The electromagnetic decays of $X(3823)$ as the $\psi_2(1^3D_2)$ state and its radial excited states

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Content

1. Background

- The bound state - charmonium of charm and anti-charm quarks is significant in the quantum chromodynamics (QCD). It is a double-heavy meson, but not heavy enough that its relativistic corrections are still large.
- In 2013, a new bound state $X(3823)$ has been observed, which is considered to be a good candidate for spin triplet D wave charmonium $\psi_2(1^3D_2)$.
- For the decay properties of this particle, since its mass is below the $D\bar{D}^*$ threshold, and the $D\bar{D}$ channel is forbidden, there is no Okubo-Zweig-Iizuka (OZI)-allowed channel. Therefore, the single photon radiation processes are important.

2. Previous research

- Other works Different models have studied the radiative decays of $X(3823)$. These studies show that the dominate decay channel of $X(3823)$ is the radiative decay to $\chi_{c1}\gamma$.

  E. J. Eichten, K. Lane *et al*, *Phys. Rev. Lett* 89, 162002 (2002)
  D. Ebert, R. N. Faustov *et al*, Phys. Rev. D 67, 014027 (2003)
  T. Barnes, S. Godfrey *et al*, Phys. Rev. D 72, 054026 (2005)
  B.-Q. Li and K.-T. Chao, Phys. Rev. D 79, 094004 (2009)

- Our works The Bethe-Salpeter (BS) equation is a relativistic dynamic equation used to describe bound state. Using Bethe-Saleter method, we get theoretical results, which agree well with the experimental data.

  Chang, Chen, and Wang, Commun. Theor. Phys. 46, 467 (2006)
  Wang and Wang, Phys. Lett. B 697, 233 (2011)
  Wang, Jiang and Wang, J. High Energy Phys. 03, 209 (2016)
  Wang, Wang and Chang, J. High Energy Phys. 05, 006 (2022)

1. Bathe-Salpeter equation

- **Bathe-Salpeter equation**

  For a fermion-antifermion system, there has the general formulation

$$(\not{p}_1 - m_1)\chi_P(q)(\not{p}_2 + m_2) = i\int \frac{d^4k}{(2\pi)^4} V(P,k,q)\chi_P(k). \tag{2.1}$$

  $\chi_P(q)$ – BS wave function, $V(P,k,q)$ – interaction kernel.

- The meson momentum P and relative momentum q:

$$p_1 = \alpha_1 P + q, \quad \alpha_1 = \frac{m_1}{m_1 + m_2},$$

$$p_2 = \alpha_2 P - q, \quad \alpha_2 = \frac{m_2}{m_1 + m_2}.$$

E.E.Salpeter and H.A.Bethe , Phys. Rev. 84 (1951)

II. Bathe-Salpeter and Salpeter equation

- Though the BS equation is the relativistic dynamic equation, it can not provide us the form of a relativistic wave function for a bound state.
- There is the difficulty about the kernel $V(P,k,q)$ of BS equation, it is a time-inspired interaction kernel.
- Reduced version is needed -(instantaneous version).

2. Salpeter equation

- Salpeter equation

The reduced (instantaneous) Bathe-Salpeter wave function

$$\varphi_p(q_\perp^\mu) \equiv i \int \frac{dq_p}{(2\pi)} \chi_P(q_\parallel^\mu, q_\perp^\mu), \quad (2.2)$$

and new integration kernel

$$\eta(q_\perp^\mu) \equiv \int \frac{k_T^2 dk_T ds}{(2\pi)^2} V(k_\perp, q_\perp) \varphi_P(k_\perp^\mu). \quad (2.3)$$

The useful notations

$$\omega_i = \sqrt{m_i^2 + q_T^2}, \qquad \Lambda_i^\pm(q_\perp) = \frac{1}{2\omega_i}\left[\frac{\not{P}}{M}\omega_i \pm J_i(m_i + \not{q}_\perp)\right],$$

$$\varphi_P^{\pm\pm}(q_\perp) = \Lambda_1^\pm(q_\perp)\frac{\not{P}}{M}\varphi_P(q_\perp)\frac{\not{P}}{M}\Lambda_2^\pm(q_\perp).$$

E. E. Salpeter, PRD 87 (1952) 328.

- Then, BS equations can be written as

$$\chi(q) = S(p_1)\eta(q_\perp)S(-p_2). \tag{2.4}$$

$S(p_1)$ and $S(-p_2)$ represent fermion and antifermion propagators, respectively.

$$-iS(p_1) = \frac{i\Lambda_1^+}{q_P + \alpha_1 M - \omega_1 + i\epsilon} + \frac{i\Lambda_1^-}{q_P + \alpha_1 M + \omega_1 - i\epsilon}$$
$$iS(-p_2) = \frac{i\Lambda_2^+}{q_P - \alpha_2 M + \omega_2 - i\epsilon} + \frac{i\Lambda_2^-}{q_P - \alpha_2 M - \omega_2 + i\epsilon}$$

