B-L in the SU(N) GUT

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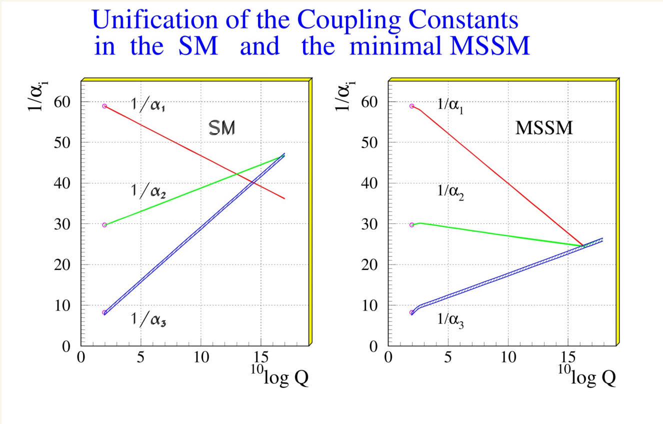
Background of GUT:

- 1974, Georgi–Glashow $SU(5)$ GUT, $3\times[\bar{5}_F \oplus 10_F]$
- 1975, Fritzsch–Minkowski $SO(10)$ GUT, $3\times 16_F$
- 1981, Dimopoulos & Georgi SUSY $SU(5)$



GUT unifies sym' & matter

$$\bar{5}_F \supset (\bar{3}, 1, +\tfrac{1}{3})_F \oplus (1, \bar{2}, -\tfrac{1}{2})_F$$

$$10_F \supset (3, 2, +\tfrac{1}{6})_F \oplus (\bar{3}, 1, -\tfrac{2}{3})_F$$

$$\oplus (1, 1, +1)_F$$

Fundamental Puzzles in the SM:

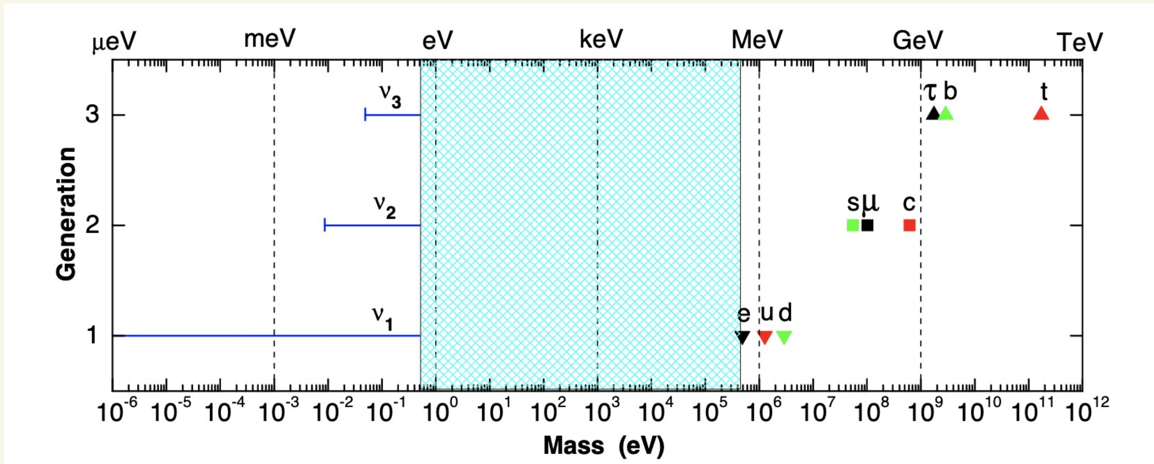
- $\mathcal{L} = \sqrt{-g}\Big[R - \frac{1}{4}(F^a_{\mu\nu})^2 - \frac{\theta}{8\pi}\alpha_s G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} + \bar{\psi} i \not{D} \psi$

$$+ Y_{ij}\psi_i\psi_j\Phi - |D_\mu \Phi|^2 - V(\Phi)\Big]$$

- Strong CP: Peccei-Quinn mechanism, but PQ quality
- Flavors: most puzzling sector, must be understood with the Higgs Yukawa couplings.
- Higgs potential: why $m_H = 125\,\mathrm{GeV}$?
- Both PQ mech & flavors are related to Higgs mech!

