# BSM investigations with Triangle Singularity

--- a kinematic perspective

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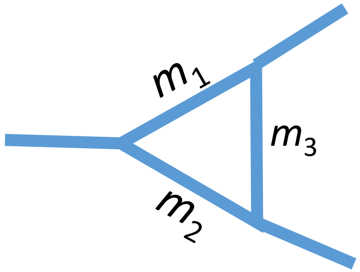


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# Outline

- Triangle `Landau' singularity
- T.S. in *s*-channel and *t*-channel scattering
- T.S. at collider: beyond a threshold?
- A new physics removal of large virtuality suppression

# Landau Singularity

*Karplus, Sommerfield, Wichmann, Phys.Rev.* 111 (1958)
*Landau, Nucl. Phys. 13, 181–192 (1959).*

- A situation when *all internal particles go on shell inside a loop*

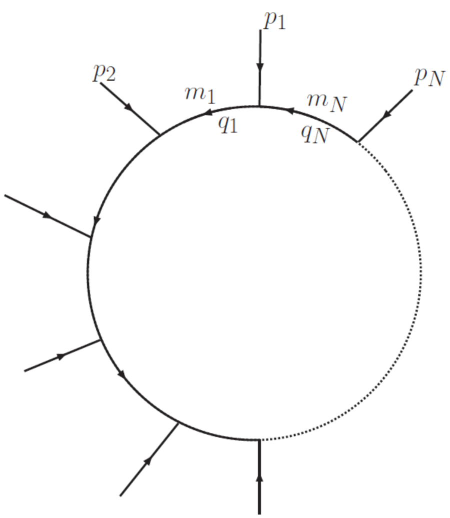



figure from 0806.1498

$$\int \frac{d^D q}{(2\pi)^D i} \frac{1}{D_1 D_2 \cdots D_N}$$

$$\sim \int_0^\infty d\alpha_1 \ldots d\alpha_N \, \delta\left(\sum_i \alpha_i - 1\right) \int \frac{d^D q}{(2\pi)^D i(\alpha_1 D_1 + \cdots + \alpha_N D_N)^N}$$

`Landau Conditions' at leading singularity

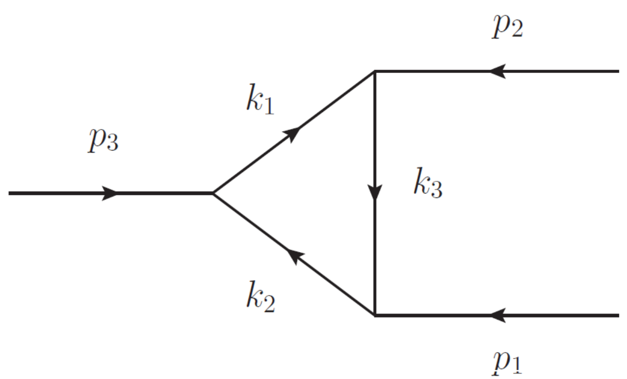
$\sum_i \alpha_i$=1 and $\alpha_i$>0; $\sum_i \alpha_i \, q_i^\mu = 0$;

$\alpha_i\left(q_i^2 - m_i^2\right) = 0$

*L.D.Landau 1958'*

@ NLO: a.k.a. the `anomalous threshold'

# N=3: triangle singularity




$$\begin{cases} \alpha_1 k_1^\mu + \alpha_2 k_2^\mu + \alpha_3 k_3^\mu = 0 \\ k_1^2 - m_1^2 = k_2^2 - m_2^2 = k_3^2 - m_3^2 = 0. \end{cases}$$

$$\alpha_{1,2,3} \in (0,1)$$

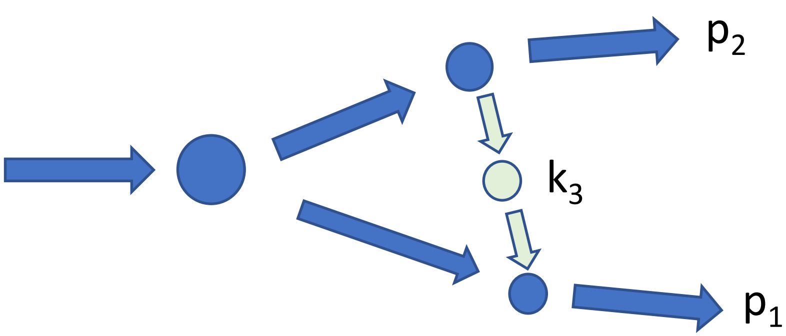
For fixed *k* and *p*, the Eq. Sys. with the internal momentum $l^\mu$ is solvable for $N \leq 4$

A well-known T.S. situation in *s*-channel

Two produced particles $p_1$,$p_2$ (in decay, etc.) **exchange another physical particle** ($k_3$)

* Singularity emerges in amplitude if the (re)scattering with $k_3$ is **inelastic**.

