FIMP dark matter mediated by massive gauge boson

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Thermal effect of the vector boson

- $$\frac{i}{k^2 - m^2}\left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2 - \xi m^2}\right)$$
- $$\frac{i}{k^2 - \xi m^2}$$

Precise cancellation of these two poles at zero temperature!

- Finite temperature.
- What's the relationship between the vector boson and the Goldstone?

“Goldstone Equivalence Gauge”

- Physical Gauge: extend the “polarization vector” with one extra Goldstone degree of freedom:
- $\epsilon^{\mu}_{\pm,L} \to \epsilon^{M}_{\pm,L}, M = 0,1,2,3,4.$
- $\epsilon^{M}_{\pm}$ does not change.

“Goldstone Equivalence Gauge”

$$P_L = \begin{pmatrix} \frac{k^2}{(n\cdot k)^2} n^\mu n^\nu & i\frac{m_A}{n\cdot k} n^\mu \\ -i\frac{m_A}{n\cdot k} n^\nu & \frac{m_A^2}{k^2+i\epsilon} \end{pmatrix},$$

$$P_G = \begin{pmatrix} 0 & 0 \\ 0 & \frac{k^2-m_A^2+i\epsilon}{k^2+i\epsilon} \end{pmatrix}.$$

$$P_L^{MN} = \epsilon_L^M \epsilon_L^{N*}$$

$$\langle (A^\mu,\phi),(A^\nu,\phi)\rangle = \frac{i}{k^2-m_A^2+i\epsilon}(P_T+P_L+P_G),$$

Transverse+Longitudinal+Goldstone

Zero temperature:
Goldstone poles in $k^2=0$
P_L**,** P_G **cancel each other.**

That is the meaning of “eating”!

$$D_0^{\text{full},MN}(k) = \frac{i}{k^2-m_A^2-\Pi_T(k)+i\epsilon}P_T + \frac{i}{k^2-m_A^2-\Pi_L(k)+i\epsilon}P_L + \frac{1}{1-\frac{\Pi_U(k)}{m_A^2}}\frac{i}{k^2+i\epsilon}\begin{bmatrix} 0_{4\times4} & 0_{4\times1} \\ 0_{1\times4} & 1 \end{bmatrix}.$$

Transverse, Longitudinal polarizations remain unchanged. However, the Goldstone poles no longer cancel!

$$\Delta^F_{\mathbf{GS}}(k) = \frac{k^2-\Pi_L(k)+i\epsilon}{k^2-m_A^2-\Pi_L(k)+i\epsilon}\frac{i}{k^2+i\epsilon}$$

Massless Goldstone with the “Renormalization Factor”

$$Z^2_{\text{GS}} = \frac{\Pi_L(k)+i\epsilon}{m_A^2+\Pi_L(k)+i\epsilon}!$$

Finite temperature

- $\Pi_L(k) = -\frac{2m_E^2 k^2}{\vec{k}^2}\left(1 - \frac{k^0}{|\vec{k}|} Q_0(\frac{k^0}{|\vec{k}|})\right)$ in HTL approximation.
- The k^2 cancels the pole in $\Delta^F_{\mathrm{GS}}(k) = \frac{k^2 - \Pi_L(k) + i\epsilon}{k^2 - m_A^2 - \Pi_L(k) + i\epsilon} \frac{i}{k^2 + i\epsilon}$!
- $\Pi_L(k)$ has a branch-cut along $k^0 = (-|\vec{k}|, |\vec{k}|)$.
- $\Delta^F_{\mathrm{GS}}(k)$ inherit this branch cut in place of the two poles.
- The branch cut peaks significantly at both $k^0 = \pm|\vec{k}|$. I call it a pair of Quasi-Pole.

Finite temperature

- When $T > T_c$, $m_A = 0$, Goldstone and the Longitudinal polarization decouples.$\Delta^F_{\text{GS}}(k) = \dfrac{i}{k^2 + i\epsilon}$.
- When $T < T_c$, two poles fragment into two a branch cut, which is still similar to two poles. I call this a “cadaver” of a Goldstone boson. Longitudinal polarization eats the Goldstone, but could not devour once in a time.
- $T = 0$, the cadaver completely disappear.
- $$\int_0^{|\vec{k}|+\delta} - \text{Im}[i\Delta^F_{\text{GS}}(k^0, \vec{k})]dk^0 = \frac{1}{\vec{k}} \int_0^{1+\delta} \text{Im}\left[\frac{x^2 - 1 + 2\gamma(x^2 - 1 + i\epsilon)(1 - xQ_0(x))}{x^2 - 1 + i\epsilon - \alpha + 2\gamma(x^2 - 1)(1 - xQ_0(x)) + i\epsilon} \frac{1}{x^2 - 1 + i\epsilon}\right] dx$$
  $$\triangleq \frac{1}{\vec{k}} R(\gamma, \alpha)\,.$$
- Regard the Goldstone as a massless boson, the “Renormalization factor” $Z_{\text{GS}} = -\dfrac{2R(\gamma, \alpha)}{\pi}$.

Applications?

- The early universe: $T \sim m_V$.
- Sterile neutrino decay: $m_W \sim m_N \sim T$, leptogenesis, Phys.Rev.D 103 (2021) 9, 095003 • e-Print: 2008.00642 [hep-ph].
- WIMP?
- No. $T \ll m_{\mathrm{WIMP}}$. For light m_V, thermal effect is negligible during the phase integration. For heavy $m_V \gtrsim m_{\mathrm{WIMP}}$, thermal effect is negligible compared with m_V.

Application

- FIMP! $T \sim m_{FIMP}$ sufficiently high temperature for the significant thermal effect!

