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Searching for Evidence of Dark Energy in Milky Way

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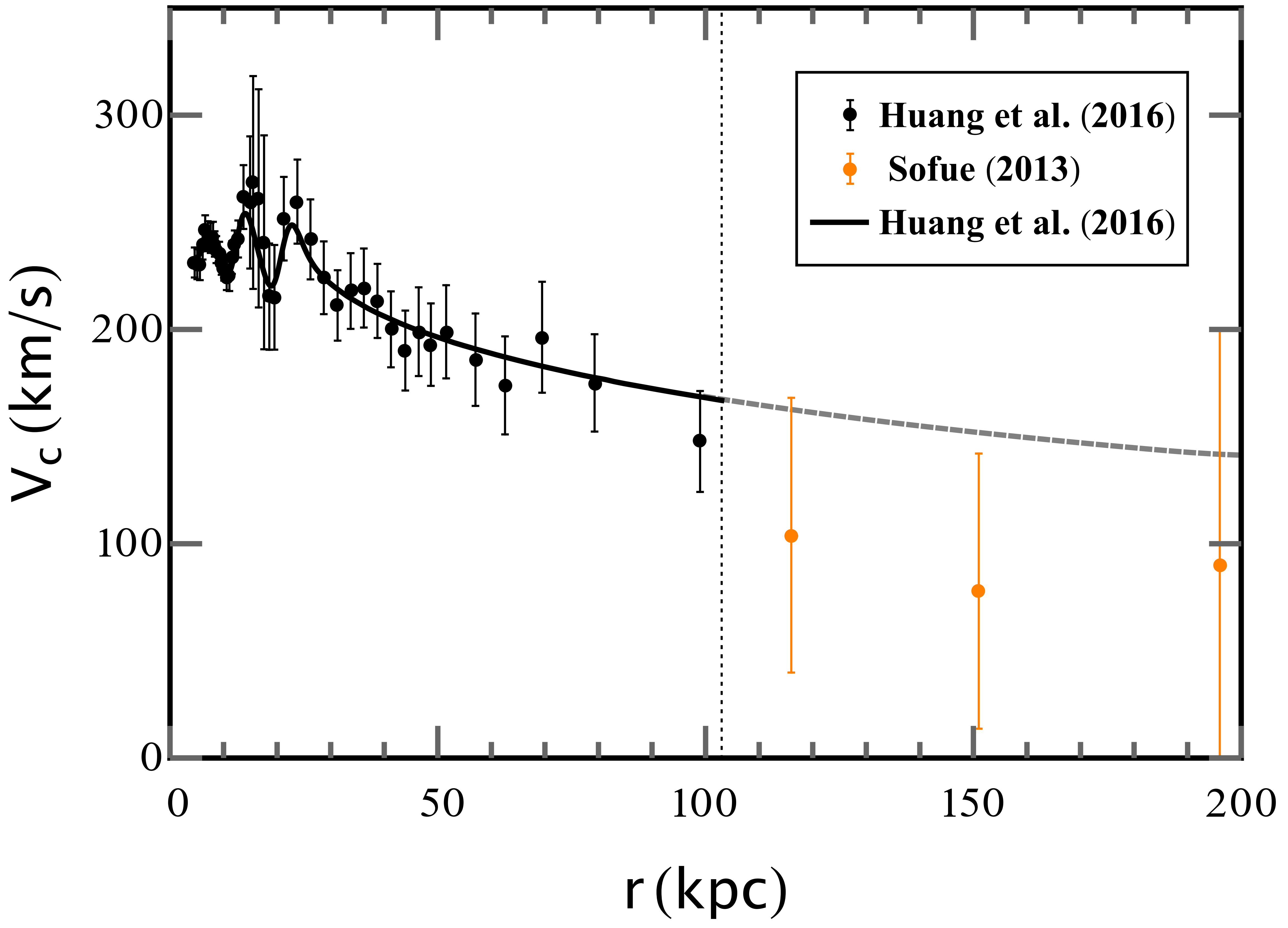
2023年8月9日

with Zhen Zhang (张镇), JCAP06(2023)031 (arxiv: 2303.14047)

Outline

- why locally probe dark energy
- theory for dark energy local effects
- searching for dark energy in Milky Way



Dark Energy (DE)

originate from cosmological constant

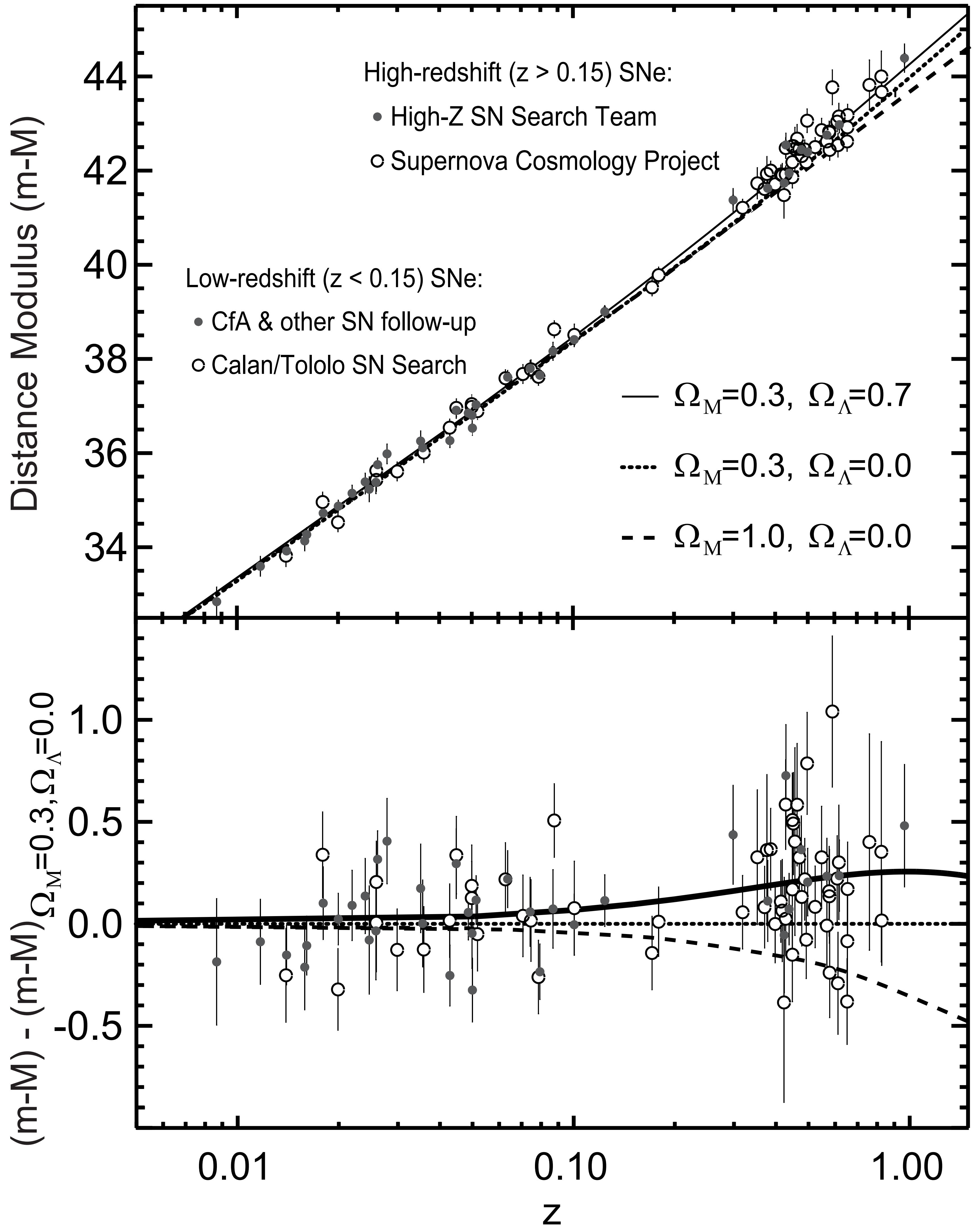
$$R^{\mu}{}_{\nu} - \frac{1}{2} g^{\mu}{}_{\nu} R = 8\pi G T^{\mu}{}_{\nu} + g^{\mu}{}_{\nu} \Lambda = 8\pi G (T^{\mu}{}_{\nu} + T^{\mu}{}_{\nu,w})$$

equation-of-state parameter

$$w \equiv \frac{T^{i}{}_{i,w}}{T^{0}{}_{0,w}} = \frac{-\Lambda}{\Lambda} = -1$$

indirect probe from SN Ia measurements on scale factor $a(t)$, since 1998 →

at cosmological scales > 100 Mpc



accelerating cosmic expansion is discovered, can be explained by cosmological constant. need direct evidence for DE.

Phenomenological Models

Quintessence (Wetterich 1988)
Phantom (Caldwell 2002)
Quintom (Feng, Wang, Zhang 2005)
Holographic (Li 2004)
Agegraphic (Wei, Cai 2008)
oscillating (Turner 1983)
coupled quintessence (Amendola 2000)
K-essense (Chiba, Okabe, Yamaguchi 2000)
cuscuton (Afshordi, Chung, Geshnizjani 2007)
kinetic gravity braiding (Deffayet 2010)
Chaplygin Gas (Kamenshchik, Moschella, Pasquier 2001)
viscous Fluid (Brevik 2002)
ghost condensation (Arkani-Hamed, Cheng, Luty, Mukohyama 2004)

higher spin (2004-2011)
- fermion (Ribas, Devecchi, Kremer 2005) (Cai, Wang 2008)(Yajnik 2011) (Tsyba 2011)
- vector (Armendariz-Picon 2004) (Zhao, Zhang 2006)
- p-form (Koivisto, Nunes 2009) (Das Gupta 2009)

Lorentz-violating (DeDeo 2006)
minimum length (Bohmer Harko 2008)
time-varying neutrino (Takahashi, Tanimoto 2006)
conformal Galilei algebra (Stichel Zakrzewski, 2009)
asymptotically AdS (Das 2009)
creation CDM (Lima, Jesus, Oliveira 2010)
entropic (Verlinde 2017)
anisotropic (Singh, Sharma 2013)

…

DE models offer ρ, p, especially w

Basic Problems of Dark Energy

- state parameter $w \equiv p_w / \rho_w$
- if $w = -1$, what is the value of Λ?
- a long story if $w \neq -1$ ...
  - if $w > -1$, maybe Quintessence; if $w < -1$, maybe Phantom
  - how to distinguish models if $w = w(z)$
  - need to fix other parameters, like $\rho(z)$

Current Experimental Results

Planck 2018

wCDM model $\quad w = w_0$

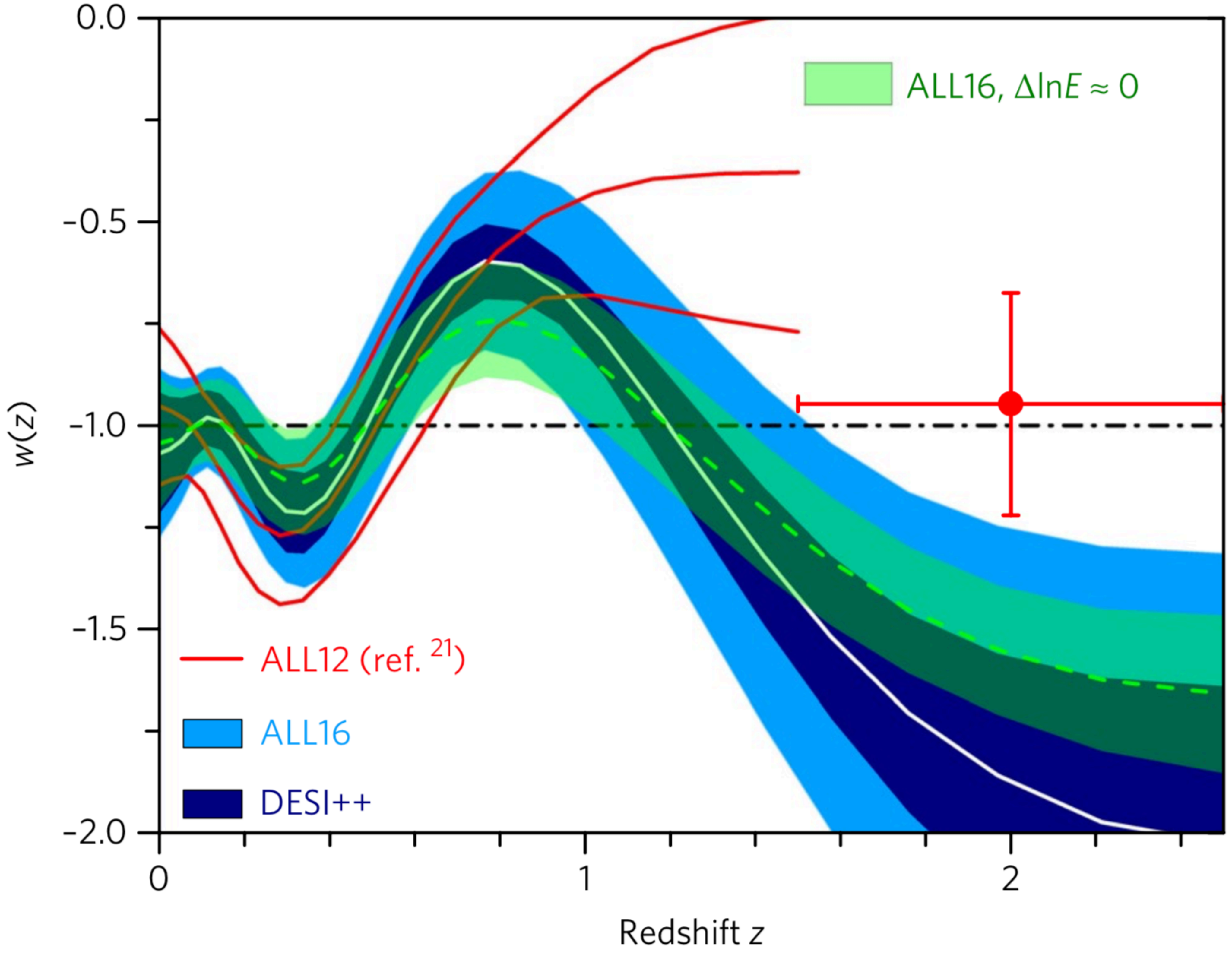
$w_0 = -1.028 \pm 0.031$

$w(z)$CDM model $\quad w = w_0 + (1-a)w_a$

$w_0 = -0.957 \pm 0.080, \quad w_a = -0.29^{+0.32}_{-0.26}$

$a(t)$ **depends on cosmological assumptions and contributions of all components**

Gong-Bo Zhao et al. Nature Astron. 1 (2017) 9, 627-632



dynamical dark energy model is 3.5 sigma preferred

Locally Probe Dark Energy

- direct search for DE, point by point

w **can be constant**

- independent with cosmological assumptions
- higher precision
- distinguish DE models (model independent method)

We propose to directly probe DE on astrophysical scales, such as galaxies or galaxy clusters, around kpc-Mpc scales.

