The study of Type II and Type III Seesaw mechanisms at muon collider

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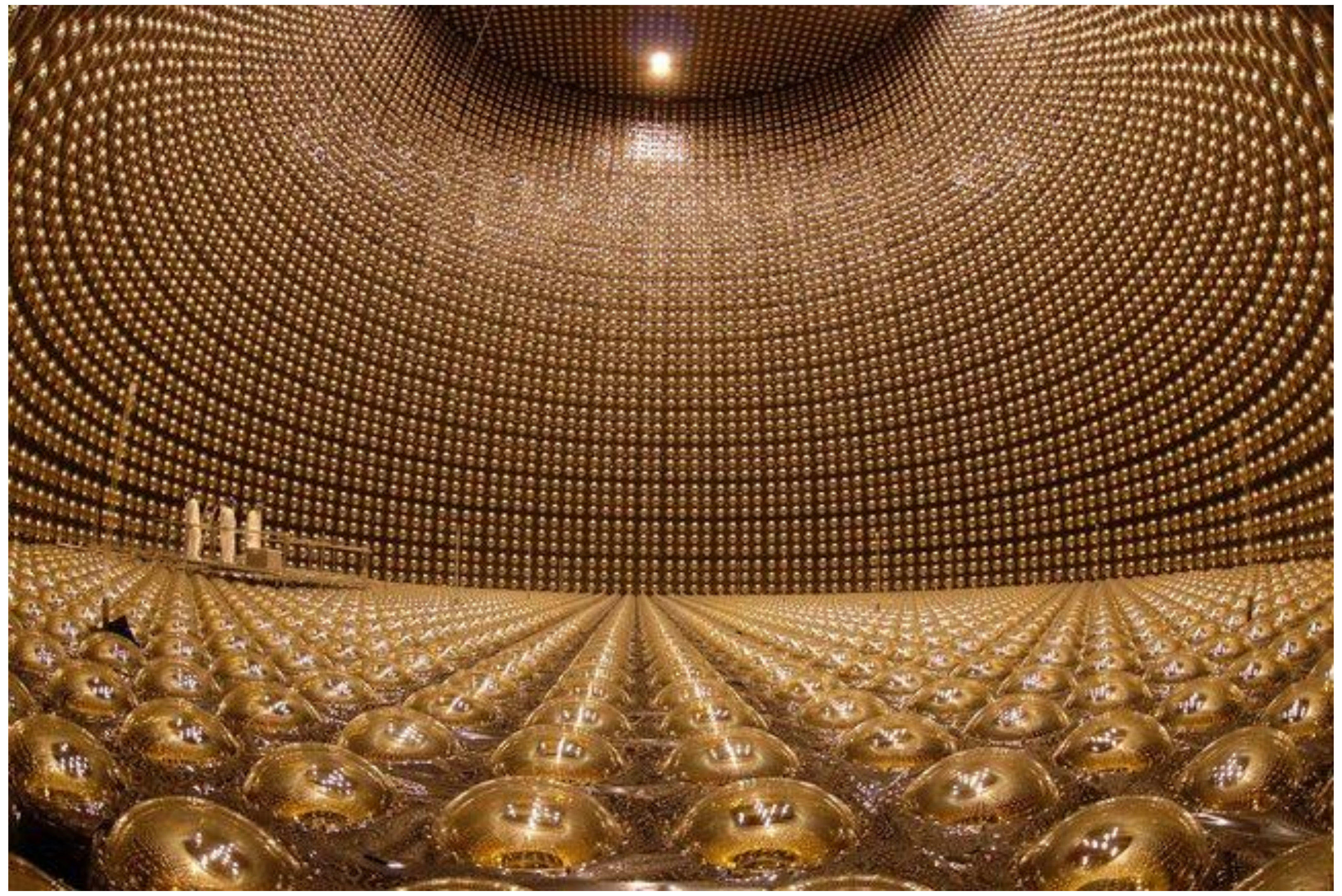
Outline

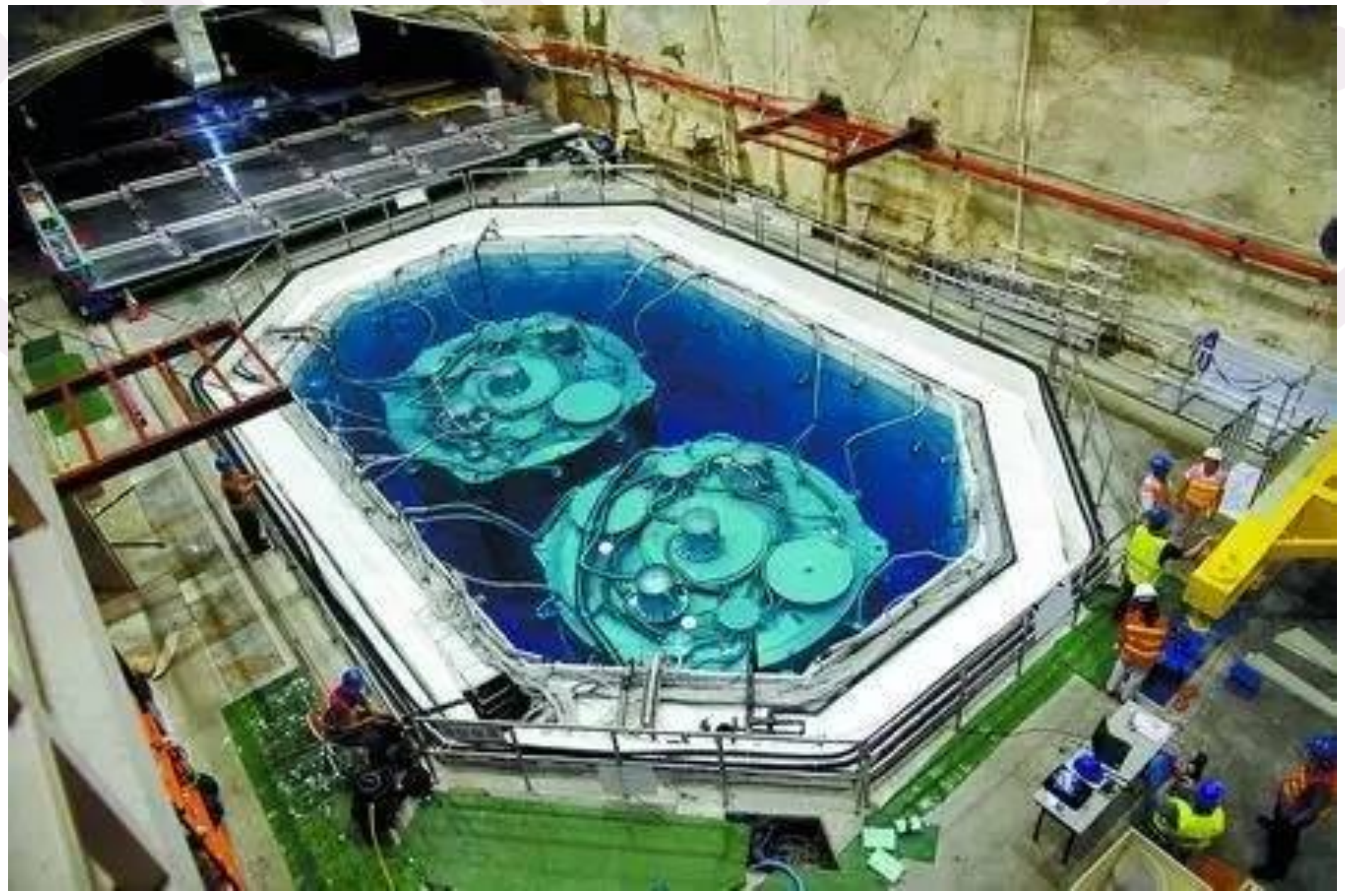
- **Motivation（Muon collider）**
- **Type II Seesaw, Type III Seesaw,**
  Heavy neutral lepton（HNL）
  2301.07274 , 2205.04214 , 2306.17368
- **Summary**

Motivation

- Since the last century， a large number of neutrino oscillation experiments provide clear and compelling evidence that neutrinos have non-zero, but very small masses. It is obviously contrary to the prediction of SM (neutrinos are massless) !!!



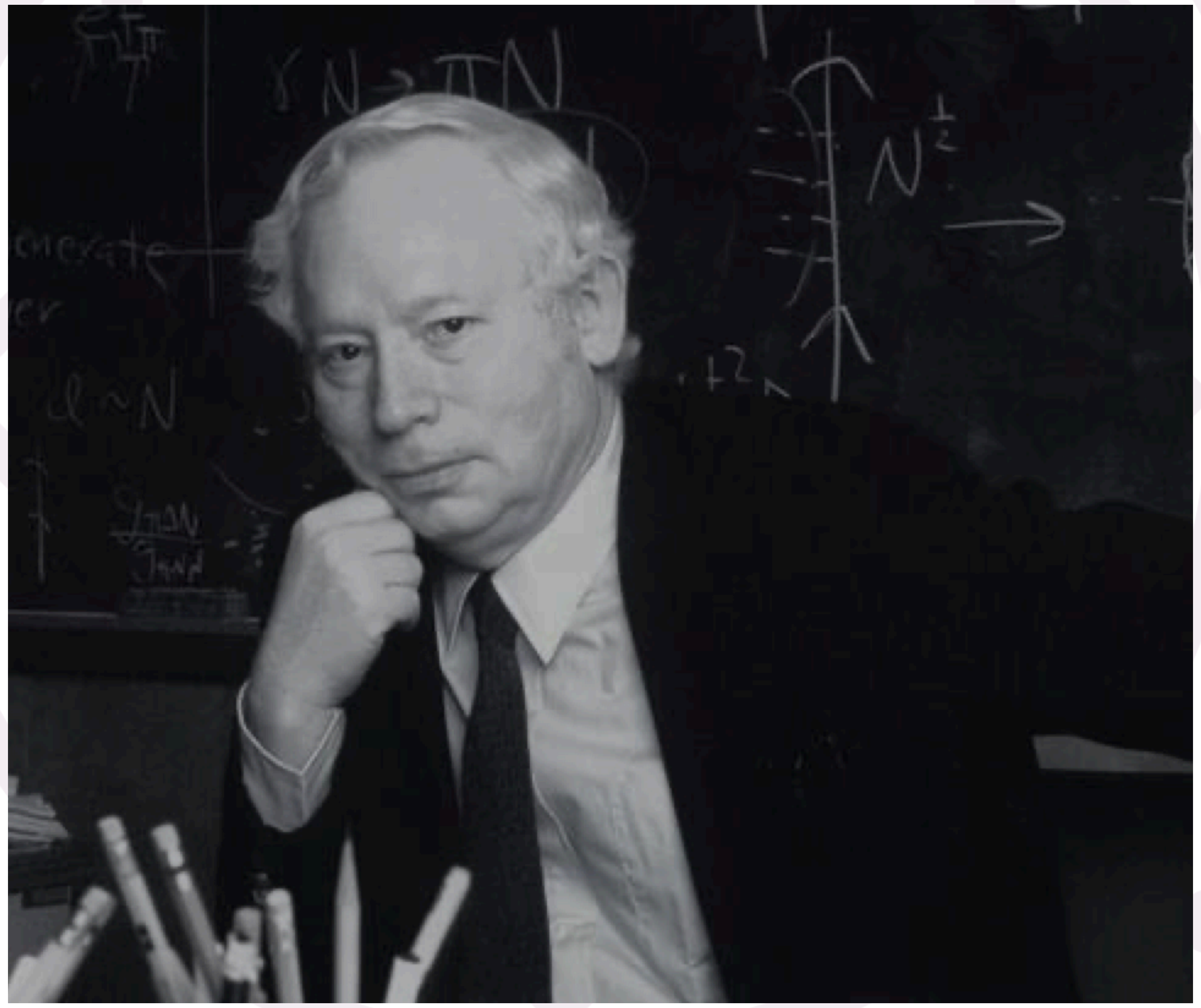
Torn between identities – tau-, electron- or muon-neutrino?



Motivation

- It is well-known that, an economical way to generate neutrino mass in the SM content is through the dim-5 effective operator ——"Weinberg operator"

$$\frac{\kappa}{\Lambda}\ell_L\ell_L HH$$



- There are only three ultraviolet (UV) completions of this "Weinberg operator" at tree level:

Type I Seesaw

Type II Seesaw → $SU(2)_L$ scalar triplet $\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$

Type III Seesaw → $SU(2)_L$ fermion triplet $\Sigma_L = \begin{pmatrix} \Sigma_L^0/\sqrt{2} & \Sigma_L^+ \\ \Sigma_L^- & -\Sigma_L^0/\sqrt{2} \end{pmatrix}$.

Motivation

- Currently, the charged Higgs ($H^{\pm\pm}$, $H^{\pm}$) in the Type-II Seesaw and heavy fermions ($E^{\pm}$, N) in the Type-III Seesaw have been extensively studied at the LHC and lepton colliders, and their mass limits are approaching 1 TeV.

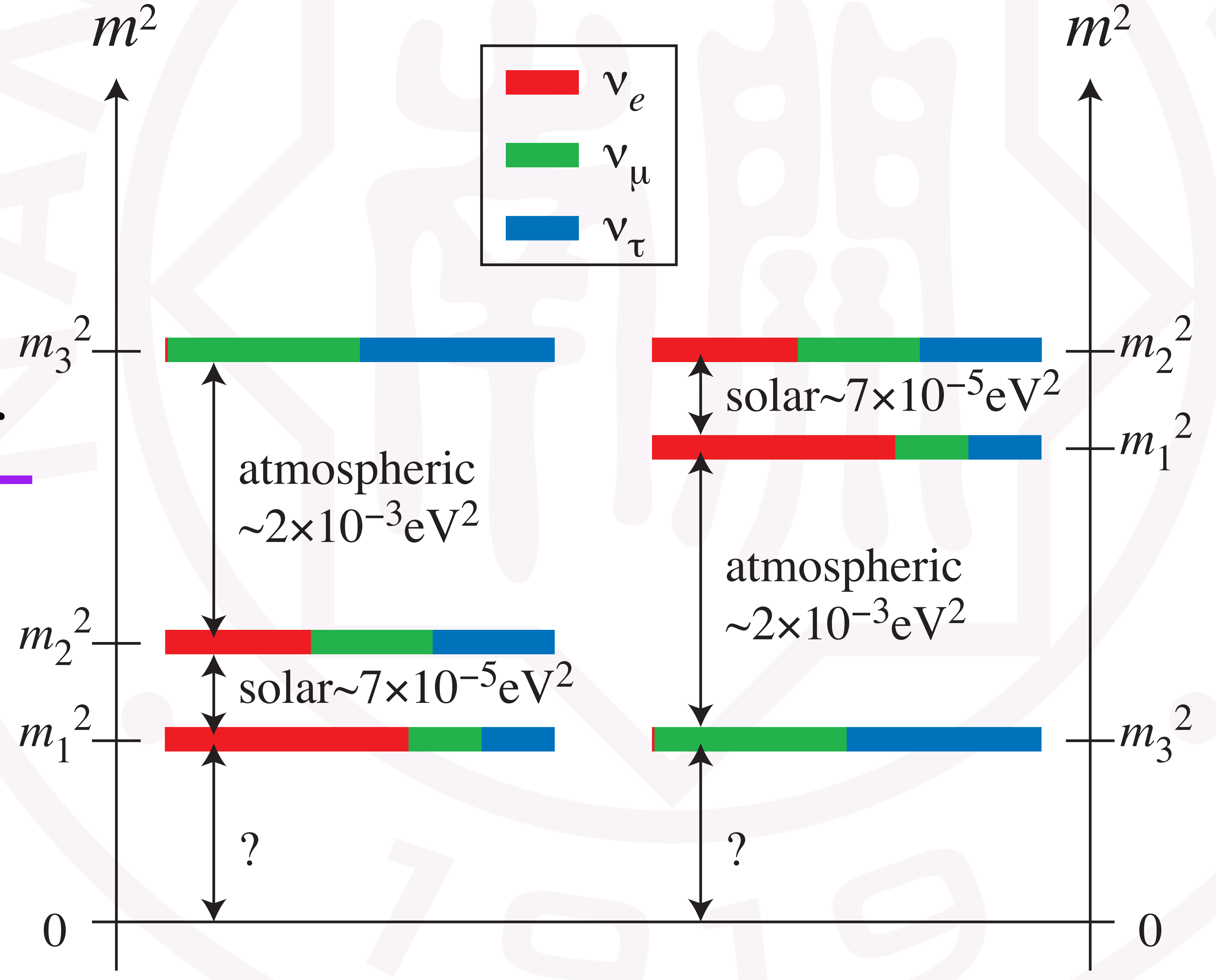
To explore the heavier charged Higgs and heavy fermions,
we need a collider with higher energy and luminosity!

Motivation

- **The hierarchy problem of neutrino masses:**

The neutrino oscillations are sensitive to the differences between the squares of the neutrino masses ($\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$).

But !!! It is presently unknown whether m_3^2 is heavier or lighter than the other two.



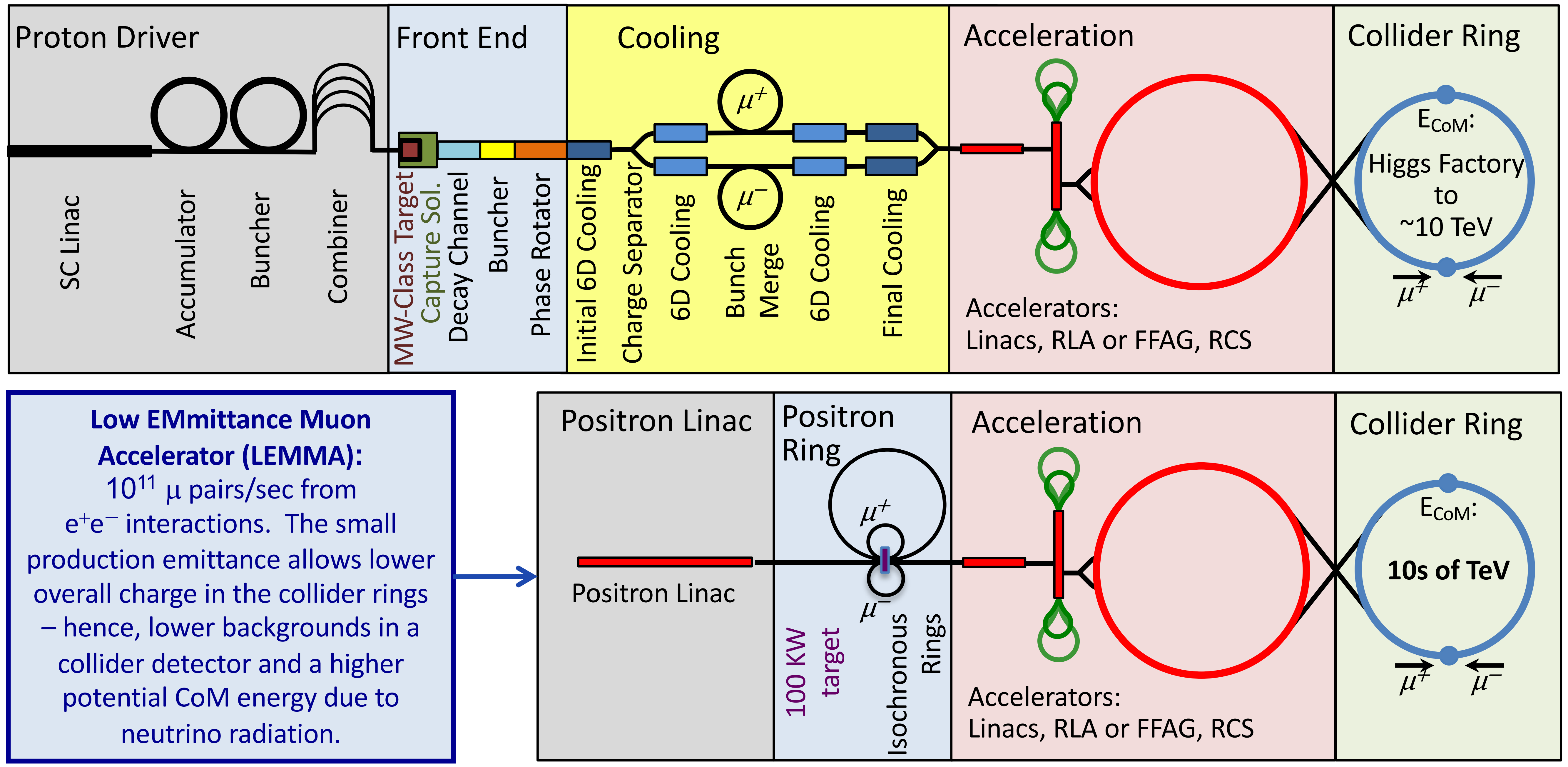
Normal Hierarchy (NH) : $m_3^2 > m_2^2 > m_1^2$

Inverted Hierarchy (IH) : $m_2^2 > m_1^2 > m_3^2$

Can the colliders provide assistance to distinguish between the mass hierarchies of neutrinos, NH or IH?