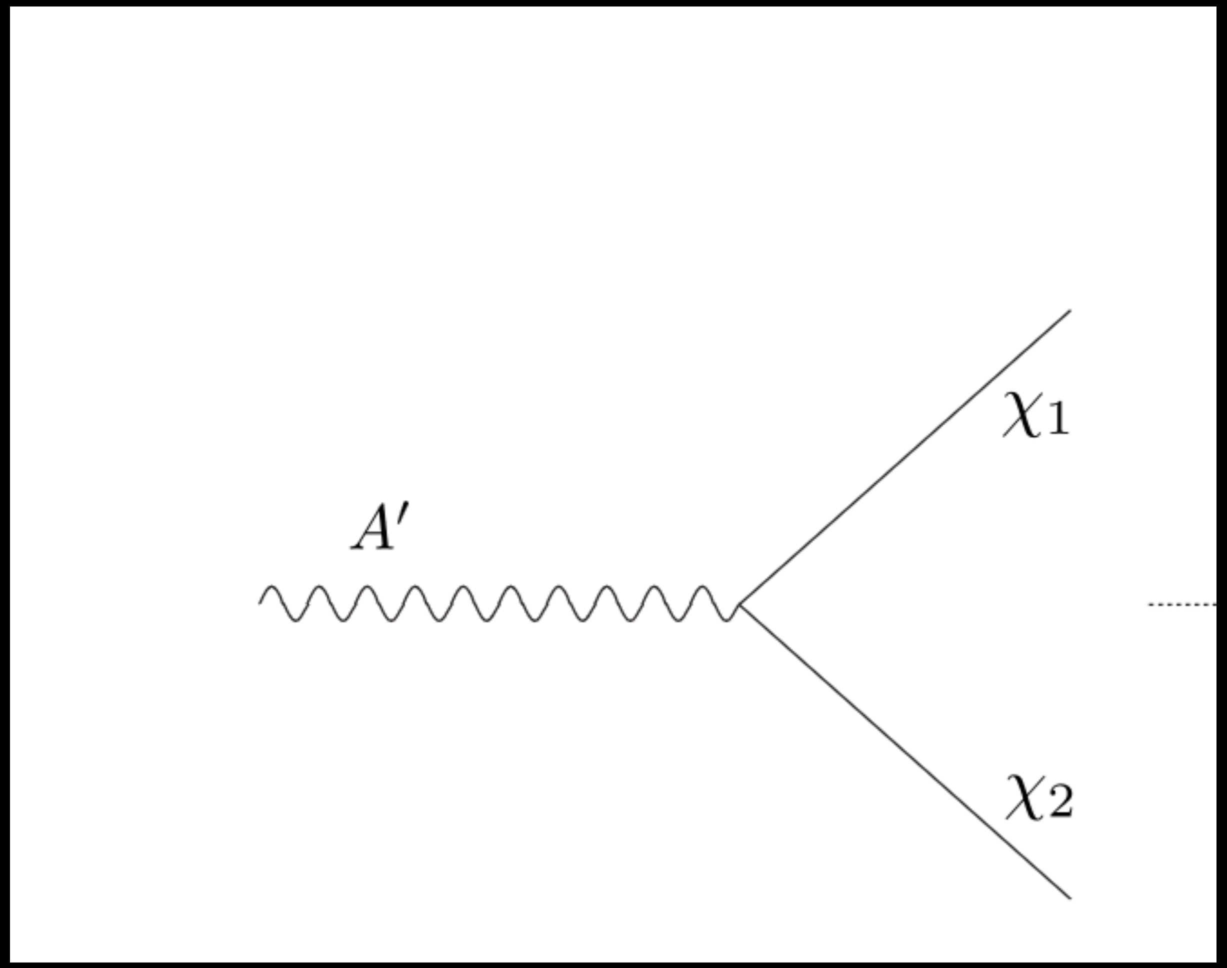
- FIMP as a vector boson? No! Out of thermal bath for the validity of the thermal calculations!

- Vector boson mediated dark matter! $T \sim m_{FIMP} \sim m_A$ Phys.Rev.D 106 (2022)3, 035028 2111.10608 [hep-ph]

