第四届粒子物理前沿研讨会
2023.08.07-12， 太原

ALP from $U(1)_L$ and its phenomenology

WEI CHAO (BNU) 2023.08.09

Based on arXiv:2210.13233, with Ying-Quan Peng、H.J. Li、M.J. Jin、Yue Wang; arXiv:2308.XXXXX with Ying-Quan Peng

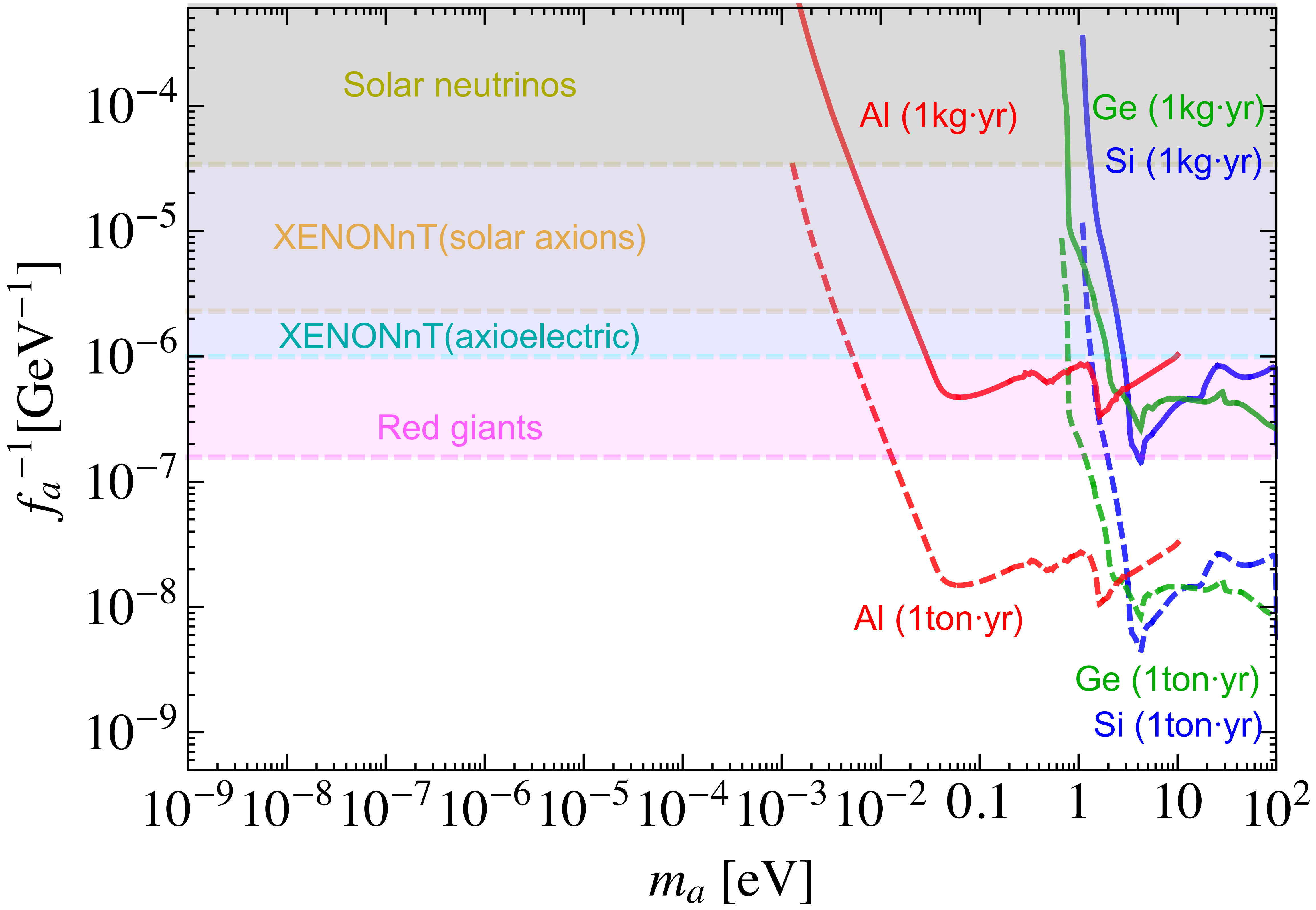
Preview

I: The origin of the ALP mass

There is no well-known mechanism for the mass generation of ALP, except the QCD axion.

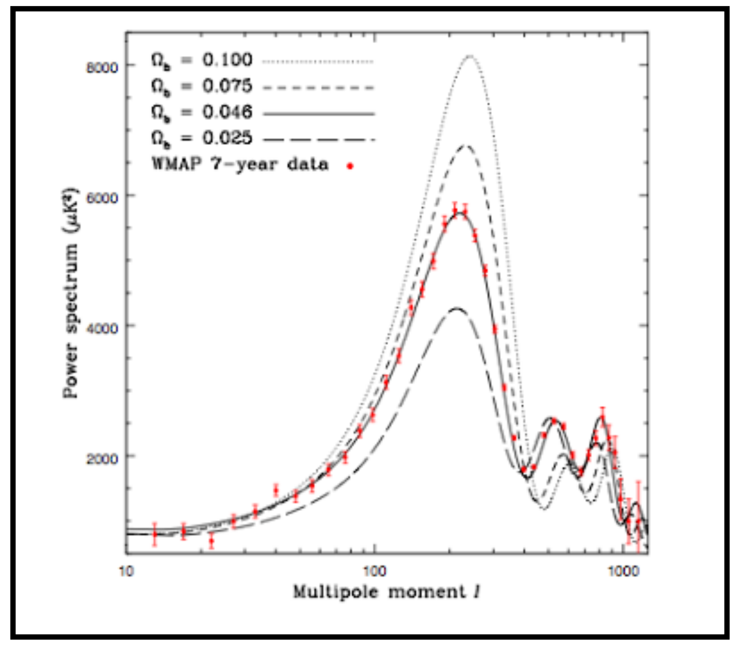
We present a mechanism for the mass generation of Majoron dark matter via the type-II seesaw mechanism!

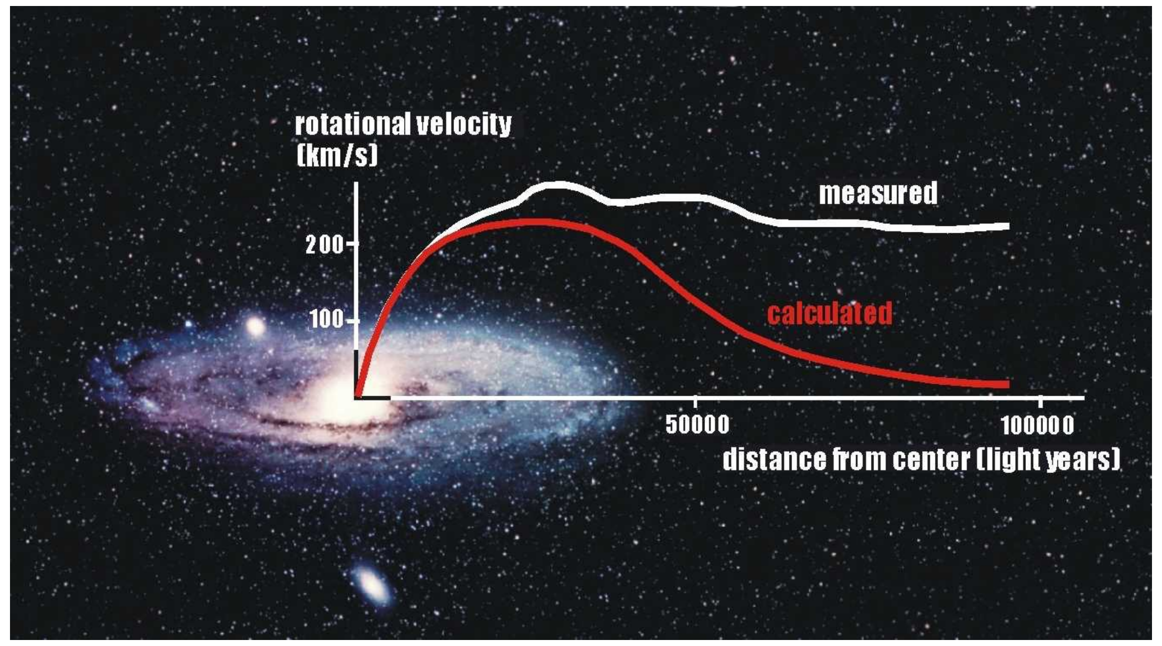
II: Constraints on the ALP parameter space