Muon collider

- Recently, high energy muon colliders have received much attention in the community due to the technological developments.



In this work, we investigate the search for the $H^{\pm\pm}$, $H^{\pm}$ (Type-II Seeaw) and $E^{\pm}$, N (Type-III Seesaw) mechanisms at high-energy muon colliders.

Muon collider

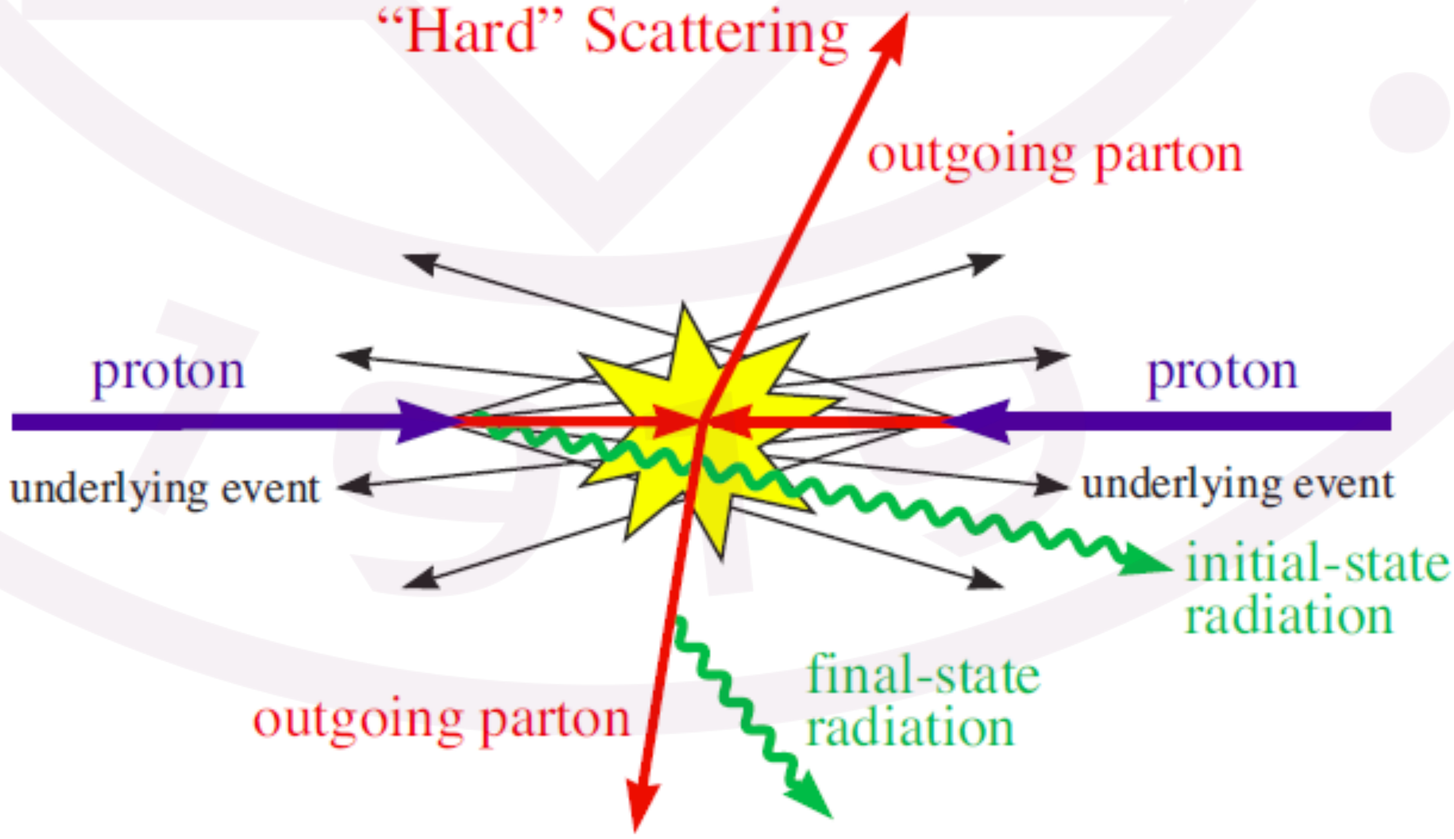
$$\Delta E \sim \gamma^4 = (\frac{E}{m_\mu})^4$$

- **The advantages of muon colliders :**

Higher energy : Compared with the e^+e^-collider, the synchrotron radiation of the muon collider is smaller. With the same size, muon collider can achieve higher energy.

Higher luminosity : Muon collider can achieve higher integrated luminosity.

$$\mathcal{L} = \left(\frac{\sqrt{s}}{10\ \text{TeV}}\right)^2 \times 10\ \text{ab}^{-1}$$



More efficient:

Compared to proton collider, muon collider has a higher energy utilization. Additionally, muon colliders have a more clean background.

Muon collider

- We simulate and study the search potential of Type-II and Type-III Seesaw mechanisms at muon collider， by using the MadGraph， Madspin， Pythia and Delphes …

MadGraph5、**Madspin** （generate parton-level envents）

MadGraph5_aMC@NLO

PYTHIA 8.3

Pythia 8 （parton-shower）

Delphes 3 （simulate the detector of collider）

DELPHES
fast simulation

Type II Seesaw

Type-II Seesaw

$H^{++}H^{--}$

1. $H^{++}H^{--} \to \mu^+\mu^+\mu^-\mu^-$
2. $H^{++}H^{--} \to W^+W^+W^-W^-$

$H^{\pm\pm}H^{\mp}$

3. $H^{\pm\pm}H^{\mp} \to \mu^{\pm}\mu^{\pm}\mu^{\mp}\nu$
4. $H^{\pm\pm}H^{\mp} \to W^{\pm}W^{\pm}W^{\mp}Z$

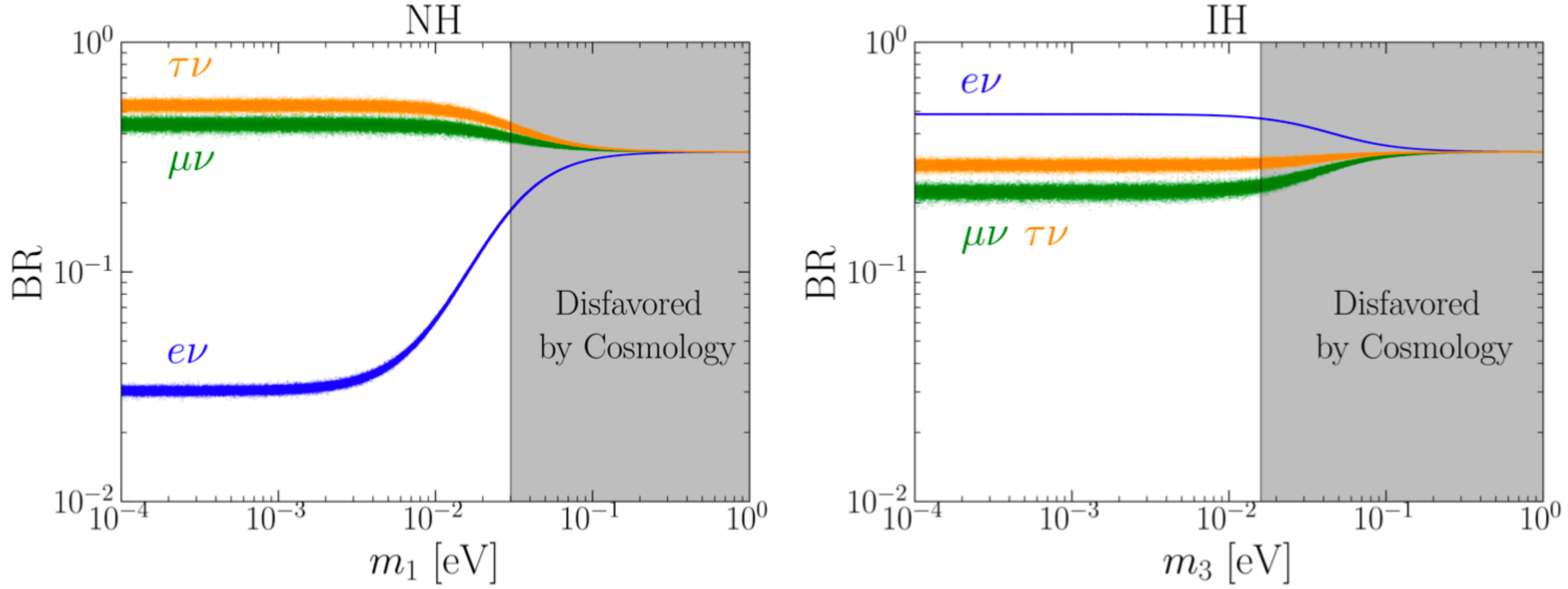
[1] T. Li, C.Y. Yao, M.Yuan "Revealing the origin of neutrino masses through the Type II Seesaw mechanism at high-energy muon colliders," JHEP 03 (2023) 137 [arXiv:2301.07274 [hep-ph]]

Type II Seesaw

We can distinguish the NH and IH by the different distributions of decay channel !

- According to the neutrino oscillation data, we scan the branching ratios of charged Higgs leptonic decay versus the lightest neutrino mass for NH and IH , with $\Phi_1 = \Phi_2 = 0$ (Majorana phase).

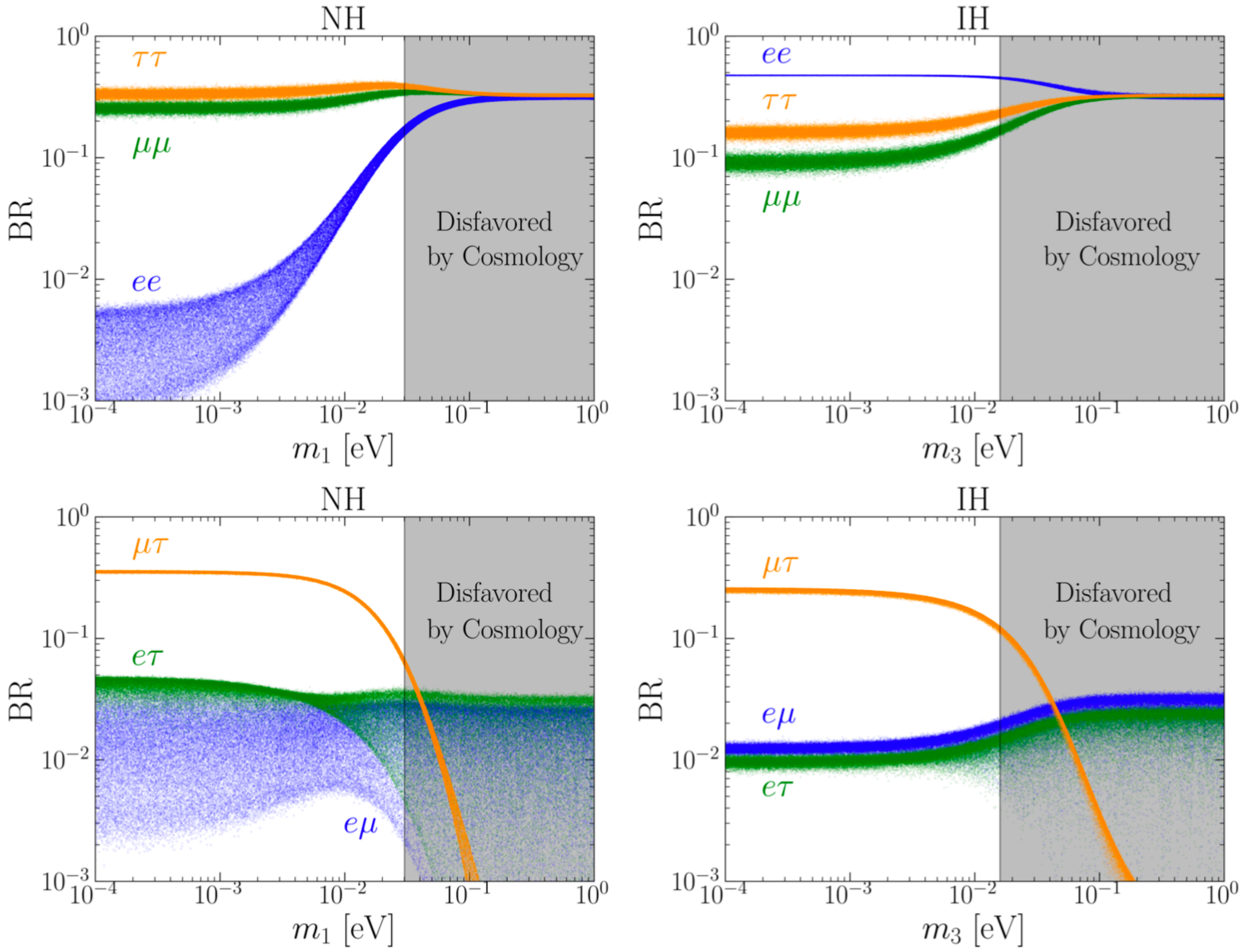
$$H^{+} \to \ell_i^{+} \nu_j$$



$$\sum m_i < 0.12 \text{ eV}$$

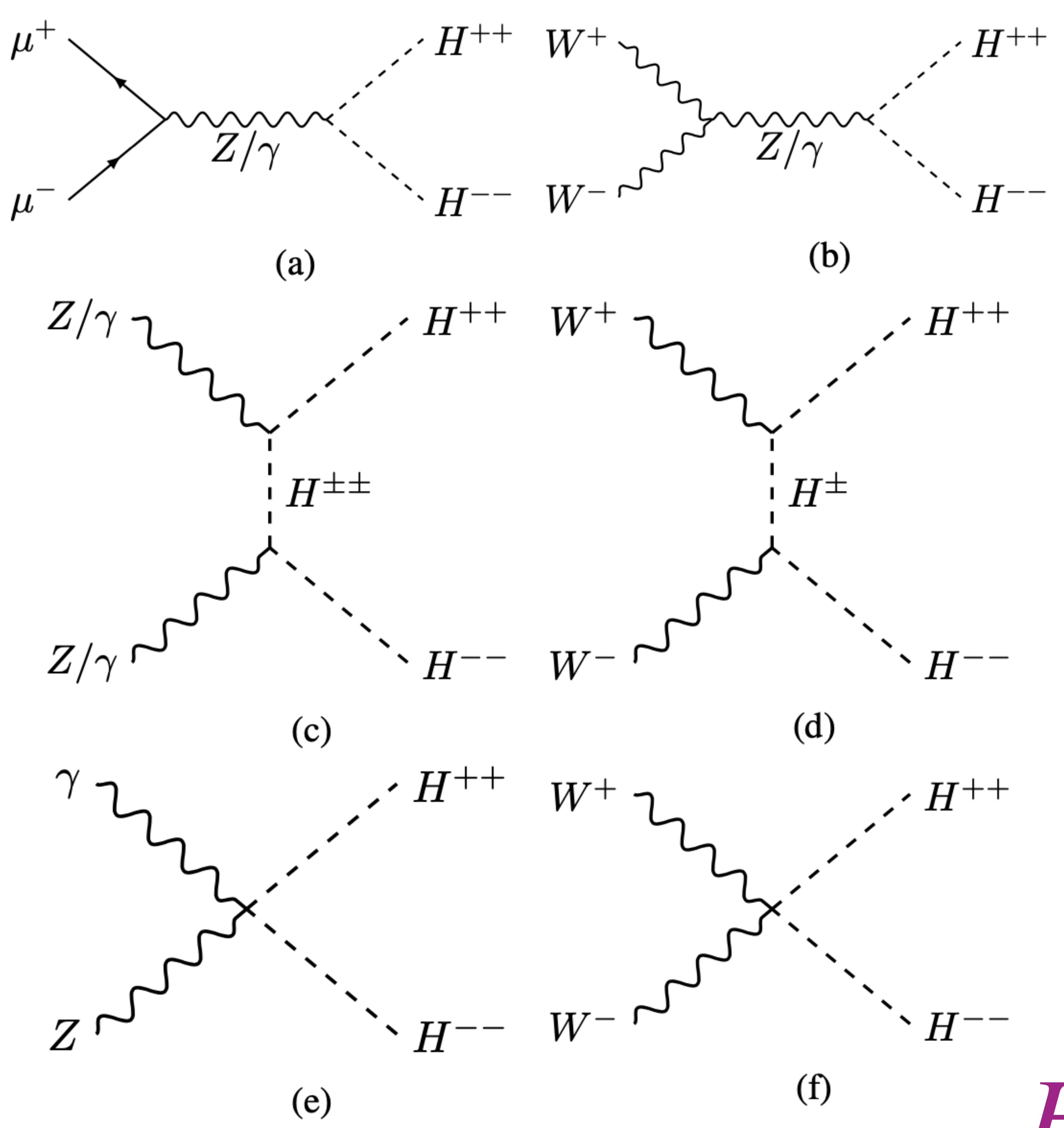
灰色区域: 根据Planck实验组结果,
对最轻中微子质量的限制。

$$H^{++} \to \ell_i^{+} \ell_j^{+}$$



Type II Seesaw

- **The pair productions of the charged Higgs ($H^{\pm\pm}H^{\mp\mp}$, $H^{\pm\pm}H^{\mp}$) at muon collider**

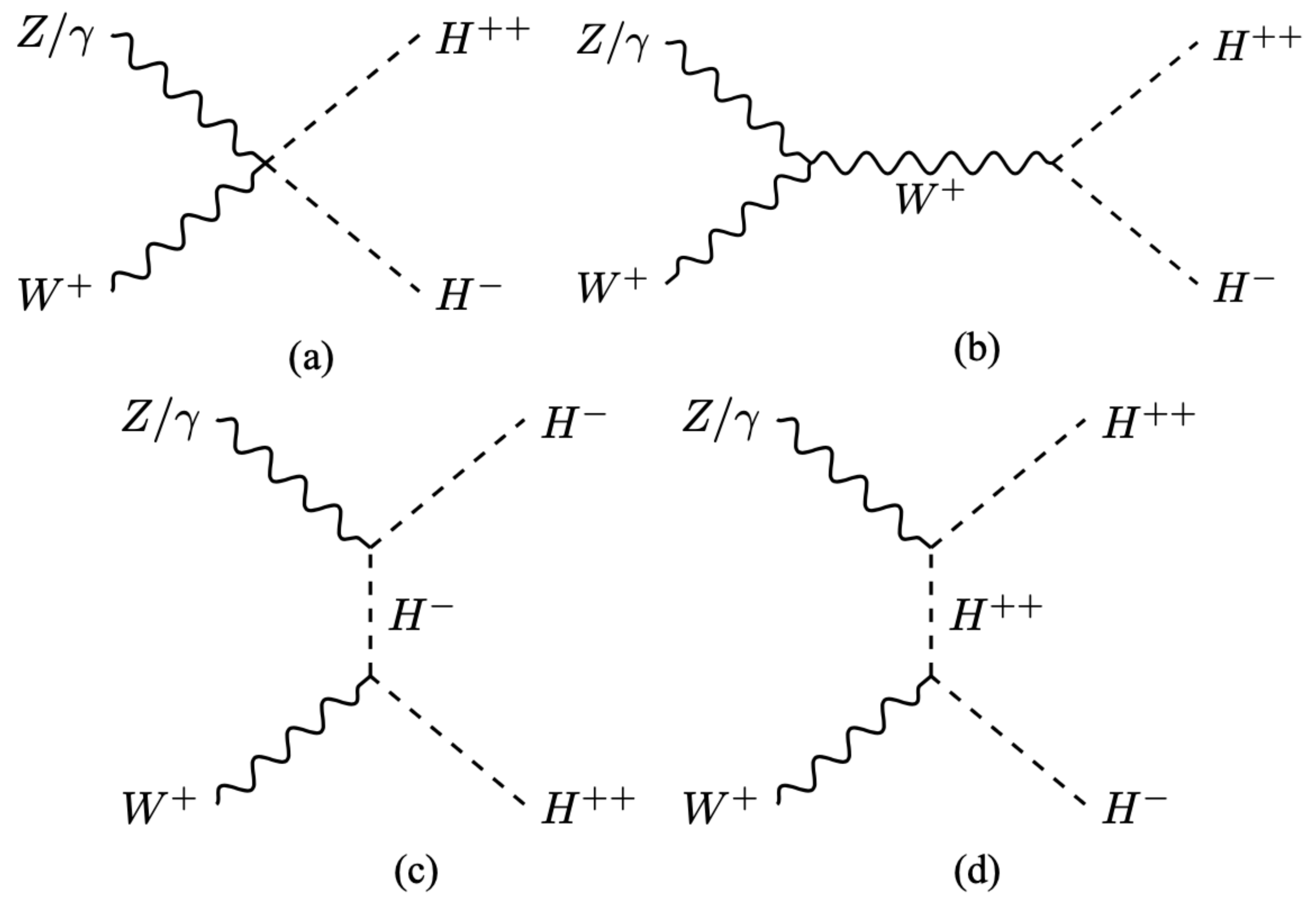


$H^{\pm\pm}H^{\mp\mp}$

$\mu^+\mu^-$ **annihilation**: $\mu^+\mu^- \to H^{++}H^{--}$

VBS : $V\,V \to H^{++}H^{--}$

(Vector boson scattering)