- Further carry out a contour integration for the time-component q_P on both sides of equation (2.4), then obtain Salpeter wave function

$$\varphi(q_\perp) = \frac{\Lambda_1^+(q_\perp)\eta(q_\perp)\Lambda_2^+(q_\perp)}{M - \omega_1 - \omega_2} - \frac{\Lambda_1^-(q_\perp)\eta(q_\perp)\Lambda_2^-(q_\perp)}{M + \omega_1 + \omega_2}. \tag{2.5}$$

II. Bathe-Salpeter and Salpeter equation

- Salpeter wave function include four parts

$$\varphi_P(q_\perp) = \varphi_P^{++}(q_\perp) + \varphi_P^{+-}(q_\perp) + \varphi_P^{-+}(q_\perp) + \varphi_P^{--}(q_\perp).$$

- Positive and negative energy wave function

  To apply the complete set of the projection operators $\Lambda_i^{\pm}(q_\perp)$, then obtain the four equations

$$(M - \omega_1 - \omega_2)\varphi_P^{++}(q_\perp) = \Lambda_1^{+}(q_\perp)\eta_P(q_\perp)\Lambda_2^{+}(q_\perp),$$

$$(M + \omega_1 + \omega_2)\varphi_P^{--}(q_\perp) = \Lambda_1^{-}(q_\perp)\eta_P(q_\perp)\Lambda_2^{-}(q_\perp),$$

$$\varphi_P^{+-}(q_\perp) = \varphi_P^{-+}(q_\perp) = 0.$$

- Normalization

$$\int \frac{d^3 q_\perp}{(2\pi)^3} tr[\bar{\varphi}_P^{++}\frac{\not{P}}{M}\varphi_P^{++}\frac{\not{P}}{M} - \bar{\varphi}_P^{--}\frac{\not{P}}{M}\varphi_P^{--}\frac{\not{P}}{M}] = 2P_0.$$

C.-H. Chang, J.-K. Chen, and G.-L.Wang, Commun. Theor. Phys. 46, 467 (2006).

II. Bathe-Salpeter and Salpeter equation

- Cornell potential

  In our model, Cornell potential is chosen as the instantaneous interaction kernel

$$V = V_0 + V_s(r) + \gamma_0 \otimes \gamma^0 V_v(r) = V_0 + \lambda r - \gamma_0 \otimes \gamma^0 \frac{4}{3}\frac{\alpha_s}{r}, \qquad (2.6)$$

  the running coupling constant $\alpha_s(\vec{q}) = \frac{12\pi}{33-2N_f} \frac{1}{log(a+\frac{\vec{q}^2}{\Lambda_{QCD}})}$.

- In momentum space

$$V(\vec{q}) = (2\pi)^3 V_s(\vec{q}) + \gamma_0 \otimes \gamma^0 V_v(\vec{q})$$

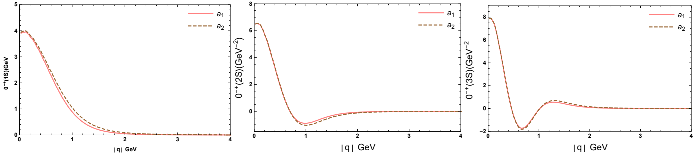
$$V_s(\vec{q}) = -(\frac{\lambda}{\alpha} + V_0)\delta^3(\vec{q}) + \frac{\lambda}{\pi^2}\frac{1}{(\vec{q}^2+\alpha^2)^2}, \quad V_v(\vec{q}) = -\frac{2}{3\pi^2}\frac{\alpha_s(\vec{q})}{\vec{q}^2+\alpha^2},$$

C.S.Kim, G-L Wang, PLB 584 (2004) 285.

1. Relativistic wave function

- For $J^{PC} = 0^{-+}$ state, the relativistic wave function

$$\varphi_{0^{-+}}(q_{f_\perp}) = M_f[\frac{\not{P}_{f_\perp}}{M_f}a_1 + a_2 + \frac{\not{P}_f \not{q}_{f_\perp}}{M_f m_c}a_2]\gamma^5, \quad (3.1)$$



The positive energy wave function

$$\varphi^{++}_{0^{-+}}(q_{f_\perp}) = [A_{f_1} + \frac{\not{P}_{f_\perp}}{M_f}A_{f_2} + \frac{\not{P}_f \not{q}_{f_\perp}}{M_f^2}A_{f_3}]\gamma^5, \quad (3.2)$$

where $A_{f_1} = \frac{M_f}{2}[\frac{\omega_f}{m_f}a_1 + a_2]$, $A_{f_2} = \frac{M_f}{2}[a_1 + \frac{m_f}{\omega_f}a_2]$, $A_{f_3} = -\frac{M_f}{\omega_f}A_{f_1}$.

