Energy-Energy-Correlations: from ee to pp then to ep colliders

朱华星
浙江大学

第六十八届强子物理在线论坛
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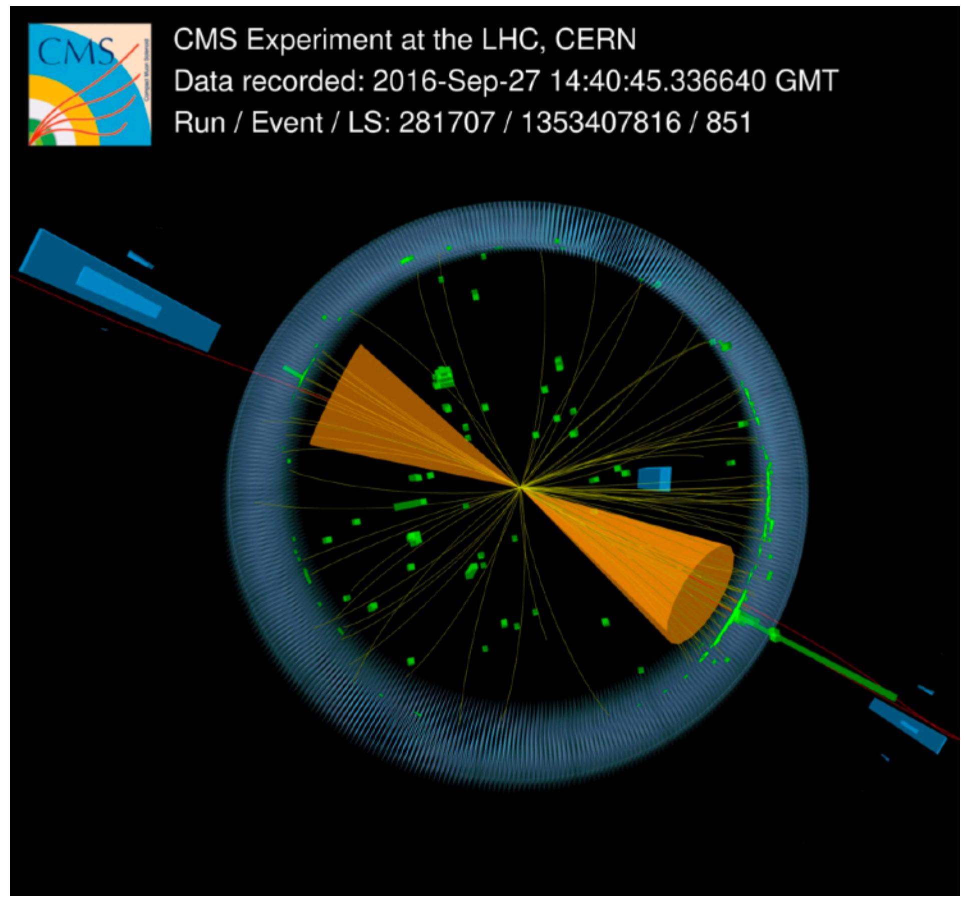
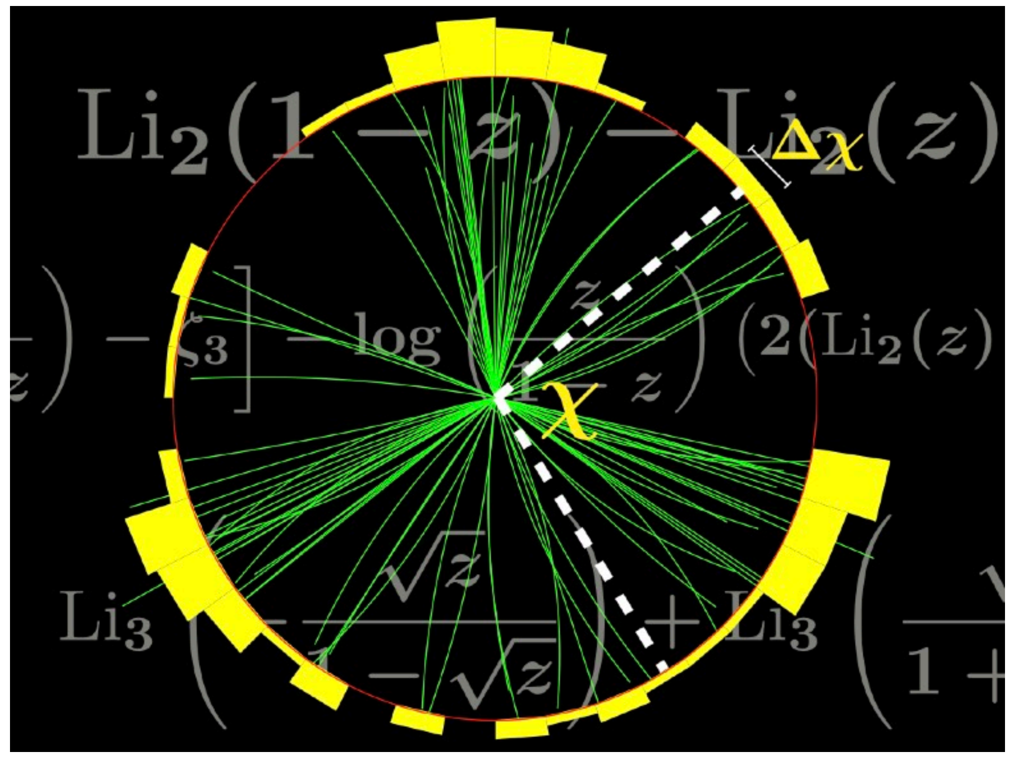


normalization calculable ✓

multi-particle correlation ✓

field theoretic definition ✗



normalization + evolution ✓

multi-particle correlation ✓

field theoretic definition ✓

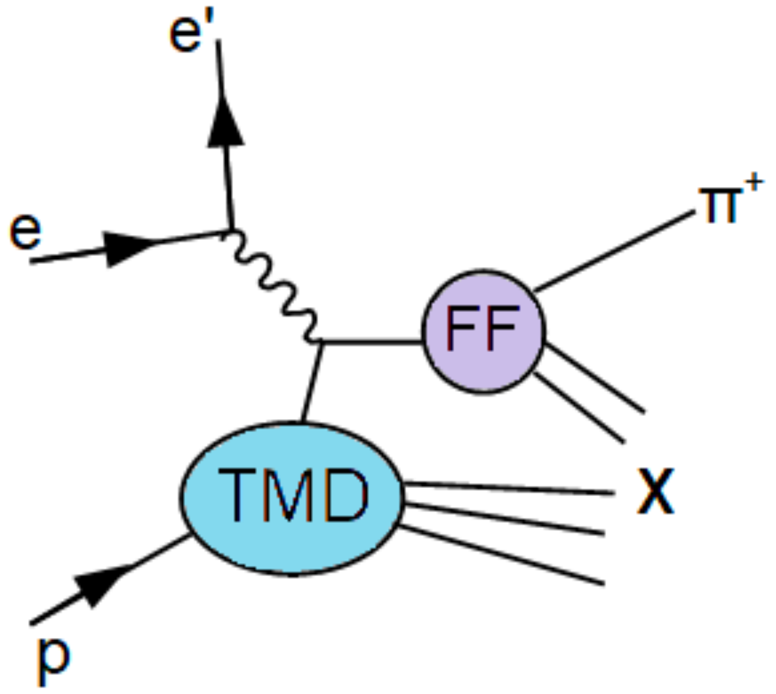
More inclusive

More exclusive

✓ evolution calculable

✗ multi-particle correlation

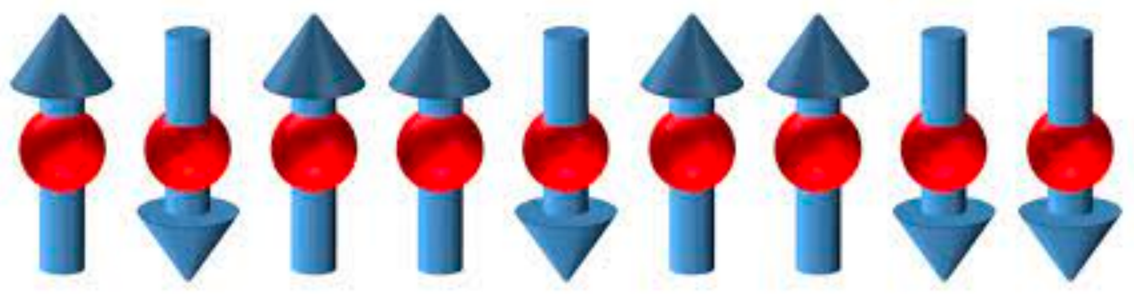
✓ field theoretic definition

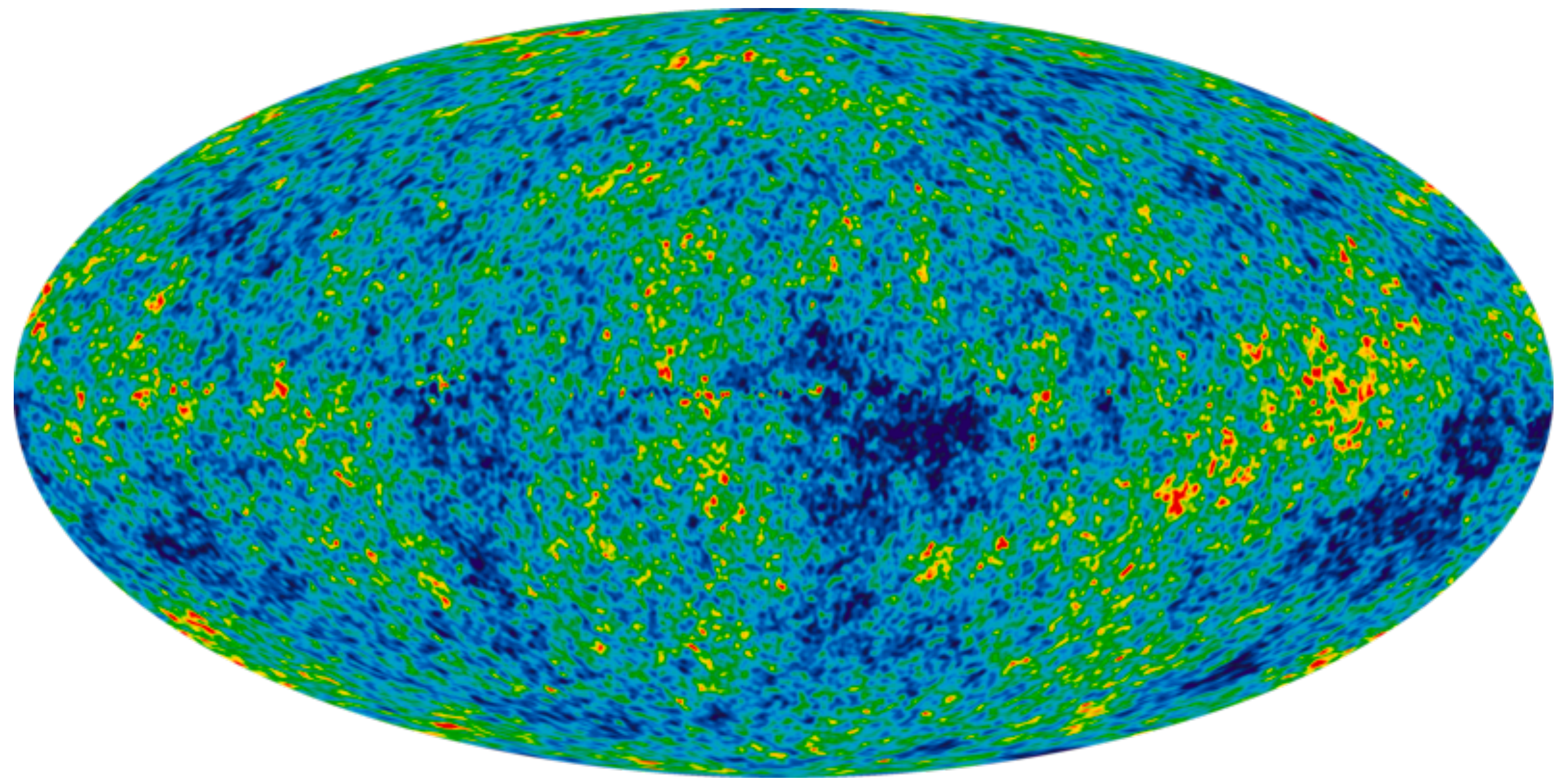


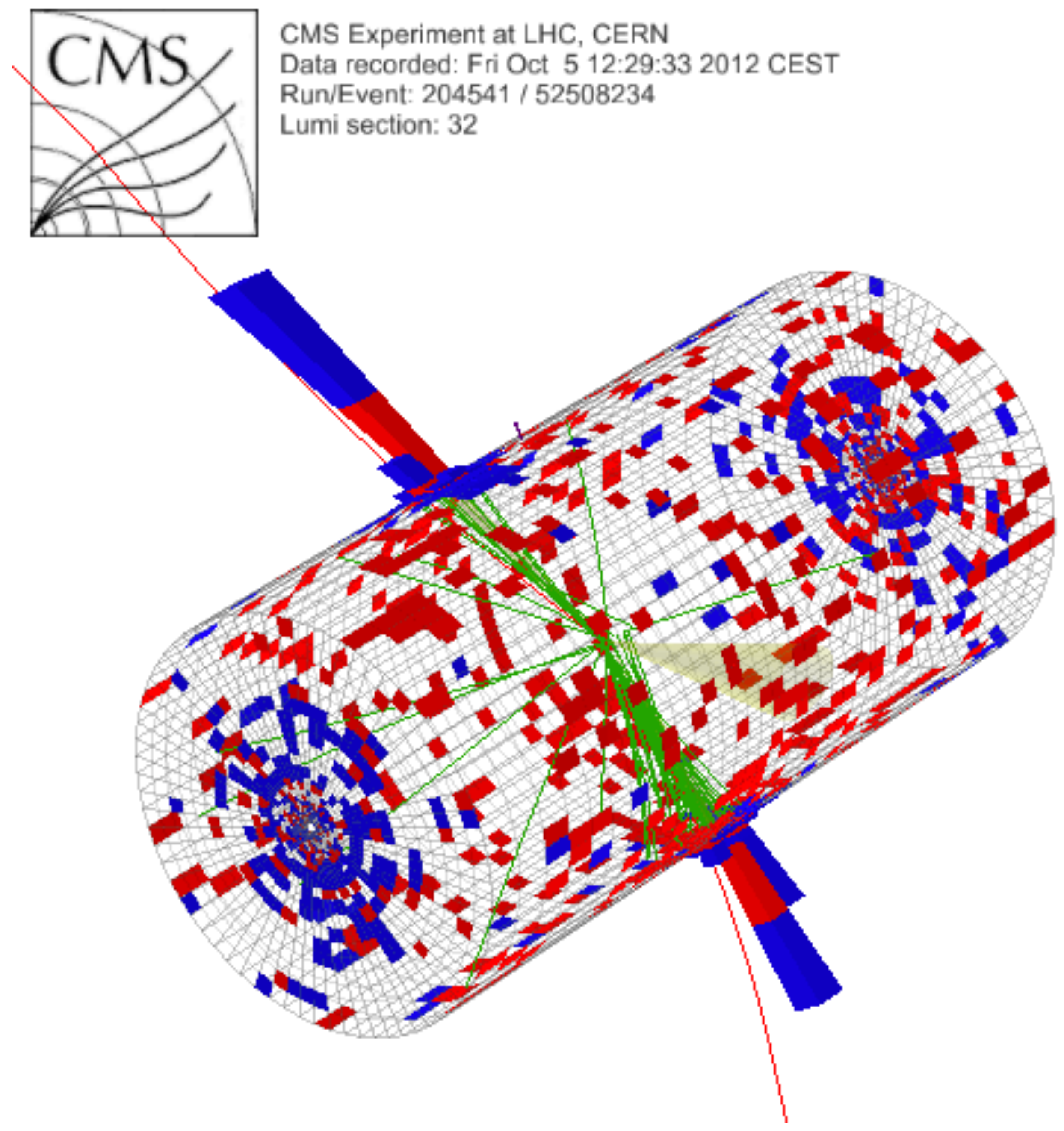
Outline of this talk

- Introduction to Energy correlators: theory foundation
- EECs in e+e- colliders: the computational frontier
- EECs in hadron colliders: “phase transition”, tracks and multi-hadron fragmentation
- EECs in ep: a new probe of nucleon structure