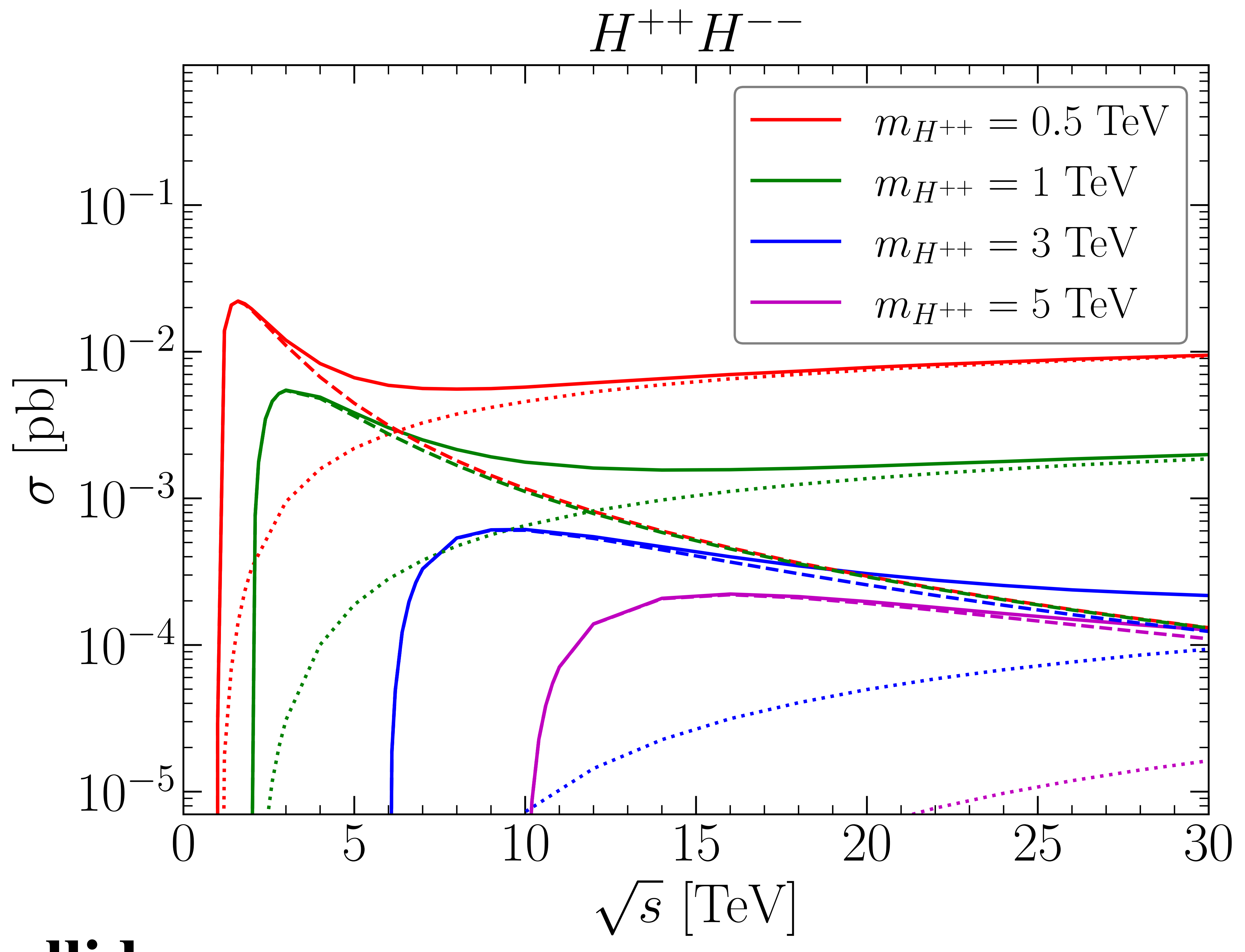
$H^{\pm\pm}H^{\mp}$

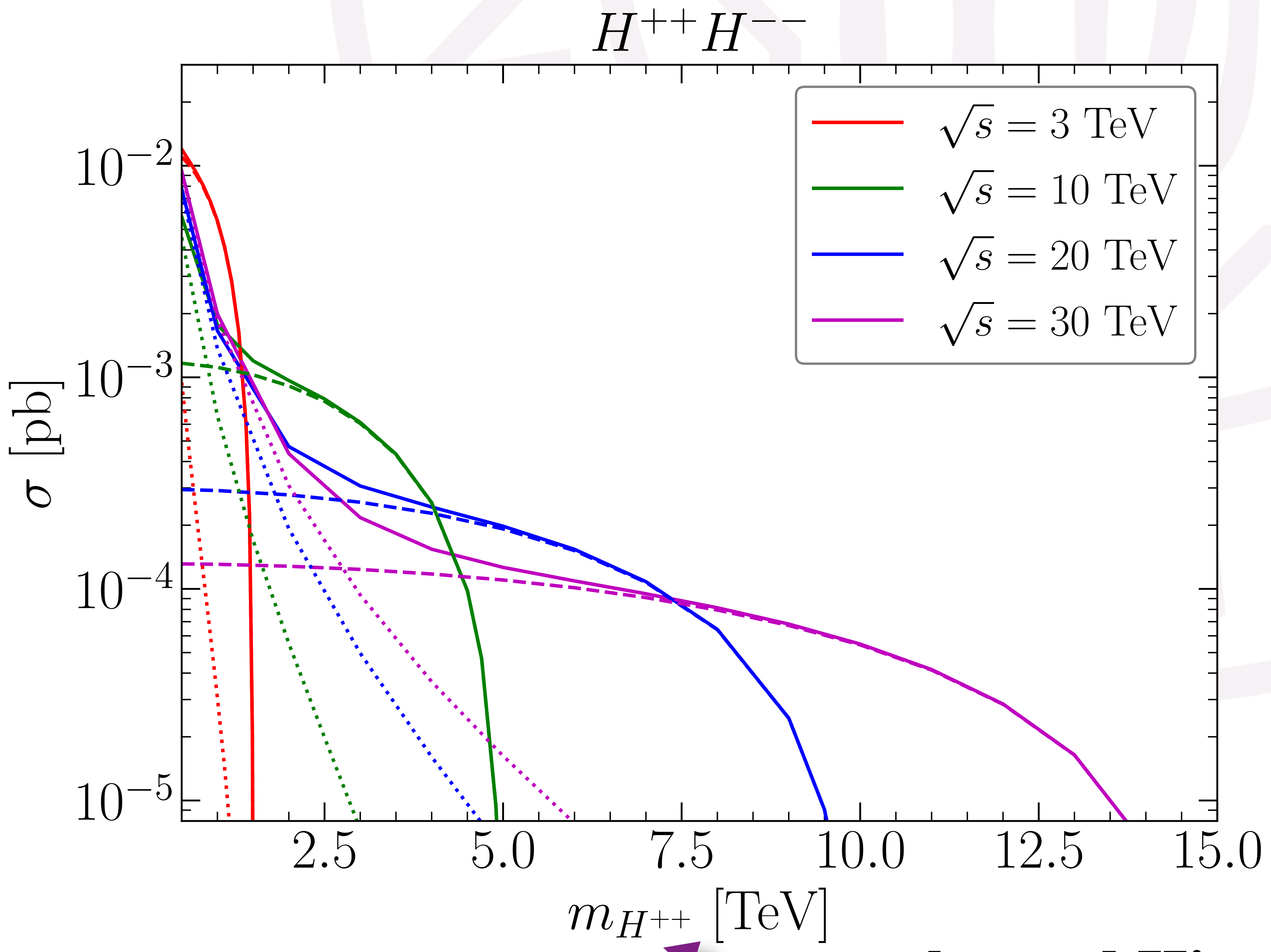
Type II Seesaw

VBS processes are more important in high-energy scale

- The cross sections of $H^{++}H^{--}$ pair production at muon collider, through $\mu^+\mu^-$ annihilation (dashed lines) and VBS (dotted lines) processes.

$$\sigma^{\mu\mu} \sim \frac{\beta}{s}, \quad \beta = \sqrt{1 - \frac{4M_E^2}{s}}$$





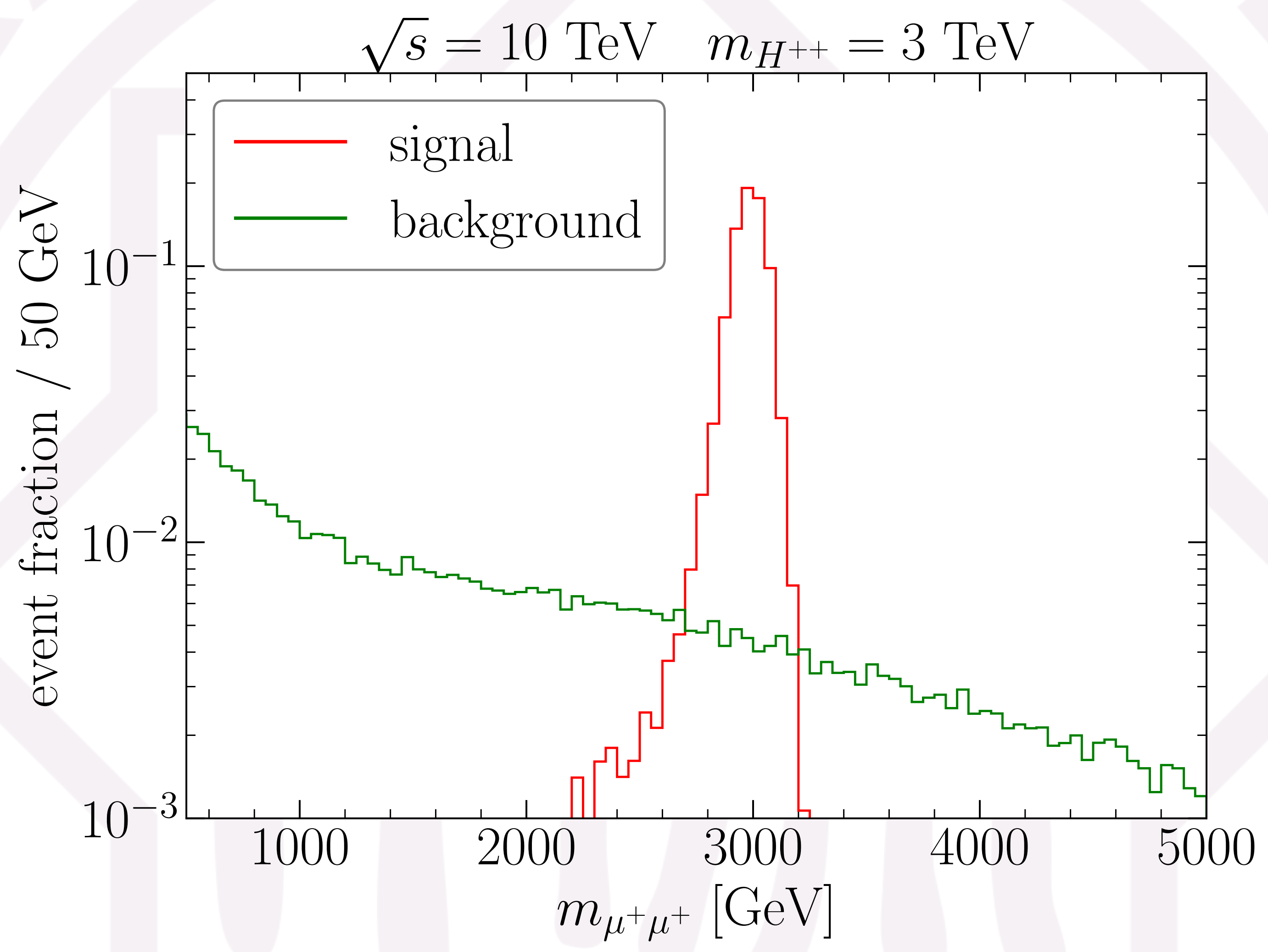
collider energy **charged Higgs mass**

Channel 1: $H^{++}H^{--} \to \mu^+\mu^+\mu^-\mu^-$

- **SM background:** $\mu^+\mu^-, VV \to \mu^+\mu^+\mu^-\mu^-$

We employ some cuts for the final states ($m_{\mu^+\mu^+}, p_T(\mu), \ldots$) to suppress the background , such as

$$|m_{\mu^\pm\mu^\pm} - m_{H^{\pm\pm}}| < m_{H^{\pm\pm}}/5\,.$$



① We use the following formula to evaluate the significance:

$$\mathcal{S} = \frac{N_S}{\sqrt{N_S + N_B}}$$

$$N_S = (\sigma_S^{\rm Ann}\epsilon_S^{\rm Ann} + \sigma_S^{\rm VBS}\epsilon_S^{\rm VBS}) \times {\rm BR}^2(H^{++} \to \mu^+\mu^+) \times \mathcal{L}\,,$$

$$N_B = (\sigma_B^{\rm Ann}\epsilon_B^{\rm Ann} + \sigma_B^{\rm VBS}\epsilon_B^{\rm VBS}) \times \mathcal{L}\,,$$

② The benchmark choices of the collider energies and the corresponding integrated luminosities are

$\sqrt{s}$	3 TeV	10 TeV	30 TeV
$\mathscr{L}$	1 ab^{-1}	10 ab^{-1}	90 ab^{-1}

$$\mathcal{L} = \left(\frac{\sqrt{s}}{10\ \text{TeV}}\right)^2 \times 10\ \text{ab}^{-1}$$

Channel 1: $H^{++}H^{--} \to \mu^+\mu^+\mu^-\mu^-$

$\sqrt{s}$	3 TeV	10 TeV	30 TeV
$\mathscr{L}$	1 ab^{-1}	10 ab^{-1}	90 ab^{-1}

- **The analyzation of the integrated luminosities**

(1) The doubly charged Higgs in NH mass pattern will be discovered.

(2) The 5σ significance can be reached in IH for $m_{H^{\pm\pm}} < 1.0,\ 3.5,\ 10$ TeV, respectively.

Channel 1: $H^{++}H^{--} \to \mu^+\mu^+\mu^-\mu^-$

- **The reachable limits of BR($H^{++} \to \mu^+\mu^+$) corresponding to 5σ or 2σ significance**

For illustration

(1) With $\sqrt{s} = 3$ TeV and $m_{H^{\pm\pm}} = 1.3$ TeV, BR($H^{++} \to \mu^+\mu^+$) can approach 15.8% (7.6%) for 5σ (2σ).

(2) With $\sqrt{s} = 10$ TeV and $m_{H^{\pm\pm}} = 3$ TeV, BR($H^{++} \to \mu^+\mu^+$) can approach 8.3% (4.2%) for 5σ (2σ).

(3) With $\sqrt{s} = 10$ TeV and $m_{H^{\pm\pm}} = 3$ TeV, BR($H^{++} \to \mu^+\mu^+$) can approach 9.3% (4.6%) for 5σ (2σ).

Channel 2: $H^{++}H^{--} \to W^+W^+W^-W^-$ $\to \mu^\pm\mu^\pm \overset{(-)}{\nu_\mu}\,\overset{(-)}{\nu_\mu}\,J\,J.$

- **SM background:**

$$\mathrm{B}_{4W,1}: \mu^+\mu^-, VV \to W^+W^+W^-W^- \to \mu^\pm\mu^\pm \overset{(-)}{\nu_\mu}\,\overset{(-)}{\nu_\mu}\,j\,j\,j\,j\,,$$

$$\mathrm{B}_{4W,2}: \quad VV \to W^\pm W^\pm W^\mp Z \quad \to \mu^\pm\mu^\pm \overset{(-)}{\nu_\mu}\,\overset{(-)}{\nu_\mu}\,j\,j\,j\,j\,,$$

$$\mathrm{B}_{4W,3}: \quad VV \to t_{(\to b\,W^+)}\,\bar{t}_{(\to \bar{b}\,W^-)}\,W^\pm \to \mu^\pm\mu^\pm \overset{(-)}{\nu_\mu}\,\overset{(-)}{\nu_\mu}\,b\,\bar{b}\,j\,j\,.$$

$\sqrt{s}$	3 TeV	10 TeV	30 TeV
$\mathscr{L}$	1 ab^{-1}	10 ab^{-1}	90 ab^{-1}

- **The analyzation of the integrated luminosities**

Channel 3: $H^{\pm\pm}H^{\mp} \to \mu^{\pm}\mu^{\pm}\mu^{\mp}\nu$

- **SM background:**

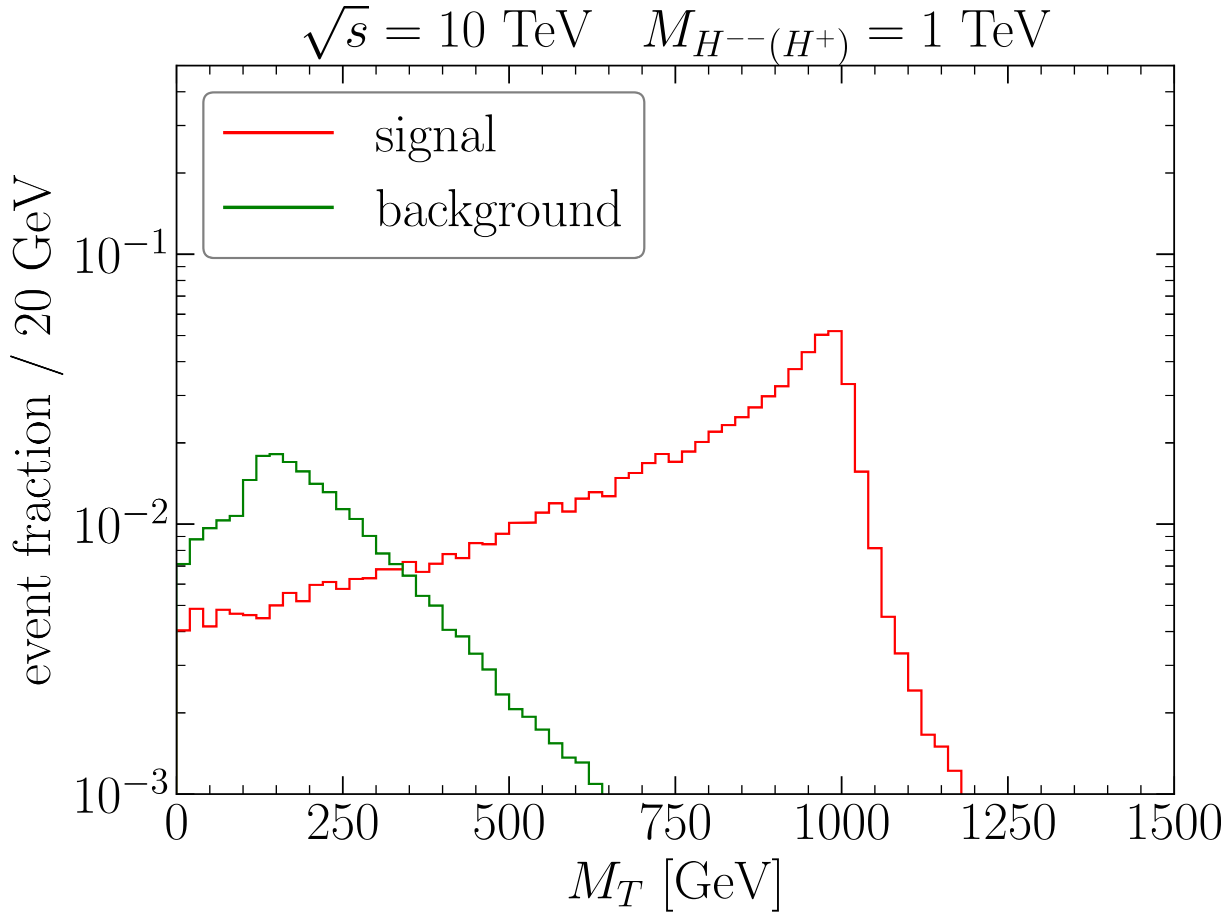
$$B_{3\ell,1}: \quad VV \to \mu^{\pm}\mu^{\pm}\mu^{\mp}\overset{(-)}{\nu}_{\mu}\,,$$