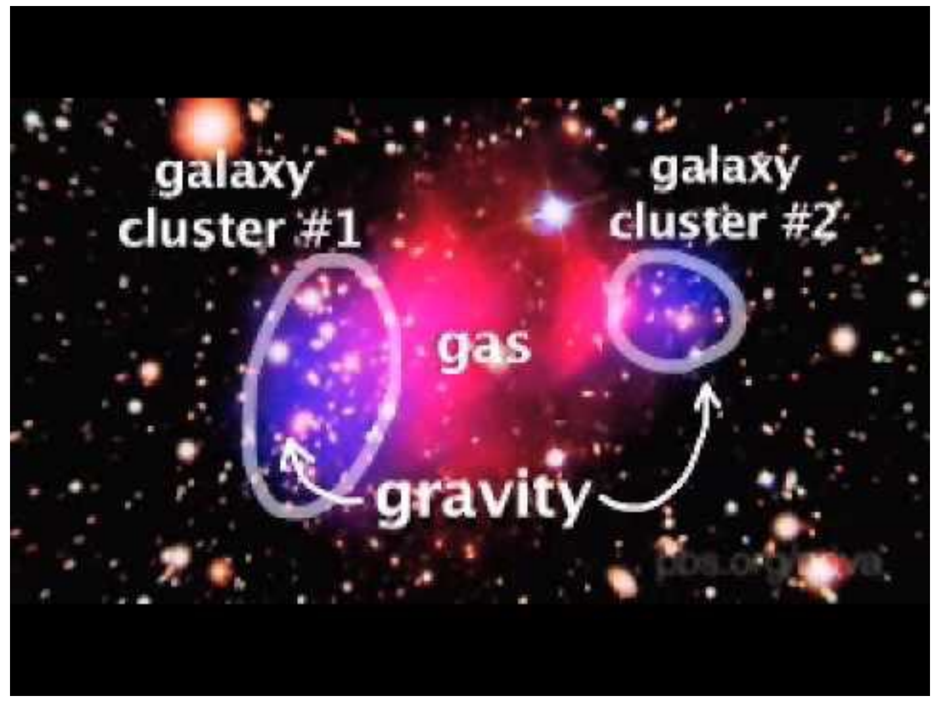


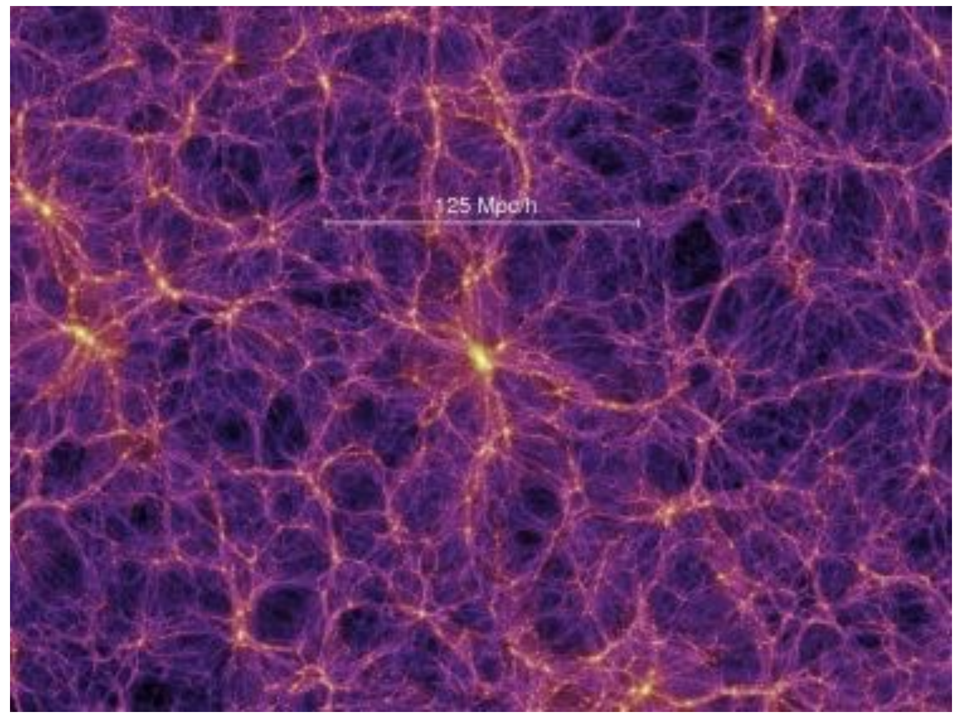
New physics beyond the SM-dark matter

Evidence of dark matter





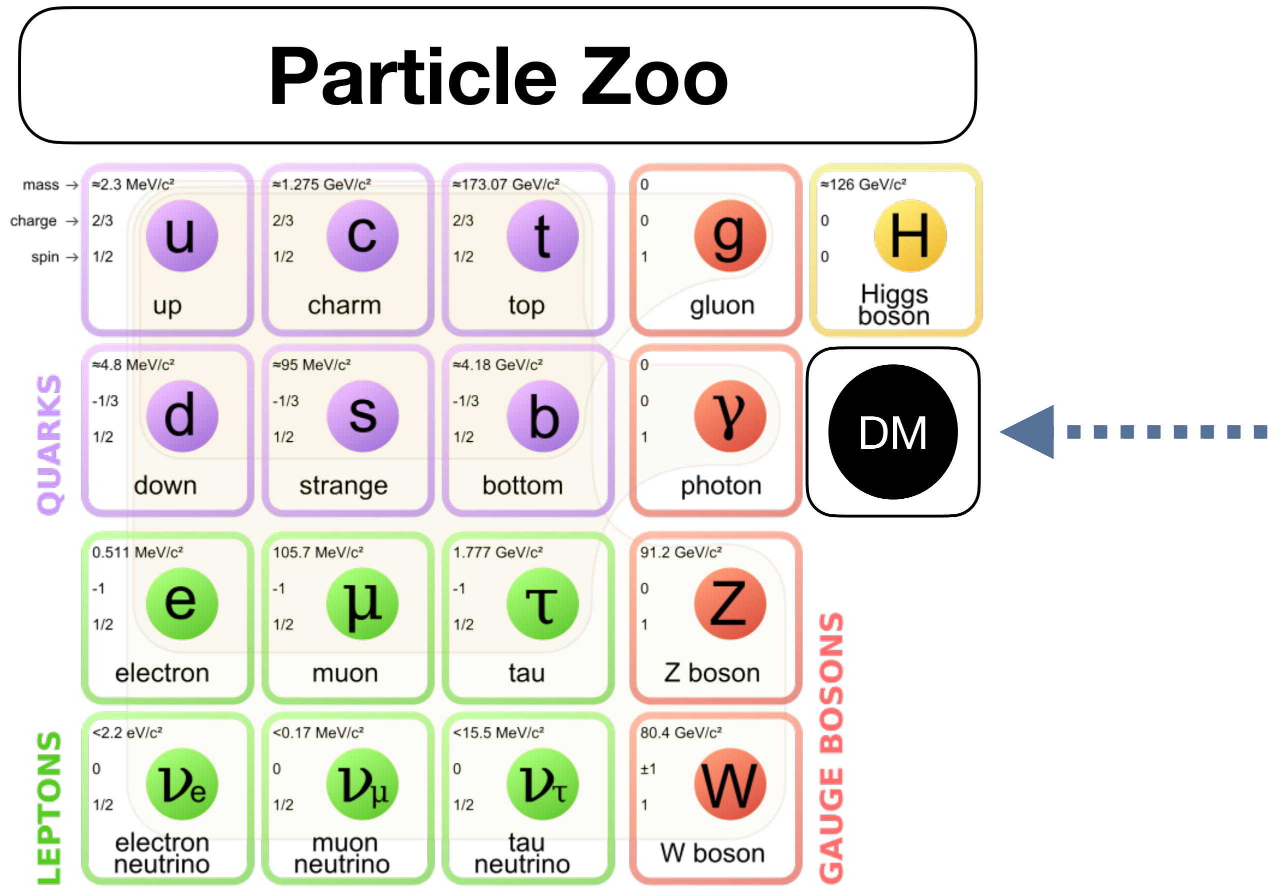


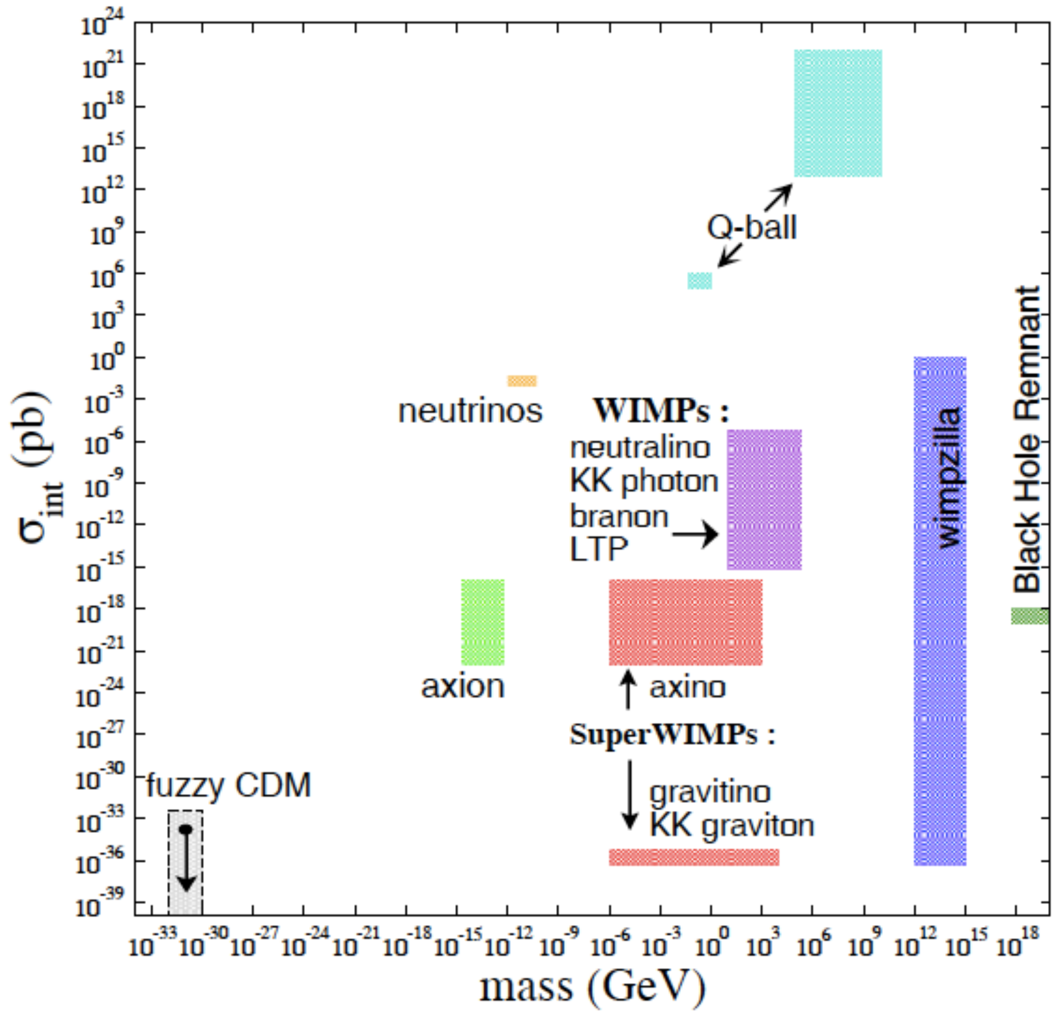


What is dark matter?

~~Mass~~ ~~Spin~~ ~~Interactions~~

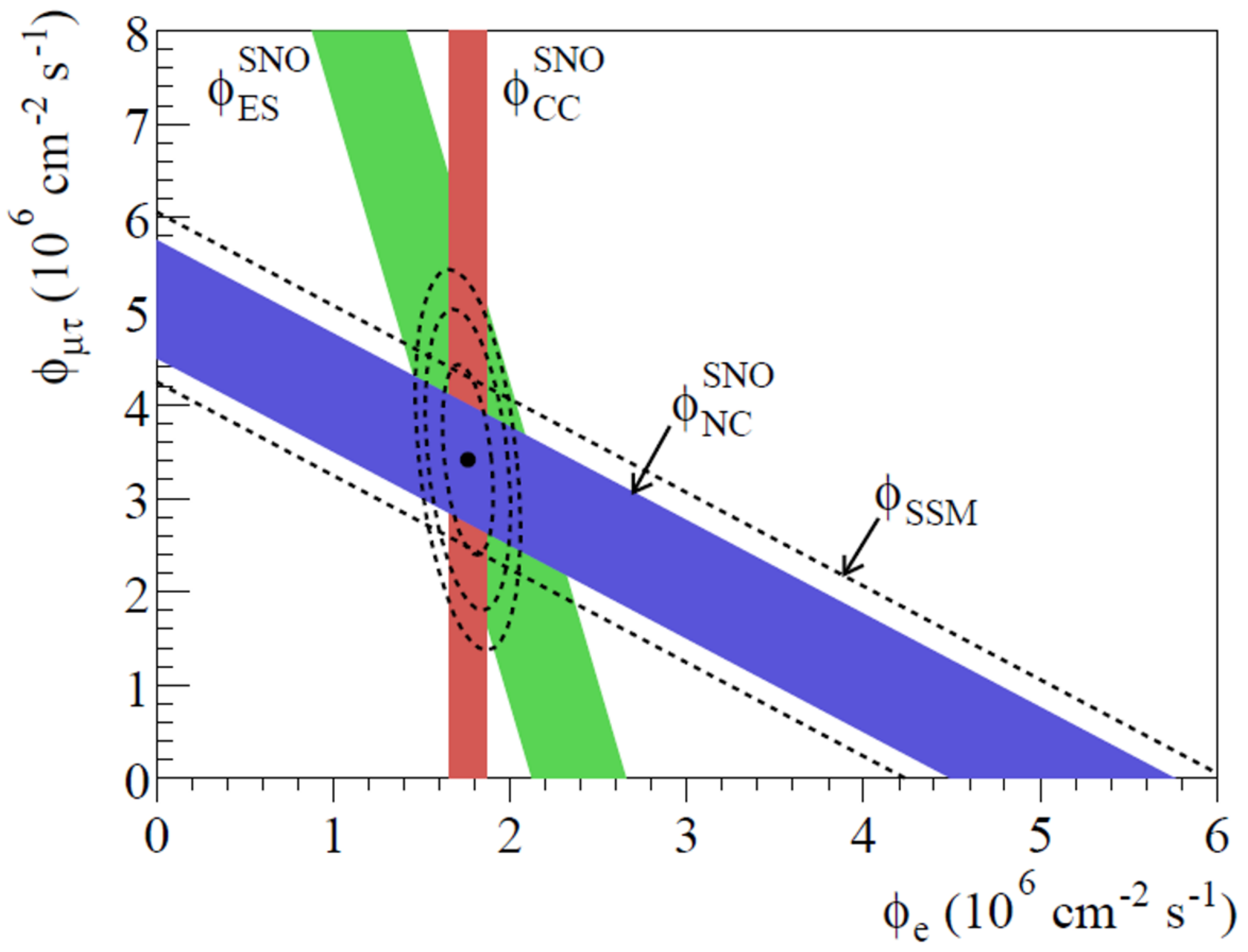
Neutral, non-baryonic, weakly interacting particle!





New physics beyond the SM-neutrino physics

Evidence of of neutrino oscillations



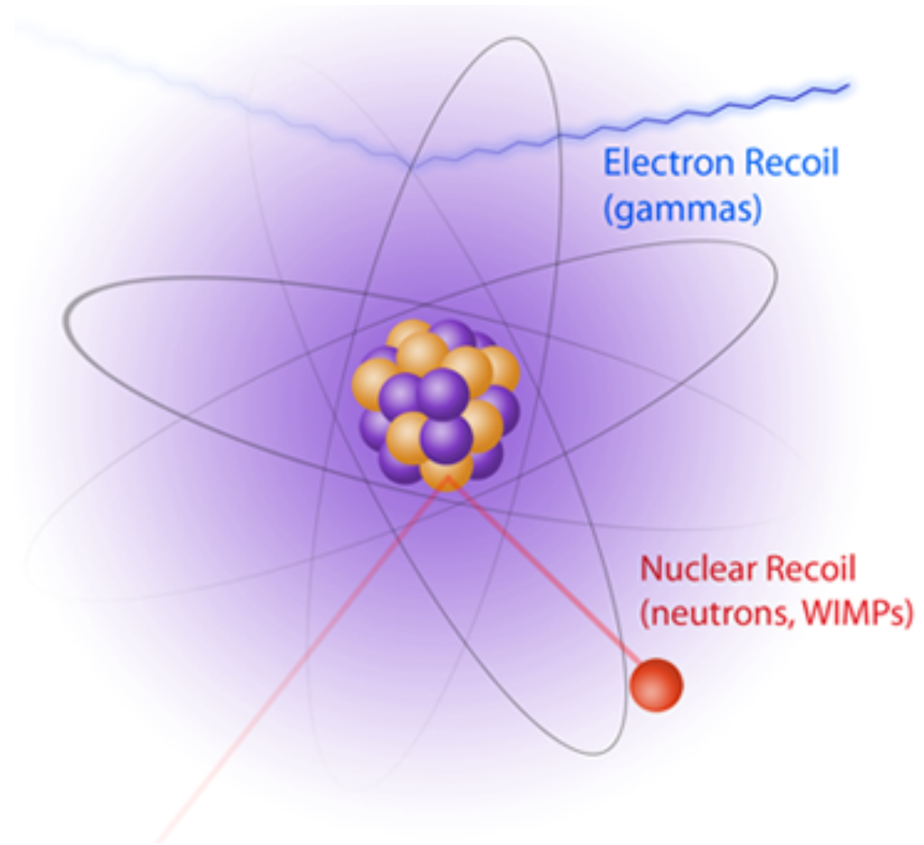
Key issues for neutrino physics

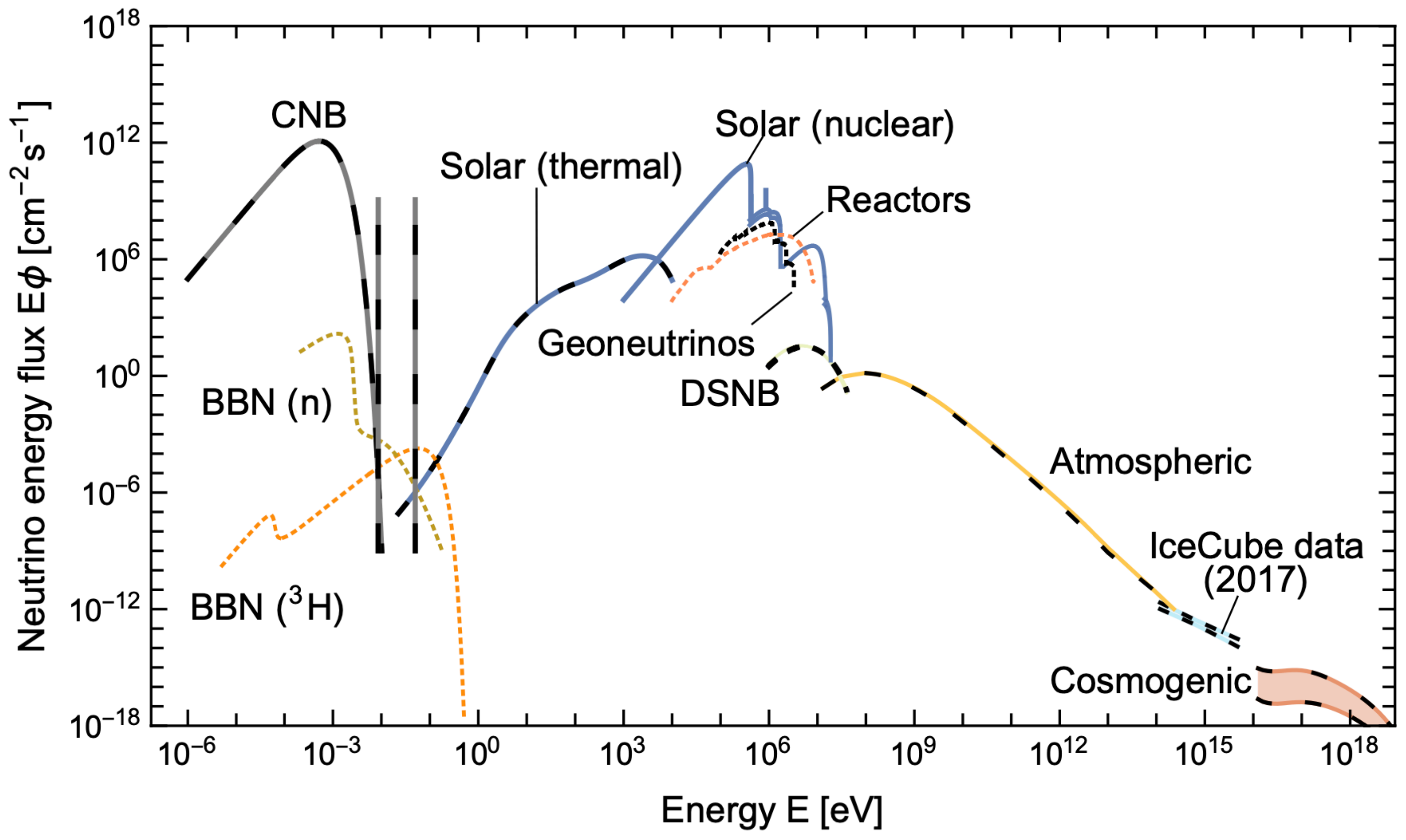
The origin of tiny but non-zero neutrino masses
Neutrino mass hierarchy
CP-violating phase in lepton mixing matrix
Is neutrino Majorana or Dirac particle?
……

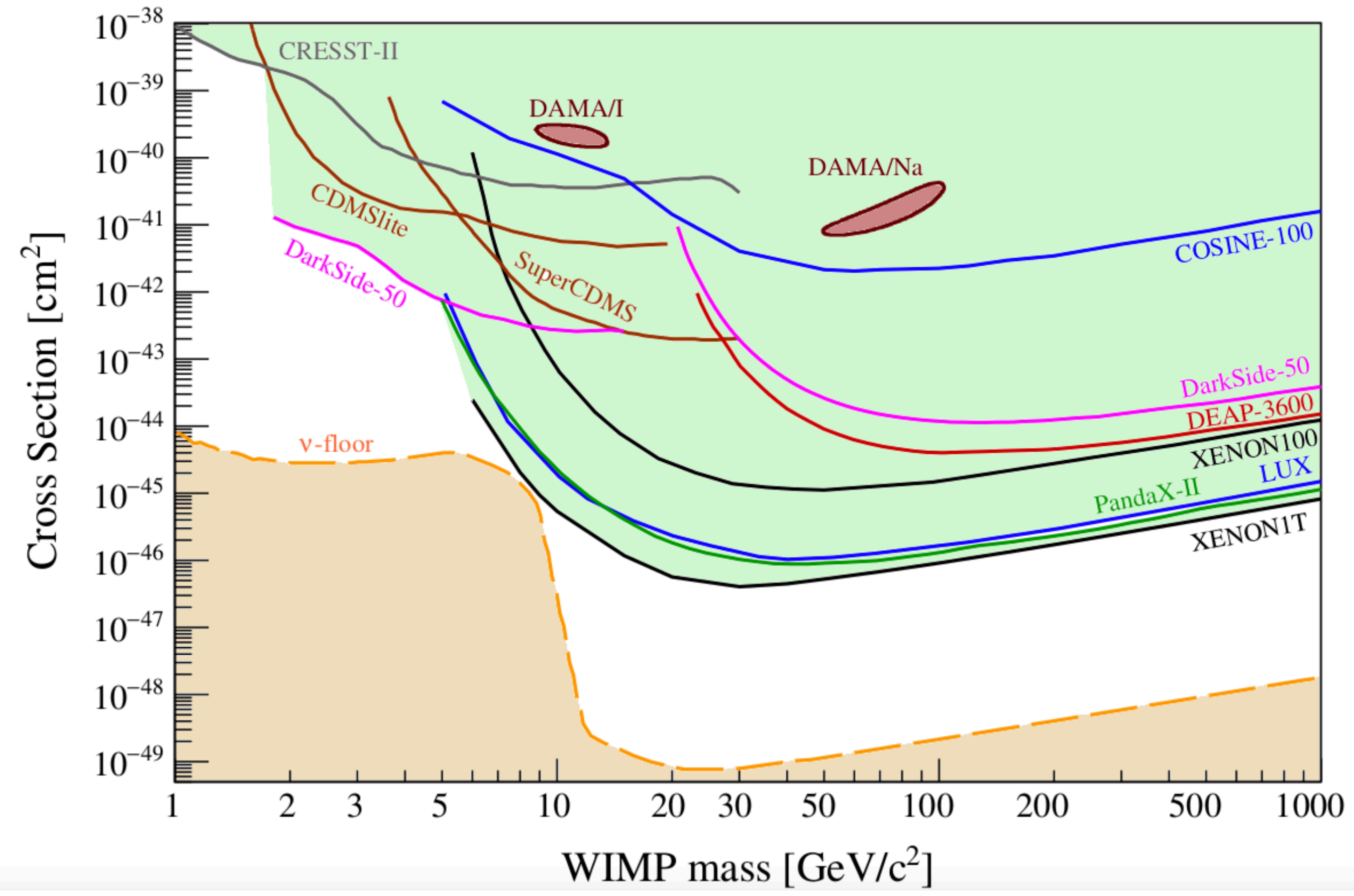
Is neutrino physics correlated with DM?