Flavor Sector:

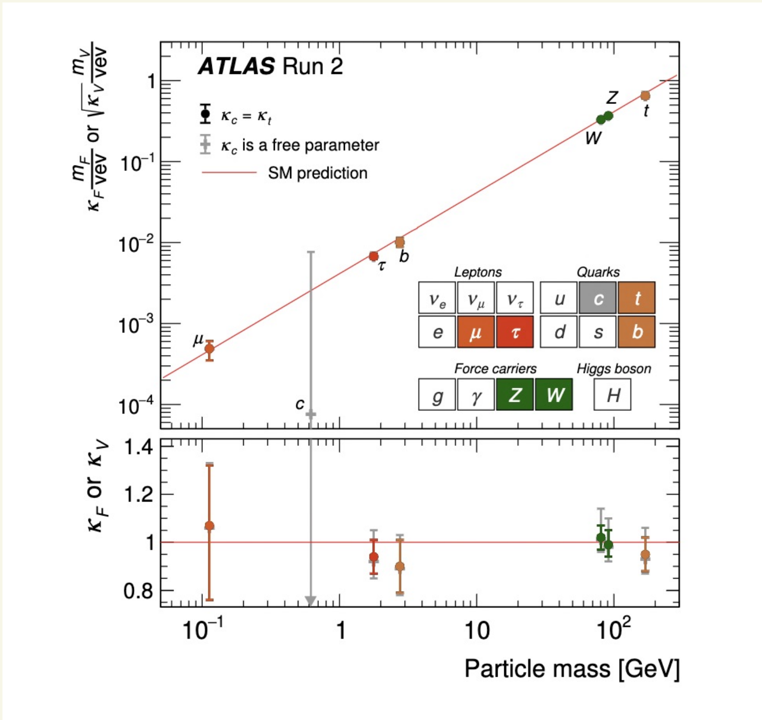
邢志忠 (1909.09610)



non-universal intragenerational mass splitting patterns:

⇒ non-universal SYMMETRY properties of SM fermions?

Flavor Sector:



(2207.00092)

SM Higgs distinguishes flavors with hierarchical Yukawa interactions.

only $y_t \sim \mathcal{O}(1)$

杨振宁 [1980]

"SYMMETRY DICTATES INTERACTIONS."

Back to GUT:

- All flavor puzzles are simply to ask WHY/How $\underline{n_g = 3}$?
- Poor imagination: $3 \times [\bar{5}_F \oplus 10_F]$ in $SU(5)$, or $3 \times 16_F$ in $SO(10)$, or $3 \times 27_F$ in E_6
- Three simple repetition is Nature's Camouflage @ ~ 100 GeV.
- Nanopoulos richmen's view [1980]: all ferm' ψ_i in inequiv' irreps of GUT group $\mathcal{G}$ with

$$\sum_i \mathrm{Anom}(\psi_i) = 0$$

No $\underline{3 \times [- -]}$ here! $\to$ an IAFFS

Georgi's 1979 proposal: flavor-unified $SU(N>5)$ theory

- Law I: $SU(3)_c$ is real (vectorial)

- Law II: $SU(2)_W \otimes U(1)_Y$ is complex (chiral)

- Law III: $\{f_L\}_{SU(N)} = \sum_k m_k [N,k]_F$, $m_k = 0$ or 1

& $\sum m_k \,\mathrm{Anom}([N,k]_F) = 0$ No repetition of any irrep

$\Rightarrow SU(11)$: $[11,4]_F \oplus [11,8]_F \oplus [11,9]_F \oplus [11,10]_F$ $\dim_F = 561$

It seems to me: we should avoid the repetitive generational structure to have $n_g = 3$ emerge naturally @ the EW scale.