C. S. Kim, Taekoon Lee, Guo-Li Wang, Phys. Lett. B 606 (2005).

III. Relativistic wave function and transition amplitude

- The partial wave of 0^{-+} state

In $\vec{P} = 0$ frame, the most general formulation of the BS wave function for the bound state $J^{PC} = 0^{-+}(^1S_0)$ may be written as the follows

$$\varphi_{0-+}(q_\perp) = \gamma^0\gamma^5 a_1 + \gamma^5 a_2 + \sqrt{\frac{4\pi}{3}}\gamma^5 \frac{q_\perp}{2M} E\gamma^0 a_3 + \sqrt{\frac{4\pi}{3}}\gamma^5 \frac{q_\perp}{2M} E a_4, \tag{3.3}$$

where

$$E \equiv [Y_{1-1}\gamma^+ + Y_{11}\gamma^- - Y_{10}\gamma^3], \quad \gamma^+ = \frac{\gamma^1 + i\gamma^2}{\sqrt{2}}, \quad \gamma^- = \frac{\gamma^1 - i\gamma^2}{\sqrt{2}},$$

and $Y_{lm} \equiv Y_{lm}(\theta_q, \phi_q)$ are spherical harmonics.

- In contrast to formula Eq.(3.1) and Eq.(3.3), we can know that A_{f_1} and $\frac{\not{P}_{f\perp}}{M_f} A_{f_2}$ terms are S $wave$, $\frac{\not{P}_f \not{q}_{f\perp}}{M_f^2} A_{f_3}$ term is P $wave$.

Chao-Hsi Chang, Jiao-Kai Chen, Xue-Qian Li, Guo-Li Wang, Commun. Theor. Phys. 43 (2005).

III. Relativistic wave function and transition amplitude

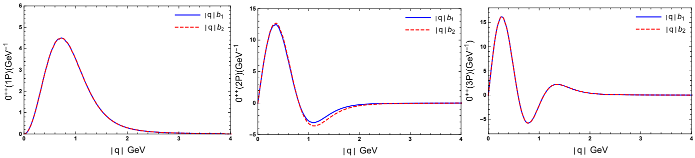
- For $J^{PC} = 0^{++}$ state, the positive energy wave function

$$\varphi^{++}_{0^{++}}(q_{f_\perp}) = B_{f_1} + \frac{\not{q}_{f_\perp}}{M_f} B_{f_2} + \frac{\not{P}_f \not{q}_{f_\perp}}{M_f^2} B_{f_3}, \tag{3.4}$$

where B_{f_2} and B_{f_3} terms are P *waves*, relativistic B_{f_1} term is S *wave*, with

$$B_{f_1} = \frac{q^2_{f_\perp}}{2m_f}[b_1 + \frac{m_f}{\omega_f} b_2], \quad B_{f_2} = \frac{M_f}{2}[b_1 + \frac{m_f}{\omega_f} b_2], \quad B_{f_3} = \frac{M_f}{2}[\frac{\omega_f}{m_f} b_1 + b_2],$$

b_1 and b_2 are independent radial wave functions.



G.-L. Wang, Phys. Lett. B 650, 15 (2007).

III. Relativistic wave function and transition amplitude

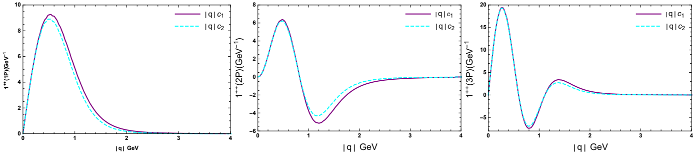
- For $J^{PC} = 1^{++}$ state, the positive energy wave function

$$\varphi^{++}_{1^{++}}(q_{f_\perp}) = i\epsilon_{\mu\nu\alpha\beta}\frac{P_f^\nu}{M_f} q_{f_\perp}^\alpha \epsilon_f^\beta \gamma^\mu [C_{f_1} + \frac{\not{P}_f}{M_f} C_{f_2} + \frac{\not{P}_f \not{q}_{f_\perp}}{M_f^2} C_{f_3}], \tag{3.5}$$

where C_{f_1} and C_{f_2} terms are $P\ waves$, relativistic C_{f_3} term is $D\ wave$, with

$$C_{f_1} = \frac{1}{2}[c_1 + \frac{\omega_f}{m_f} c_2], \quad C_{f_2} = -\frac{1}{2}[\frac{m_f}{\omega_f} c_1 + c_2], \quad C_{f_3} = -\frac{M_f}{\omega_f} C_{f_1},$$

c_1 and c_2 are independent radial wave functions.



G.-L. Wang, Phys. Lett. B 650, 15 (2007).