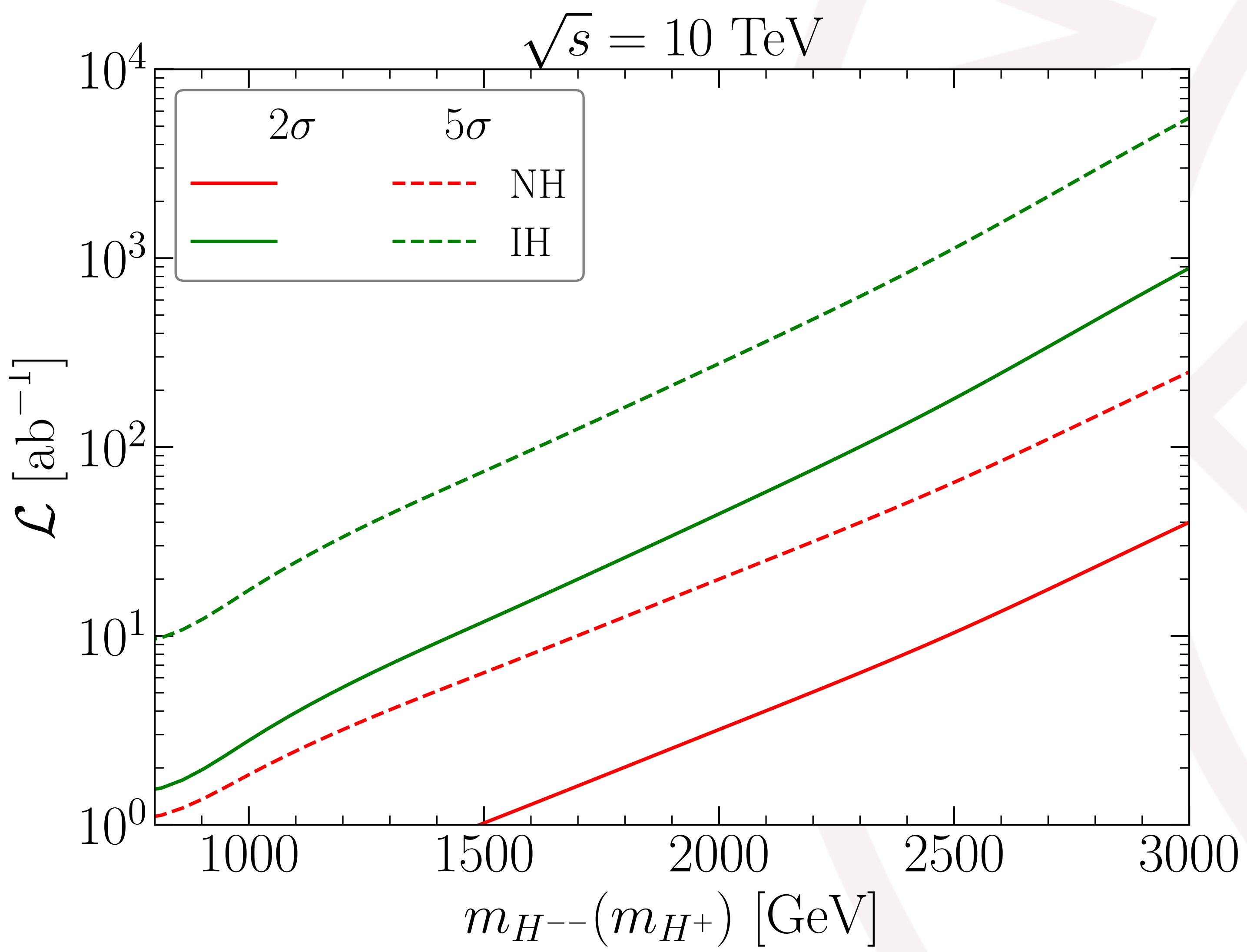
$$B_{3\ell,2}: \quad VV \to W^{\pm}W^{\pm}W^{\mp} \to \mu^{\pm}\mu^{\pm}\mu^{\mp}\overset{(-)}{\nu}_{\mu}\overset{(-)}{\nu}_{\mu}\overset{(-)}{\nu}_{\mu}\,,$$

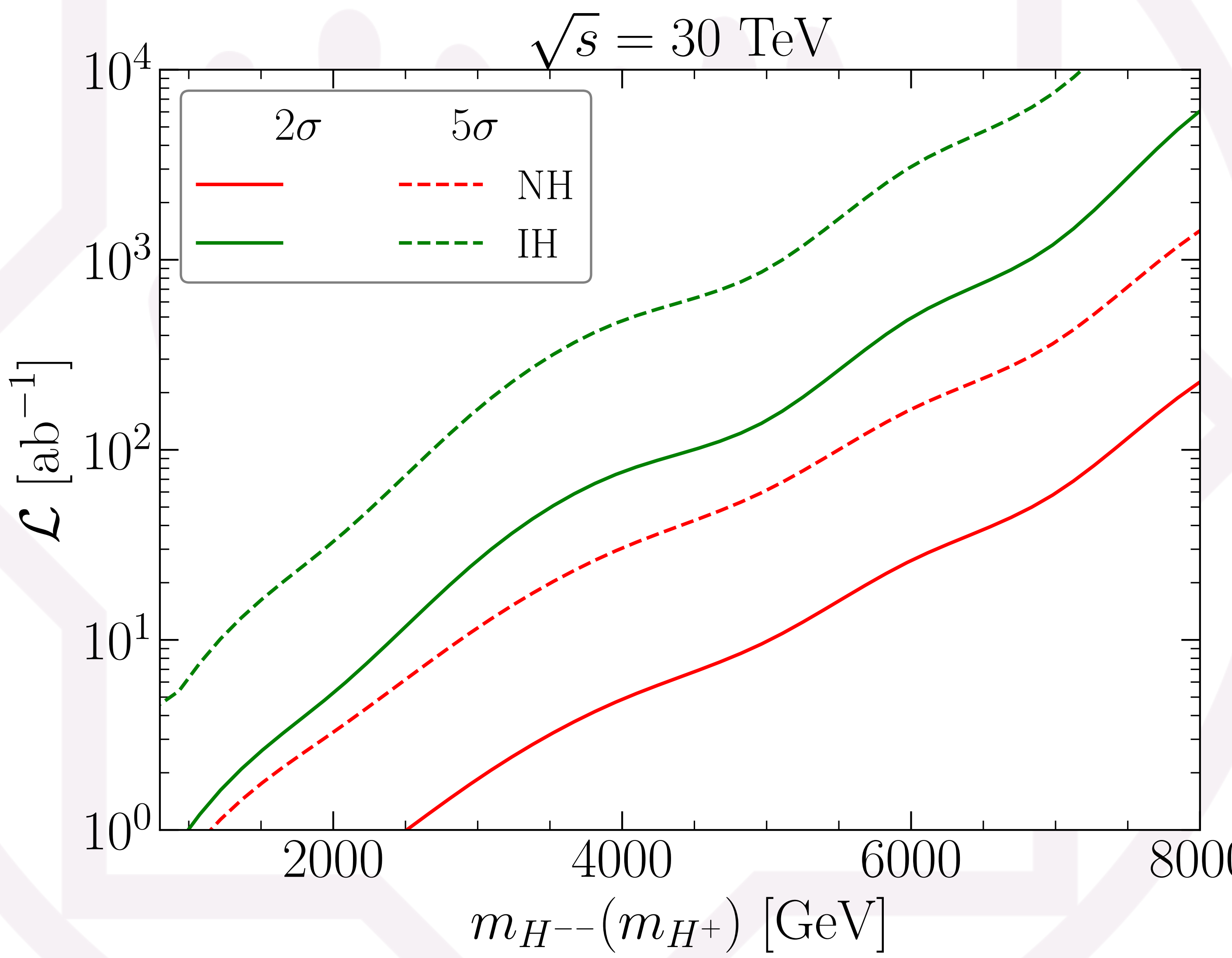
$$B_{3\ell,3}: \quad VV \to ZZW^{\pm} \to \mu^{\pm}\mu^{\pm}\mu^{\mp}\overset{(-)}{\nu}_{\mu}\,\nu\,\overline{\nu}\,.$$

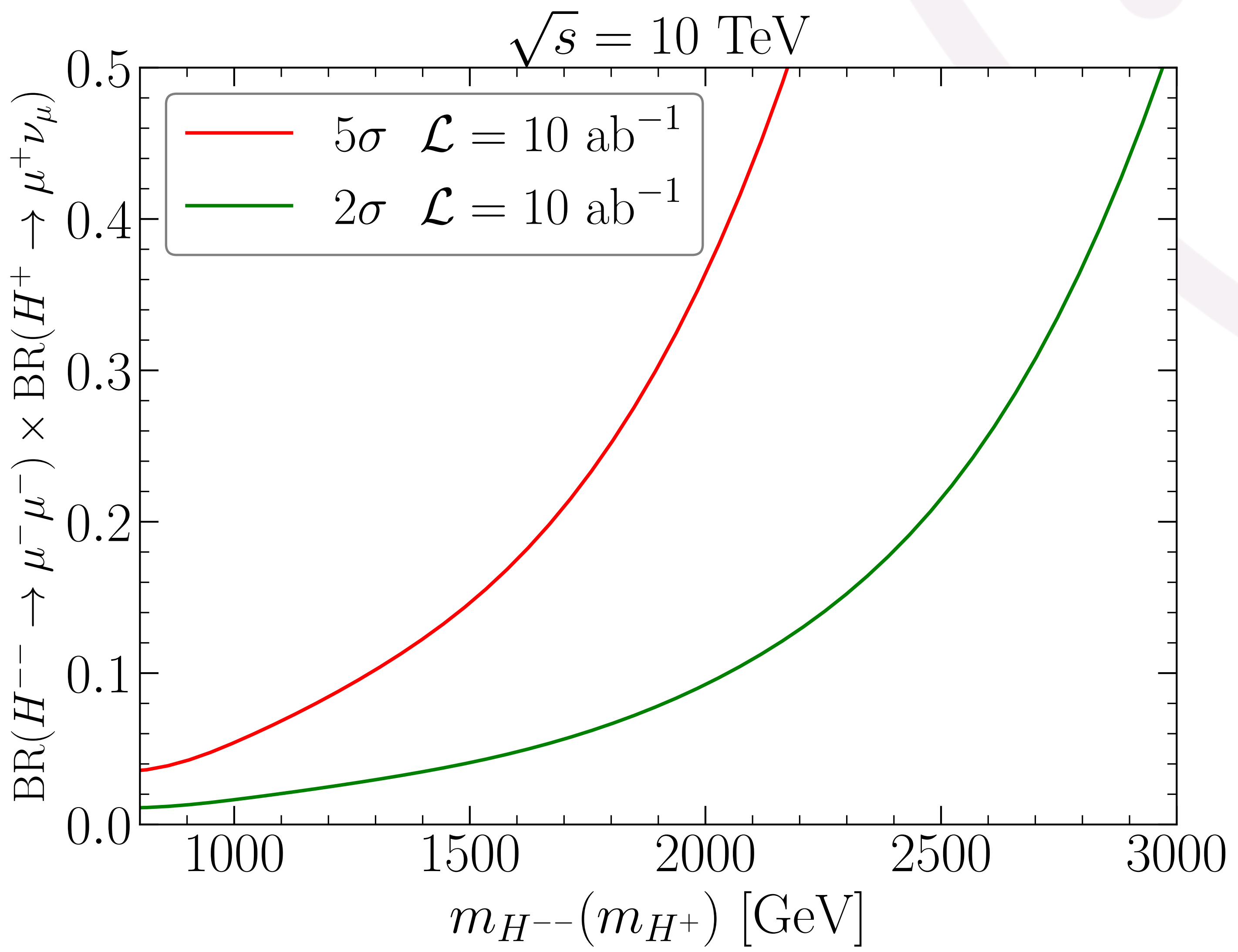
(1) Only be generated by VBS processes.

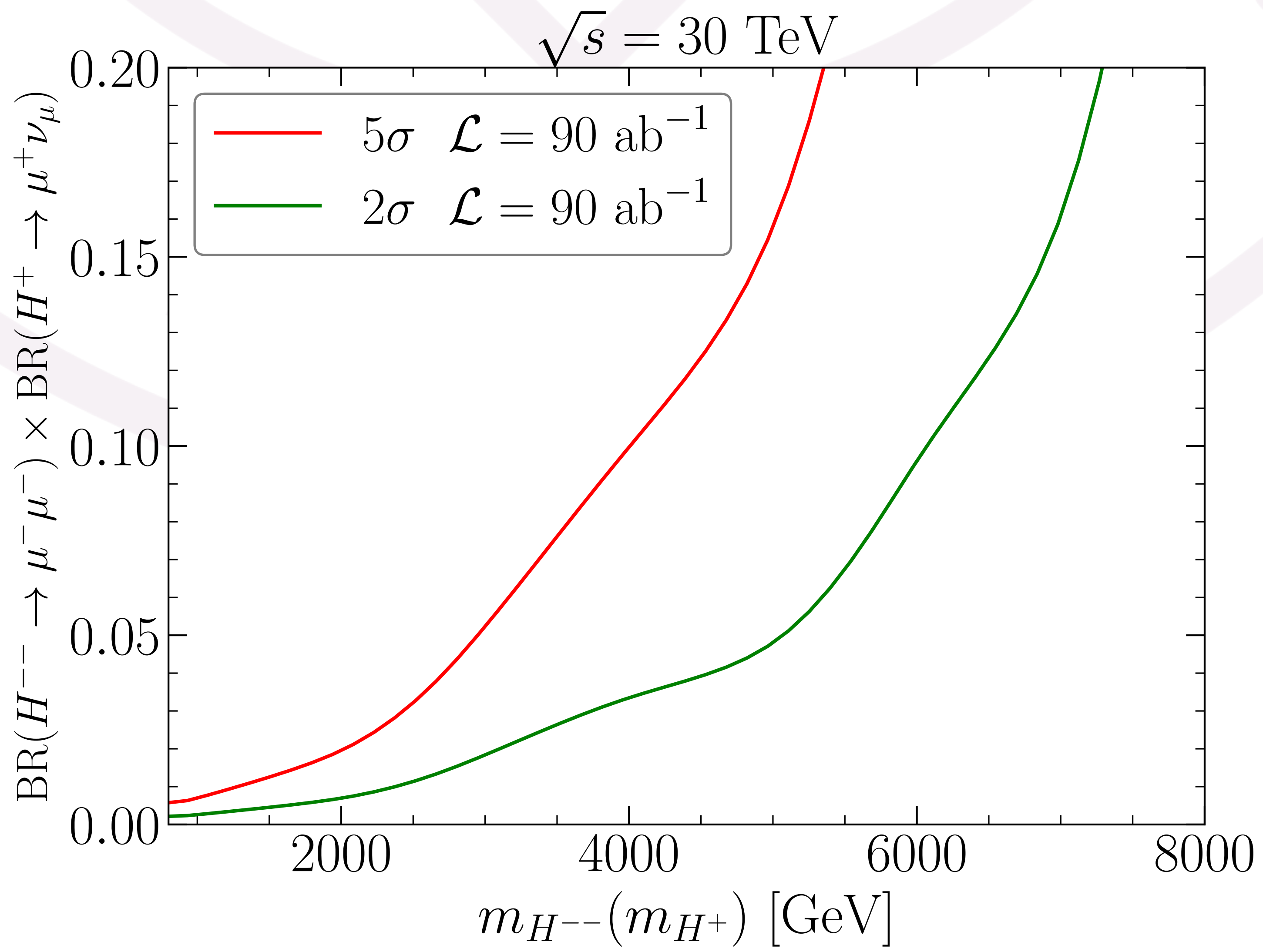
(2) Only consider the case of 10 and 30TeV.











Channel 4: $H^{++}H^{-} \to W^{+}W^{+}W^{-}Z$

- **SM background:**

$$\mathrm{B}_{4W,1}: \mu^+\mu^-, VV \to W^+W^+W^-W^- \to \mu^\pm\mu^\pm \overset{(-)}{\nu_\mu} \overset{(-)}{\nu_\mu} j\, j\, j\, j\,,$$

$$\mathrm{B}_{4W,2}: \quad VV \to W^\pm W^\pm W^\mp Z \to \mu^\pm\mu^\pm \overset{(-)}{\nu_\mu} \overset{(-)}{\nu_\mu} j\, j\, j\, j\,,$$

$$\mathrm{B}_{4W,3}: \quad VV \to t_{(\to b\, W^+)}\, \bar{t}_{(\to \bar{b}\, W^-)}\, W^\pm \to \mu^\pm\mu^\pm \overset{(-)}{\nu_\mu} \overset{(-)}{\nu_\mu} b\, \bar{b}\, j\, j\,.$$

- Although the cuts are efficient for suppressing the backgrounds, the rates of backgrounds are still far larger than the signal. As we have seen, **it is not optimistic to probe the charged Higgs at muon collider through this channel.**

σ (pb) $\times\, \epsilon$ $\times$BRs	no cuts ($\sigma \times 100\% \times$ BRs)	basic cuts	$M_{W,Z}$ rec. $M_{W,Z} \pm 15$ GeV	m_{WZ} Eq. (3.11)	m_T Eq. (3.13)
S	1.15×10^{-7}	8.72×10^{-8}	5.04×10^{-8}	4.85×10^{-8}	4.80×10^{-8}
$\mathrm{B}_{4W,1}$	2.60×10^{-3}	1.41×10^{-3}	5.78×10^{-4}	1.16×10^{-4}	4.88×10^{-5}
$\mathrm{B}_{4W,2}$	4.70×10^{-4}	1.33×10^{-4}	5.12×10^{-5}	1.01×10^{-5}	4.54×10^{-6}
$\mathrm{B}_{4W,3}$	1.78×10^{-4}	8.38×10^{-6}	5.09×10^{-8}	–	–

Type III Seesaw

Type-III Seesaw

E^+E^- → 1. $E^+E^- \to W^+W^-\nu\bar{\nu}$

E^+E^- → 2. $E^+E^- \to W^+W^-Z\,Z$

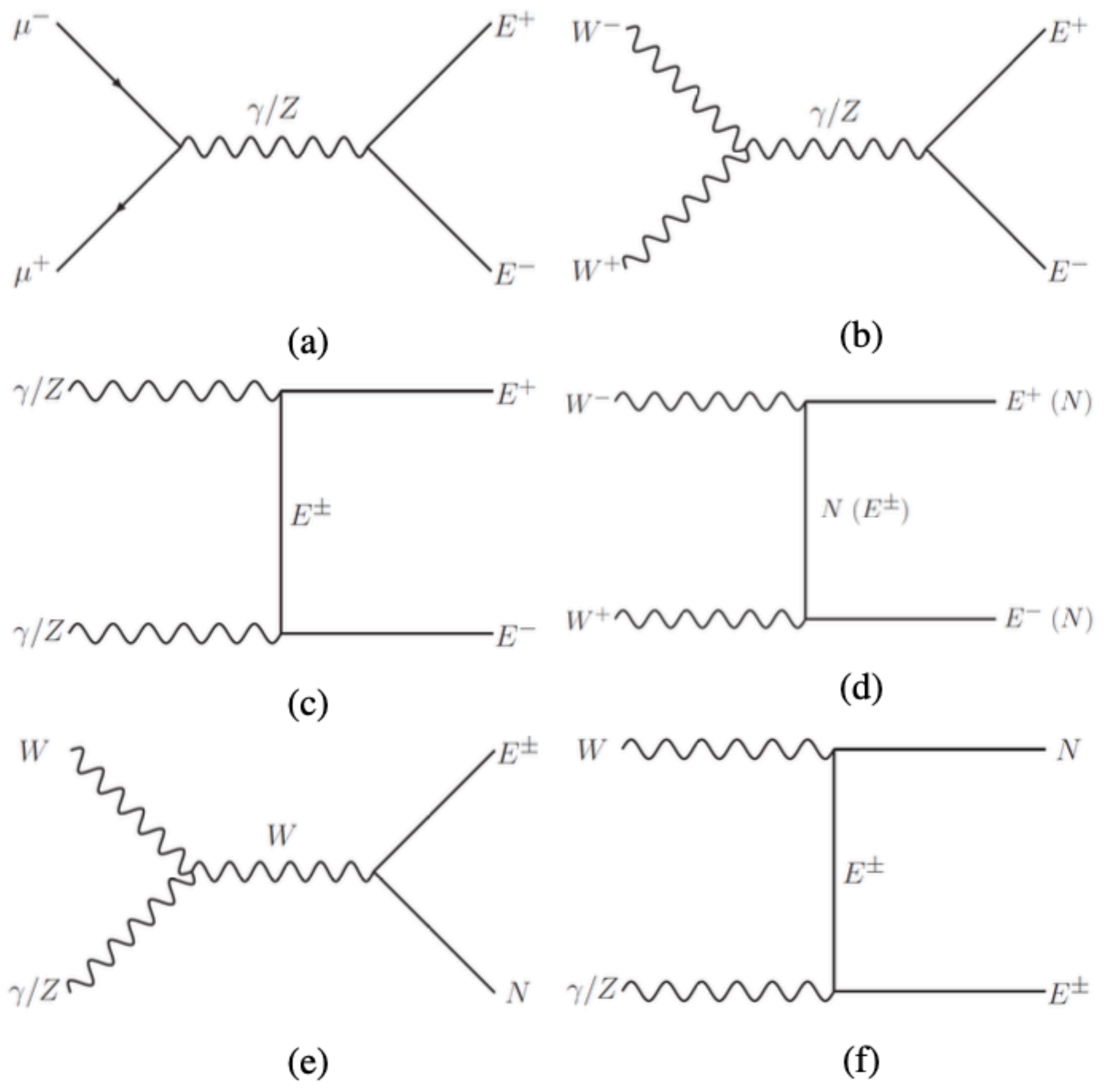
$E^\pm N$ → 3. $E^\pm N \to ZW^\mp\mu^\pm\mu^\pm$