Georgi's Counting Rule: decompose irreps into the SU(5) irreps

$[N,1]_F = (N-5) \times \mathbf{1}_F \oplus 5_F$ and etc.

$\Rightarrow$ A set of $(\mathbf{1}_F, 5_F, 10_F, \overline{10}_F, \overline{5}_F)$

Anomaly-free: $\nu_{5_F} + \nu_{10_F} = \nu_{\overline{5}_F} + \nu_{\overline{10}_F}$

# of total generation: $n_g = \nu_{10_F} - \nu_{\overline{10}_F} = \nu_{\overline{5}_F} - \nu_{5_F}$

- In the SU(N), one has: $\nu_{10_F}[N,2]_F - \nu_{\overline{10}_F}[N,2]_F \equiv 1$

$\nu_{10_F}[N,3]_F - \nu_{\overline{10}_F}[N,3]_F = N-6$, and etc.

$\Rightarrow$ rank-2 SU(N) theory alone is insufficient!

Def: irreducible anomaly-free fermion set (IAFFS)

$\sum_R m_R f_L(R)$, $m_R \in \mathbb{Z}$, s.t. $\sum_R m_R \,\mathrm{Anom}(f_L(R)) = 0$.

They satisfy 3 conditions:

(1) $GCD\{m_R\} = 1$;

(2) Not any of $f_L(R)$ can be removed, otherwise Gauge Anom $\neq 0$;

(3) No singlet, self-conjugate, adjoint fermions.

— Law III (CHEN): No simple repetition of any IAFFS in the GUT. (2209.11446, 2307.07921)

Examples:

1) One-generational SM: $(3,2,+\frac{1}{6})_F \oplus (\bar{3},1,-\frac{2}{3})_F \oplus (\bar{3},1,+\frac{1}{3})_F$

$\oplus (1,\bar{2},-\frac{1}{2})_F \oplus (1,1,+1)_F$, an LAFFS

2) SU(5): $3\times[\bar{5}_F \oplus 10_F]$, fails the Law-III.

3) SU(7) [Frampton]: $7\times\bar{7}_F \oplus 21_F \oplus 2\times 35_F$

Ans: This fails the Law-III, by rewriting fermions into

$$[3\times\bar{7}_F \oplus 21_F] \oplus \underbrace{[4\times\bar{7}_F \oplus 2\times 35_F]}_{\text{Simple repetition.}}$$

- Minimal Flavor-unified theory: $SU(8)$

$\mathrm{Anom}(\bar{8}_F) = 1 \qquad \mathrm{Anom}(28_F) = 4 \qquad \mathrm{Anom}(56_F) = 5$

$$\bar{8}_F = 3 \times \mathbb{1}_F \oplus \bar{5}_F$$

$$28_F = 3 \times \mathbb{1}_F \oplus 3 \times \bar{5}_F \oplus 10_F \quad \Rightarrow \quad \mathrm{I}\ \text{gen}'$$

$$56_F = \mathbb{1}_F \oplus 3 \times \bar{5}_F \oplus 3 \times 10_F \oplus \overline{10}_F \quad \Rightarrow \quad 3 - \underline{\underline{1}} = 2\ \text{gen}'$$

↳ mirror ferm.

$$\{f_L\}_{SU(8)} = [\bar{8}_F^{\,\lambda} \oplus 28_F] \oplus [\bar{8}_F^{\,\dot{\lambda}} \oplus 56_F]$$

$\lambda = 3, \underline{\mathrm{IV}}, \underline{\mathrm{V}}, \underline{\mathrm{VI}}$: rank-2 IAFFS

$\dot{\lambda} = \dot{1}, \dot{2}, \underline{\dot{\mathrm{VII}}}, \underline{\dot{\mathrm{VIII}}}, \underline{\dot{\mathrm{IX}}}$: rank-3 IAFFS

Two distinctive IAFFSs w.o. repetition.