- For $J^{PC}=2^{++}$ state, the positive energy wave function

$$\varphi^{++}_{2++}(q_{f_\perp})=\epsilon_{f,\mu\nu}q^{\mu}_{f_\perp}q^{\nu}_{f_\perp}[D_{f_1}+\frac{\not P_f}{M_f}D_{f_2}+\frac{\not q_{f_\perp}}{M_f}D_{f_3}+\frac{\not P_f\not q_{f_\perp}}{M_f^2}D_{f_4}]$$
$$+M_f\epsilon_{f,\mu\nu}\gamma^{\mu}q^{\nu}_{f_\perp}[D_{f_5}+\frac{\not P_f}{M_f}D_{f_6}+\frac{\not P_f\not q_{f_\perp}}{M_f^2}D_{f_7}], \quad (3.6)$$

where D_{f_5} and D_{f_6} terms are P partial waves, D_{f_1}, D_{f_2} and D_{f_7} terms are D partial waves, while D_{f_3} and D_{f_4} terms are F partial waves, with

$$D_{f_1}=\frac{1}{2M_fm_f\omega_f}[\omega_fq^2_{f_\perp}d_3+m_fq^2_{f_\perp}d_4+M_f^2\omega_fd_5-M_f^2m_fd_6],\ D_{f_2}=\frac{M_f}{2m_f\omega_f}[m_fd_5-\omega_fd_6],$$

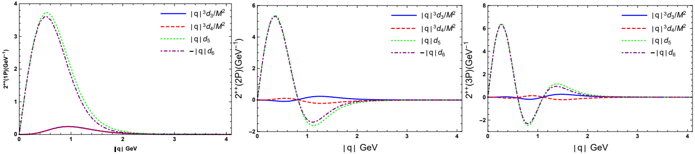
$$D_{f_3}=\frac{1}{2}[d_3+\frac{m_f}{\omega_f}d_4-\frac{M_f^2}{m_f\omega_f}d_6],\ D_{f_4}=\frac{1}{2}[\frac{\omega_f}{m_f}d_3+d_4-\frac{M_f^2}{m_f\omega_f}d_5],\ D_{f_5}=\frac{1}{2}[d_5-\frac{\omega_f}{m_f}d_6],$$

$$D_{f_6}=\frac{1}{2}[-\frac{m_f}{\omega_f}d_5+d_6],\quad D_{f_7}=\frac{M_f}{2\omega_f}[-d_5+\frac{\omega_f}{m_f}d_6],$$

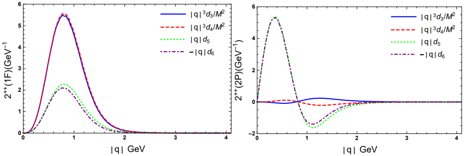
d_i are independent radial wave functions.

G.-L. Wang, Phys. Lett. B 650, 15 (2007).

- For $J^{PC} = 2^{++}$ (nD) state,



- For $J^{PC} = 2^{++}$ (nF) state,

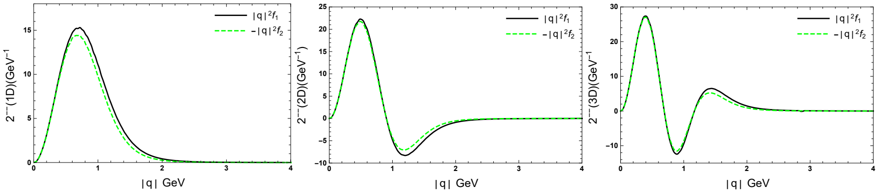


G.-L. Wang, Phys. Lett. B 674, 172 (2009).

III. Relativistic wave function and transition amplitude

- For $J^{PC} = 2^{--}$ state, the relativistic wave function

$$\varphi_{2^{--}}(q_\perp) = i\epsilon_{\mu\nu\alpha\beta}\frac{P^\nu}{M}q_\perp^\alpha\epsilon^{\beta\delta}q_{\perp\delta}\gamma^\mu\left(f_1 + \frac{\not{P}}{M}f_2 + \frac{\not{P}\not{q}_\perp}{Mm_c}f_2\right), \quad (3.7)$$



The positive energy wave function for a 2^{--} state is

$$\varphi^{++}_{2^{--}}(q_\perp) = i\epsilon_{\mu\nu\alpha\beta}\frac{P^\nu}{M}q_\perp^\alpha q_{\perp\delta}\epsilon^{\beta\delta}\gamma^\mu\left[F_1 + \frac{\not{P}}{M}F_2 + \frac{\not{P}\not{q}_\perp}{M^2}F_3\right], \quad (3.8)$$

where $F_1 = \frac{1}{2}[f_1 - \frac{\omega_c}{m_c}f_2]$, $F_2 = -\frac{1}{2}[\frac{m_c}{\omega_c}f_1 - f_2]$, $F_3 = -\frac{M}{\omega_c}F_1$.

T.-H. Wang and G.-L. Wang *et. al*, Int. J. Mod. Phys. A 32, 1750035 (2017).