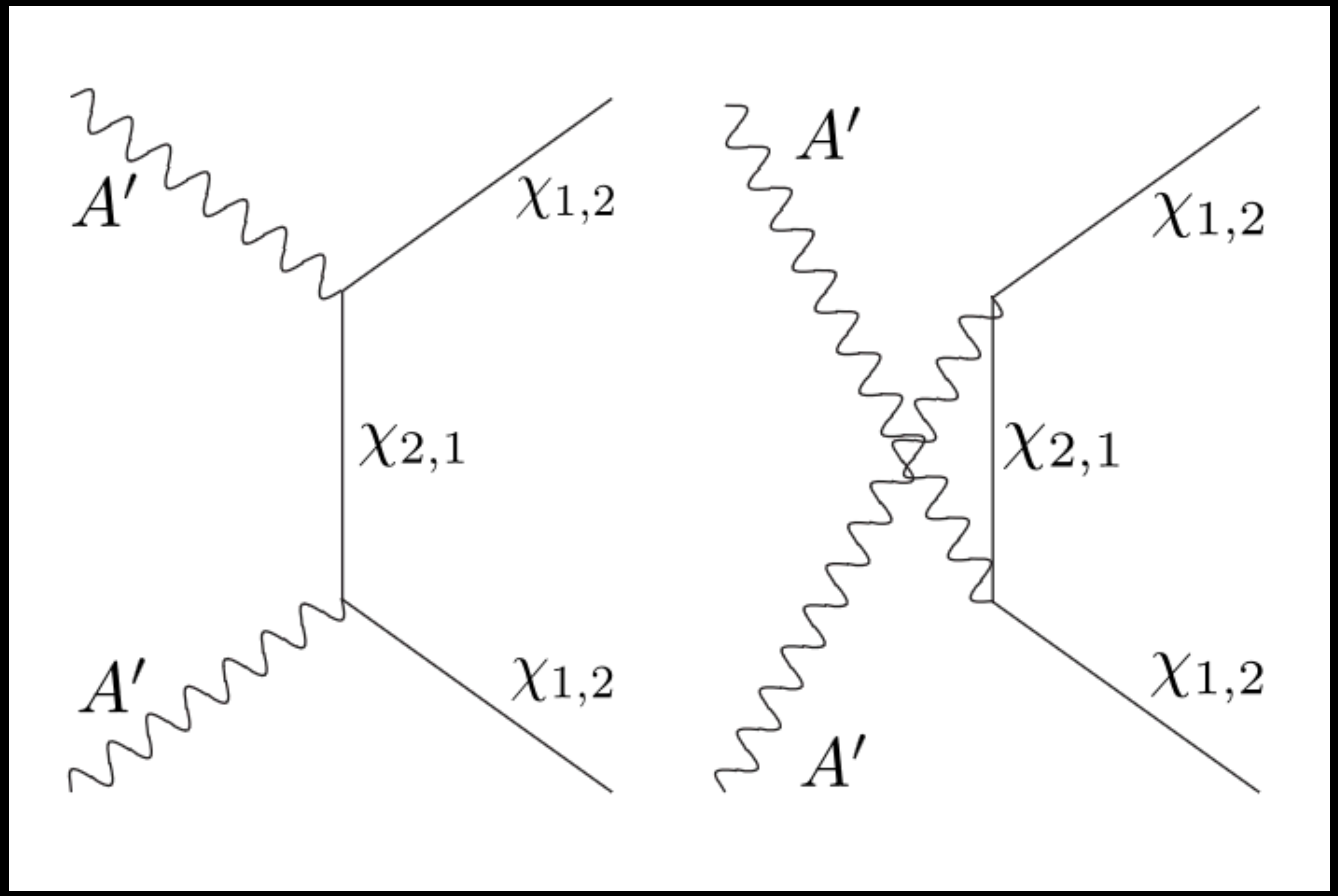
Comparison with the literature

- $\gamma \leftrightarrow \gamma' \leftrightarrow$ DM.
- Thermal corrections only for SM photon.

FIMP mediated by vector boson



- Simplest model with massive A':
- Phys.Rev.D 98 (2018) 3, 035038 1806.00016
- V<->FF
- VV<->FF
- Longitudinal vector boson?



FIMP mediated by vector boson

Switch off Φ_w if someone wants a minimal model!

$$
\begin{aligned}
\mathcal{L}_{\rm kin} &= -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + D_\mu\Phi_s(D^\mu\Phi_s)^\dagger + D_\mu\Phi_w(D^\mu\Phi_w)^\dagger + i\bar{\chi}D_\mu\gamma^\mu\chi,\\
\mathcal{L}_{\chi m} &= m_\chi\overline{\chi}\chi,\\
\mathcal{L}_Y &= \frac{\sqrt{2}y_\chi}{2}\Phi_w\bar{\chi}\chi^C + h.c..
\end{aligned}
$$

Usual Higgs Mechanism

Feebly interacting Higgs

Provide a DM-longitudinal interaction term

FIMP mediated by vector boson

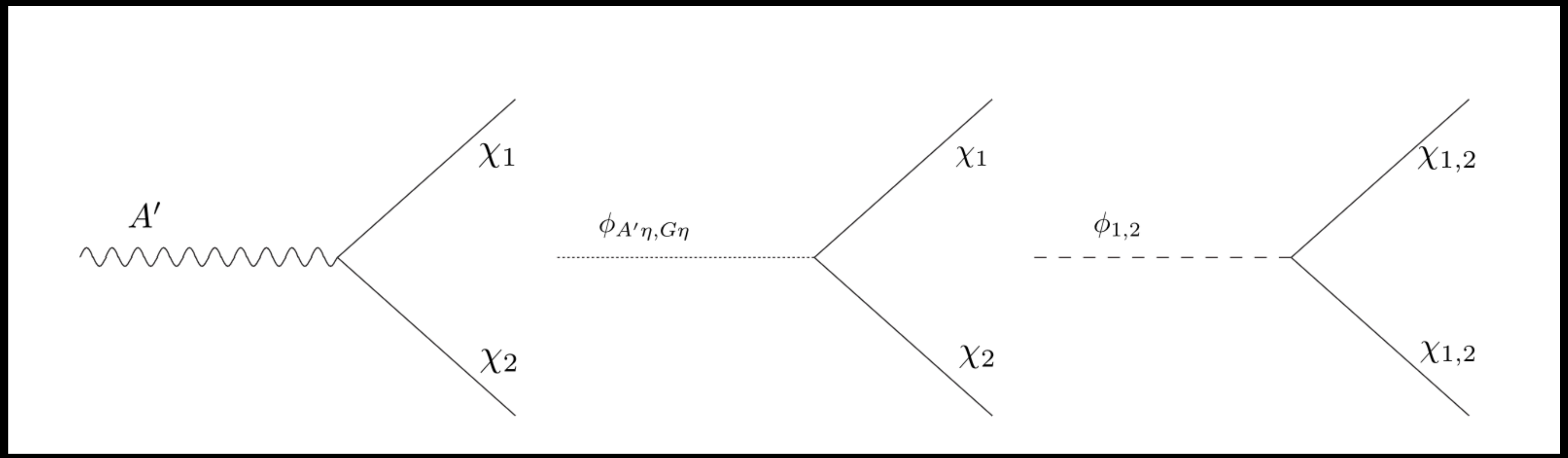
- Scenario I: $\Phi_w \approx \Phi_s \sim 1\mathrm{TeV}$。Complicated phase transition, while the interaction of the dark matter with the Goldstone/longitudinal polarized vector bosons is too small.

