Higgs Mechanism under the language of on-shell scattering amplitude
在壳散射振幅语言下的希格斯机制

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- Significant progresses have been achieved in the study of scattering amplitude. Maximum Helicity Violated(MHV) amplitude for gluons can be expressed regularly by spinor inner products. The final amplitudes are independent of the gauge choice. Obtain the helicity amplitudes directly in the gauge theory.
- In 4-dimension of spacetime, the Lorenzt group SO(3,1) is locally isomorph to SL(2,C). So Dirac spinor can be replaced by Weyl spinor and one can use Weyl spinor to write the amplitudes.

- Two advantages:
  (1). 3-point amplitudes with certain heilicites can be written conveniently and they can be building blocks to construct multi-point amplitudes.
  (2). Evade the trouble of gauge redundancy.
- Deeper understanding of symmetry breaking from the point of view of the scattering amplitude. How gauge boson obtain its mass from symmetry breaking.

Early spinor amplitude methods

- For massless photon and gluon, by choosing polarization vectors particularly, the amplitudes can only depend on the momentum of the fermions in a Dirac algebra form(*Kleiss 1985; Causmaecker et al,1982*).
- Differ from the 4-component Fermion Dirac spinor, ref(*Dittmaier 1998*) choose 2-component Weyl spinor to describe the momentum, polarization vectors and spinors.

Formal theory of the scattering amplitude

- Construct tree level amplitude by MHV amplitudes by Cachazo-Svrcekk-Witten(CSW).
- Britto-Cachazo-Feng-Witten(BCFW) on-shell recursion relation.
- Relations among locality, unitary and geometry(*Nima et al, 2021*).

Further studies

- Generalize to scattering amplitudes with arbitrary mass and spin(*Conde2016, Nima et al, 2021*).
- Researches on SM(*Bachu2020*), EFT(*Shadmi2019*) by using on-shell scattering amplitude method.

Higgs Mechanism in SM

Adopting unitary gauge

$$
\begin{aligned}
|(D^{\mu}\Phi)|^2 &= |(\partial_\mu - ig_2\frac{\tau_a}{2}W_\mu^a - ig_1\frac{1}{2}B_\mu)\Phi|^2 \\
&= \frac{1}{2}\left|\begin{pmatrix} \partial_\mu - \frac{i}{2}(g_2W_\mu^3 + g_1B_\mu) & -\frac{ig_2}{2}(W_\mu^1 - iW_\mu^2) \\ -\frac{ig_2}{2}(W_\mu^1 + iW_\mu^2) & \partial_\mu + \frac{i}{2}(g_2W_\mu^3 - g_1B_\mu) \end{pmatrix}\begin{pmatrix} 0 \\ (v + H(x)) \end{pmatrix}\right|^2 \\
&= \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{8}g_2^2(v+H)^2|(W_\mu^1 + iW_\mu^2)|^2 + \frac{1}{8}(v+H)^2|(g_2W_\mu^3 - g_1B_\mu)|^2,
\end{aligned}
\tag{1}
$$

Redefine

$$
W^{\pm} = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2),\ Z_\mu = \frac{g_2W_\mu^3 - g_1B_\mu}{\sqrt{g_2^2 + g_1^2}},\ A_\mu = \frac{g_2W_\mu^3 + g_1B_\mu}{\sqrt{g_2^2 + g_1^2}},
\tag{2}
$$

Mass term

$$
M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu + \frac{1}{2}M_A^2 A_\mu A^\mu
\tag{3}
$$

Definitions and Conventions

Little Group: Subgroup that makes momentum unchanged under Lorentz transformation. By using the classification of spinors under the representation of Little Group, for arbitrary angle (θ, ϕ) in space, the spinor

massless

$$\lambda_\alpha = \sqrt{2E}\begin{pmatrix} -s^* \\ c \end{pmatrix}, \ \tilde{\lambda}_{\dot{\alpha}} = \sqrt{2E}\begin{pmatrix} -s & c^* \end{pmatrix}, \tag{4}$$

$$s \equiv sin\frac{\theta}{2}e^{\frac{i}{2}\phi}, \ c \equiv cos\frac{\theta}{2}e^{\frac{i}{2}\phi} \tag{5}$$