2. Amplitude and form factors

- The Feynman diagrams for the transition $X(3823) \to \chi_{cJ}\gamma$

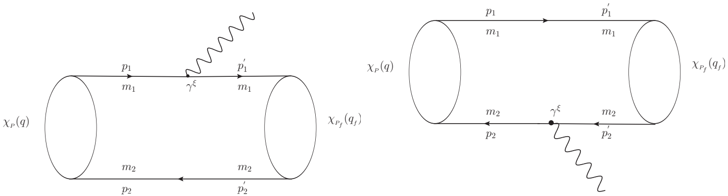


Figure: Feynman diagrams for the transition $X(3823) \to \chi_{cJ}\gamma$. The two diagrams show that photons come from the quark and the anti-quark, respectively.

- The amplitude can be written as

$$\mathcal{M}^\xi = \int \frac{d^4q}{(2\pi)^4}\frac{d^4q_f}{(2\pi)^4} Tr[\bar{\chi}_{P_f}(q_f)Q_1 e\gamma^\xi \chi_P(q)(2\pi)^4\delta^4(p_2 - p_2')$$
$$S_2^{-1}(-p_2) + \bar{\chi}_{P_f}(q_f)(2\pi)^4\delta^4(p_1 - p_1')S_1^{-1}(p_1)\chi_P(q)Q_2 e\gamma^\xi], \qquad (3.9)$$

- The φ^{++}, is much bigger than the φ^{--}, so the contribution of the φ^{++} is much larger than others. Therefore, the decay amplitude can be written as

$$\mathcal{M}^{\xi} = \int \frac{d^3 q_\perp}{(2\pi)^3} Tr[Q_1 e \frac{\not{P}}{M} \bar{\varphi}_f^{++}(q_\perp + \alpha_2 P_{f_\perp}) \gamma^{\xi} \varphi_i^{++}(q_\perp) + Q_2 e \bar{\varphi}_f^{++}(q_\perp - \alpha_1 P_{f_\perp}) \frac{\not{P}}{M} \varphi_i^{++}(q_\perp) \gamma^{\xi}]. \quad (3.10)$$

Chao-Hsi Chang, Jiao-Kai Chen, Xue-Qian Li, Guo-Li Wang, Commun. Theor. Phys. 43 (2005)

- The form factors expression of amplitude

Integrate internal $q_\perp$ over the initial and final state wave functions, then obtain the amplitude described using form factors.

(1) For the channel $X(3823) \to \eta_c(^1S_0)\gamma$,

$$\mathcal{M}_1^\xi = P^\xi \epsilon_{\mu\nu} P_f^\mu P_f^\nu h_1 + \epsilon_\mu^\xi P_f^\mu h_2, \tag{3.11}$$

(2) For $X(3823) \to \chi_{c0}(^3P_0)\gamma$,

$$\mathcal{M}_2^\xi = i\epsilon^{\beta\xi\mu\nu} \epsilon_{\beta\alpha} P_\mu P_{f,\nu} P_f^\alpha t_1, \tag{3.12}$$

(3) For $X(3823) \to \chi_{c1}(^3P_1)\gamma$,

$$\begin{aligned}\mathcal{M}_3^\xi = {} & \epsilon_{\mu\nu} P^\xi P_f^\mu P_f^\nu P\cdot\epsilon_f s_1 + \epsilon_\nu^\xi P_f^\nu P\cdot\epsilon_f s_2 + \epsilon_{\mu\nu} P^\xi \epsilon_f^\mu P_f^\nu s_3 \\ & + \epsilon_{\mu\nu}\epsilon_f^\xi P_f^\mu P_f^\nu s_4 + \epsilon_\mu^\xi \epsilon_f^\mu s_5,\end{aligned} \tag{3.13}$$

- The form factors expression of amplitude

(4) For $X(3823) \to \chi_{c2}(^3P_2)\gamma$ or $X(3823) \to \chi_{c2}(^3F_2)\gamma$,

$$\begin{aligned}\mathcal{M}_4^{\xi} = & \, i\epsilon^{\beta\lambda P_f P}\left(\epsilon_{\beta P_f}\epsilon_{f,\lambda P}P^{\xi}g_1 + \epsilon_{\beta}^{\phi}\epsilon_{f,\lambda\phi}P^{\xi}g_2 + \epsilon_{\beta}^{\xi}\epsilon_{f,\lambda P}g_3 + \epsilon_{\beta P_f}\epsilon_{f,\lambda}^{\xi}g_4\right) \\ & + i\epsilon^{\beta\xi P_f P}\left(\epsilon_{\beta P_f}\epsilon_{f,PP}g_5 + \epsilon_{\beta}^{\phi}\epsilon_{f,\phi P}g_6\right) + i\epsilon^{\beta\xi\lambda P}\left(\epsilon_{\beta P_f}\epsilon_{f,\lambda P}g_7 + \epsilon_{\beta}^{\phi}\epsilon_{f,\lambda\phi}g_8\right),\end{aligned} \tag{3.14}$$

where $\epsilon_{f,\mu\nu}$ is the polarization tensor of $\chi_{c2}(^3P_2)$, and we have used some abbreviations, for example, $\epsilon^{\beta\lambda P_f P}\epsilon_{\beta P_f}\epsilon_{f,\lambda P} = \epsilon^{\beta\lambda\mu\nu}P_{f,\mu}P_{\nu}\epsilon_{\beta\alpha}P_f^{\alpha}\epsilon_{f,\lambda\rho}P^{\rho}$.