$N\,N$ → 4. $N\,N \to W^\mp W^\mp\mu^\pm\mu^\pm$

[2] T. Li, H. Qin, C.Y. Yao, M.Yuan "Probing heavy triplet leptons of the type-III Seesaw mechanism at future muon colliders," Phys.Rev.D 106 (2022) 3, 035021 [arXiv:2205.04214 [hep-ph]]

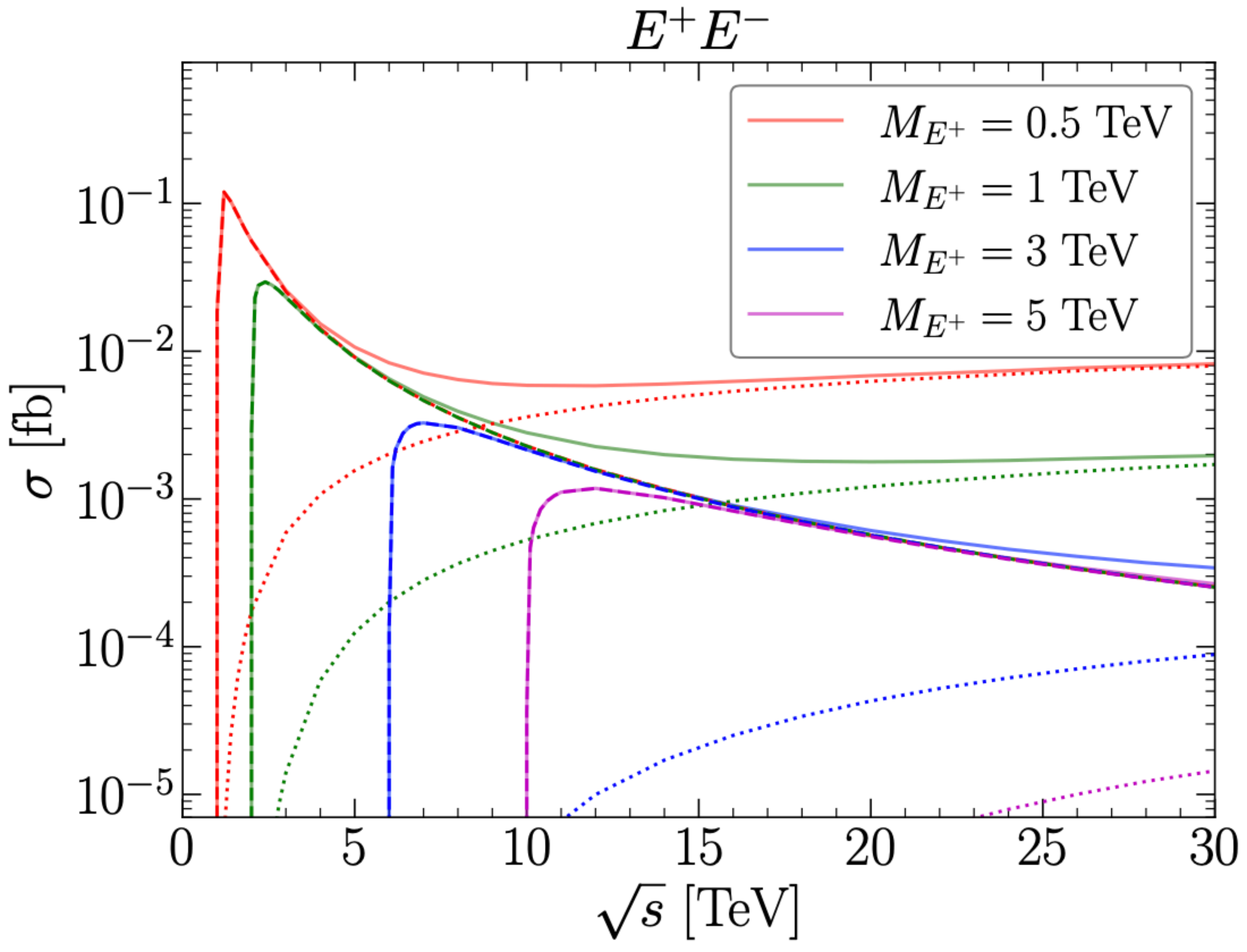
Type III Seesaw

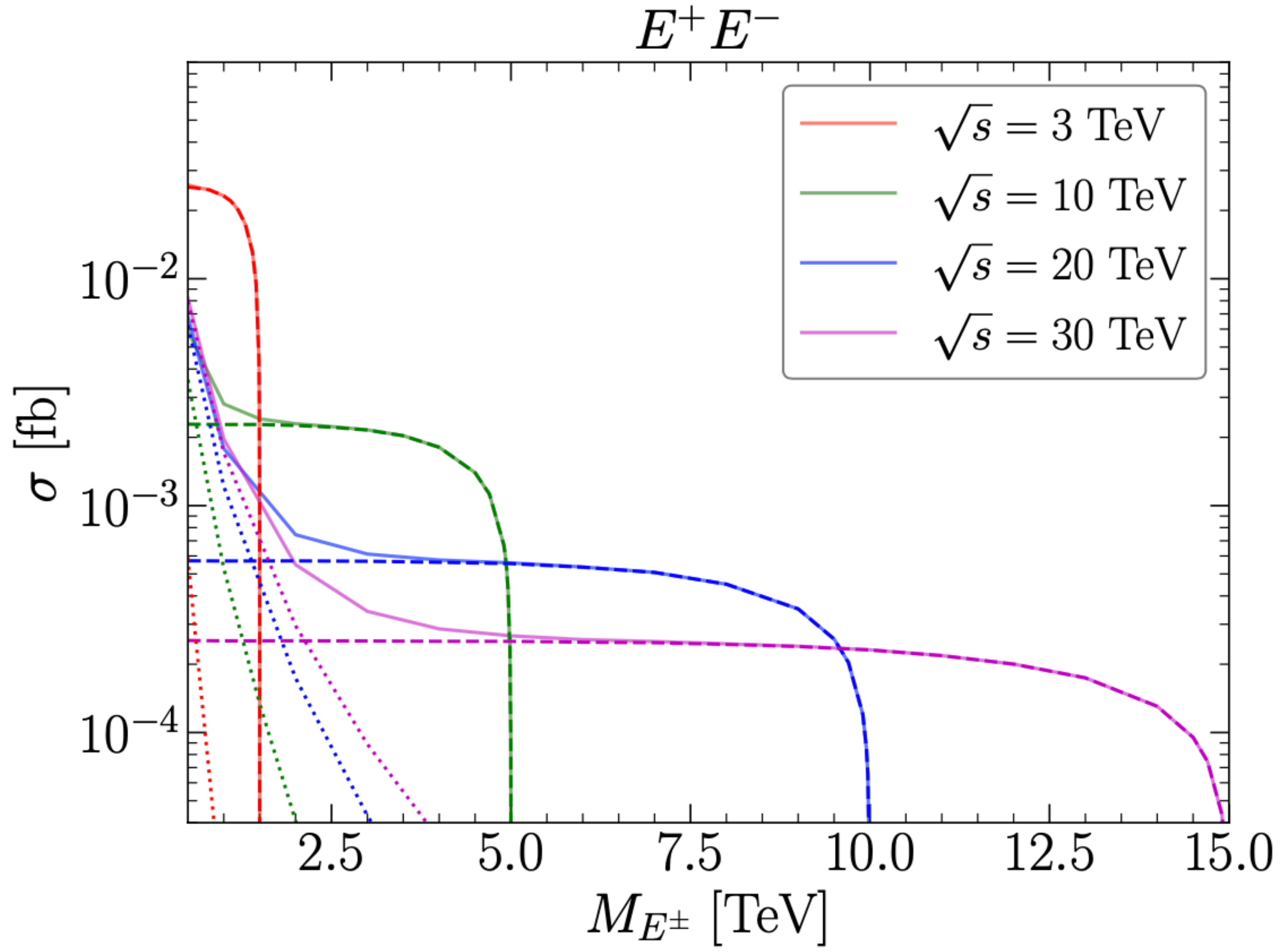
- **The pair productions of the heavy fermions (E^+E^-, $E^\pm N$, NN) at muon collider.**



$\mu^+\mu^-$ **annihilation**： $\mu^+\mu^- \to E^+E^-$

VBS： $V\,V \to E^+E^-, E^\pm E, NN$





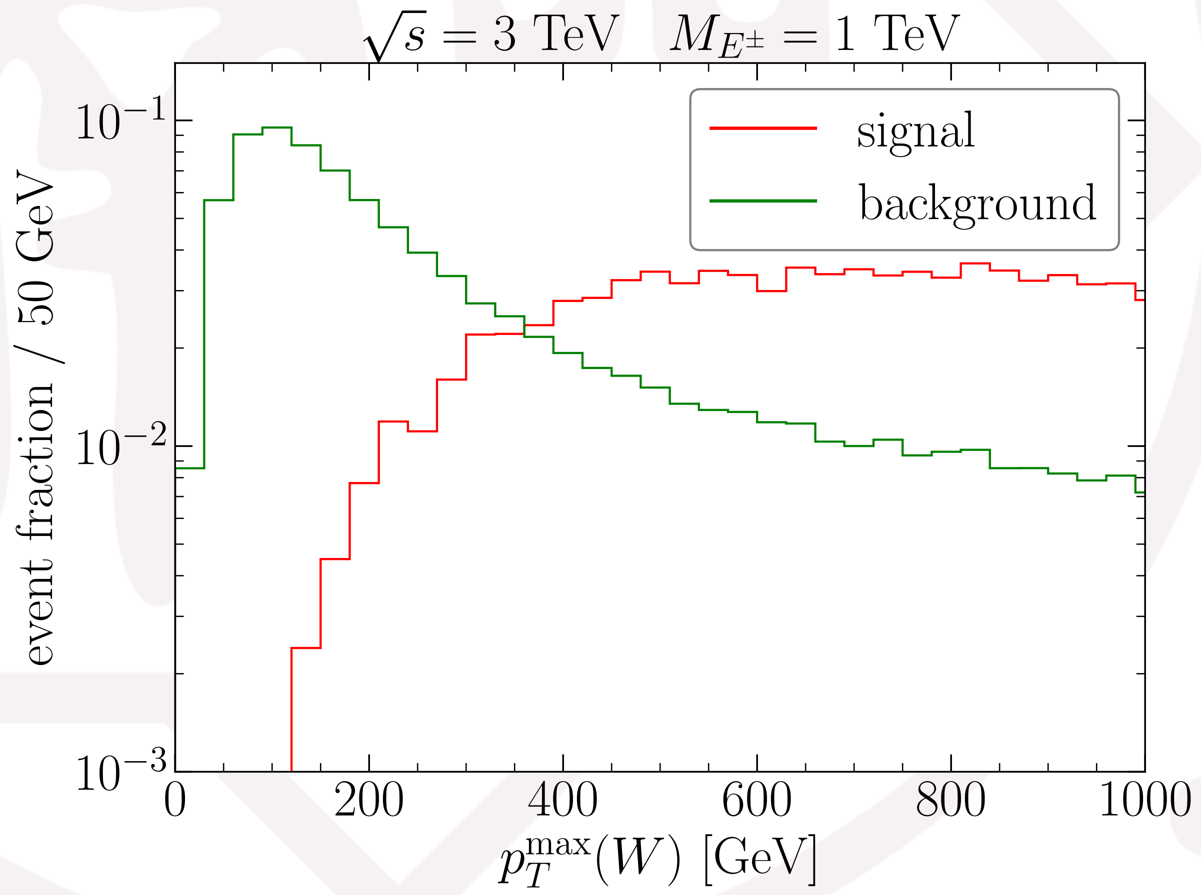
A. E^+E^- pair production

$\sqrt{s}$	3 TeV	10 TeV	30 TeV
$\mathscr{L}$	1 ab^{-1}	10 ab^{-1}	90 ab^{-1}

Channel 1: $E^+E^- \to W^+W^-\nu\bar{\nu}$

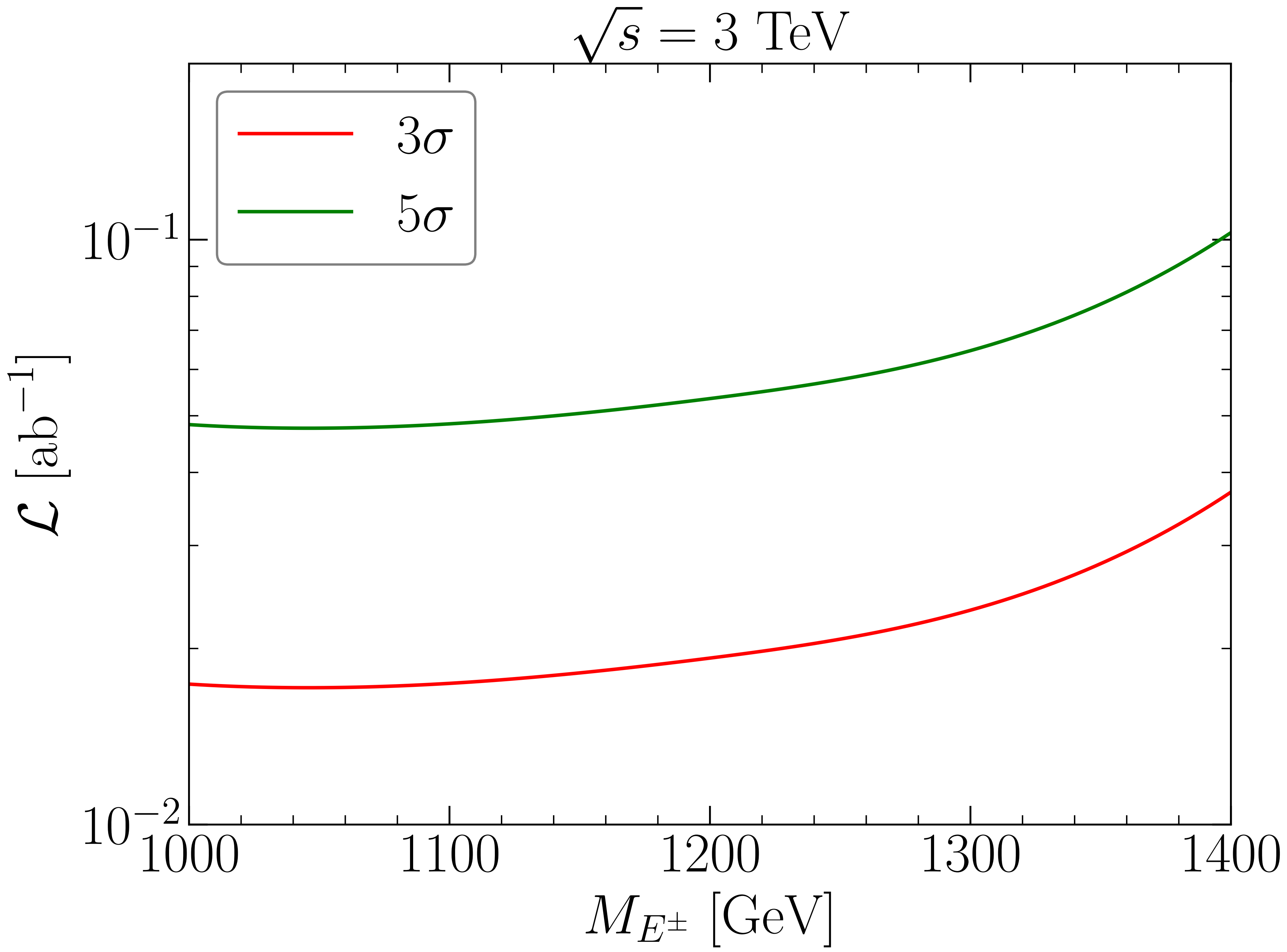
$W^\pm \to j\, j$

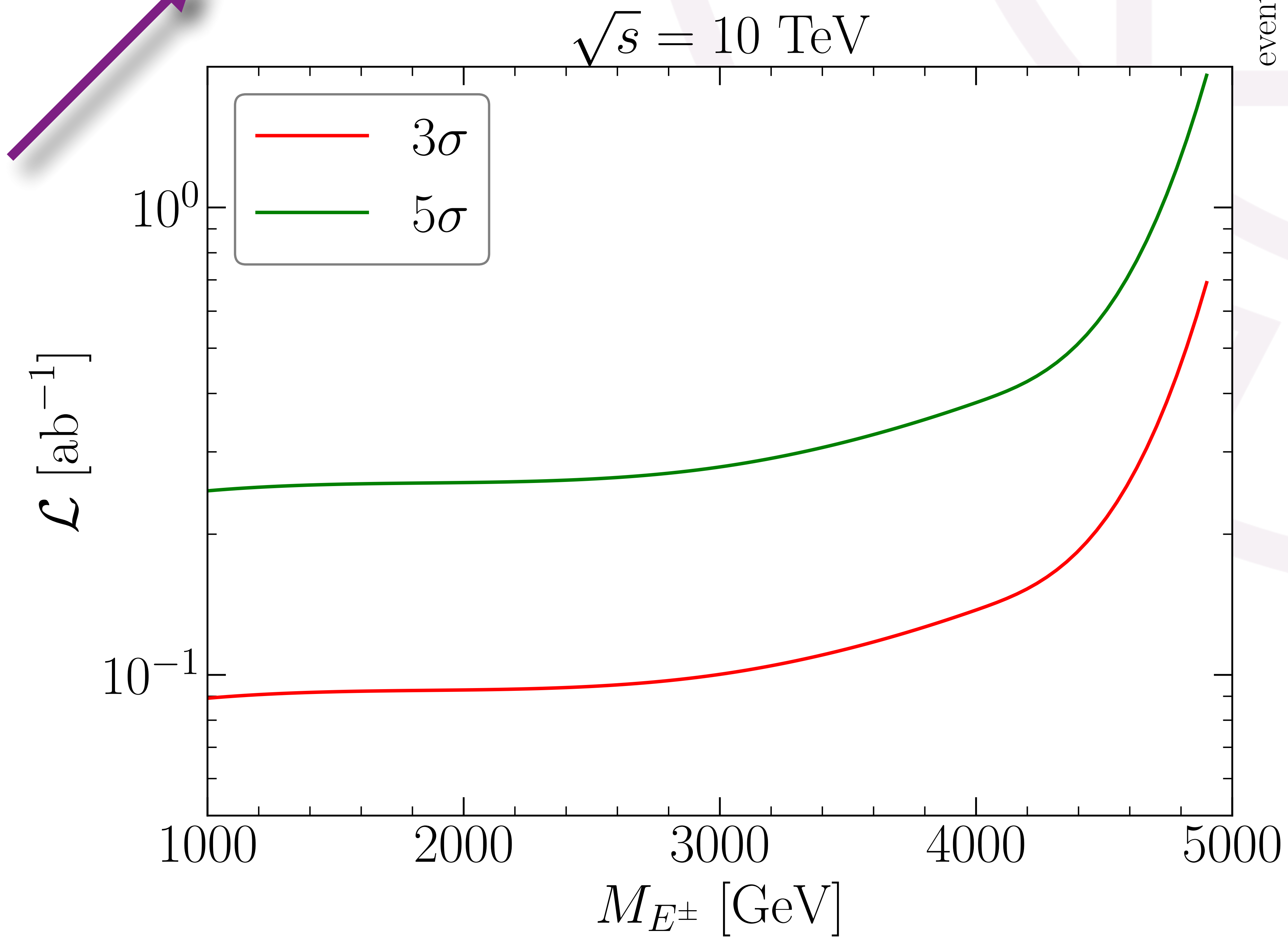
- **SM backgrounds**: $\mu^+\mu^-$, $VV \to W^+W^-Z \to W^+W^-\nu\bar{\nu}$



- **The integrated luminosities**

It is obviously an optimistic result, but it is impossible to reconstruct $E^\pm$.