massive

$$\begin{aligned} \lambda_\alpha^{I=1} &\equiv |r\rangle = \eta_\alpha = \sqrt{E-p}\begin{pmatrix} c^* \\ s \end{pmatrix}, \quad & \tilde{\lambda}_{\dot{\alpha}I=1} &\equiv [r| = \tilde{\eta}_{\dot{\alpha}} = \sqrt{E-p}\begin{pmatrix} c & s^* \end{pmatrix}, \\ \lambda_\alpha^{I=2} &\equiv |k\rangle = \lambda_\alpha = \sqrt{E+p}\begin{pmatrix} -s^* \\ c \end{pmatrix}, \quad & \tilde{\lambda}_{\dot{\alpha}I=2} &\equiv [k| = \tilde{\lambda}_{\dot{\alpha}} = \sqrt{E+p}\begin{pmatrix} -s & c^* \end{pmatrix}. \end{aligned} \tag{6}$$

High energy limit

$$\eta_\alpha, \tilde{\eta}_{\dot{\alpha}} \propto \sqrt{E-p} = \frac{\sqrt{E^2-p^2}}{\sqrt{E+p}} = \frac{m}{2E} \xrightarrow{m\to 0} 0$$

Choose $\vec{k}||\vec{r}$, massive spinors $|\boldsymbol{p}^I\rangle$ and $|\boldsymbol{p}^I]$ will degenerate to massless spinor $|p^I\rangle$.

$I=1 \ \ (I=2)$ corresponds to positive(negative) helicity.

$$\begin{aligned}
\varepsilon^-_{\alpha\dot{\alpha}} &= \frac{\lambda_\alpha \tilde{\eta}_{\dot{\alpha}}}{[\tilde{\lambda}\tilde{\eta}]} = \frac{|k\rangle[r|}{[kr]} \xrightarrow{m\to 0} \frac{\lambda_\alpha \tilde{\mu}_{\dot{\alpha}}}{[\tilde{\lambda}\tilde{\mu}]} = \frac{|p\rangle[q|}{[pq]}, \\
\varepsilon^+_{\alpha\dot{\alpha}} &= -\frac{\eta_\alpha \tilde{\lambda}_{\dot{\alpha}}}{\langle\lambda\eta\rangle} = -\frac{|r\rangle[k|}{\langle kr\rangle} \xrightarrow{m\to 0} \frac{\mu_\alpha \tilde{\lambda}_{\dot{\alpha}}}{\langle\mu\lambda\rangle} = \frac{|q\rangle[p|}{\langle qp\rangle}.
\end{aligned} \tag{7}$$

Gauge related problem

Photon propagator under R_ξ gauge.

$$\langle A^\mu(x)A^\nu(y)\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2}\sum_{r=0}^{3}\left(\varepsilon_\mu^r(k)\varepsilon_\mu^{r*}(k)\right) e^{-ik\cdot(x-y)}. \tag{8}$$

where the sum of the polarization vectors include unphysical polarization states

$$\sum_{r=0}^{3}\left(\varepsilon_\mu^r(k)\varepsilon_\nu^{r*}(k)\right) = -\left(g_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right). \tag{9}$$

Introduce virtual particle to cancel gauge dependence term to maintain the gauge invariance. Eg. ghost field. Extra Feynman diagram

Gauge related problem

Contribution from pure physical states under axial gauge.

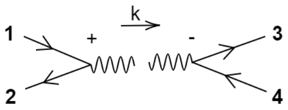
$$\sum_{\lambda=\pm} \varepsilon_\mu^\lambda \left(\varepsilon_\nu^\lambda\right)^* = -g_{\mu\nu} + \frac{k_\mu q_\nu + q_\mu k_\nu}{k \cdot q}, \tag{10}$$

q_μ is reference momentum
Contributions from Goldstone and scalar polorization cancels under unitary gauge

$$\sum_{i=+,-,0} \varepsilon_\mu^i \left(\varepsilon_\nu^i\right)^* = -\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right), \tag{11}$$

Including contribution from Goldstone under Goldstone equivalent gauge, similarly like the upper massless case,

$$\sum_{\lambda=\pm,0} \varepsilon_M^\lambda \left(\varepsilon_N^\lambda\right)^* = -g_{MN} + \frac{p_M n_N + n_M p_N}{p \cdot n}. \tag{12}$$



Eg. amplitude with massless particles
Split the 4-pt amplitude into 3-pt amplitudes

$$M = (\bar{u}_3(-ie\gamma^{\mu})\bar{v}_4)\frac{-g_{\mu\nu} + \frac{k_\mu q_\nu + q_\mu k_\nu}{k\cdot q}}{k^2 + i\epsilon}(u_2(-ie\gamma^{\nu})v_1). \quad (13)$$

Eg. Helicity $1,3-,2,4+$

$$M = \frac{1}{k^2}\left(\langle 3|\gamma_\nu|4]\langle 2|\gamma^\nu|1] + \frac{k_\mu q_\nu + q_\mu k_\nu}{k\cdot q}\langle 3|\gamma^\mu|4]\langle 2|\gamma^\nu|1]\right). \quad (14)$$

second term =0 due to momentum conservation.