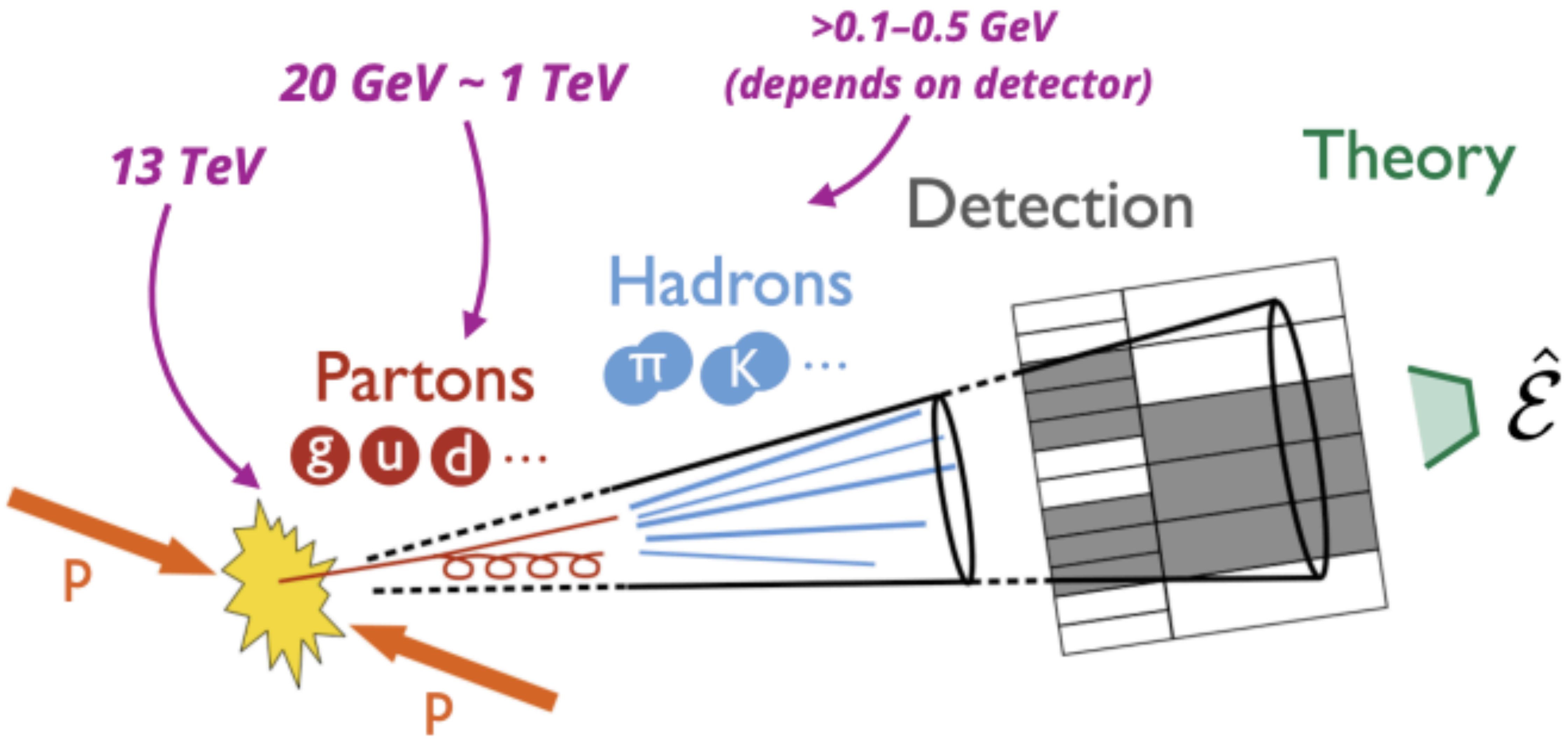
The world of correlation functions





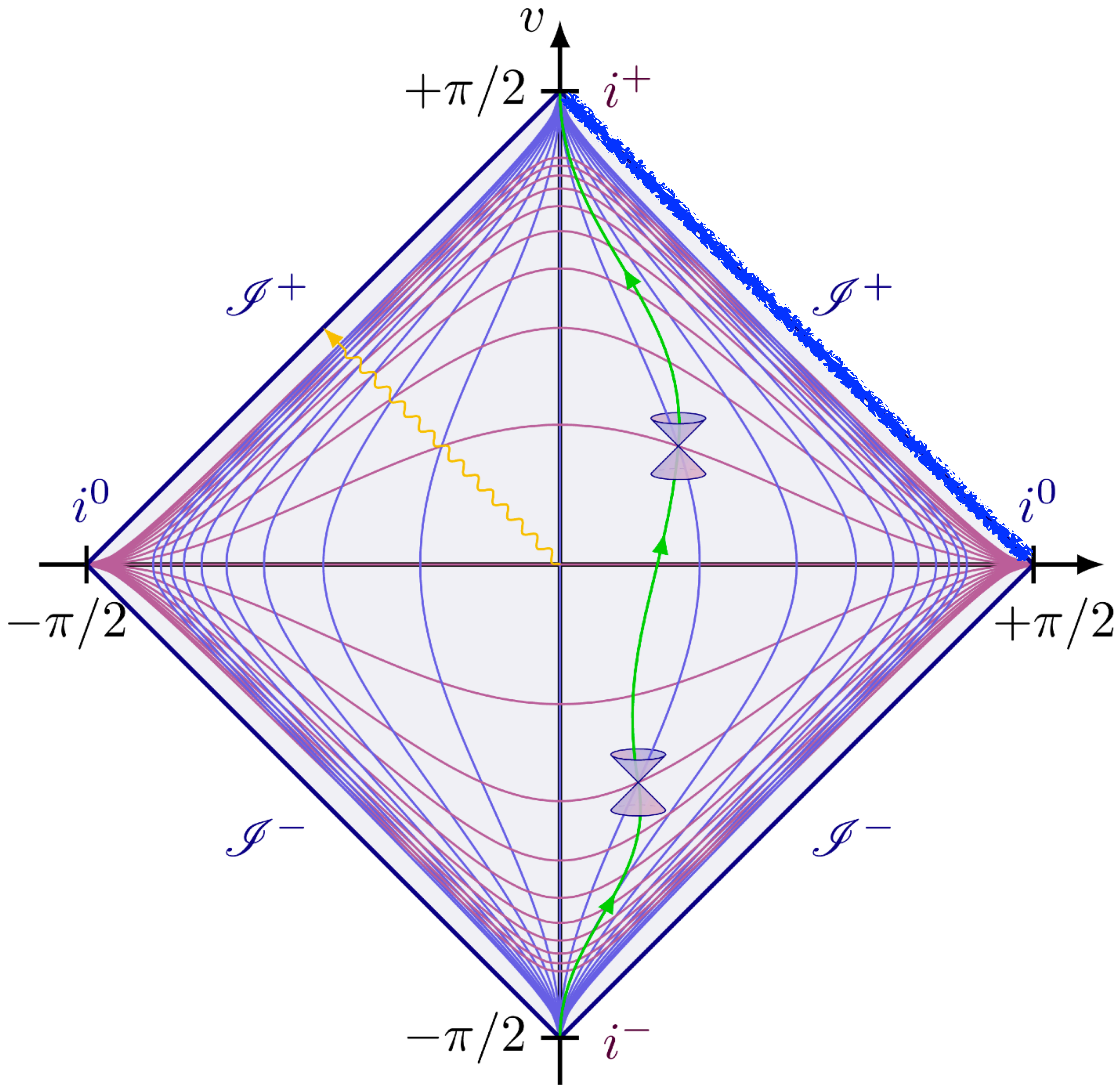


Energy flow operator



$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n})$$

Tkachov, 1995
Morris, Thorne, Yurtsever, 1988

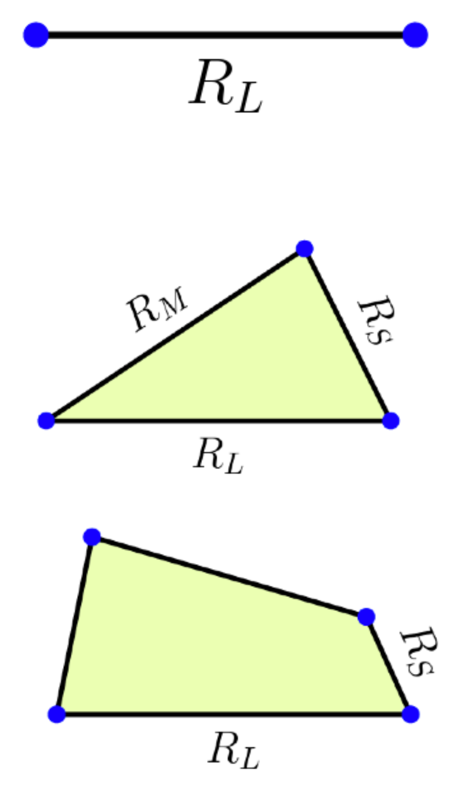


$$r + t = \tan(u+v)$$
$$r - t = \tan(u-v)$$

https://tikz.net/relativity_penrose_diagram/

Energy correlators (EECs)

- Two point energy correlator $\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_\Psi$
- three point $\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\mathcal{E}(n_3)\rangle_\Psi$
- four point $\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\mathcal{E}(n_3)\mathcal{E}(n_4)\rangle_\Psi$



- …

Ψ: excitation of color sources from vacuum or medium

- Conjecture: EECs consist of a complete basis of all IRC safe observables

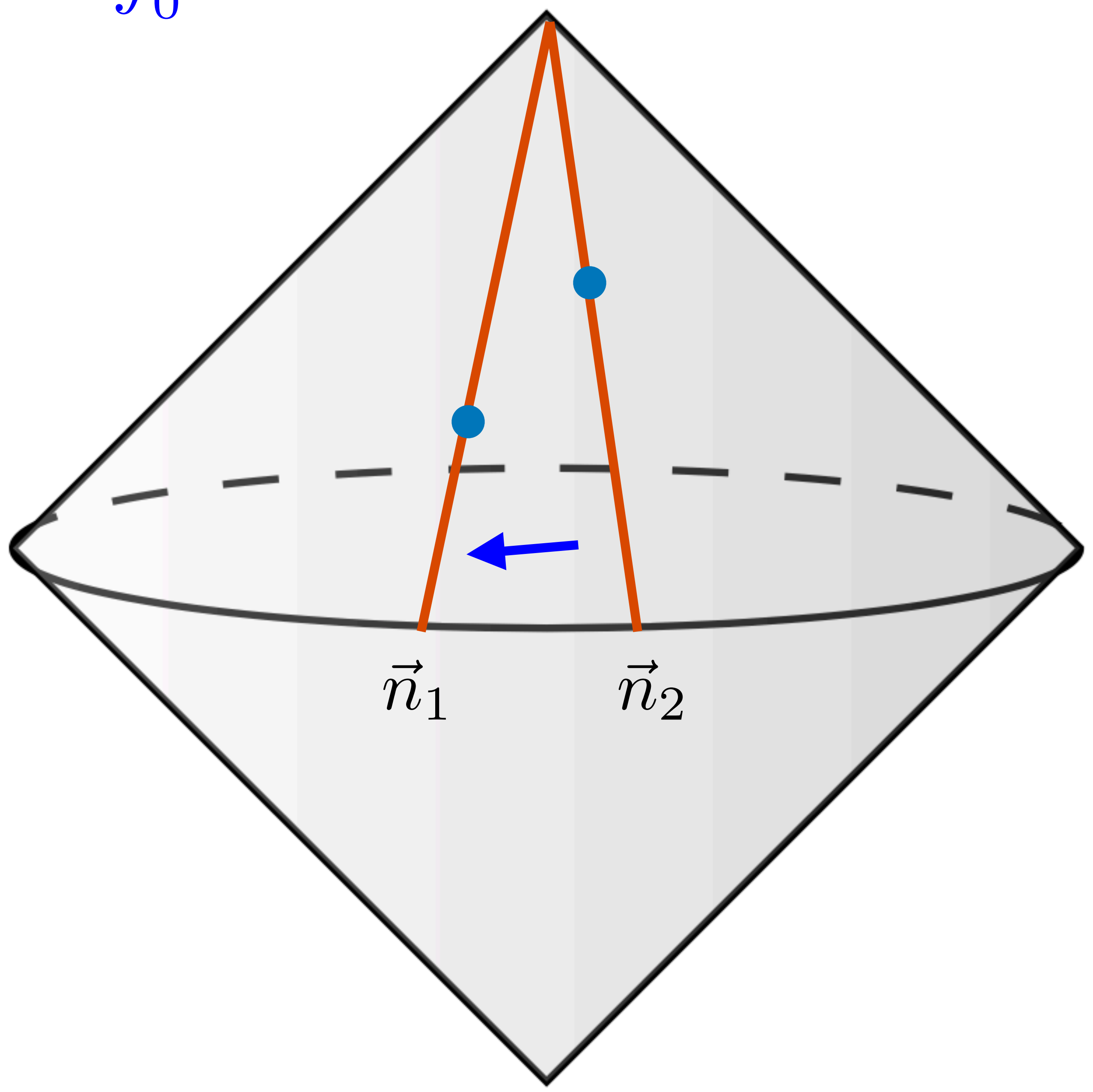
moment of EEC related to average of C parameter
moment of EEEC related to average of D parameter

Chen, Moult, Zhang, HXZ, 2004.11381

A spacelike-timelike puzzle

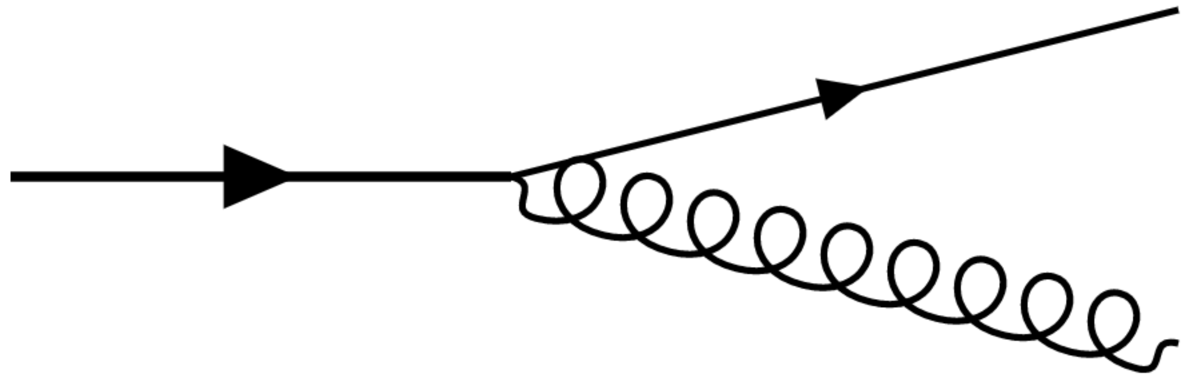
What happen if $\lim_{n_2 \to n_1} \langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_\Psi$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n})$$



$$\lim_{\vec{n}_2 \to \vec{n}_1} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \sum_i \theta_{12}^{\gamma_i} \mathbb{O}_i(\vec{n}_1)$$

spacelike OPE



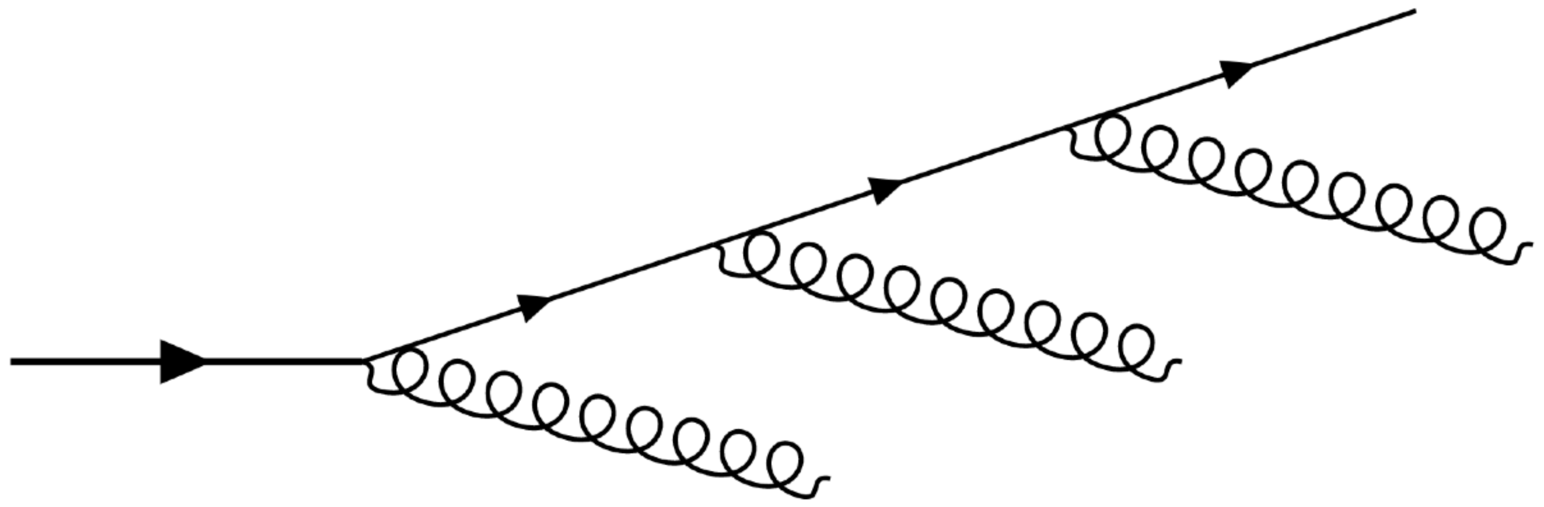
$$P_{gq}(x) = C_F\left(\frac{1+(1-x)^2}{x} - \epsilon x\right),$$

$$P_{gg}(x) = 2C_A \frac{(1-x+x^2)^2}{x(1-x)},$$

$$P_{qg}(x) = \frac{1}{2}\left(1 - \frac{2x(1-x)}{1-\epsilon}\right).$$

timelike fragmentation

A spacelike-timelike duality



All order resummation

Konishi, Ukawa, Veneziano, 1978
Dixon, Moult, HXZ, 1905.01310

$$\frac{d}{d\ln\mu^2}\vec{J}\left(\ln\theta^2,\mu\right) = \int_0^1 dy\, y^2 \vec{J}\left(\ln(y\theta^2),\mu)\right)\cdot P_T(y,\mu)$$

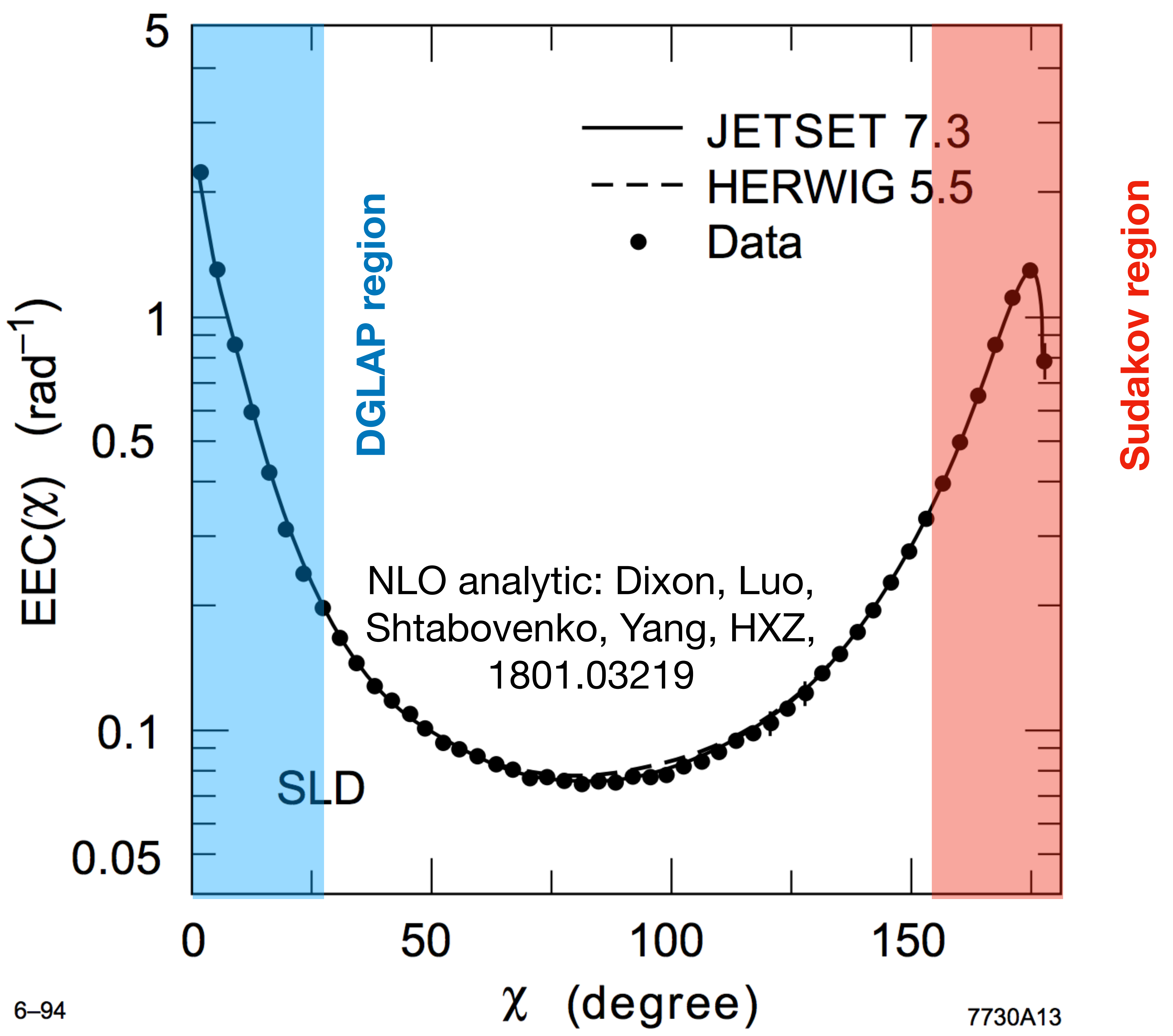
$$\gamma_S(N) \Leftarrow \gamma_T(N+\gamma_S(N))$$

spacelike DGLAP at integer spin = timelike DGLAP at non-integer spin

Gribov-Lipatov-Basso-Korchemsky reciprocity

Chen, Yang, HXZ, Zhu, 2006.10534

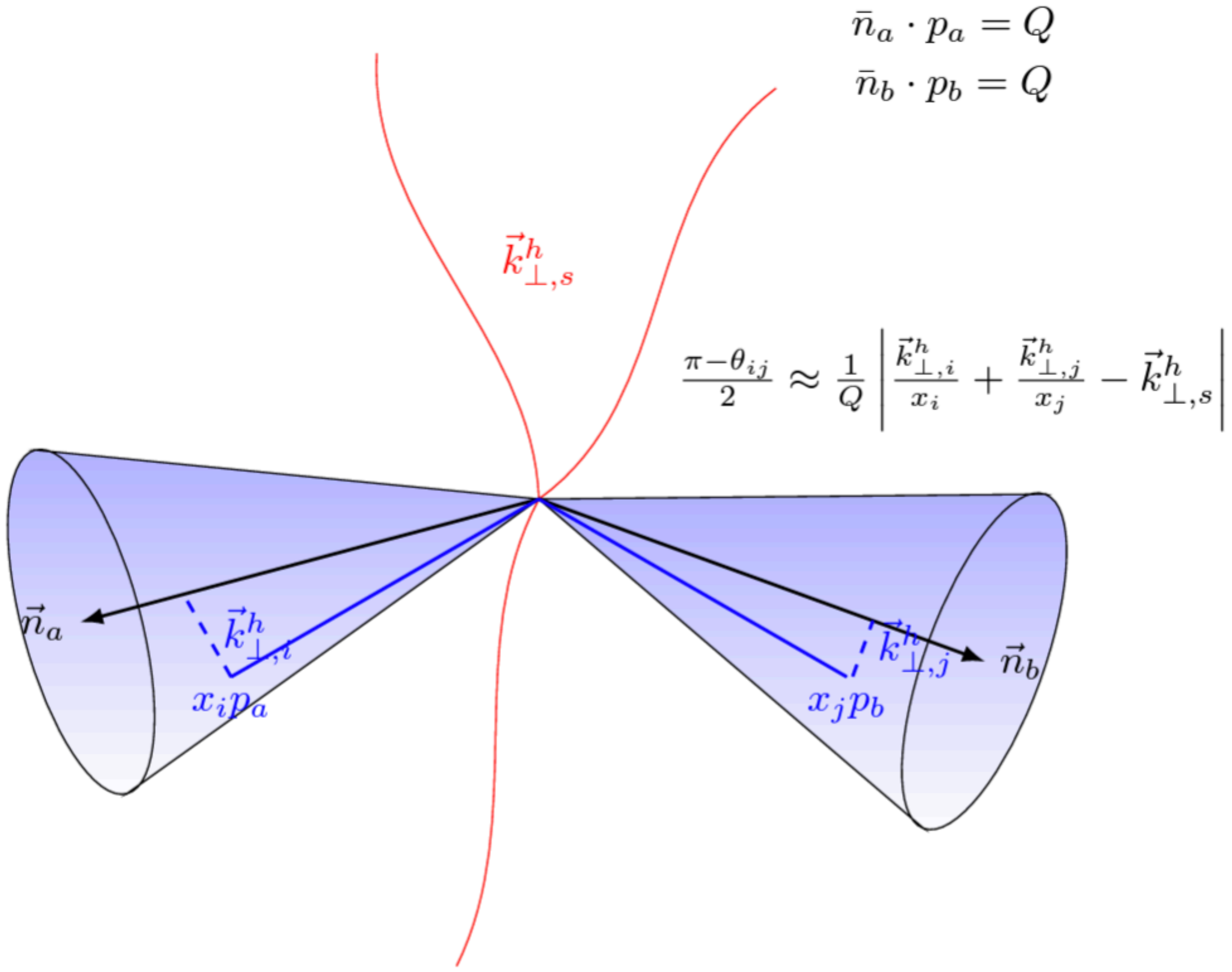
e+e- collider



5
1
0.5
0.1
0.05
EEC(χ) (rad⁻¹)
0
50
100
150
χ (degree)
JETSET 7.3
HERWIG 5.5
Data
DGLAP region
Sudakov region
SLD
NLO analytic: Dixon, Luo, Shtabovenko, Yang, HXZ, 1801.03219
6–94
7730A13

EEC and TMD fragmentation

$$\alpha_s \ln^2(\pi-\theta) + \alpha_s^2 \ln^4(\pi-\theta) + \alpha_s^3 \ln^6(\pi-\theta)$$



Collins, Soper, 1982
Moult, HXZ, 1801.02627

All logarithmic order resummation formula

$$\lim_{\theta\to\pi} \mathrm{EEC}(\theta) = H(Q) J(\theta) \otimes J(\theta) \otimes S(\theta)$$