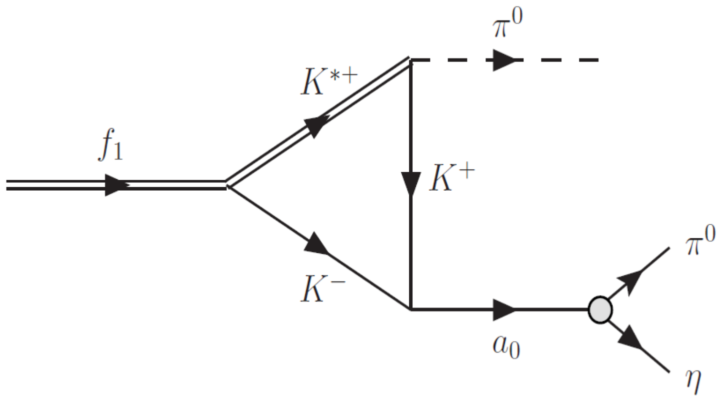
C.Schmid,1967
I. Aitchison, C. Kacser,1968



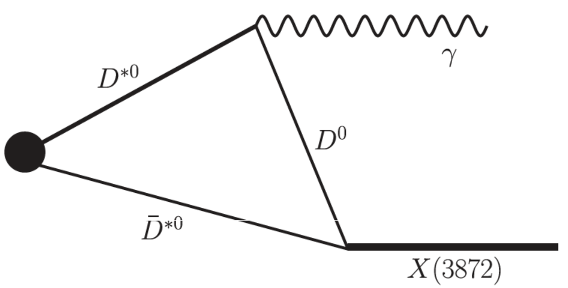
# Hadron interactions

Hadron spectroscopy review:
Guo, Liu, Sakai, 1912.07030

Singularity may be mis-identified as new resonances.




One TS diagram in f(1285) decay
Aceti, Dias, Oset, 1501.06505

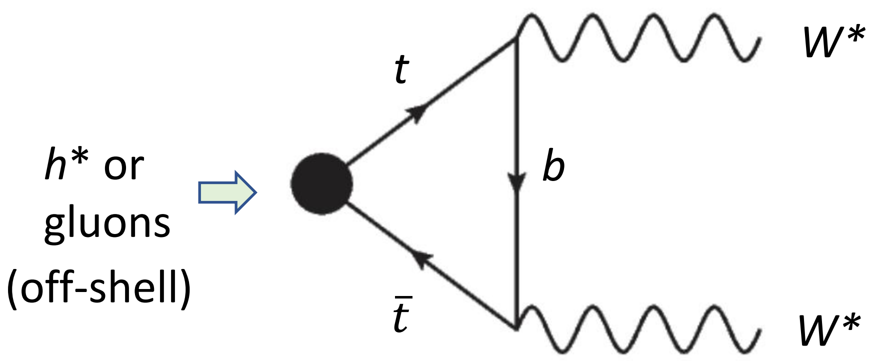



X(3872) production with a TS diagram
F.K.Guo, 1902.11221

- Large number of hadronic states opt to meet Landau conditions
- Mild (compared to HE) virtuality suppression for external momenta

# @ weak / higher energy scale?

- Known example in the SM: tt -> X*X*




Virtual final-state $W$ bosons:
Can trigger T.S. for $350 < \sqrt{s} < 750$ GeV
No physical solution for two on-shell Ws.

leads to an anomalous threshold:
finite correction in xsec.

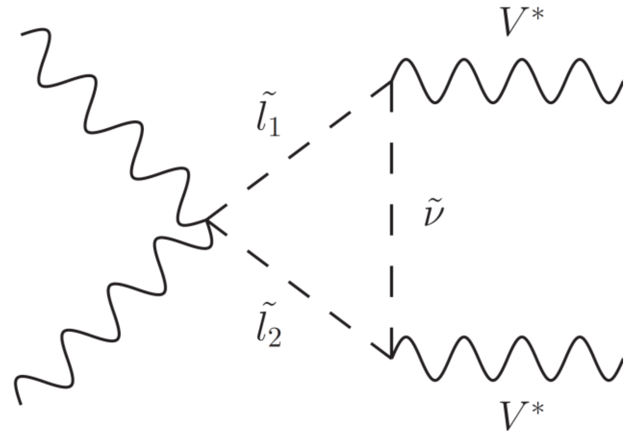
Other discussions in (NLO) SM, see:

Landau singularity in Z Z-> Z Z, Denner,Dittmaier,Hahn, 9612390
N=4 (box) Landau singularity in gg-> h bb, Boudjema and Duc Ninh, 0806.1498

Anomalous threshold in MSSM slepton pair production:

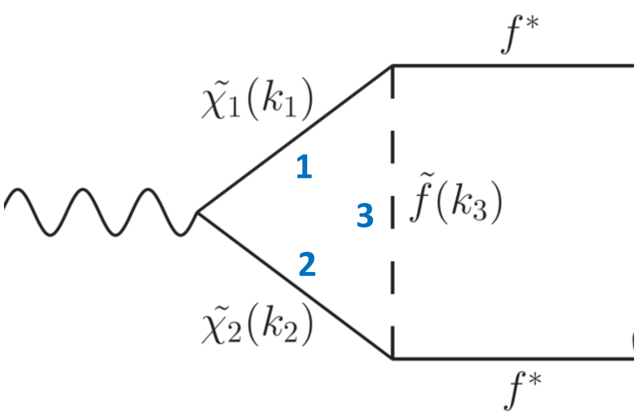
# Why not try BSM?

- A kaleidoscope of new particles.
- Extra bosons (a blessing).
- Can have a purely SM final state.