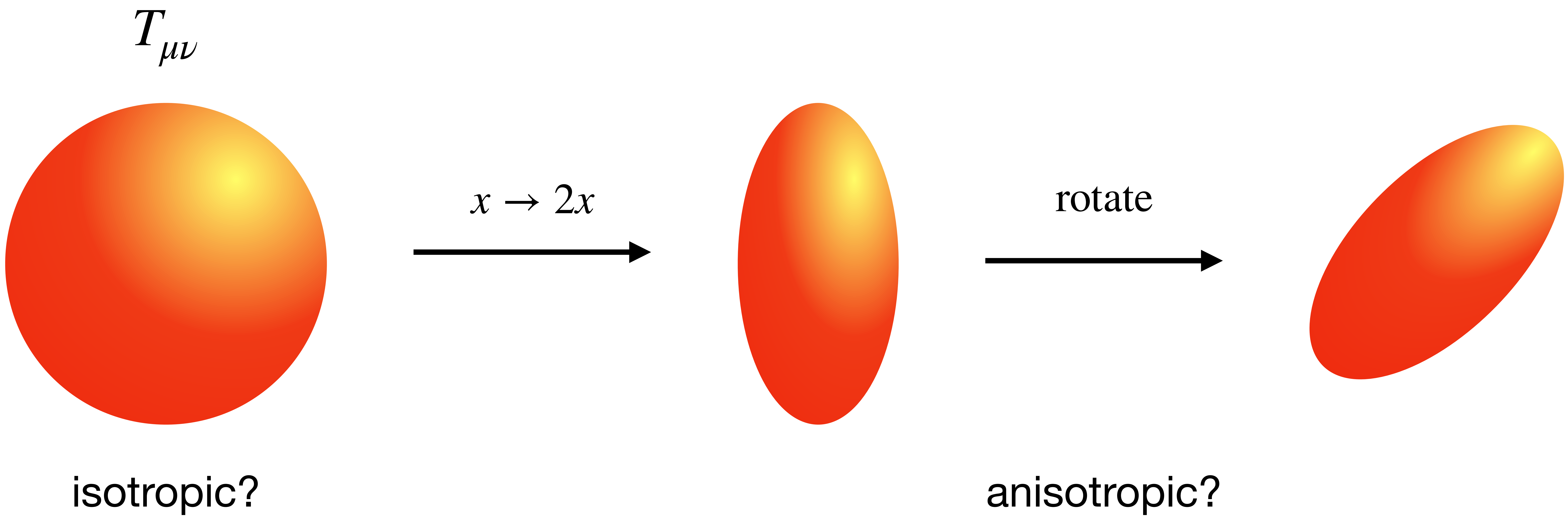
Difficulties on Astrophysical Scales

- Locally define isotropic dark fluid: How to coordinate-independently define isotropic energy-momentum tensor?
- Relate to dark energy model: How to coordinate-independently define energy and pressure?
- Determine dark force: How to deal with realistic astrophysical system?

Coordinate-Independent Energy-Momentum Tensor

Energy momentum tensor $\widetilde{T} = T_{\mu\nu}\, \mathrm{d}x^{\mu} \otimes \mathrm{d}x^{\nu}$ should be coordinate-independent

component $T_{\mu\nu}$ is coordinate-dependent



Static Isotropic Metric

(Steven Weinberg)
Gravitation and cosmology: principles and applications of the general theory of relativity

- quasi-Minkowskian coordinates
  $$x^1 = r \sin\theta \cos\phi\,, \quad x^2 = r\sin\theta\sin\phi\,, \quad x^3 = r\cos\theta$$
- static isotropic invariants
  $$x \cdot x, \quad x \cdot \mathrm{d}x, \quad \mathrm{d}x \cdot \mathrm{d}x$$
- general metric tensor
  $$\mathrm{d}s^2 = \Big(1+2\Phi(r)\Big)\mathrm{d}t^2 - \Big(1+2\Phi(r)\Big)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

Perfect Dark Fluid

general expression for DE in Weinberg's form

$$T^t{}_t = A(r), \quad T^t{}_i = 0, \quad T^i{}_j = B(r)\delta^i{}_j + C(r)x^i x_j\,.$$

$$T^i{}_j \mathrm{d}x_i \mathrm{d}x^j = -B(r)\mathrm{d}x^i\mathrm{d}x^i - C(r)(x^i\mathrm{d}x^i)^2$$

$\mathrm{d}x \cdot \mathrm{d}x$ $\qquad$ $(x \cdot \mathrm{d}x)^2$

Pressure from $T^{\mu}{}_{\nu}$

component $T_{\mu\nu}$ is coordinate-dependent

$$T^{\mu}{}_{\nu} = (\rho, -p, -p, -p) \quad \Rightarrow \quad T^{i}{}_{j} = -p\delta^{i}{}_{j} + 0 r^{i} r_{j}$$

isotropic in Minkowski space

How to relate it to local system?

1. define pressure from $C(r) r^{i} r_{j}$ term
2. must be coordinate-independent
3. compatible with cosmological-scale description

Coordinate-Independent Pressure from $T^{\mu}{}_{\nu}$

The coordinate-independent observables can be only defined by:

$$T^{0}{}_{0} = \rho\,, \quad \sum T^{i}{}_{i} = -3p$$

Definition: Any energy-momentum tensor with $T^{t}{}_{i} = 0$ (comoving observer)

$$\rho_w \equiv T^{t}{}_{t}\,, \quad p_w \equiv -\frac{\mathrm{Tr}\left[T^{i}{}_{j}\right]}{3}$$

Relation to Cosmological Description

Extended Robertson-Walker Metric

$$(\mathrm{d}s)^2 = (\mathrm{d}t)^2 - a^2(t)R^2(x,y,z)\left[(\mathrm{d}r)^2 + r^2(\mathrm{d}\theta)^2 + r^2\sin^2\theta(\mathrm{d}\phi)^2\right]$$

general metric describe inhomogeneous universe

inhomogeneity from primordial quantum fluctuations or catastrophic astrophysical events

Friedmann Equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}T^0{}_0 - \frac{K(x,y,z)}{a^2}$$

$K(x,y,z)$: local Gaussian curvature

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(T^0{}_0 - T^i{}_i) \qquad \Rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p)$$

Conservation Equation

$T^i{}_j$ always in trace

$$0 = \frac{\mathrm{d}T^0{}_0}{\mathrm{d}t} + \frac{3\dot{a}}{a}(T^0{}_0 - \frac{1}{3}T^i{}_i) \qquad \Rightarrow \rho \propto a^{-(3w+3)}$$

Solution of Dark Energy

SdS$_w$ metric: $$\mathrm{d}S_w^2 = \left[1 - 2\frac{M}{r} - 2\left(\frac{r_O}{r}\right)^{3w+1}\right]\mathrm{d}t^2 \frac{1}{\left[1 - 2\frac{M}{r} - 2\left(\frac{r_O}{r}\right)^{3w+1}\right]}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right)$$

$$\Phi(r) = \frac{a}{r} + \frac{b}{r^{3w+1}} \equiv -\frac{M}{r} - \left(\frac{r_O}{r}\right)^{3w+1}$$

- $3w + 1 < 0$ at 22σ on cosmological scales (planck2018)
- $3w + 1 > 0$ cause strong force at $r \rightarrow 0$
- $3w + 1 > 0 \longleftrightarrow 3w + 1 < 0$ strong phase transition

require $w < -1/3$

Realistic Astrophysical Systems

exact result of Einstein equation

$$R^0{}_0 = 8\pi\left(T^0{}_0 - \frac{1}{2}T^\mu{}_\mu\right)$$

$$\partial_r\partial_r\Phi_w + \frac{2}{r}\partial_r\Phi_w = 8\pi\left(\frac{1+3w}{2}\rho_w\right)$$

DE

$$\nabla^2\Phi_w = 4\pi\left(\rho_w + 3p_w\right)$$

$$\Phi_w = -\left(\frac{r_{\rm O}}{r}\right)^{3w+1}$$

astrophysical matters

$$\nabla^2\Phi_{\rm m} = 4\pi\left(\rho_{\rm m} + 3p_{\rm m}\right)$$

$$\Phi_{\rm m} = -\int\frac{\rho_{\rm m}(\overrightarrow{r'})}{|\,\vec{r} - \overrightarrow{r'}\,|}\mathrm{d}\overrightarrow{r'}$$

add matters by post-Newtonian approximation

$$\nabla^2\Phi = 4\pi(\rho_w + 3p_w + \rho_{\rm m} + 3p_{\rm m})$$

$$\Phi = -\int\frac{\rho_{\rm m}(\overrightarrow{r'})}{|\,\vec{r} - \overrightarrow{r'}\,|}\mathrm{d}\overrightarrow{r'} - \left(\frac{r_{\rm O}}{r}\right)^{3w+1}$$ as traditional gravitational potential

Repulsive Local Force

$$\vec{F} = -\vec{\nabla}\Phi = -\vec{\nabla}\Phi_{\rm m} - (3w+1)\frac{1}{r}\left(\frac{r_{\rm O}}{r}\right)^{3w+1}\hat{e}_r$$ **!!!**

weak field condition guaranteed by $|\Phi_w| \lesssim |\Phi_{\rm m}| \sim \frac{M}{r} \sim v^2 \ll 1$

Scale of Dark Force: Critical Radius

$$|3w+1|\frac{1}{r_{\rm cri}}\left(\frac{r_{\rm O}}{r_{\rm cri}}\right)^{3w+1} \approx |\vec{\nabla}\Phi_{\rm m}| \approx \frac{V_0^2}{r_{\rm cri}}$$

$$\frac{r_{\rm cri}}{r_{\rm O}} = \left(\frac{V_0^2}{|3w+1|}\right)^{\frac{1}{|3w+1|}} \approx \frac{500\ {\rm kpc}}{7.71\times 10^6\ {\rm kpc}}$$

power law behavior and precise rotation velocity reduce the scale

Gravitational-Bound System

only valid here

galaxy's center

$r \ll r_{\text{cri}}$

weak-field/post-Newtonian approximation

$r \approx r_{\text{cri}}$

$r = r_{\text{cri}}$

$r \gg r_{\text{cri}}$

conformally flat

$$ds^2 = (1 + 2\Phi)dt^2 - (1 + 2\Phi)^{-1}dr^2 - r^2 d\Omega$$

$$r' = R(r)r$$

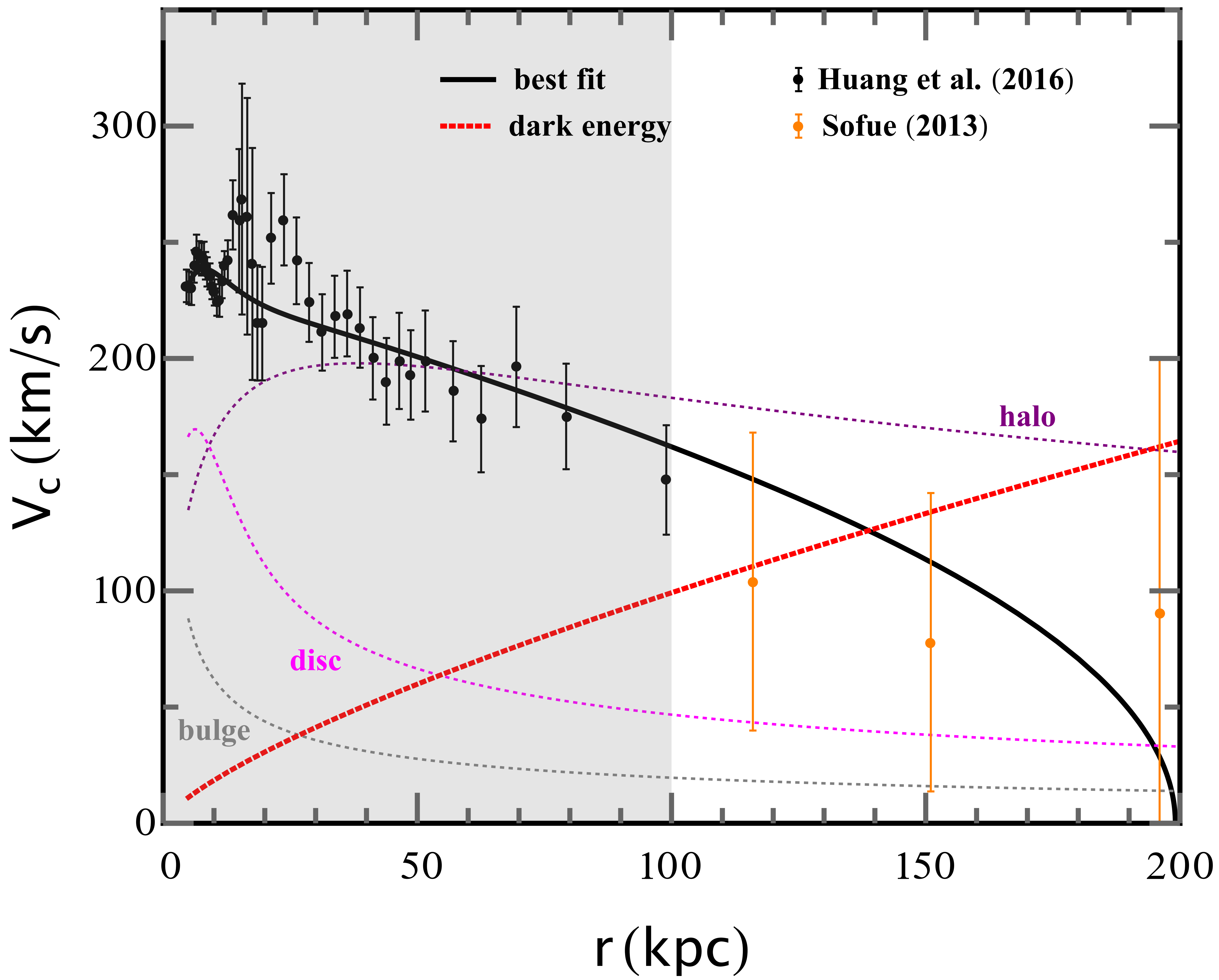
$$ds^2 = R^2(dt'^2 - dr'^2 - r'^2 d\Omega)$$