Yes!

- **Properties of neutrinos are similar to these of dark matter**
- **Neutrino is a hot dark matter candidate**
- **Sterile neutrino is typical warm/cold dark matter candidate**
- **The signal of neutrino in direct detection experiments is similar to that of DM**







The issue we concern in this talk

Standard model: Lepton number is accidental global U(1) symmetry

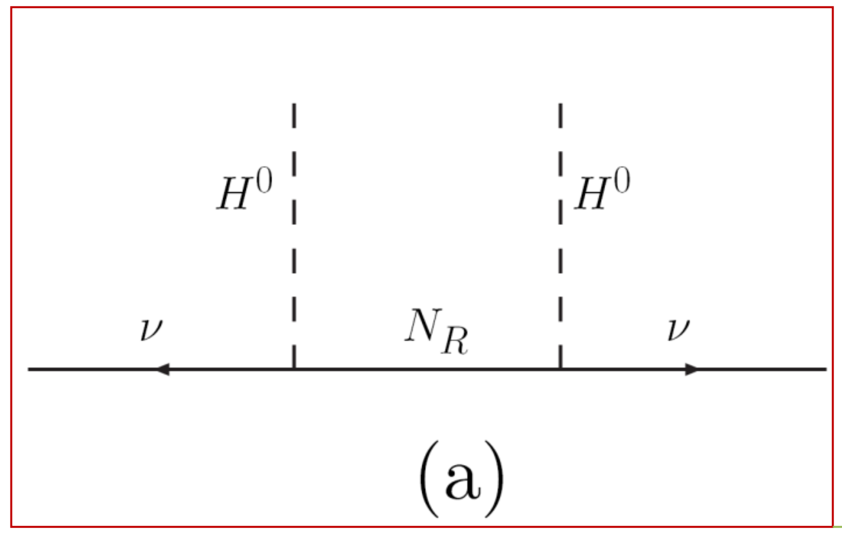
Traditional seesaw mechanisms: $U(1)_L$ is explicitly broken at the tree-level

Is lepton number spontaneously broken, or explicitly broken, or both ?

Connections between neutrino mass and the ALP physics!

Neutrino mass generations

History： Majorana neutrino mass from the dim-5 Weinberg operator $\kappa_{\alpha\beta}\overline{\ell_L^{\alpha}}\tilde{H}\tilde{H}^T\ell_L^{\beta C}$

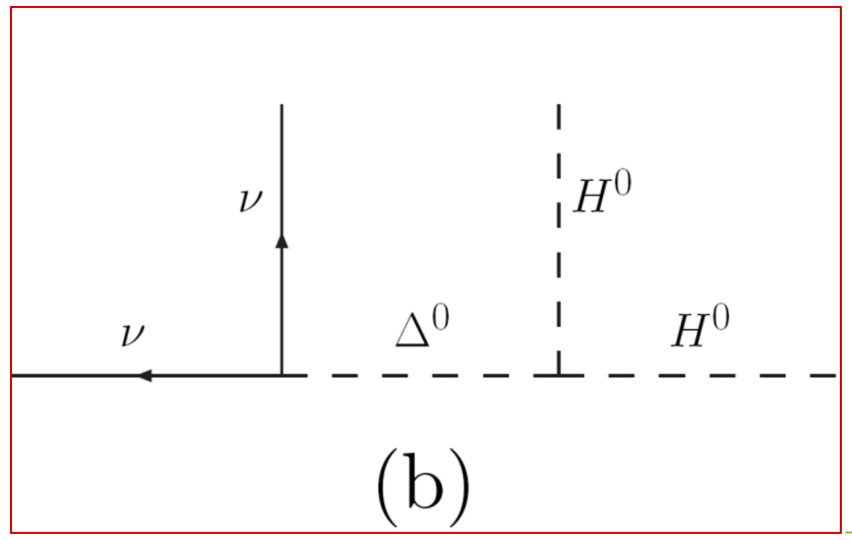


$$-\mathcal{L} = \overline{\ell_L} Y_\nu N_R \tilde{H} + \frac{1}{2}\overline{N_R^C} M_R N_R + \text{h.c.}$$

SU(2)_L fermion singlet

$$M_\nu = -M_D M_R^{-1} M_D^T$$

Minkowski（77）

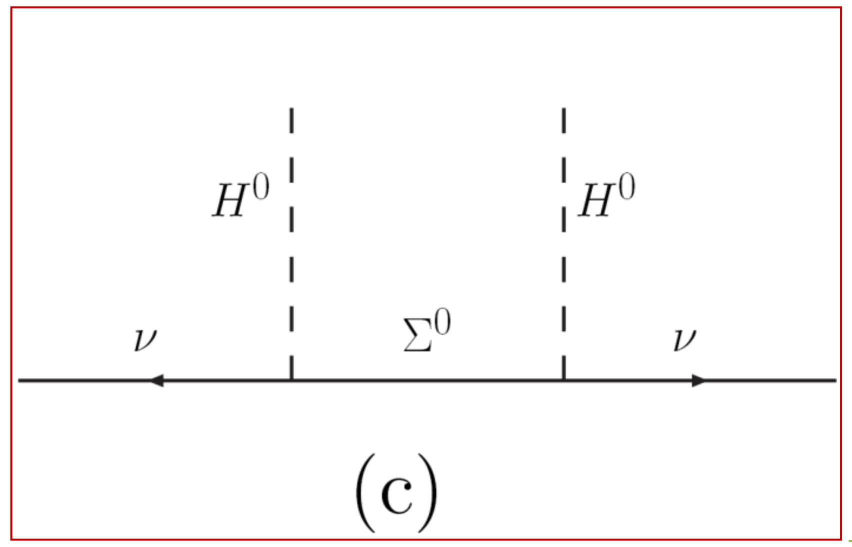


$$-\mathcal{L} = \frac{1}{2}\overline{\ell_L} Y_\Delta \Delta \varepsilon \ell_L^C - \lambda_\Delta M_\Delta H^T \varepsilon \Delta H + \text{h.c.}$$

SU(2)_L scalar triplet

$$M_\nu = Y_\Delta v_\Delta$$

Magg， Wetterich（80）

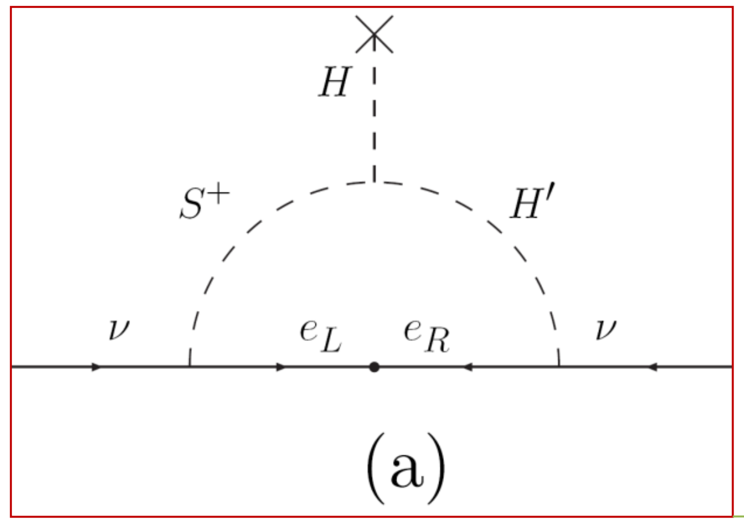


$$-\mathcal{L} = \frac{1}{2}\text{Tr}\left[\Sigma M_\Sigma \Sigma^C\right] + \sqrt{2}\overline{\ell_L} Y_\Sigma \Sigma \tilde{H} + \text{h.c.}$$

SU(2)_L fermion triplet

$$M_\nu = -M_D M_\Sigma^{-1} M_D^T$$

Foot，Lew， He，Joshi（89）

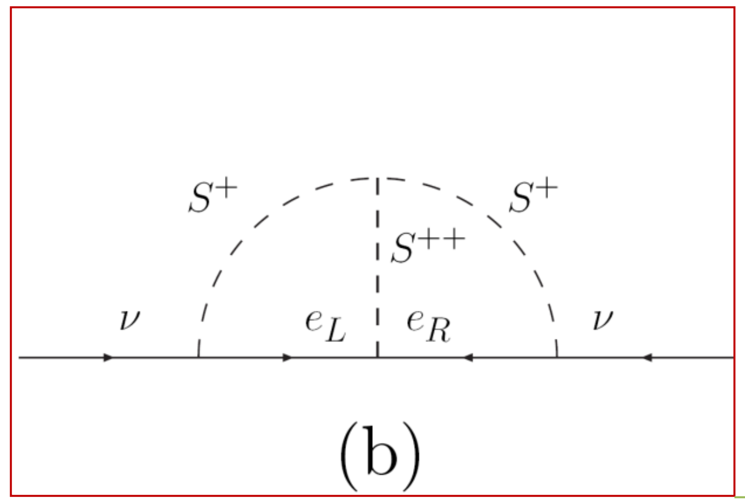


$$-\mathcal{L} = \ell_L^T Y_S \varepsilon \ell_L S^+ - \mu H^T \varepsilon H' S^- + \text{h.c.}$$

Zee（80）

$$(M_\nu)_{\alpha\beta} = A\left(Y_S\right)_{\alpha\beta}\left(m_\alpha^2 - m_\beta^2\right)$$

Charged scalar singlet+ doublets

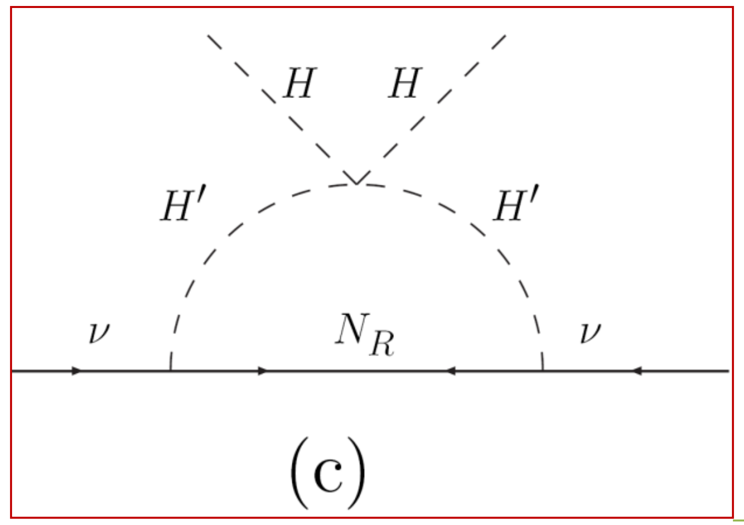


$$-\mathcal{L} = \ell_L^T Y_S \varepsilon \ell_L S^+ + e_R^T F_S e_R S^{++} + \mu S^- S^- S^{++} + \text{h.c.}$$

Babu（88）

$$(M_\nu)_{\alpha\beta} = 8\mu \sum_{\kappa\lambda} m_\kappa m_\lambda \left(Y_S\right)_{\alpha\kappa}\left(F_S\right)_{\kappa\lambda}\left(Y_S\right)_{\lambda\beta} I_{\kappa\lambda}$$

Two charged scalar singlets



$$-\mathcal{L} = \overline{\ell_L} Y_\nu N_R \tilde{H}' + \frac{1}{2}\overline{N_R^C} M_R N_R + \frac{1}{2}\lambda_5 (H^\dagger H')^2 + \text{h.c.}$$

Ma （98）

$$(M_\nu)_{\alpha\beta} = \sum_i (Y_\nu)_{\alpha i} M_i^{-1} (Y_\nu)_{\beta i} I(M_i^2/m_0^2)$$

Higgs doublets + Majorana neutrino+ Z2 symmetry

● ● ●

Y. Cai, T. Han, T. Li, 1711.02180

Neutrino mass generations

Neutrino mass from higher dimensional effective operators

Majorana neutrino mass the tree-level:

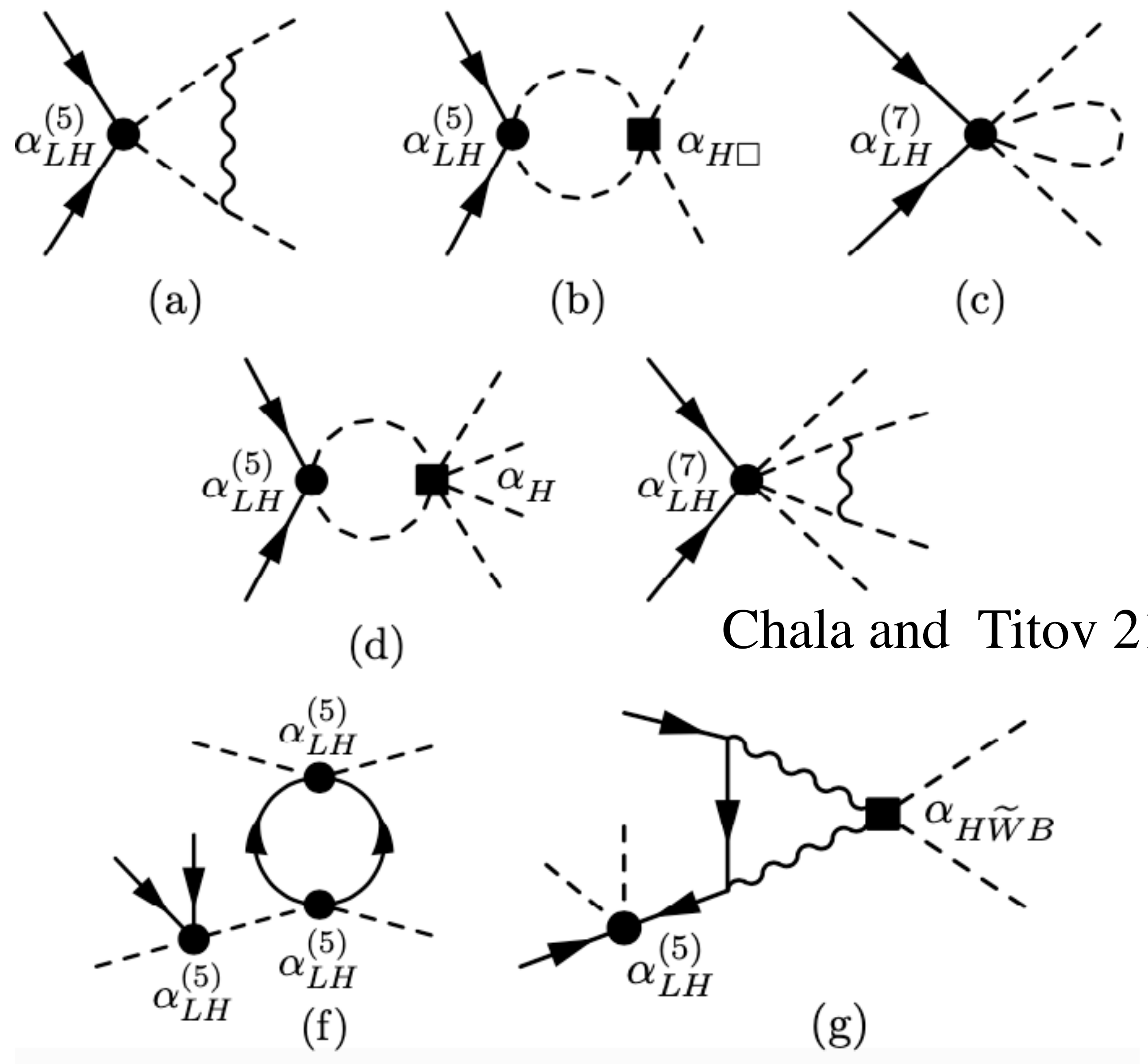
(F. Bonnet et al, 2009; Y. Liao, 2011)

The unique operator of dim 2n+5, that can give neutrino masses at the tree level is

$$\mathcal{O}^{2n+5} = \mathcal{O}_{\text{weinberg}} \times \frac{(H^\dagger H)^n}{\Lambda^{2n}}$$

Neutrino mass from loop corrections:

- *Dimension-5: Weinberg operator for neutrino-masses (S. Weinberg 1979)*
- *Dimension-6: W. Buchmuller and D. Wyler, 1986; B. Grzadkowski et al, 2010; ……*
- *Dimension-7: L. Lehman, 2014; Y. Liao and X.D. Ma, 2016; ……*
  Dimension-8: C.W. Murphy, 2020; H.L. Li et al., 2020; … …
- *Dimension-9: Y. Liao and X.D. Ma, 2020; H.L. Li et al, 2020, 2021; … …*