Jet function

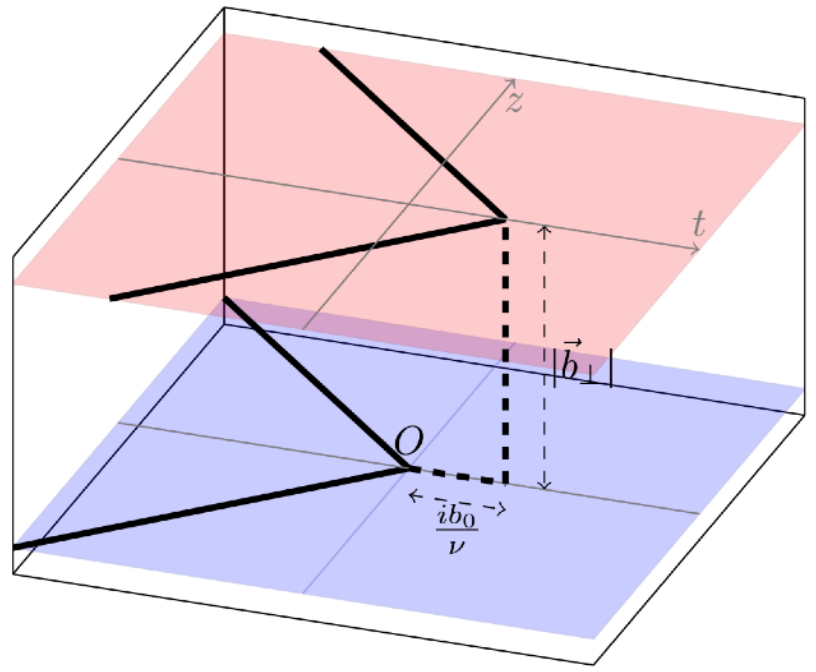
$$J(b_\perp) = \sum_N \int_0^1 dz\, z \mathcal{D}_{N/q}(z, b_\perp)$$

first moment of TMD fragmentation function

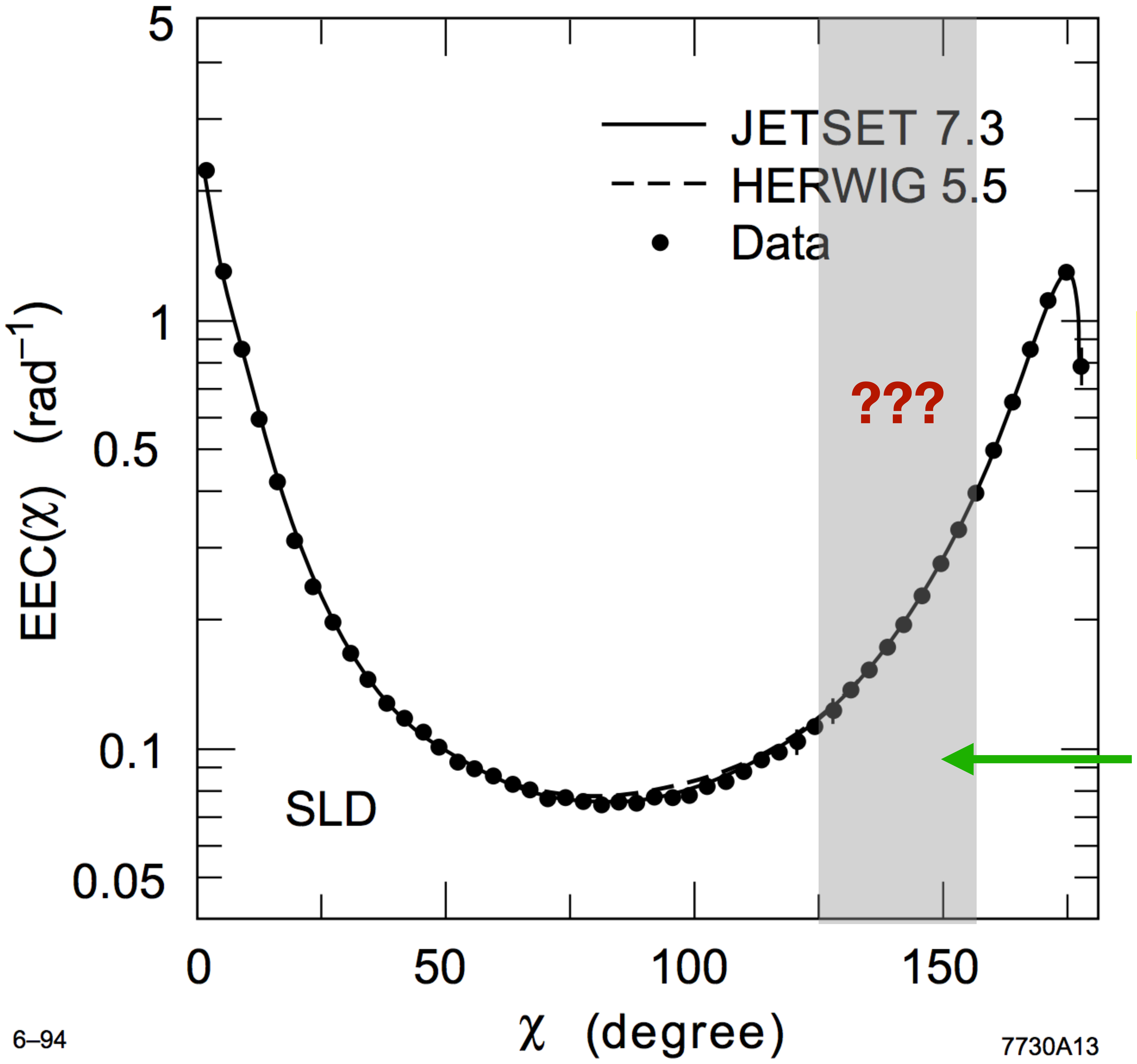
$$\mathcal{D}^{\text{bare}}_{N/q}(z, b_\perp) = \frac{1}{z}\sum_X \int \frac{db^-}{2\pi} e^{iP^+b^-/z} \langle 0|\bar{\chi}_n(0, b^-, b_\perp)|N(P), X\rangle \frac{\bar{\not{n}}}{2} \langle N(P), X|\chi_n(0)|0\rangle$$

$$S_{\text{EEC}}(\vec{b}_\perp, \mu, \nu) = \lim_{\nu\to+\infty} \frac{1}{N_c} \text{tr}\langle 0|T\left[S^\dagger_{\bar{n}+}(0) S_{n-}(0)\right] \bar{T}\left[S^\dagger_{n+}\left(y_\nu(\vec{b}_\perp)\right) S_{\bar{n}-}\left(y_\nu(\vec{b}_\perp)\right)\right]|0\rangle$$

TMD soft function



How to transit from resummation to fixed order?



Requires Sudakov resummation of power corrections, a hard problem.

Power corrections are important

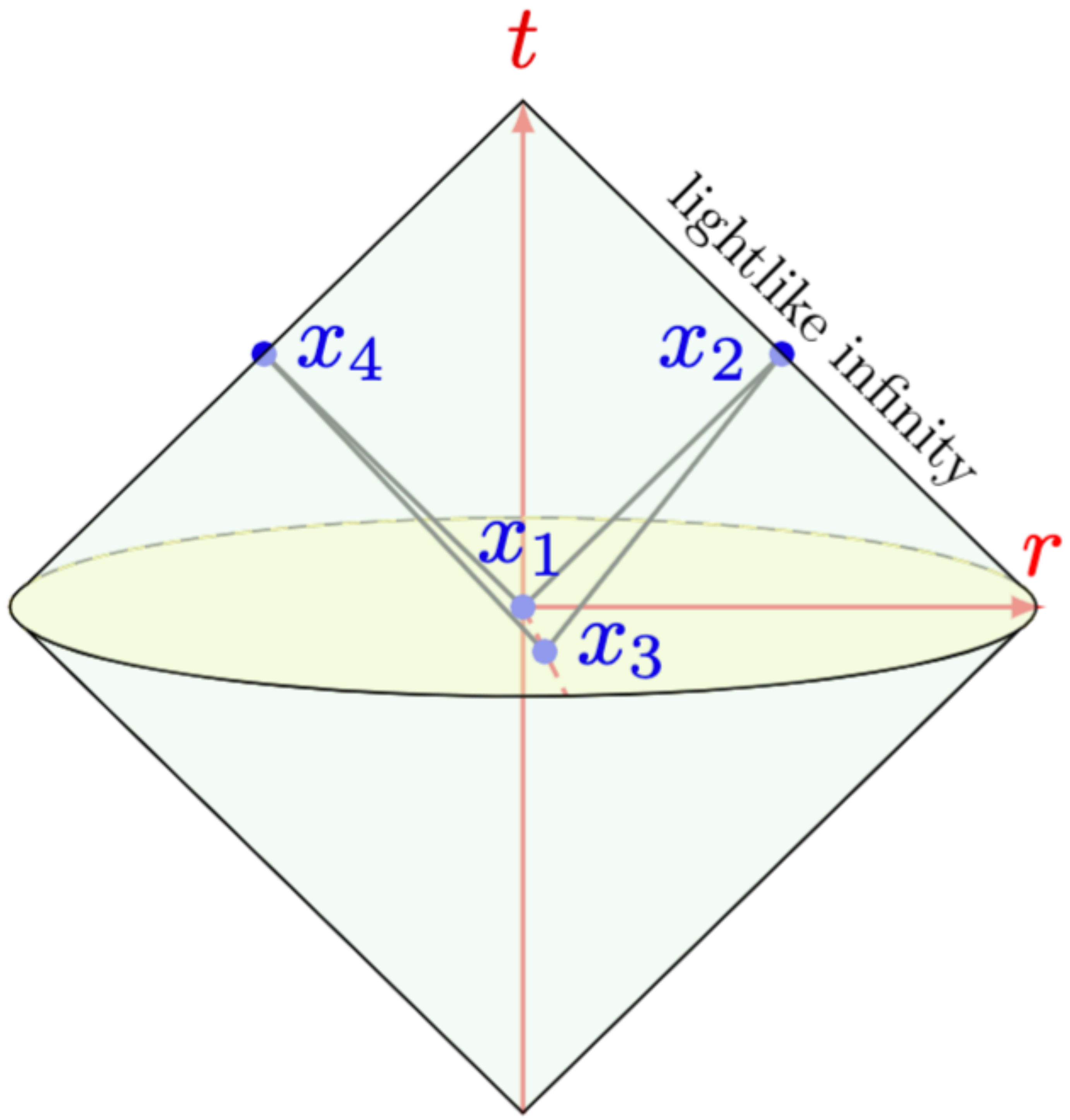
Spacetime picture of Sudakov divergences

Chen, Zhou, HXZ, 2301.03616

$\int d^4x_{13} e^{iq\cdot x_{13}}$ + detector time integral $\langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3)\rangle$

q: virtual photon momentum

$x_1 = 0, \quad x_2 = (t_2, r\vec{n}_2), \quad x_4 = (t_4, r\vec{n}_4)$



Sudakov limit ⇔ Double lightcone limit

$$x_{12}^2,\ x_{23}^2 \ll x_{13}^2$$

Origin of double logarithms:

1. Ordinary lightcone OPE $x_{12}^2 \to 0$
2. Summing over infinite number of operator exchange

$$\bar{\psi}\gamma^+(D^+)^{J-1}\psi$$

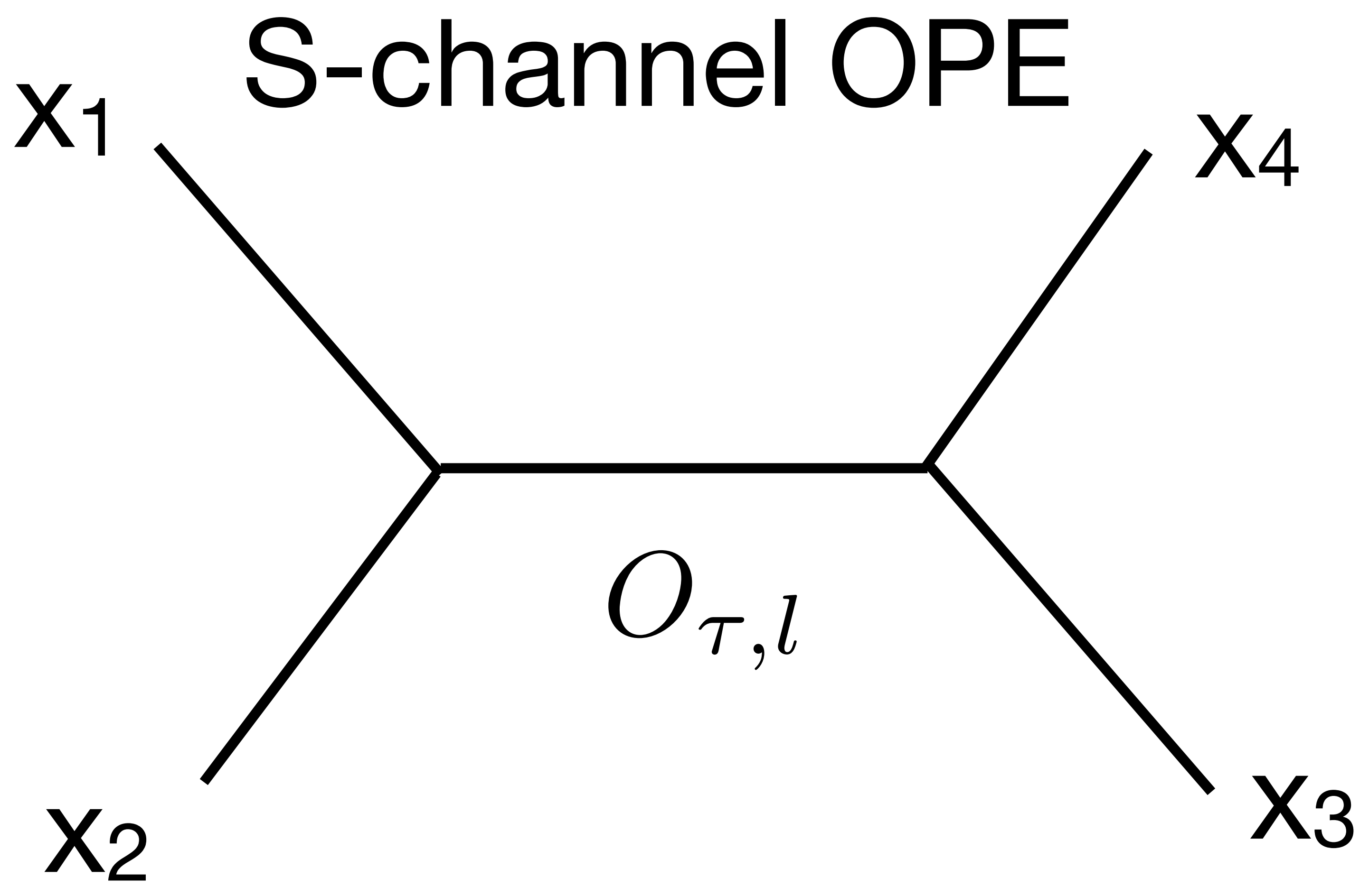
Resumption through infinite operator summation

Chen, Zhou, HXZ, 2301.03616

$$\langle J_\mu(x_1) T^{\rho\sigma}(x_2) T^{\lambda\kappa}(x_4) J^\mu(x_3)\rangle$$

$$\sum_{\tau,l}$$

truncated in τ, but infinite in l



partial wave decomposition of 4-pt function

$$= \sum_{\tau,l} a_{\tau,l} G_{\Delta,l}(x_1,x_2,x_3,x_4)$$

4-pt conformal block

$$G_{\Delta,\ell}(u,v) = \frac{z\bar{z}}{\bar{z}-z}\left[k_{\Delta-\ell-2}(z)k_{\Delta+\ell}(\bar{z}) - (z \leftrightarrow \bar{z})\right]$$

$$k_\beta(x) = x^{\beta/2}\,{}_2F_1(\beta/2,\beta/2,\beta;x)$$

Two ways to sum over infinite spin:

1. Geometric approach: approximate the spin sum as continuous integral. Use integral representation of G and saddle approximation
2. **Algebraic approach: Expand G around large spin + recursion relation in spin expansion**

$$\mathcal{C}_\tau G_{\Delta,l}(z,\bar{z}) = J^2_{\tau,l} G_{\Delta,l}(z,\bar{z}) \qquad J^2_{\tau,\ell} = (\ell+\frac{\tau}{2})(\ell+\frac{\tau}{2}-1)$$

First TMD-like Sudakov resummation beyond leading power and leading log (in a toy model)

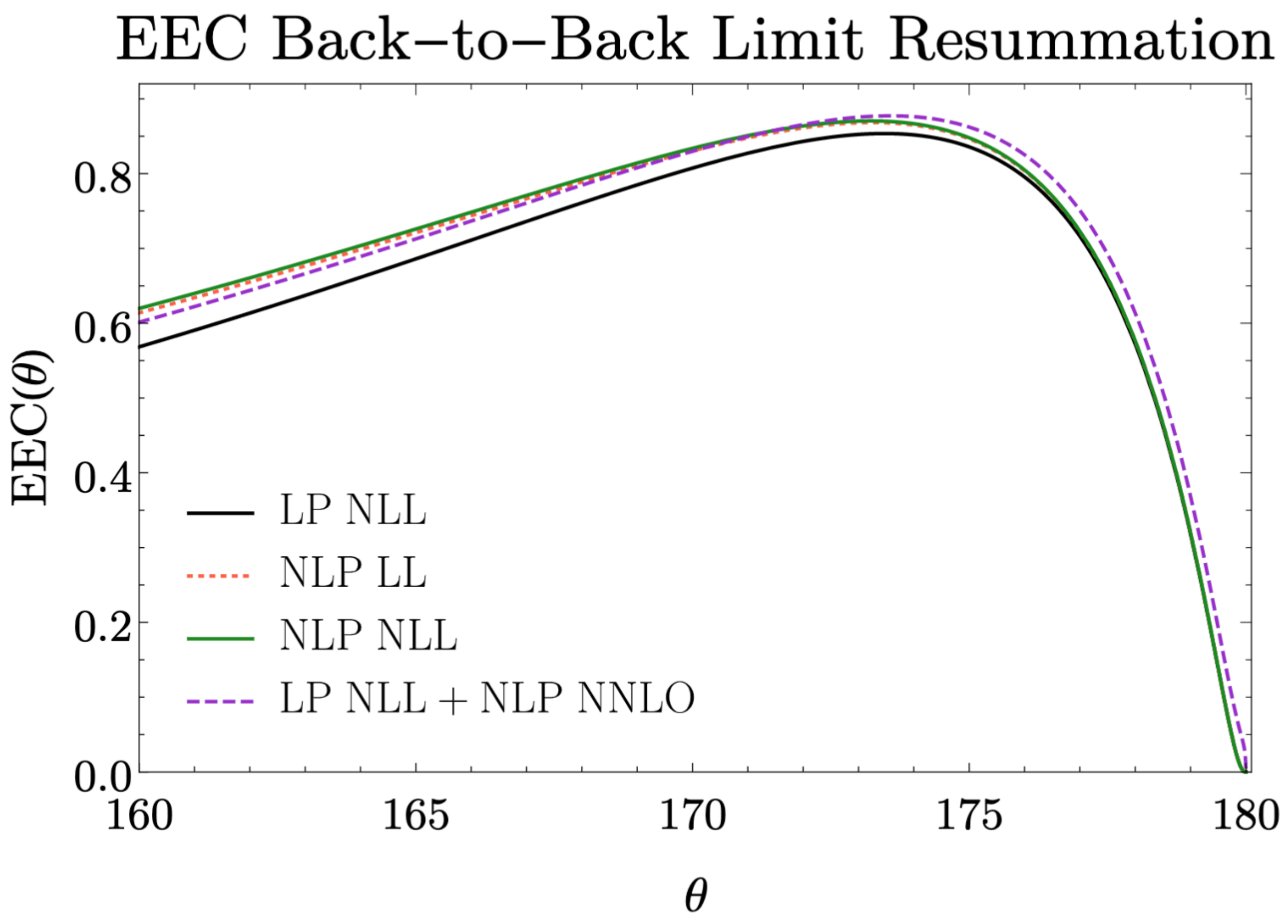
Chen, Zhou, HXZ, 2301.03616

$$y = \frac{1+\cos\theta}{2}$$

$$\mathrm{EEC}(y) = -\frac{aL_y e^{-\frac{aL_y^2}{2}}}{4y} - \frac{1}{4}\left[\sqrt{\frac{\pi}{2}}\sqrt{a}\,\mathrm{erf}\left(\sqrt{\frac{a}{2}}L_y\right) + aL_y e^{-\frac{aL_y^2}{2}}\right] + \frac{a}{48}(7aL_y^2-4)e^{-\frac{aL_y^2}{2}} + \frac{a}{12} + \cdots$$