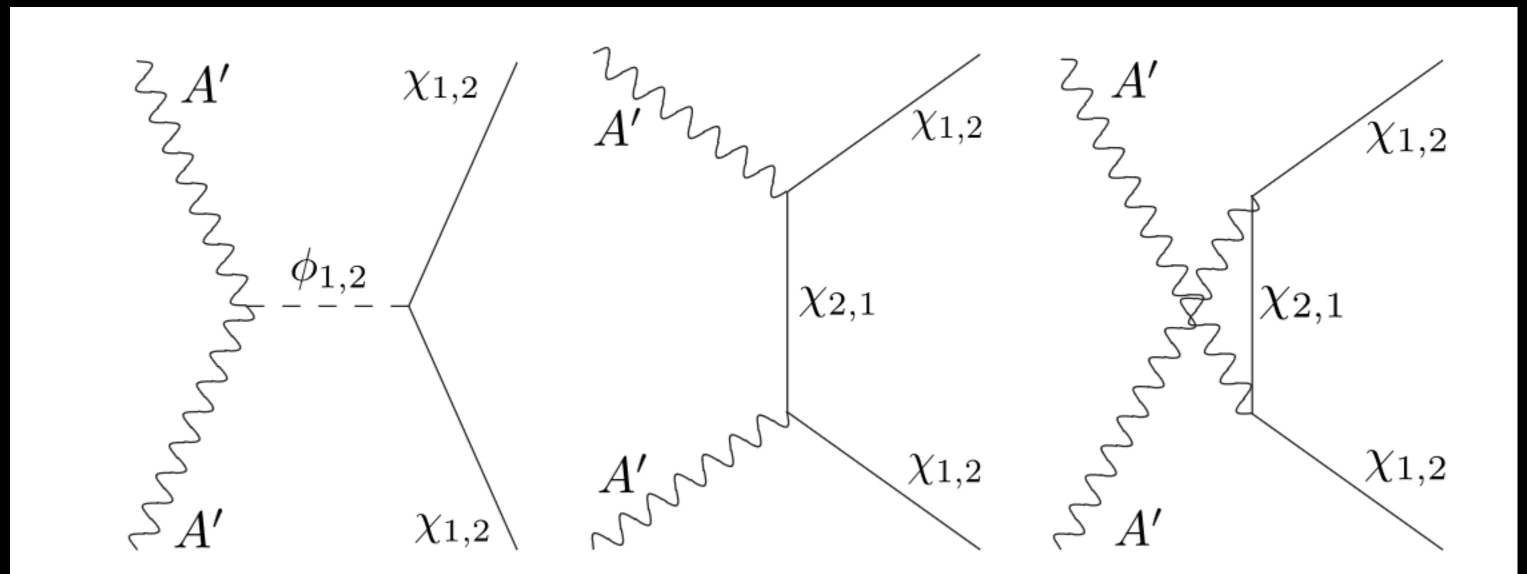
- Scenario II: $\Phi_w \gg \Phi_s \sim 1\mathrm{TeV}$。$\Phi_w$ looks like a “dummy field”, or equivalently becomes a “Stückelberg field” somehow. Phase transition becomes simple, since the two steps of both Higgs boson acquires VEVs can be discussed separately.

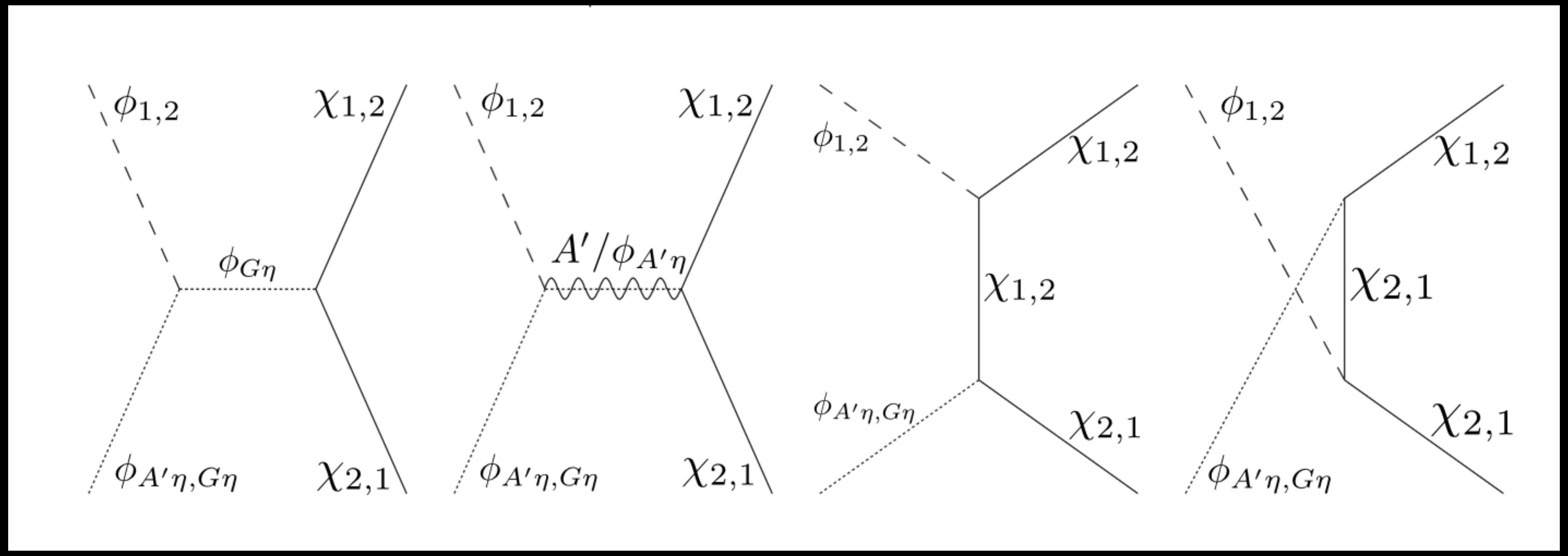
- Significant DM-longitudinal interactions since Φ_w contributes a significant part of the vector boson’s mass.