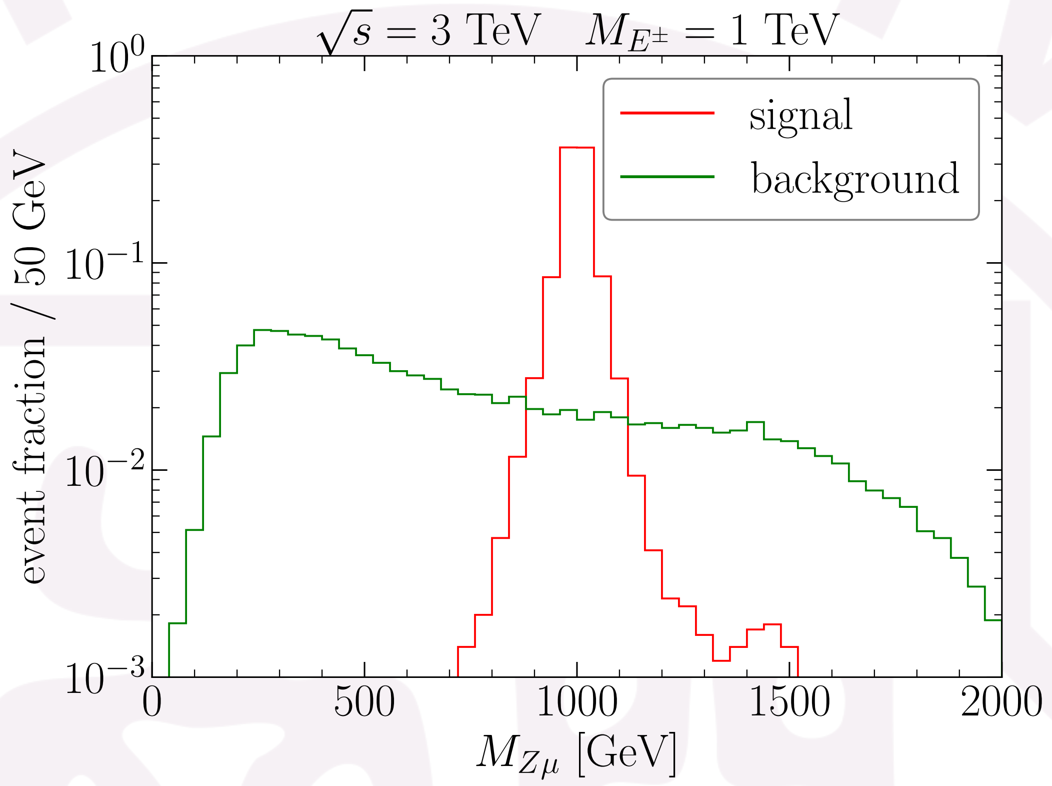




A. E^+E^- pair production

Channel 2: $E^+E^- \to Z\,Z\,\mu^+\mu^-$

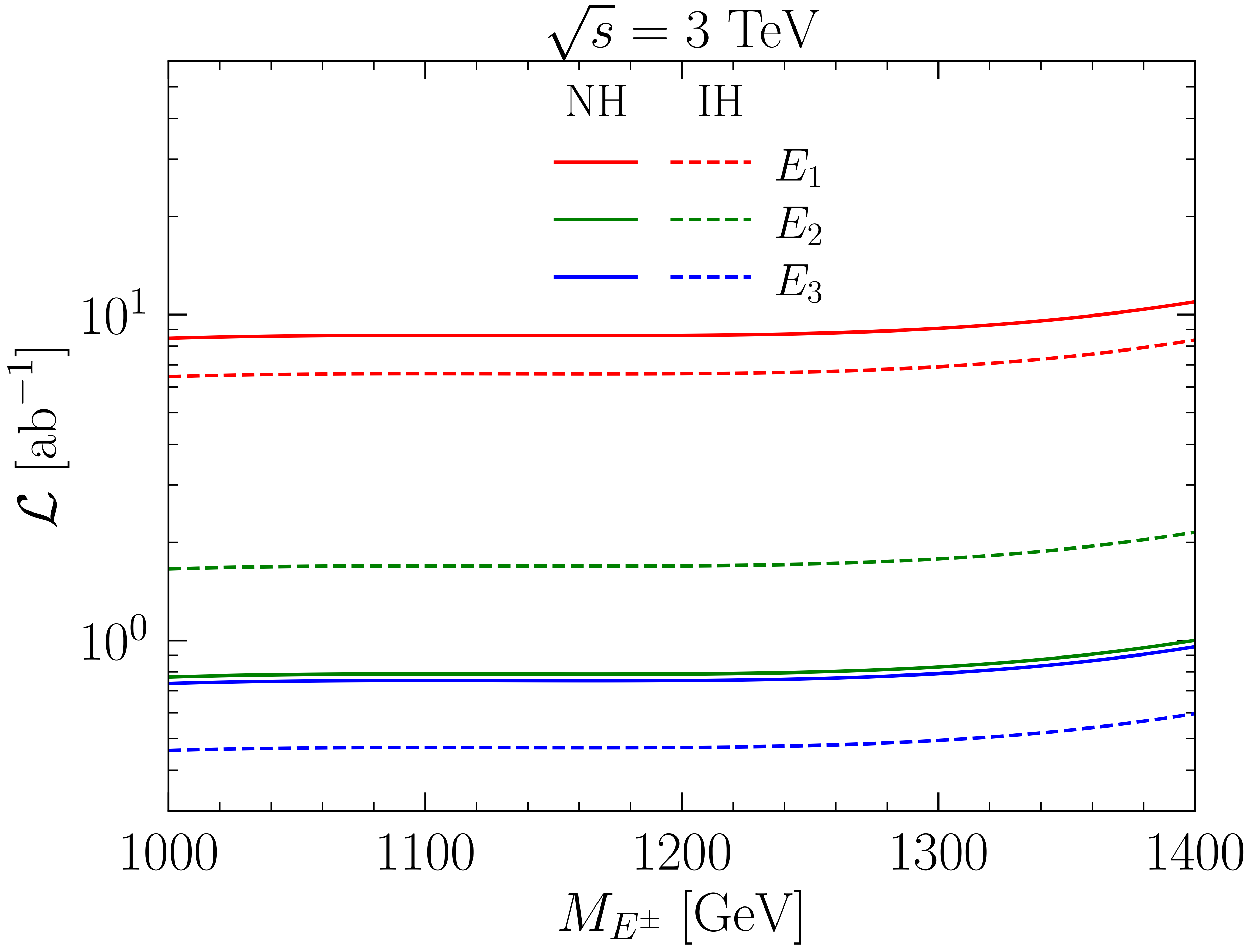
$Z \to j\,j$

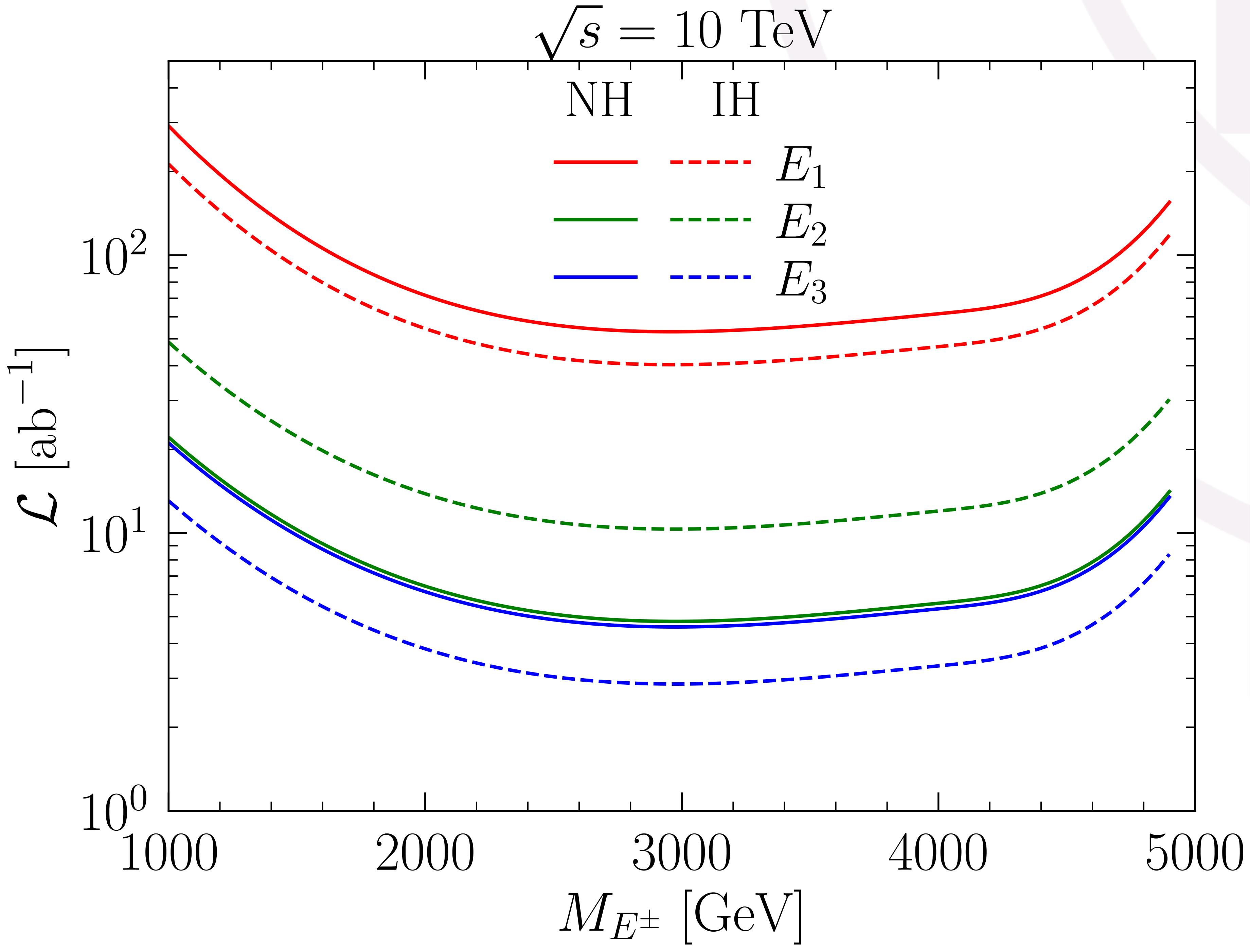


- **SM backgrounds**: $\mu^+\mu^-, VV \to Z\,Z\,\mu^+\mu^-$

- **The integrated luminosities**

$\sqrt{s}$	3 TeV	10 TeV	30 TeV
$\mathscr{L}$	1 ab^{-1}	10 ab^{-1}	90 ab^{-1}





(1) The E_2 and E_3 heavy leptons can be discovered.

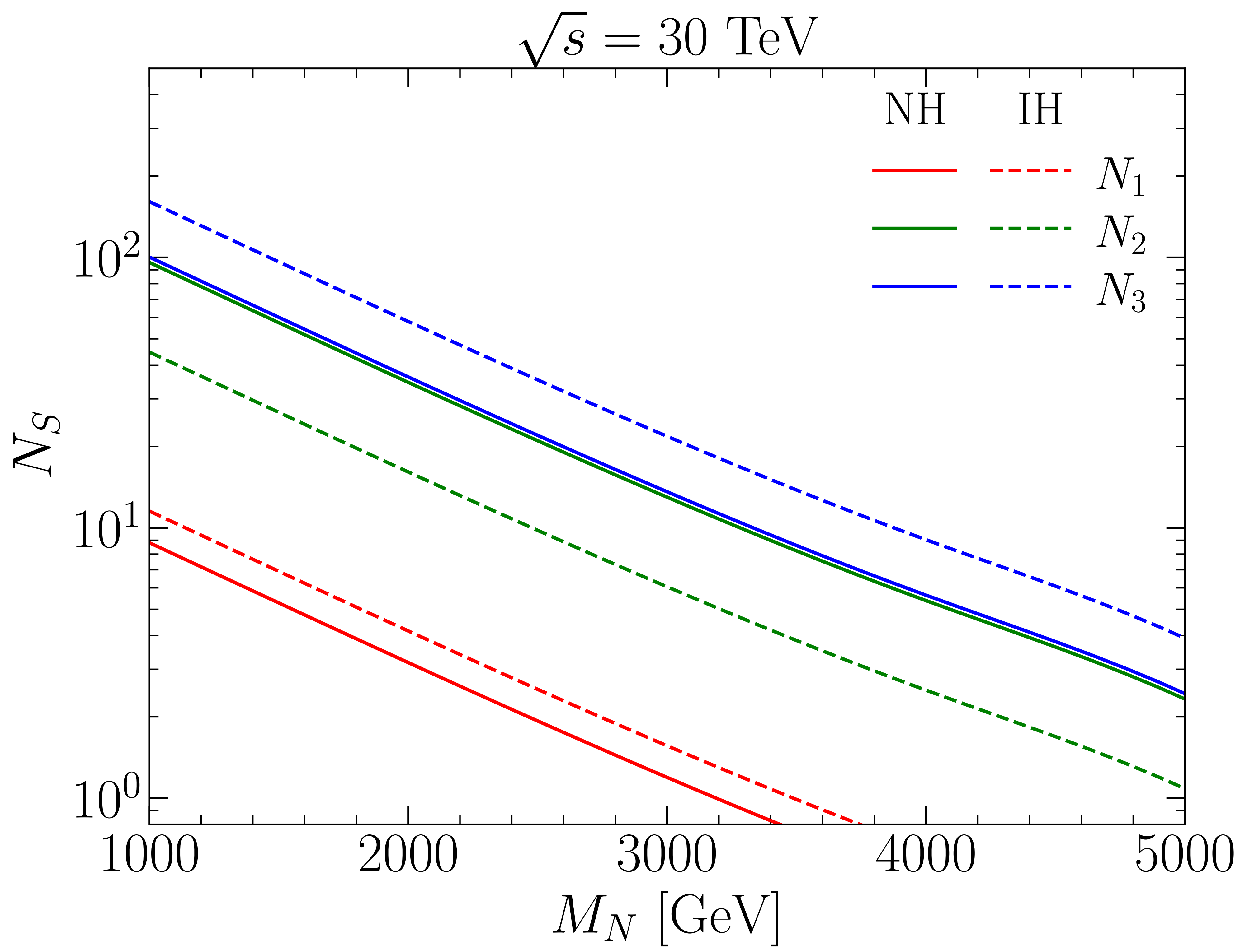
(2) To discover E_1, we need higher integrated luminosities.

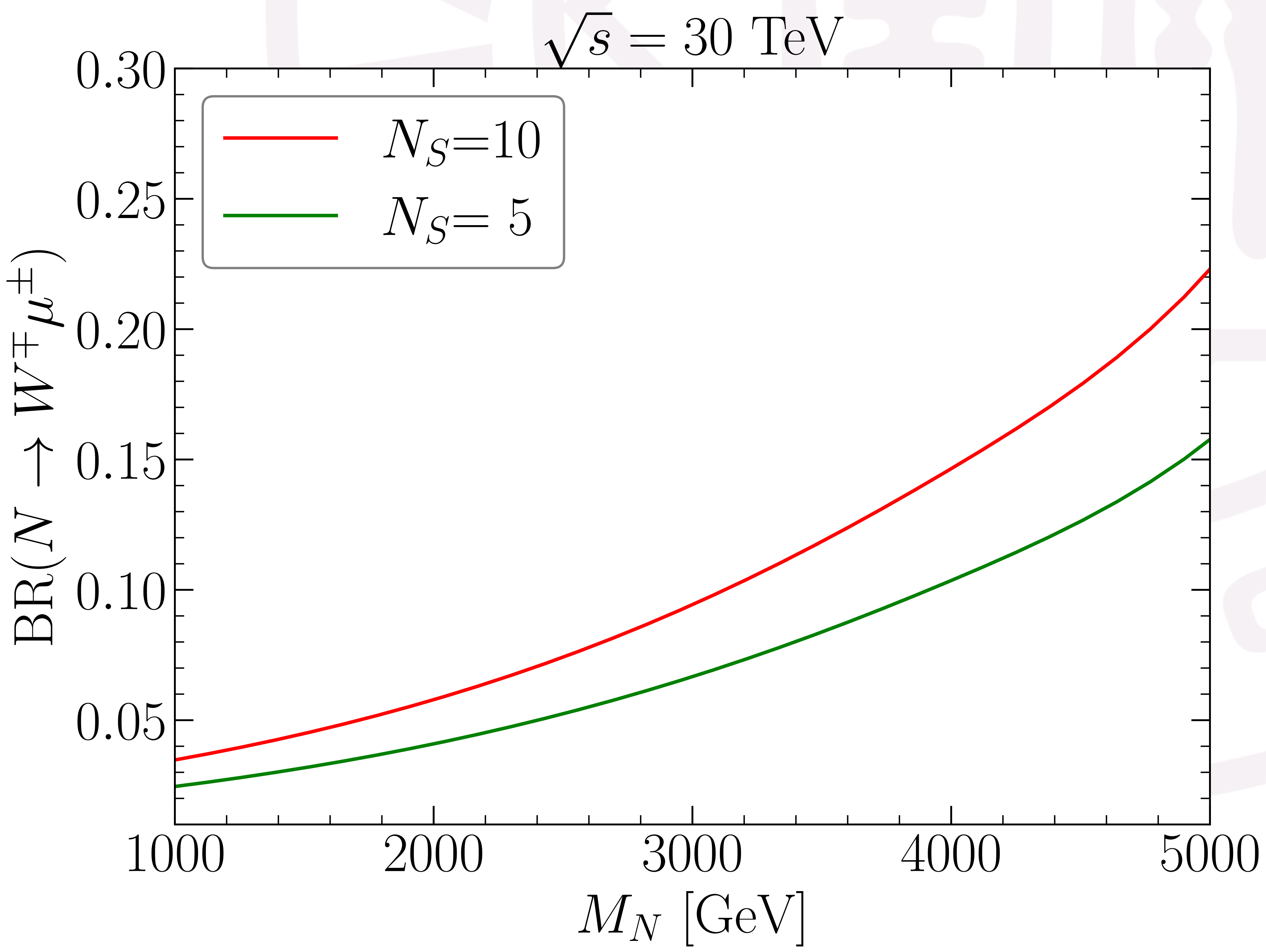
B. $E^{\pm}N$ pair production

- $E^{\pm}N \to ZW^{\mp}\mu^{\pm}\mu^{\pm}$ **LNV**

SM backgrounds: $ZW^{\mp}W^{\pm}W^{\pm} \to ZW^{\mp}\mu^{\pm}\mu^{\pm}\nu\ \nu$

When $M_{E(N)} > 6$ TeV, it is difficult to generate more than one signal event.