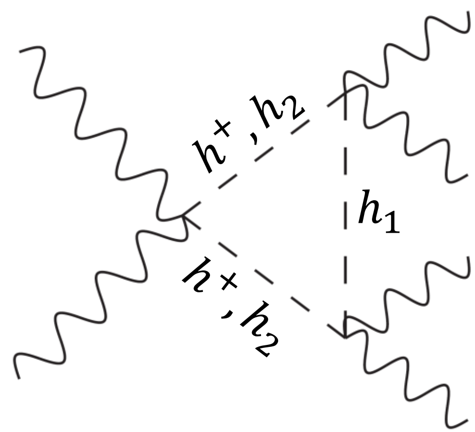

VBF diagram (slepton loop, MSSM)

[SM final state: visible for a `compressed' MSSM spectrum, when $m_{\tilde{l}} \approx m_{\tilde{\nu}}$]




Drell-Yan like (gaugino+sfermion loop, MSSM)

[GIM theorem may apply when $k_1$ and $k_2$ assume different particles]




VBF – all bosons (2HDM)

[$k_1$, $k_2$ can assume identical particles]

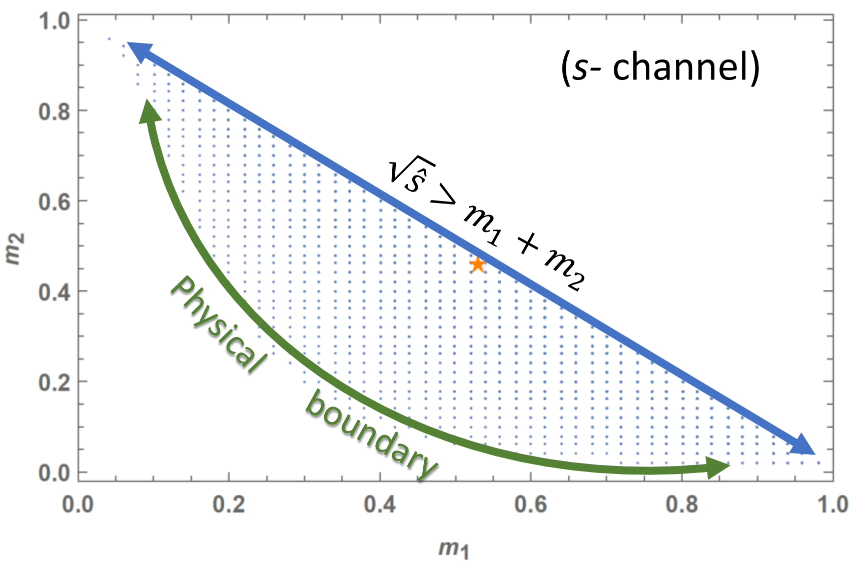
# Kinematic region

For given internal line masses & $p_B^2$, physical solutions require

Assume $k_3$ direction: $B \to A$

$$p_C^2 \in \left[(m_1+m_2)^2, m_1^2+m_2^2+m_2m_3+\frac{m_2}{m_3}\left(m_1^2-p_B^2\right)\right]$$ ⇨ init. state inv. $m^2$

$$p_A^2 \in \left[(m_2+m_3)^2, m_2^2+m_3^2+m_1m_2+\frac{m_2}{m_1}\left(m_3^2-p_B^2\right)\right]$$ ⇨ at least one final-state system is massive



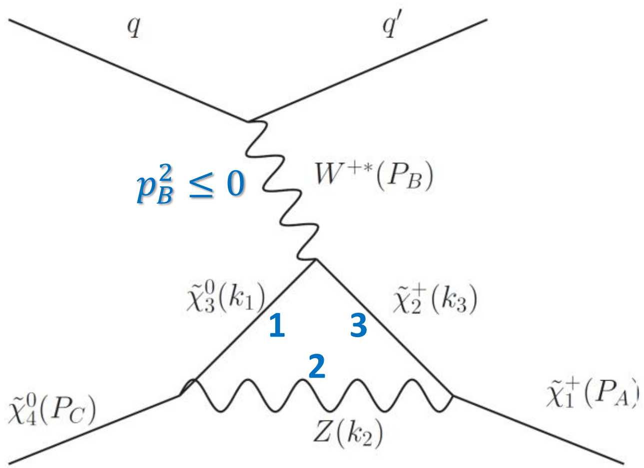

(1) above pair-production threshold
(2) Satisfy physical boundary ($\alpha_i > 0$)

T.S. region on $\{m_1, m_2\}$ plane at given $\sqrt{p_C^2}$, and $p_A^2, p_B^2 > 0$

# A singularity in *t*-channel

- $p_A^2, p_C^2$ are likely massive when internal lines are at BSM mass scale.
- NOTE: Solutions exist for a negative $p_B^2 \leq 0$ when $p_A^2, p_C^2 > 0$.

A negative $p_B^2$ stands for a momentum exchange in a scattering process (MSSM)




massive init. state particle (can not be a LSP dark matter candidate)

massive final state particle or particle system

Extendable to (soft) $\sqrt{|p_B^2|} << m_{BSM}$ region.

$$\begin{cases} m_1 \to m_3 \\ p_C^2 \to p_A^2 \end{cases} \quad \text{for } p_B^2 \to 0$$

Ideal in case a model contains light boson(s).

Not a spontaneous decay when $p_B^2 < 0$.

Need a massive initial state particle for 2->2 scattering (difficult at collider for a BSM particle)

The massive initial state is unstable (so unlikely a vanilla dark matter)

# An MSSM benchmark point.

| Loop particles | Process | $m_1(\Gamma_1)$ (GeV) | $m_2(\Gamma_2)$ (GeV) | $m_3(\Gamma_3)$ (GeV) | $p_A^2$ (GeV$^2$) | $p_B^2$ (GeV$^2$) | $p_C^2$ (GeV$^2$) |
|---|---|---|---|---|---|---|---|
| $\{\tilde{\chi}_2^-, \tilde{\chi}_1^+, \tilde{\chi}_1^0\}$ | DY-like | 1528(13) | 359(0.9) | 306(0) | $700^2$ | $800^2$ | $1943^2$ |
| $\{\tilde{l}_1, \tilde{l}_2, \tilde{\nu}\}$ | WW fusion | 1701(1.56) | 1500(0.34) | 1499(0.16) | $3000^2$ | $100^2$ | $3202^2$ |
| $\{\tilde{\chi}_3^0, Z, \tilde{\chi}_2^+\}$ | $t$-channel | 1351(3.79) | 91(2.45) | 1528(13) | $1621^2$ | $-2500^2$ | $1528^2$ |