How to def' the B-L ? S. Weinberg, Wilczek-Zee [1979]

- $SU(5)$: $\bar{5}_F \oplus 10_F$ $\tilde{\hat{u}}(1)_{\square} \otimes \hat{u}(1)_{\square} \Rightarrow$ non-anomalous $\tilde{\hat{u}}(1)_{T_2}$

$$T_2(\bar{5}_F) = -3t_2 \quad T_2(10_F) = +t_2 \quad \text{s.t.} \quad [SU(5)]^2 \cdot \tilde{\hat{u}}(1)_{T_2} = 0$$

$$-\mathcal{L}_Y = \bar{5}_F 10_F 5_H^{\dagger} + 10_F 10_F 5_H + \text{H.c.} \qquad T_2(5_H) = -2t_2$$

- $B-L \equiv \tilde{\hat{a}}_1 T_2 + \tilde{\hat{a}}_2 Y$ with $\hat{a}_1 = 1$ from the 't Hooft

anomaly matching (1980): $[\tilde{\hat{u}}(1)_{T_2}]^3 = [\tilde{\hat{u}}(1)_{B-L}]^3$

- $\tilde{\hat{u}}(1)_{B-L}$-neutral: $10_F 10_F 5_H + \text{H.c.} \supset \underbrace{(3, 2, +\tfrac{1}{6})_F}_{+1/3} \otimes \underbrace{(\bar{3}, 1, -\tfrac{2}{3})_F}_{-1/3} \otimes \underbrace{(1, 2, +\tfrac{1}{2})_H}_{0.}$

$$(B-L)(1, 2, +\tfrac{1}{2})_H = -2t_2 + \tfrac{1}{2}\tilde{\hat{a}}_2 = 0 \quad \Rightarrow \quad \tilde{\hat{a}}_2 = 4t_2$$

$$B-L \equiv T_2 + 4t_2 Y \quad \text{in the } SU(5).$$

- In the $SU(8)$:

$$\{f_L\} = [\, \overline{8}_F^{\lambda} \oplus 28_F \,] \oplus [\, \overline{8}_F^{\dot\lambda} \oplus 56_F \,]$$

Global Dimopoulos – Raby – Susskind (DRS) sym' of [1980].

$$[\widetilde{SU}(4)_\lambda \otimes \widetilde{U}(1)_\lambda \otimes \widetilde{U}(1)_{A_2}] \otimes [\widetilde{SU}(5)_{\dot\lambda} \otimes \widetilde{U}(1)_{\dot\lambda} \otimes \widetilde{U}(1)_{A_3}]$$

- Find the non-anomalous $\widetilde{U}(1)_\lambda \otimes \widetilde{U}(1)_{A_2} \to \widetilde{U}(1)_{T_2}$

& $\widetilde{U}(1)_{\dot\lambda} \otimes \widetilde{U}(1)_{A_3} \to \widetilde{U}(1)_{T_3}$

	$\overline{8}_F^{\lambda}$	28_F
T_2:	$-3t_2$	$+2t_2$

	$\overline{8}_F^{\dot\lambda}$	56_F
T_3:	$-3t_3$	$+t_3$

$$\boxed{t_2 = t_3}$$

in the end.