Chala and Titov 2104.08248

$\mathcal{O}_{2B}$	$-\frac{1}{2}(\partial_\mu B^{\mu\nu})(\partial^\rho B_{\rho\nu})$
$\mathcal{O}_{2W}$	$-\frac{1}{2}(D_\mu W^{I\mu\nu})(D^\rho W^I_{\rho\nu})$
$\mathcal{O}_{BDH}$	$\partial_\nu B^{\mu\nu}(H^\dagger i\overleftrightarrow{D}_\mu H)$
$\mathcal{O}_{WDH}$	$D_\nu W^{I\mu\nu}(H^\dagger i\overleftrightarrow{D}^I_\mu H)$
$\mathcal{O}_{DH}$	$(D_\mu D^\mu H)^\dagger(D_\nu D^\nu H)$
$\mathcal{O}'_{HD}$	$(H^\dagger H)(D_\mu H)^\dagger(D^\mu H)$
$\mathcal{O}''_{HD}$	$(H^\dagger H)D_\mu(H^\dagger i\overleftrightarrow{D}^\mu H)$
$\mathcal{O}_{LD}$	$\frac{i}{2}\overline{L}\{D_\mu D^\mu, \not{D}\}L$
$\mathcal{O}'^{(1)}_{HL}$	$(H^\dagger H)(\overline{L}i\overleftrightarrow{\not{D}}L)$
$\mathcal{O}''^{(1)}_{HL}$	$\partial_\mu(H^\dagger H)(\overline{L}\gamma^\mu L)$
$\mathcal{O}'^{(3)}_{HL}$	$(H^\dagger\sigma^I H)(\overline{L}i\overleftrightarrow{\not{D}}^I L)$
$\mathcal{O}''^{(3)}_{HL}$	$D_\mu(H^\dagger\sigma^I H)(\overline{L}\gamma^\mu\sigma^I L)$
$\mathcal{O}^{(R)}_{LHD}$	$\epsilon_{ij}\epsilon_{mn}L^iCL^mH^j\Box H^n$

Majoron & neutrino mass via type-II seesaw

Type-II seesaw + spontaneous breaking $U(1)_L$ symmetry

LNV term!

$$V(S,\Phi,\Delta) = V(\Phi,\Delta) - \mu_S^2(S^\dagger S) + \lambda_6 (S^\dagger S)^2$$
$$+\lambda_7(S^\dagger S)(\Phi^\dagger\Phi) + \lambda_8(S^\dagger S)\mathrm{Tr}(\Delta^\dagger\Delta) + \mu\Phi^T i\tau_2\Delta^\dagger\Phi + \lambda S\Phi^T i\tau_2\Delta^\dagger\Phi + \mathrm{h.c.},$$

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v_\phi + \phi + i\chi}{\sqrt{2}} \end{pmatrix} \qquad \Delta = \begin{pmatrix} \frac{\Delta^+}{\sqrt{2}} & \Delta^{++} \\ \frac{v_\Delta + \delta + i\xi}{\sqrt{2}} & \frac{\Delta^+}{\sqrt{2}} \end{pmatrix} \qquad S = \frac{v_s + \tilde{s} + i\tilde{a}}{\sqrt{2}}$$

$\tilde{a}$: **Majoron**

Yukawa Interaction $\quad -\mathscr{L}_\Delta = Y_{\alpha\beta}\overline{\ell_L^{\alpha C}} i\sigma^2 \Delta \ell_L^\beta + h.c.$

Key term: $\quad \mu\Phi^T i\sigma^2\Delta\Phi + h.c.$

Majoron & neutrino mass via type-II seesaw

Gauge boson masses

$$m_W^2 = \frac{g^2}{4}\left(v_\phi^2 + 2v_\Delta^2\right)\,, \quad m_Z^2 = \frac{g^2}{4\cos^2\theta_W}\left(v_\phi^2 + 4v_\Delta^2\right)\,. \qquad \rho \equiv \frac{m_W^2}{m_Z^2\cos^2\theta_W} = \frac{1+\frac{2v_\Delta^2}{v_\phi^2}}{1+\frac{4v_\Delta^2}{v_\phi^2}}\,.$$

Scalar mixings and masses

$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \mathscr{R}(\beta)\begin{pmatrix} \phi^\pm \\ \Delta^\pm \end{pmatrix}\,, \quad \begin{pmatrix} G \\ A \\ a \end{pmatrix} = \mathscr{V}(\beta_1', \beta_2', \beta_3')\begin{pmatrix} \chi \\ \xi \\ \tilde{a} \end{pmatrix}\,, \quad \begin{pmatrix} h \\ H \\ s \end{pmatrix} = \mathscr{U}(\alpha_1, \alpha_2, \alpha_3)\begin{pmatrix} \phi \\ \delta \\ \tilde{s} \end{pmatrix}\,,$$

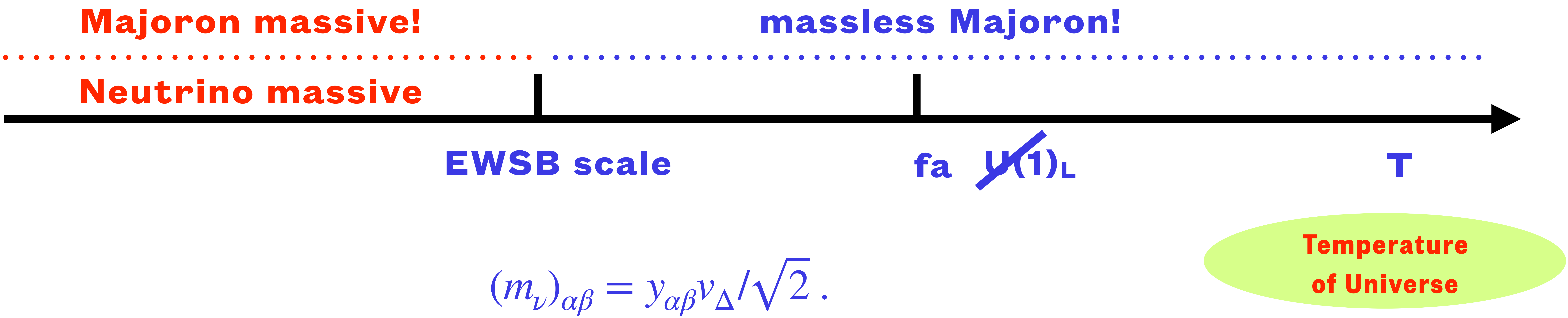
Mixing angl for pseudo-scalars

Majoron gets non-zero mass from the mixing!

$$\tan\beta = \frac{\sqrt{2}v_\Delta}{v_\phi}\,, \quad \tan\beta_1' = \frac{2v_\Delta}{v_\phi}\,, \quad \tan\beta_2' = 0\,, \quad \tan 2\beta_3' = \frac{2\lambda v_\Delta v_s v_\phi\sqrt{v_\phi^2 + 4v_\Delta^2}}{v_\phi^2\left(-\lambda v_\Delta^2 + \lambda v_s^2 + \sqrt{2}\mu v_s\right) + 4v_\Delta^2 v_s\left(\sqrt{2}\mu + \lambda v_s\right)}\,.$$

Majoron & neutrino mass via type-II seesaw

Sequential breaking of various symmetries



$$(m_\nu)_{\alpha\beta} = y_{\alpha\beta} v_\Delta/\sqrt{2}\,.$$

$$m_a^2 = \frac{\sqrt{2}\mu v_\phi^2 v_\Delta (v_\phi^2 + 4v_\Delta^2)}{2v_\phi^2(v_\Delta^2 + v_s^2) + 8v_\Delta^2 v_s^2} \simeq \frac{\mu v_\phi^2 v_\Delta}{\sqrt{2}v_s^2}\,,$$

For experts of axion physics

Majoron mass should arise from cosine like potential!

$$\ell_L \to e^{-\frac{ia}{2f}}\ell_L \qquad S \to e^{+\frac{ia}{f}}S$$

$$E_R \to e^{-\frac{ia}{2f}}E_R \qquad \Delta \to e^{-\frac{ia}{f}}\Delta$$

$$H \to H$$

$$-\mathscr{L}_{\rm int} \supset \mu e^{i\frac{a}{f_a}}\Phi^T i\tau_2 \Delta^\dagger \Phi + {\rm h.c.}.$$

After electroweak symmetry breaking

$$-\mathscr{L}_{\rm int} \supset \frac{\mu v_\Phi^2 v_\Delta}{\sqrt{2}} \cos\left(\frac{a}{f_a}\right)$$

Non-zero Majoron mass

Majoron & neutrino mass via type-I seesaw

Lagrangian of the type-I seesaw mechanism with LNV

$$\mathscr{L} \sim (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) + \mu_\Phi^2 \Phi^\dagger \Phi - \lambda_1 (\Phi^\dagger \Phi)^2 - \lambda_2 (\Phi^\dagger \Phi)(H^\dagger H)$$

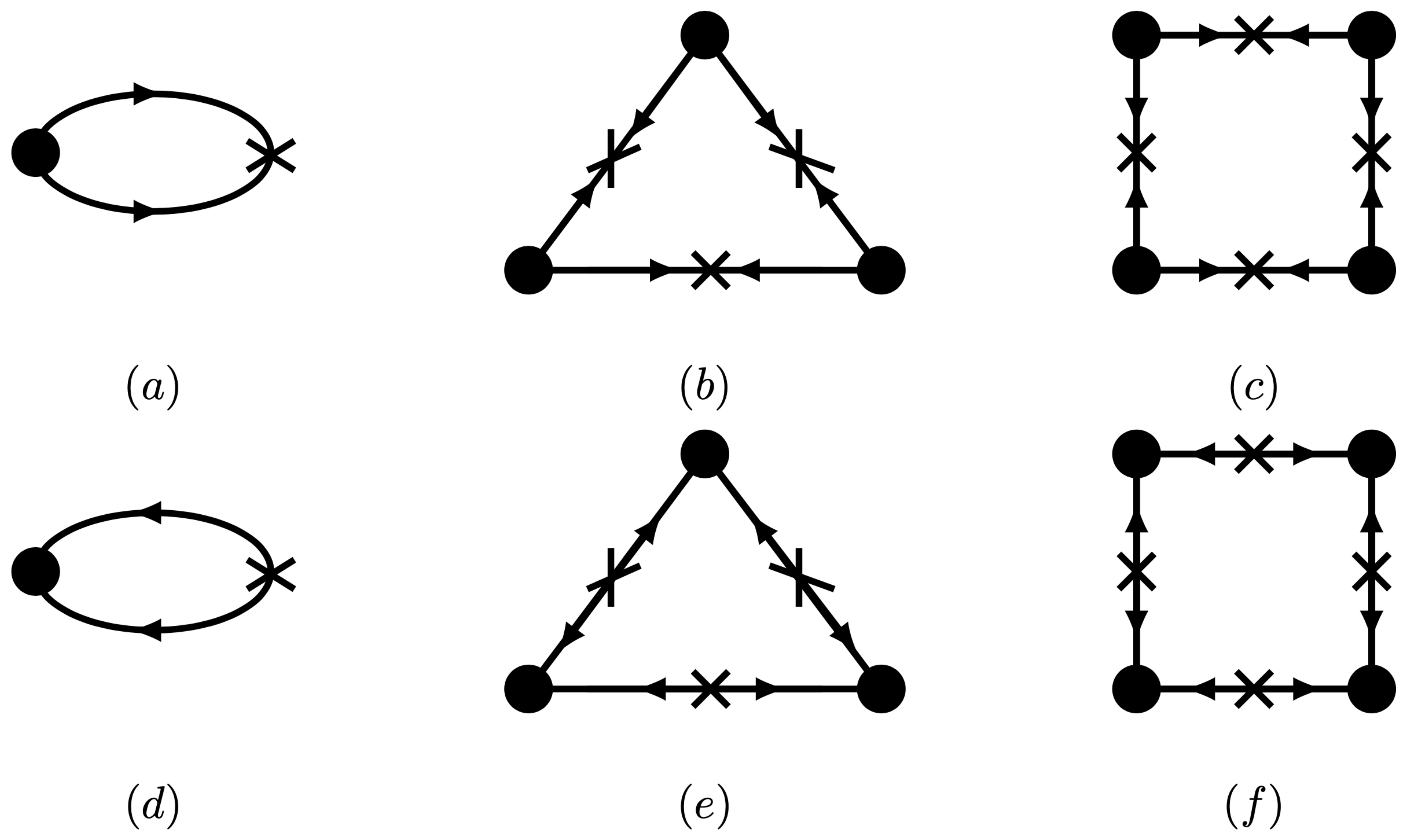
$$-\left[Y_{\rm N} \overline{\ell_L} \tilde{H} N_R + \frac{1}{2} \overline{N_R^C} \left(Y_{\rm M} \Phi + m \right) N_R + \mathrm{h.c.} \right]$$

$$\left.\begin{array}{l} \Phi = \dfrac{1}{\sqrt{2}} \phi e^{i\theta} = \dfrac{1}{\sqrt{2}} \phi e^{ia/f} \\[2ex] f_{L,R} \to e^{-i\theta/2} f_{L,R} \end{array}\right\} \longrightarrow \frac{1}{2} e^{-i\theta} \overline{N_R^C} m N_R + \mathrm{h.c.}$$

Majoron neutrino interaction

Majoron & neutrino mass via type-I seesaw

Majoron neutrino interaction $\frac{1}{2}e^{-i\theta}\overline{N_R^C}mN_R + \mathrm{h.c.}$



$$V \sim -\frac{1}{16\pi^2}\sum_{n=1}^{4} a_n \cos n\theta\,.$$

a_1	a_2	a_3	a_4
$mM^3\left(1+\log\frac{m^2}{\mu^2}\right)$	$2m^2M^2\log\frac{m^2}{\mu^2}$	m^3M	m^4

- Dark matter phenomenology
- Matter antimatter asymmetry of the universe
- …

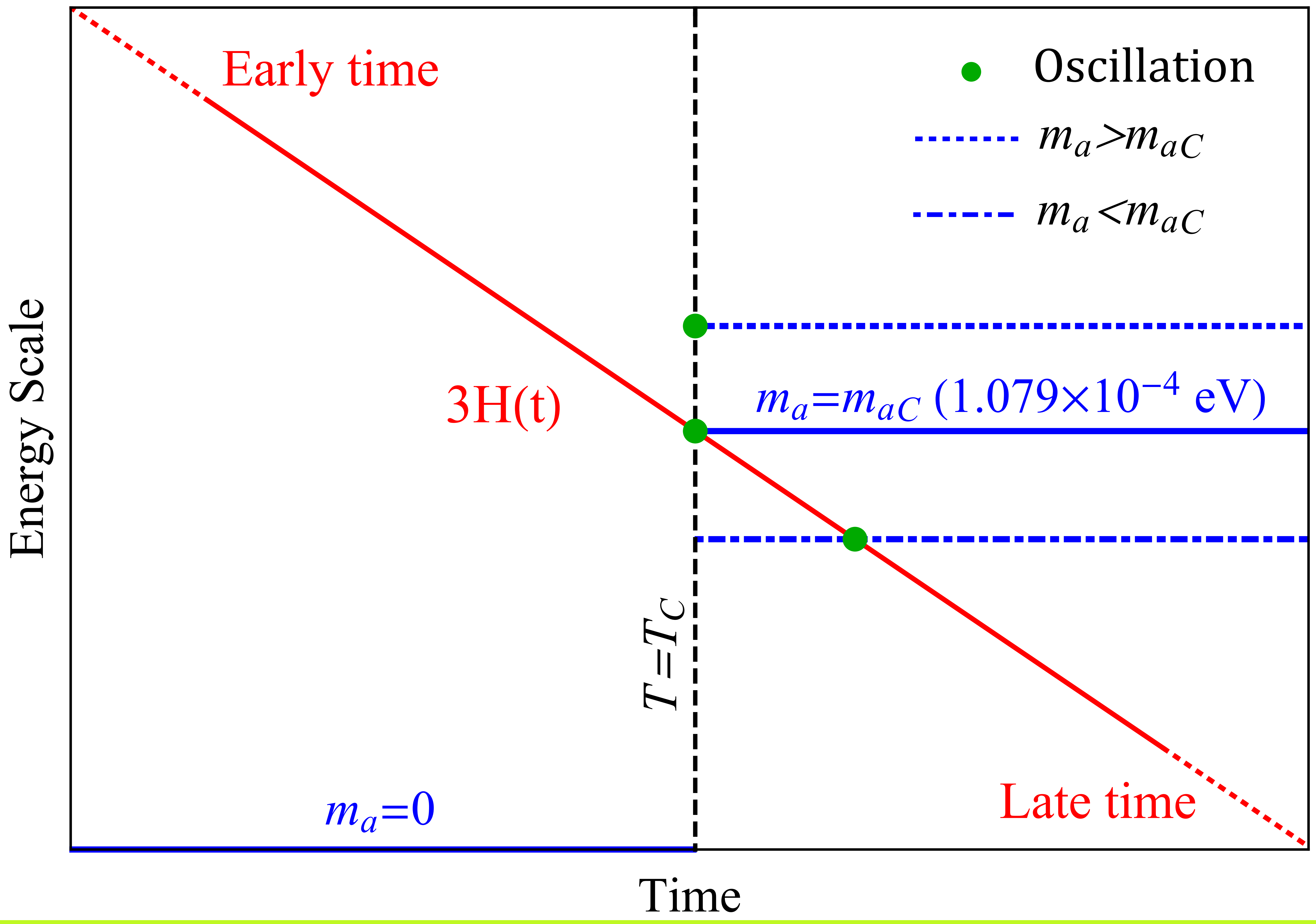
Wei Chao, Yingquan Peng, arXiv:2308.XXXXX