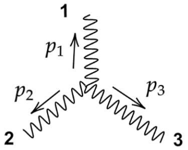
- Tree level amplitude can be expressed as two sub-amplitude times a Feynman propagator.
  BCFW recursion relation

$$M_n = \sum_{r,h} M^{h}_{r+1} \frac{1}{P^2} M^{-h}_{n-r+1}, \tag{15}$$

- How to get the 3pt amplitudes

Matching at the amplitude level—*WWZ* 3pt amplitude

W, Z obtain their mass by electroweak symmetry breaking. Their coupling strength determines the coupling constant of the EW theory.



WWZ 3pt amplitude by on-shell scattering method

$$M_{WWZ} \sim \left[\eta_{\mu\nu}\left(p_1 - p_2\right)_\rho + \eta_{\nu\rho}\left(p_2 - p_3\right)_\mu + \eta_{\rho\mu}\left(p_3 - p_1\right)_\nu\right] \varepsilon_1^\mu \varepsilon_2^\nu \varepsilon_3^\rho$$
$$= -\frac{\sqrt{2}}{m_1 m_2 m_3}\left(m_1 [\mathbf{12}][\mathbf{13}]\langle\mathbf{23}\rangle + m_2 [\mathbf{12}]\langle\mathbf{13}\rangle[\mathbf{23}] + m_3 \langle\mathbf{12}\rangle[\mathbf{13}][\mathbf{23}]\right)$$
$$= -\frac{\sqrt{2}}{m_1 m_2 m_3}\left(m_1 \langle\mathbf{12}\rangle\langle\mathbf{13}\rangle[\mathbf{23}] + m_2 \langle\mathbf{12}\rangle[\mathbf{13}]\langle\mathbf{23}\rangle + m_3 [\mathbf{12}]\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle\right), \quad (16)$$

The upper calculation has used Schouten Identity.

$$\begin{aligned}\langle \mathbf{313}][\mathbf{12}] &= m_1\langle \mathbf{13}\rangle[\mathbf{23}] + m_2[\mathbf{13}]\langle \mathbf{23}\rangle - m_3[\mathbf{13}][\mathbf{23}], \\ [\mathbf{313}\rangle\langle \mathbf{12}\rangle &= m_1[\mathbf{13}]\langle \mathbf{23}\rangle + m_2\langle \mathbf{13}\rangle[\mathbf{23}] - m_3\langle \mathbf{13}\rangle\langle \mathbf{23}\rangle.\end{aligned} \tag{17}$$

$$\begin{aligned}&\lambda_\alpha^{I=1} \equiv |r\rangle = \eta_\alpha = \sqrt{E-p}\begin{pmatrix} c^* \\ s \end{pmatrix}, \qquad \tilde{\lambda}_{\dot{\alpha}I=1} \equiv [r| = \tilde{\eta}_{\dot{\alpha}} = \sqrt{E-p}\begin{pmatrix} c & s^* \end{pmatrix}, \\ &\lambda_\alpha^{I=2} \equiv |k\rangle = \lambda_\alpha = \sqrt{E+p}\begin{pmatrix} -s^* \\ c \end{pmatrix}, \qquad \tilde{\lambda}_{\dot{\alpha}I=2} \equiv [k| = \tilde{\lambda}_{\dot{\alpha}} = \sqrt{E+p}\begin{pmatrix} -s & c^* \end{pmatrix}, \\ &s \equiv sin\tfrac{\theta}{2}e^{\frac{i}{2}\phi}, \qquad c \equiv cos\tfrac{\theta}{2}e^{\frac{i}{2}\phi}\end{aligned} \tag{18}$$

In principle, every 3pt amplitude corresponds to a certain coupling constant. Information of coupling constant can not be given by the above calculations.