III. Relativistic wave function and transition amplitude

- The form factors expression of amplitude

In here, these form factors are not independent. Due to the Ward identity $(P_\xi - P_{f,\xi})\mathcal{M}_i^\xi = 0$ $(i = 1, 2, 3, 4)$, they are linked by the following constrain conditions:

$$h_2 = (M^2 - ME_f)h_1, \tag{3.15}$$

$$s_2 = (M^2 - ME_f)s_1 + s_4, \quad s_5 = (M^2 - ME_f)s_3, \tag{3.16}$$

$$g_3 = (M^2 - ME_f)g_1 + g_4 + g_7, \quad g_8 = -(M^2 - ME_f)g_2. \tag{3.17}$$

Other form factors such as t_1, g_5 and g_6 are independent and have no such constraints.

- Defining the decay width

The amplitude square for the electromagnetic (EM) decay of $X(3823)$ is

$$\overline{|\mathcal{M}|^2} = \frac{1}{2J+1} \sum_{\gamma} \varepsilon_{\xi}^{(\gamma)} \varepsilon_{\xi'}^{(\gamma)} \mathcal{M}^{\xi} \mathcal{M}^{\xi'}, \tag{3.18}$$

where, $\varepsilon_{\xi}^{(\gamma)}$ is the polarization vector of the final state photon γ, J is the total angular momentum of the initial state.

Finally, the two-body decay width formulation can be written as

$$\Gamma = \frac{|\vec{P}_f|}{8\pi M^2} \overline{|\mathcal{M}|^2}, \tag{3.19}$$

where, $|\vec{P}_f| = \left(M^2 - M_f^2\right)/2M$.

1. EM decay widths of $X(3823)$ and its radial excited states

- **EM decay widths of $X(3823)$ as the $\psi_2(1^3D_2)$ state**

Considering $X(3823)$ as the $\psi_2(1^3D_2)$ state, the final state are $\eta_c(1S)\gamma$ and $\chi_{c0}(1P)\gamma$, the decay widths are

$$\Gamma[X(3823) \to \chi_{c0}(1S)\gamma] = 1.22 \text{ keV},$$

$$\Gamma[X(3823) \to \eta_c(1S)\gamma] = 1.30 \text{ keV}.$$

The EM decay results of other channels are

$$\Gamma[X(3823) \to \chi_{\{c1,\ c2\}}(1P)\gamma] = \{265,\ \ 57\} \text{ keV},$$

and

$$\Gamma[X(3823) \to \eta_c(2S)\gamma] = 0.069 \text{ keV}.$$

III. Results and discussions

- EM decay widths of $X(3823)$ as the $\psi_2(1^3D_2)$ state

TABLE I. The decay widths (keV) of the radiative transition $X(3823) \to \chi_{cJ}(1P)\gamma$ ($J = 0, 1, 2$), $X(3823) \to \eta_c(1S, 2S)\gamma$, and the ratio of $\frac{\Gamma(\psi_2(1D)\to\chi_{c2}(1P)\gamma)}{\Gamma(\psi_2(1D)\to\chi_{c1}(1P)\gamma)}$.

	[20]	[25]	[26]				[27]		[28]			[29]		Ours	
	RE	*RE*	*NR*	*RV*	*RS*	*RVS*	*NR*	*GI*	NR_1	NR_2	*RE*	NR_1	NR_2	*RE*	EX [31]
$\Gamma(\psi_2(1D) \to \chi_{c1}(1P)\gamma)$	250	260	297	215	215	215	307	268	307	342	208	285	296	265	
$\Gamma(\psi_2(1D) \to \chi_{c2}(1P)\gamma)$	60	56	62	55	51	59	64	66	64	70	55	91	96	57	
$\frac{\Gamma(\psi_2(1D)\to\chi_{c2}(1P)\gamma)}{\Gamma(\psi_2(1D)\to\chi_{c1}(1P)\gamma)}$ %	24	22	21	26	24	27	21	25	21	20	26	32	32	22	$28^{+14}_{-11} \pm 2$
$\Gamma(\psi_2(1D) \to \chi_{c0}(1P)\gamma)$														1.2	
$\Gamma(\psi_2(1D) \to \eta_c(1S)\gamma)$														1.3	
$\Gamma(\psi_2(1D) \to \eta_c(2S)\gamma)$														0.069(0.067)	

$$\frac{\mathcal{B}[X(3823) \to \chi_{c2}\gamma]}{\mathcal{B}[X(3823) \to \chi_{c1}\gamma]} = 22\ \%, \quad \frac{\mathcal{B}[X(3823) \to \chi_{c0}\gamma]}{\mathcal{B}[X(3823) \to \chi_{c1}\gamma]} = 0.46\ \%.$$

This result is within the range of current experimental value $0.28^{+0.14}_{-0.11} \pm 0.02$ and < 0.24.