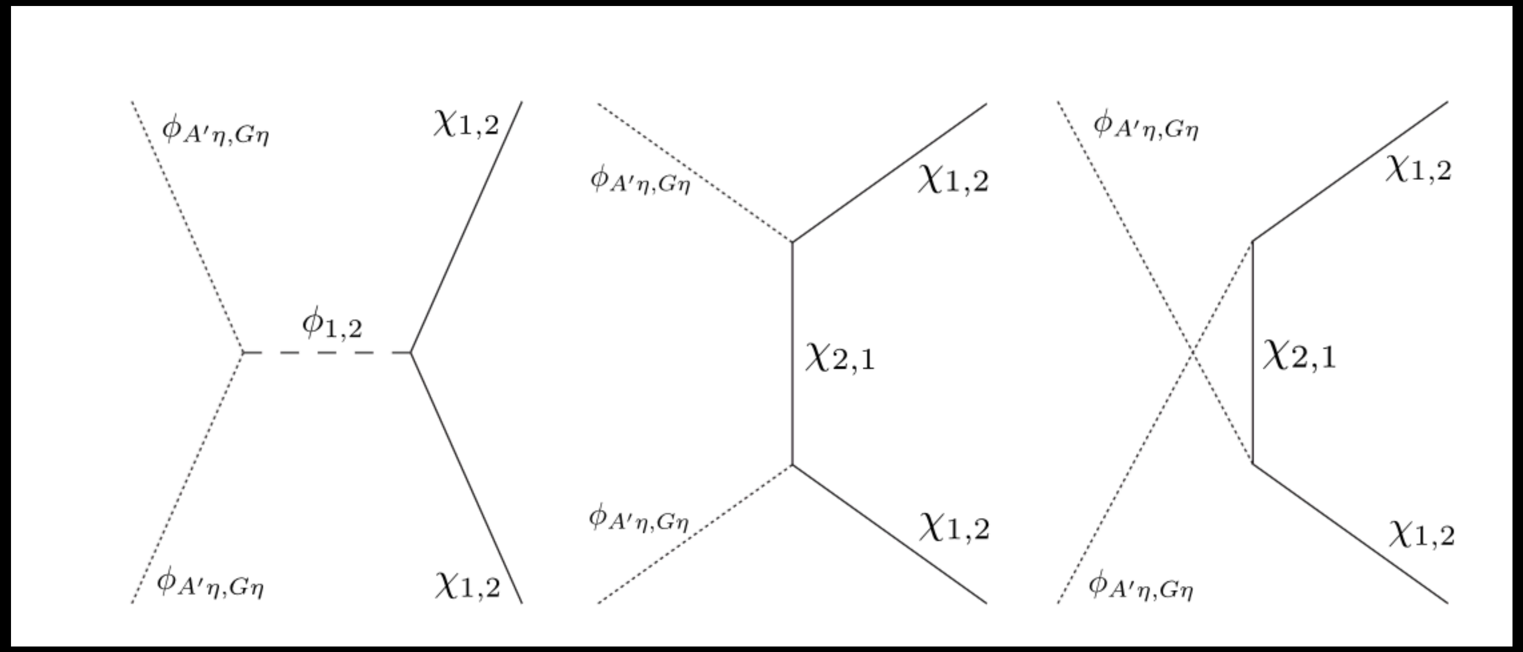
FIMP mediated by vector boson

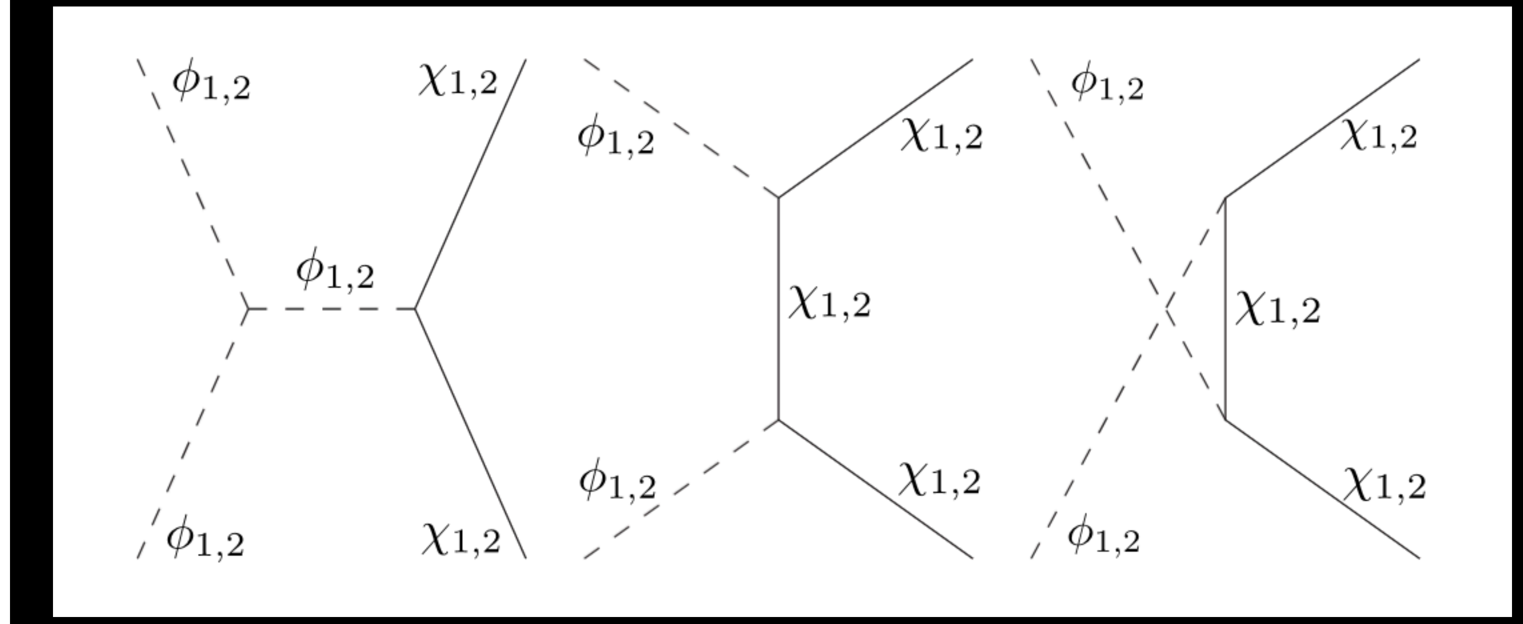
L,T,GS

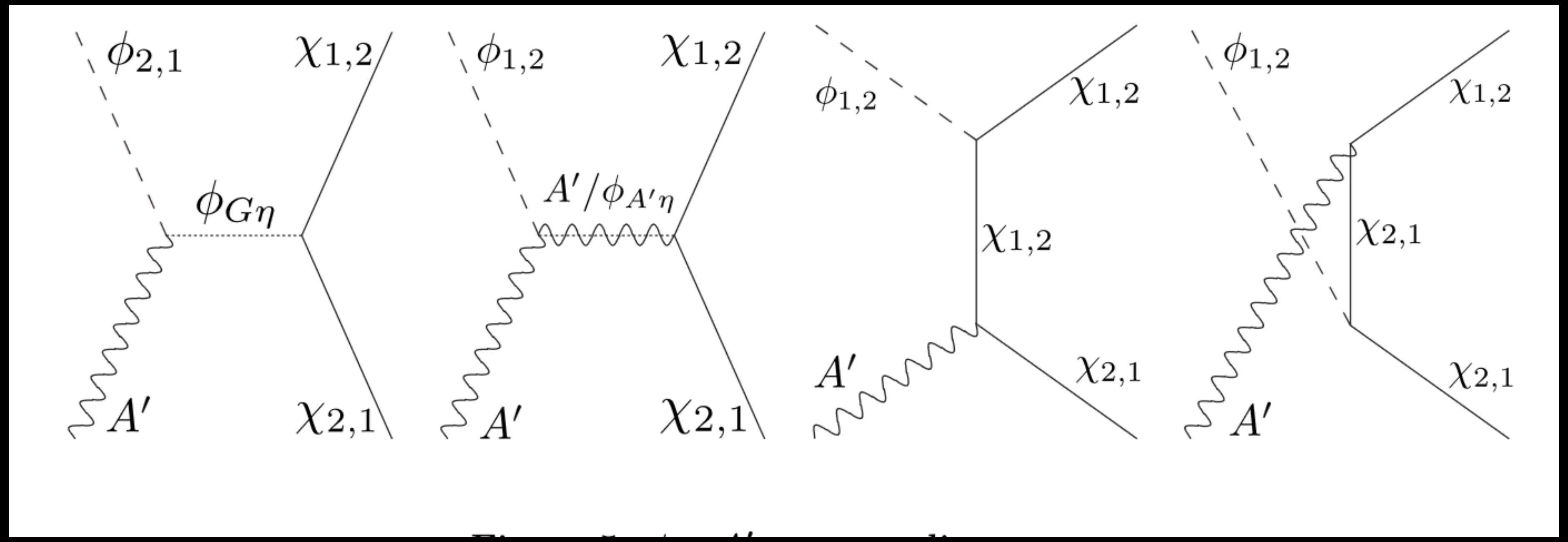