He and Zhang
JCAP 08 (2017) 036
arxiv: 1701.03418

$r > r_O$

unknown

Results



$$r_O = \sqrt{\frac{6}{\Lambda}} = 7.71 \times 10^6 \text{ kpc}$$

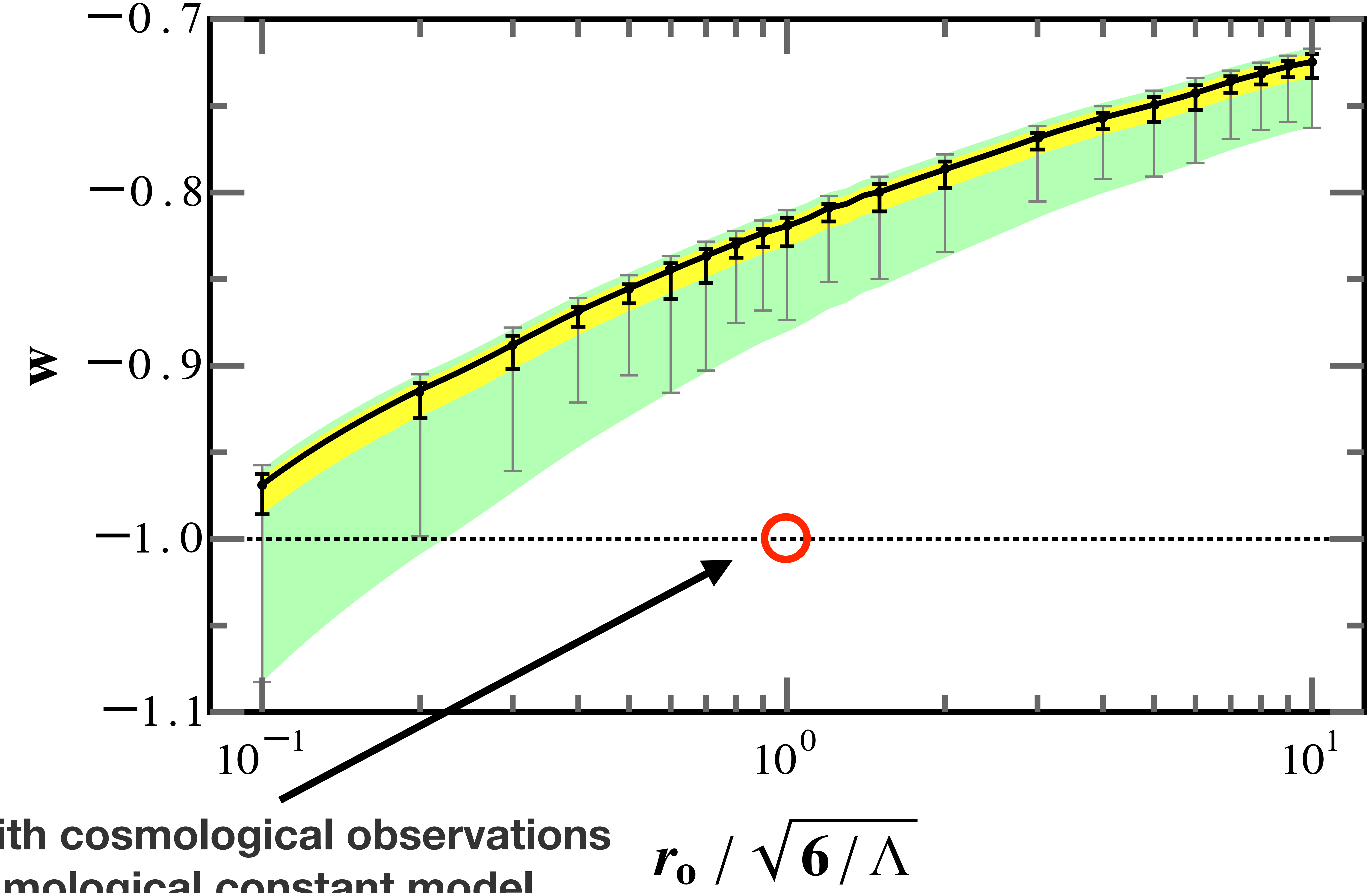
fixed by cosmological measurements

range	r_d(kpc)	$\rho_{h,0}$ $(M_\odot \text{ pc}^{-3})$	r_h(kpc)	w	χ^2_{red}
4.5-200 kpc	$2.9^{+0.2}_{-0.1}$	$0.011^{+0.002}_{-0.003}$	18^{+1}_{-3}	$-0.82^{+0.01}_{-0.01}$	0.85
4.5-100 kpc	$2.8^{+0.1}_{-0.1}$	$0.006^{+0.004}_{-0.002}$	24^{+8}_{-6}	$-0.79^{+0.01}_{-0.02}$	0.85

$w(z=0) = -1.07^{+0.21}_{-0.20}$ **Gong-Bo Zhao et al. (2017)**

$w_\Lambda = -1$ **tension with cosmological constant model**

r_O Dependence



tension with cosmological observations
under cosmological constant model

$$\frac{r_{cri}}{r_O} = \left(\frac{V_0^2}{|3w+1|} \right)^{\frac{1}{|3w+1|}}$$

Let

$V_0 = 200$ km/s

$-1 < w < -0.8$

$260 \text{ kpc} < r_{cri} < 1 \text{ Mpc}$

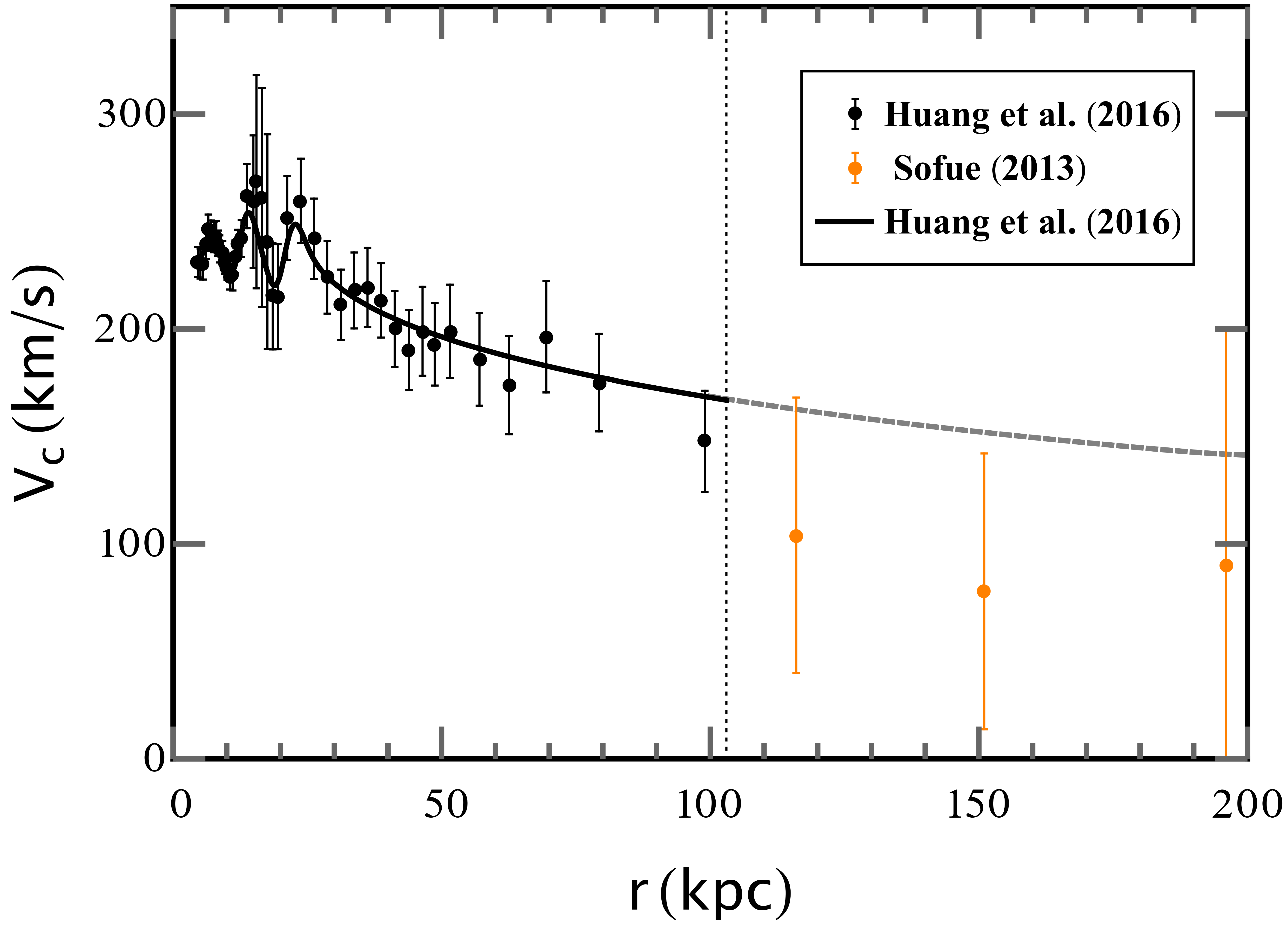
$r_O \sim (0.1 - 10)\sqrt{6/\Lambda}$

Conclusion

- We introduce concept of perfect dark fluids in curved spacetime, and associated with dark energy models.
- We offer the basic theory for dark energy local effects: dark force.
- Dark force can be probed on the scale of a galaxy.
- Results consistent with dynamical dark energy model, not cosmological constant model.

Backup

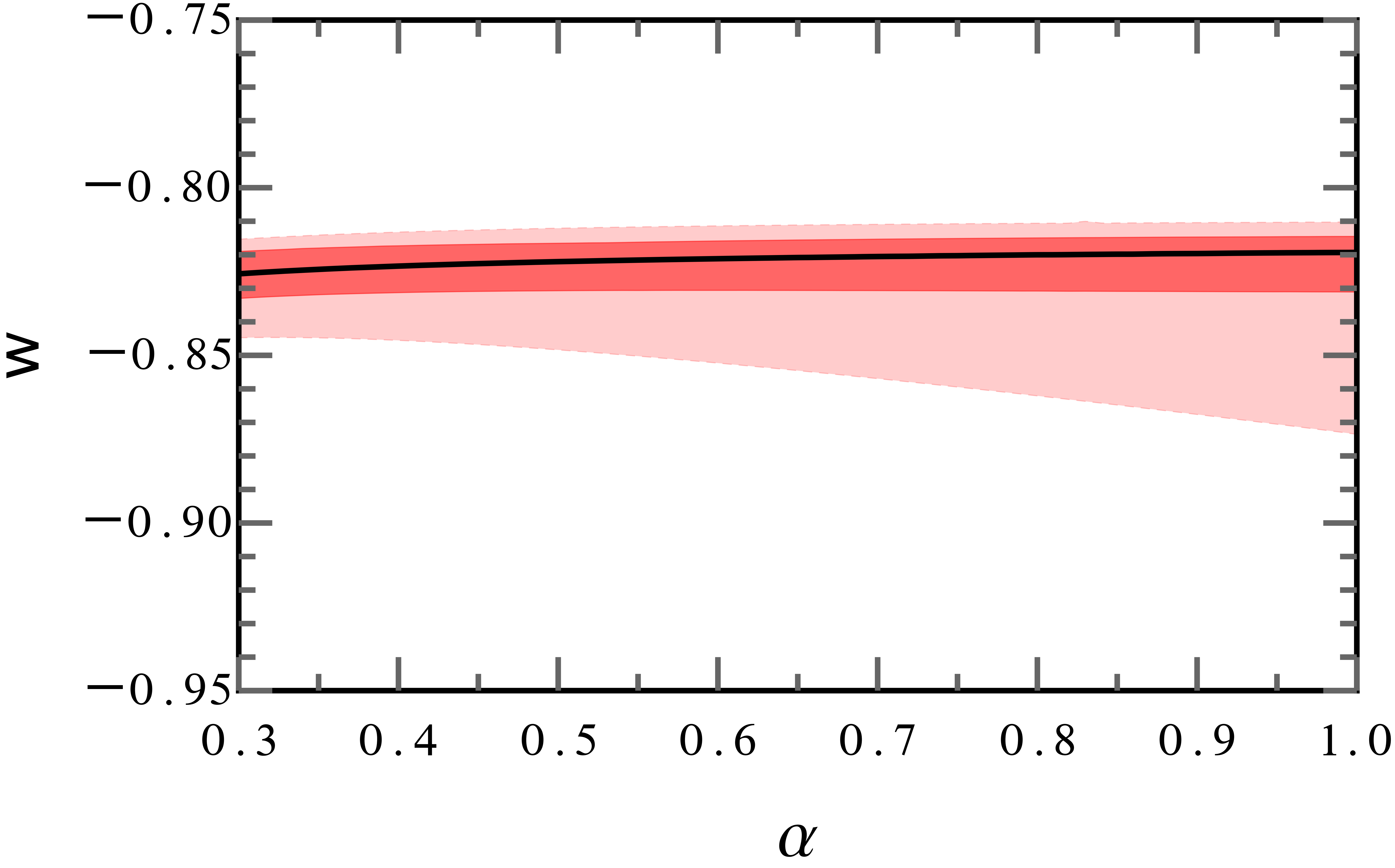
Anomalous Drop



$$\overrightarrow{F}_w = \frac{V_0^2}{r}\left(\frac{r_{cri}}{r}\right)^{3w+1}\hat{e}_r$$

