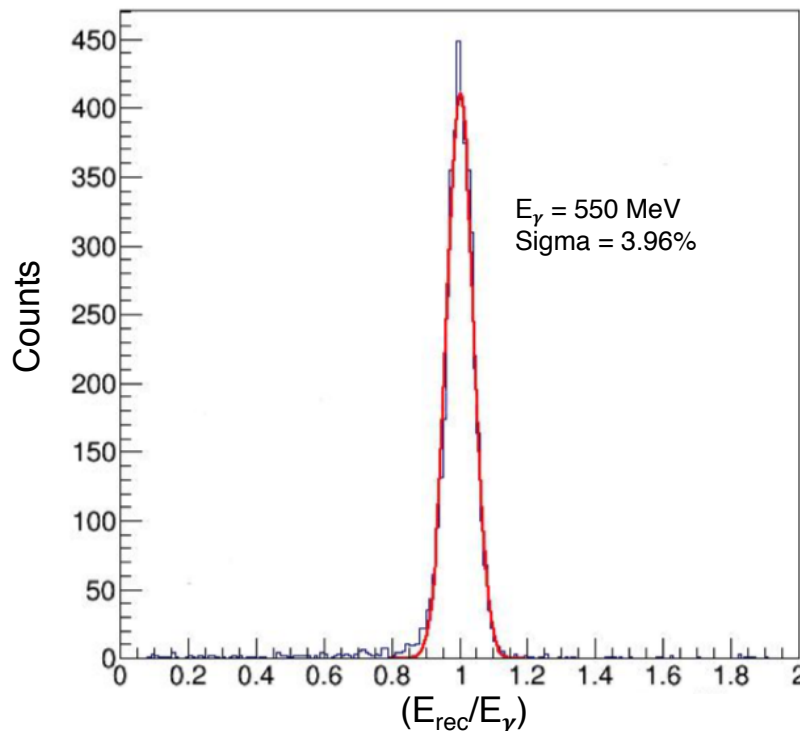
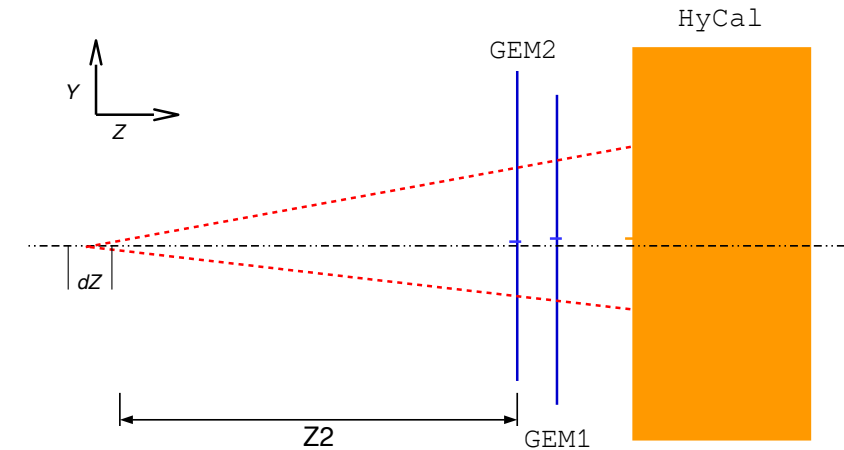
Item	r_p uncertainty [fm]	n_1 uncertainty	n_2 uncertainty
Event selection	0.0070	0.0002	0.0006
Radiative correction	0.0069	0.0010	0.0011
Detector efficiency	0.0042	0.0000	0.0001
Beam background	0.0039	0.0017	0.0003
HyCal response	0.0029	0.0000	0.0000
Acceptance	0.0026	0.0001	0.0001
Beam energy	0.0022	0.0001	0.0002
Inelastic ep	0.0009	0.0000	0.0000
G_M^p parameterization	0.0006	0.0000	0.0000
Total	0.0115	0.0020	0.0013