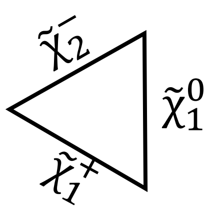
\`EWSBMSSM' parameters

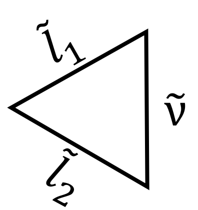
( SUSPECT2 )

| Parameter | Value |
|---|---|
| $\tan\beta$ | 10 |
| $\mu$ | 300 GeV |
| $m_1$ | 1.5 TeV |
| $m_2$ | 1.4 TeV |
| ml3 | 1.5 TeV |
| mr3 | 1.7 TeV |

(modified pars. only) pass LHC search bounds

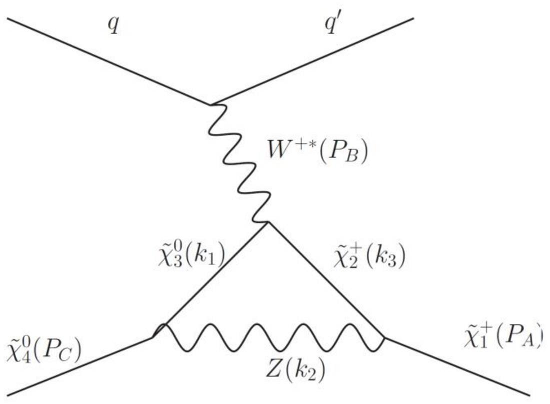
s-channel: Drell-Yan & VBF, into SM V*V* final states.



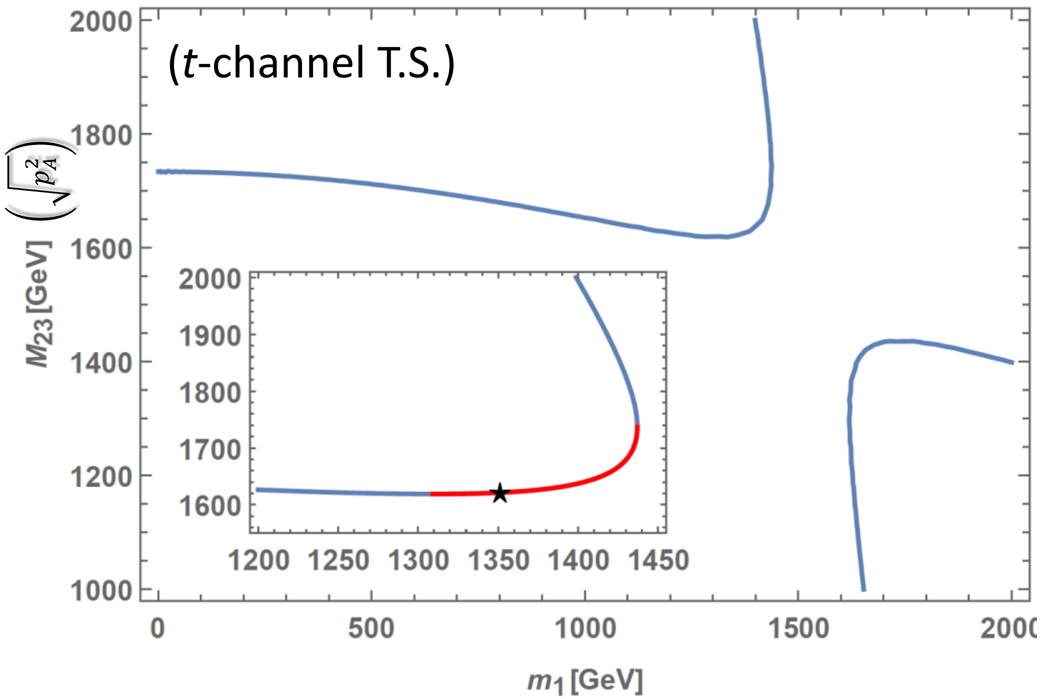




t-channel: $p_A^2 < p_C^2$,

system becomes (more) excited after scattering



# Dalitz plot




Conventional choice:

fix $m_2$, $m_3$ and $p_B^2$, $p_C^2$

m1 and as $m_{23} = \sqrt{p_A^2}$ as variables

**blue**: trajectory of det$|y_{ij}|$=0 (*t*-channel)

**red**: physical solutions

**asterisk**: MSSM benchmark

Landau conditions equiv. as

$$\beta_i + \sum_j^{j \neq i} \beta_j y_{ij} = 0,$$

$$\text{where} \quad y_{ij} \equiv \frac{m_i^2 + m_j^2 - p_k^2}{2 m_i m_j},$$

$$\text{and } \beta_i \equiv \alpha_i m_i$$

Solutions require

$$\det \begin{vmatrix} 1 & y_{12} & y_{13} \\ y_{12} & 1 & y_{23} \\ y_{13} & y_{23} & 1 \end{vmatrix} = 0$$