- Higgs sector of the SU(8):

$$-\mathcal{L}_Y = \overline{8_F}^{\lambda}\, 28_F\, \overline{8_{H,\lambda}} + 28_F\, 28_F\, 70_H + \cancel{56_F\, 56_F\, 28_H} \equiv 0$$

$$+ \overline{8_F}^{\dot\lambda}\, 56_F\, \overline{28_{H,\dot\lambda}} + \frac{1}{M_{pl}}\, 56_F\, 56_F (\overline{28_{H,\dot\lambda}})^{\dagger}\, 63_H + \cancel{28_F\, 56_F\, 56_H} + h.c.$$

to rule out by B−L

- $\{H\} = \overline{8_{H,\lambda}} \oplus \overline{28_{H,\dot\lambda}} \oplus 70_H \oplus \underbrace{63_H}_{\text{real, adjoint}}$

- SSB pattern: determined by Higgs irreps, L-F Li [1974]

- Any GUT must undergo a max SSB stage & is only by its adjoint Higgs & as high as possible for "proton decay"

$$SU(8) \xrightarrow{63_H} \mathcal{G}_{441} \qquad \langle 63_H \rangle = \frac{1}{4}\, \mathrm{diag}(-\mathbb{1}_4, +\mathbb{1}_4)\, v_U$$

- SSB pattern of the SU(8):

$$SU(8) \xrightarrow{63_H} \mathcal{G}_{441X_0} \xrightarrow{\overline{8}_{H,\lambda}} \mathcal{G}_{341X_1} \xrightarrow{\overline{8}_{H,\lambda},\ \overline{28}_{H,\dot\lambda}} \mathcal{G}_{331X_2}$$

$$\xrightarrow{\overline{8}_{H,\lambda},\ \overline{28}_{H,\dot\lambda}} \mathcal{G}_{SM} \xrightarrow{70_H} SU(3)_c \otimes U(1)_{EM}.$$

- Decomposition rule: $8_H = (4, 1, -\frac{1}{4})_H \oplus (1, 4, +\frac{1}{4})_H$

$$(\overline{4}, 1, +\frac{1}{4})_H = (\overline{3}, 1, +\frac{1}{3})_H \oplus \underline{(1, 1, 0)_H}$$
$$(1, \overline{4}, -\frac{1}{4})_H = (1, \overline{3}, -\frac{1}{3})_H \oplus \underline{(1, 1, 0)_H}$$

singlets, can develop VEVs

To decompose the Higgs fields to find the SSB directions, with the LieART (1912.10969)

- Decompositions of the SU(8) fermions

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\overline{\mathbf{8}_{\mathbf{F}}}^{\Lambda}$	$(\overline{\mathbf{4}},\mathbf{1},+\frac{1}{4})_{\mathbf{F}}^{\Lambda}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Lambda}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Lambda}$	$(\overline{\mathbf{3}},\mathbf{1},+\frac{1}{3})_{\mathbf{F}}^{\Lambda}\ :\ \mathcal{D}_R^{\Lambda\,c}$
		$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda}\ :\ \check{\mathcal{N}}_L^{\Lambda}$
	$(\mathbf{1},\overline{\mathbf{4}},-\frac{1}{4})_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\overline{\mathbf{4}},-\frac{1}{4})_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\overline{\mathbf{3}},-\frac{1}{3})_{\mathbf{F}}^{\Lambda}$	$(\mathbf{1},\overline{\mathbf{2}},-\frac{1}{2})_{\mathbf{F}}^{\Lambda}\ :\ \mathcal{L}_L^{\Lambda}=(\mathcal{E}_L^{\Lambda},-\mathcal{N}_L^{\Lambda})^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda'}\ :\ \check{\mathcal{N}}_L^{\Lambda'}$
			$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda''}$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}^{\Lambda''}\ :\ \check{\mathcal{N}}_L^{\Lambda''}$