Leading power

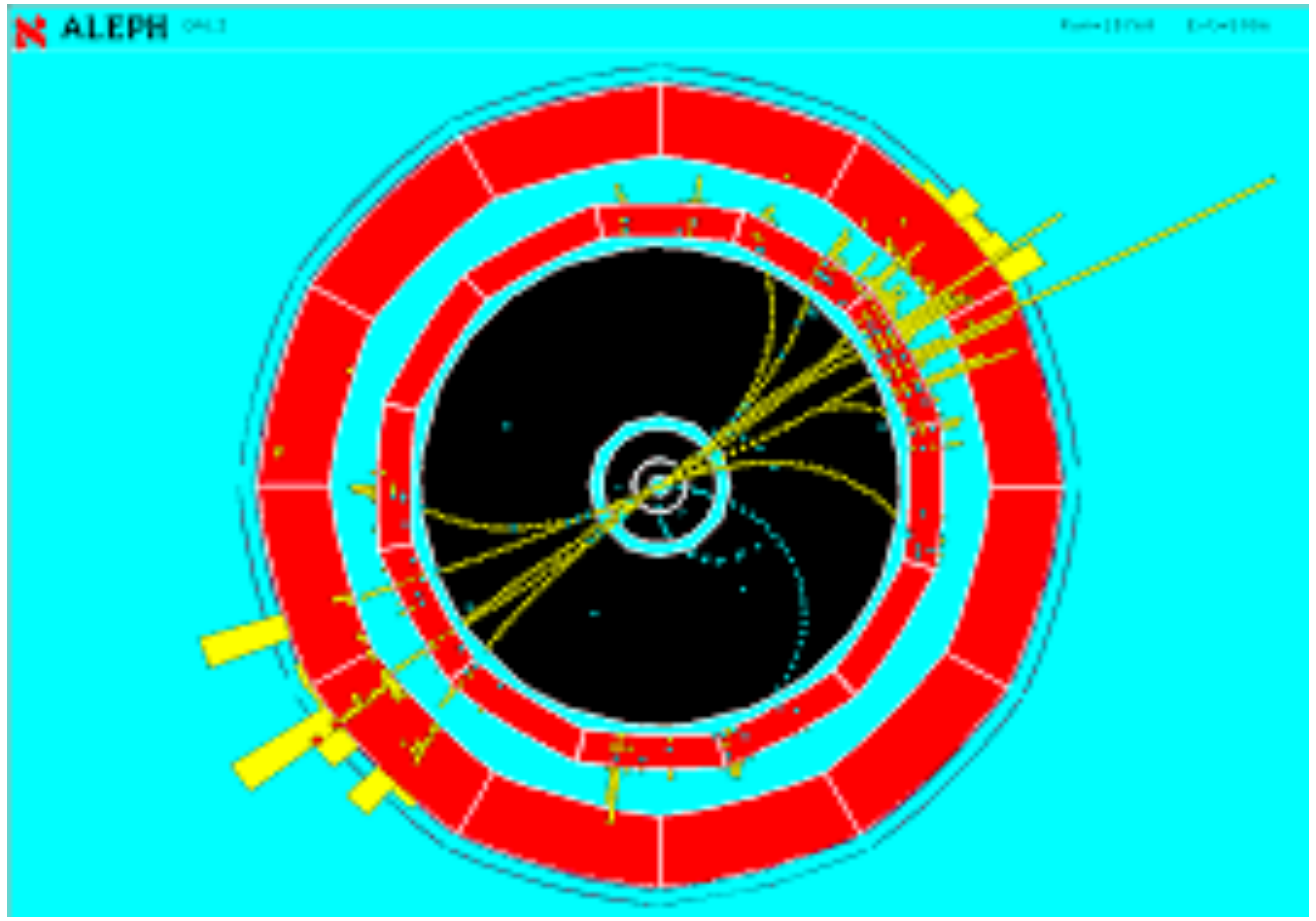
Next-to-leading power $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt$



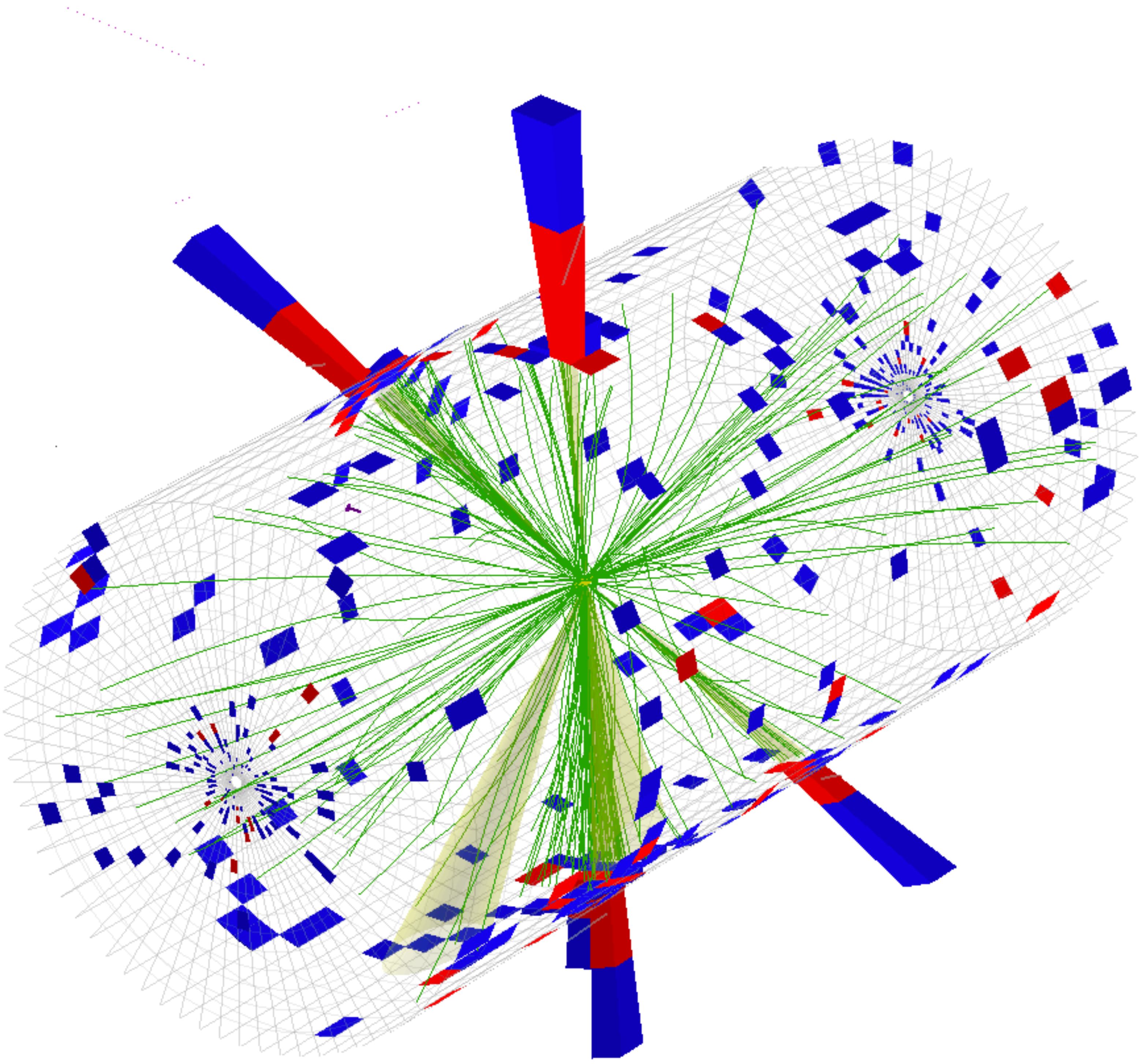
Currently only N=4 SYM, but promising for QCD

Many interesting things to explored, stay tune!

From ee to pp collider



Jet at the LEP ~ 50 GeV
short window of QCD evolution
small number of particles

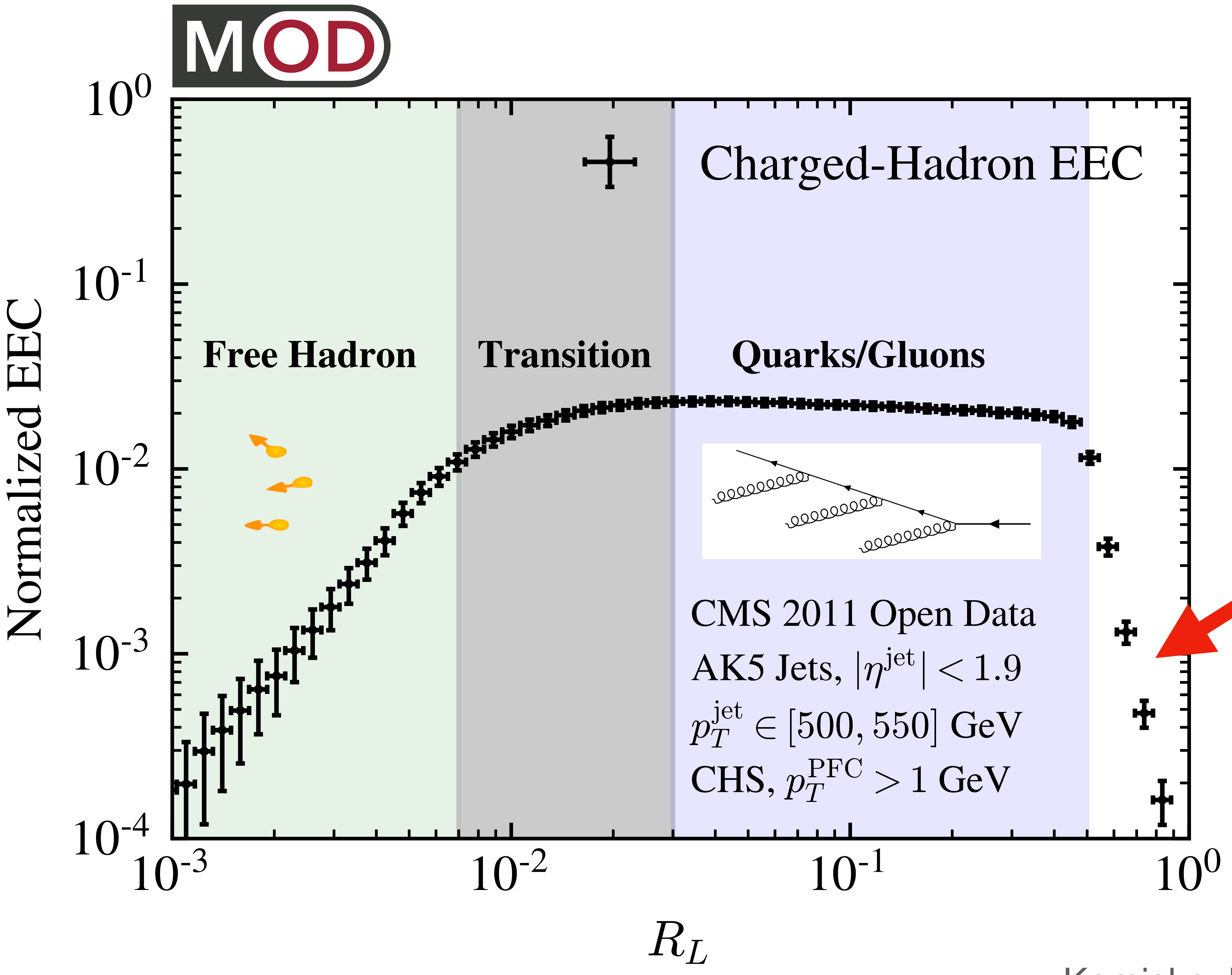


Jet at the LHC ~ TeV
large window of evolution
large number of particles
perturbation works very well

Probing the time evolution of jet through CMS open data

$$\langle \mathcal{E}(n_1)\mathcal{E}(n_2)\rangle_\Psi\,, \quad \vec{n}_1\cdot\vec{n}_2 = \cos R_L$$

NOT REAL EXPERIMENTAL MEASUREMET



Correlation energy scale:

$$P_T R_L$$

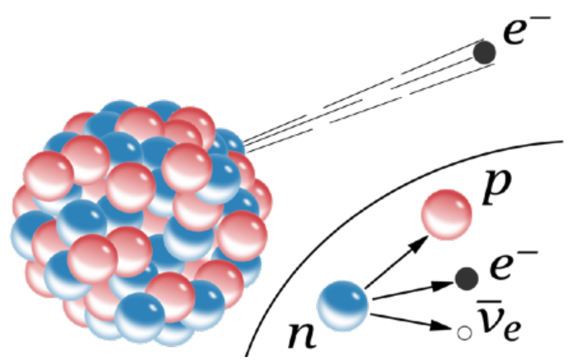
unphysical from jet clustering algorithm

Komiske, Moult, Thaler, HXZ, 2201.07800

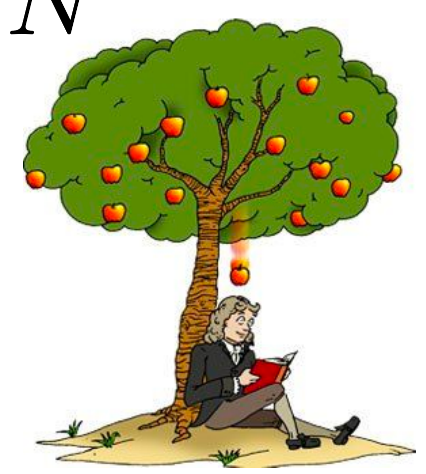
Precision measurement for α_s

$$\frac{\delta\alpha}{\alpha} \simeq 10^{-10}$$

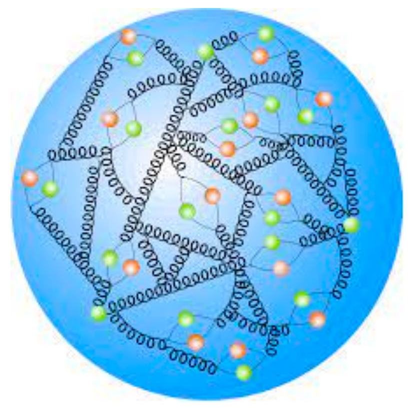
$$\frac{\delta G_F}{G_F} \simeq 10^{-8}$$



$$\frac{\delta G_N}{G_N} \simeq 10^{-5}$$



$$\frac{\delta \alpha_S}{\alpha_S} \simeq 0.01$$



Largest uncertainty!

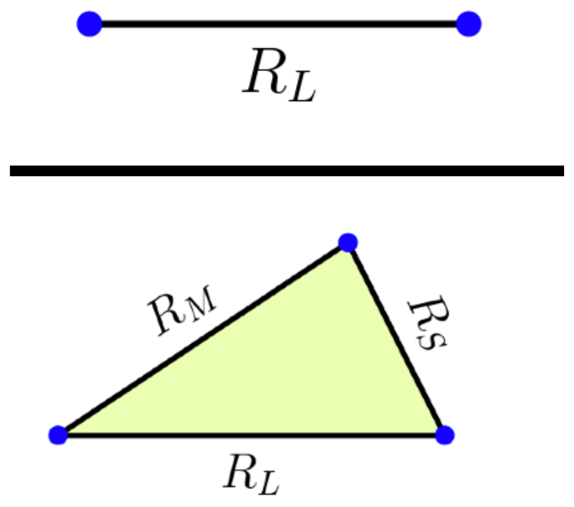
Can we precisely measure α_s from jet substructure?

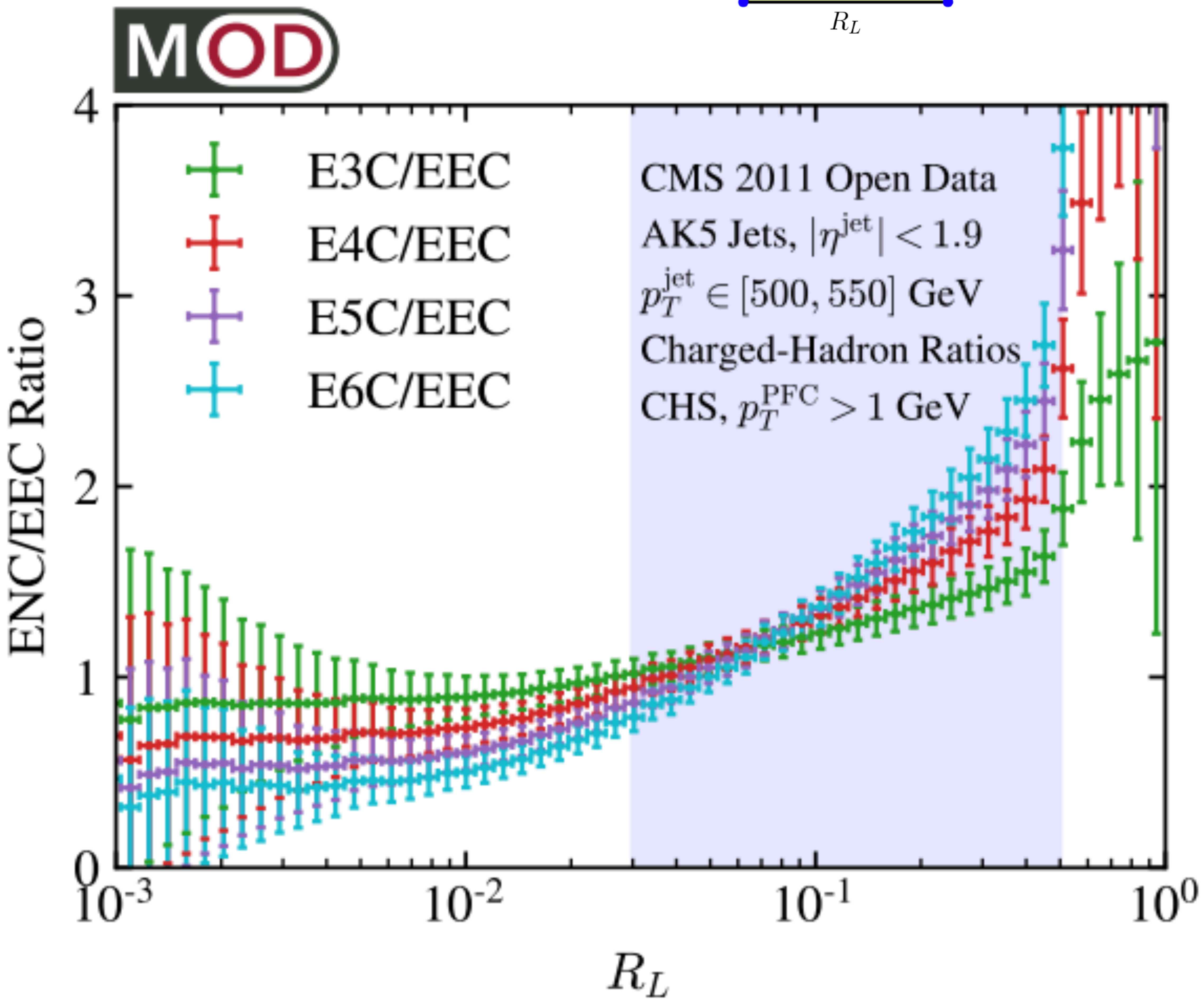
Prospects for strong coupling measurement at hadron colliders using soft-drop jet mass

Holmfridur S. Hannesdottir,[a,b] Aditya Pathak,[c,d,e] Matthew D. Schwartz,[b] Iain W. Stewart[f]

and emphasize that experimentally distinguishing quark and gluon jets is not required for an α_s measurement. We conclude that measuring α_s to the 10% level is feasible now, and with improvements in theory a 5% level measurement is possible. Getting down to the 1% level to be competitive with other state-of-the-art measurements will be challenging.

Projected energy correlators

E3C/EEC = 



- Slope related to anomalous dimension of twist 2 operator $\gamma(\alpha_s)$
- Many non-perturbation effects cancel out:
  - PDFs
  - hadronization corrections
  - hard scattering scale dependence
  - Experiment can achieve sub-percent accuracy. The only bottleneck: theory prediction!

Chen, Moult, Zhang, HXZ, 2004.11381

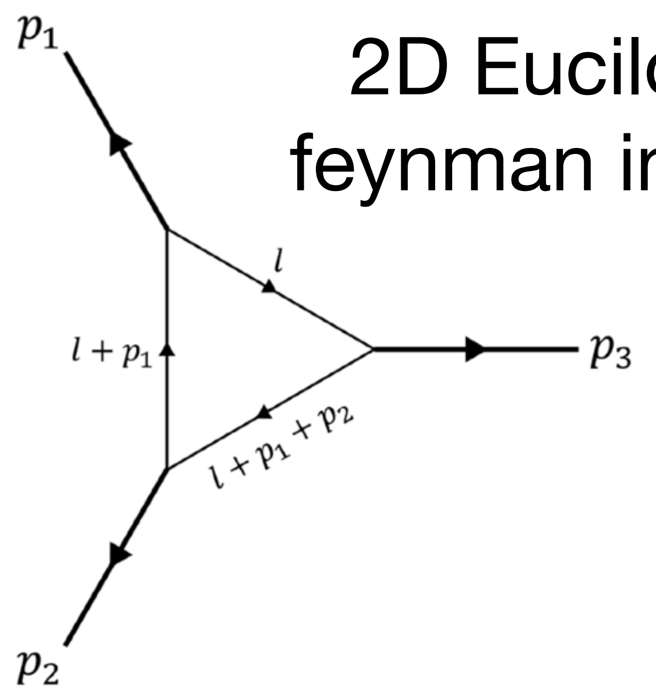
NNLL$_{\text{approx}}$ now available!