Checking Systematics – Stability vs. Runs



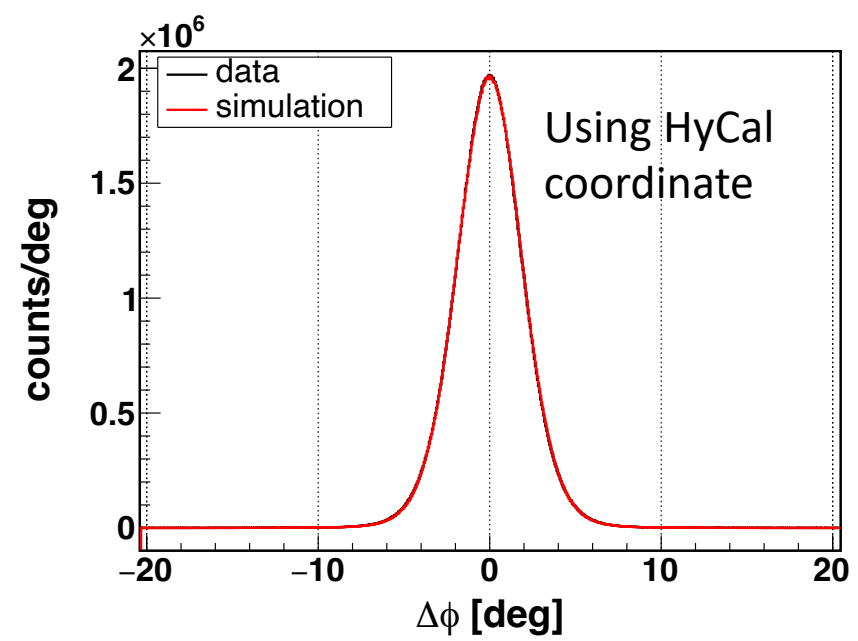
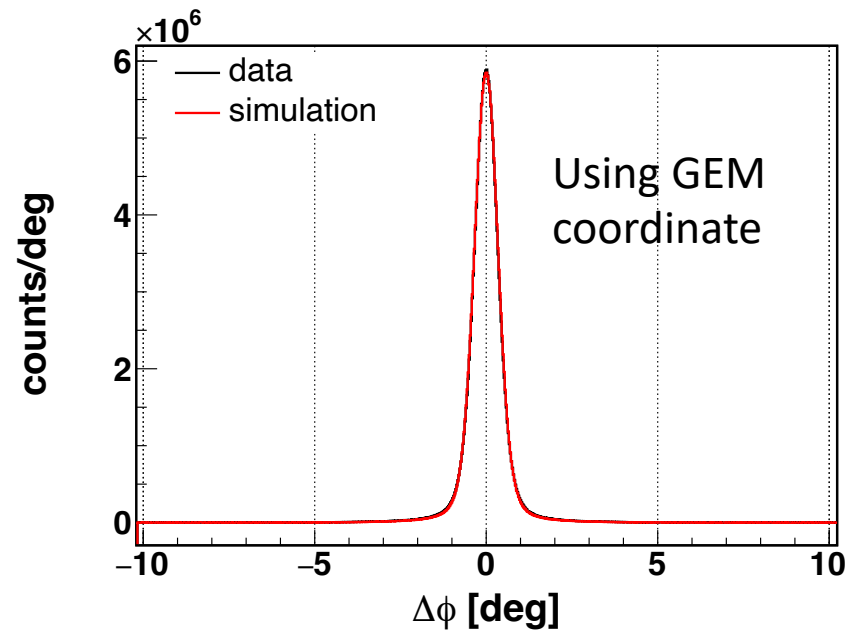
Detector Calibration

- HyCal energy calibration:
 1. Tagged photon beam calibration
 2. Calibration using elastic ep and ee events
- Detector position calibration
 1. Detector position surveyed by JLab survey group
 2. Using double arm Møller events