To obtain information about the coupling constant, 3pt amplitude at UV can be calculated by two ways.

- Amplitude of massive particle at IR limit

$$M(\mathbf{V_1 V_2 V_3}) = C_{a_1 a_2 a_3} \left(\frac{[\mathbf{12}]\langle\mathbf{23}\rangle[\mathbf{31}]}{m_2 m_3} + cyc. \right). \tag{19}$$

As the coupling in 3pt amplitude will not change at UV and IR limit, amplitude of massless particles for certain helicities can be obtained by shrinking little group index of the amplitude of massive particles.

$$M^{+00}_{V_1 V_2 V_3} = \frac{C(m_1^2 - m_2^2 - m_3^2)}{m_2 m_3} \frac{[12][13]}{[23]} \tag{20}$$

- Amplitude of massless particle at UV limit

$$M_{V^+ \phi\phi} = -C_{V\phi\phi} \frac{[12][13]}{[23]} \tag{21}$$

Amplitude at UV limit can be wrote directly according on-shell scattering method.

- There are 1 longitudinal polarization component and 2 transverse polarization components for a massive particle. There are only 2 transverse polarization components for a massless particle.
- Goldstone equivalent theorem: At UV energy limit, the scalar polarization component(label by index 0) for a spin 1 vector particle has the same behavior as the Goldstone boson ϕ that corresponds to the gauge boson .

By matching the coupling coefficients of the amplitude at IR energy limit with that of the amplitude written directly at UV limit, one can obtain the relation of the coefficients. Eg.

$$\begin{pmatrix} Z \\ \gamma \end{pmatrix} = \begin{pmatrix} cos\theta_w & -sin\theta_w \\ sin\theta_w & cos\theta_w \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}, \tag{22}$$

$$M(WWZ) = \cos\theta_W M(WWW^3) - \sin\theta_W M(WWB) \tag{23}$$

$$M_{WWZ} = C_{WWZ}\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle} \quad M_{WWW^3} = C_{WWW^3}\frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle} \quad M_{WWB} = 0 \tag{24}$$

Because there does not exist WWB 3pt amplitude, coefficient of 3W at UV $C_{WWW^3} = g$. $C_{WWZ} = g\cos\theta_W$.
Matching of the amplitudes at different energy scales exhibits the electroweak symmetry breaking from the amplitude level.
Connecting the massless 3pt amplitude at UV with the massive 3pt amplitude at IR.

Matching at the amplitude level — general case for 3pt amplitude

1. 3pt amplitudes form the building block to construct high pt amplitudes. Classify all possible 3pt amplitude in EW theory according to IR and UV.
2. Obtain UV 3pt amplitude from two ways.
3. By matching the coupling coefficients of the amplitude at IR energy limit between that of the amplitude written directly at UV limit, one can obtain the relations between coefficients.

(a). Let $V_1, V_2 = W_{\pm}$

$$M(WWZ) = c_w M(WWW^3) - s_w M(WWB), \tag{25}$$

$$\Longrightarrow \quad C_{WWZ} = g c_w$$

$$M(WW\gamma) = s_w M(WWW^3) + c_w M(WWB), \tag{26}$$

$$\Longrightarrow \quad C_{WW\gamma} = e = g s_w$$

The longitudinal mode of the vector boson and the Higgs particle are regarded as different linear combination of scalar field ϕ at UV. Note the coefficient matrix of the linear combination as $C_{V,h}$.

The four degree of freedom of Higgs doublet: one scalar Higgs particle + 3 goldstone bosons to be eaten by 3 gauge bosons
Introduce a constant vector $V_{vev} = \{v_1, v_2, v_3, v_4\}^T$ to diagonalize the mass matrix $U_M^a = m_a U^a$. $V^T t^i t^j V = (m^2)^{ij}$, $a = W, Z, h$.
t^s: generator for certain Lie group. Real representation of $C(0,3)$.
Isomorphism between Pauli matrices and Clifford algebra.

$$t^1 = i\begin{pmatrix} 0 & 0 & i & -1 \\ 0 & 0 & -1 & -i \\ -i & 1 & 0 & 0 \\ 1 & i & 0 & 0 \end{pmatrix}, \quad t^2 = i\begin{pmatrix} 0 & 0 & -i & -1 \\ 0 & 0 & -1 & i \\ i & 1 & 0 & 0 \\ 1 & -i & 0 & 0 \end{pmatrix},$$
$$t^3 = i\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad t^B = i\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{27}$$