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{28}_{\mathbf{F}}$	$(\mathbf{6},\mathbf{1},-\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}\ :\ \mathfrak{D}_L$
		$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}\ :\ t_R{}^c$
	$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})_{\mathbf{F}}\ :\ (\mathfrak{e}_R{}^c,\mathfrak{n}_R{}^c)^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}\ :\ \check{\mathfrak{n}}_R^c$
			$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})_{\mathbf{F}}'\ :\ (\mathfrak{n}'_R{}^c,-\mathfrak{e}'_R{}^c)^T$
				$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}\ :\ \tau_R{}^c$
	$(\mathbf{4},\mathbf{4},0)_{\mathbf{F}}$	$(\mathbf{3},\mathbf{4},-\frac{1}{12})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}\ :\ (t_L,b_L)^T$
				$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}'\ :\ \mathfrak{D}'_L$
			$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}''$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}''\ :\ \mathfrak{D}''_L$
		$(\mathbf{1},\mathbf{4},+\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})_{\mathbf{F}}''$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})_{\mathbf{F}}''\ :\ (\mathfrak{e}''_R{}^c,\mathfrak{n}''_R{}^c)^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}'\ :\ \check{\mathfrak{n}}_R'^c$
			$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}''$	$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}''\ :\ \check{\mathfrak{n}}_R''^c$

SU(8)	$\mathcal{G}_{441}$	$\mathcal{G}_{341}$	$\mathcal{G}_{331}$	$\mathcal{G}_{\rm SM}$
$\mathbf{56}_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{4}},+\frac{3}{4})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{4}},+\frac{3}{4})_{\mathbf{F}}$	$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}'$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})_{\mathbf{F}}'''\ :\ (\mathfrak{n}_R'''^c,-\mathfrak{e}_R'''^c)^T$
				$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}'\ :\ e_R{}^c$ or $\mu_R{}^c$
			$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}''$	$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}''\ :\ \mathfrak{E}_R{}^c$
	$(\overline{\mathbf{4}},\mathbf{1},-\frac{3}{4})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'\ :\ u_R{}^c$ or $c_R{}^c$
		$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}$	$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}$	$(\mathbf{1},\mathbf{1},-1)_{\mathbf{F}}\ :\ \mathfrak{E}_L$
	$(\mathbf{4},\mathbf{6},+\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{6},+\frac{1}{6})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}'$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}'\ :\ (u_L,d_L)^T$
				or $(c_L,s_L)^T$
				$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}'''\ :\ \mathfrak{D}_L'''$
			$(\mathbf{3},\overline{\mathbf{3}},+\frac{1}{3})_{\mathbf{F}}$	$(\mathbf{3},\overline{\mathbf{2}},+\frac{1}{6})_{\mathbf{F}}''\ :\ (\mathfrak{d}_L,-\mathfrak{u}_L)^T$
				$(\mathbf{3},\mathbf{1},+\frac{2}{3})_{\mathbf{F}}\ :\ \mathfrak{U}_L$
		$(\mathbf{1},\mathbf{6},+\frac{1}{2})_{\mathbf{F}}'$	$(\mathbf{1},\mathbf{3},+\frac{1}{3})_{\mathbf{F}}'$	$(\mathbf{1},\mathbf{2},+\frac{1}{2})_{\mathbf{F}}''''\ :\ (\mathfrak{e}_R''''^c,\mathfrak{n}_R''''^c)^T$
				$(\mathbf{1},\mathbf{1},0)_{\mathbf{F}}'''\ :\ \check{\mathfrak{n}}_R'''^c$
			$(\mathbf{1},\overline{\mathbf{3}},+\frac{2}{3})_{\mathbf{F}}''$	$(\mathbf{1},\overline{\mathbf{2}},+\frac{1}{2})_{\mathbf{F}}'''''\ :\ (\mathfrak{n}_R'''''^c,-\mathfrak{e}_R'''''^c)^T$
				$(\mathbf{1},\mathbf{1},+1)_{\mathbf{F}}'''\ :\ \mu_R{}^c$ or $e_R{}^c$
	$(\mathbf{6},\mathbf{4},-\frac{1}{4})_{\mathbf{F}}$	$(\mathbf{3},\mathbf{4},-\frac{1}{12})_{\mathbf{F}}'$	$(\mathbf{3},\mathbf{3},0)_{\mathbf{F}}''$	$(\mathbf{3},\mathbf{2},+\frac{1}{6})_{\mathbf{F}}'''\ :\ (c_L,s_L)^T$
				or $(u_L,d_L)^T$
				$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}''''\ :\ \mathfrak{D}_L''''$
			$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}'''''$	$(\mathbf{3},\mathbf{1},-\frac{1}{3})_{\mathbf{F}}'''''\ :\ \mathfrak{D}_L'''''$
		$(\overline{\mathbf{3}},\mathbf{4},-\frac{5}{12})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{3},-\frac{1}{3})_{\mathbf{F}}$	$(\overline{\mathbf{3}},\mathbf{2},-\frac{1}{6})_{\mathbf{F}}\ :\ (\mathfrak{d}_R{}^c,\mathfrak{u}_R{}^c)^T$
				$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}''\ :\ \mathfrak{U}_R{}^c$
			$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'''$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})_{\mathbf{F}}'''\ :\ c_R{}^c$ or $u_R{}^c$