$$\frac{|\,\Delta_w V^2(r)\,|}{V_0^2} = \left(\frac{r}{r_{cri}}\right)^{-3w-1}$$

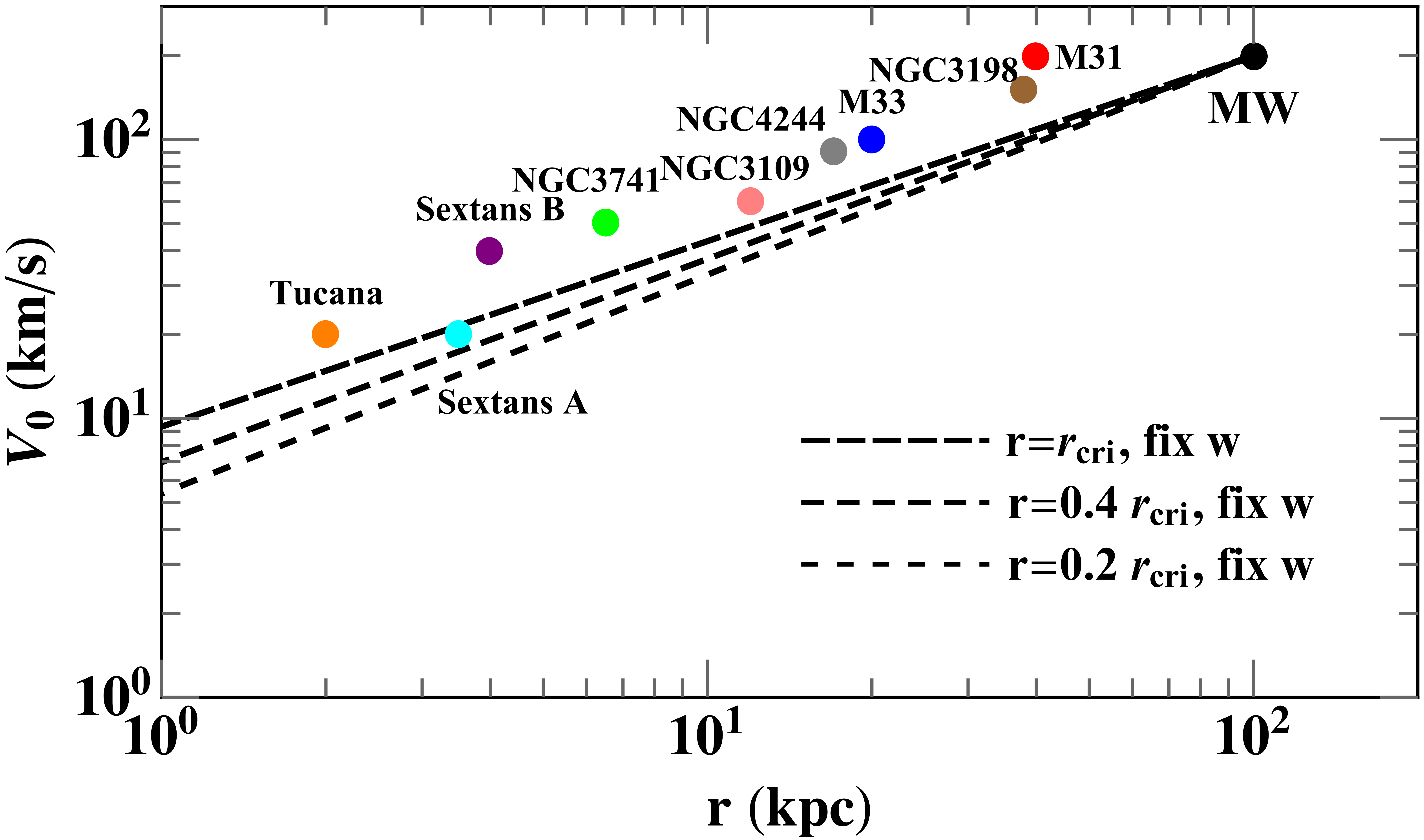
Expect Higher-Precision Data



$\Delta v_{\rm th}(r) = \alpha \times \Delta v_{\rm exp}(r)$

for 100-200 kpc data

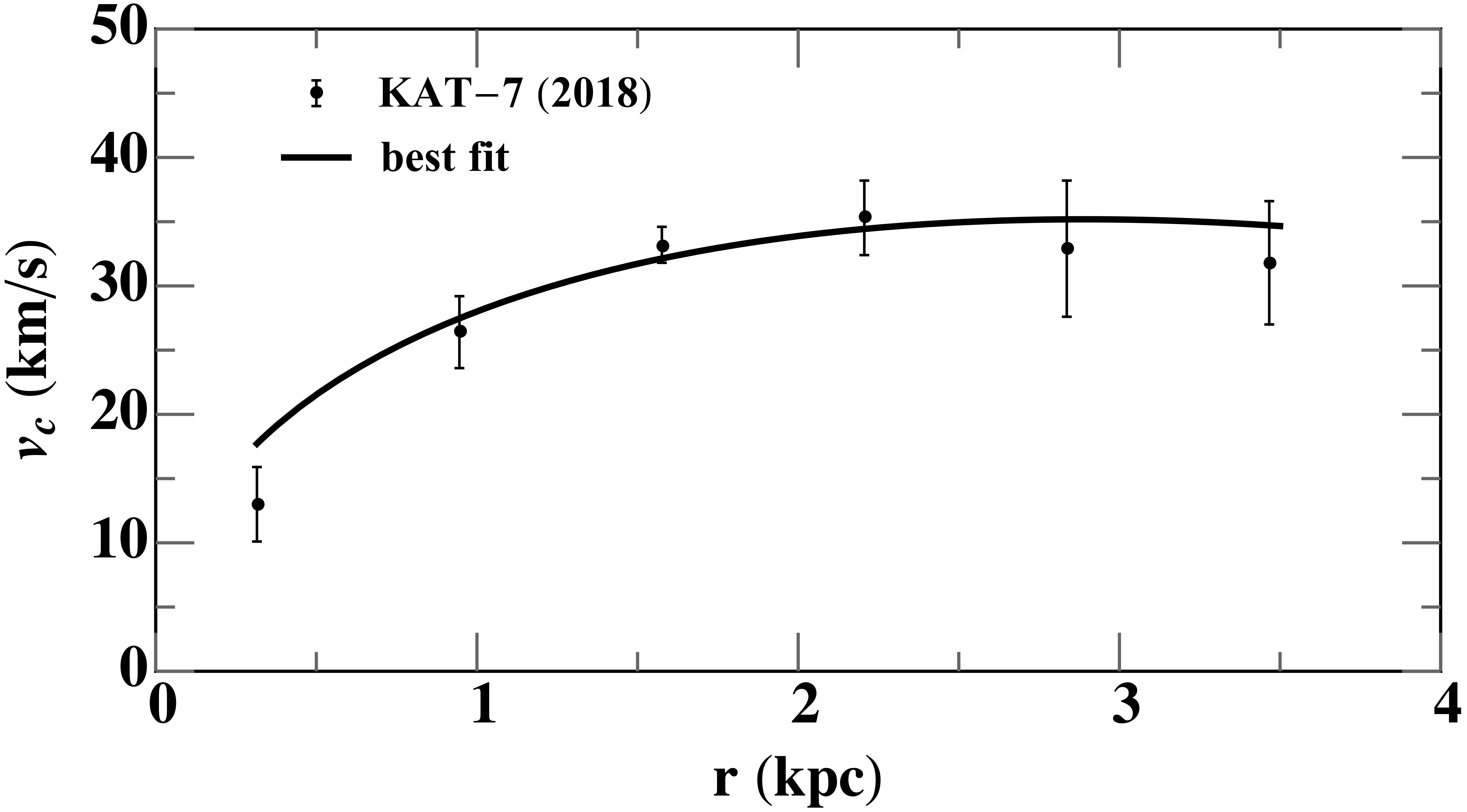
Expect More Data



$$V_0^2 = |3w+1|(\frac{r_O}{r_{cri}})^{3w+1}$$

$$\frac{|\Delta_w V^2(r)|}{V_0^2} = \left(\frac{r}{r_{cri}}\right)^{-3w-1}$$

Rotation Velocity of Sextans A



data from
B. Namumba et cl.
MNRAS 478 (2018) 1, 487-500
arxiv: 1804.07730

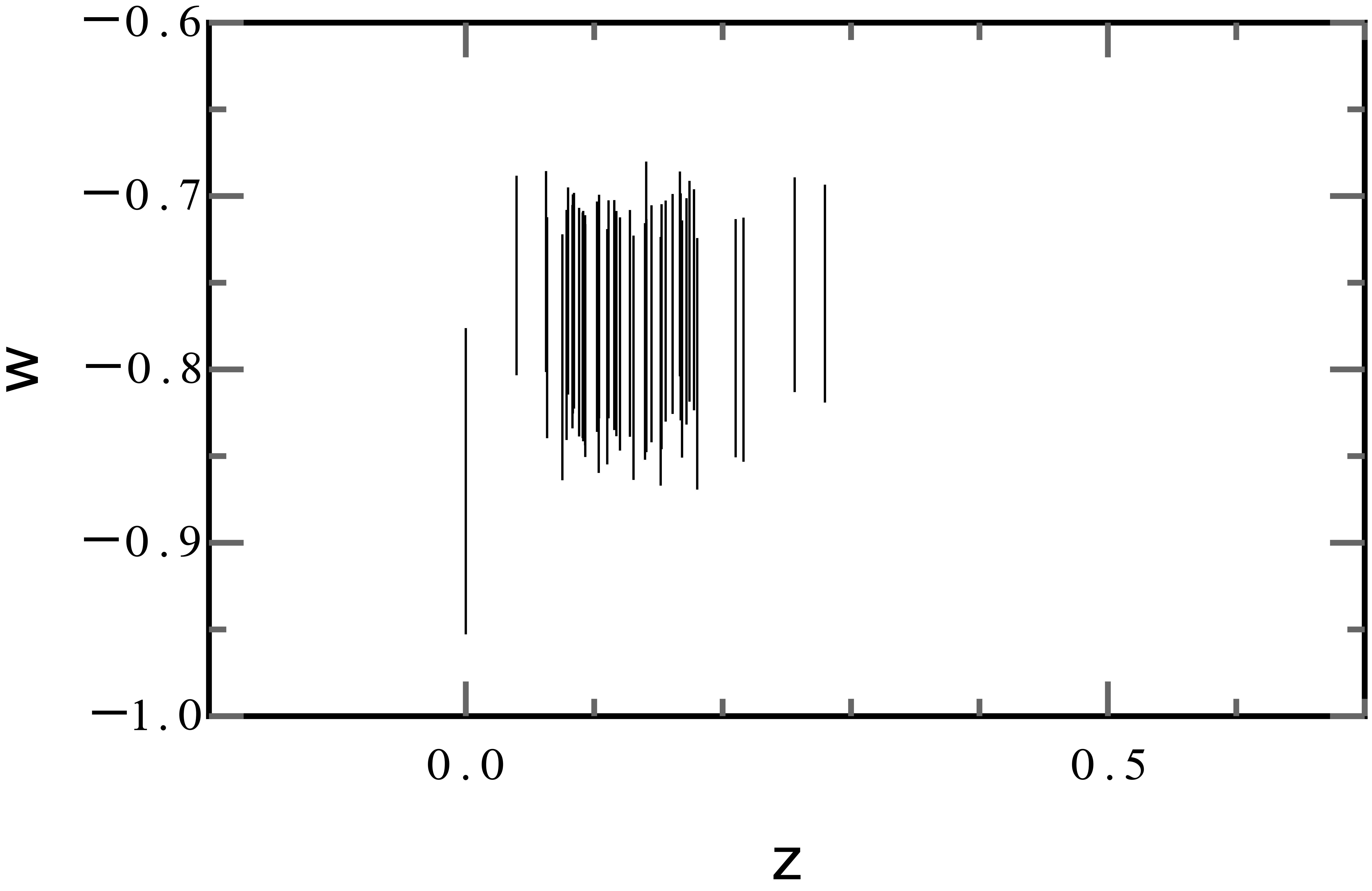
$\rho_{h,0} = 0.005\ M_\odot\,\mathrm{pc}^{-3}$

$r_h = 9.9$ kpc

$w = -0.74$

3.5 kpc still sensitive to DE

Expect More Data



data from E. Teodoro et cl.
MNRAS 507 (2021) 4, 5820-5831
arxiv: 2109.03828

$$V_0^2 = |3w+1|(\frac{r_O}{r_{cri}})^{3w+1}$$

$$1 < r_{cri}/r < 20$$

Scale of Dark Energy

$$(\mathrm{d}s)^2 = \left(1 + 2\Phi(r)\right)(\mathrm{d}t)^2 - \left(1 + 2\Phi(r)\right)^{-1}(\mathrm{d}r)^2 - r^2(\mathrm{d}\theta)^2 - r^2\sin^2\theta(\mathrm{d}\phi)^2$$

$$\Phi = \frac{a}{r} + \frac{b}{r^{3w+1}} \equiv -\frac{M}{r} - \left(\frac{r_\mathrm{O}}{r}\right)^{3w+1}$$

$$\Phi_w\Big|_\Lambda = -\frac{1}{6}\Lambda r^2 \equiv -\left(\frac{r_\mathrm{O}}{r}\right)^{-2}, \; r_\mathrm{O} = 7.71 \times 10^6 \text{ kpc}$$

Benchmark

fix $r_\mathrm{O} = 7.71 \times 10^6$ kpc !

Static Isotropic Metric

(Steven Weinberg)
Gravitation and cosmology: principles and applications of the general theory of relativity

$$\mathrm{d}s^2 = F(r)\mathrm{d}t^2 - 2rE(r)\mathrm{d}t\,\mathbf{x}\cdot\mathrm{d}\mathbf{x} - r^2D(r)(\mathbf{x}\cdot\mathrm{d}\mathbf{x})^2 - C(r)\mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x}$$

$$\mathbf{x}\cdot\mathrm{d}\mathbf{x} = r\mathrm{d}r \qquad \mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x} = \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2$$

step 1: $t \to t + \Phi(r), \quad \frac{\mathrm{d}\Phi}{\mathrm{d}r} = -\frac{rE(r)}{F(r)}$

$$\mathrm{d}s^2 = F(r)\mathrm{d}t^2 - r^2\left(D(r) + \frac{E^2(r)}{F(r}\right)\mathrm{d}r^2 - C(r)(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2)$$

step 2: $r^2 \to C(r)r^2$

$$\mathrm{d}s^2 = \beta(r)\mathrm{d}t^2 - \alpha(r)\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

step 3: Newtonian gauge $\alpha(r)\cdot\beta(r) = 1$

$$\mathrm{d}s^2 = (1+2\Phi(r))\mathrm{d}t^2 - (1+2\Phi(r))^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

Proof for $T^t{}_i = 0$

$$g_{ii} = -a^2(t)\alpha^2(x,y,z)\,, \quad g^{ii} = -a^{-2}\alpha^{-2}\,, \quad g_{00} = \beta^2(x,y,z)\,, \quad g^{00} = \beta^{-2}\,, \quad g_{ij} = g^{ij} = 0 (i \neq j)\,.$$

$$\Gamma^0{}_{ii} = -\frac{1}{2}g^{00}\partial_0 g_{ii}\,, \quad \Gamma^i{}_{i0} = \Gamma^i{}_{0i} = \frac{1}{2}g^{ii}\partial_0 g_{ii}\,, \quad \Gamma^i{}_{ii} = \frac{1}{2}g^{ii}\partial_i g_{ii}\,, \quad \Gamma^i{}_{jj} = -\frac{1}{2}g^{ii}\partial_i g_{jj}\,, \quad \Gamma^i{}_{ij} = \Gamma^i{}_{ji} = \frac{1}{2}g^{ii}\partial_j g_{ii}\,.$$

$$R_{0i} = \Gamma^\nu{}_{0i,\nu} - \Gamma^\nu{}_{\nu i,0} + \Gamma^\lambda{}_{0i}\Gamma^\nu{}_{\lambda\nu} - \Gamma^\lambda{}_{\nu i}\Gamma^\nu{}_{\lambda 0}$$

$$= \Gamma^0{}_{0i,0} + \Gamma^i{}_{0i,i} - \Gamma^0{}_{0i,0} - \Gamma^j{}_{ji,0} + \Gamma^0{}_{0i}\Gamma^j{}_{0j} + \Gamma^i{}_{0i}\Gamma^0{}_{i0} + \Gamma^i{}_{0i}\Gamma^j{}_{ij} - \Gamma^j{}_{ji}\Gamma^j{}_{j0} - \Gamma^0{}_{ii}\Gamma^i{}_{00} - \Gamma^i{}_{0i}\Gamma^0{}_{i0}$$

$$= \Gamma^i{}_{0i,i} - \Gamma^j{}_{ji,0} + \Gamma^0{}_{0i}\Gamma^j{}_{0j} + \Gamma^i{}_{0i}\Gamma^j{}_{ij} - \Gamma^j{}_{ji}\Gamma^j{}_{j0} - \Gamma^0{}_{ii}\Gamma^i{}_{00}$$