- The result of channel C. $N\ N \to W^{\mp}W^{\mp}\mu^{\pm}\mu^{\pm}$ is similar

Heavy Neutral Lepton

heavy neturino

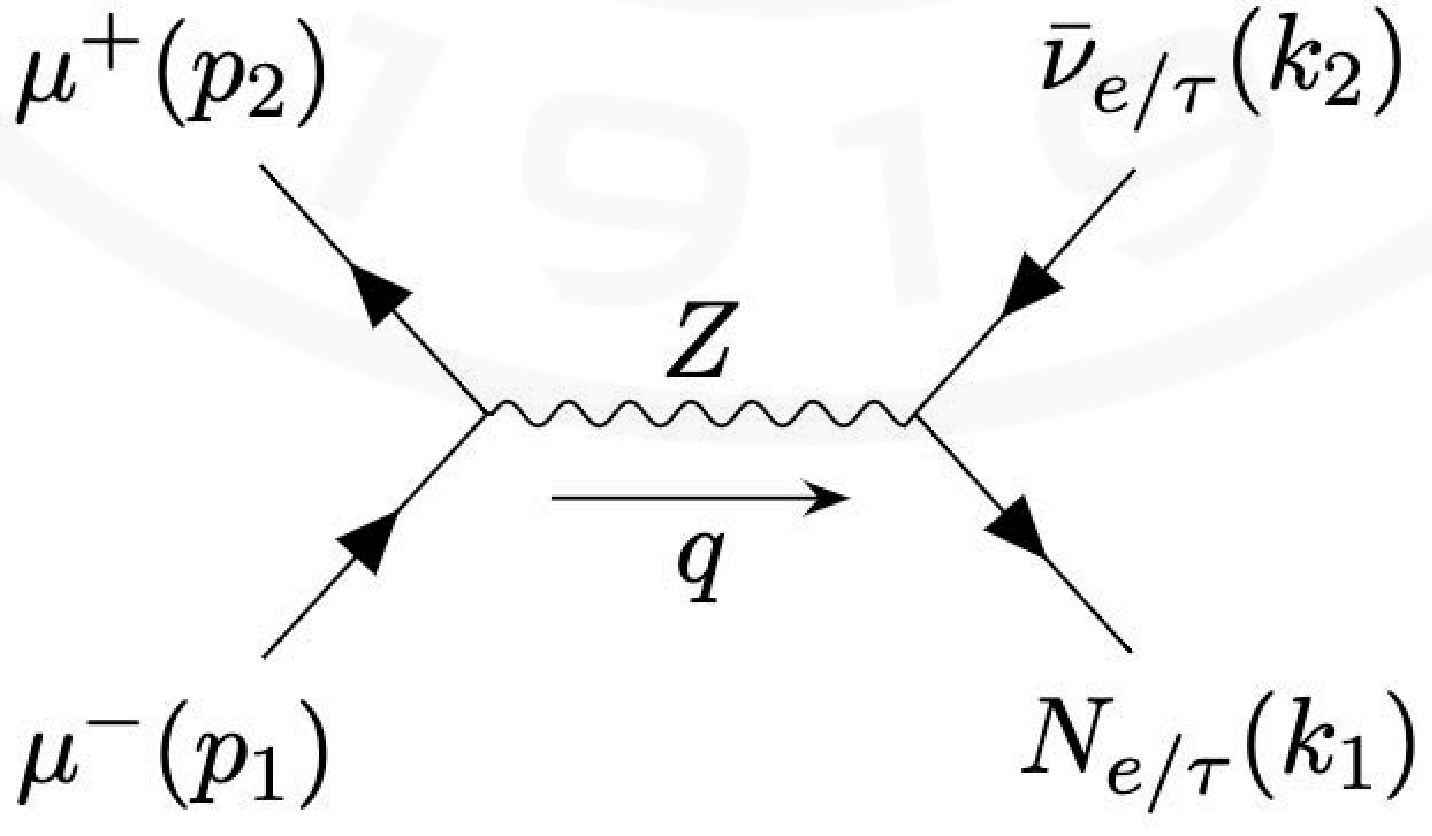
Type I Seeaw
Type III Seesaw

- Recently, we study the search potential of heavy neutral lepton (HNL) at muon collider. (The mixing matrix $V_{\ell N}$ transiting HNL to charged leptons in the mass basis)

[1] K.Mekala, J.Reuter, A.F.Zarnecki 2301.02602
[2] T.H.Kwok, L.Li, T.Liu, A.Rock 2301.05177
[3] P.Li, Z.Liu, K.F.Lyu 2301.07117

There are a few studies of searching for HNL at muon collider， they proposed that an HNL can be produced together with a light neutrino.

$$\mu^+\mu^- \to N_\ell \bar{\nu}_\ell \to \ell^\pm \bar{\nu}_\ell q \bar{q}'$$

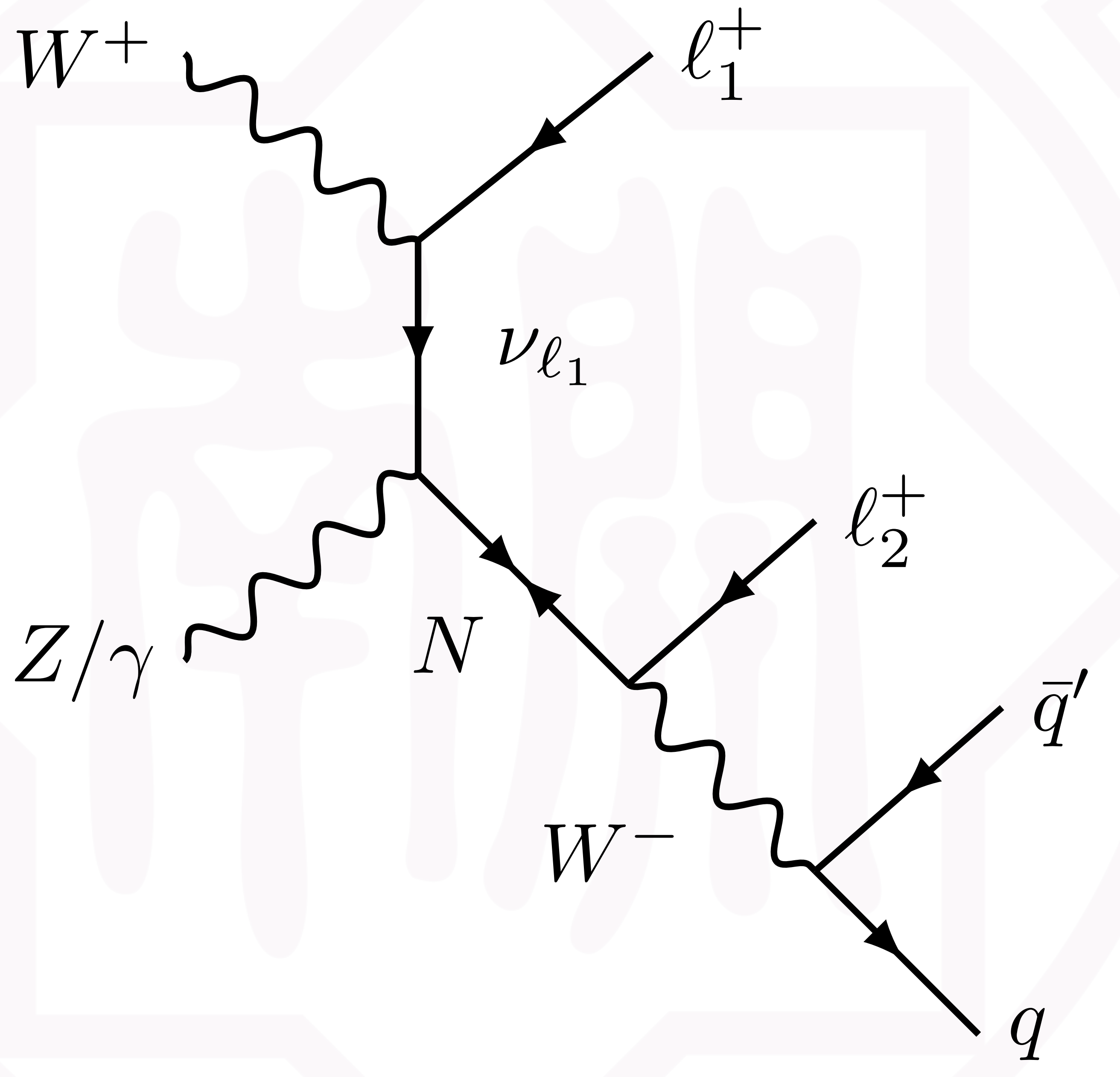


This channel cannot directly tell whether the HNL is Dirac or Majorana fermion

Heavy Neutral Lepton

- We propose a clear way to search for heavy Majorana neutrino through LNV signature at muon collider.

VBS



$$W^\pm Z/\gamma \to \ell^\pm N \to \ell^\pm \ell^\pm W^\mp \to \ell^\pm \ell^\pm q\bar{q}'$$

[3] T. Li, C.Y. Yao, *M.Yuan* arXiv: 2306.17368

- We separate $|V_{\ell N}|^2$ from the cross section and select some cuts to suppress the SM background as mentioned earlier.

$$\sigma\left(V_i V_j \to \ell_1^\pm N \to \ell_1^\pm \ell_2^\pm q\bar{q}'\right) \approx \sigma\left(V_i V_j \to \ell_1^\pm N\right) \times \mathrm{BR}(N \to \ell_2^\pm q\bar{q}') \times (2 - \delta_{\ell_1\ell_2})$$
$$\equiv \frac{|V_{\ell_1 N} V_{\ell_2 N}|^2}{\sum_{\ell=e,\mu,\tau} |V_{\ell N}|^2} \times \sigma_0 \times (2 - \delta_{\ell_1\ell_2}) ,$$

$\sqrt{s}$ **= 3, 10 and 30 TeV**

Heavy Neutral Lepton

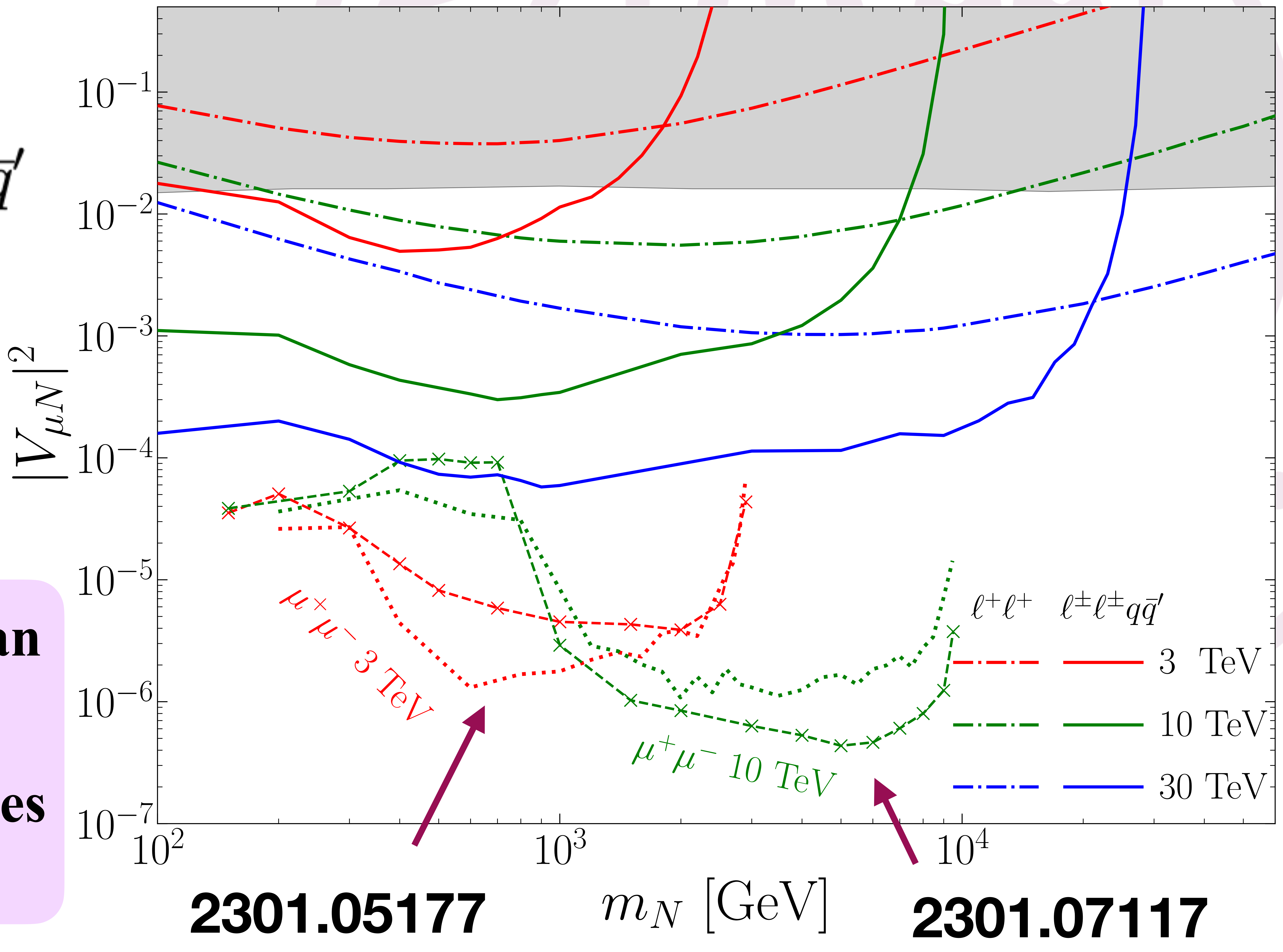
- **The 2σ exclusion limits for $|V_{\mu N}|^2$.**

$$W^{\pm}Z/\gamma \to \ell^{\pm}N \to \ell^{\pm}\ell^{\pm}W^{\mp} \to \ell^{\pm}\ell^{\pm}q\bar{q}'$$

As a comparison, the results of channel $\mu^+\mu^- \to N\nu$ in other studies are shown.

The probing potential of $|V_{\mu N}|^2$ is worse than that from other annihilation channel.

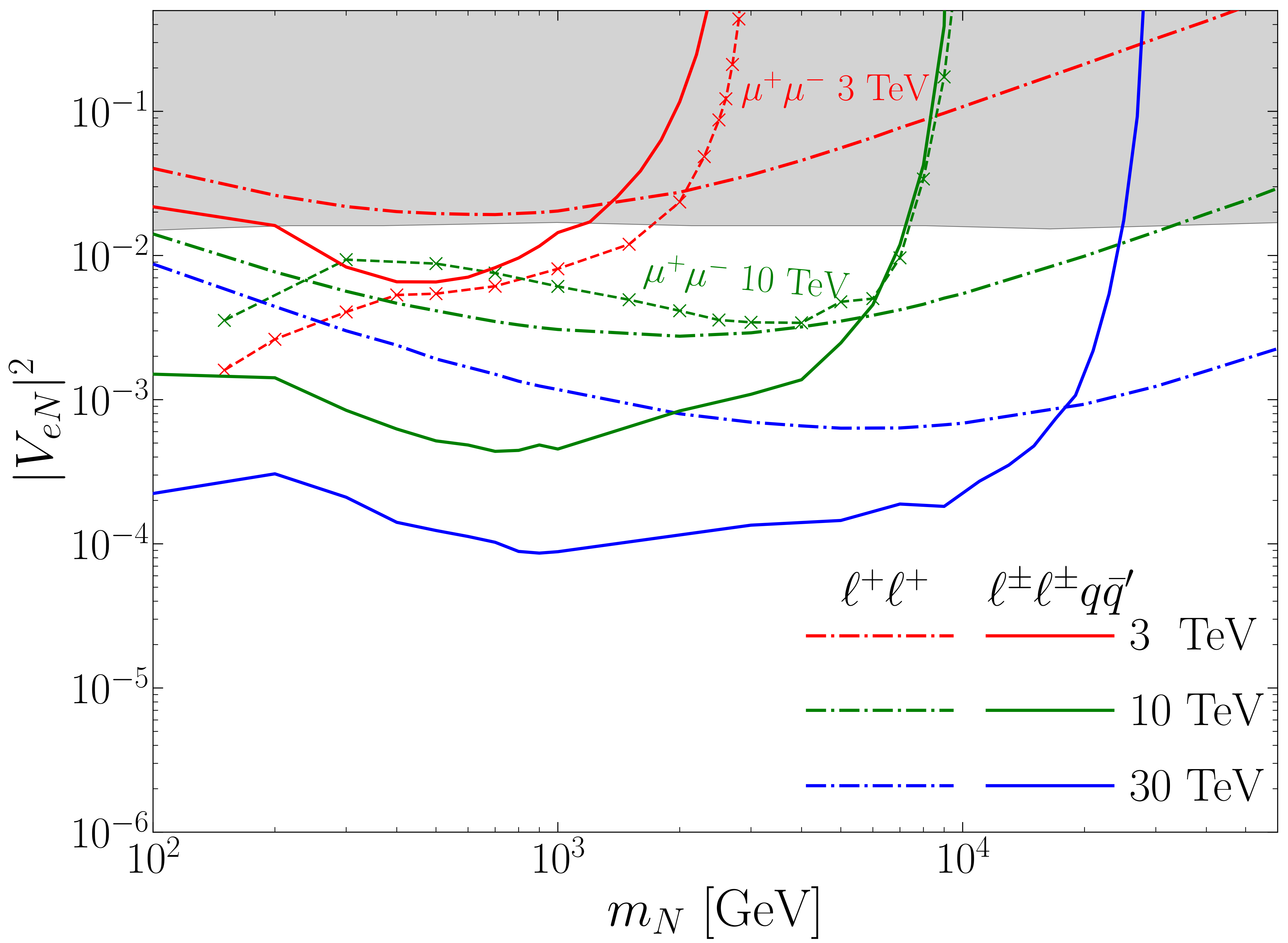
But we provide smoking-gun LNV signatures as a complement through VBS process.

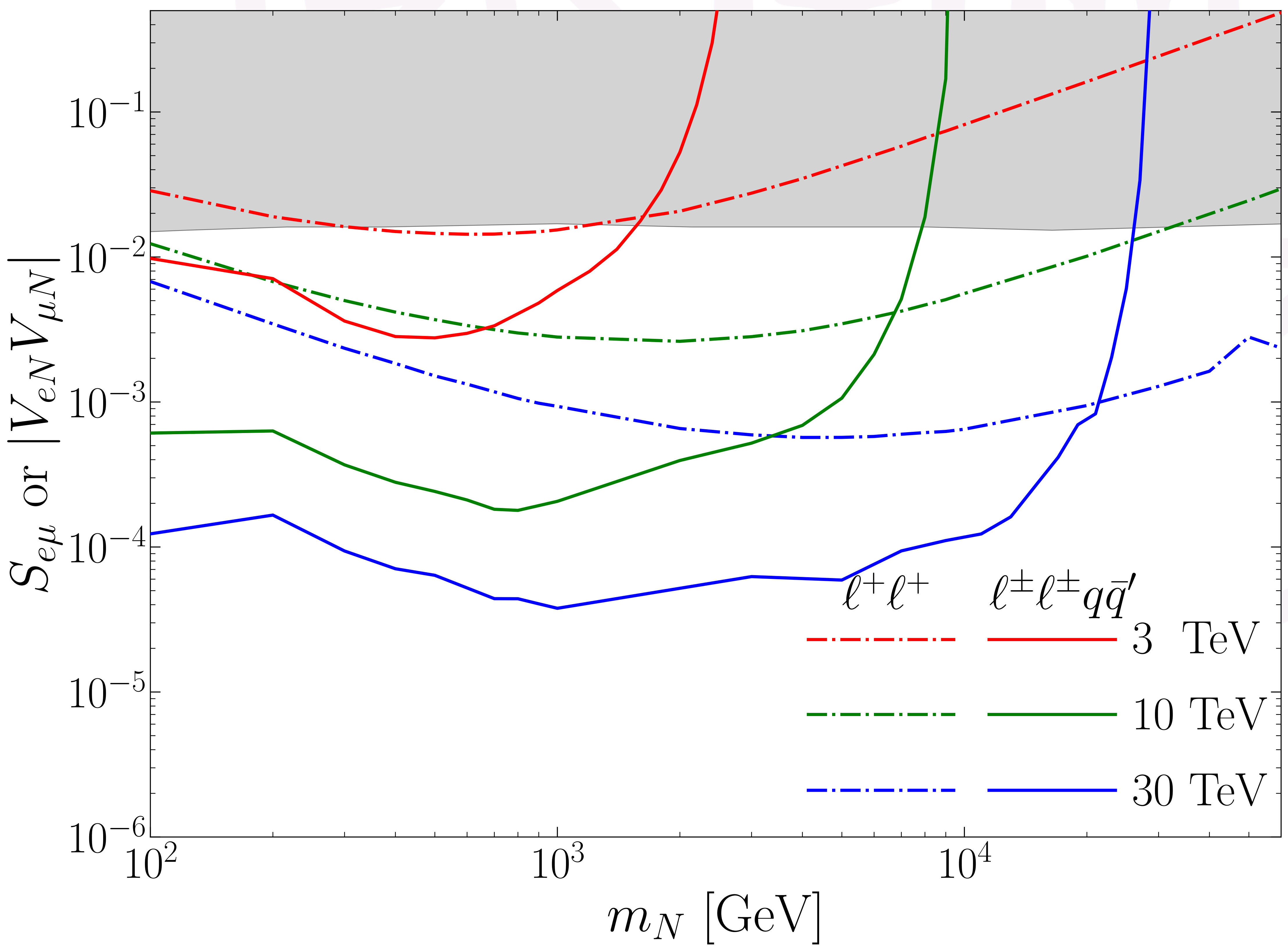


Heavy Neutral Lepton

- **The 2σ exclusion limits for $|V_{eN}|^2$ and $|V_{eN}\,V_{\mu N}|$.**

The probing potential of $|V_{eN}|^2$ is stronger than that from $\mu^+\mu^- \to N\,\nu$ channel.