Majoron DM—oscillation time

$$m_a^2(T) = \begin{cases} \dfrac{\mu v_\phi^2(T) v_\Delta(T)}{\sqrt{2} f_a^2}\,, & T \le T_C \\ 0\,, & T > T_C \end{cases}$$

$$T_{\rm osc} = \begin{cases} T_*\,, & m_a < m_{aC} \\ T_C\,, & m_a \ge m_{aC} \end{cases}$$

$$m_{aC} = 1.079 \times 10^{-4}\,\mathrm{eV}$$



Majoron DM—simulations

Equation of motion

$$\ddot{\theta} + 3H\dot{\theta} + m_a^2(T)\theta = 0$$

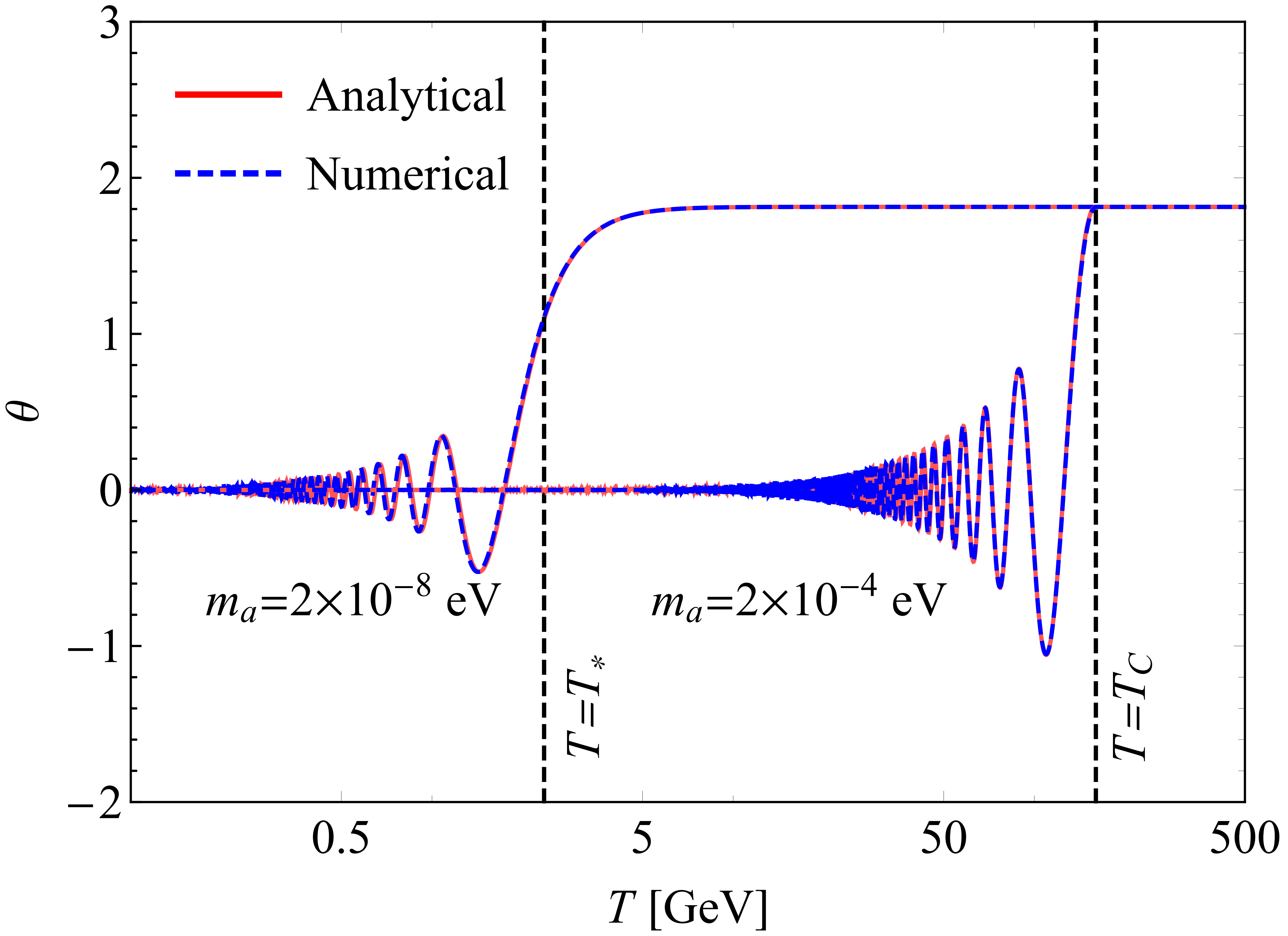
Analytical results

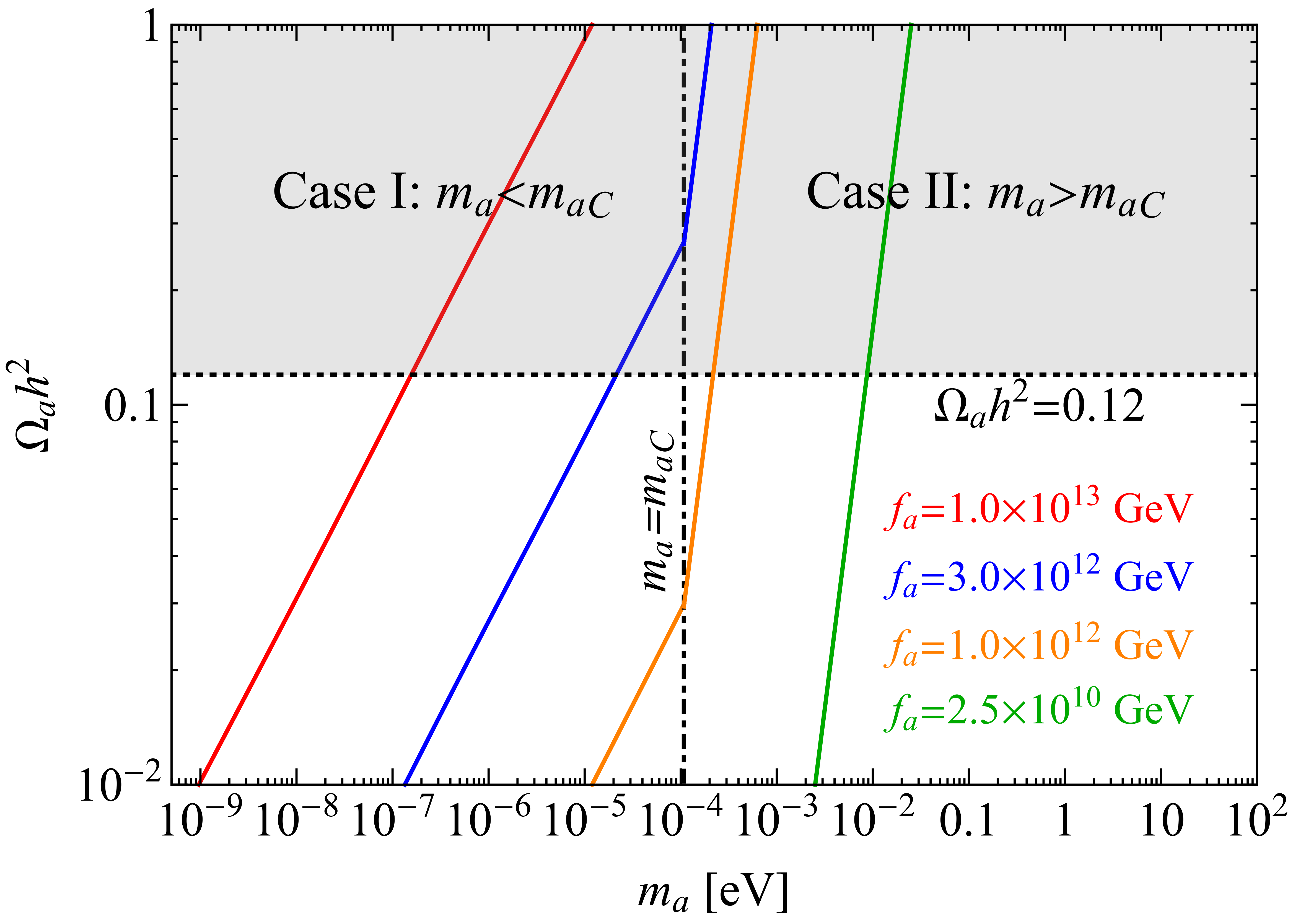
$$\begin{aligned}\theta(t) = -\pi\Big[& -2m_a t_i J_{\frac{1}{4}}(m_a t) Y_{-\frac{3}{4}}(m_a t_i) + 2m_a t_i Y_{\frac{1}{4}}(m_a t) J_{-\frac{3}{4}}(m_a t_i) \\ & -Y_{\frac{1}{4}}(m_a t) J_{\frac{1}{4}}(m_a t_i) - 2m_a t_i Y_{\frac{1}{4}}(m_a t)_{\frac{5}{4}}(m_a t_i) + J_{\frac{1}{4}}(m_a t) Y_{\frac{1}{4}}(m_a t_i) \\ & +2m_a t_i J_{\frac{1}{4}}(m_a t) Y_{\frac{5}{4}}(m_a t_i)\Big] \Big/ \Big\{ 2\sqrt{3}\, t^{\frac{1}{4}} t_i^{\frac{3}{4}} \Big[J_{\frac{1}{4}}(m_a t_i) Y_{-\frac{3}{4}}(m_a t_i) - J_{-\frac{3}{4}}(m_a t_i) Y_{\frac{1}{4}}(m_a t_i) \\ & +J_{\frac{5}{4}}(m_a t_i) Y_{\frac{1}{4}}(m_a t_i) - J_{\frac{1}{4}}(m_a t_i) Y_{\frac{5}{4}}(m_a t_i)\Big]\end{aligned}$$

Majoron energy density

$$\rho_a(T_0) = \frac{1}{2} m_a^2 f_a^2 \langle \theta_{a,i}^2 \rangle \frac{g_{*s}(T_0)}{g_{*s}(T_{\rm osc})} \left(\frac{T_0}{T_{\rm osc}}\right)^3$$

Majoron DM—Relic Density





Majoron interactions from mixings

Interactions with scalars

Vertices	Coefficients
a^4	$\frac{1}{4}\lambda_1 V_{13}^4 + \frac{1}{4}\lambda_4 V_{13}^2 V_{23}^2 + \frac{1}{4}\lambda_5 V_{13}^2 V_{23}^2 + \frac{1}{2}\lambda V_{13}^2 V_{23} V_{33} + \frac{1}{4}\lambda_2 V_{23}^4 + \frac{1}{4}\lambda_3 V_{23}^4 + \frac{1}{4}\lambda_6 V_{33}^4$
$a^3 G$	$\lambda_1 V_{11} V_{13}^3 + \frac{1}{2}\lambda_4 V_{11} V_{13} V_{23}^2 + \frac{1}{2}\lambda_5 V_{11} V_{13} V_{23}^2 + \lambda V_{11} V_{13} V_{23} V_{33} + \frac{1}{2}\lambda_4 V_{13}^2 V_{21} V_{23}$ $+\frac{1}{2}\lambda_5 V_{13}^2 V_{21} V_{23} + \frac{1}{2}\lambda V_{13}^2 V_{21} V_{33} + \frac{1}{2}\lambda V_{13}^2 V_{23} V_{31} + \lambda_2 V_{21} V_{23}^3 + \lambda_3 V_{21} V_{23}^3 + \lambda_6 V_{31} V_{33}^3$
$a^2 h^2$	$\frac{1}{2}\lambda_1 U_{11}^2 V_{13}^2 + \frac{1}{4}\lambda_4 U_{11}^2 V_{23}^2 + \frac{1}{4}\lambda_5 U_{11}^2 V_{23}^2 - \frac{1}{2}\lambda U_{11}^2 V_{23} V_{33} + \lambda U_{11} U_{21} V_{13} V_{33}$ $-\lambda U_{11} U_{31} V_{13} V_{23} + \frac{1}{4}\lambda_4 U_{21}^2 V_{13}^2 + \frac{1}{4}\lambda_5 U_{21}^2 V_{13}^2 + \frac{1}{2}\lambda_2 U_{21}^2 V_{23}^2 + \frac{1}{2}\lambda_3 U_{21}^2 V_{23}^2 + \frac{1}{2}\lambda U_{21} U_{31} V_{13}^2 + \frac{1}{2}\lambda_6 U_{31}^2 V_{33}^2$
$a^2 h$	$\lambda_1 U_{11} v_\phi V_{13}^2 + \frac{1}{2}\lambda_4 U_{11} v_\phi V_{23}^2 + \frac{1}{2}\lambda_5 U_{11} v_\phi V_{23}^2 - \lambda U_{11} v_\phi V_{23} V_{33} - \sqrt{2}\mu U_{11} V_{13} V_{23}$ $-\lambda U_{11} V_{13} V_{23} v_s + \lambda U_{11} V_{13} V_{33} v_\Delta + \lambda U_{21} v_\phi V_{13} V_{33} + \frac{1}{\sqrt{2}}\mu U_{21} V_{13}^2 + \frac{1}{2}\lambda_4 U_{21} V_{13}^2 v_\Delta + \frac{1}{2}\lambda_5 U_{21} V_{13}^2 v_\Delta$ $+\frac{1}{2}\lambda U_{21} V_{13}^2 v_s + \lambda_2 U_{21} V_{23}^2 v_\Delta + \lambda_3 U_{21} V_{23}^2 v_\Delta - \lambda U_{31} v_\phi V_{13} V_{23} + \frac{1}{2}\lambda U_{31} V_{13}^2 v_\Delta + \lambda_6 U_{31} V_{33}^2 v_s$
$a^2 G^2$	$\frac{3}{2}\lambda_1 V_{11}^2 V_{13}^2 + \frac{1}{4}\lambda_4 V_{11}^2 V_{23}^2 + \frac{1}{4}\lambda_5 V_{11}^2 V_{23}^2 + \frac{1}{2}\lambda V_{11}^2 V_{23} V_{33} + \lambda_4 V_{11} V_{13} V_{21} V_{23} + \lambda_5 V_{11} V_{13} V_{21} V_{23}$ $+\lambda V_{11} V_{13} V_{21} V_{33} + \lambda V_{11} V_{13} V_{23} V_{31} + \frac{1}{4}\lambda_4 V_{13}^2 V_{21}^2 + \frac{1}{4}\lambda_5 V_{13}^2 V_{21}^2 + \frac{1}{2}\lambda V_{13}^2 V_{21} V_{31} + \frac{3}{2}\lambda_2 V_{21}^2 V_{23}^2 + \frac{3}{2}\lambda_3 V_{21}^2 V_{23}^2 + \frac{3}{2}\lambda_6 V_{31}^2 V_{33}^2$
$a^2 G^+ G^-$	$\lambda_1 V_{13}^2 \cos^2\beta + \frac{1}{2}\lambda_4 V_{13}^2 \sin^2\beta + \frac{1}{4}\lambda_5 V_{13}^2 \sin^2\beta + \frac{1}{\sqrt{2}}\lambda_5 V_{13} V_{23} \sin\beta\cos\beta$ $+\sqrt{2}\lambda V_{13} V_{33} \sin\beta\cos\beta + \lambda_2 V_{23}^2 \sin^2\beta + \frac{1}{2}\lambda_3 V_{23}^2 \sin^2\beta + \frac{1}{2}\lambda_4 V_{23}^2 \cos^2\beta$
$a G^3$	$\lambda_1 V_{11}^3 V_{13} + \frac{1}{2}\lambda_4 V_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda_5 V_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda V_{11}^2 V_{21} V_{33} + \frac{1}{2}\lambda V_{11}^2 V_{23} V_{31}$ $+\frac{1}{2}\lambda_4 V_{11} V_{13} V_{21}^2 + \frac{1}{2}\lambda_5 V_{11} V_{13} V_{21}^2 + \lambda V_{11} V_{13} V_{21} V_{31} + \lambda_2 V_{21}^3 V_{23} + \lambda_3 V_{21}^3 V_{23} + \lambda_6 V_{31}^3 V_{33}$
$a h^2 G$	$\lambda_1 U_{11}^2 V_{11} V_{13} + \frac{1}{2}\lambda_4 U_{11}^2 V_{21} V_{23} + \frac{1}{2}\lambda_5 U_{11}^2 V_{21} V_{23} - \frac{1}{2}\lambda U_{11}^2 V_{21} V_{33} - \frac{1}{2}\lambda U_{11}^2 V_{23} V_{31}$ $+\lambda U_{11} U_{21} V_{11} V_{33} + \lambda U_{11} U_{21} V_{13} V_{31} - \lambda U_{11} U_{31} V_{11} V_{23} - \lambda U_{11} U_{31} V_{13} V_{21} + \frac{1}{2}\lambda_4 U_{21}^2 V_{11} V_{13}$ $+\frac{1}{2}\lambda_5 U_{21}^2 V_{11} V_{13} + \lambda_2 U_{21}^2 V_{21} V_{23} + \lambda_3 U_{21}^2 V_{21} V_{23} + \lambda U_{21} U_{31} V_{11} V_{13} + \lambda_6 U_{31}^2 V_{31} V_{33}$
$a\bar{\nu}\nu$	$V_{23} m_\nu / v_\Delta$