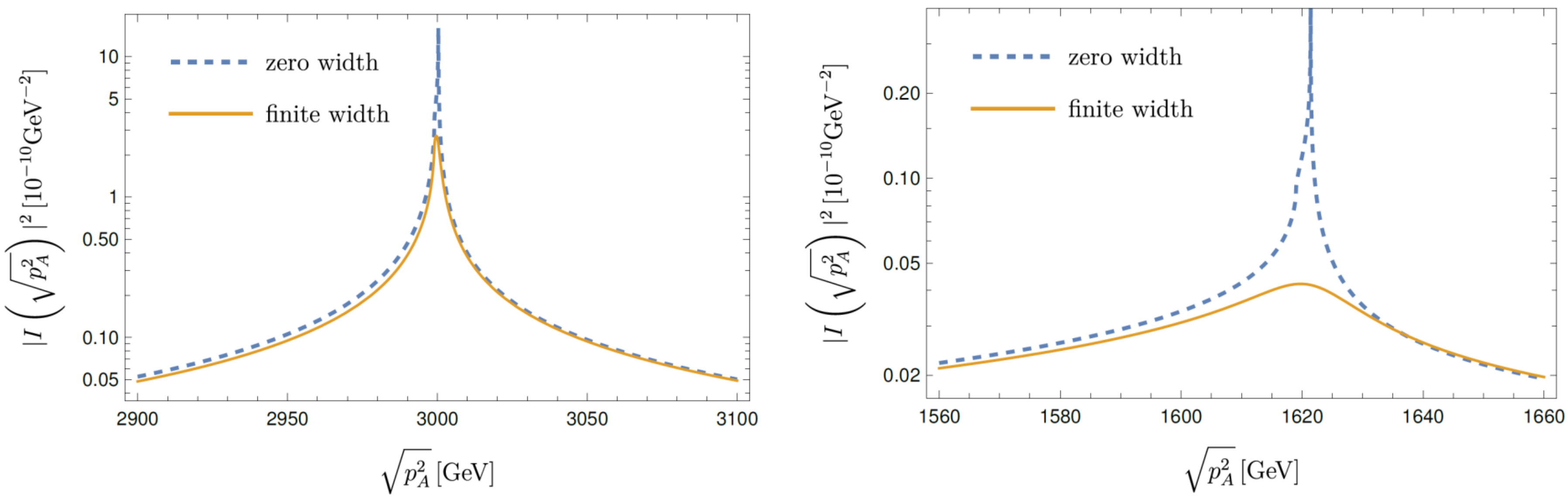
$$1 + 2 y_{12} y_{23} y_{13} - y_{12}^2 - y_{23}^2 - y_{13}^2 = 0$$

containing 6 kinematic parameters.

$\alpha_i > 0$ select a small section (red) of physical solutions.

# Peak in amplitude ($C_0$)

Singularity encoded in $$I\left(\sqrt{p_A^2}\right) = \int \frac{d^4 l}{i\pi^2}\left[\frac{1}{l^2 - m_3^2}\cdot\frac{1}{(l+p_A)^2 - m_2^2}\cdot\frac{1}{(l+p_A+p_C)^2 - m_1^2}\right]$$




(a) $WW$ fusion (b) $t$-channel

Finite width of internal particles gives a small Im part.
Singularity -> broadened peak
Plots: VBF & $t$-channel at MSSM benchmark scenarios, with $p_C^2$ fixed.

# Reconstruction of T.S. ?

$p_A$, $p_B$ can be particle systems

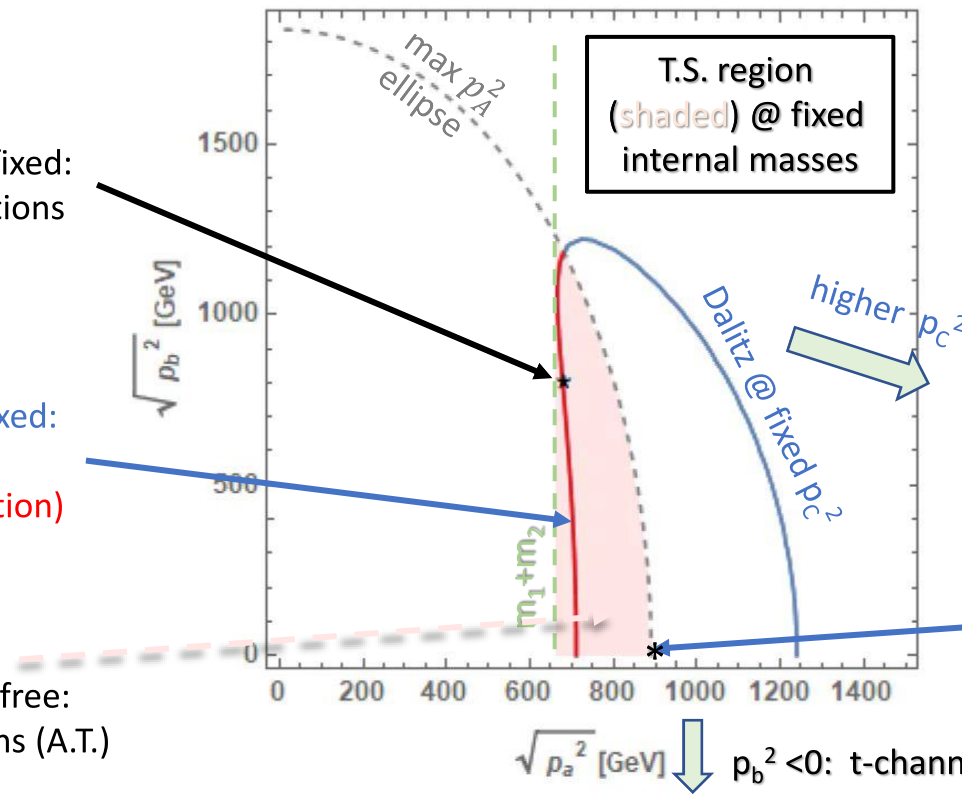
(e.g. if particle states lay inside $p^2{}_A$ *and* $p^2{}_B$ physical ranges.)