Summary

- **Muon collider combines the advantages of e^+e^- and pp colliders with higher energy and luminosity. It has tremendous potential in high-energy physics research and provides many opportunities to explore new physics beyond the SM.**

- **For the Type-II Seesaw, we study the search potential of charged Higgs at muon collider by simulating the four purely leptonic and bosonic decay channels.**

$$H^{++}H^{--} \to \mu^+\mu^+\mu^-\mu^- \qquad H^{\pm\pm}H^{\mp} \to \mu^\pm\mu^\pm\mu^\mp\nu$$

$$H^{++}H^{--} \to W^+W^+W^-W^- \qquad H^{\pm\pm}H^{\mp} \to W^\pm W^\pm W^\mp Z$$

Summary

- **For the Type-III Seesaw, we adopt the similar simulation approach for heavy fermions at muon collider.**

$$E^+E^- \to W^+W^-\nu\bar{\nu} \qquad E^\pm N \to ZW^\mp\mu^\pm\mu^\pm$$

$$E^+E^- \to Z\,Z\,\mu^+\mu^- \qquad N\,N \to W^\mp W^\mp\mu^\pm\mu^\pm$$

- **For the HNL, we provide smoking-gun LNV signatures through VBS process, and search the exclusion limit on mixing** $|V_{\mu N}|^2$, $|V_{eN}|^2$ **and** $|V_{eN}\,V_{\mu N}|$.

$$V\,V \to N\,l^\pm \to l^\pm l^\pm W^\mp$$

- **In the future, we can study other new physics models at the muon collider. Welcome to join us for discussion, if you're interested in it .**

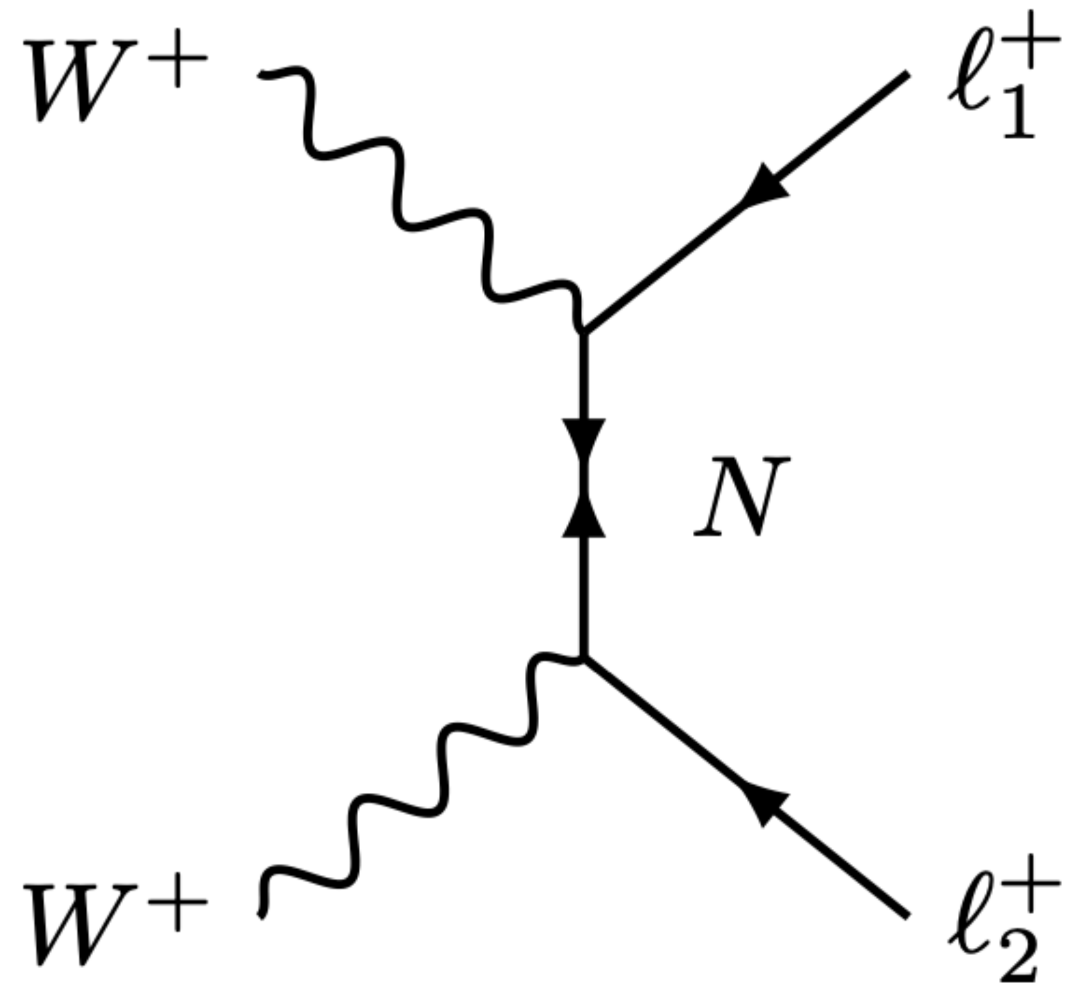
Thanks for your attention！

Appendix 1

- We consider the LNV signature through VBS at same-sign muon collider

$$W^+W^+ \to \ell^+\ell^+$$

VBS

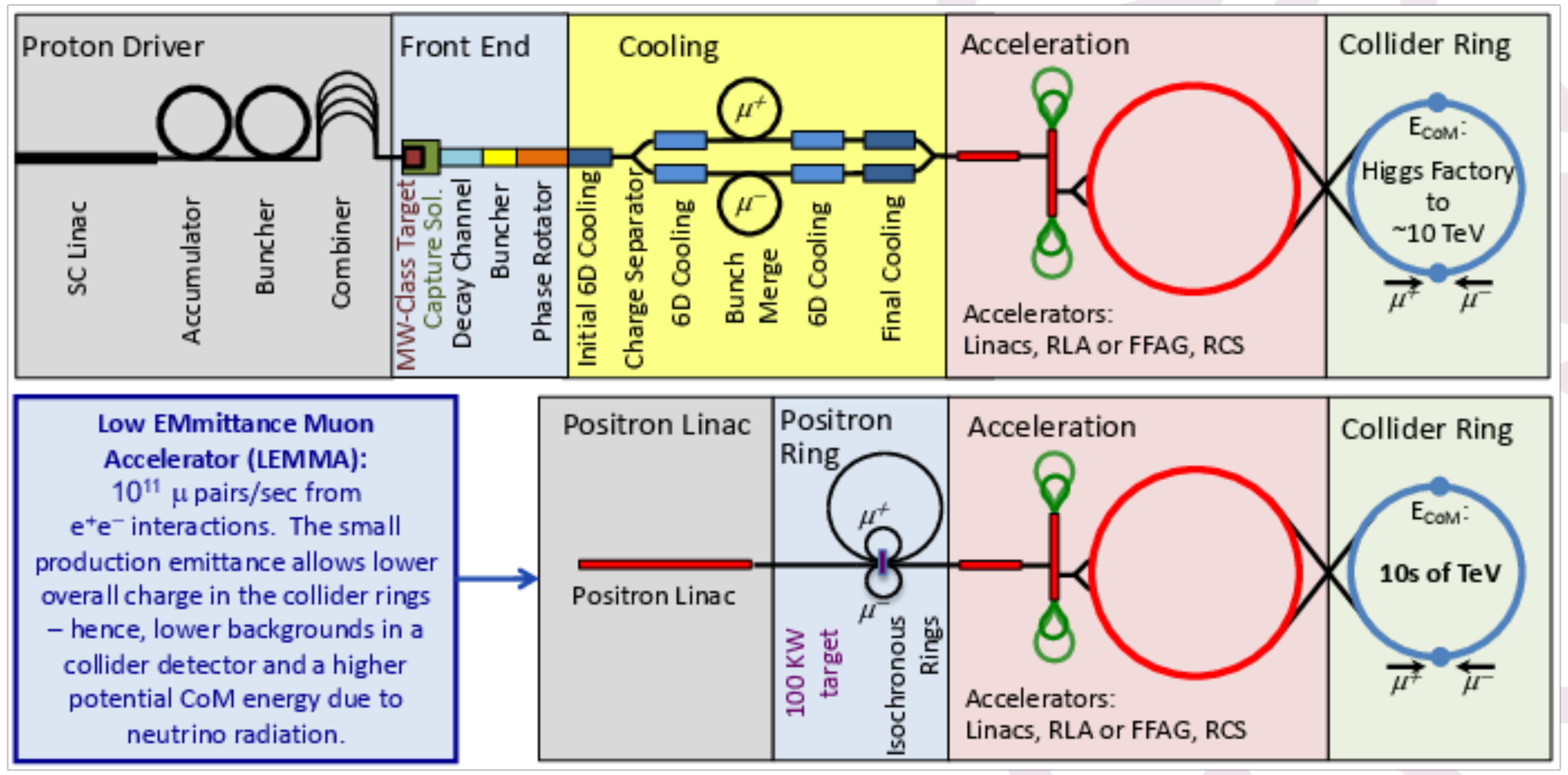


[3] T. Li, C.Y. Yao, *M.Yuan* arXiv: 2306.17368

Cooling

质子打靶 → π介子 → 缪子 → 加速器

Appendix 2



Collider Ring

- 质子打靶：利用高强度的质子束，打靶产生大量π介子。
- 产生缪子：然后π介子在衰变通道中衰变出大量缪子，再将衰变出的缪子通过聚束和相位旋转系统，形成缪子束流。
- 冷却阶段：在高强度磁场下利用一系列的吸收器和射频腔降低纵向和横向发射度。
- 加速阶段：利用一台直线加速器或两台再循环直线加速器进行低能加速，加速到60GeV的能量。在此之后缪子束流分成正反缪子束流分别进入到环形加速器继续加速，直到满足目标能量。
- 准备对撞：最后两束缪子束流注入对撞环，进行对撞。

Appendix 3

- The Lagrangian of the Type II Seesaw

$$\mathcal{L}_{\text{TypeII}} = (D_\mu H)^\dagger (D^\mu H) + \text{Tr}\left[(D_\mu \Delta)^\dagger (D^\mu \Delta)\right] - V(H,\Delta) + \left(-Y_\nu \ell_L^T C i\sigma_2 \Delta \ell_L + \text{h.c.}\right) .$$

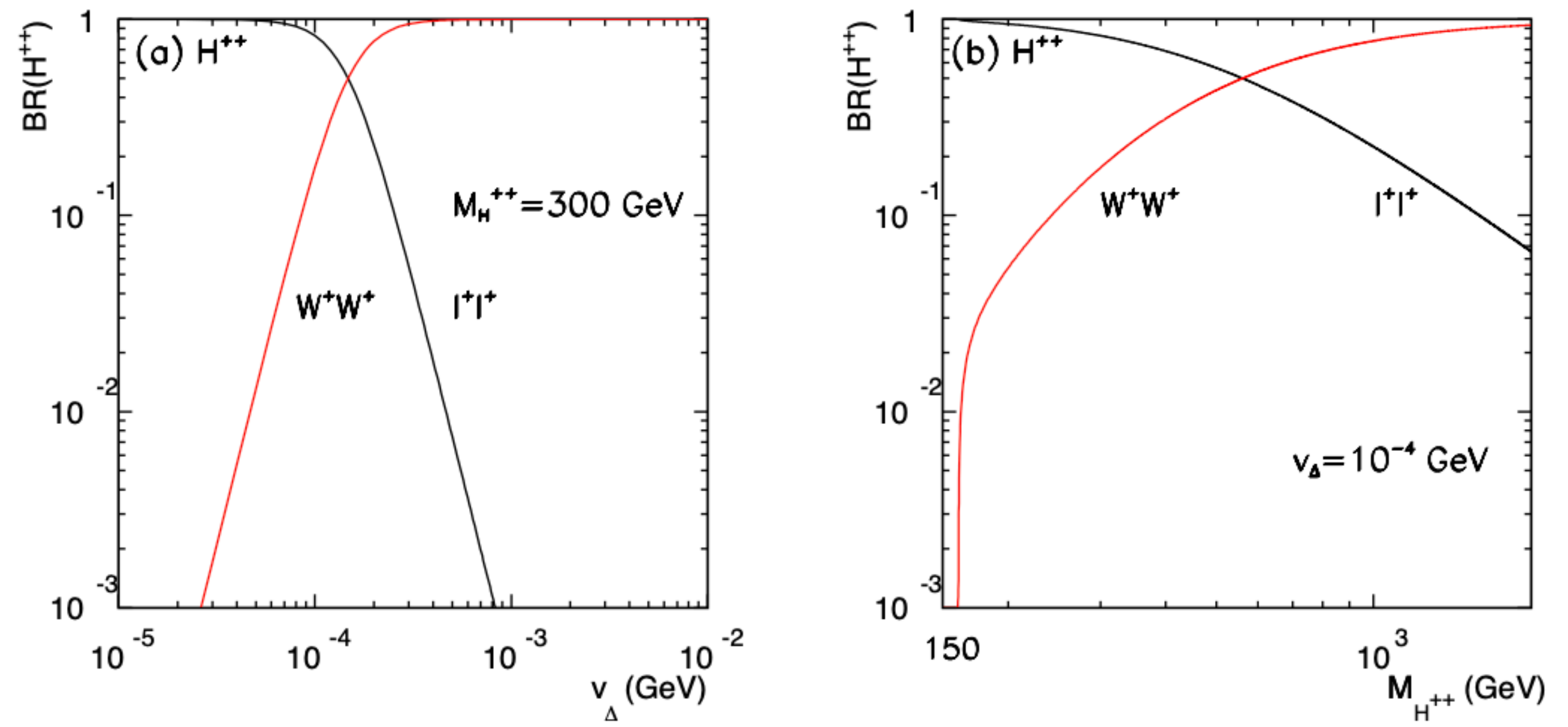


FIG. 4: Branching fractions of the doubly charged Higgs boson decay versus (a) v_Δ for $M_{H^{++}} = 300$ GeV, and (b) $M_{H^{++}}$ for $v_\Delta = 10^{-4}$ GeV.

- 根据带电Higgs的衰变宽度以及最新中微子振荡参数，考虑带电Higgs轻子型衰变的branching ratios.

$$\Gamma(H^{++} \to \ell_i^+ \ell_j^+) = \frac{1}{4\pi(1+\delta_{ij})} |(Y_\nu^{++})_{ij}|^2 M_{H^{++}}$$

$BR(H^{++})$	ee	$e\mu$	$e\tau$	$\mu\mu$	$\mu\tau$	$\tau\tau$
NH	0.28%	1.25%	4.27%	25.57%	35.43%	33.20%
IH	47.49%	1.23%	0.96%	8.84%	25.63%	15.85%

$$\Gamma(H^{+} \to \ell_i^+ \bar{\nu}_j) = \frac{1}{16\pi} |(Y_\nu^{+})_{ij}|^2 M_{H^{+}}$$

$BR(H^{+})$	$e\bar{\nu}$	$\mu\bar{\nu}$	$\tau\bar{\nu}$
NH	3.04%	43.91%	53.05%
IH	48.59%	22.27%	29.14%

Parameter	Normal Hierarchy	Inverted Hierarchy
$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.304^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.570^{+0.016}_{-0.022}$
$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02241^{+0.00074}_{-0.00062}$
$\delta_{\text{CP}}[^\circ]$	230^{+36}_{-25}	278^{+22}_{-30}
$\Delta m^2_{21}[10^{-5}\,\text{eV}^2]$	$7.42^{+0.21}_{-0.20}$	$7.42^{+0.21}_{-0.20}$
$\Delta m^2_{3\ell}[10^{-3}\,\text{eV}^2]$	$+2.510^{+0.027}_{-0.027}$	$-2.490^{+0.026}_{-0.028}$

这里利用了中微子振荡参数的最佳拟合值，假定最轻的中微子质量为10^{-4} eV 以及忽略Majorana相位。

Appendix 4

- Type III Seesaw 的拉氏量

$$\mathcal{L}_{\text{TypeIII}} = \text{Tr}\left[\overline{\Sigma}_L i \not{D} \Sigma_L\right] - \left(\frac{1}{2}\text{Tr}\left[\overline{\Sigma}_L^c M_\Sigma \Sigma_L\right] + Y_\Sigma \bar{\ell}_L \widetilde{H} \Sigma_L^c + \text{h.c.}\right).$$

- 根据重费米子的衰变宽度以及最新中微子振荡参数，得到其衰变分支比。

BR(N_i)	$e^\pm W^\mp$	$\mu^\pm W^\mp$	$\tau^\pm W^\mp$
N_1	17% (17%)	3.26% (3.73%)	4.74% (4.27%)
N_2	7.43% (7.43%)	10.75% (7.34%)	6.82% (10.23%)
N_3	0.56% (0.56%)	11% (13.93%)	13.44% (10.51%)

Appendix 5

- Type III Seesaw 中重粒子的混合参数以及衰变宽度

这里假设$\Omega = I$ (单位矩阵)

The general solution of the $V_{\ell N}$ in Eq. (9) can be parameterized in terms of an arbitrary orthogonal complex matrix Ω in the Casas-Ibarra parametrization [50]

$$V_{\ell N} = U_{\rm PMNS}(m_\nu^{diag})^{1/2}\Omega(M_N^{diag})^{-1/2}, \tag{10}$$

with the orthogonality condition $\Omega\Omega^T = I$. Using the SM electroweak current for heavy Majorana neutrinos N_i, in the mixed mass-flavor basis, one can obtain the partial width of their decay into charged lepton [51]

$$\Gamma(N_i \to \ell^\pm W^\mp) = \frac{G_F}{8\sqrt{2}\pi}|V_{\ell N_i}|^2 M_{N_i}(M_{N_i}^2 + 2M_W^2)\left(1 - \frac{M_W^2}{M_{N_i}^2}\right)^2, \tag{11}$$

Appendix 6

- Type III Seesaw 中重粒子的衰变宽度以及Branching ratios

where PMNS refers to Pontecorvo-Maki-Nakagawa-Sakata matrix, and all fields are mass eigenstates.

The above gauge interactions between SM leptons and heavy triplet leptons are all given by $V_{\ell N}$ [26,45] and the partial widths of both heavy charged leptons and heavy neutrinos are proportional to $|V_{\ell N}|^2$. In the limit of $M_E \approx M_N \gg M_W, M_Z$, and M_h, the partial widths become [45,46]

$$\frac{1}{2}\Gamma\left(N \to \sum_{\ell} \ell^+ W^- + \ell^- W^+\right) \approx \Gamma\left(N \to \sum_{\nu} \nu Z + \bar{\nu} Z\right) \approx \Gamma\left(N \to \sum_{\nu} \nu h + \bar{\nu} h\right)$$
$$\approx \frac{1}{2}\Gamma\left(E^\pm \to \sum_{\nu} \overset{(-)}{\nu} W^\pm\right) \approx \Gamma\left(E^\pm \to \sum_{\ell} \ell^\pm Z\right) \approx \Gamma\left(E^\pm \to \sum_{\ell} \ell^\pm h\right)$$
$$\approx \frac{G_F}{8\sqrt{2}\pi} \sum_{\ell} |V_{\ell N}|^2 M_\Sigma^3. \tag{7}$$

Thus, the decay branching ratios (BRs) of heavy leptons exhibit asymptotic behavior consistent with the Goldstone equivalence theorem [47,48] and are given by the following relations [25,45,46,49]:

$$\frac{1}{2}\mathrm{BR}\left(N \to \sum_{\ell} \ell^+ W^- + \ell^- W^+\right) \approx \mathrm{BR}\left(N \to \sum_{\nu} \nu Z + \bar{\nu} Z\right) \approx \mathrm{BR}\left(N \to \sum_{\nu} \nu h + \bar{\nu} h\right)$$
$$\approx \frac{1}{2}\mathrm{BR}\left(E^\pm \to \sum_{\nu} \overset{(-)}{\nu} W^\pm\right) \approx \mathrm{BR}\left(E^\pm \to \sum_{\ell} \ell^\pm Z\right) \approx \mathrm{BR}\left(E^\pm \to \sum_{\ell} \ell^\pm h\right) \approx \frac{1}{4}. \tag{8}$$