Interactions with fermions

$$\overline{\nu_L^C} i a \lambda_{a\bar{\nu}\nu} \nu_L + \mathrm{h.c.}$$

$$\to \lambda_{a\bar{\nu}\nu} : V_{23} m_\nu / v_\Delta ,$$

$$Y_E \overline{\ell_L} H E_R + \mathrm{h.c.} \to$$

$$\lambda_{aee} \bar{e} i \gamma_5 e$$

$$\to \lambda_{aee} : V_{13} \frac{m_e}{v_h} ,$$

Majoron interactions from anomaly

Schemas

$$-\mathscr{L}_{\rm int} \supset \frac{\lambda}{\sqrt{2}} f_a e^{i\frac{a}{f_a}} \Phi^T i\tau_2 \Delta^\dagger \Phi + {\rm h.c.}..$$

$$\Delta \to \Delta e^{-i\frac{a}{f_a}}$$

$$-\mathscr{L}_{\rm Yukawa} = y_{\alpha\beta} \overline{\ell_L^{\alpha c}} i\tau_2 \Delta' e^{i\frac{a}{f_a}} \ell_L^\beta + \text{h.c.}$$

$$-\mathscr{L}_{\rm Yukawa} = y^E_{\alpha\beta} \overline{\ell_L^{\alpha'}} H e^{\frac{ia}{2f_A}} E_R^\beta + \text{h.c.}$$

$$\left.\begin{aligned} \ell_L &\to e^{\frac{-ia}{2f}} \ell_L \\ E_R &\to e^{\frac{-ia}{2f}} E_R \end{aligned}\right\} \longrightarrow$$

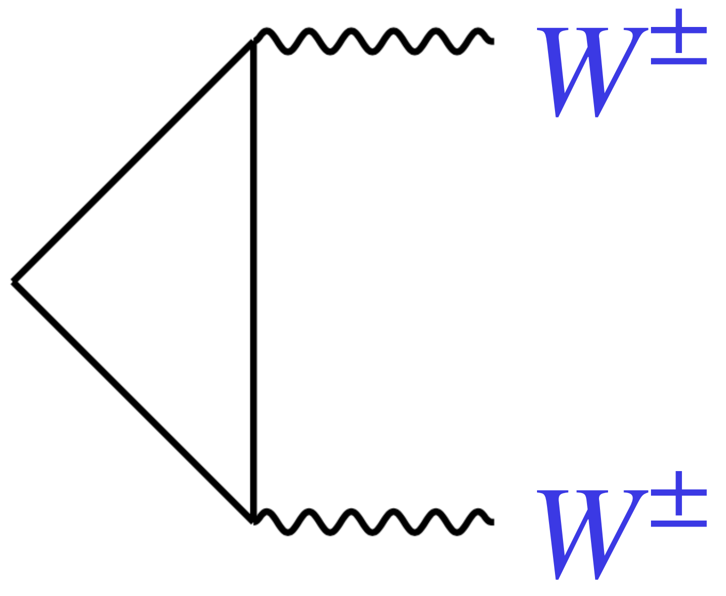
$$\mathscr{L} \to \mathscr{L} - \frac{a}{2f} \partial_\mu \left(\overline{\ell_L} \gamma^\mu \ell_L + \overline{E_R} \gamma^\mu E_R \right)$$

$$= \mathscr{L} - \frac{a}{2f} \partial_\mu J^L_\mu$$

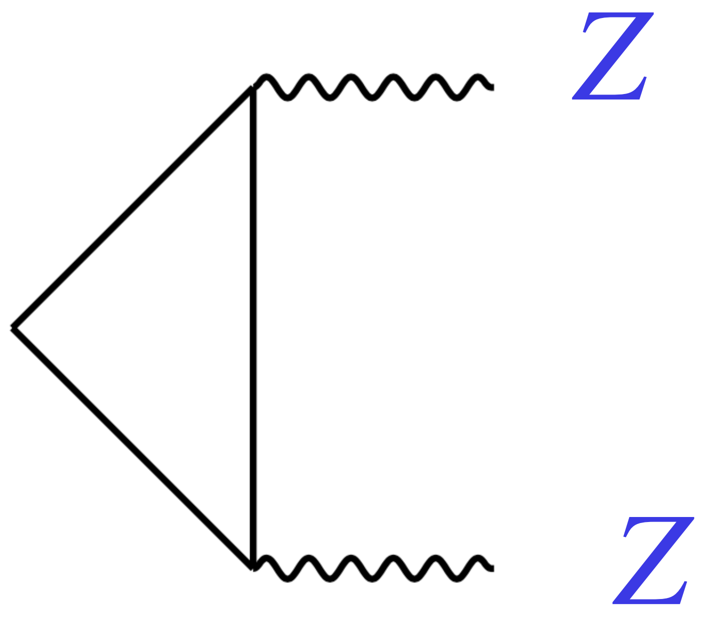
$$= \mathscr{L} + \frac{a}{2f} \frac{N_f}{32\pi^2} \left(g^2 W^a_{\mu\nu} \widetilde{W}^{\mu\nu,a} - g'^2 B_{\mu\nu} \widetilde{B}^{\mu\nu} \right)$$

Majoron interactions from anomaly

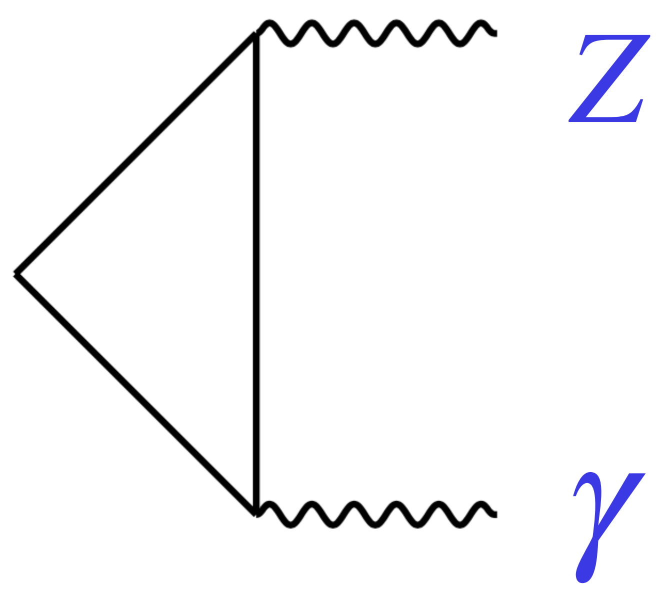
Interactions in mass eigenstates



$$\frac{\alpha N_f}{16\pi f_a} a W_{\mu\nu}\widetilde{W}^{\mu\nu}$$

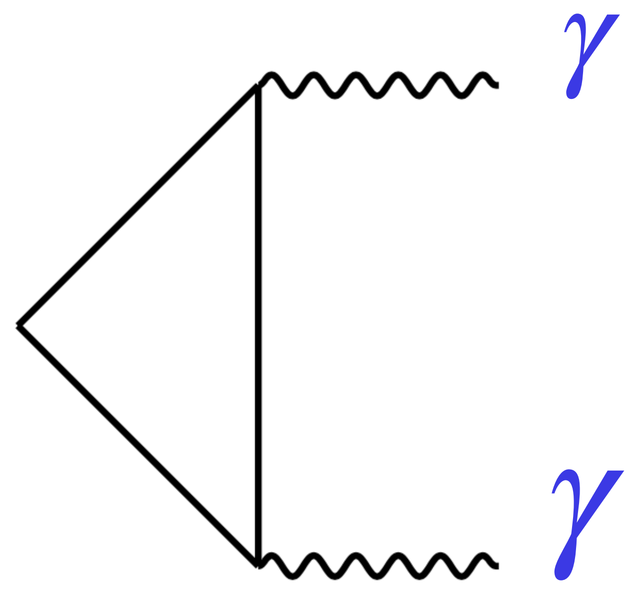


$$\frac{\alpha}{8\pi\cos^2\theta_w f_a}\left(\frac{1}{2}-\sin^2\theta_w\right) aZ_{\mu\nu}\widetilde{Z}^{\mu\nu}$$



$$\frac{\alpha\tan\theta_W}{32\pi f_a} a\left(Z_{\mu\nu}\widetilde{F}^{\mu\nu}+F_{\mu\nu}\widetilde{Z}^{\mu\nu}\right)$$

$$\boxed{\alpha=\frac{g^2}{4\pi}}$$



$$\boxed{0\times aF_{\mu\nu}\widetilde{F}^{\mu\nu}}$$

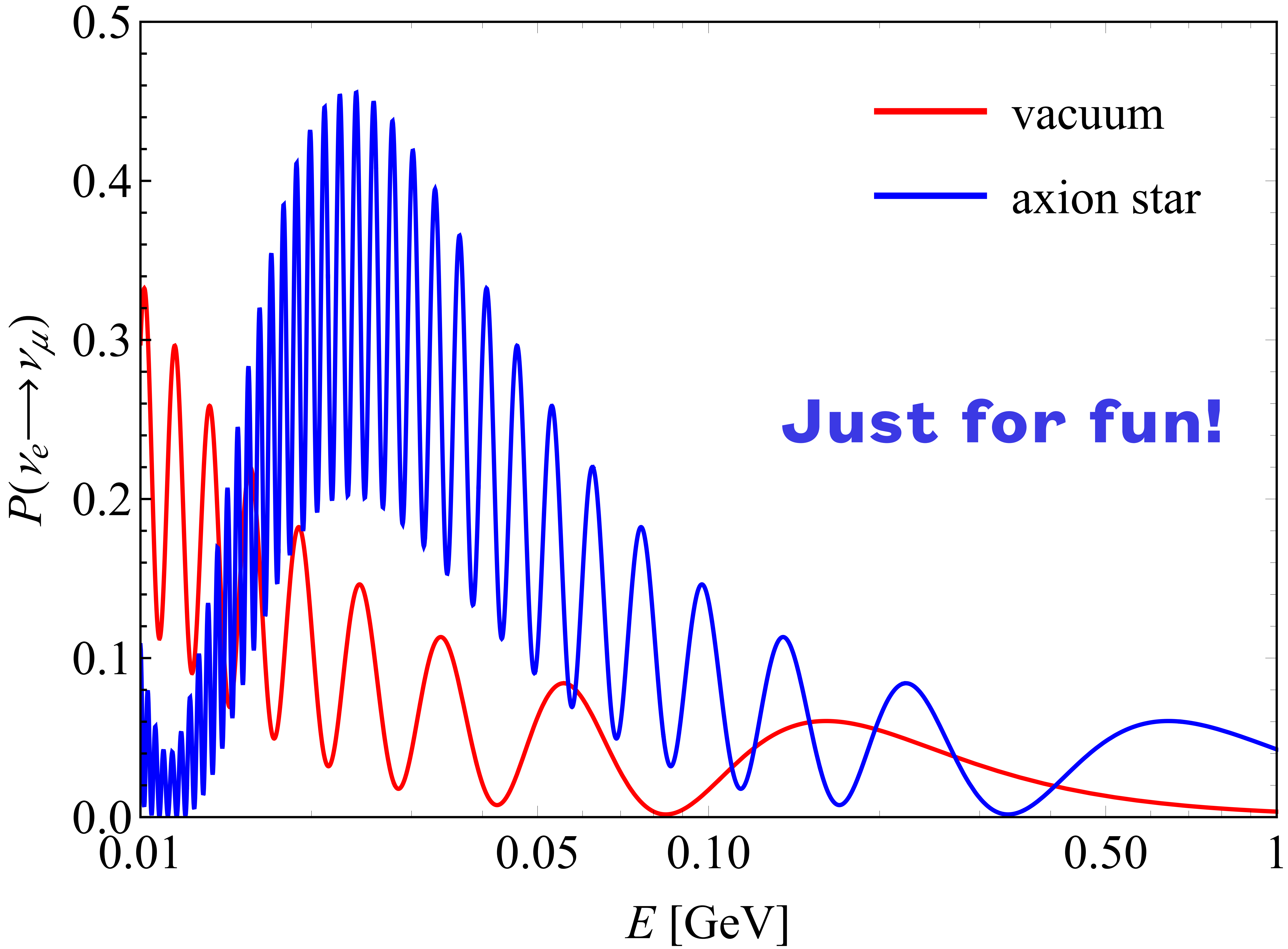
Neutrino oscillation in Majoron star

Effective potential

$$V_{\text{eff}} = i\sqrt{2\rho_a} V_{23} m_a^{-1} v_\Delta^{-1} \cos(m_a t) \overline{\nu_L^C} m_\nu \nu_L + \mathrm{h.c.}$$

Amplitude:

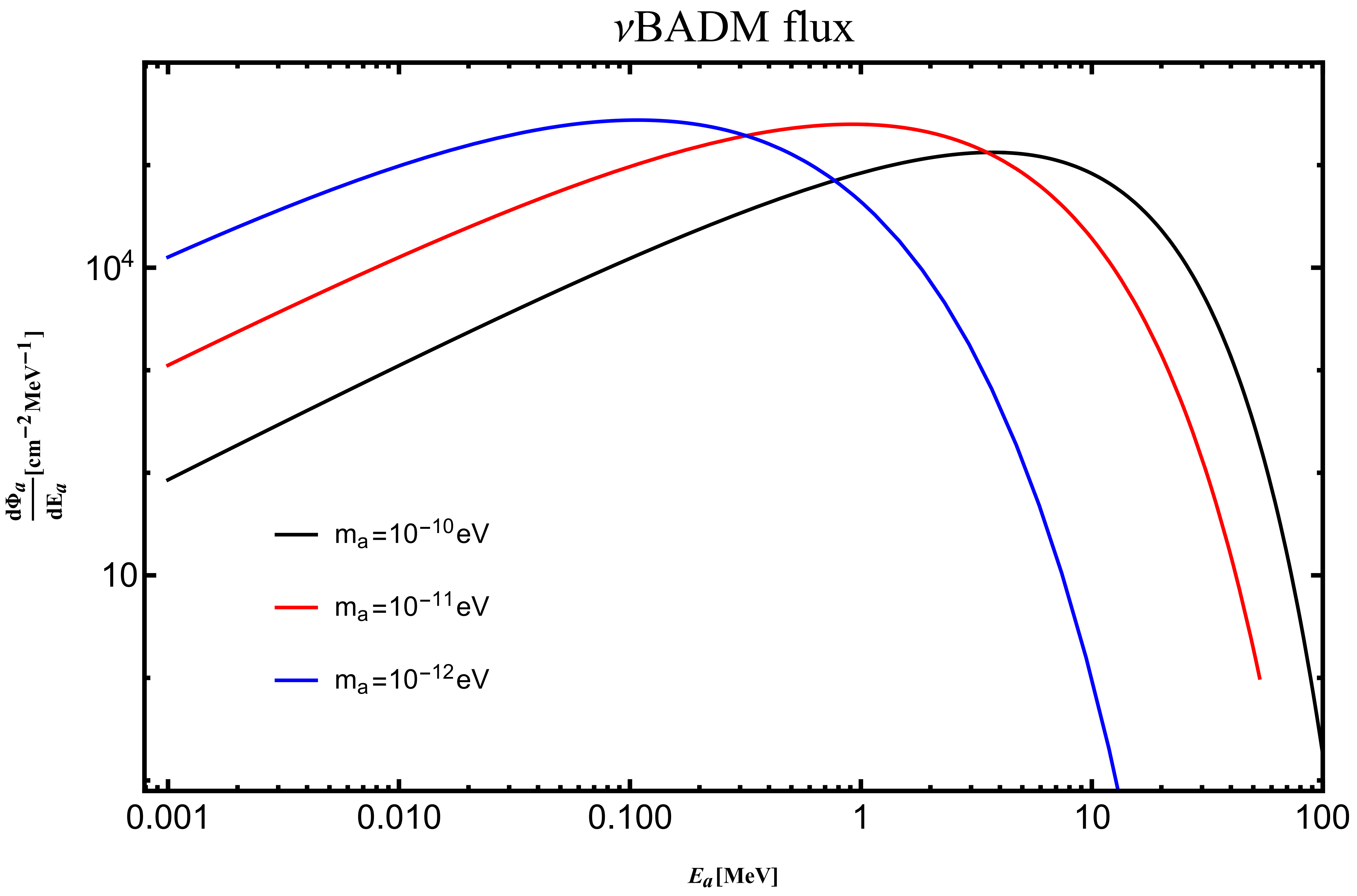
$$A_{\alpha\to\beta} = \sum_i \widehat{U}_{\beta i} \widehat{U}^*_{\alpha i} \exp\left[-i\frac{m_i^2 x}{2E}\left(1 + \frac{\rho_a V_{23}^2}{m_a^2 v_\Delta^2} + \frac{\rho_a V_{23}^2 \cos 2m_a x}{2x m_a^3 v_\Delta^2}\right)\right]$$