$$= \partial_i[\frac{1}{2}g^{ii}\partial_0 g_{ii}] - \partial_0[\frac{1}{2}g^{jj}\partial_i g_{jj}] + \frac{1}{2}g^{00}\partial_i g_{00}\frac{1}{2}g^{jj}\partial_0 g_{jj} + \frac{1}{2}g^{ii}\partial_0 g_{ii}\frac{1}{2}g^{jj}\partial_i g_{jj} - \frac{1}{2}g^{jj}\partial_i g_{jj}\frac{1}{2}g^{jj}\partial_0 g_{jj} - \frac{1}{2}g^{00}\partial_0 g_{ii}\frac{1}{2}g^{ii}\partial_i g_{00}$$

$$= \frac{1}{2}\partial_i[g^{ii}\partial_0 g_{ii}] - \frac{3}{2}\partial_0[g^{ii}\partial_i g_{ii}] + \frac{3}{4}g^{00}\partial_i g_{00}g^{ii}\partial_0 g_{ii} + \frac{3}{4}g^{ii}\partial_0 g_{ii}g^{ii}\partial_i g_{ii} - \frac{3}{4}g^{ii}\partial_i g_{ii}g^{ii}\partial_0 g_{ii} - \frac{1}{4}g^{ii}\partial_0 g_{ii}g^{00}\partial_i g_{00}$$

$$= \frac{1}{2}g^{ii}\partial_0\partial_i g_{ii} - \frac{1}{2}(g^{ii}\partial_i g_{ii})(g^{ii}\partial_0 g_{ii}) - \frac{3}{2}g^{ii}\partial_0\partial_i g_{ii} + \frac{3}{2}(g^{ii}\partial_0 g_{ii})(g^{ii}\partial_i g_{ii}) + \frac{3}{4}(g^{00}\partial_i g_{00})(g^{ii}\partial_0 g_{ii}) - \frac{1}{4}(g^{ii}\partial_0 g_{ii})(g^{00}\partial_i g_{00})$$

$$= -g^{ii}\partial_0\partial_i g_{ii} + (g^{ii}\partial_i g_{ii})(g^{ii}\partial_0 g_{ii}) + \frac{1}{2}(g^{00}\partial_i g_{00})(g^{ii}\partial_0 g_{ii})$$

$$= -4\frac{\dot{a}}{a}\frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}x^i} + 4\frac{\dot{a}}{a}\frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}x^i} + 2\frac{\dot{a}}{a}\frac{1}{\beta}\frac{\mathrm{d}\beta}{\mathrm{d}x^i} = 2\frac{\dot{a}}{a}\frac{1}{\beta}\frac{\mathrm{d}\beta}{\mathrm{d}x^i}$$

Apply to Dark Energy

a little difference when considering g_{rr}

$$(\mathrm{d}s)^2 = g_{tt}(\mathrm{d}t)^2 - g_{rr}(\mathrm{d}r)^2 - r^2\left[(\mathrm{d}\theta)^2 + \sin^2\theta(\mathrm{d}\phi)^2\right] \quad \Leftrightarrow \quad (\mathrm{d}s)^2 = g_{tt}(\mathrm{d}t)^2 - \left[\left(g_{rr} - 1\right)\frac{x^i\,x^j}{r^2} + \delta^{ij}\right]\mathrm{d}x_i\mathrm{d}x_j$$

$$\begin{aligned}\widetilde{T} = {} & g_{tt}(r)\,A(r)\,\mathrm{d}t \otimes \mathrm{d}t \\ & - \left[\left(g_{rr}(r) - 1\right)B(r) + g_{rr}(r)\,C(r)\,r^2\right]\mathrm{d}r \otimes \mathrm{d}r \\ & - B(r)\left(\mathrm{d}r \otimes \mathrm{d}r + r^2\mathrm{d}\theta \otimes \mathrm{d}\theta + r^2\sin^2\theta\,\mathrm{d}\varphi \otimes \mathrm{d}\varphi\right)\end{aligned}$$

Solution of Dark Energy

$$(\mathrm{d}s)^2 = g_{tt}(\mathrm{d}t)^2 - g_{rr}(\mathrm{d}r)^2 - r^2\left[(\mathrm{d}\theta)^2 + \sin^2\theta(\mathrm{d}\phi)^2\right]$$

$$\begin{aligned}\widetilde{T} = {} & g_{tt}(r)\,A(r)\,\mathrm{d}t \otimes \mathrm{d}t \\ & - \left[\Big(g_{rr}(r) - 1\Big)B(r) + g_{rr}(r)\,C(r)\,r^2\right]\mathrm{d}r \otimes \mathrm{d}r \\ & - B(r)\left(\mathrm{d}r \otimes \mathrm{d}r + r^2\mathrm{d}\theta \otimes \mathrm{d}\theta + r^2\sin^2\theta\,\mathrm{d}\varphi \otimes \mathrm{d}\varphi\right)\end{aligned}$$

$$T^t{}_t = \rho, \quad T^t{}_i = 0, \quad T^i{}_j = 3p\left[B\delta^i_j - (1+3B)\frac{r^i r_j}{r^2}\right], \quad B = -\frac{1+3w}{6w}\,.$$

Relation to Cosmological Description

Extended Robertson-Walker Metric

$$(\mathrm{d}s)^2 = (\mathrm{d}t)^2 - a^2(t)R^2(x,y,z)\left[(\mathrm{d}r)^2 + r^2(\mathrm{d}\theta)^2 + r^2\sin^2\theta(\mathrm{d}\phi)^2\right]$$

general metric describe inhomogeneous universe

inhomogeneity from primordial quantum fluctuations or catastrophic astrophysical events

Friedmann Equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}T^0{}_0 - \frac{K(x,y,z)}{a^2} \qquad K(x,y,z) = \frac{1}{3}\left[-\frac{2}{R^2}\frac{\partial_i^2 R}{R} + \frac{1}{R^2}\left(\frac{\partial_i R}{R}\right)^2\right]$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(T^0{}_0 - T^i{}_i) \qquad \Rightarrow \frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p)$$

Conservation Equation

$T^i{}_i$ always in sum

$$0 = \frac{\mathrm{d}T^0{}_0}{\mathrm{d}t} + \frac{3\dot{a}}{a}(T^0{}_0 - \frac{1}{3}T^i{}_i) \qquad \Rightarrow \rho \propto a^{-(3w+3)}$$

Sectional Curvature I

Metric and curvature tensor

$$\mathrm{d}\sigma^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j \equiv R^2(x,y,z)\left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$

$$^{(3)}R_{ijkm} = \gamma_{is}\,^{(3)}R^s_{jkm} = \gamma_{is}\left[^{(3)}\Gamma^s{}_{jm,k} - {}^{(3)}\Gamma^s{}_{jk,m} + {}^{(3)}\Gamma^p{}_{jm}{}^{(3)}\Gamma^s{}_{pk} - {}^{(3)}\Gamma^p{}_{jk}{}^{(3)}\Gamma^s{}_{pm}\right]$$

Gaussian curvature

$$K_p = K_p[i,j,k,m] = -\frac{^{(3)}R_{ijkl}}{\gamma_{ik}\gamma_{jm} - \gamma_{im}\gamma_{jk}}$$

$$K_p^i = K_p[j,k,j,k] = -\frac{1}{R^2}\left[\left(\frac{\partial_j R}{R}\right)^2 - \frac{\partial_j^2 R}{R} + \left(\frac{\partial_k R}{R}\right)^2 - \frac{\partial_k^2 R}{R} - \left(\frac{\partial_i R}{R}\right)^2\right]$$

$$K\left(x,y,z\right) \equiv \frac{\sum K_p^i}{3} = \frac{1}{3}\left[-\frac{2}{R^2}\frac{\partial_i^2 R}{R} + \frac{1}{R^2}\left(\frac{\partial_i R}{R}\right)^2\right]$$

Sectional Curvature II

$$\mathrm{d}\sigma^2 = -\frac{\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2}{\left[1 + \frac{1}{4}\kappa\left(x^2 + y^2 + z^2\right)\right]^2}$$

$$R(x, y, z) = \frac{1}{1 + \frac{1}{4}\kappa\left(x^2 + y^2 + z^2\right)}$$

$$K\left(x, y, z\right) = \frac{1}{3}\left[-\frac{2}{R^2}\frac{\partial_i^2 R}{R} + \frac{1}{R^2}\left(\frac{\partial_i R}{R}\right)^2\right] = \kappa$$

Energy-Momentum Tensor for DE

$$\mathrm{d}s^2 = \beta(r)\mathrm{d}t^2 - \alpha(r)\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2)$$

$T^t{}_t = T^r{}_r$ protected by $T^t{}_t - T^r{}_r = -\dfrac{1}{r\alpha}\left(\dfrac{\alpha'}{\alpha} + \dfrac{\beta'}{\beta}\right)$

$$T^t{}_t = \rho, \quad T^t{}_i = 0, \quad T^i{}_j = 3w\rho\left[B\delta^i_j - (1+3B)\frac{r^i r_j}{r^2}\right], \quad B = -\frac{1+3w}{6w}.$$

We need to solve the big problem for $w \neq -1$!

Solution to Einstein Equation

$$\mathrm{d}s^2 = \Big(1 + 2\Phi(r)\Big)\mathrm{d}t^2 - \Big(1 + 2\Phi(r)\Big)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)$$

$$8\pi T^t{}_t = 8\pi T^r{}_r = -\frac{2\Phi + 2r\partial_r\Phi}{r^2} = \rho$$

$$8\pi T^\theta{}_\theta = 8\pi T^\varphi{}_\varphi = -\partial_r\partial_r\Phi - \frac{2}{r}\partial_r\Phi = -\frac{3w+1}{2}\rho$$

$$\Phi = \frac{a}{r} + \frac{b}{r^{3w+1}} \equiv -\frac{M}{r} - \left(\frac{r_\mathrm{O}}{r}\right)^{3w+1}$$

Proof for b>0

$$\Phi = \frac{a}{r} + \frac{b}{r^{3w+1}}$$

$$8\pi T^t{}_t = 8\pi T^r{}_r = -\frac{2\Phi + 2r\partial_r\Phi}{r^2} = 8\pi\rho$$

$$8\pi\rho = \frac{6wb}{r^{3w+3}} > 0$$

$$\Phi = -\frac{M}{r} \pm \left(\frac{r_O}{r}\right)^{3w+1} \quad + \text{ for } w > 0, \quad - \text{ for } w < 0$$

Proof for Weak Field Condition

$r_{\rm L} = 200$ kpc, $\quad {\rm r_S} = 100$ kpc, $\quad {\rm v(r_S)} = 200$ km/s

$$\Delta_w v^2(r) \equiv -|3w+1|(\frac{r_0}{r})^{3w+1}$$

$$\frac{M(r_{\rm L})}{r_{\rm L}} > |\Delta_w v^2(r_{\rm L})|$$

$$v^2(r_{\rm S}) = \frac{M(r_{\rm S})}{r_{\rm S}} - |\Delta_w v^2(r_{\rm S})| > \frac{M(r_{\rm S})}{r_{\rm S}} - \frac{M(r_{\rm L})}{r_{\rm L}}(\frac{r_{\rm L}}{r_{\rm S}})^{3w+1} \approx \frac{M(r_{\rm S})}{r_{\rm S}}\left[1-(\frac{r_{\rm L}}{r_{\rm S}})^{3w+1}\right]$$

$$|\Phi_w(r_{\rm L})| = (\frac{r_{\rm O}}{r_{\rm L}})^{3w+1} \lesssim \frac{v^2(r_{\rm S})}{1-(\frac{r_{\rm L}}{r_{\rm S}})^{3w+1}} < 10^{-6} \text{ for } w < -0.6$$

Cosmic Expansion vs Dark Force I

Cosmological scales

cosmic scale factor $a(t)$

$$(\mathrm{d}s)^2 = (\mathrm{d}t)^2 - a^2(t)\left[\frac{(\mathrm{d}r)^2}{1-kr^2} + r^2(\mathrm{d}\theta)^2 + r^2\sin^2\theta(\mathrm{d}\phi)^2\right]$$

Astrophysical scales

local dark force: $-\vec{\nabla}\Phi$

$$(\mathrm{d}s)^2 = (1+2\Phi)(\mathrm{d}t)^2 - (1+2\Phi)^{-1}(\mathrm{d}r)^2 - r^2(\mathrm{d}\theta)^2 - r^2\sin^2\theta(\mathrm{d}\phi)^2$$

$$\Phi = -\int\frac{\rho_\mathrm{m}(\vec{r'})}{|\vec{r}-\vec{r'}|}\mathrm{d}\vec{r'} - \left(\frac{r_\mathrm{o}}{r}\right)^{3w+1}$$

$$\nabla^2\Phi = 4\pi(\rho_\mathrm{m} + 3p_\mathrm{m} + \rho_w + 3p_w)$$

Cosmic Expansion vs Dark Force II

Cosmic Expansion

$$\nabla^2\Phi_w = -3\frac{\ddot{a}}{a} \qquad \Phi_w = -\frac{\ddot{a}}{a}r^2$$

Balaguera-Antolinez et al.
CQG 24 (2007) 2677-2688
arxiv: 0704.1871

$z \gtrsim 0.1 \quad r \gtrsim 400$ Mpc

Dark Force

$$\nabla^2\Phi_w = 4\pi(\rho_w + 3p_w) \qquad \Phi_w = -\left(\frac{r_o}{r}\right)^{3w+1}$$