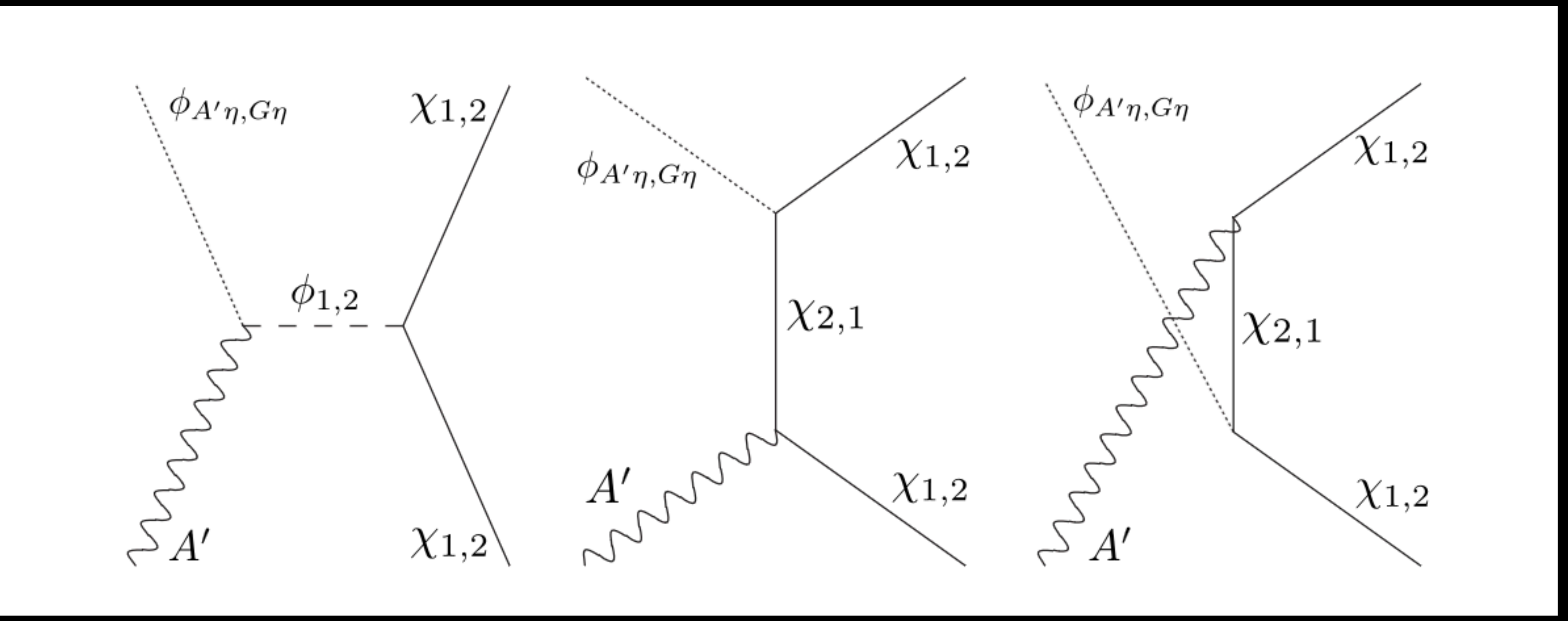












FIMP mediated by vector boson

- Besides, the phase integral is extremely cumbersome due to the Lorentz violation (by thermal plasma).