Diagonalize $C_{W_1}(g(t^3)c_w - g'(t^B)s_w)C_{W_2} \quad \Longrightarrow$
$V^T\left(gc_w t^2 t^3 t^1 - g' t^2 t^B t^1 s_w\right) V = -(m_Z^2 - 2m_W^2)$

(b). Consider amplitude $V^0 V^0 V$ at UV

$$M(W^0 W^0 Z) = C_{W_1} C_{W_2}\left(c_w M(\phi\phi W^3) - s_w M(\phi\phi B)\right), \qquad (28)$$

$$M^{+00}_{V_1 V_2 V_3} = \frac{C(m_1^2 - m_2^2 - m_3^2)}{m_2 m_3}\frac{[12][13]}{[23]} \qquad (29)$$

$$\Longrightarrow \quad C_{WWZ}\frac{(m_Z^2 - 2m_W^2)}{m_W^2} = -C_{W_1}(g(t^3)c_w - g'(t^B)s_w)C_{W_2}$$

$$M(W^0 W^0 \gamma) = C_{W_1} C_{W_2}\left(s_w M(\phi\phi W^3) + c_w M(\phi\phi B)\right), \qquad (30)$$

$$\Longrightarrow \quad -C_{WW\gamma} = -C_{W_1}(g(t^3)s_w + g'(t^B)c_w)C_{W_2}$$

Collect above equations

$$
\begin{aligned}
C_{WWZ} &= g c_w , \\
-C_{WW\gamma} &= -C_{W_1}\left(g(t^3)s_w + g'(t^B)c_w\right)C_{W_2} , \\
C_{WW\gamma} &= e = g s_w , \\
C_{WWZ}\frac{(m_Z^2 - 2m_W^2)}{m_W^2} &= -C_{W_1}\left(g(t^3)c_w - g'(t^B)s_w\right)C_{W_2} .
\end{aligned}
\tag{31}
$$

$$
\Longrightarrow \quad m_Z^2 = \frac{m_W^2\left(2g s_w c_w - g'(c_w^2 - s_w^2)\right)}{g s_w c_w + g' c_w^2} .
$$

(c).

$$M(W^0 Z^0 \gamma) = 0 = C_{W_1} \left(s_w M(\phi\phi W^3) + c_w M(\phi\phi B) \right) C_Z, \quad (32)$$

$$\Longrightarrow \quad 0 = C_{W_1} (g(t^3) s_w + g'(t^B) c_w) C_Z$$

$$M(V^0 h\gamma) = 0 = C_V \left(s_w M(\phi\phi W^3) + c_w M(\phi\phi B) \right) C_h. \quad (33)$$

Define Weinberg angle by requiring the unitary matrix satisfy $C_{W_1}(t^3)C_Z = -C_{W_1}(t^B)C_Z \neq 0$ or $C_V(t^3)C_h = -C_V(t^B)C_h \neq 0$

$$tan\theta_w = \frac{g'}{g}. \quad (34)$$

$$m_Z^2 c_w^2 = m_W^2. \quad (35)$$

$$
\begin{aligned}
&C_{W_2}(t^2)C_Z = C_{W_1}(t^1)C_Z = \frac{1}{2},\\
&C_{WWh} = gC_{W_2}(t^1)C_h,\\
&C_{ZZh} = C_Z(g(t^3)c_w - g'(t^B)s_w)C_h,\\
&C_{W_1}(t^3)C_Z = -C_{W_1}(t^B)C_Z \neq 0,\\
&C_V(t^3)C_h = -C_V(t^B)C_h \neq 0.
\end{aligned}
\tag{36}
$$

Summary and Prospect

- Amplitude can be written by helicity spinors in modern scattering amplitude theory. By introducing conventions of "bra-ket" and "Bold", the amplitudes can be expressed in a more compact form.
- 3pt amplitudes for given helicities can be easily obtained and they form a series of bases to construct high point amplitude by BCFW recursion relation.
- By taking WWZ 3pt amplitude as an example, Higgs mechanism of EW symmetry breaking can be reformulated by the matching relation between 3pt amplitudes at UV and IR scales. New point of view for Higgs mechanism.
- We merely consider 3pt amplitude of W, Z bosons in electroweak part, Fermion particles are not included here.

Thanks!