He and Zhang
JCAP 08 (2017) 036
arxiv: 1701.03418

$r \approx 1 - 20$ Mpc

This work
arxiv: 2303.14047

$r \approx 100 - 200$ kpc

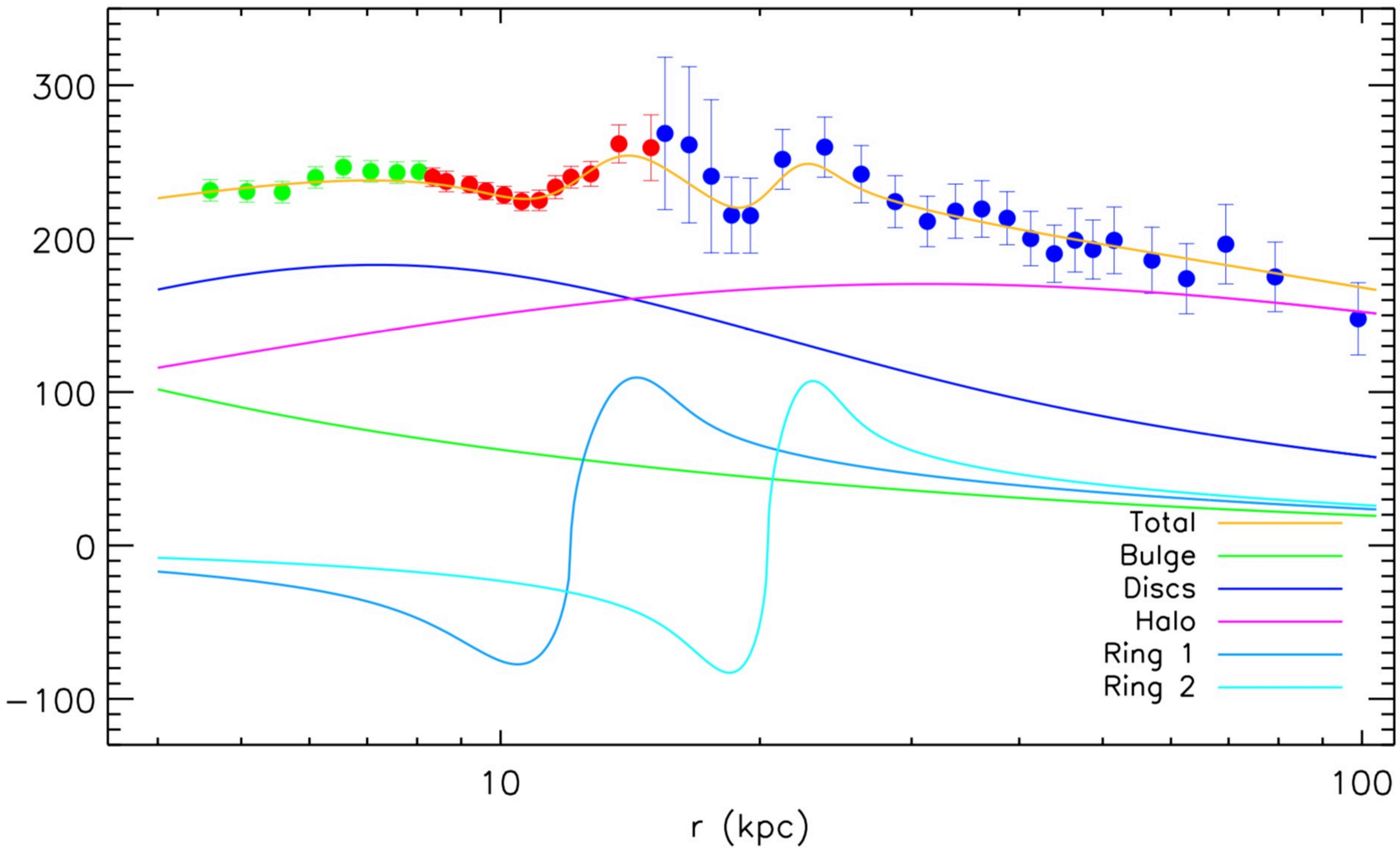
Cosmological constant model

$$\nabla^2\Phi_w = -\Lambda \qquad \Phi_w = -\frac{1}{6}\Lambda r^2$$

Ho and Hsu
Astropart.Phys. 74 (2016) 47-50
arxiv: 1501.05952

$r \sim 500$ kpc

Data Sample I

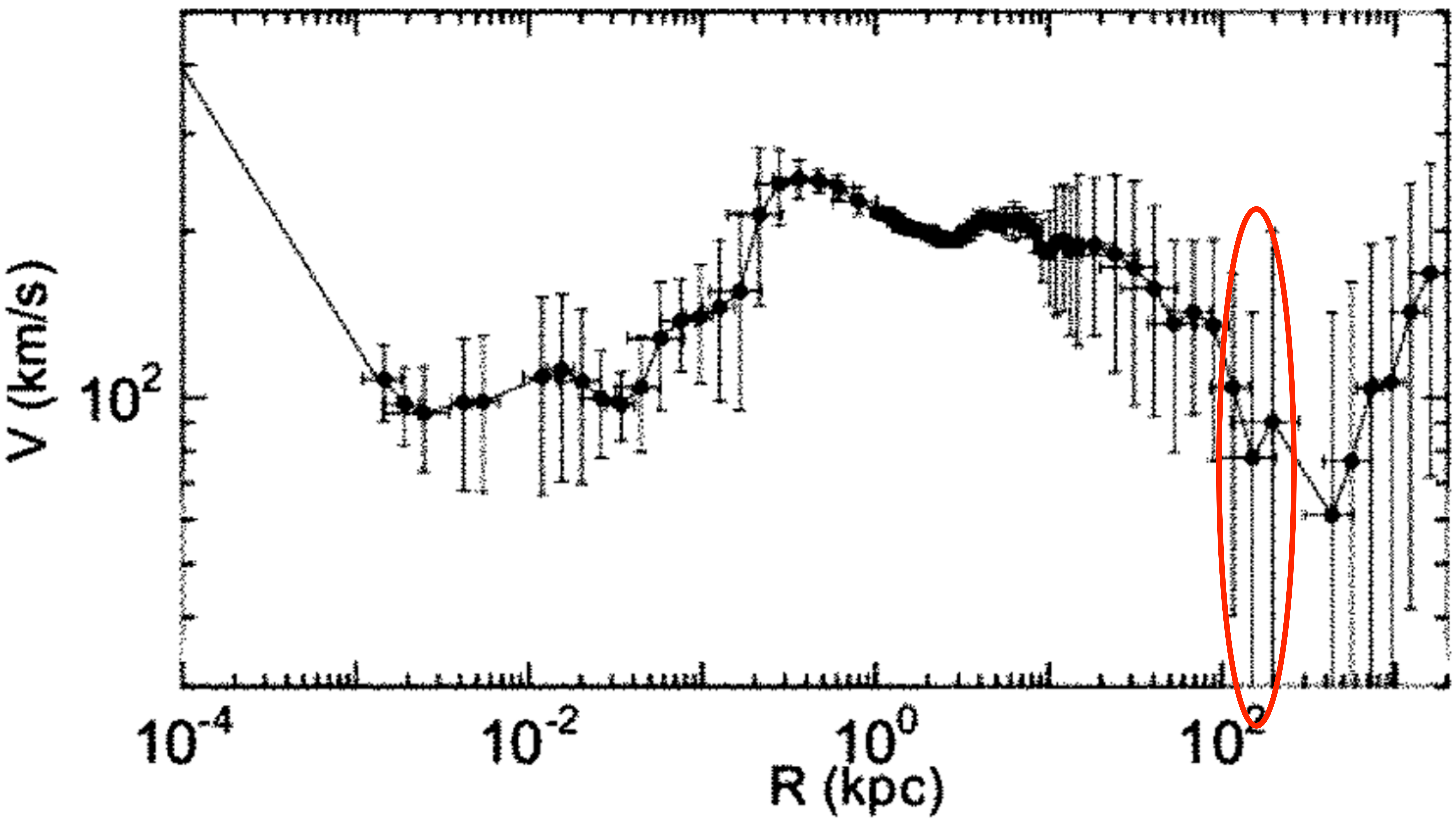


Yang Huang et al.
MNRAS 463 (2016) 2623
arxiv: 1604.01216

LAMOST 4.5-100 kpc

$r_{200} = 255.69 \pm 7.67$ kpc

Data Sample II



Yoshiaki Sofue
Publ.Astron.Soc.Jap. 65 (2013) 118
arxiv: 1307.8241

3 points 100-200 kpc

Scale of Dark Force I

$$\vec{F} = -\vec{\nabla}\Phi = -\frac{1}{r^2}\left[M + (3w+1)\frac{r_0^{3w+1}}{r^{3w}}\right]\hat{e}_r$$

$$M_{\rm cri} = |\,3w+1\,|\,\frac{r_0^{3w+1}}{r_{\rm cri}^{3w}} \qquad r_{\rm cri} = r_{\rm O}\left(\,|\,3w+1\,|\,\frac{r_{\rm O}}{M_{\rm cri}}\right)^{\frac{1}{3w}}$$

For MW and cosmological constant model, $r_{\rm cri}\Big|_{\Lambda} = \left(\frac{3GM_{\rm cri}(r_{\rm cri})}{\Lambda}\right)^{\frac{1}{3}} = 500\ {\rm kpc}$

Scale of Dark Force II

critical radius

$$|3w+1|\left(\frac{r_{\rm O}}{r'_{\rm cri}}\right)^{3w+1} \approx \frac{M(r'_{\rm cri})}{r'_{\rm cri}} \approx V_0^2$$

$w=-1$ $\quad r'_{\rm cri} = 3.6\times10^3\ {\rm kpc}$

$w=-0.8$ $\quad r'_{\rm cri} = 176\ {\rm kpc}$

$$r'_{\rm cri} > 200\ {\rm kpc} \Rightarrow w < -0.808$$

Dark Force Scale: Summary

$$\Phi = -\int \frac{\rho_{\mathrm{m}}(\overrightarrow{r'})}{|\vec{r} - \overrightarrow{r'}|} \mathrm{d}\overrightarrow{r'} - \left(\frac{r_0}{r}\right)^{3w+1}$$

- scale of dark energy $r_O = 7.71 \times 10^6$ kpc
- critical radius $r_{\mathrm{cri}} \approx 500$ kpc
- $r_{200} = 255.69 \pm 7.67$ kpc

Rotation Curve Sensitivity I

$$\vec{F} = -\vec{\nabla}\Phi = -\frac{1}{r^2}\left[M + (3w+1)\frac{r_0^{3w+1}}{r^{3w}}\right]\hat{e}_r$$

$$r = 0.4 r_{\rm cri}$$

$$\frac{|\Delta_w V^2(r)|}{V_{\rm N}^2} \equiv |3w+1|\frac{\frac{r_0^{3w+1}}{r^{3w}}}{M} = |3w+1|\left(\frac{r}{r_{\rm cri}}\right)^{-3w} = 12.8\,\%$$

$$w = -1$$

$$\frac{|\Delta_w V^2(r)|}{V_{\rm N}^2} = |3w+1|\frac{\frac{r_0^{3w+1}}{r^{3w}}}{M(r_{\rm cri})\frac{r}{r_{\rm cri}}} = |3w+1|\left(\frac{r}{r_{\rm cri}}\right)^{-3w-1} = 32\,\%$$

Rotation Curve Sensitivity II

$$\vec{F} = -\vec{\nabla}\Phi = -\frac{1}{r^2}\left[M + (3w+1)\frac{r_0^{3w+1}}{r^{3w}}\right]\hat{e}_r$$

$\Delta v/v$	$r = 0.2 r_{\text{cri}}$	$r = 0.4 r_{\text{cri}}$
$w = -1$	$4\,\%$	$16\,\%$
$w = -0.8$	$7\,\%$	$19\,\%$

current experimental precision is $3-8\%$ at 4-100 kpc

Galactic Mass Model

$$v^2 = v_b^2 + v_d^2 + v_h^2 + \Delta_w v^2$$

- bulge — $v_b(r) = 196\ \text{km/s}\ (\frac{r}{\text{kpc}})^{-1/2}$
- disk — $\Sigma_d(r) = \Sigma_{d,0} \exp(-r/r_d)$ $\Sigma_d(r_\odot) = 54.4\ M_\odot\,\text{pc}^{-2}$
- dark matter halo — NFW $\rho_h(r) = \rho_{h,0}(r/r_h)^{-1}(1 + r/r_h)^{-2} \sim r^{-3}$
- dark energy

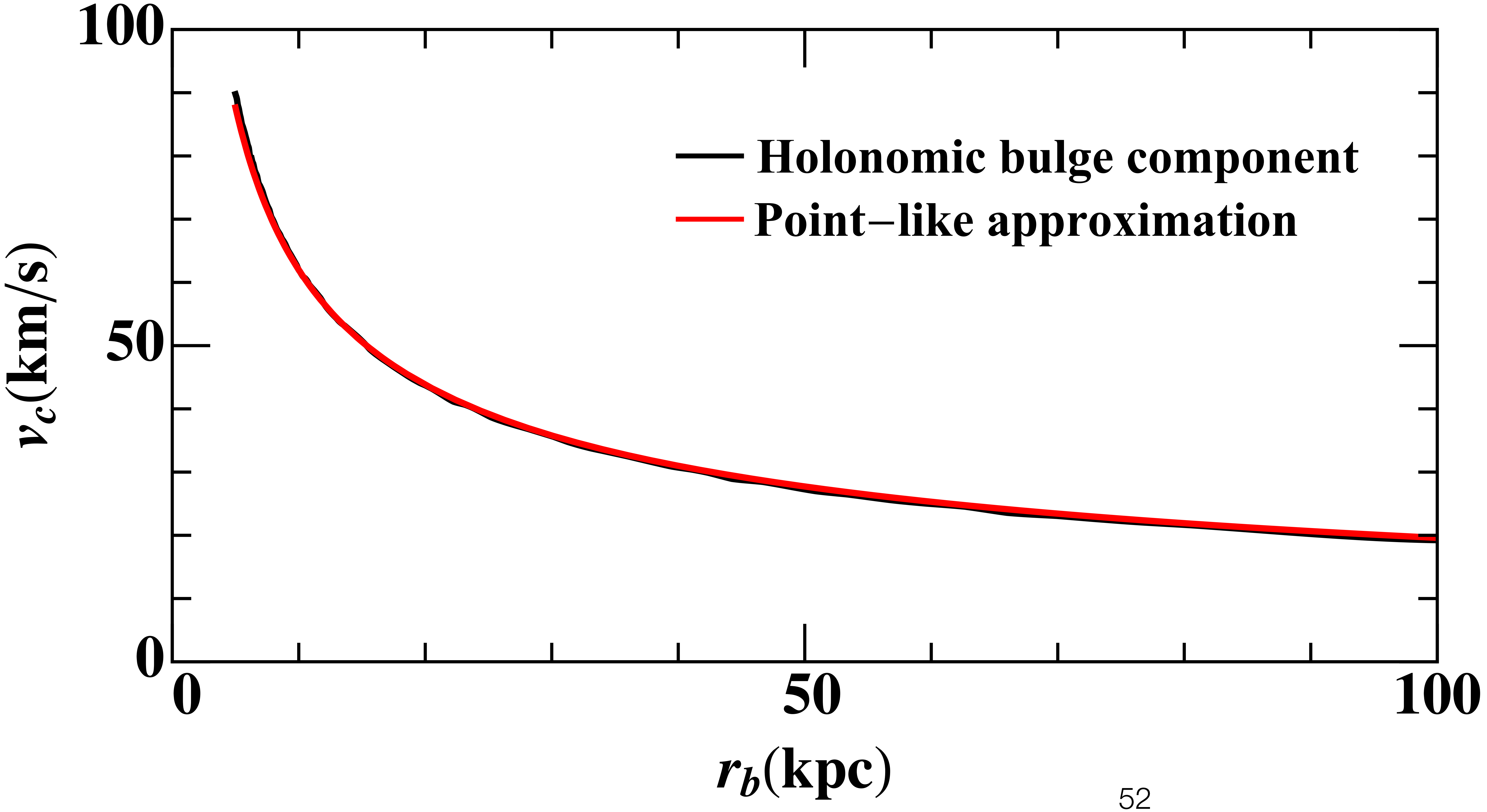
Galactic Mass Model: bulge

$$\rho(R,Z) = \frac{\rho_0}{m^\gamma(1+m)^{\beta-\gamma}} \exp\left[-(mr_0/r_t)^2\right], \quad m(R,Z) = \sqrt{(R/r_0)^2 + (Z/qr_0)^2}$$

$$\gamma = 0, \beta = 1.8, r_0 = 0.075\ \text{kpc}, r_t = 2.1\ \text{kpc}, q = 0.5, \rho_0 = 9.93 \times 10^{10}\ M_\odot \text{kpc}^{-3}$$

Galactic Mass Model: bulge

$$v = 196\ \text{km/s}\ \left(\frac{r}{\text{kpc}}\right)^{-1/2}$$



0.1% overestimation on velocity

Galactic Mass Model: disc

$$\Sigma_d(r) = \Sigma_{d,0} \exp(-r/r_d) \qquad \Sigma_d(r_\odot) = 54.4 \ M_\odot \, \mathrm{pc}^{-2}$$

$$v_c^2(r) = 4\pi G \Sigma_{d,0} r_d y^2 \left[I_0(y) K_0(y) - I_1(y) K_1(y) \right]$$

- total local stellar surface density $38.0 \ M_\odot \, \mathrm{pc}^{-2}$ [J. Bovy, H. Rix, ApJ 779 (2013) 115]
- subtract stellar halo surface density $0.6 \ M_\odot \, \mathrm{pc}^{-2}$ [C. Flynn et al. MNRAS 372 (2006)1149-1160]
- H_2 gas $2.0 \ M_\odot \, \mathrm{pc}^{-2}$ and warm gas $3.0 \ M_\odot \, \mathrm{pc}^{-2}$ [ibid.]
- HI gas $12.0 \ M_\odot \, \mathrm{pc}^{-2}$ [P. Kalberla, L. Dedes, A&A 487 (2008) 951]

Galactic Mass Model: NFW halo

$$\rho_h(r) = \rho_{h,0}(r/r_h)^{-1}(1 + r/r_h)^{-2} \sim r^{-3}$$

$$v_h^2 = \frac{4\pi\rho_{h,0}r_h^3}{r}\left(\ln\frac{r_h + r}{r_h} - \frac{r}{r + r_h}\right)$$

Planck 2018

This value is our "best estimate" of H_0 from *Planck*, assuming the base-ΛCDM cosmology.