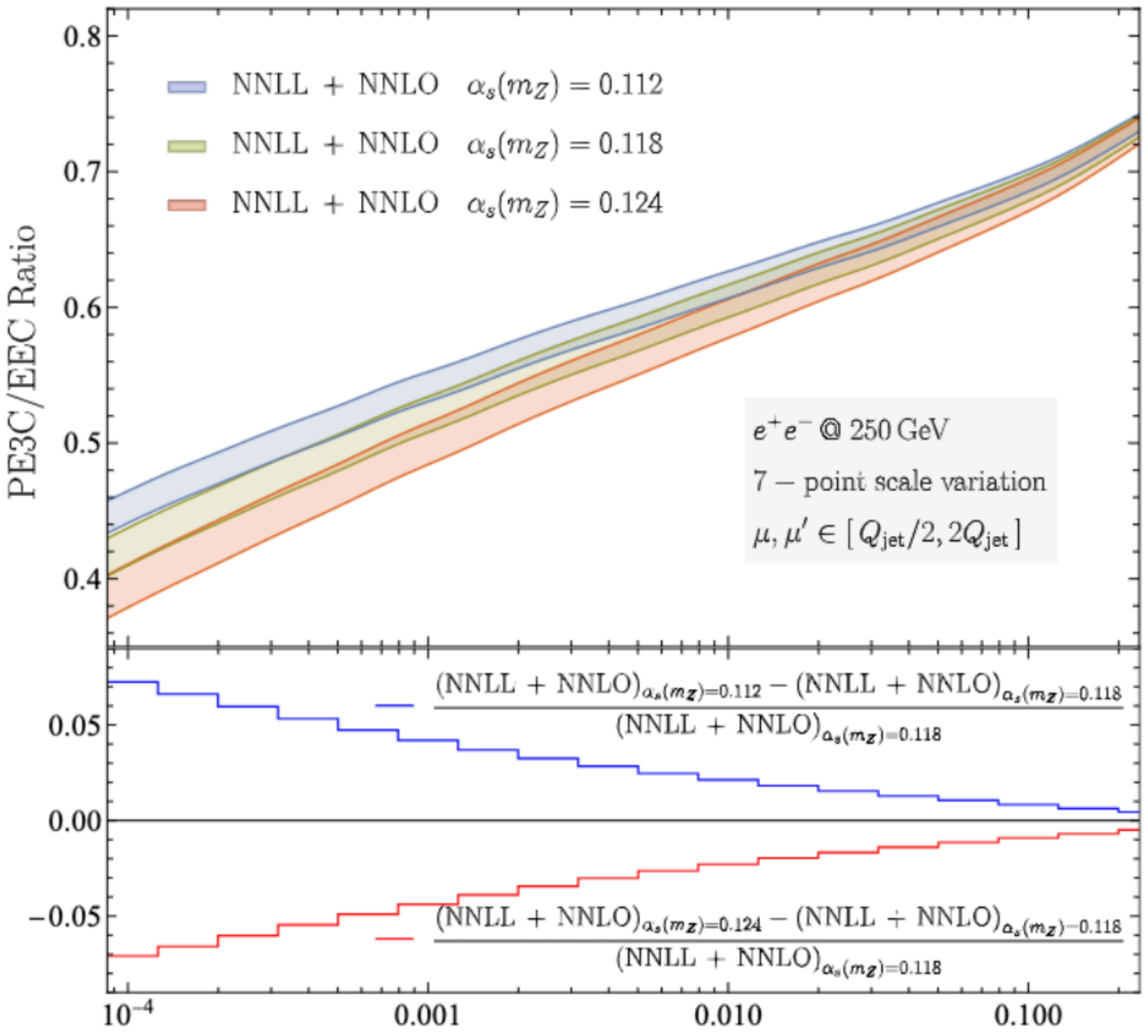
$$\frac{d\vec{J}^{[N]}(\ln\frac{x_L Q^2}{\mu^2})}{d\ln\mu^2} = \int_0^1 dy\, y^N \vec{J}^{[N]}(\ln\frac{x_L y^2 Q^2}{\mu^2})\cdot\widehat{P}(y)$$

jet function need to know beyond NLO!

$$J_q(x_1,x_2,x_3,Q,\mu^2) = \int\frac{dl^+}{2\pi}\frac{1}{2N_C}\mathrm{Tr}\int d^4x e^{il\cdot x}\langle 0|\frac{\not{\bar{n}}}{2}\chi_n(x)\widehat{\mathcal{M}}_{\mathrm{EEEC}}\,\delta(Q+\bar{n}\cdot\mathcal{P})\delta^2(\mathcal{P}_\perp)\bar{\chi}_n(0)|0\rangle$$

4D Minkowskian phase space integral ⟷ 2D Eucildean feynman integral

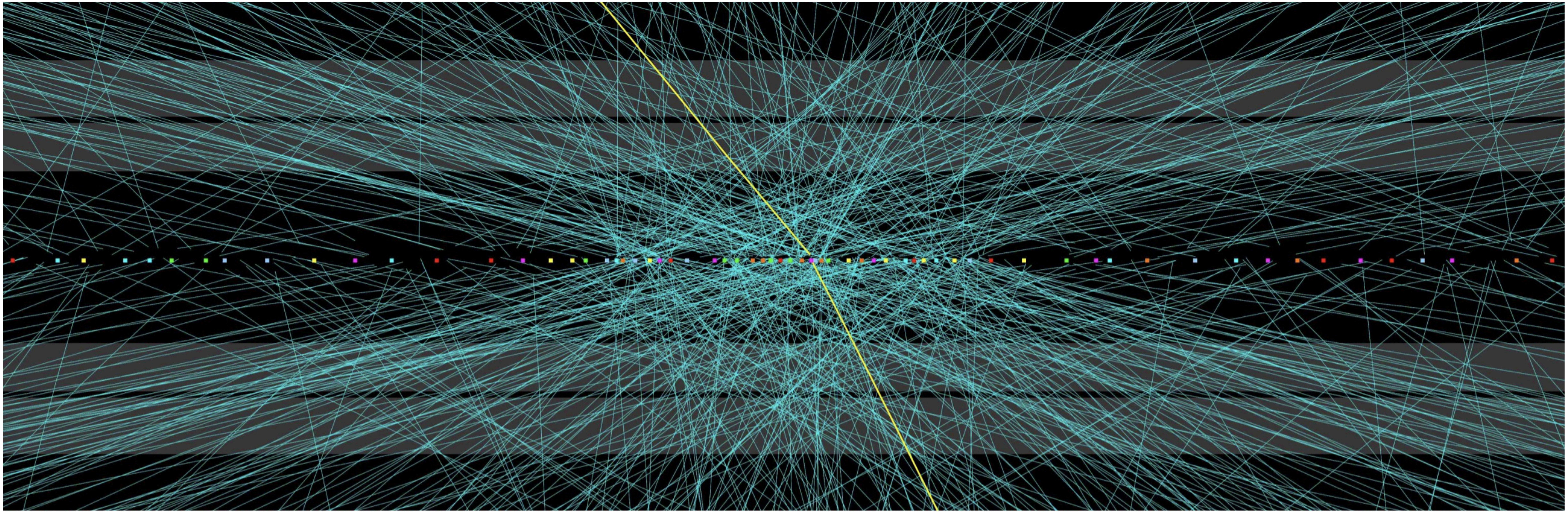




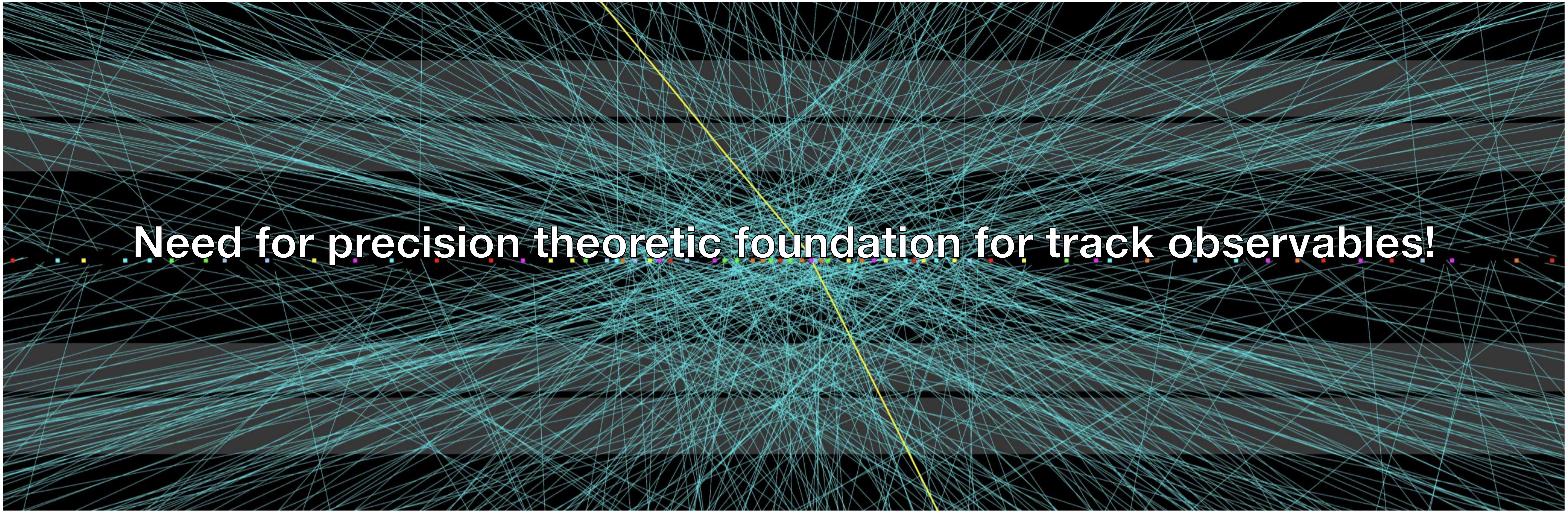
5% accuracy on α_s from jet substructure feasible now
1% possible with further theory improvement

Chen, Gao, Li, Xu, Zhang, HXZ, 2306.xxxxx

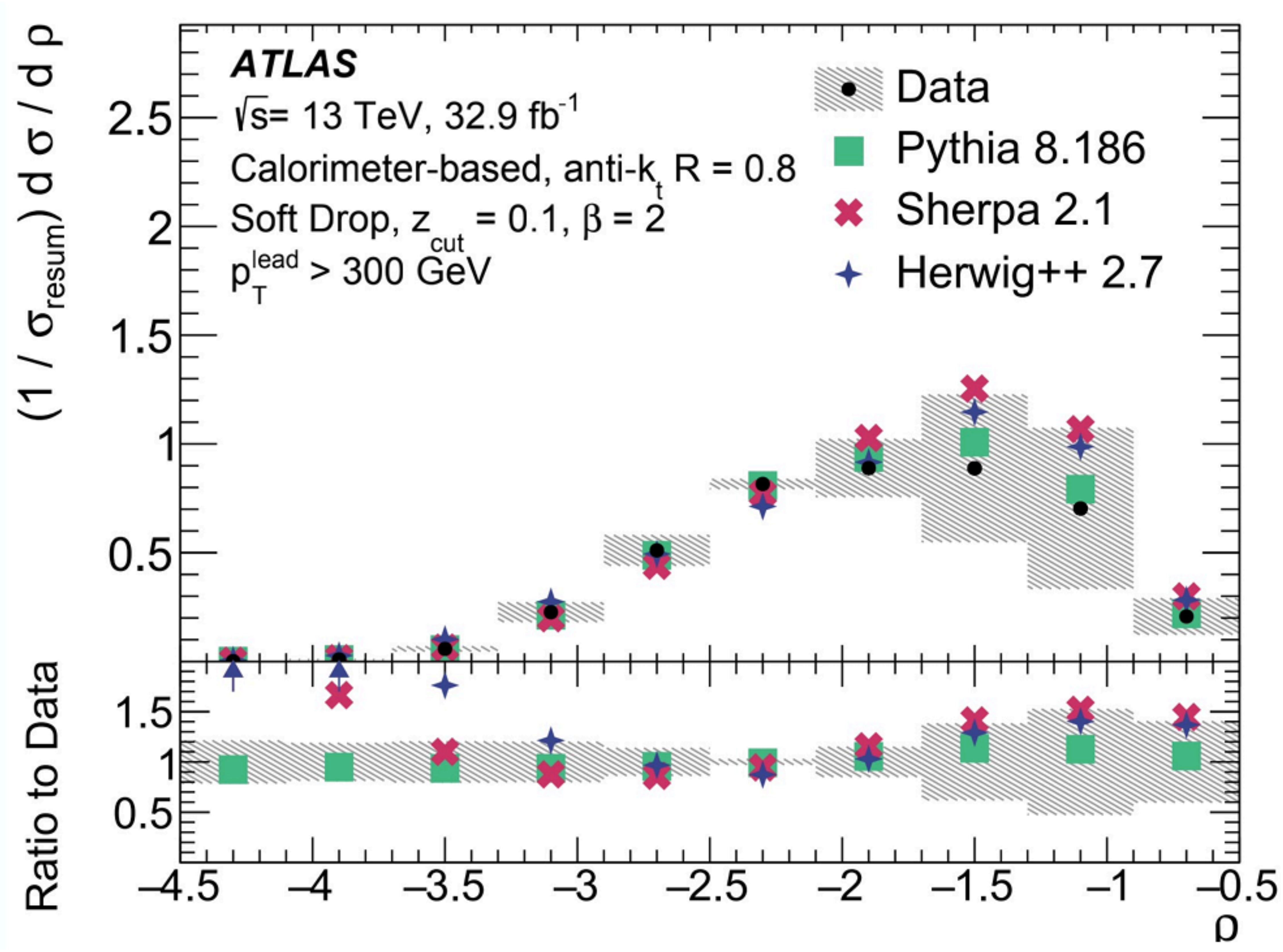
High luminosity LHC



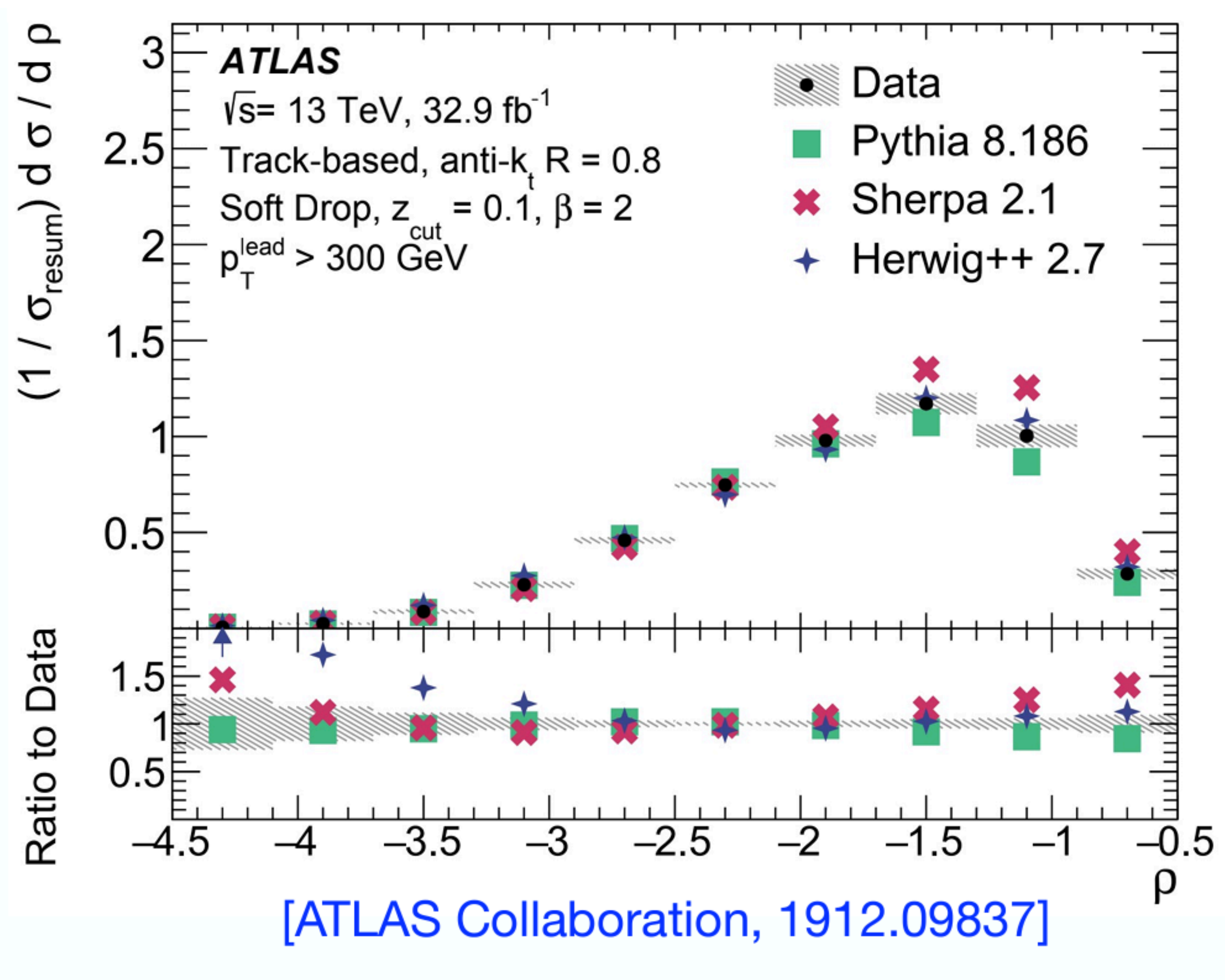
High luminosity LHC



Superior accuracy from track-based observable

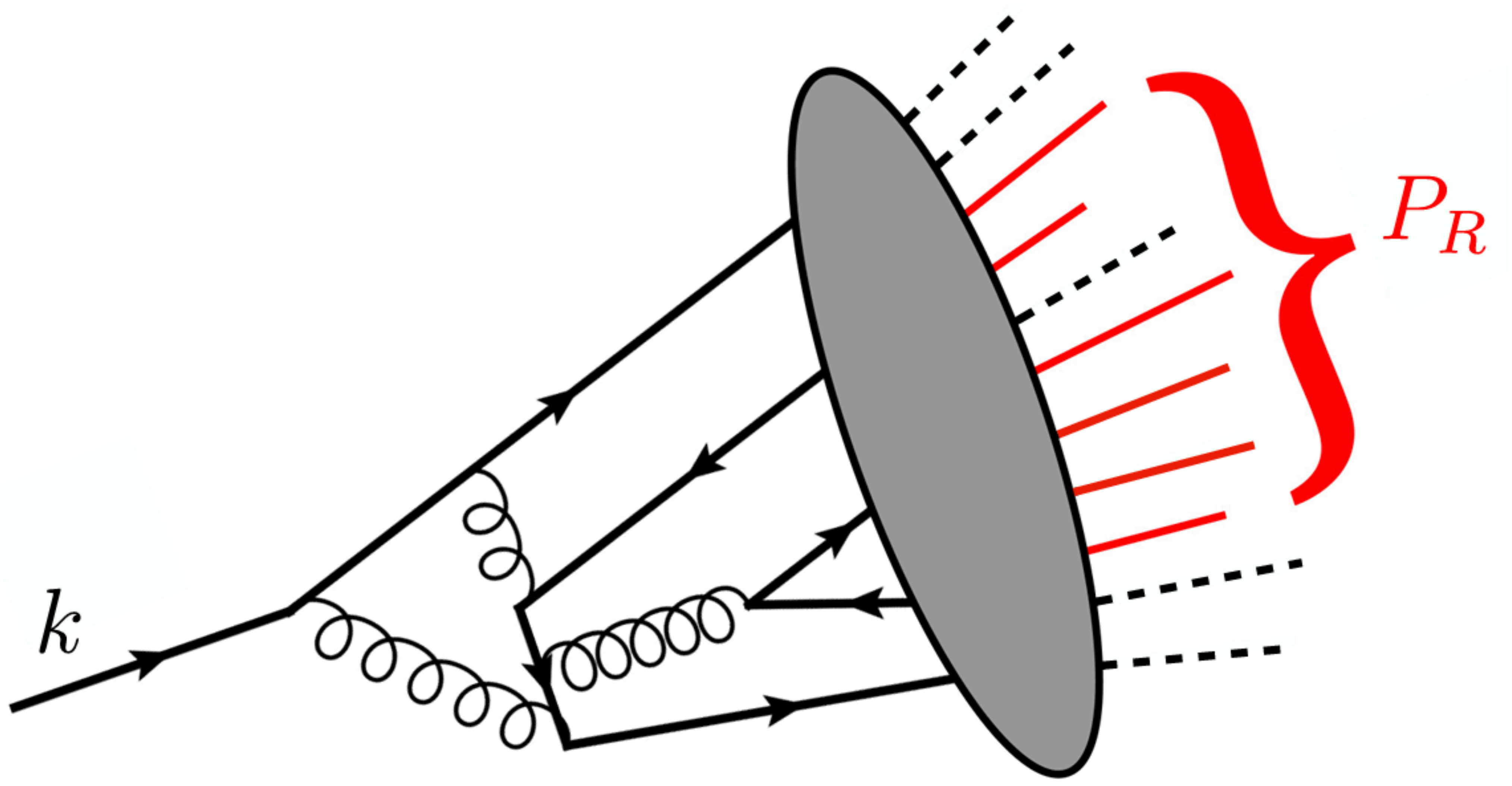


calorimeter-based observable



track-based observable

Track function



$$T_q(x) = \int \mathrm{d}y^+ \mathrm{d}^{d-2} y_\perp e^{ik^- y^+/2} \sum_X \delta\left(x - \frac{P_R^-}{k^-}\right)$$
$$\frac{1}{2N_c} \mathrm{tr}\left[\frac{\gamma^-}{2} \langle 0|\psi(y^+, 0, y_\perp)|X\rangle\langle X|\bar{\psi}(0)|0\rangle\right]$$

- Involving multi-particle correlation
- Straightforward generalization to other quantum number
- Complex evolution equation, previously only known at LO

$$\frac{d}{d\ln\mu^2} T_i(x,\mu) = a_s(\mu) \sum_{j,k} \int dz dx_1 dx_2 P^{(0)}_{i\to jk}(z_1, z_2)\delta(1 - z_1 - z_2)$$
$$\times T_j(x_1,\mu) T_k(x_2,\mu) \delta[x - z_1 x_1 - z_2 x_2] \; .$$