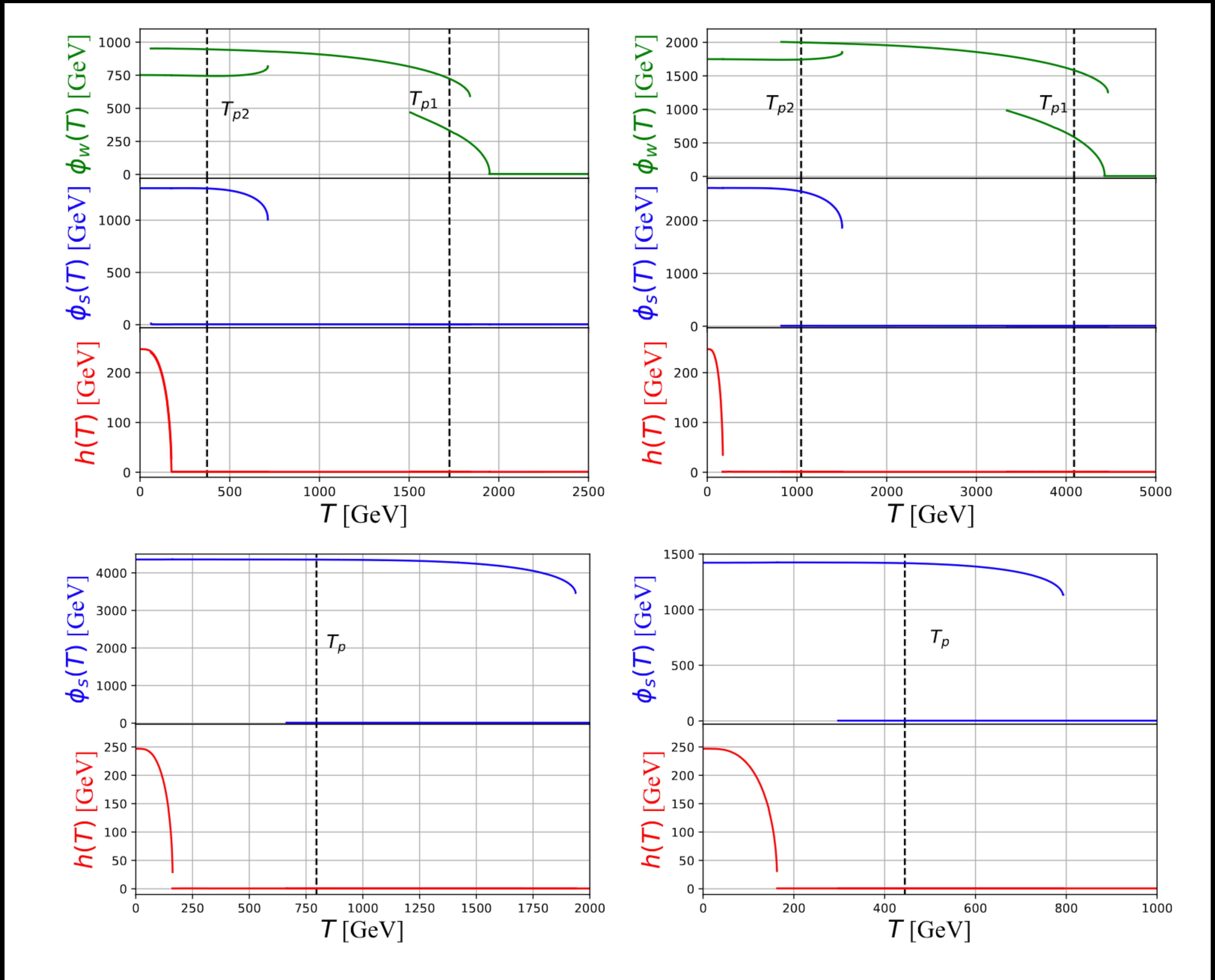
$$p_{A'}^2 - m_{A'}^2 - \Pi_{T,L}(p_{A'}) = 0,$$

No boost symmetry!!!