Flavor ID ?

$\check{\mathcal{N}}_L$ $\check{\mathfrak{n}}_R$: sterile neutrinos.

- Def' $\widetilde{U}(1)_{B-L}$ in the $SU(8)$:

(1) 't Hooft anomaly matching @ each SSB stage

(2) Higgs fields that can develop VEVs are $\widehat{U}(1)_T$ - neutral

- $SU(8) \to \mathcal{G}_{441X_0}$: $\mathcal{T}_2' = \mathcal{T}_2 - 4t_2 \mathcal{X}_0$ $\quad$ $\mathcal{T}_3' = \mathcal{T}_3 - 4t_3 \mathcal{X}_0$

$\mathcal{G}_{441} \to \mathcal{G}_{341X_1}$: $\mathcal{T}_2'' = \mathcal{T}_2' + 8t_2 \mathcal{X}_1$ $\quad$ $\mathcal{T}_3'' = \mathcal{T}_3' + 8t_3 \mathcal{X}_1$

$\mathcal{G}_{341} \to \mathcal{G}_{331X_2}$: $\mathcal{T}_2''' = \mathcal{T}_2''$ $\quad$ $\mathcal{T}_3''' = \mathcal{T}_3''$

$\mathcal{G}_{331} \to \mathcal{G}_{SM}$: $B-L = \mathcal{T}_2''' = \mathcal{T}_3'''$

$t_2 = t_3 = +\frac{1}{4}$

- Non-anomalous $\widetilde{U}(1)_{\mathcal{T}_2} \otimes \widetilde{U}(1)_{\mathcal{T}_3}$ charges

	$\overline{8_F}^{\lambda}$	28_F	$\overline{8}_{H,\lambda}$	56_H	70_H
$\mathcal{T}_2$:	$-3t_2$	$+2t_2$	$+t_2$	$-2t_2$	$-4t_2$

	$\overline{8_F}^{\dot\lambda}$	56_F	$\overline{28}_{H,\dot\lambda}$	56_H
$\mathcal{T}_3$:	$-3t_3$	$+t_3$	$+2t_3$	$-t_3$

- $28_F\, 56_F\, 56_H + H.c.$

$$56_H \supset \cdots \supset (1, \bar{3}, +\tfrac{2}{3})_H \oplus \left[(1, 3, +\tfrac{1}{3})_H'' \oplus (1, \bar{3}, +\tfrac{2}{3})_H'' \right]$$

$$\begin{array}{cccc} \mathcal{T}_2''': & +t_2 & +t_2 & +t_2 \\ \mathcal{T}_3''': & +2t_3 & +2t_3 & +2t_3 \end{array}$$