Chang, Procura, Thaler, Waalewijn, 1303.6637

EEC on tracks

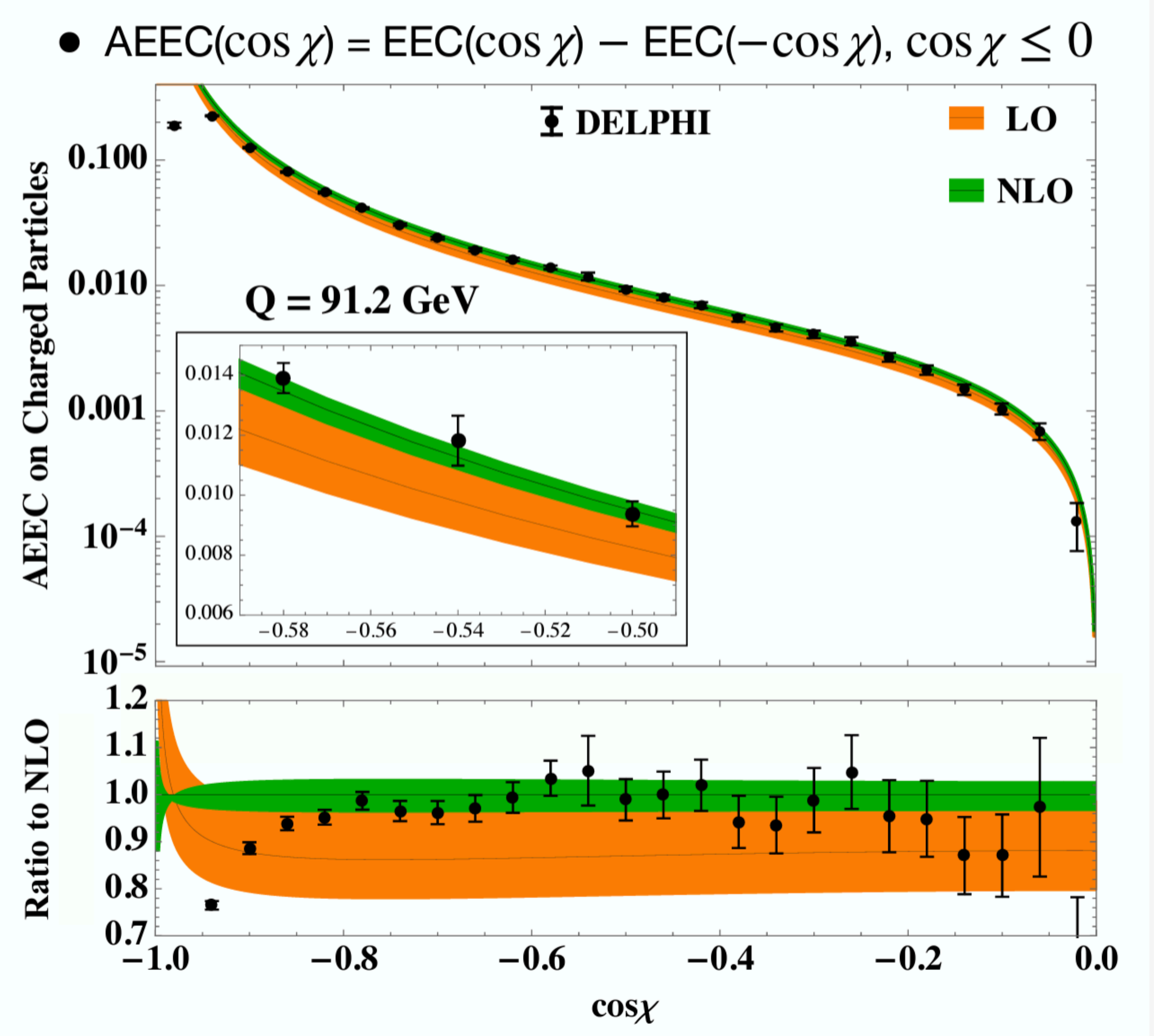
$$\frac{d}{d\ln\mu^2}T_i(x,\mu) = a_s(\mu)\sum_{j,k}\int dz\,dx_1\,dx_2\,P^{(0)}_{i\to jk}(z_1,z_2)\,\delta(1-z_1-z_2)$$
$$\times\, T_j(x_1,\mu)T_k(x_2,\mu)\,\delta[x - z_1x_1 - z_2x_2]\,.$$

$$\int_0^1 dx\, x \qquad\qquad \int_0^1 dx_1 dx_2\,(x_1z_1+x_2z_2)$$

$$\frac{d}{d\ln\mu^2}T_g(1) = -\gamma^{(1)}_{gg}(2)T_g(1) + \sum_q\left(-2\gamma^{(1)}_{qg}(2)\right)T_q(1)\,,$$

$$\frac{d}{d\ln\mu^2}T_g(2) = -\gamma^{(1)}_{gg}(3)T_g(2) + \sum_q\left(-2\gamma^{(1)}_{qg}(3)\right)T_q(2) + \left[C_A^2\left(-8\zeta_3+\frac{2158}{675}+\frac{26\pi^2}{45}\right)-\frac{4}{9}C_A n_f T_F\right]T_g(1)T_g(1)+\cdots\,,$$

$$\frac{d}{d\ln\mu^2}T_g(3) = -\gamma^{(1)}_{gg}(4)T_g(3) + \sum_q\left(-2\gamma^{(1)}_{qg}(4)\right)T_q(3) + \left[C_A^2\left(24\zeta_3+\frac{767263}{4500}-\frac{278\pi^2}{15}\right)-\frac{2}{3}C_A n_f T_F\right]T_g(2)T_g(1)$$
$$+\sum_q\left[C_A T_F\left(\frac{23051}{1125}-\frac{28}{15}\pi^2\right)-C_F T_F\frac{28}{15}\right]T_g(1)T_q(1)T_q(1)+\cdots\,.$$

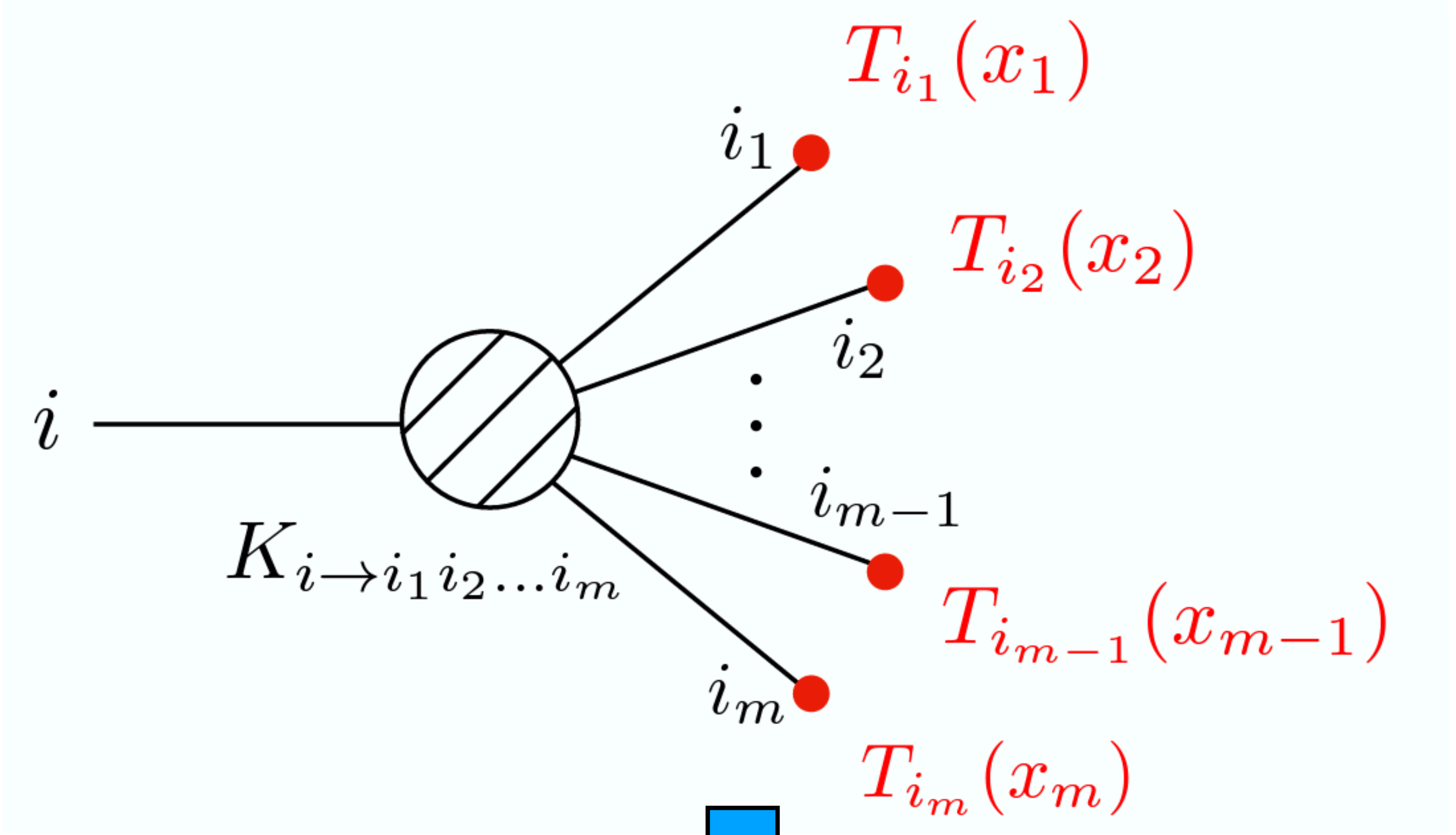


Li, Moult, van Velzen, Waalewijn, HXZ, 2108.01674

Generalized DGLAP evolution

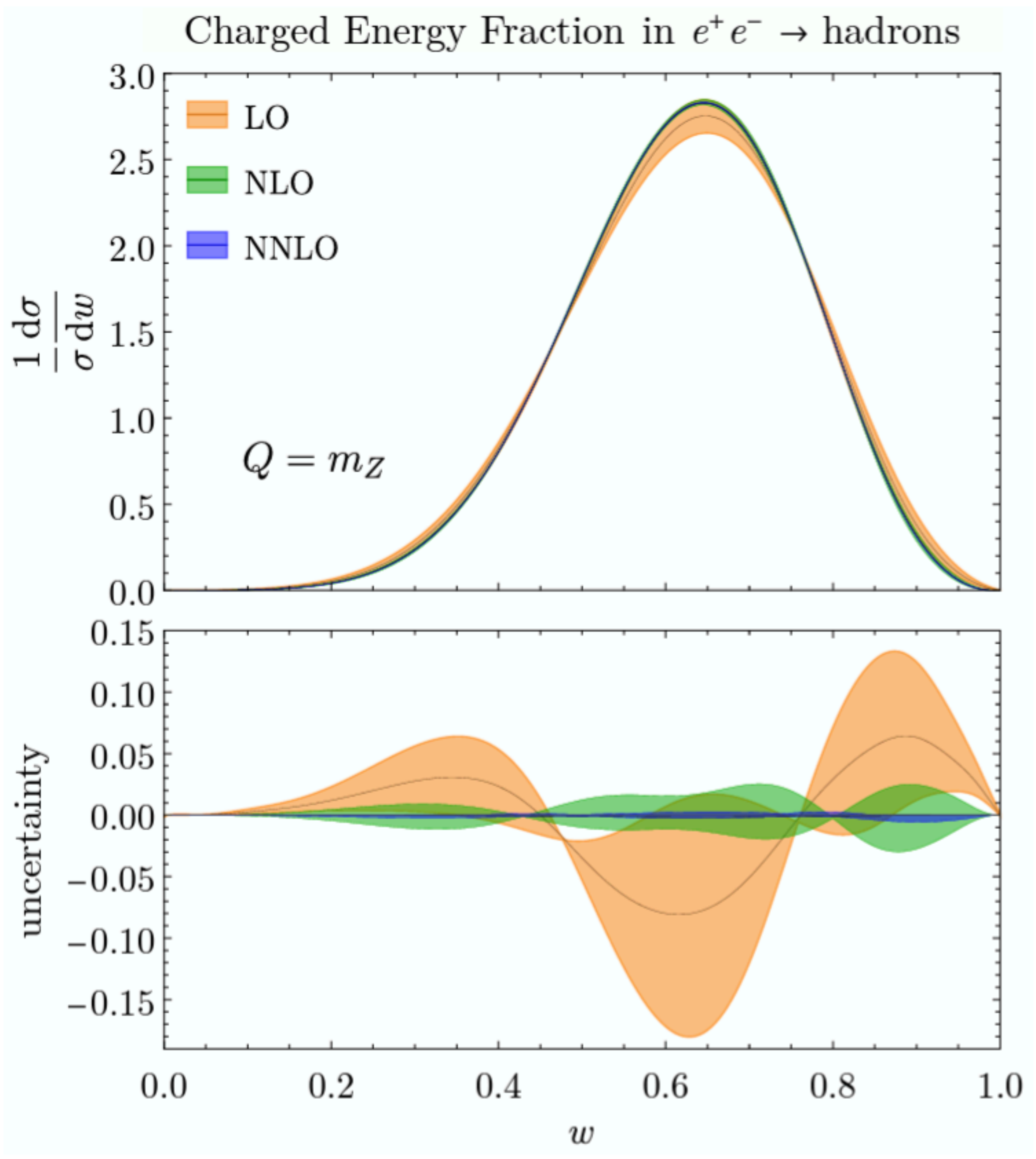
The full NLO evolution has recently been calculated, using knowledge from moment

Highly nonlinear evolution equation



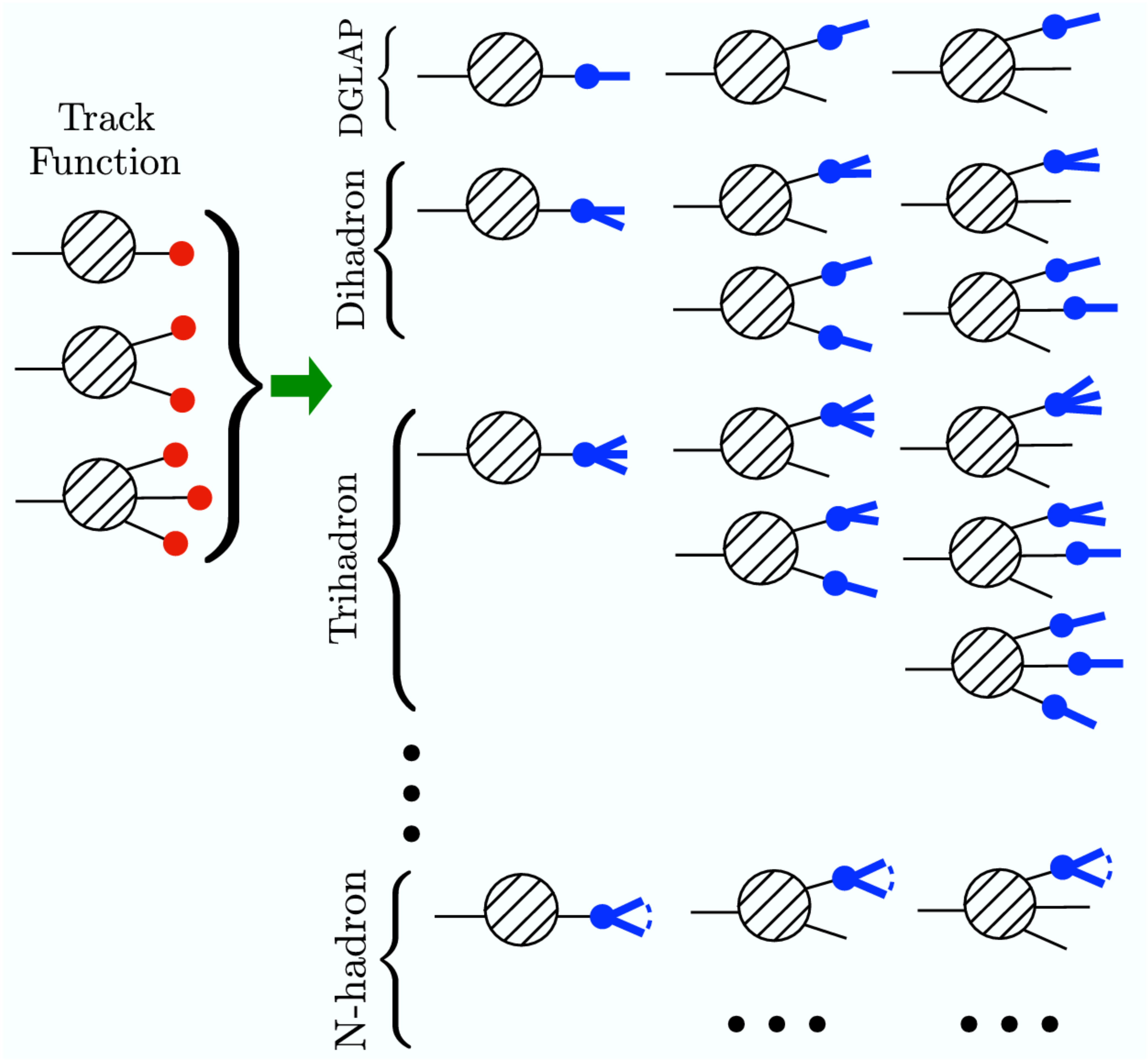
truncated to at most
1->3 kernel

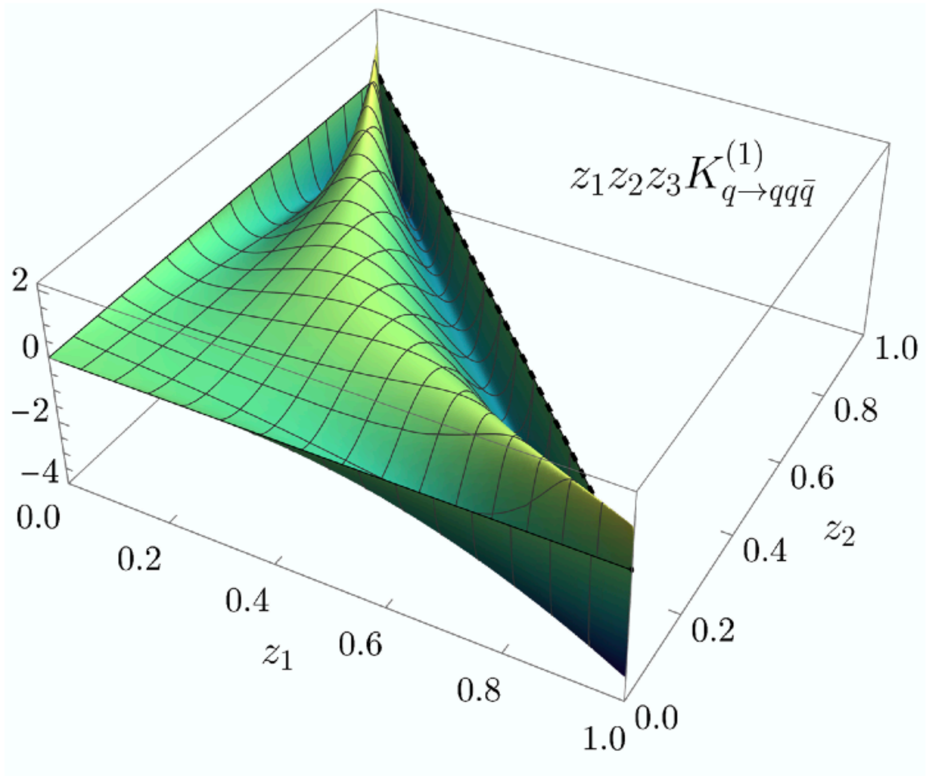
$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2}T_i(x) = a_s\Big[K^{(0)}_{i\to i}T_i(x) + K^{(0)}_{i\to i_1 i_2}\otimes T_{i_1}T_{i_2}(x)\Big] + a_s^2\Big[K^{(1)}_{i\to i}T_i(x) + K^{(1)}_{i\to i_1 i_2}\otimes T_{i_1}T_{i_2}(x) + K^{(1)}_{i\to i_1 i_2 i_3}\otimes T_{i_1}T_{i_2}T_{i_3}(x)\Big] + \mathcal{O}(a_s^3)\,,$$