M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D 106, 052012 (2022).

- EM decay widths of $\psi_2(2^3D_2)$ state

The dominant decay channel is $\psi_2(2D) \to \chi_{c1}(2P)\gamma$

$$\Gamma[\psi_2(2D) \to \chi_{c1}(2P)\gamma] = 237 \text{ keV}.$$

TABLE II. The decay widths (keV) of the radiative transition of the $\psi_2(2D) \to \chi_{cJ}\gamma$ $(J = 0,\ 1,\ 2)$ and $\psi_2(2D) \to \eta_c\gamma$.

	[27]		[47]			[29]		Ours
	NR	GI	NR_1	NR_2	NR_3	NR_1	NR_2	RE
$\Gamma(\psi_2(2D) \to \chi_{c0}(1P)\gamma)$								0.16
$\Gamma(\psi_2(2D) \to \chi_{c1}(1P)\gamma)$	26	23	17	26	10	68	68	33
$\Gamma(\psi_2(2D) \to \chi_{c2}(1P)\gamma)$	7.2	0.62	6.7	10	3.8	20	20	7.3
$\Gamma(\psi_2(2D) \to \chi_{c2}(1F)\gamma)$								6.2
$\Gamma(\psi_2(2D) \to \chi_{c0}(2P)\gamma)$								1.13
$\Gamma(\psi_2(2D) \to \chi_{c1}(2P)\gamma)$	298	225	140	178	92	223	188	237 (230)
$\Gamma(\psi_2(2D) \to \chi_{c2}(2P)\gamma)$	52	65	39	64	19	115	64	58
$\frac{\Gamma(\psi_2(2D)\to\chi_{c1}(1P)\gamma)}{\Gamma(\psi_2(2D)\to\chi_{c1}(2P)\gamma)}(\%)$	8.7	10	12	15	11	30	36	14
$\frac{\Gamma(\psi_2(2D)\to\chi_{c2}(2P)\gamma)}{\Gamma(\psi_2(2D)\to\chi_{c1}(2P)\gamma)}(\%)$	17	29	28	36	21	52	34	25
$\Gamma(\psi_2(2D) \to \eta_c(1S)\gamma)$								2.1
$\Gamma(\psi_2(2D) \to \eta_c(2S)\gamma)$								0.33 (0.32)
$\Gamma(\psi_2(2D) \to \eta_c(3S)\gamma)$								0.092

IV. Results and discussions

- EM decay widths of $\psi_2(3^3D_2)$ state

The dominant decay channel is $\psi_2(3D) \to \chi_{c1}(3P)\gamma$

$$\Gamma[\psi_2(3D) \to \chi_{c1}(3P)\gamma] = 218\ \text{keV}.$$

TABLE III. The EM decay widths (keV) of the excited state $\psi_2(3D)$.

Initial state	Final state	$\Gamma_{(\text{our})}$	Final state	$\Gamma_{(\text{our})}$	Final state	$\Gamma_{(\text{our})}$
$\psi_2(3D)$	$\chi_{c0}(1P)\ \gamma$	0.26	$\chi_{c0}(2P)\ \gamma$	0.54	$\chi_{c0}(3P)\ \gamma$	1.1
$\psi_2(3D)$	$\chi_{c1}(1P)\ \gamma$	38	$\chi_{c1}(2P)\ \gamma$	40(41)	$\chi_{c1}(3P)\ \gamma$	218
$\psi_2(3D)$	$\chi_{c2}(1P)\ \gamma$	6.8	$\chi_{c2}(2P)\ \gamma$	8.3	$\chi_{c2}(3P)\ \gamma$	41
$\psi_2(3D)$	$\chi_{c2}(1F)\ \gamma$	8.3	$\chi_{c2}(2F)\ \gamma$	11		
$\psi_2(3D)$	$\eta_c(1S)\ \gamma$	4.6	$\eta_c(2S)\ \gamma$	2.55(2.44)	$\eta_c(3S)\ \gamma$	0.24

The dominant EM decay channel for $\psi_2(nD)$ is $\chi_{c1}(nP)\gamma$, and the second is $\chi_{c2}(nP)\gamma$, where $n = 1, 2, 3$, respectively, while $\chi_{c0}(nP)\gamma$ and $\eta_c(nS)\gamma$ channels always have small contributions.

IV. Results and discussions

2. Contributions of different partial waves

- For $\psi_2(1D) \to \eta_c(1S)\gamma$, the contribution of $D\ wave \to S\ wave$ transition is suppressed, indicates that the major contribution of this decay process is due to relativistic effect.