Since we are considering a flat universe in this section, a constraint on Ω_m translates directly into a constraint on the dark-energy density parameter, giving

ΛCDM model

$$\Omega_\Lambda = 0.6847 \pm 0.0073 \quad (68\,\%, \text{TT,TE,EE+lowE+lensing}). \quad (15)$$

In terms of a physical density, this corresponds to $\Omega_\Lambda h^2 = 0.3107 \pm 0.0082$, or cosmological constant $\Lambda = (4.24 \pm 0.11) \times 10^{-66}\,\text{eV}^2 = (2.846 \pm 0.076) \times 10^{-122}\, m_{\text{Pl}}^2$ in natural units (where m_{Pl} is the Planck mass).

To test a time-varying equation of state we adopt the functional form

$w(z)$ CDM model

$$w(a) = w_0 + (1 - a)w_a\,, \quad (49)$$

where w_0 and w_a are assumed to be constants. In ΛCDM, $w_0 = -1$ and $w_a = 0$. We use the parameterized post-Friedmann (PPF)

Fixing the evolution parameter $w_a = 0$, we obtain the tight constraint

$$w_0 = -1.028 \pm 0.031 \quad (68\,\%, \textit{Planck}\ \text{TT,TE,EE+lowE+lensing+SNe+BAO}), \quad (50)$$

Table 6. Marginalized values and 68 % confidence limits for cosmological parameters obtained by combining *Planck* TT,TE,EE +lowE+lensing with other data sets, assuming the (w_0, w_a) parameterization of $w(a)$ given by Eq. (49). The $\Delta\chi^2$ values for best fits are computed with respect to the ΛCDM best fits computed from the corresponding data set combination.

Parameter	*Planck*+SNe+BAO	*Planck*+BAO/RSD+WL
w_0	-0.957 ± 0.080	-0.76 ± 0.20
w_a	$-0.29^{+0.32}_{-0.26}$	$-0.72^{+0.62}_{-0.54}$
H_0 [km s^{-1}Mpc^{-1}]	68.31 ± 0.82	66.3 ± 1.8
σ_8	0.820 ± 0.011	$0.800^{+0.015}_{-0.017}$
S_8	0.829 ± 0.011	0.832 ± 0.013
$\Delta\chi^2$	-1.4	-1.4

Relation to RW Universe

$$(\mathrm{d}s)^2 = Z^2(x,y,z)(\mathrm{d}t)^2 - a^2(t)R^2(x,y,z)\left[(\mathrm{d}r)^2 + r^2(\mathrm{d}\theta)^2 + r^2\sin^2\theta(\mathrm{d}\phi)^2\right]$$

Conservation Equation

$$0 = D_\nu T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma^\mu{}_{\nu\rho}T^{\rho\nu} + \Gamma^\nu{}_{\nu\rho}T^{\mu\rho}$$

$$0 = \frac{\mathrm{d}T^0{}_0}{\mathrm{d}t} + \frac{3\dot{a}}{a}(T^0{}_0 - \frac{1}{3}T^i{}_i) \qquad \Rightarrow \rho \propto a^{-(3w+3)}$$

$$0 = \partial_j T^i{}_j + \frac{1}{R}\frac{\partial R}{\partial x^i}(T^i{}_i - T^j{}_j) + \frac{2}{R}\frac{\partial R}{\partial x^j}T^i{}_j \qquad \Rightarrow \partial_i p = 0$$

Conservation Equation for Newtonian gauge

$$0 = D_\nu T^{\mu\nu} = \partial_\nu T^{\mu\nu} + \Gamma^\mu{}_{\nu\rho} T^{\rho\nu} + \Gamma^\nu{}_{\nu\rho} T^{\mu\rho}$$

protected by Bianchi identity

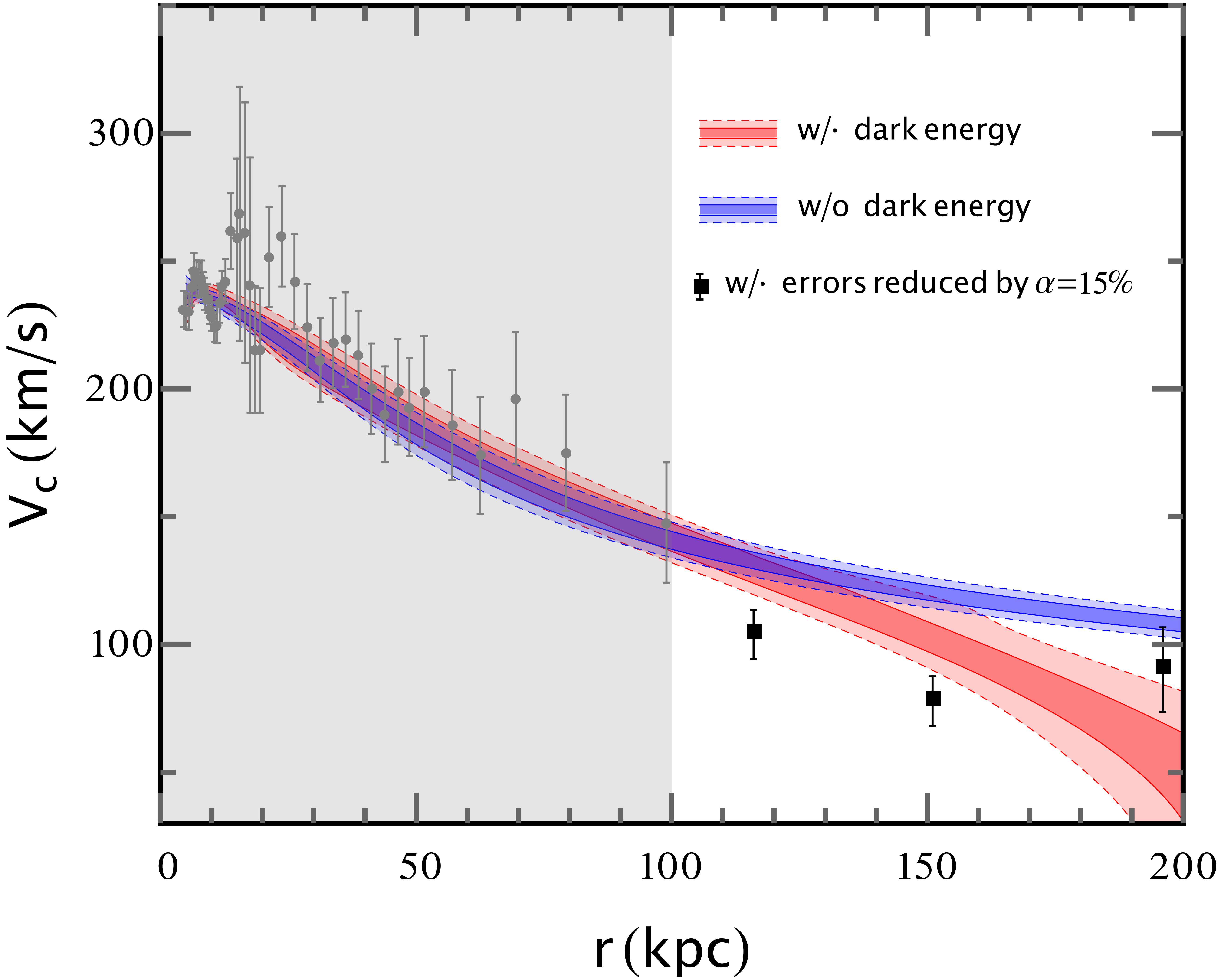
$$T^t{}_t = T^r{}_r = -\frac{2\Phi + 2r\partial_r\Phi}{r^2} \qquad T^\theta{}_\theta = T^\varphi{}_\varphi = -\partial_r\partial_r\Phi - \frac{2}{r}\partial_r\Phi$$

spherical polar coordinates

$$0 = D_\nu T^{\mu\nu} = \partial_\mu T^{\mu\mu} + \Gamma^\mu{}_{\nu\nu} T^{\nu\nu} + \Gamma^\nu{}_{\nu\mu} T^{\mu\mu}$$

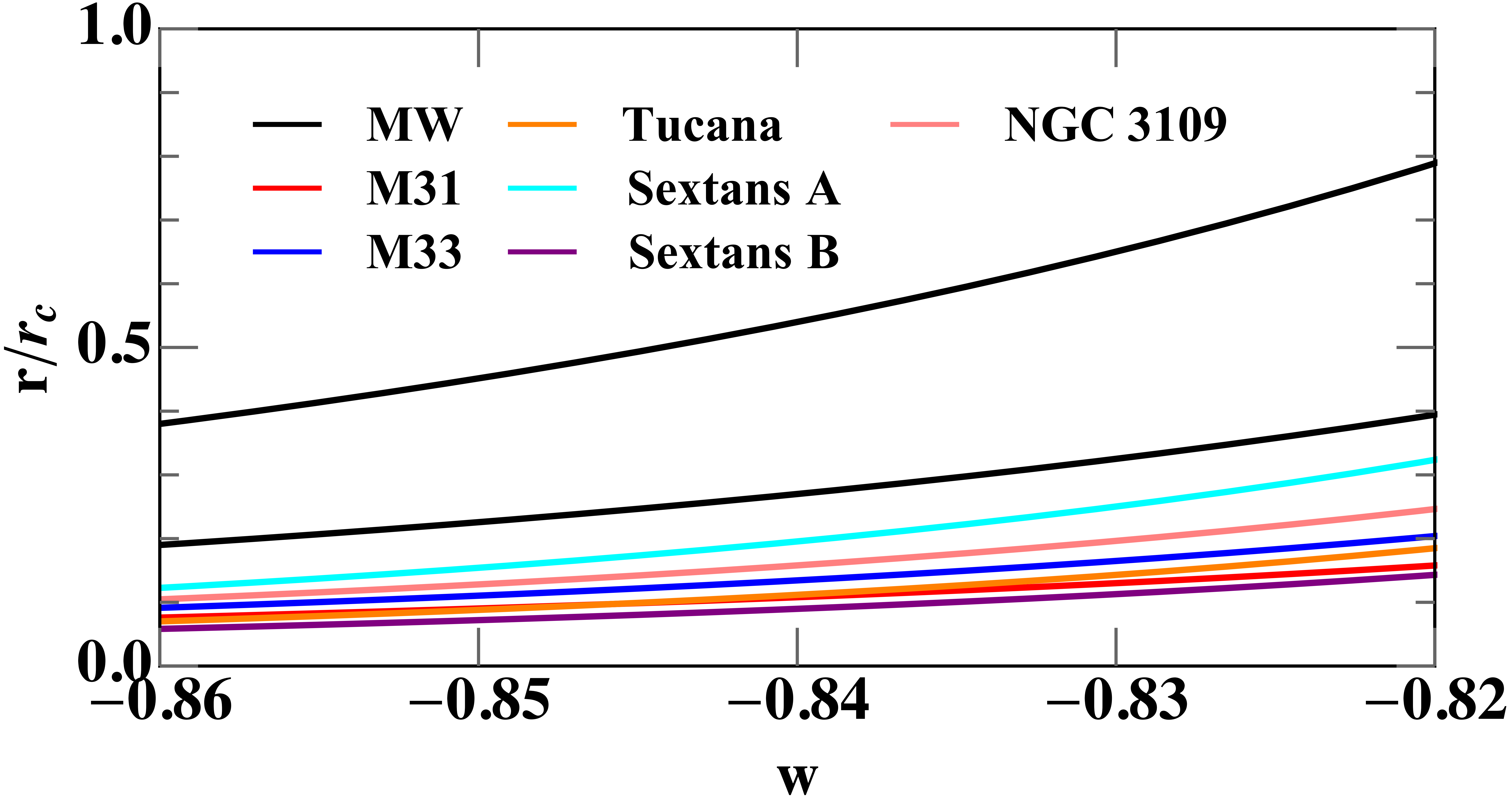
	$\partial_\mu T^{\mu\mu}$	$\Gamma^\mu{}_{\nu\nu} T^{\nu\nu}$	$\Gamma^\nu{}_{\nu\mu} T^{\mu\mu}$	$\partial_\mu T^{\mu\mu} + \Gamma^\mu{}_{\nu\nu} T^{\nu\nu} + \Gamma^\nu{}_{\nu\mu} T^{\mu\mu}$
$\mu = t$	0	0	0	0
$\mu = r$	$\partial_r(g^{rr} T^t{}_t)$	$2T^t{}_t \partial_r\Phi - 2T^\theta{}_\theta \frac{g^{rr}}{r}$	$\frac{2}{r} g^{rr} T^t{}_t$	$g^{rr}\left(\partial_r(T^t{}_t) + \frac{2}{r}(T^t{}_t - T^\theta{}_\theta)\right) = 0$
$\mu = \theta$	0	$\Gamma^\theta{}_{\varphi\varphi} g^{\varphi\varphi} T^\varphi{}_\varphi$	$\Gamma^\varphi{}_{\varphi\theta} g^{\theta\theta} T^\theta{}_\theta$	0
$\mu = \varphi$	0	0	0	0

Prospect



w/· dark energy
w/o dark energy
w/· errors reduced by α=15%
300
200
100
0
50
100
150
200
V_c (km/s)
r (kpc)