- Physical range(s) of external inv. masses: peak → threshold
- $\sqrt{s}$ at T.S. shrinks to a peak only if final-state inv. $p^2{}_{A,B}$ become *both fixed*.
- Extension toward higher $\sqrt{s}$ from beam pdf / ISR.

Two exterior p² fixed: Peaked distributions

One exterior p² fixed: 2D-correlation
Red: (Physical section)

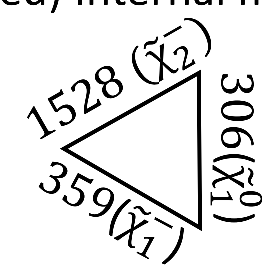
All exterior p² left-free: threshold distributions (A.T.)




$p_b^2<0$: t-channel T.S.

$p_C^2$ maximizes at max $p_A^2$, the physical section reduces to one point.

(fixed) Internal mass:



# More than a threshold?

Low inv-mass-sq system may identify with a particle.

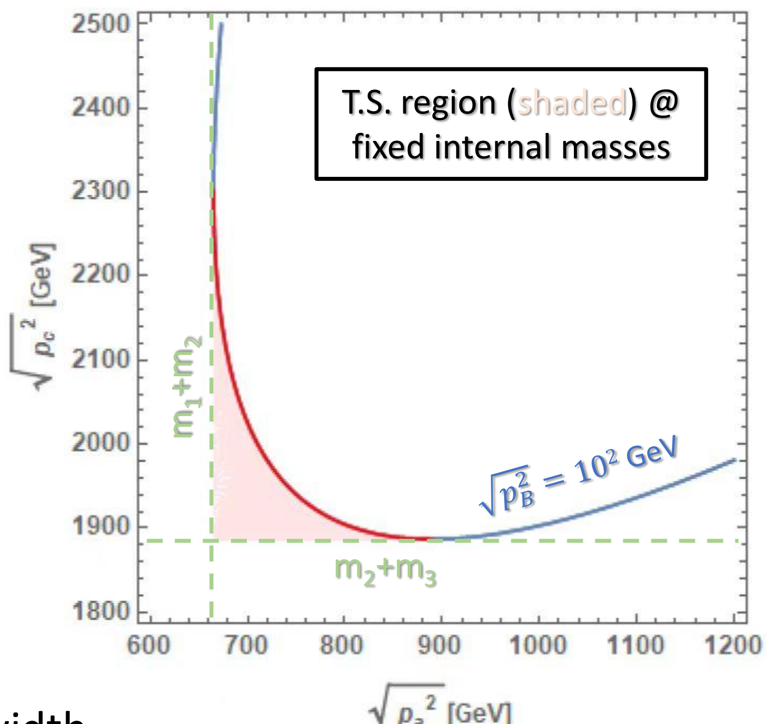
(e.g. if particle states lay inside $p^2{}_A$ *and* $p^2{}_B$ physical ranges.)

- Physical range(s) of external inv. masses: peak $\rightarrow$ threshold
- $\sqrt{s}$ at T.S. shrinks to a peak only if final-state inv. $p^2{}_{A,B}$ become *both fixed*.
- Smeared toward higher $\sqrt{s}$ from beam pdf / ISR.

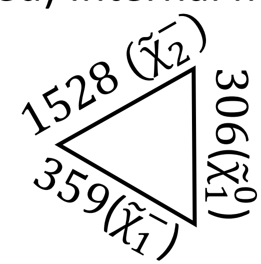
One FS mass extends to lower (EW) range:

**One physical FS particle:**

Correlate COM energy and FS par.sys. invariant mass at singularity



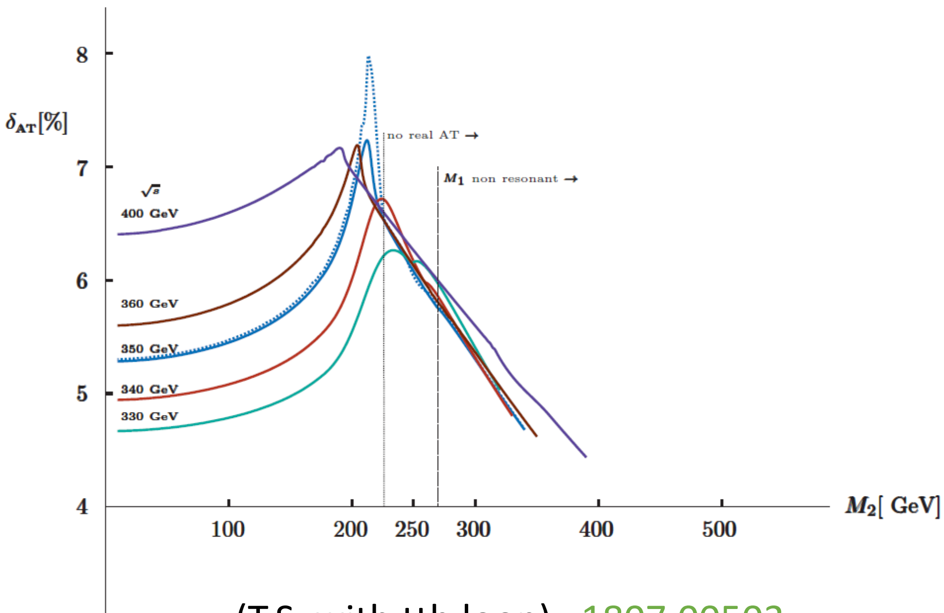

(fixed) Internal mass:



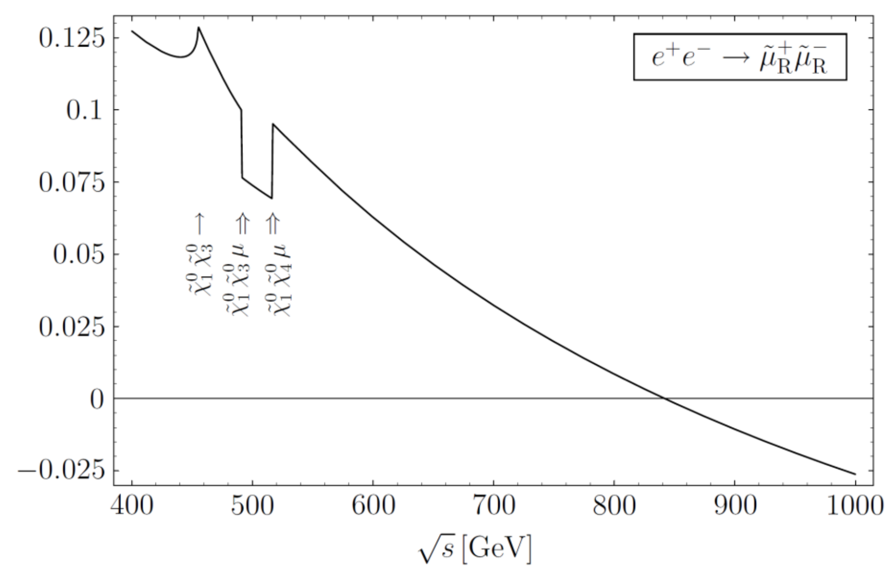

* par. sys. should be visible
* F.S. combinatorics
* broadening by external width

# Visibility (threshold-case)

- EW/BSM scale loop correction to SM-> SM
- Apply to `compressed scenarios'

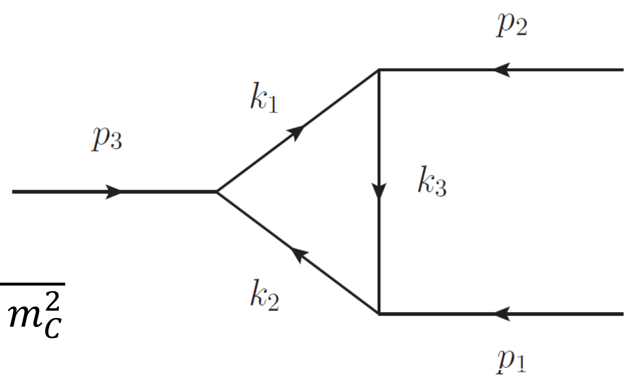



(T.S. with ttb loop) 1807.00503
Correction percentage around threshold for $H^*(Q) \to W^* W^* \to \mu^+ e^- \nu\bar{\nu}$




(T.S. with neutralino-muon loop)
hep-ph/0310182: slepton production: correction from multiple thresholds

# Less suppressed scenario?




$$\sim \frac{1}{p_B^2 - m_B^2}$$

$$\sim \frac{1}{p_C^2 - m_C^2}$$

$$\sim \frac{1}{p_A^2 - m_A^2}$$

A not small virtuality suppression as

$$\sqrt{p_C^2} > \; m_1 + m_3$$

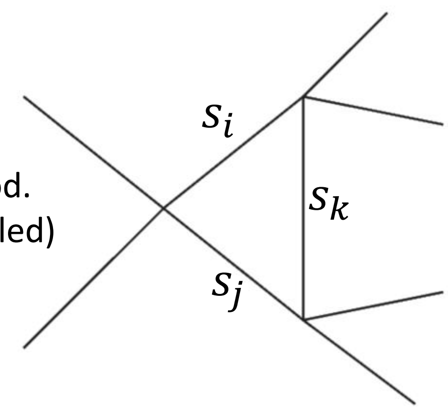
$$\sqrt{p_A^2} > \; m_2 + m_3$$

when BSM particles are heavy. e.g. in an *s*-channel process: XX > scalar> triangle

IS: 4-particle vertex allows for TS in a *boson-fusion* process, avoids a large 1/s.

FS: 4-particle vertices replaces a highly virtual propagator with two *more identifiable,* on-shell particles.

VBF prod.
(if coupled)




~ four boson coupling. w/o heavy prop.

on-shell final particles more identifiable

# `Everything on shell': bosonic couplings

- Gauge couplings

  *SSV* : $ig(k_1 - k_2)^\mu \rightarrow igk_V{}^\mu$ ⇨

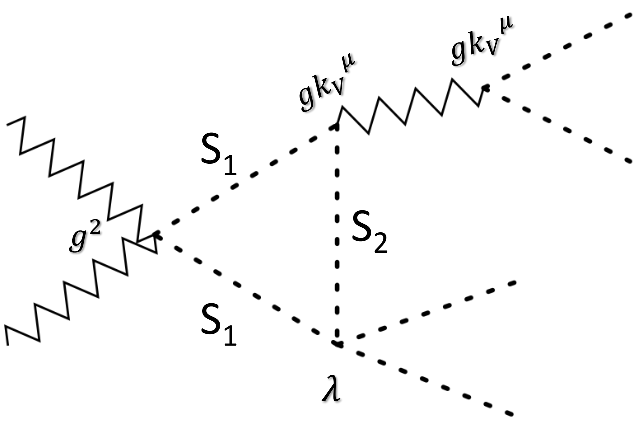
  Derivative 3-pnt coupling: $k_V . k_V$ may lift the large propagator suppression

  *SSVV*: $ig^2 \rightarrow const$

- Scalar couplings

  *SSSS*: quartic couplings → $\lambda = const$

- Fermions: have no ren. 4-point vertices; three-point vertices do not cancel propagator suppression.