56_H has no $\widetilde{U}(1)_T$-neutral components, hence

$$\cancel{28_F\, 56_F\, 56_H} + H.c.$$

• 70_H only contains the EWSB components:

$$70_H \supset (4, \bar{4}, +\tfrac{1}{2})_H \oplus (\bar{4}, 4, -\tfrac{1}{2})_H \supset \cdots$$

$$\supset \underbrace{(1, \bar{2}, +\tfrac{1}{2})_H^{'''}}_{B-L=0} \oplus \underbrace{(1, 2, -\tfrac{1}{2})_H^{'''}}_{B-L=-2}$$

$\Rightarrow$ $(1, \bar{2}, +\tfrac{1}{2})_H^{'''}$ is the SM Higgs doublet

Yukawa: $$28_F \, 28_F \, 70_H \supset \cdots \supset \underbrace{(\bar{3}, 1, -\tfrac{2}{3})_F}_{t_R{}^c} \otimes \underbrace{(3, 2, +\tfrac{1}{6})_F}_{(t_L, b_L)^T} \otimes (1, \bar{2}, +\tfrac{1}{2})_H^{'''}$$

@ tree-level: SM Higgs doublet of $(1, \bar{2}, +\tfrac{1}{2})_H^{'''}$ only gives the top quark mass.

$\Rightarrow$ 28_F only contains the (t_L, b_L), $t_R{}^c$, $\tau_R{}^c$ (3^{rd} generation)

$1^{st}/2^{nd}$ generations of (u_L, d_L), (c_L, s_L), $u_R{}^c$, $c_R{}^c$, $e_R{}^c$, $\mu_R{}^c$ are in the 56_F

- The flavor-unified $SU(N)$ theory contains sterile neutrinos, e.g., in $\overline{8_F}^{\lambda}$ & $\overline{8_F}^{\dot\lambda}$ of $SU(8)$. 27 copies of $\check{\mathcal{N}}_L^{\Lambda,\Lambda',\Lambda''}$

- only 4 right-handed sterile neutrinos from the 28_F & 56_F, they obtain Dirac masses with their left-handed $\check{\mathcal{N}}_L^{\Lambda,\Lambda',\Lambda''}$

- Masses of the remaining 23 $\check{\mathcal{N}}_L^{\Lambda,\Lambda',\Lambda''}$? W.o. any detail of the seesaw mech' and/or Weinberg Op', they cannot be massive above the EW scale, by 't Hooft anomaly matching s.t. $[\widetilde{U}(1)_{T_2/T_3}]^3 = \cdots = [\widetilde{U}(1)_{B-L}]^3$

- Summary

1. The Higgs sector of SU(8): $\bar{8}_{H,\lambda} \oplus \overline{28}_{H,\dot\lambda} \oplus 70_H \oplus \underline{63_H}$

$$V \supset \mu^2 |\Phi|^2 + \lambda |\Phi|^4 + \frac{1}{M_{pl}} \left(\mathcal{E} \cdot \bar{8}_{H,\lambda}^{4} \right) \cdot 70_H + \cdots + H.c.$$

2. # of "massless" sterile neutrinos counted precisely.

3. Non-trivial embedding of three generational SM fermions

$\Rightarrow$ flavor non-universality under the heavy $\mathcal{G}_{441}$ & $\mathcal{G}_{341}$ flavor-conserving neutral GBs, e.g. in the SU(7) toy model (2209.11446)

- Outlook

1. $SU(8)$ is the only AF flavor-unified theory ($b_1 = -9$)

2. The next-minimal theory? Frampton's $SU(9)$ [1980, 2009]:

~~$9 \times \bar{9}_F \oplus 84_F$~~ since $\frac{1}{M_{pl}} 84_F\, 84_F\, 80_H\, 84_H$ leads to suppressed top quark Yukawa coupling

3. My proposal: $\underbrace{[5 \times \bar{9}_F \oplus 36_F]}_{\text{rank-2 IAFFS}} \oplus \underbrace{[5 \times \bar{9}_F \oplus 126_F]}_{\text{rank-4 IAFFS}}$

4. The ultimate goal: SM fermion masses & gauge coupling unification & pheno implication.