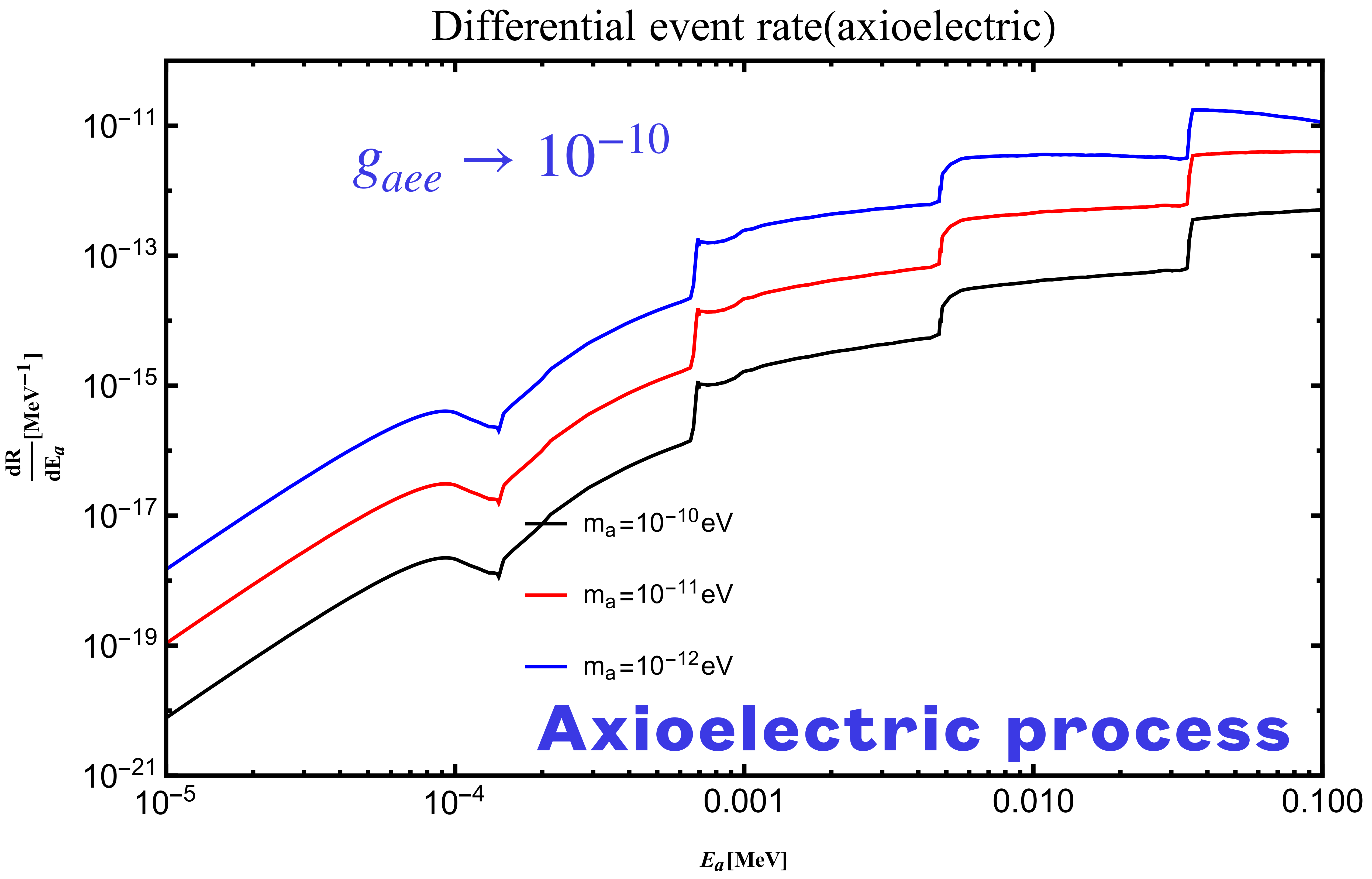
Just for fun!

Direct detections of Majoron DM

Boosted Majoron by supernova ν



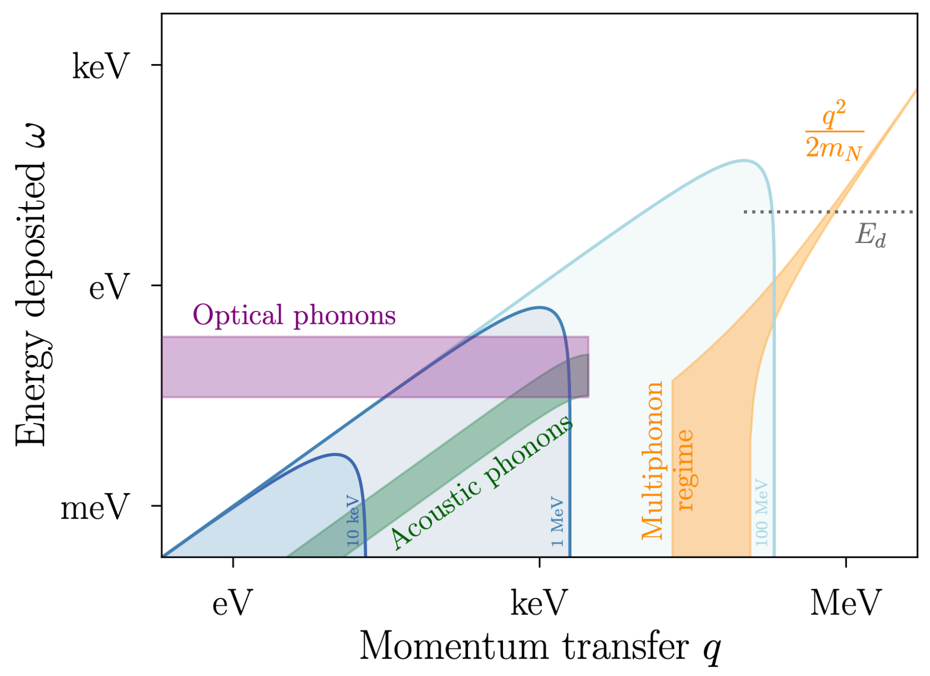
Differential event rate



Direct detections of Majoron DM

Direct detections in condensed matter systems

DM mass	DM energy or momentum	CM scale
50 MeV	$p_\chi \sim 50$ keV	zero-point ion momentum in lattice
20 MeV	$E_\chi \sim 10$ eV	atomic ionization energy
2 MeV	$E_\chi \sim 1$ eV	semiconductor band gap
100 keV	$E_\chi \sim 50$ meV	optical phonon energy

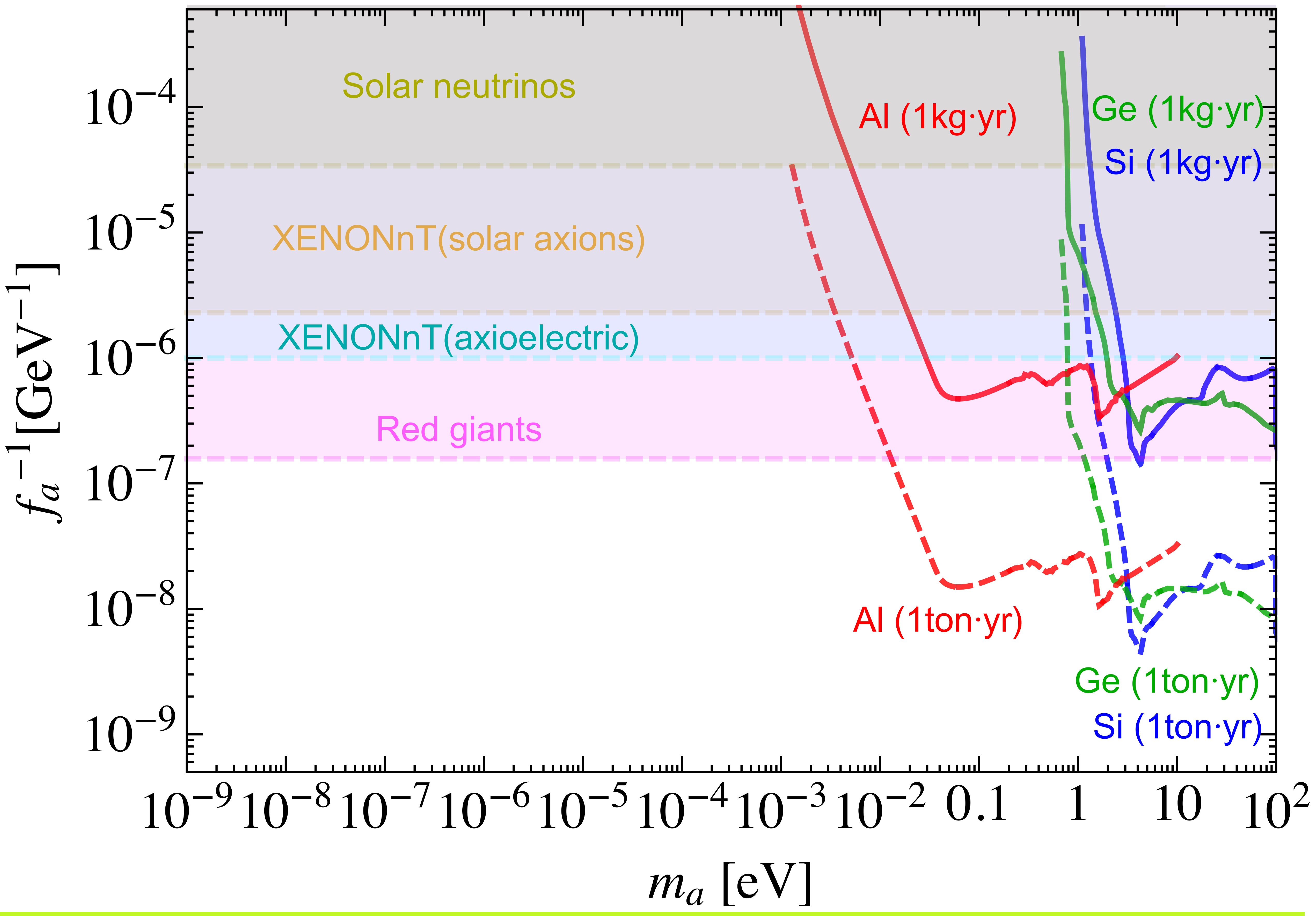


$$R \sim \frac{1}{\rho}\frac{\rho_a}{m_a}\frac{3m_a^2}{4m_e^2}\frac{g_{aee}^2}{e^2}\langle n_e\sigma_{abs}v_{rel}\rangle_\gamma$$

$$\langle n_e\sigma_{abs}v_{rel}\rangle_\gamma = -\frac{\mathrm{Im}\Pi(\omega)}{\omega}$$

Absorption rate for photon in material

Combined Constraints



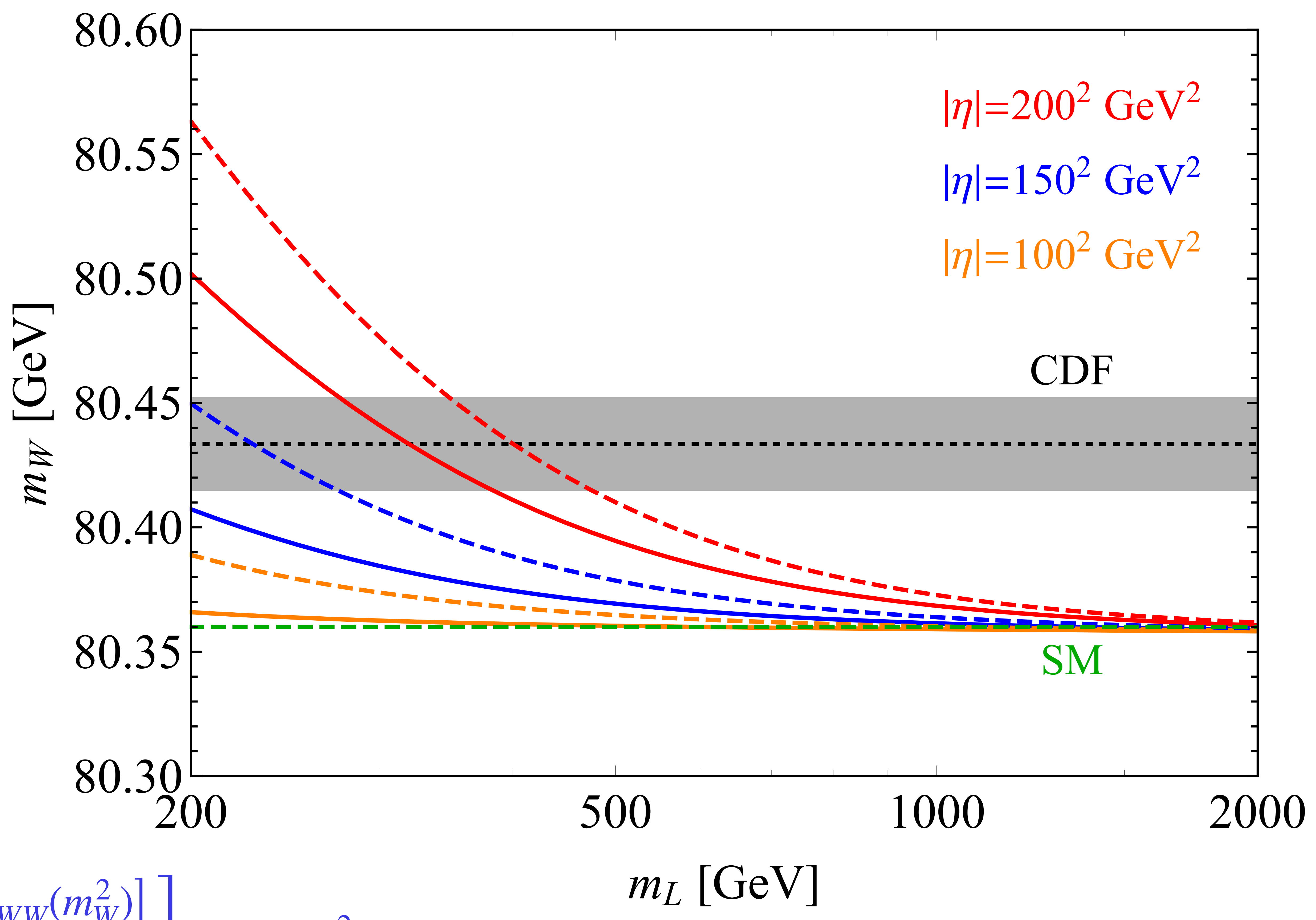
Relevant pheno: W-boson mass anomaly

$$m_W^2 = \frac{m_Z^2}{2}\left[1+\sqrt{1-\frac{4\pi\alpha_{\rm em}}{\sqrt{2}G_F m_Z^2}(1+\Delta r)}\right],$$

$$\Delta r = \Delta\alpha_{\rm em} - c_W^2/s_W^2\Delta\rho_{\rm loop} + \Delta r_{\rm rem}$$

$$\Delta\alpha_{\rm em} = \Pi'_{\gamma\gamma}(0) - \Pi'_{\gamma\gamma}(m_Z^2)\,,$$