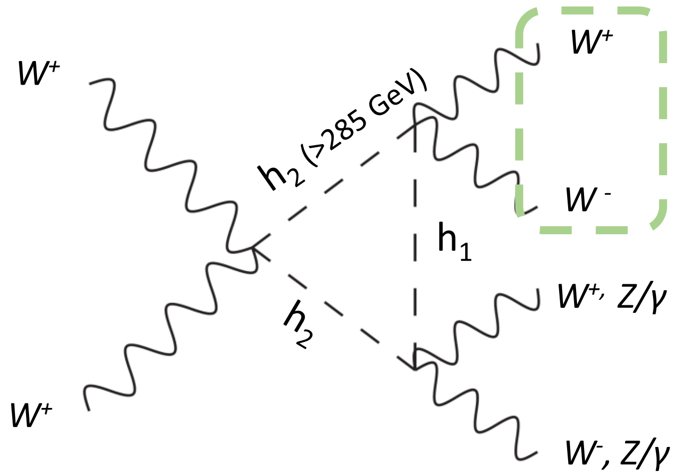
# `EOS': not in the SM?

- The SM weak-scale bosons are too few;
  the mass splits are also insufficient (w tree level vertices):

one particle must be heavier than two/three coupled particles.

- BSM: `EOS' just needs *one extra boson*.

In 2HDM: $h_2 h_2 h_1$ loop, $m_{h_2} > m_{h_1} + m_{V_1} + m_{V_2}$, similar in other BSM models.

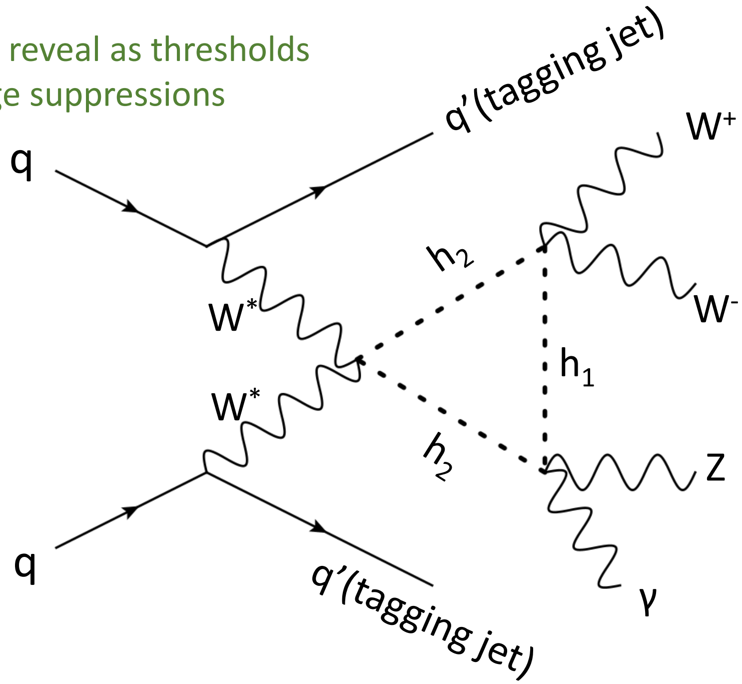



Can be replaced by a single particle X, if $m_X = m_{h2} - m_{h1}$
(A.T. $\longrightarrow \sqrt{\hat{s}} - \sqrt{p_A^2}$ correlation)

<GIM generally evaded in extended Higgs/scalar sectors>

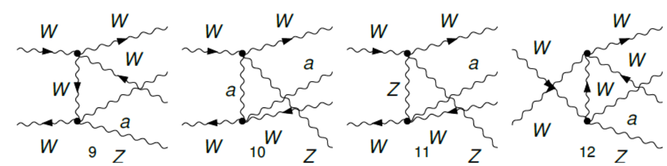
# A physical process at collider: VBF

T.S. may reveal as thresholds
w/o large suppressions




An attempt with FeynArts:
WW → WaWZ yields
468 tree diagrams and
5468 1-loop diagrams
VBF: 2->6 process with pdf.

Loop diagrams involving fermions have UV divergences and need renormalization work.




(a few non-TS, interfering examples)

- Everything on shell (except for the WW pair, but w/o large virtually)
- UV non-divergent loop integral ($C_0$), demonstrating a Landau singularity.
- Purely SM, fully visible and reconstructible, final state.
- Asymmetric kinematics: one bi-particle mass can be much larger.
- A non-resonant initial state: lots of interfering diagrams, need careful cut selection.

# Summary

- BSM spectra should, and *often*, trigger triangle singularities.
- Unlike with hadrons, physical range(s) of external inv. momenta lead to
  * Anomalous thresholds at collide. (no on-shell state.)
  * Kinematic correlations (one on-shell state.)
  * Peak (both states on shell)
- `Everything on shell’ T.S. possible with bosonic couplings, by replacing a off-shell momentum with a bi-particle system.
- EW (or higher scale ) `EOS’ needs BSM, likely appears as A.T.
- Triangle singularity possible in a *t*-channel ($q^2 < 0$) scattering, and it can approach to a soft realm ($|q^2| << m^2$).