Phase evolution

BP_S1_1

BP_S1_2



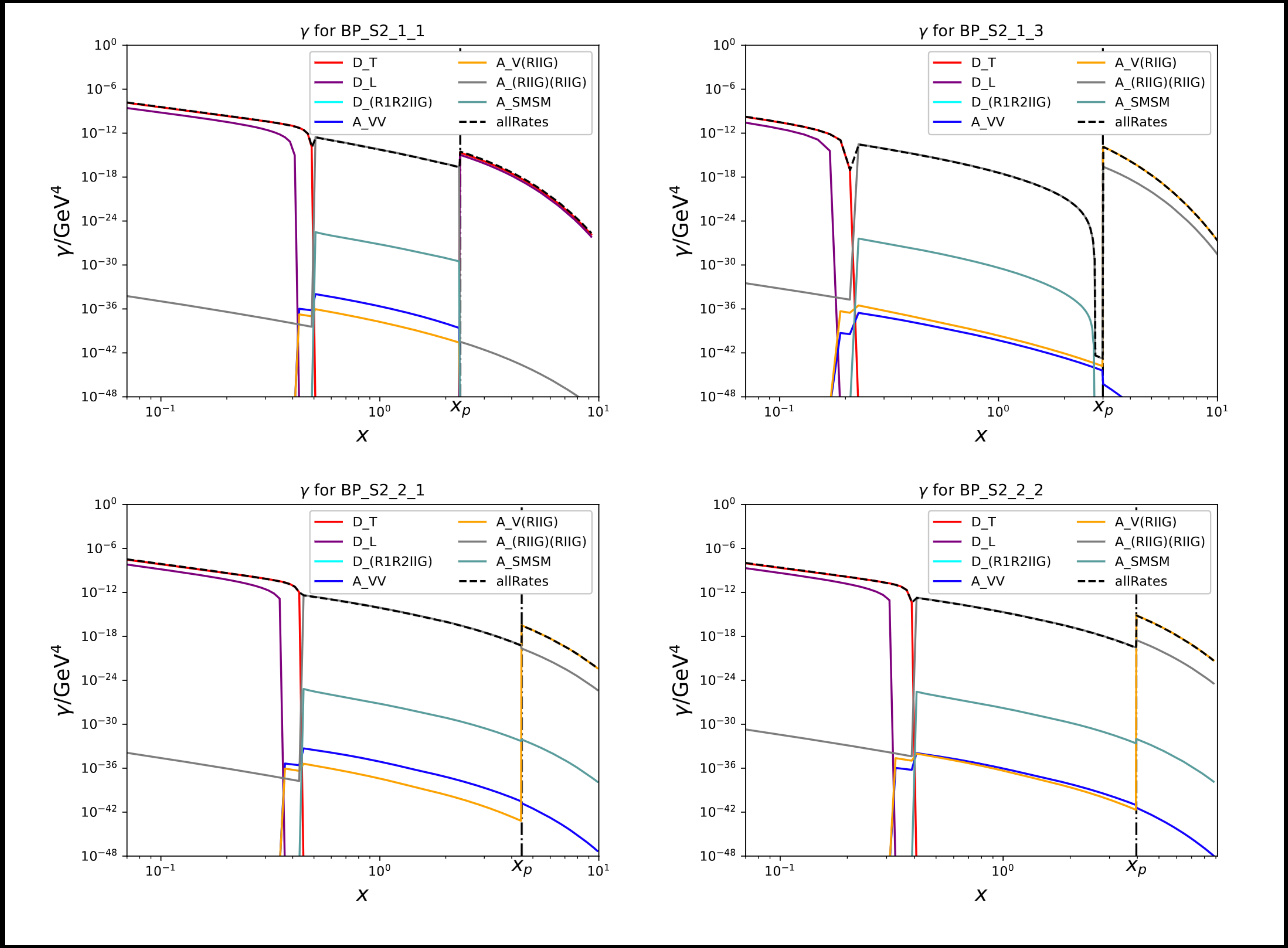
BP_S2_1

BP_S2_2

DM relic evolution

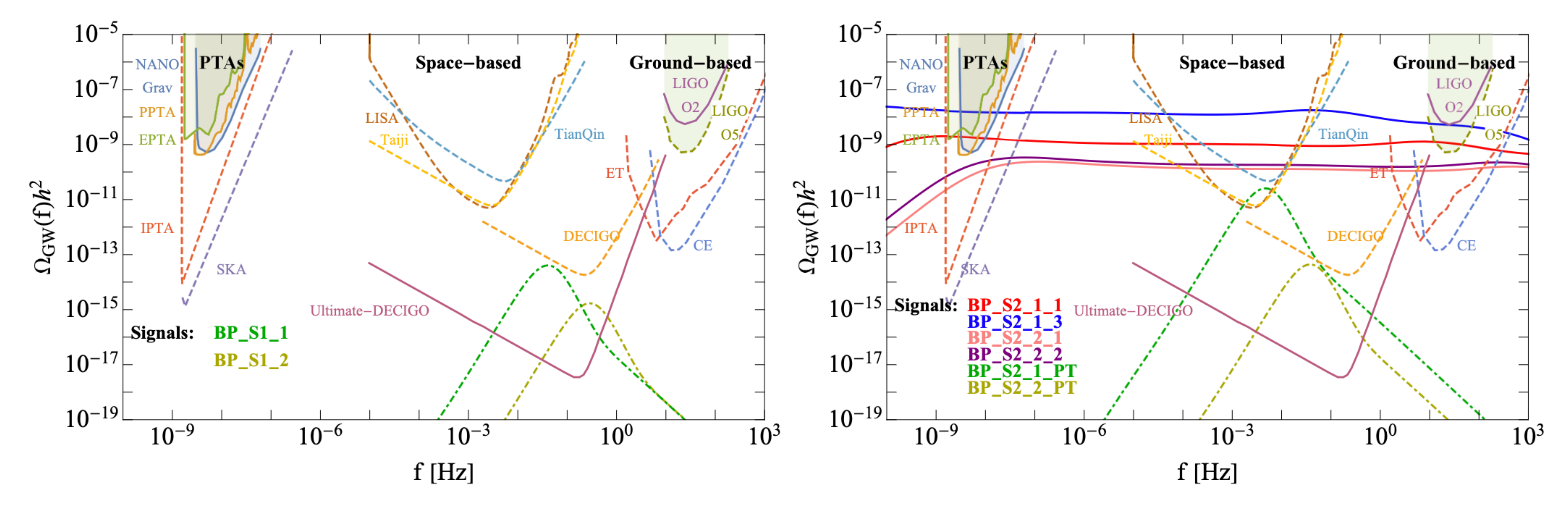
Parameters	BP_S2_1_1		BP_S2_1_3	BP_S2_2_1	BP_S2_2_2	
m_χ/GeV	2000		2000	2000	2000	
m_{χ_1}/GeV	1853.8		-2385.6	1975.9	1759.5	
g_χ	5.00×10^{-12}		3.64×10^{-13}	5.64×10^{-12}	4.03×10^{-12}	
y_χ	$0.1 g_\chi$		$3 g_\chi$	$0.1 g_\chi$	g_χ	

Only BP_S2_1 benchmark points are plotted.



Gravitational wave spectrum

BP_S1 **BP_S2**



Summary

- When $T \sim m_{FIMP} \sim m_A$, the thermal effect of the vector boson during the freeze-in processes cannot be neglected.
- Transverse, longitudinal, and goldstone remains should be computed separately, and the phase space integration is complicated. Evolution of the thermal masses affect the production rate through the modification of the phase space.
- For significant coupling with the longitudinal/goldstone elements, large v_w is acquired, and significant gravitational wave emission through the cosmic string effects can arise.