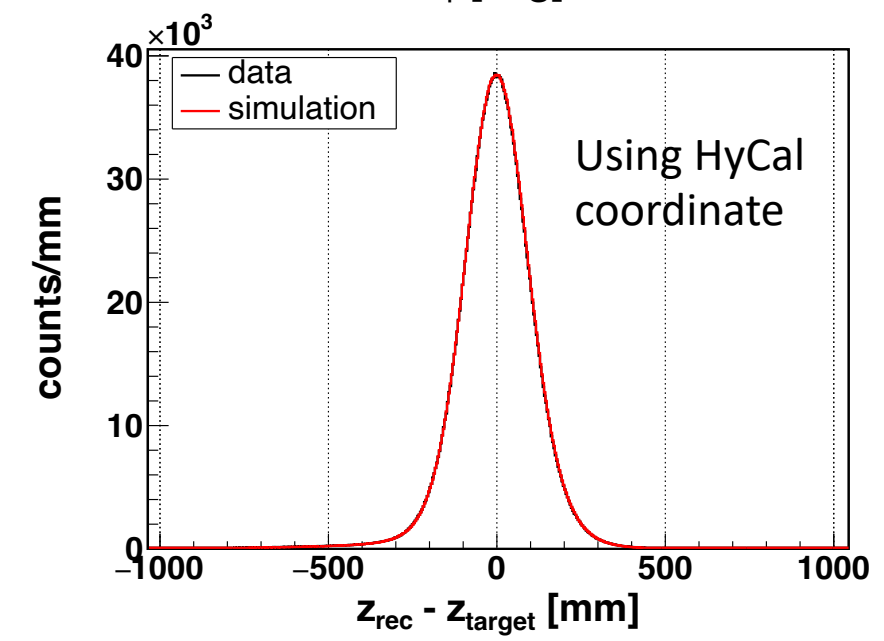
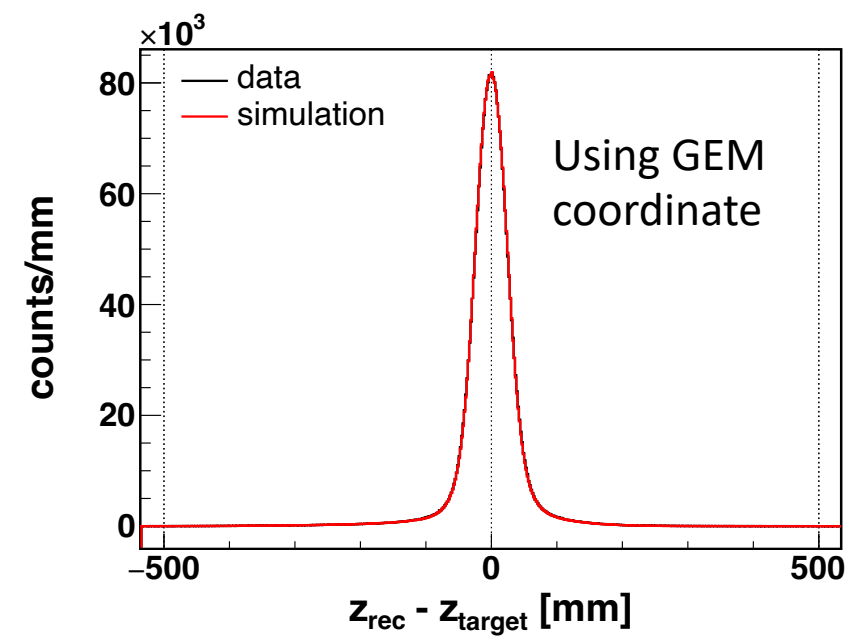


Detector Calibration

Co-planarity



Vertex-z



Quality of the Fit

- $\chi^2/\text{ndf} \sim 1.3$ (statistical uncertainty only, 33 data points from 1.1 GeV, 38 data points from 2.2 GeV)
- Furthestmost outlier about 2.5σ from the fit
- **67%** and **58%** data points within 1σ for **1.1** and **2.2** GeV data respectively