Chen, Jaarsma, Li, Moult, Waalewijn, HXZ, 2210.10058, 2210.10061

NLO kernel of di-hadron fragmentation from track evolution





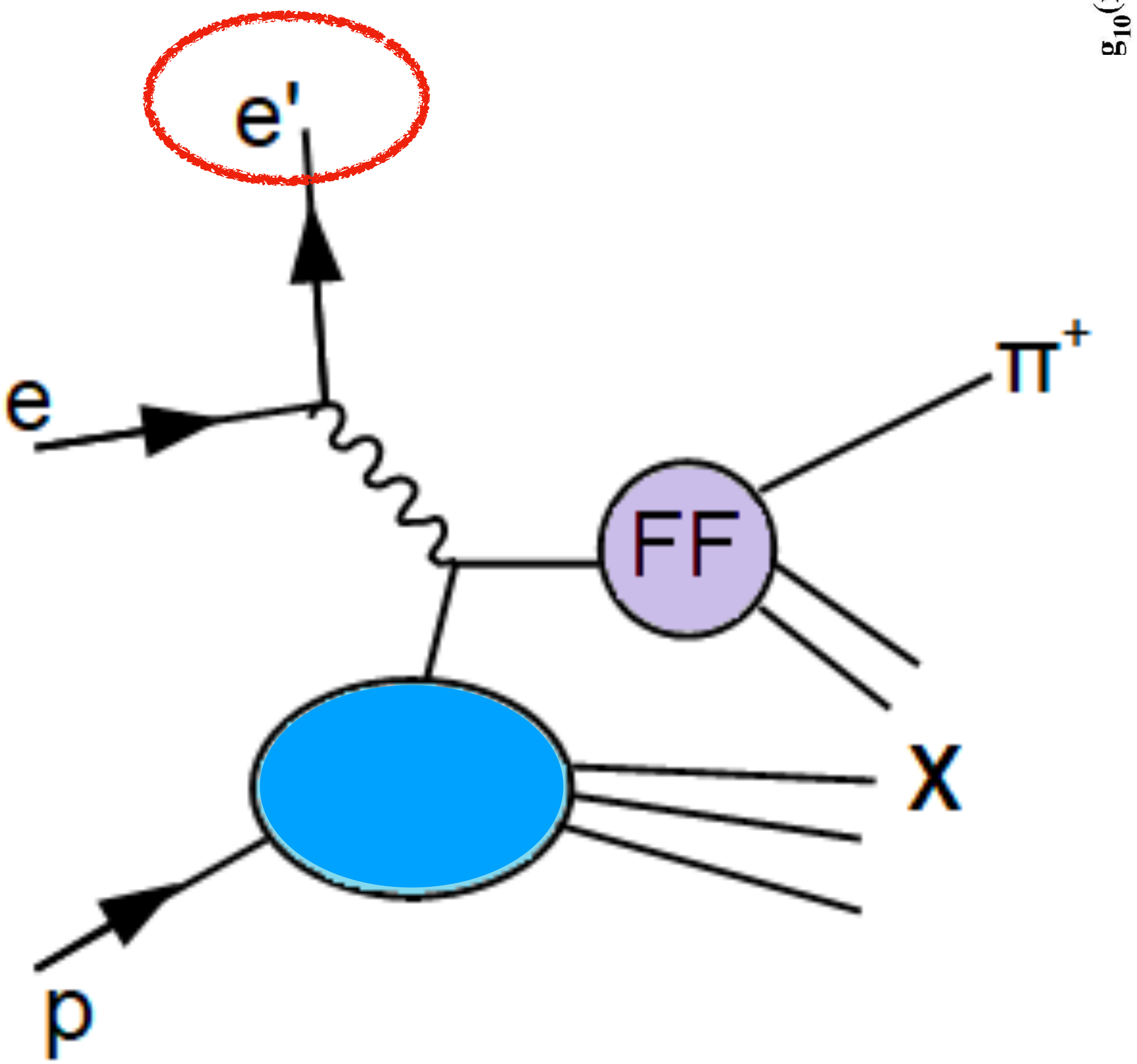
$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}\ln\mu^2} D_{i\to h_1h_2}(y_1,y_2) \\
&= a_s^2 \Big\{ K^{(1)}_{i\to i} D_{i\to h_1h_2}(y_1,y_2) \\
&+ \sum_{\{i_1i_2\}} \sum_n \int_0^1 \mathrm{d}z\, {}^nK^{(1)}_{i\to i_1i_2}(z) \Big[\int \mathrm{d}y'_{11}\mathrm{d}y'_{12} D_{i_1\to h_1h_2}(y'_{11},y'_{12}) \delta(y_1 - {}^nz_1\, y'_{11}) \delta(y_2 - {}^nz_1\, y'_{12}) \\
&+ \int \mathrm{d}y'_{21}\mathrm{d}y'_{22} D_{i_2\to h_1h_2}(y'_{21},y'_{22}) \delta(y_1 - {}^nz_2\, y'_{21}) \delta(y_2 - {}^nz_2\, y'_{22}) \Big] \\
&+ \sum_{\{i_1i_2i_3\}} \sum_n \int \mathrm{d}z\mathrm{d}t\, {}^nK^{(1)}_{i\to i_1i_2i_3}(z,t) \Big[\int \mathrm{d}y'_{11}\mathrm{d}y'_{12} D_{i_1\to h_1h_2}(y'_{11},y'_{12}) \delta(y_1 - {}^nz_1\, y'_{11}) \delta(y_2 - {}^nz_1\, y'_{12}) \\
&+ \int \mathrm{d}y'_{21}\mathrm{d}y'_{22} D_{i_2\to h_1h_2}(y'_{21},y'_{22}) \delta(y_1 - {}^nz_2\, y'_{21}) \delta(y_2 - {}^nz_2\, y'_{22}) \\
&+ \int \mathrm{d}y'_{31}\mathrm{d}y'_{32} D_{i_3\to h_1h_2}(y'_{31},y'_{32}) \delta(y_1 - {}^nz_3\, y'_{31}) \delta(y_2 - {}^nz_3\, y'_{32}) \Big] \Big\}
\end{aligned}
$$

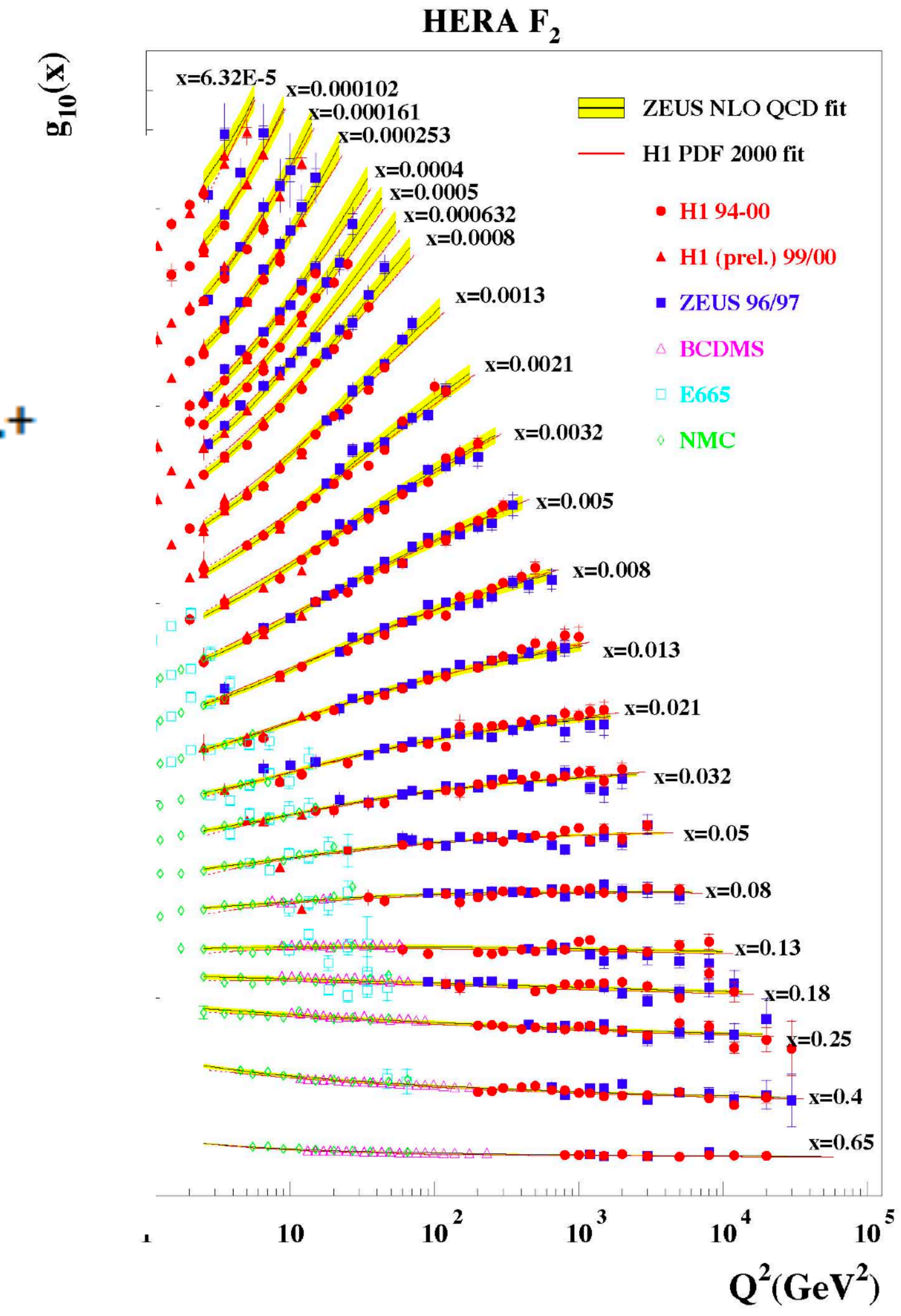
+ single hadron fragmentation evolution

Majumder, Wang, hep-ph/0402245

Chen, Jaarsma, Li, Moult, Waalewijn, HXZ, 2210.10058, 2210.10061

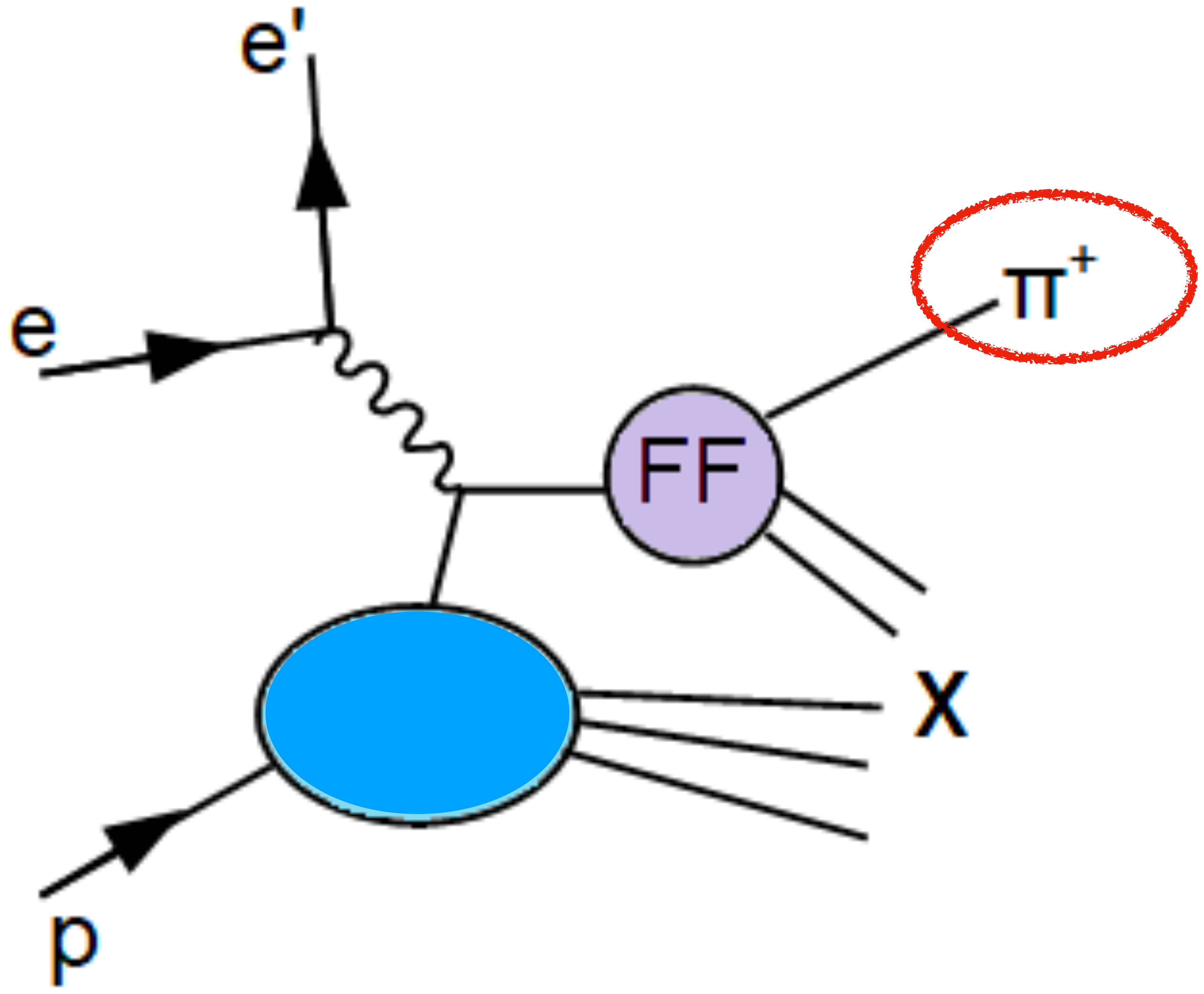
ep and nucleon structure

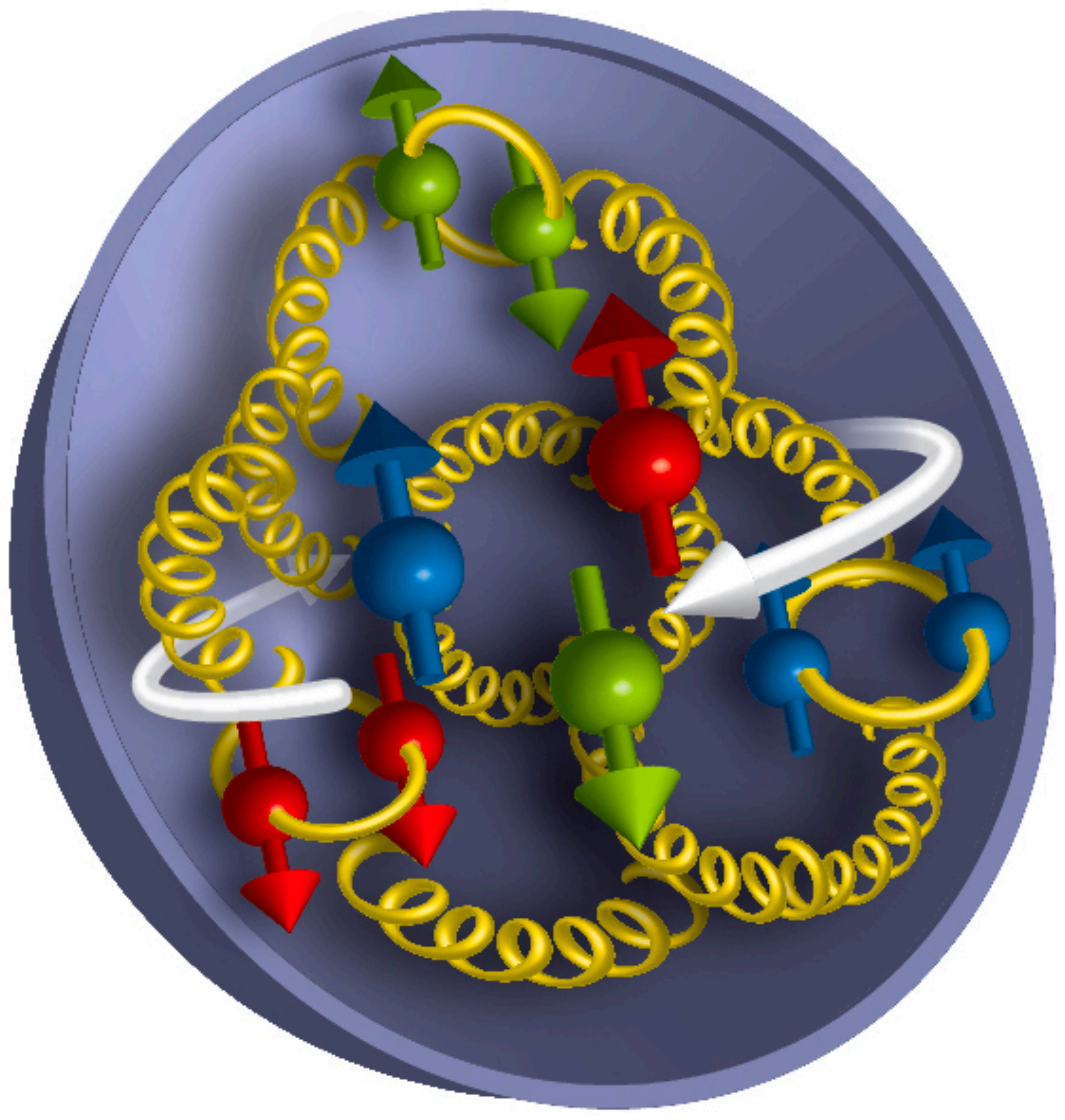




- Structure function measurement: PDFs(x)

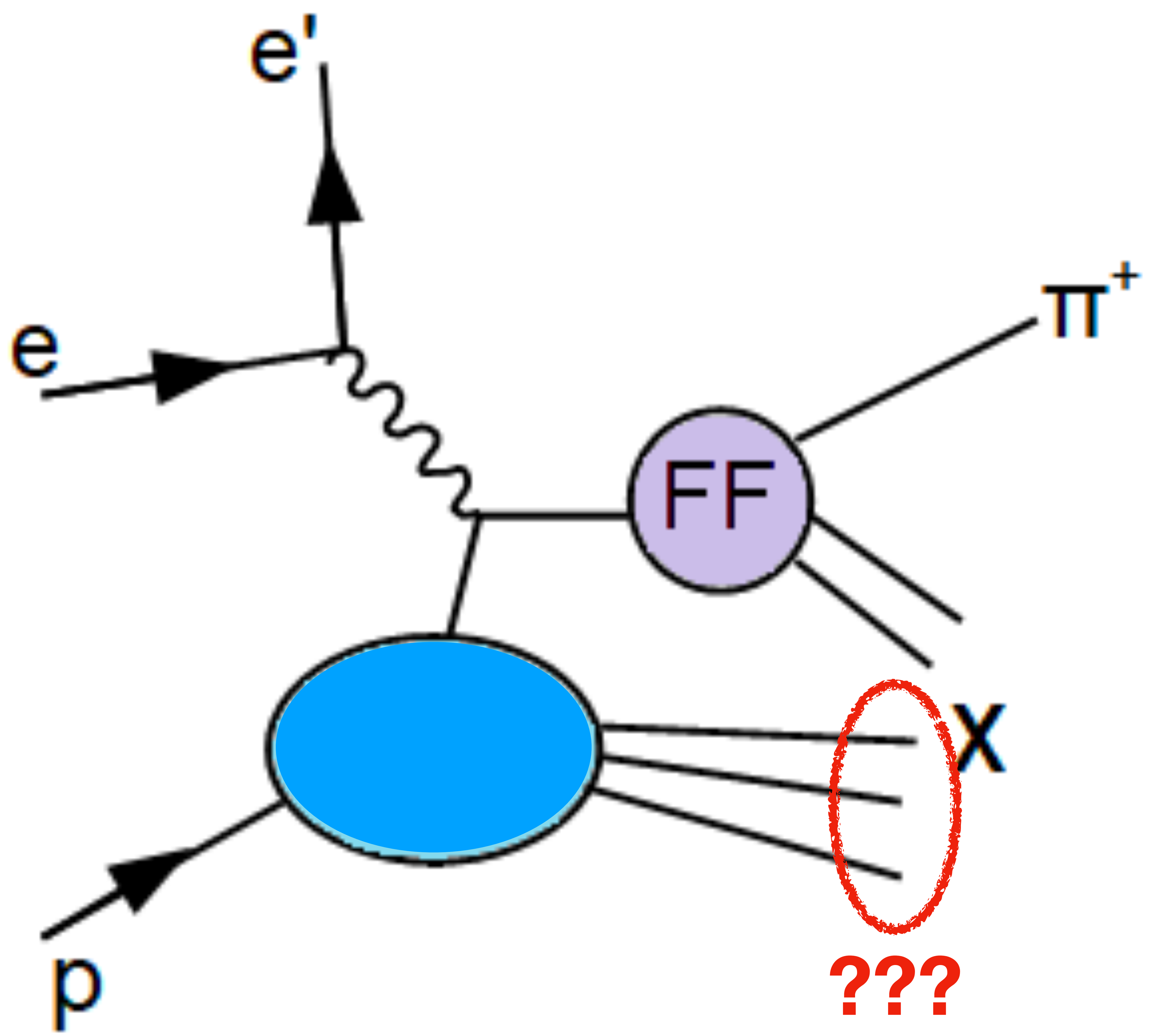
ep and nucleon structure





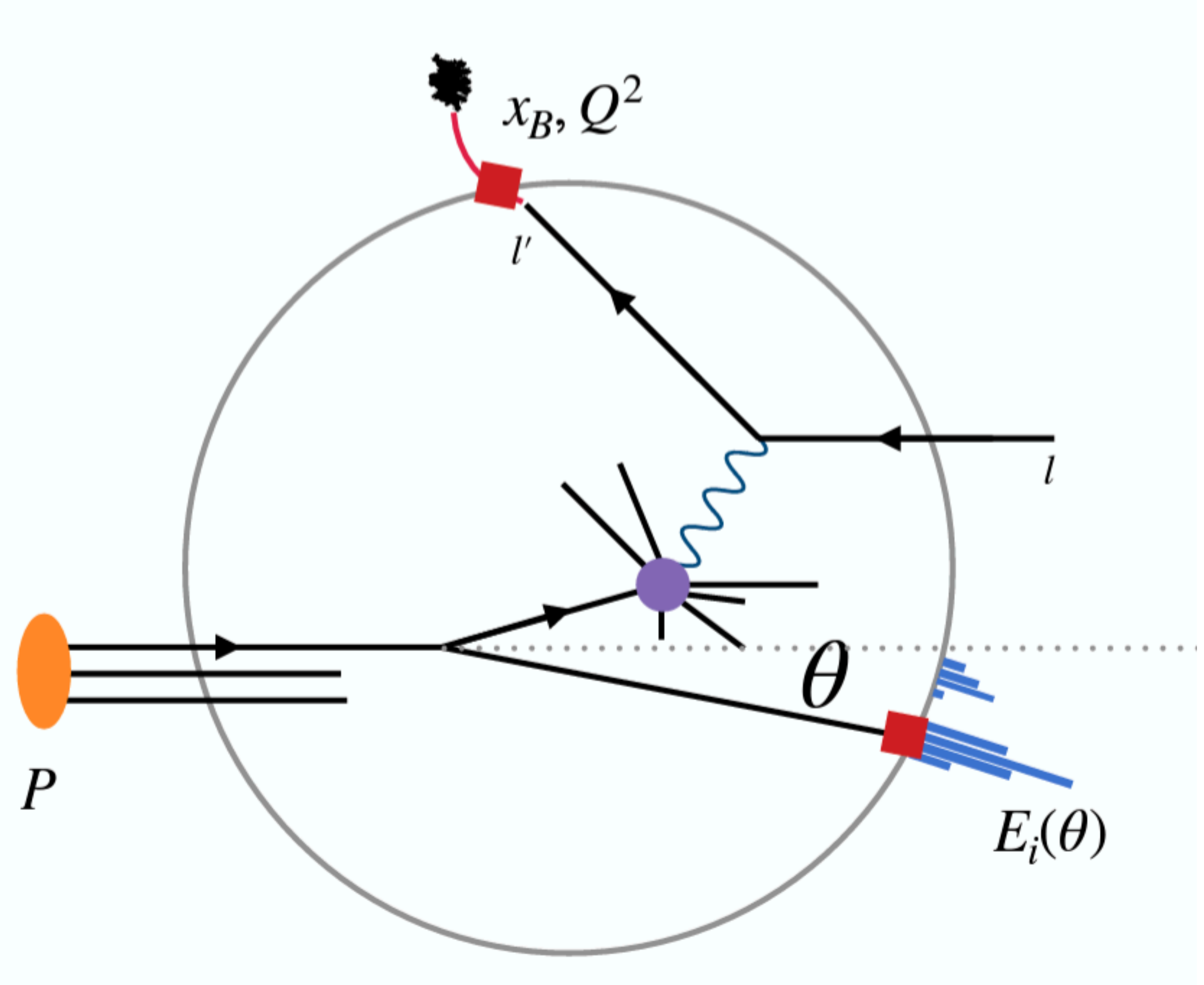
- Structure function measurement: PDFs(x)
- SIDIS:
  - TMD
  - spin

ep and nucleon structure



- Structure function measurement: PDFs(x)
- SIDIS:
  - TMD
  - spin
- How can we utilize the forward information and what does it probes?