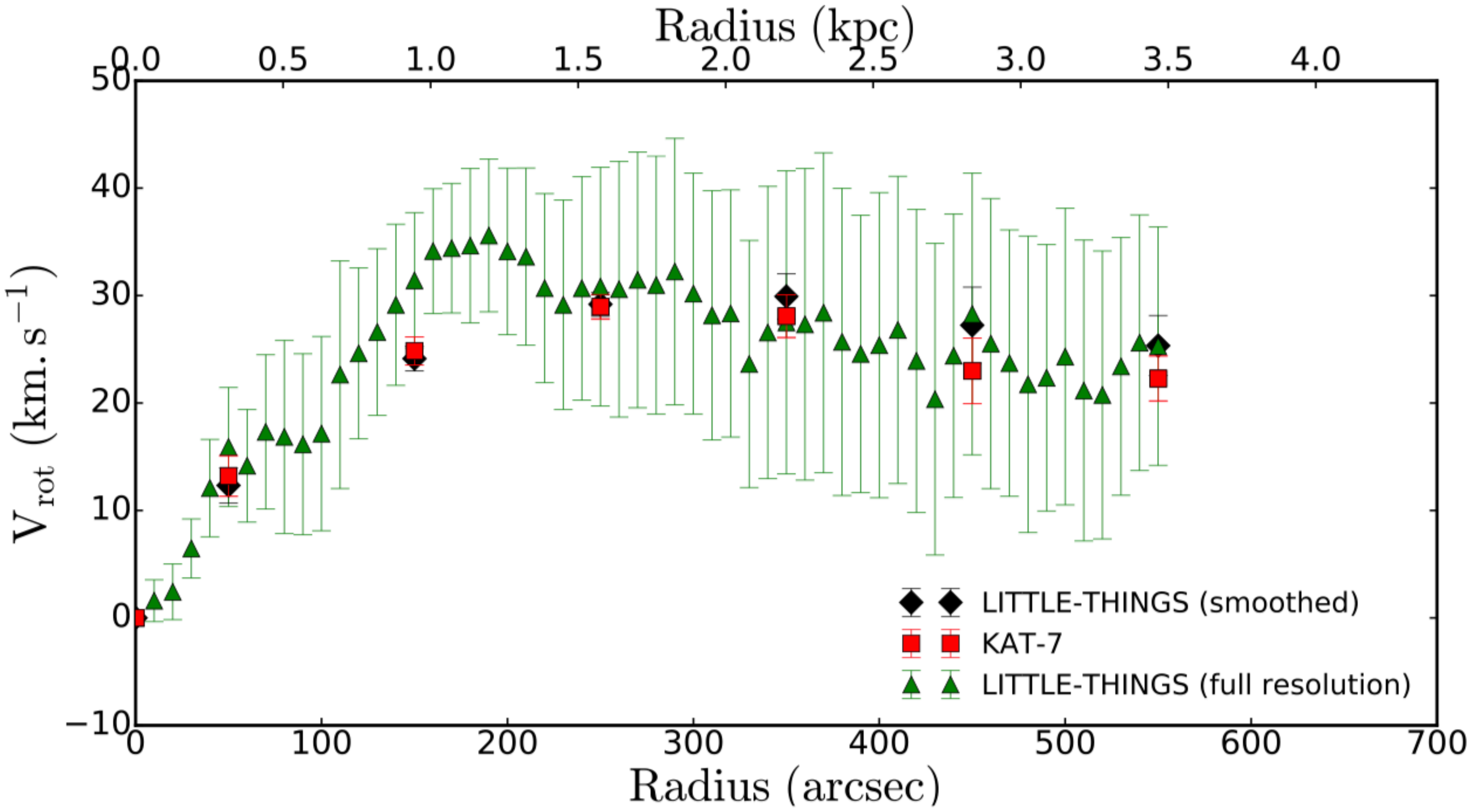
Prospect



$$|3w+1|\left(\frac{r_O}{r_{\rm cri}}\right)^{3w+1} = V_0^2$$

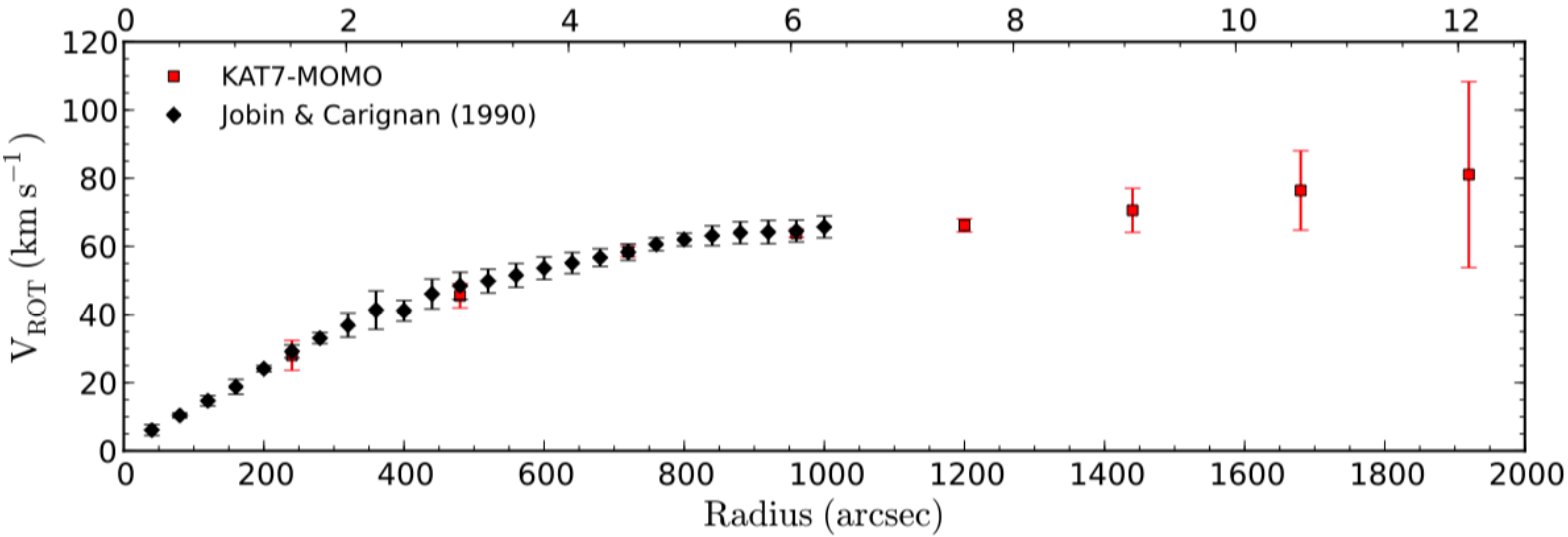
rotation velocity of Sextans A

B. Namumba et cl.
MNRAS 478 (2018) 1, 487-500
arxiv: 1804.07730



rotation velocity of NGC 3109

C. Carignanet cl.
ApJ 146 (2013) 48
arxiv: 1306.3227



rotation velocity of M31

Y. Sofue
Publ.Astron.Soc.Jap. 67 (2015) 4, 75
arxiv: 1504.05368

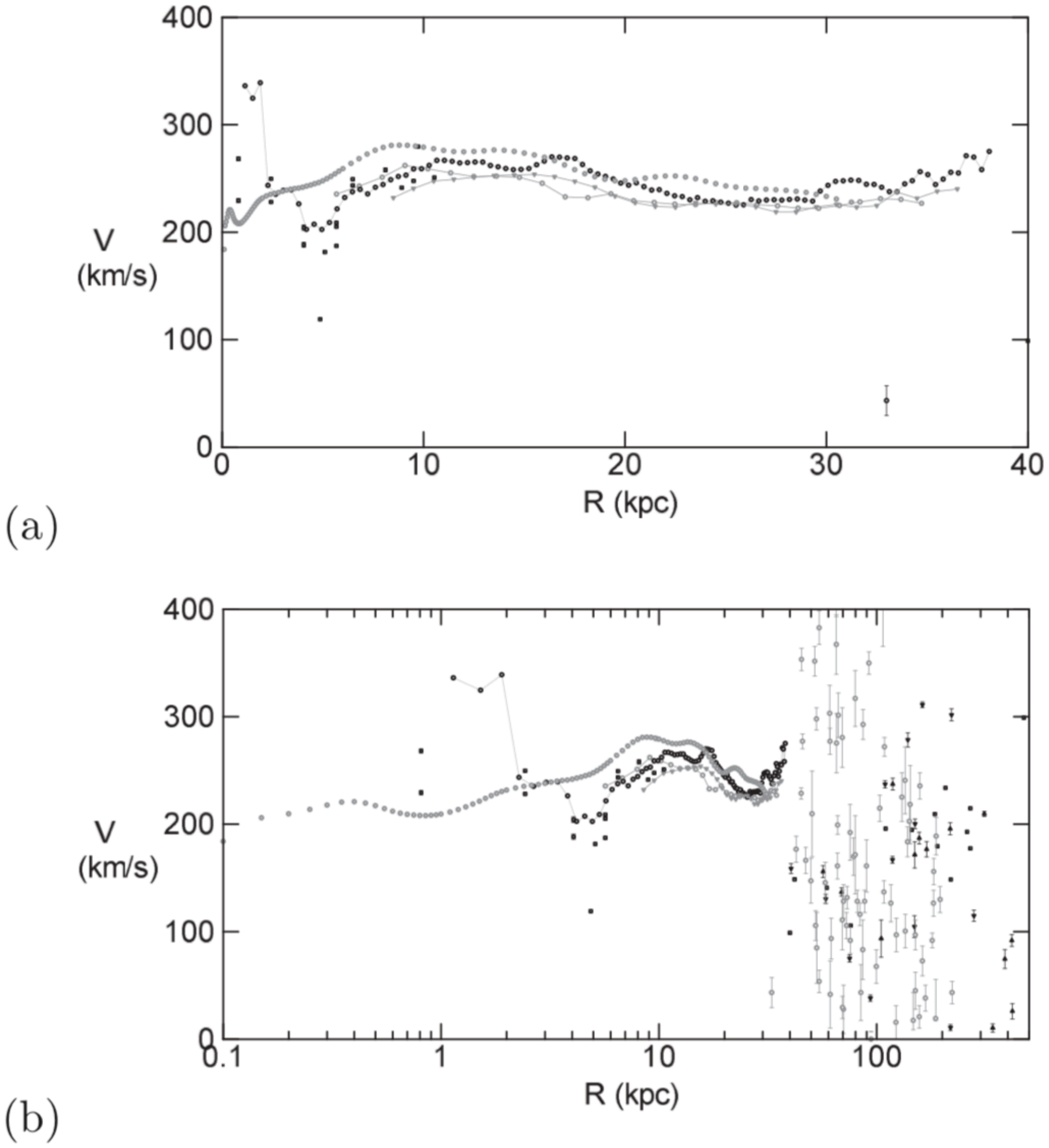
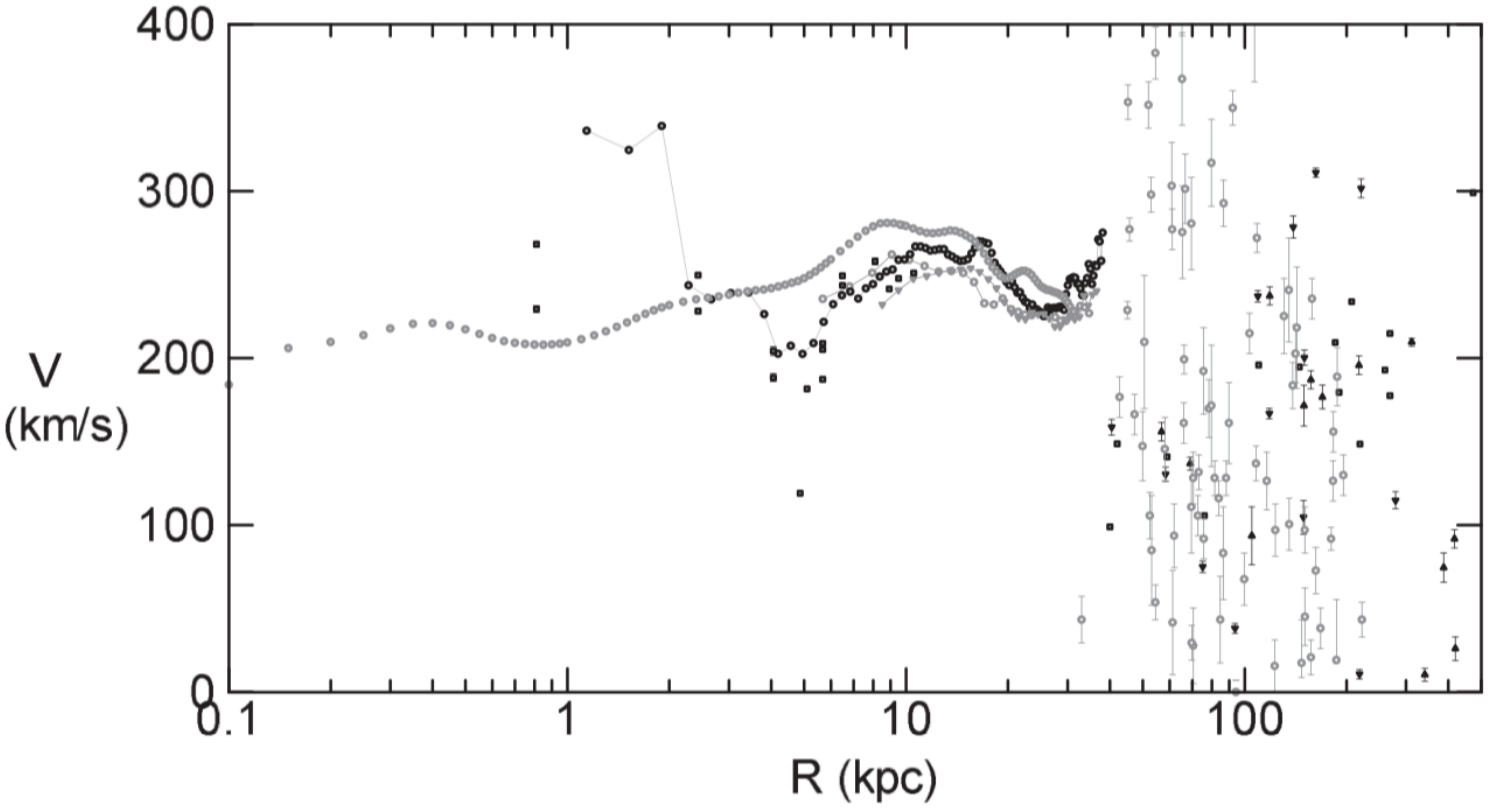


Fig. 1. Rotation velocities of the disk and pseudo rotation velocities of non-coplanar objects in M31 (a) in linear and (b) semi-logarithmic scaling. References to the data are listed in table 1.



Prospect

P. Bhattacharjee et cl.
ApJ 785 (2014) 63
arxiv: 1310.2659