TABLE IV. The decay width (keV) of different partial waves for $\psi_2(1D) \to \eta_c(1S)\gamma$.

2^{--} \ 0^{-+}	$Complete$	$S\,wave(A_{f_1}, A_{f_2})$	$P\,wave(A_{f_3})$
$Complete$	1.3	0.0035	1.3
$D\,wave(F_1, F_2)$	3.1	0.41	1.3
$F\,wave(F_3)$	0.39	0.39	0

2. Contributions of Different Partial Waves

- Partial wave contribution

For $\psi_2(1D) \to \chi_{c0}(1P)\gamma$, its result is similar to the case of $\psi_2(1D) \to \eta_c(1S)\gamma$, the contribution of dominant P wave in final state is very small, while the contribution of the small component of S wave is large.

TABLE V. The EM decay width (keV) of different partial waves for $\psi_2(1D) \to \chi_{c0}(1P)\gamma$.

2^{--} \ 0^{++}	$Complete$	$S\,wave(B_{f_1})$	$P\,wave(B_{f_2}, B_{f_3})$
$Complete$	1.2	1.3	0.19
$D\,wave(F_1, F_2)$	1.3	1.4	0.19
$F\,wave(F_3)$	0.14	0.14	~0

- Partial wave contribution

For $\psi_2(1D) \to \chi_{c1}(1P)\gamma$, the main contribution of the final state come from the dominant P partial wave which provides the non-relativistic result, and the relativistic correction (D partial wave in 1^{++} state) contribute very small.

TABLE VI. The EM decay width (keV) of different partial waves for $\psi_2(1D) \to \chi_{c1}(1P)\gamma$.

2^{--} \ 1^{++}	$Complete$	$P\,wave(C_{f_1}, C_{f_2})$	$D\,wave(C_{f_3})$
$Complete$	265	204	4.0
$D\,wave(F_1, F_2)$	209	211	4.2
$F\,wave(F_3)$	3.4	0.17	0.0056

IV. Results and discussions

- Partial wave contribution

For $\psi_2(1D) \to \chi_{c2}(1P)\gamma$, the main contribution of the final state come from the dominant P partial wave which provides the non-relativistic result, and the relativistic correction (D and F partial wave in 2^{++} state) contribute very small.

TABLE VII. The EM decay width (keV) of different partial waves for $\psi_2(1D) \to \chi_{c2}(1P)\gamma$.

2^{--} \ 2^{++}	$Complete$	$P\,wave(D_{f_5}, D_{f_6})$	$D\,wave(D_{f_1}, D_{f_2}, D_{f_7})$	$F\,wave(D_{f_3}, D_{f_4})$
$Complete$	57	18	1.5	0.23
$D\,wave(F_1, F_2)$	75	44	4.9	0.70
$F\,wave(F_3)$	1.7	6.1	1.4	0.0057

3. Discussions

- From these tables, we can see that in all the decays, the main contribution of 2^{--} state ψ_2 comes from its dominant partial wave, namely D wave, which is also its non-relativistic term, and its relativistic correction term, namely F partial wave, has a relatively small contribution.

- Compared with the complete relativistic results, the relativistic effects make up 68 %, 84 %, 20 %, 23 % of $X(3823) \to \eta_c(1S)\gamma$, $X(3823) \to \chi_{cJ}(1P)\gamma$ ($J = 0, 1, 2$), respectively.

V. Summary

- **We study the EM decays of $\psi_2(n^3D_2)$ ($n = 1, 2, 3$) by using the relativistic Bethe-Salpeter method, where the new particle $X(3823)$ is treated as $\psi_2(1^3D_2)$ in this paper. And the dominant EM decay channel is $\psi_2(n^3D_2) \to \chi_{c1}(nP)\gamma$.**
- **Our results show that $\Gamma[X(3823) \to \chi_{c1}\gamma] = 265$ keV, this is the dominant decay channel. The decay ratio $\mathcal{B}[X(3823) \to \chi_{c2}\gamma]/\mathcal{B}[X(3823) \to \chi_{c1}\gamma] = 22\%$ is consistent with the observation $0.28^{+0.14}_{-0.11} \pm 0.02$, and the decay ratio $\mathcal{B}[X(3823) \to \chi_{c0}\gamma]/\mathcal{B}[X(3823) \to \chi_{c1}\gamma] \simeq 0.46\%$ is also less than experimental upper limit 0.24.**
- **In addition, we calculated the contributions of different partial waves. For the decays $X(3823) \to \eta_c(1S)\gamma$ and $X(3823) \to \chi_{c0}(1P)\gamma$, the main contribution comes from the relativistic effect, while for the $X(3823) \to \chi_{cJ}(1P)\gamma$ ($J = 1, 2$) decay, the non-relativistic contribution is the dominant one.**

Thanks