The nucleon EEC



Liu, HXZ, 2209.02080

Energy weighted correlation of forward hadron with beam

$$f_{\rm EEC}(x,\theta) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle P|\bar{\psi}(0)\mathcal{E}(\theta)\psi(y^-)|P\rangle$$

Insertion of energy flow operator between lightcone separated field

Naturally generalize to N energy flow operator insertion

Compare with

collinear PDF: $$f_q(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle P|\bar{\psi}(0)\psi(y^-)|P\rangle$$

TMD PDF: $$f_q(x) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle P|\bar{\psi}(0)\psi(y_\perp, y^-)|P\rangle$$

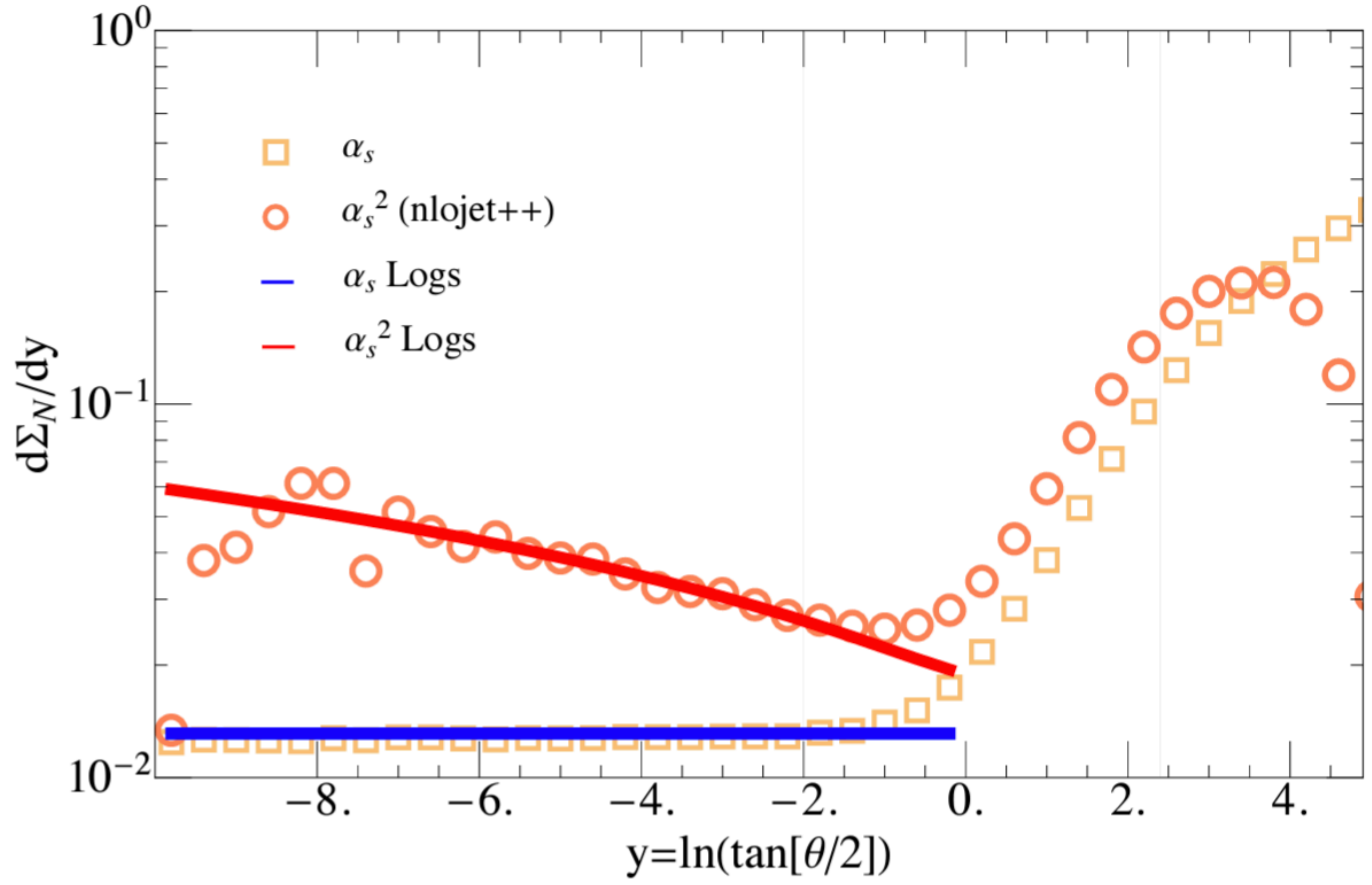
What can we learn from the nucleon EEC?

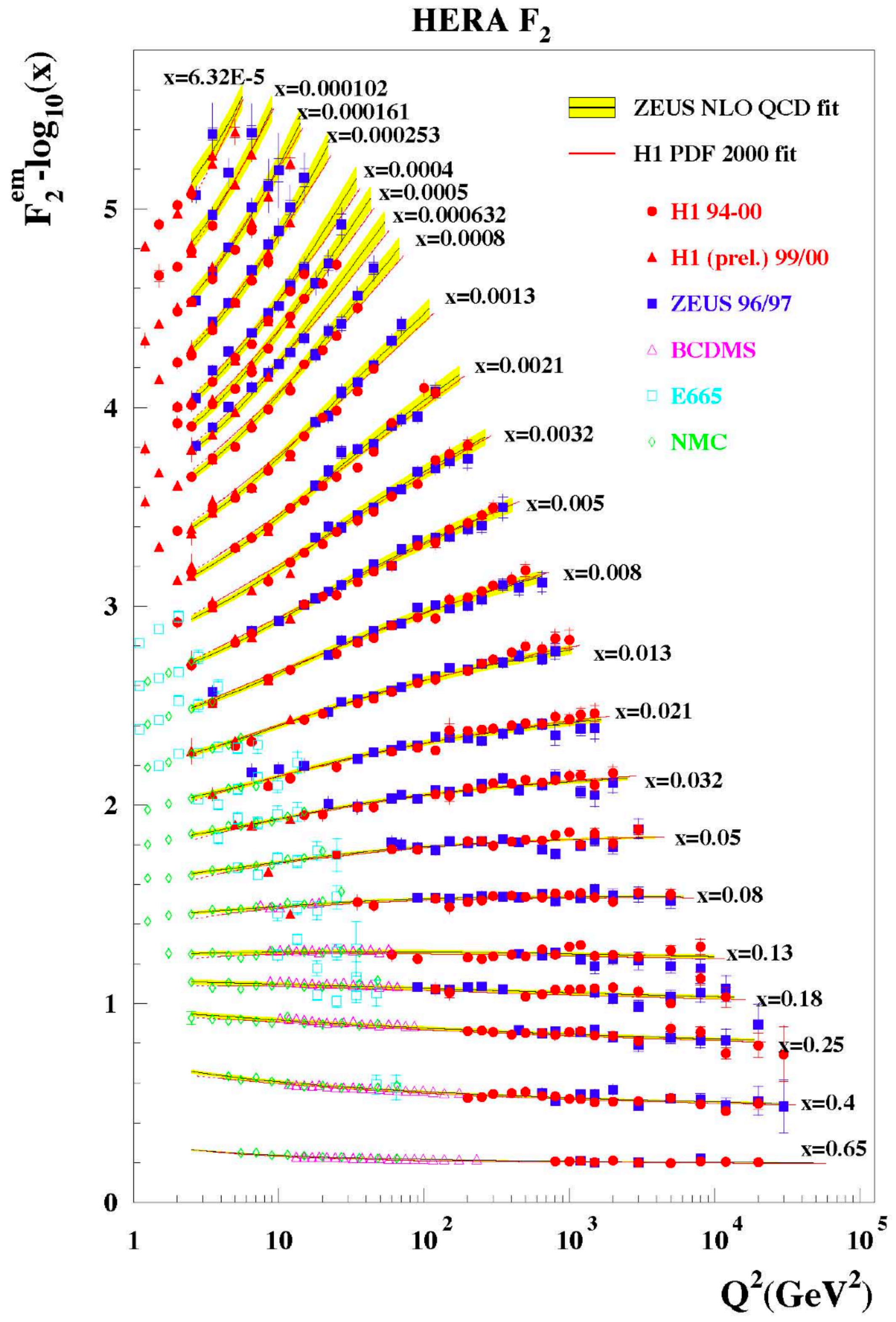
Gauge link is suppressed for brevity

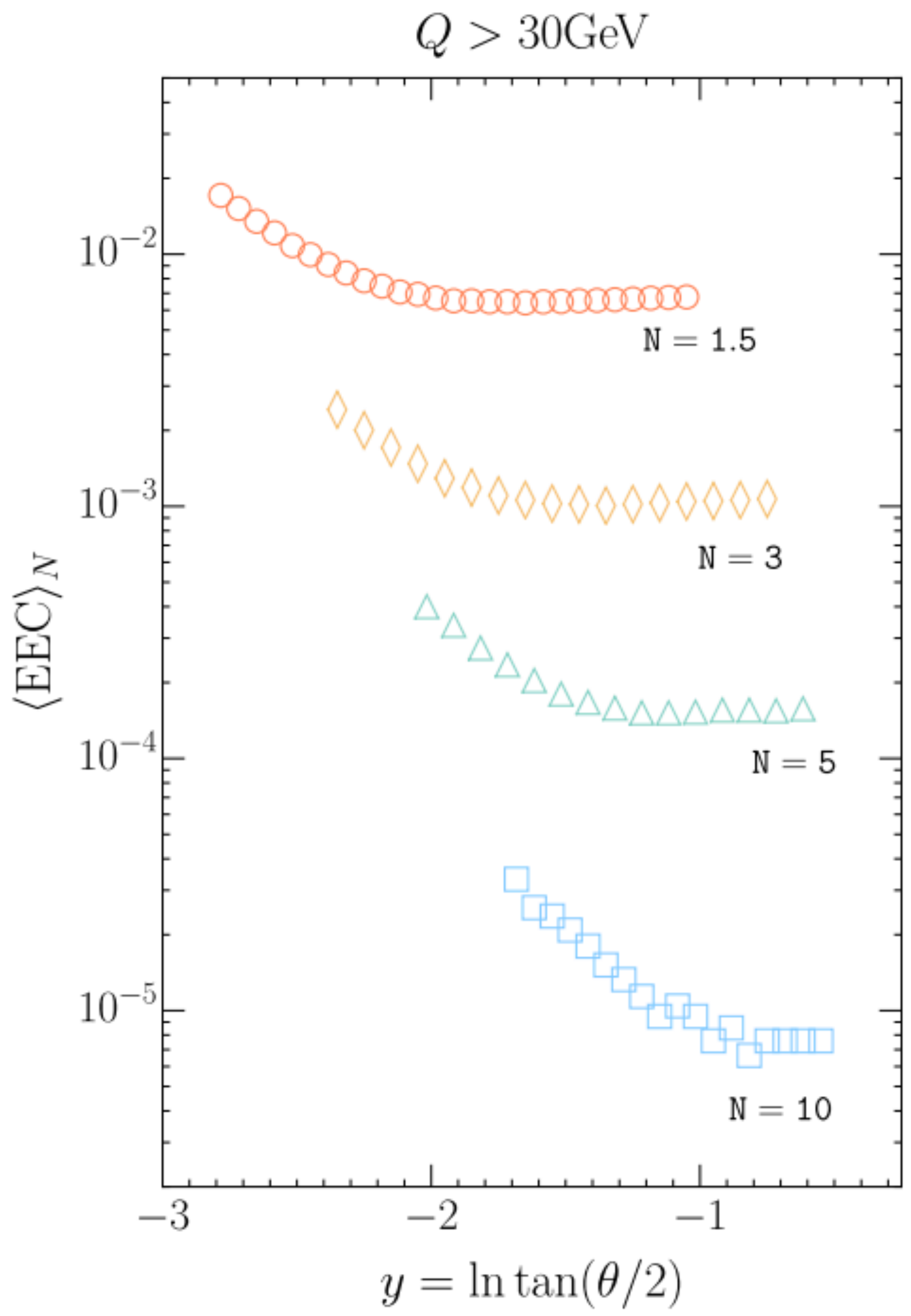
Modified DGLAP evolution

Liu, HXZ, 2209.02080
Cao, Liu, HXZ, 2303.01530

$$\frac{d}{d\ln\mu^2} f_{i,\mathrm{EEC}}\left(N, \ln\frac{Q\theta}{u\mu}\right) = \sum_j \int d\xi\, \xi^{N-1} P_{ij}(\xi) f_{j,\mathrm{EEC}}\left(N, \ln\frac{Q\theta}{\xi u\mu}\right)$$

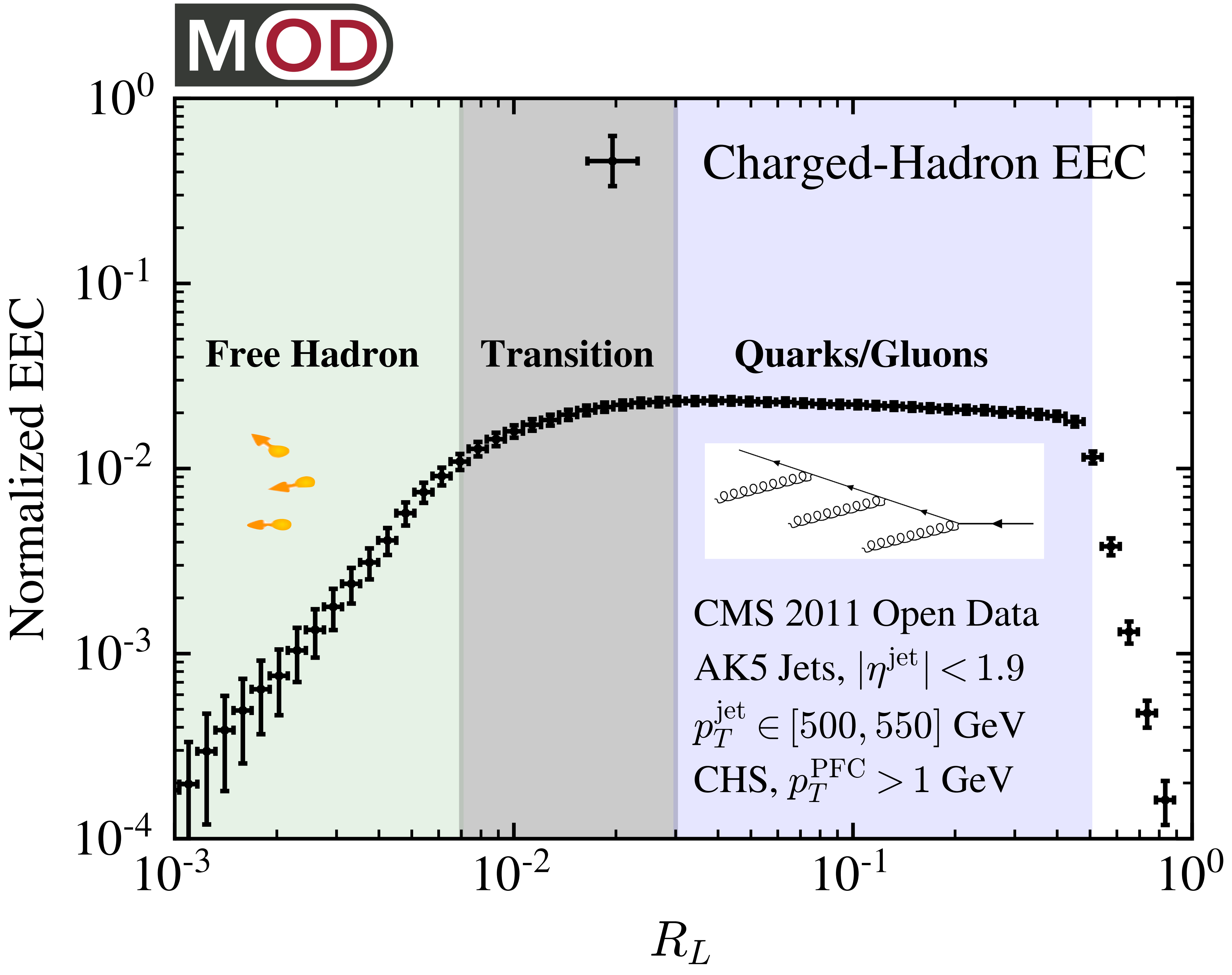




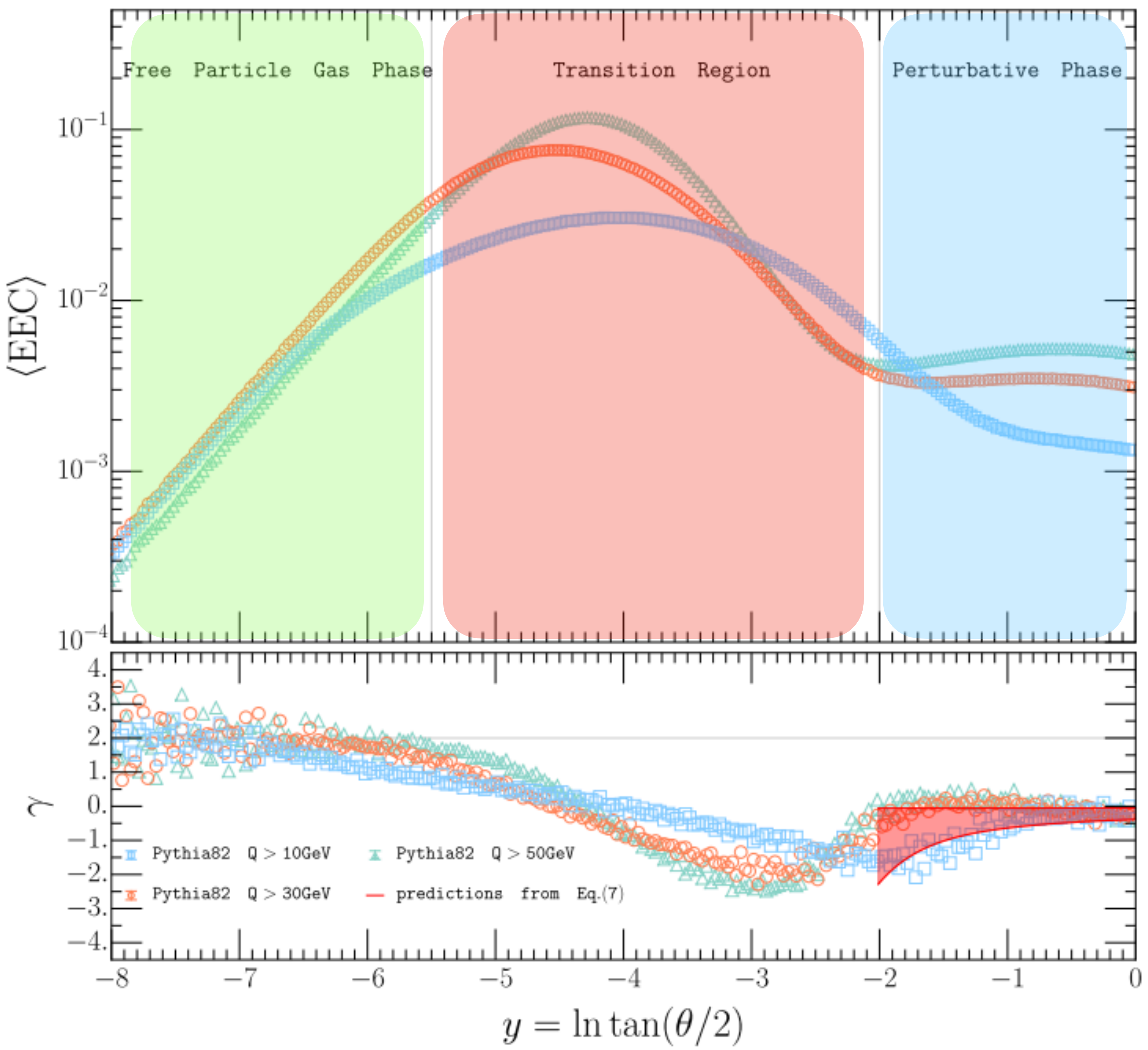


Bjorken scaling and scaling violation, not in Q evolution but in angle!

Quark fragmentation v.s. nucleon structure



EEC in jet fragmentation
imaging the time evolution of quark to hadrons through angle



The nucleon EEC
Imaging the structure of nucleon through angle

Pythia simulation only

Summary and outlook

- Energy correlators provide new perspectives on old problems in QCD
- Solid theory foundation with operator definition
- Many different applications from ee to pp then to ep colliders
    - Sudakov resummation and TMD;
    - Probing the time evolution of quark fragmentation and nucleon structure
    - Precision measurement of strong coupling constant
    - Application to track observables and generalized DGLAP evolution
    - Gluon spin double slit interference in jet (not covered)
    - Probing gluon saturation (not covered)

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Many exciting theory and phenomenology waiting for exploration. Stay tune!

Thank you for your attention!