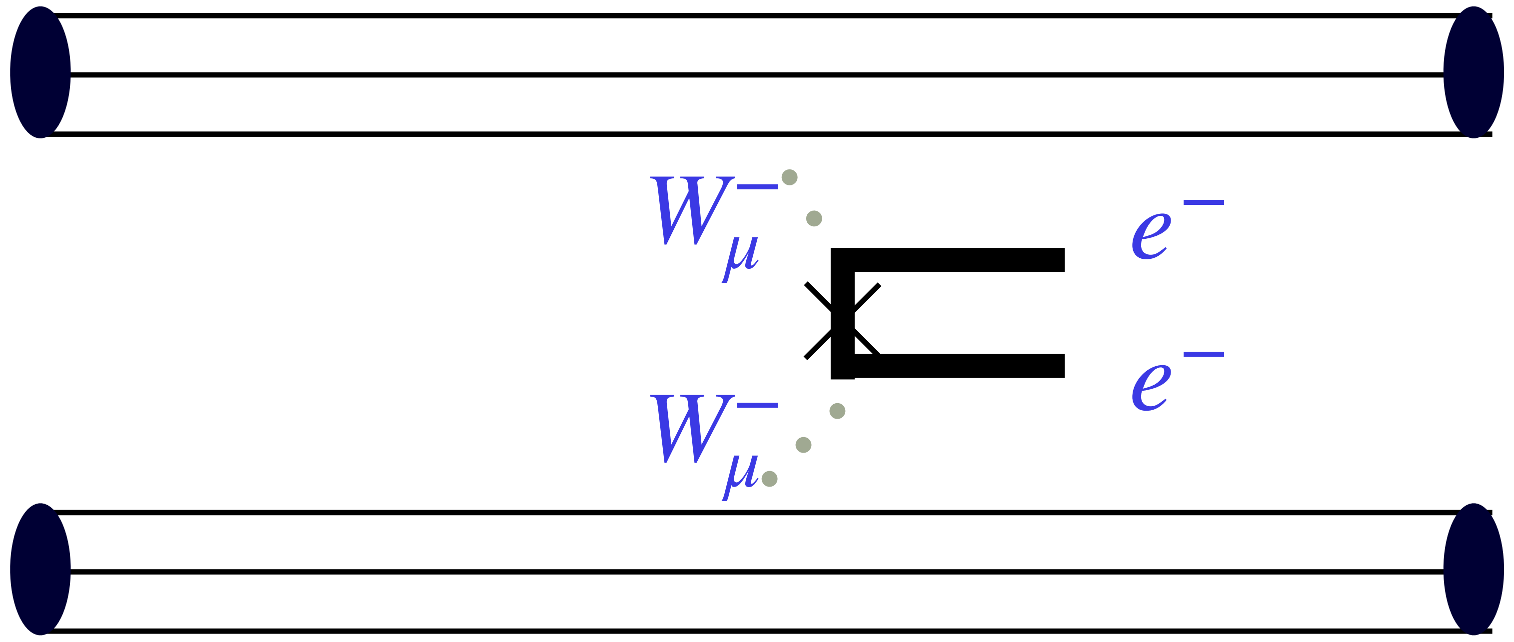
$$\Delta\rho_{\rm loop} = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} + \frac{2s_W}{c_W}\frac{\Pi_{Z\gamma}(0)}{m_Z^2}\,,$$

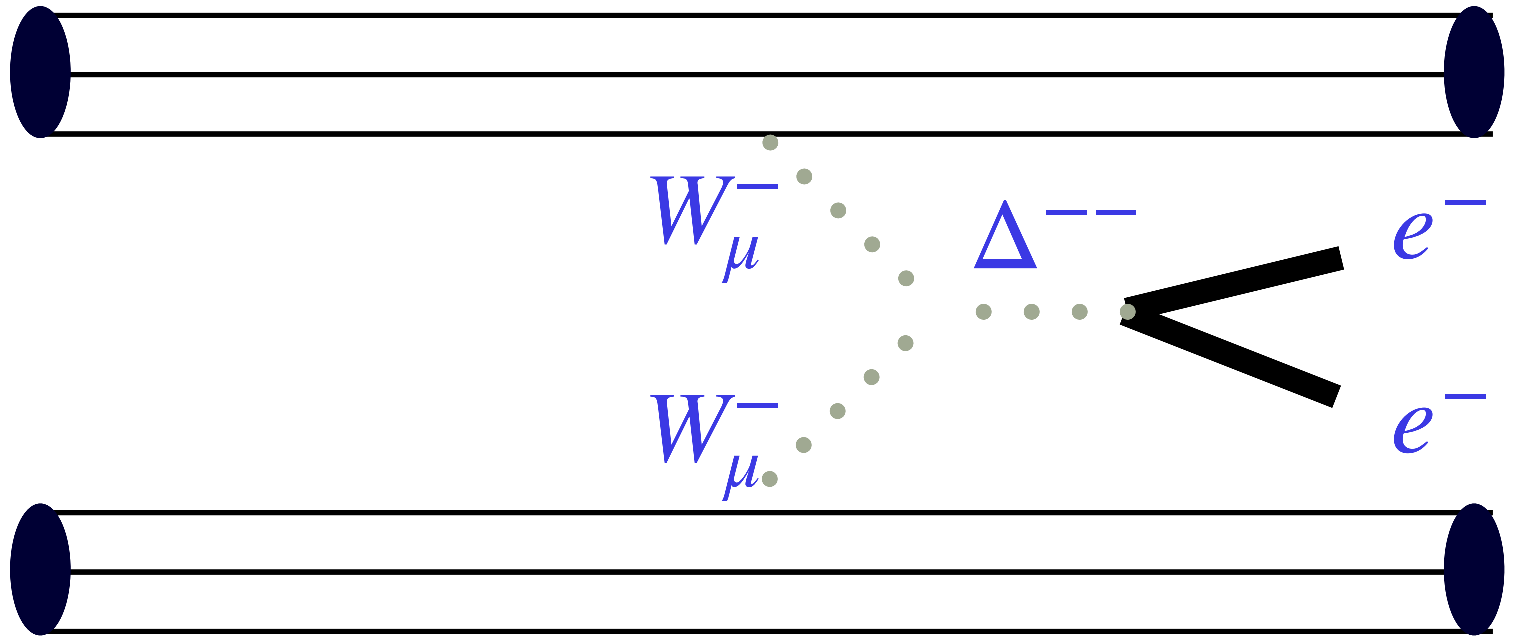


$$\Delta r_{\rm rem} = \frac{c_W^2}{s_W^2}\left[\frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{{\rm Re}\left[\Pi_{ZZ}(m_Z^2)\right]}{m_Z^2}\right] + \left(1-\frac{c_W^2}{s_W^2}\right)\left[\frac{\Pi_{WW}(0)}{m_W^2} - \frac{{\rm Re}\left[\Pi_{WW}(m_W^2)\right]}{m_W^2}\right] + \Pi'_{\gamma\gamma}(m_Z^2) + \delta_{\rm VB}\,.$$

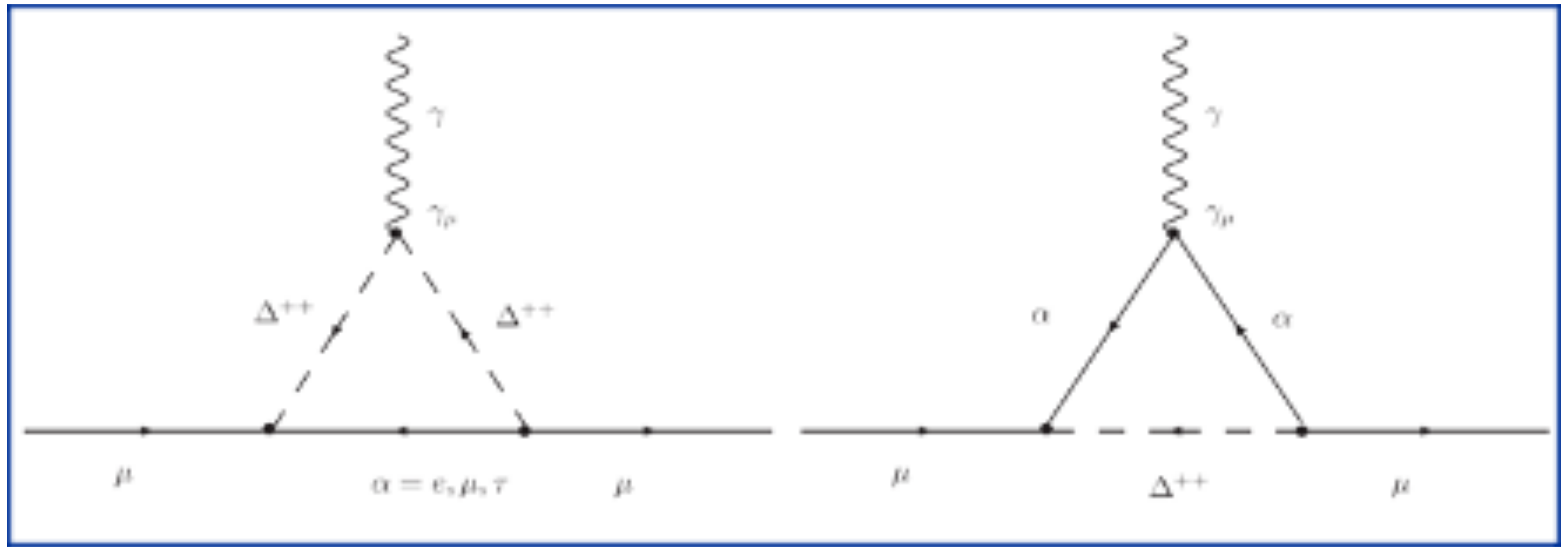
Relevant pheno: $0\nu\beta\beta$ process and muon g-2

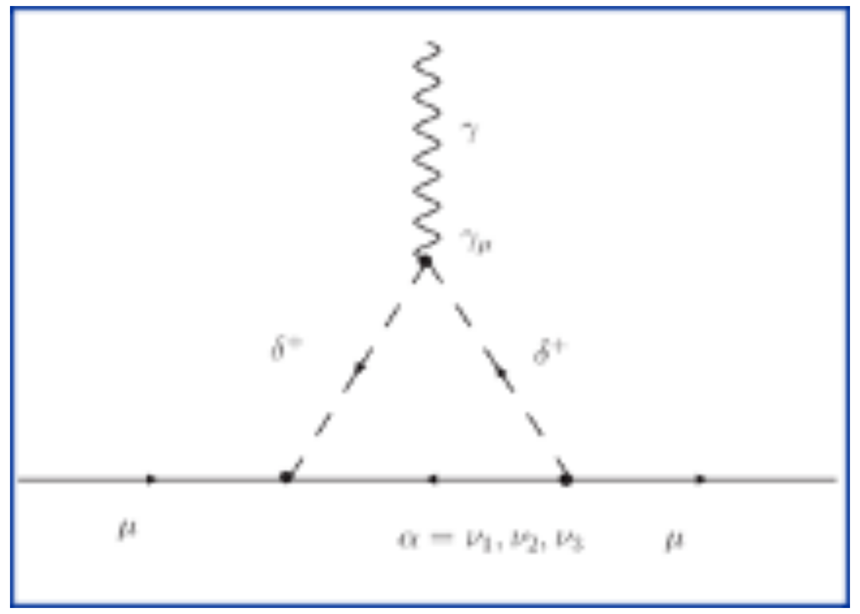
$0\nu\beta\beta$ process in type-II seesaw

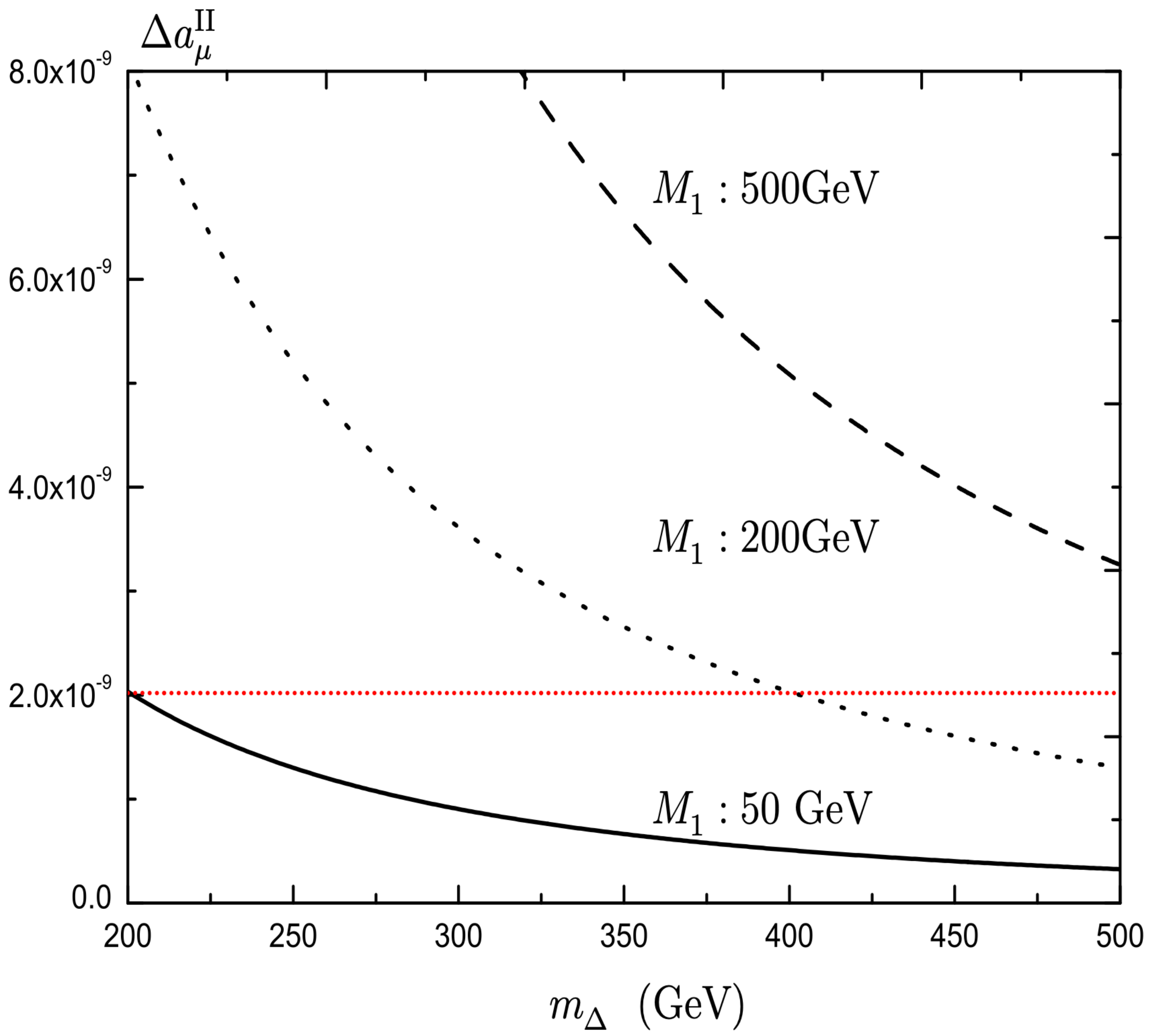




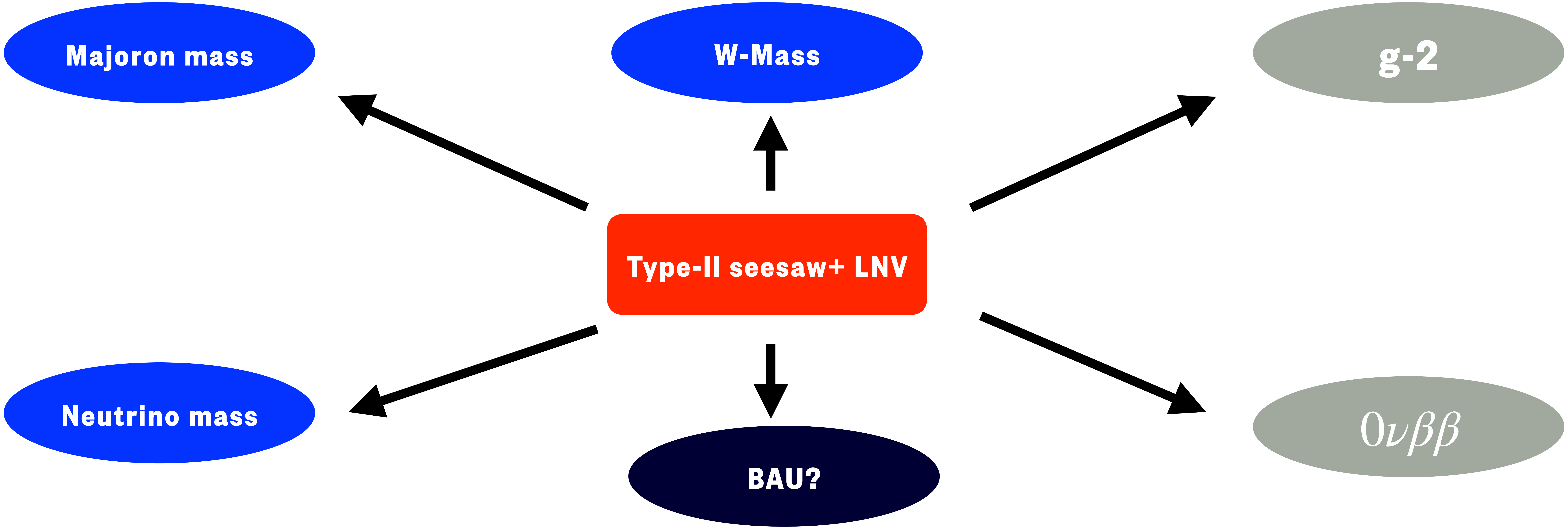
Muon g-2







Concluding Remarks



Thank you!